21/10/19 Quantum Mechanics. (Module-III)

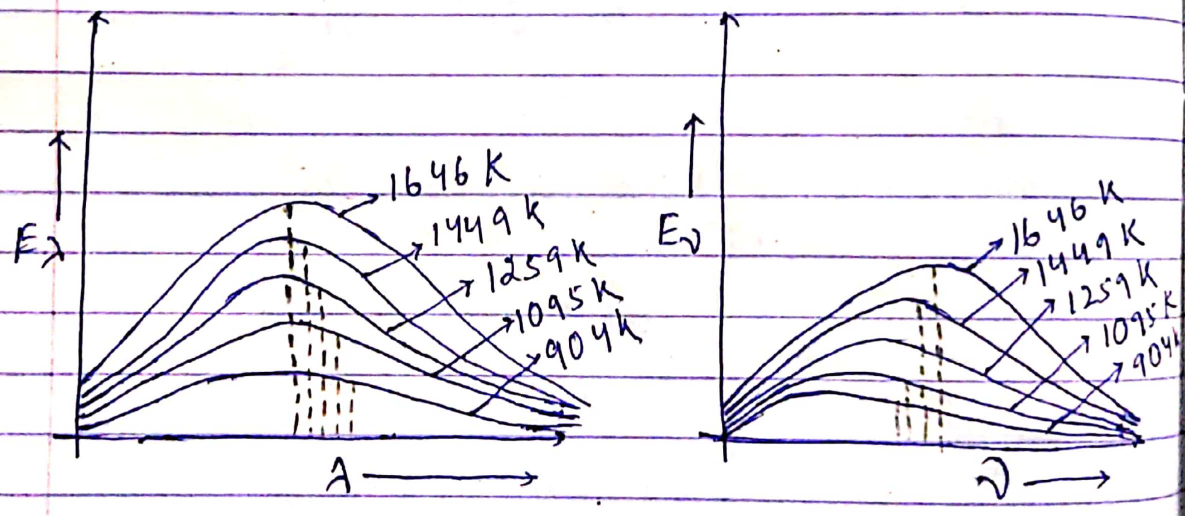
The quantum theory is developed which couldn't explain by the help of classical theory / classical physics.

(i) Planck's theory of black body radiation (1900)
(ii) Einstein's theory of photoelectric effect (1905)
(iii) Bohr's theory of hydrogen spectrum (1913)
(iv) Compton's theory of X-rays scattering (1922)
(v) De-Broglie's hypothesis (1924)
(vi) Heisenberg's uncertainity principle (1927)
(v) S. ' wave eq^n (1928)

· **Planck's theory of black body radiations-**

Experimental Observation of black body radiati

E_λ or E_ν → Radiant Energy density i.e. energy radiated per unit volume. [$\lambda = c/\nu$]



(i) At a given temp. the energy density has max. value corresponding to a value of frequency or wavelength.

(ii) The frequency corresponding to max. energy density increases with increase of temp.

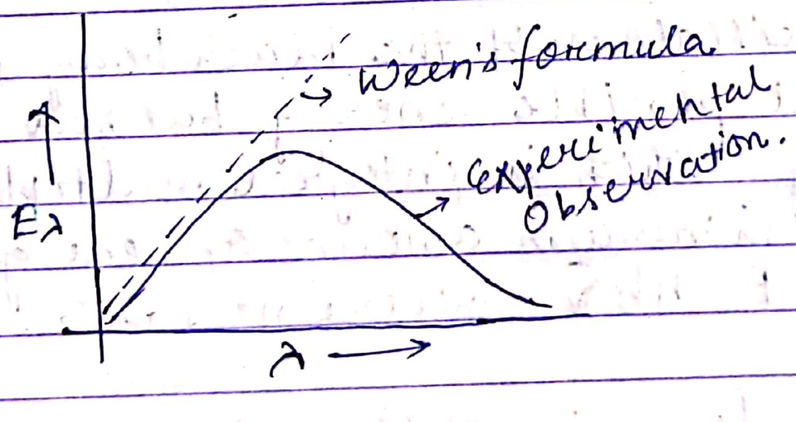
(iii) The energy density decreases to zero for both higher and lower value of frequency or wavelength.

(iv) The energy density corresponding to a given frequency or wavelength increases with increase of temp.

(v) The total energy radiated at any temp. is given by the area b/w the curve corresponding to that temp. and the horizontal axis.

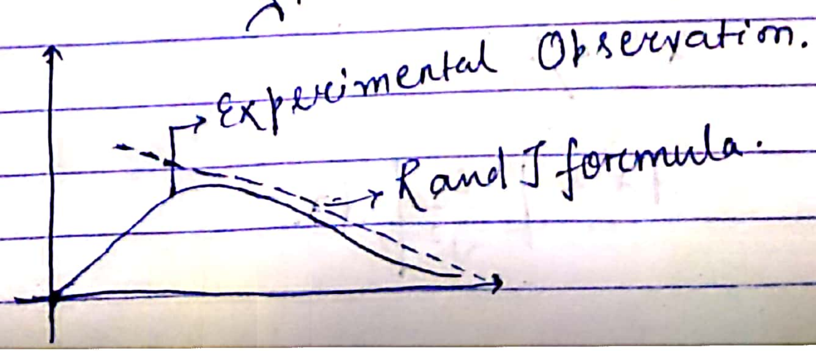
Ween's formula for Black body radiation-

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda KT} d\lambda$$



Rayligh and Jean (1900)

$$E_\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda$$



Planck's Quantum theory of Black Body Radiation -

In order to explain the experimental observ. distribution of energy planck in 1900 suggested some assumptions which is called as planck's quantum theory.

→ A chamber containing black body radiation contain some simple harmonic oscillators. of molecular dimensions which can vibrate. with all possible frequencies.

→ The classical principle of equipartition of energy is not applicable to the black body radiation.

→ The frequency of radiation emitted by an oscillator. is the same as that of the frequency of vibration.

→ The oscillators of the black body cannot have all possible energy but has discrete energy which is integral multiple of some minimum amount of energy.

$$E = nh\nu \quad \text{where } n = 1, 2, 3 \ldots, n \neq 0$$

Planck's formula of black body radiation.

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT} - 1} d\lambda \qquad (1)$$

$$\lambda = c/\nu$$

$$d\lambda = \left| \frac{-c}{\nu^2} \right| d\lambda.$$

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$$E_\nu \, d\nu = \frac{8\pi h \nu^3}{c^3} \cdot \frac{1}{e^{h\nu/KT} - 1} d\nu$$

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Planck's formula reduced to Ween's formula for low wavelength region.

For low wavelength.

λT is very small.

$e^{hc/\lambda KT} >>> 1$

$$e^{hc/\lambda KT} - 1 \simeq e^{hc/\lambda KT}.$$

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \times \frac{1}{e^{hc/\lambda KT}} d\lambda.$$

$$= \frac{8\pi hc}{\lambda^5} \times e^{-hc/\lambda KT}.$$

So, Planck's formula has a very good fitting to the experimental result at low wavelength region.

Planck's formula reduced to Rayleigh's formula for high wavelength region:

λT is very large.

$$e^{hc/\lambda KT} = 1 + \frac{hc}{\lambda KT} + \frac{1}{2!}\left(\frac{hc}{\lambda KT}\right)^2 + \cdots$$

$$= 1 + \frac{hc}{\lambda KT}. \text{ (Neglecting higher order).}$$

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \times \frac{1}{\frac{hc}{\lambda KT}}$$

$$= \frac{8\pi KT}{\lambda^4} d\lambda \rightarrow \text{Rayleigh and Jean formula.}$$

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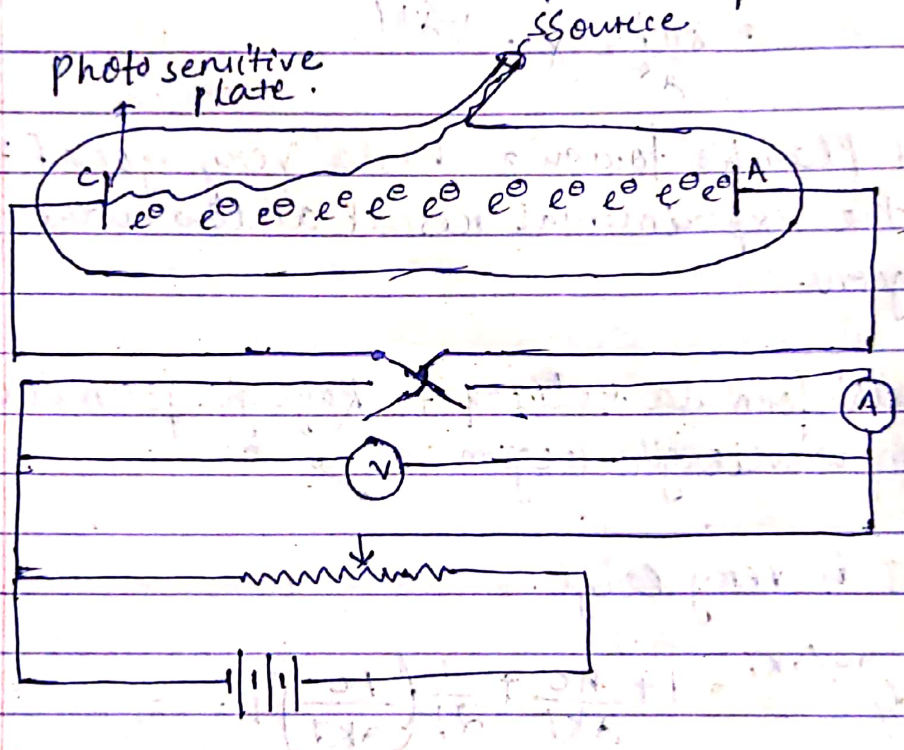
So Plank's formula has a very good fitting to the experimental result at high wavelength region.
So, it concludes this formula agrees for to the experimental result for all ranges of wavelengths

Photoelectric Effect :-

The emission of $e^{\ominus}$s from a metal plate when illuminated by a light radiation of suitable frequency or wavelength is called as photoelectric effect.
The emitted $e^{\ominus}$s are called as photoelectrons.



Experimental Observation -

→ Photoelectric effect is an instantaneous process. As soon as the radiation is incident on the metal plate, photo electrons are emitted. The time lag b/w them is 10^{-8} sec.

→ For a given frequency and given photo-potential diff. b/w anode and photocathode, the photocurrent is directly proportional to the intensity of incident radiations.

→ The emitted photoelectrons do not have same kinetic energy that varies from zero to a maximum value.

For a given photocathode and given potential difference the maximum kinetic energy depends upon the frequency of radiation but independent of intensity.

An increase in intensity increases the no. of photoelectrons but not the maximum kinetic energy.

→ For a given photocathode, the photoelectric effect can take place only if the frequency of radiation must have a minimum value called as threshold frequency, below which no photoelectrons are emitted. even the intensity is made very high.

→ If the anode made -ve and cathode made +ve then photocurrent decreases even the electrons has max. kinetic energy but could not reach the anode.

So, the minimum potential (Negative potential) to get zero photocurrent is called as stopping potential

→ For a given metal surface stopping potential is directly proportional to the frequency of incident radiation but independent of intensity.

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Einstein's Explanation Of Photoelectric Effect -

Einstein extended Planck's idea to explain photoelectric effect. So, ~~we~~ he assumed that electromagnetic wave of frequency ν can be regarded as a stream of particles of energy '$h\nu$' each and this particle are called as photons.

The energy of photons is used in two parts -

(i) A part of it is used to free the electrons from the atom and make away from the metal surface. This energy is called as workfunction of the metal and denoted by "W_0"

(ii) The rest part of energy of photons is used in giving the kinetic energy to the electrons.

$$\therefore h\nu = W_0 + \tfrac{1}{2}mv^2 \quad \text{--- (1)}$$

where, $W_0 \rightarrow$ workfunction of metal surface

v - velocity of the photoelectron.

Eqn (1) is the Einstein's photoelectric eqn.

$W_0 = h\nu_0$

ν_0 - minimum frequency required for which photoelectric effect can take place i.e.

ν_0 - ~~thres~~ threshold frequency.

$$h\nu = h\nu_0 + \tfrac{1}{2}mv^2$$

$$\tfrac{1}{2}mv^2 = h\nu - h\nu_0$$

$$KE = h(\nu - \nu_0) \quad \text{--- (2)}$$

$$\tfrac{1}{2}mv^2 \propto \nu \quad \text{and} \quad v^2 \propto \nu$$

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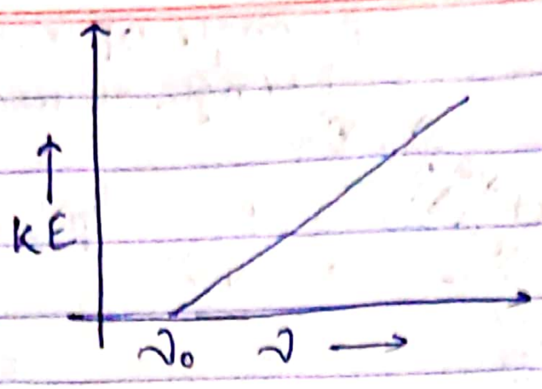


Stopping Potential-

$\frac{1}{2}mv^2 = qV.$

q - Charge particle.

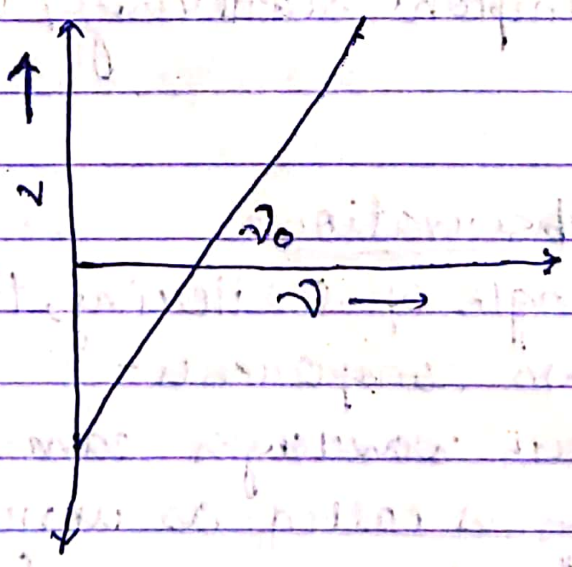
V - negative potential given in b/w cathode & anode.

$KE = \frac{1}{2}mv^2 = eV$

so, eq^n (2) can be written as

$eV = \frac{e}{e} h(\nu - \nu_0)$

So, $\boxed{V \propto \nu}$



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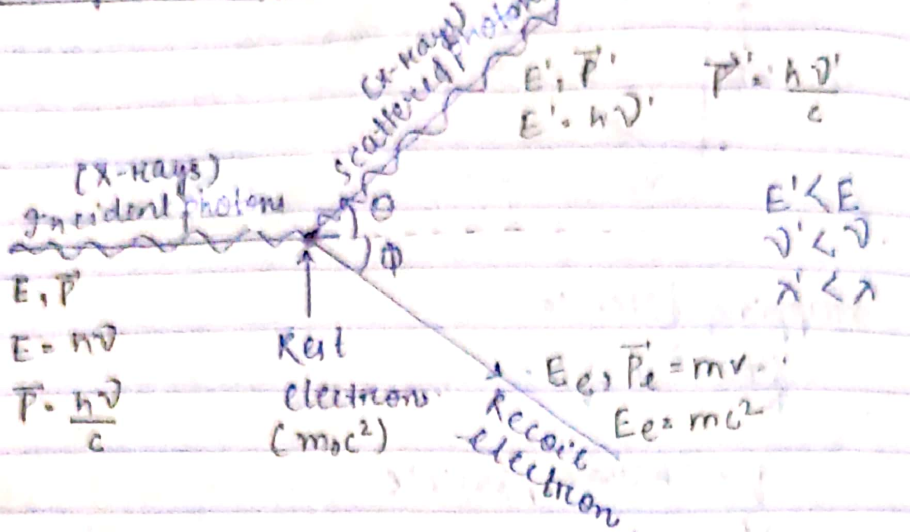
Compton's Effect:

θ - Angle of scattering
φ - Angle of recoil electron



When a monochromatic beam of high frequency radiation such as X-rays collides with the rest electron, a part of energy given to that electron. Due to this $e^{\ominus}$ will gain K.E and scattering photon (X-rays) will have lower energy. so lower frequency or greater wavelength than the incident one which is called as Compton's Scattering / Compton's Effect.

Experimental Observation :-

→ For a given angle of scattering, the scattered X-rays have two components -

(i) A component with wavelength same as that of the incident X-rays is called as unmodified component

(ii) Another component having lower frequency and greater wavelength is called as modified component or Compton's Component.

→ The increase in wavelength in Compton's component is called as Compton's shift which depends upon the angle of scattering.

→ The Compton's shift is independent of the wavelength of the incident radiation.

Classical Theory failed to explain experimental observations by considering the radiations (X-rays) as wave nature.

Compton's Explaination-
An adequate explaination of this effect was provided by Compton in 1992 on the basis of Planck's quantum theory of radiation. According to that quantum concept the radiation is constituted by energy packets called photon having energy $h\nu$.
The incident rays with frequency 'ν' are regarded as a stream of particles with energy '$h\nu$'.

Before Collision-
The energy of the incident photon,
$$E = h\nu$$
Momentum of the incident photon,
$$\vec{p} = h\nu/c$$
Energy of the rest $e^{\ominus}$ $= m_0 c^2$
Momentum of the rest electron $= 0$

After Collision-
Energy of the scattered photon,
$$E' = h\nu'$$
Momentum of the scattered photon.
$$\vec{p}' = h\nu'/c$$
Energy of the recoil $e^{\ominus} = mc^2 = E_e$.
Momentum of the recoil $e^{\ominus}$,
$$\vec{P_e} = mv$$

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$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\Rightarrow m^2 = \frac{m_0^2}{1-\frac{v^2}{c^2}}$$

$$\Rightarrow m^2 = \frac{m_0^2 c^2}{c^2 - v^2}$$

$$\Rightarrow m^2c^2 - m^2v^2 = m_0^2c^2$$

Multiplying c^2 both sides.

$$\Rightarrow m^2c^4 - m^2c^2v^2 = m_0^2c^4$$

$$\Rightarrow E_e^2 = P_e^2c^2 + m_0^2c^4 \quad \text{——— (1)}$$

Applying Law Of Conservation Of Energy.

$$E + m_0c^2 = E' + E_e$$

$$E + m_0c^2 = E' + \sqrt{P_e^2c^2 + m_0^2c^4}$$

$$\Rightarrow E + m_0c^2 - E' = \sqrt{P_e^2c^2 + m_0^2c^4} \quad \text{——— (2)}$$

Applying Law Of Conservation Of Momentum

$$\vec{P} = \vec{P}' + \vec{P_e}$$

$$\vec{P_e} = \vec{P} - \vec{P}'$$

$$\Rightarrow \vec{P_e}^2 = \vec{P_e}\cdot\vec{P_e} = (\vec{P} - \vec{P}')\cdot(\vec{P} - \vec{P}')$$

$$P_e^2 = P^2 + P'^2 - 2PP'\cos\theta \quad \text{—— (3)}.$$

Squaring eq^n (2) both the sides.

$$(E - E')^2 + \cancel{m_0^2c^4} + 2(E - E')m_0c^2 = P_e^2c^2 + \cancel{m_0^2c^4} \quad \text{—— (4)}.$$

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Substitute the value of P_e from eqⁿ ③ in eqⁿ ④

$$(E-E')^2 + 2(E-E')m_0c^2 = p^2c^2 + p'^2c^2 - 2pp'c^2\cos\theta.$$

$$\Rightarrow (E-E')^2 + 2(E-E')m_0c^2 = E^2 + E'^2 - 2EE'\cos\theta.$$

Add and substract $2EE'$ in RHS

$$(E-E')^2 + 2(E-E')m_0c^2 = E^2 + E'^2 - 2EE'\cos\theta + 2EE' - 2EE'$$

$$\Rightarrow \cancel{(E-E')^2} + 2(E-E')m_0c^2 = \cancel{(E-E')^2} + 2EE' - 2EE'\cos\theta.$$

$$\Rightarrow \cancel{2}(E-E')m_0c^2 = \cancel{2}EE'(1-\cos\theta).$$

$$\Rightarrow \frac{E}{EE'} - \frac{E'}{EE'} = \frac{1-\cos\theta}{m_0c^2}.$$

$$\Rightarrow \frac{1}{E'} - \frac{1}{E} = \frac{1-\cos\theta}{m_0c^2}$$

$$E = h\nu = \frac{hc}{\lambda}, \quad E' = h\nu' = \frac{hc}{\lambda'}$$

$$\Rightarrow \frac{\lambda'}{hc} - \frac{\lambda}{hc} = \frac{1-\cos\theta}{m_0c^2}$$

$$\Rightarrow \frac{\lambda'}{h} - \frac{\lambda}{h} = \frac{1-\cos\theta}{m_0c}$$

$$\lambda' - \lambda = \frac{h(1-\cos\theta)}{m_0c}$$

$$\Rightarrow \boxed{\Delta\lambda = \lambda' - \lambda = \frac{h(1-\cos\theta)}{m_0c}}$$

$$\boxed{\Delta\lambda = \lambda_c(1-\cos\theta)}$$

Compton's shift

λ_c - Compton's wavelength.

$$= \frac{h}{m_0c}$$

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The compton's shift '$\Delta\lambda$' only depends upon the angle of scattering 'θ' and independent of the wavelength of the incident radiation.

Different cases -

Case I - $\theta = 0$

$$\Delta\lambda = 0$$

i.e. $\lambda' = \lambda$ i.e. no Compton's shift

Case II - $\theta = \pi/2$

$$\Delta\lambda = \lambda_c$$

$$\lambda' - \lambda = \lambda_c$$

$$\lambda' = \lambda_c + \lambda$$

Case III - $\theta = \pi$

$$\Delta\lambda = 2\lambda_c$$

which is the maximum value of Compton's shift

Q.1. Find the Compton's shift for X-rays of wavelength 1.5 A° scattered by 60°. Also find the wavelength of the scattered X-rays.

Q.2. X-rays of ~~0~~ 1A° are scattered from a carbon block find the wavelength of the scattered beam in a direction making 90° with incident beam How much K.E is imparted to the recoil $e^{\ominus}$?

Q.3. X-rays of wavelength 1.2 A° under the Compton's scattering due to $e^{\ominus}$s. What is max. possible value of Compton's shift if the compton's wavelength of the $e^{\ominus}$ is 0.0242 A°.

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Q.4. Calculate the velocity of photon $e^{\ominus}$ if the workfunction of the target material is 1·24e and the wavelength of incident rays is 4.36×10^{-7} m. What is the retarding potential necessary to stop the emission of this electron?

Q.5. Electrons are emitted with zero velocity from a certain metal surface when it is exposed to a radiation of 6800 A°. Calculate the threshold frequency and workfunction of the metal.

Q.6. The workfunction of the aluminium is 4·2 eV. Calculate the K.E of the fastest and slowest $e^{\ominus}$, the stopping potential. and the cut-off wavelength when light of wavelength 2000 A° falls on the clean aluminium surface.

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Distinguish b/w the photoelectric effect and Compton's Scattering.

Photoelectric Effect.	Compton Scattering.

Pair Production -

Impt Dual Nature Of Radiation (de-Broglie hypothesis).

Matter Waves.

$$\boxed{\lambda = \frac{h}{p}}$$

λ - wavelength
p - momentum, (mv).

Looking to the discuss given by photoelectric effect, Compton's Scattering and Pair production Louis de-Broglie proposed that all material particles are associated with waves called as matter waves or de-Broglie's wave. According to his hypothesis the wavelength λ of the matter wave associated with a moving particle of linear momentum 'p' is given by

$$\boxed{\lambda = \frac{h}{p}}$$

where h - planck's constant.

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Calculation of de-Broglie's Wavelength-.

1. Free Particle - $KE = \frac{1}{2}mv^2$

~~Tota~~ No potential Energy.

Total Energy = KE

$$KE = \frac{p^2}{2m}.$$

$$p = \sqrt{2mE}$$

$$\boxed{\lambda = \frac{h}{\sqrt{2mE}}}$$

2. For any particle-

Let the particle moves under potential 'V'

Total Energy = E + V. T - total energy

$E = T - V$ V - Potential energy.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \Rightarrow \boxed{\lambda = \frac{h}{\sqrt{2m(T-V)}}}$$

3. For a charge particle.-

Let the charge particle having charge 'q' accelerated with the potential difference 'V".

KE of the charge particle = qV.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}.$$

Particular Case-

For electron

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{12.27}{\sqrt{V}} \, A^\circ$$

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4. For thermal particle.

KE of the thermal particle $= \frac{3}{2}KT$. T - Temp. in Kelvin
K - Bolt's constant

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{3mkT}}.$$

Q.1 Calculate the de-Broglie wavelength associated with a $e^{\ominus}$ subjected to the potential diff of 10 V.

Q.2. A ball of mass 0.1 g. has a speed of 300 m/s Calculate the de-Broglie wavelength associated with.

Heisenberg's Uncertainity Principle -
Acc. to this principle it is not possible to measure simultaneously the position and momentum of particle with 100% accuracy.

$$\Delta x \cdot \Delta p \geq \hbar/2 \qquad \hbar = \frac{h}{2\pi}$$

$$\Delta t \cdot \Delta E \geq \hbar/2.$$

Properties -
i) Non existance of electron in the nucleus.
ii) Ground state energy of harmonic oscillator is non zero.
(iii) Ground state energy of H-atom is non-zero.

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Proofs:

(i) Let us assume the $e^{\ominus}$ exist in the nucleus

dimension of nucleus $\propto 10^{-14}$ m.

$\Delta x \simeq 10^{-14}$ m.

Use Heisenberg's Uncertainity principle.

$$\Delta x \cdot \Delta p \simeq \frac{\hbar}{2}.$$

$$\hbar = \frac{h}{2\pi} = 1.052 \times 10^{-34}$$

$$\Delta p = \frac{\hbar}{2\Delta x}$$

$$= \frac{1.052 \times 10^{-34}}{2 \times 10^{-14}} = 5.3 \times 10^{-21} \text{ kg m/s}$$

$$E = \sqrt{m_0^2 c^4 + p^2 c^2} \quad \text{[Crompton's Scattering]}$$

$m_0 = 9.1 \times 10^{-31}$ kg.

$c = 3 \times 10^8$ m/s.

$\Delta p \approx p$.

$$E = \sqrt{(9.1 \times 10^{-31})^2 \times (3 \times 10^8)^4 + (5.3 \times 10^{-21})^2 (3 \times 10^8)^2}$$

$$E = 1.6 \times 10^{-12} \text{ J} \simeq 10 \text{ MeV}.$$

This means if the $e^{\ominus}$ exist inside the nucleus then ~~thes~~ its energy must be in the order of 10 MeV

But the $e^{\ominus}$s emitted from the radio active β-decay has K.E about 1 MeV which is much smaller than the predicted value by uncertainity principle. So, $e^{\ominus}$s do not exist inside the nucleus.

$$m \Delta v = 5.3 \times 10^{-21} \text{ kg m/s}.$$

$$\Delta v = \frac{5.3 \times 10^{-21}}{9.1 \times 10^{-31}}.$$

↓

Uncertainity in velocity.

(ii) The energy of the 1-D harmonic oscillator is $E = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$ —— ①.

Use Heisenberg's Uncertainity Principle,

$$\Delta x \cdot \Delta p \simeq \frac{\hbar}{2}$$

Let $\Delta x \approx x$

$\Delta p \approx p$.

$$\Rightarrow x \cdot p \approx \frac{\hbar}{2}$$

$$p = \frac{\hbar}{2x}$$

Substitute the values.

$$E = \frac{\hbar^2}{8mx^2} + \frac{1}{2}m\omega^2x^2 \quad —— ②.$$

As the ground state energy is minimum for $x = x_0$, so we can write

$$\left.\frac{\partial E}{\partial x}\right|_{x=x_0} = 0.$$

$$\frac{\partial E}{\partial x} = \frac{-2\hbar^2}{8mx^3} + \frac{2}{2}m\omega^2 x$$

$$\left.\frac{\partial E}{\partial x}\right|_{x=x_0} = \frac{-2\hbar^2}{8mx_0^3} + m\omega^2 x_0 = 0.$$

$$x_0^4 = \frac{\hbar^2}{4m^2\omega^2}$$

$$\boxed{x_0^2 = \frac{\hbar}{2m\omega}}$$

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E_0 = $\frac{\hbar\omega}{4} + \frac{1}{2} m \omega^2 \times \frac{\hbar}{2m\omega}$

Energy at ground state $= \frac{\hbar\omega}{4} + \frac{\hbar\omega}{4} = \frac{\hbar\omega}{2}$

(iii) The energy of H-atom,

$$E = \frac{p^2}{2m} - \frac{e^2}{4\pi\epsilon_0 r}.$$

r - radius of the orbit of the atom.

Use H.U.P.

$$\Delta x \cdot \Delta p \simeq \hbar$$

$$\Delta x \approx r$$

$$\Delta p \approx p.$$

$$p = \frac{\hbar}{r}.$$

Substitute the value.

$$E = \frac{\hbar^2}{2mr^2} - \frac{e^2}{4\pi\epsilon_0 r}$$

$$\left.\frac{\partial E}{\partial r}\right|_{r=r_0} = 0$$

$$\frac{\partial E}{\partial r} = -\frac{2\hbar^2}{2mr^3} + \frac{e^2}{4\pi\epsilon_0 r^2}$$

$$\left.\frac{\partial E}{\partial r}\right]_{r=r_0} = 0.$$

$$\Rightarrow \frac{e^2}{4\pi\epsilon_0 r_0^{\cancel{2}}} = \frac{\cancel{2}\hbar^2}{\cancel{2}m r_0^{\cancel{3}}}$$

$$\Rightarrow r_0 = \frac{\hbar^2 4\pi\epsilon_0}{e^2 m}$$

$$E_0 = \frac{\hbar^2 \times e^4 m^2}{2m\hbar^4 16\pi^2\epsilon_0^2} + \frac{e^2 \times e^4 m^2}{4\pi\epsilon_0 \hbar^4 16\pi^2\epsilon_0^2}$$

$$E_0 = \frac{1}{\hbar^4 16\pi^2\epsilon_0^2}\left[\frac{\hbar^2 e^4 m^2}{2m} + \frac{e^6 m^2}{4\pi\epsilon_0}\right]$$

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Q.1. An $e^{\ominus}$ and a proton are accelerated through the same potential if their masses are m_e & m_p respectively then calculate the ratio of de-Broglie wavelength.

Q.2. Estimate the de-Broglie wavelength whose Energy is 45 eV.

Q.3. An $e^{\ominus}$ is accelerated by potential of V volt has a de-Broglie wavelength 'λ' if the $e^{\ominus}$ is accelerated by again by p.d of 4V find its de-Broglie wavelength.

Q.4 A proton and α-particle are accelerated by same p.d find the ratio of their de-Broglie wavelength

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Transition from Deterministic to Probabilistic -
(From book or google it)

In quantum mechanics every

Laser and Fibre Optics (Module-V)

Light amplification by stimulated emission of radiation. (LASER)

Laser-

It is a device which amplifies electromagnetic radiation and generates extreme intense, coherent, monochromatic and directional radiation.

Characteristics Of Laser Light-

→ They are highly directional.
→ They are monochromatic
→ Laser beam is spatially and temporarily coherent to one extra-ordinary degree. so, interference and diffraction effect can be observed by taking two independent laser
→ Laser beam are highly intense as compared to ordinary light.

Q. Write the diff. b/w ordinary light and laser

Spontaneous emission & stimulated emission.
Meta stable state.
Population inversion.
Pumping.

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Atomic Transition.

There are 3 types of atomic transition

i) Spontaneous Emission
ii) Stimulated Absorption
iii) Stimulated Emission.

Diff. b/w Spontaneous Emission and Stimulated Emission

	Stimulated Emission
→ Atoms in the excited state at higher energy level may drop to lower energy level by emitting photon without any external provocation	→ The atoms in the excited state at higher energy level may force to drop to lower energy level by emitting photons with external provocation (by incident photon)
Atoms emits Electromagnetic wave has no definite phase or directional relation with photons emitted by another atom.	The emitted photon is in phase with that of the incident photon and also in the same direction with incident photons.
No coherent sources can be produced by this method.	Coherent sources are produced by this method.

Meta Stable State -

· It is an intermediate state in between excited state and ground state, and it is more close to the excited state.

The life time of atoms in this state is 10^{-5} sec to 10^{-3} sec.

In excited state the life time of atom is 10^{-8} sec. So it is insufficient for the interaction of incident photons with the atoms before stimulated emission.

So, for production of laser meta stable state is require

Population Inversion-
In a normal condition or thermal equilibrium the density of atoms or population of atoms decreases if we go from lower to higher energy level.
But when the population of higher energy level is more than that of lower energy level then it is called as population inversion for the system or it is called as a active system.

Probability of spontaneous emission rate vs stimulated emission rate and Einstein's coeff. -
Let us consider a system or assembly of atoms in thermal equilibrium i.e. at temp. T (kelvin scale) with radiation frequency 'ν' and energy density '$f(\nu)$'
Under this condition let all the transition process occur.

(i) Stimulated absorption (Upward direction)
(ii) Spontaneous emission. (Downward direction).
Stimulated emission (Downward direction).

N_1 - no. of atoms per unit volume in lower energy level
E_1 - Energy of the atoms in lower energy level.
N_2 - no. of atoms per unit volume in higher energy level.
E_2 - Energy of atoms in higher energy level.

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$E_2 - E_1 = h\nu$.

If $\rho(\nu)$ = Energy density of the interacting photons.
$= n h\nu$.

n - no. of interacting photons per unit volume.

Case I. Stimulated absorption.

Stimulated absorption rate $\propto N_1$
$\propto \rho(\nu)$
$= B_{12} N_1 \rho(\nu)$

B_{12} = Proportionality constant and known as Einstein's absorption coeff.

Case II. Spontaneous emission.

Spontaneous emission rate $\propto N_2$
$= A_{21} N_2$

A_{21} - Proportionality constant / known as Einstein's spontaneous emission coeff.

Case III. Stimulated emission

Stimulated emission rate $\propto N_2$
$\propto \rho(\nu)$
$= B_{21} N_2 \rho(\nu)$

B_{21} - proportionality constant / known as Einstein's stimulated ~~rate~~ emission coeff.

Under thermal equilibrium total upward transition is equal to total downward transition

$$B_{12} N_1 \rho(\nu) = B_{21} N_2 \rho(\nu) + A_{21} N_2.$$

$$\boxed{\rho(\nu) = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2} = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}}\left(\frac{N_1}{N_2}\right) - 1}} \quad \text{--- (1)}$$

By Boltzman distribution ~~level~~ law..
The population density at different energy level

$$N_i = N_0 e^{-E_i/KT} \quad \text{--- (2)}$$

N_i - Population density in the i^{th} energy level.
N_0 - Population density at the ground state.

$$N_1 = N_0 e^{-E_1/KT}, \quad N_2 = N_0 e^{-E_2/KT}.$$

$$\frac{N_1}{N_2} = e^{(E_2-E_1)/KT}.$$

$$\Rightarrow \frac{N_1}{N_2} = e^{h\nu/KT}.$$

$$\rho(\nu) = \frac{A_{21}/B_{21}}{\left(\frac{B_{12}}{B_{21}} e^{h\nu/KT} - 1\right)} \quad \text{--- (3)}$$

From planck's law of black body radiation.
the energy density

$$\rho(\nu) = \frac{8\pi h\nu^3}{c^3} \times \frac{1}{e^{h\nu/KT} - 1} \quad \text{--- (4)}$$

Comparing eqn (3) and (4).

$$\frac{B_{12}}{B_{21}} = 1 \quad \Rightarrow B_{12} = B_{21}.$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}.$$

Let R be the ratio of spontaneous emission rate to stimulated emission rate.

$$R = \frac{N_2 A_{21}}{N_2 B_{21} \rho(\nu)} = \frac{A_{21}}{B_{21}} \frac{1}{\rho(\nu)}.$$

$$R = e^{h\nu/KT} - 1.$$

$$\Rightarrow R = e^{(E_2-E_1)/KT} - 1.$$

Explain why we cannot have transition b/w any two consecutive states.

Explain how population inversion is a negative temp. state

As $E_2 - E_1 = h\nu$ and that is positive the probability of spontaneous emission increases rapidly with the energy diff. of two states. So, under thermal eq. spontaneous emission is a dominant process.

$$\frac{N_2}{N_1} = e^{-(E_2-E_1)/kT}$$

$$\frac{N_2}{N_1} - 1 = e^{-(E_2-E_1)/kT} - 1$$

$$N_2 - N_1 = N_1\left(e^{-(E_2-E_1)/kT} - 1\right)$$

$$\Rightarrow \boxed{\Delta N = N_1\left(e^{-(E_2-E_1)/kT} - 1\right)}$$

> +ve for population inversion.

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RHS can be +ve when T is -ve.
So, population inversion is refered as a negative temperature state

As $N_1 > N_2$ so, probability of stimulate absorption is more than that of stimulated emission.
[As $N_1 > N_2$ so, interaction of photons in the lower level is more than that of the higher level. So, stimulated absorption dominates]

So, the conclusion drawn the higher energy state can never be more populated than the lower energy state or ground state.
So, population inversion is not possible by direct excitation from a lower energy level to a higher energy level.

This only can be possible when we consider atleast a 3 energy level state.

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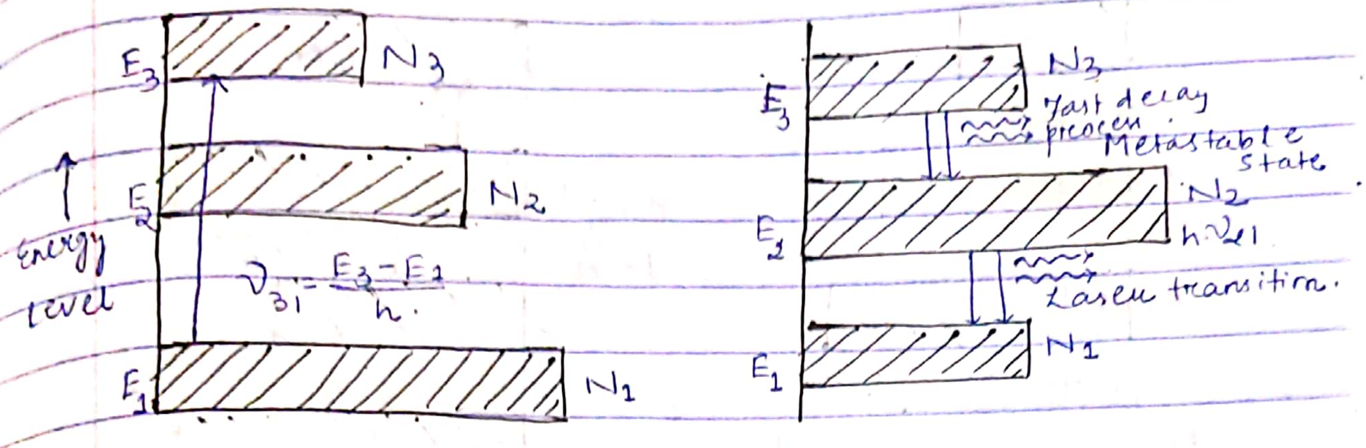
Three energy level-



→ Stimulated absorption excites the atoms from ground state E_1 to the highest level E_3 by incident radiation of frequency, $\nu_{31} = \frac{E_3 - E_1}{h}$

→ But as the life time of the excited atoms in the level E_3 is very short so they decay quickly to E_2.

→ It will remain for a longer period in E_2 than E_3. So, E_2 is called as a metastable state.

→ We intense pumping from E_1 to E_3 and rapid decay from E_3 to E_2, the population of E_2 exceeds E_1 and at this condition the system referred as a active system. So, suitable for emitting laser radiation of frequency ν_{21}/ν_{12}

i.e. $\nu_{21} = \frac{E_2 - E_1}{h}$

demerit of 3 energy state is merit of 4 energy state.

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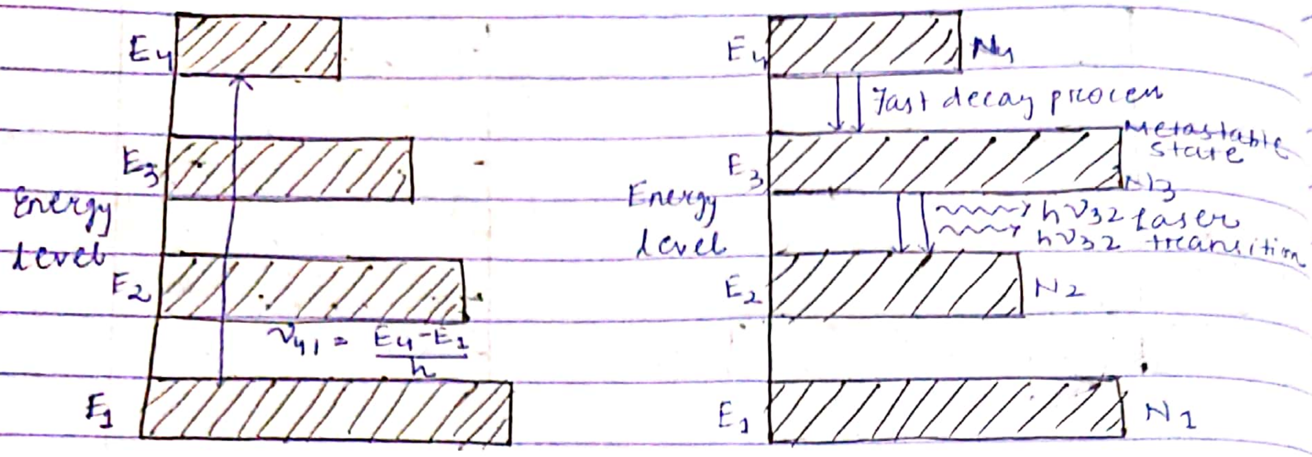
Four energy level state -



Population inversion achieved in b/w N_3 & N_2

Merits /.

→ Here stimulated absorption excite atoms from ground state E_1 to the highest E_4 level by incident radiation of frequency $\nu_{41} = \frac{E_4 - E_1}{h}$.

→ As the life time of the excited atom in the energy level is very short, they decay quickly to E_3 level which is the upper level of meta-stable state.

→ So population inversion achieved in b/w E_3 and E_2 level. (the atoms present for the time $\sim 10^{-3}$ sec

→ If the light radiation of frequency ν_{32} is incident on the active system it stimulates the radiation of same frequency.

→ Both the original incident radiation and and stimulated radiation are coherent.

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Merits/Advantages of 4 energy level with respect to 3 energy level. (de-merit of 3 energy level)-

→ As the ground level is not the lower level for laser transition so there is no need to pump more than one half of the population from the ground level., so less energy is required for pumping

→ As E_2 is the lower ~~Rue~~ laser level so it is easy to maintain the population inversion in b/w E_3 and E_2.

→ So, the laser transition is continuous emission, in this case.

Pumping-

It is a method of achieving population inversion externally.

There are various types of pumping-

(i) Optical pumping. (External source is a lamp, ~~sou~~ or led)

(ii) Electric field pumping (~~Ruby~~ He-Ne laser is an Ex).

(iii) Electron beam pumping.

(iv) Chemical pumping.

Components of Laser-

(i) Energy source-

(ii) Optical feedback.

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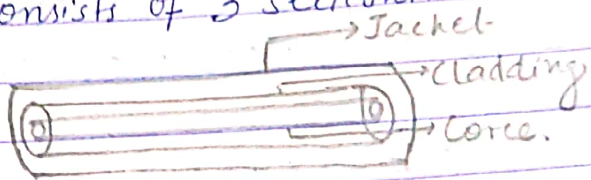
Advantages of Optical Fibre Lines over conventional Line (wire)-

- Long distance signal transmission.
- Large band width, light weight and small diam.
- Easy installation and easy transportation
- Non-conductivity.
- Dielectric in nature.
- Used for security concern.
- Cheap.

Structure-

Optical fibre mainly consists of 3 sections.-

(i) Core
(ii) Cladding
(iii) Outer Core / Jacket



(i)

It is the inner most section and is made up of glass or plastic. Its diam. is about 5µm to 100µm. Remarkable property of conducting an optical beam.

(ii)

Core is surrounded by cladding which is made up of plastic or glass and its diam. is about 125 µm. It has the diff. optical property than that of the core. Such that the refractive index of core is greater than the refractive index of the cladding

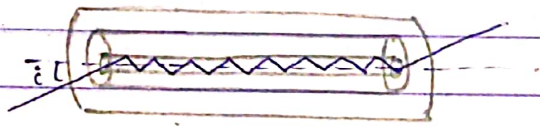
(iii)

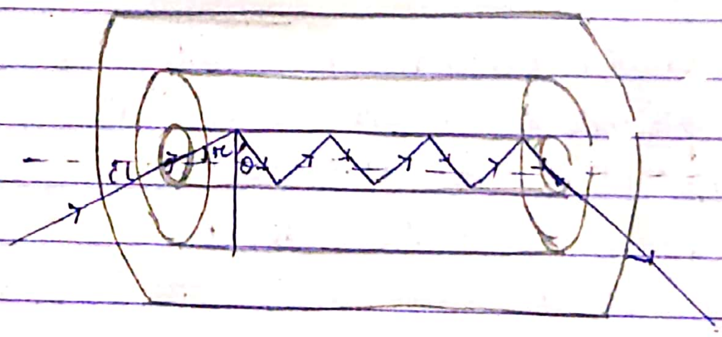
It is the outer most section made up of plastic, polymer or some suitable material and mainly used.

for the protection against moisture, crushing or any type of environmental hazards and its diamr is about 250 μm.

Classification Of Optical Fibre-
There are 2 methods of classification of optical fibres-
(i) One method is based on the variation of refractive index of the ~~rate~~ core.
(a) Step index fibre.
(b) Graded index fibre.
(ii) ~~Q~~ Other method is based on the core diam. or mode of transmission of light. Those are-
(a) Single mode fibre.
(b) Multi mode fibre.

Principle Of Optical Fibre-





i - Angle of incidence at the core.
r - Angle of refraction with axis
θ - Angle of incidence at core and cladding interface.

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The total internal reflection is the main principle of optical fibre communication. The light signal entering at one end of fibre has to travel through the entire length and emerge with other end without much loss.

Derivation for Numerical apperature and acceptance angle -

Condition for propagation of light signal through optical fibre -

(i) Core must have higher refractive index than that of cladding.

$$\mu_1 > \mu_2$$

μ_1 - R·I of core
μ_2 - R·I of cladding

(ii) Angle of incidence at cladding should be more than that of the critical angle.

$$\theta > \theta_c$$

$$\text{or } \theta \geq \theta_c$$

$$\sin\theta \geq \sin\theta_c \quad \text{— (1)}$$

$$\sin\theta_c = \frac{\mu_2}{\mu_1} \quad \text{— (2)}$$

From fig., $\theta = 90 - r$.

$$\sin\theta = \sin(90 - r) = \cos r \quad \text{— (3)}$$

from eq^n (1), (2) and (3).

$$\cos r \geq \frac{\mu_2}{\mu_1} \quad \text{— (4)}$$

From Snell's law.

$$\frac{\sin i}{\sin r} = \mu_1.$$

$$\sin r = \frac{\sin i}{\mu_1}$$

$$\cos r = \sqrt{1-\sin^2 r}$$

$$\Rightarrow \cos r = \sqrt{1 - \frac{\sin^2 i}{\mu_1^2}} \quad \text{--- (5)}$$

Equating eq^n (4) and (5)

$$\sqrt{1-\frac{\sin^2 i}{\mu_1^2}} \geq \frac{\mu_2}{\mu_1}$$

$$1 - \frac{\sin^2 i}{\mu_1^2} \geq \frac{\mu_2^2}{\mu_1^2}$$

$$\Rightarrow \mu_1^2 - \sin^2 i \geq \mu_2^2$$

$$\Rightarrow \mu_1^2 - \mu_2^2 \geq \sin^2 i$$

$$\Rightarrow \sin i \leq \sqrt{\mu_1^2 - \mu_2^2}$$

Take the maximum value of i and let it be i_m.

$$\boxed{\sin i_m = \sqrt{\mu_1^2 - \mu_2^2}}$$

This is the numerical apperature

$$\sin i_m = \sqrt{(\mu_1 - \mu_2)(\mu_1 + \mu_2)}$$

$$\sin i_m = \sqrt{2\mu_1 \Delta\mu}$$

Limiting case.

Upper limit $i_m = 90°$

$$1 = \sqrt{\mu_1^2 - \mu_2^2}$$

$$\mu_1^2 = 1 + \mu_2^2$$

This is not a practically physible optical fibre through which the communication can take place.

In all the practical cases,
$$\mu_1^2 < (1+\mu_2^2)$$

Lower limit: $i_m = 0$.

$\Rightarrow \mu_1 = \mu_2$.

Acceptance Angle-

It is the twice of the critical angle $(\neq \theta_c)$ and the critical angle is the max. value of the incidence angle (i)

$$i_c = 2 i_m$$

Acceptance angle.

Q1: The refractive index of the core and cladding for a step index fibre are 1.52 and 1.41 resp. Calculate the numerical apperature of fibre.

Q2. Advantages of Optical Fibre Communication System -

FOCS - Fibre Optics. Communication System

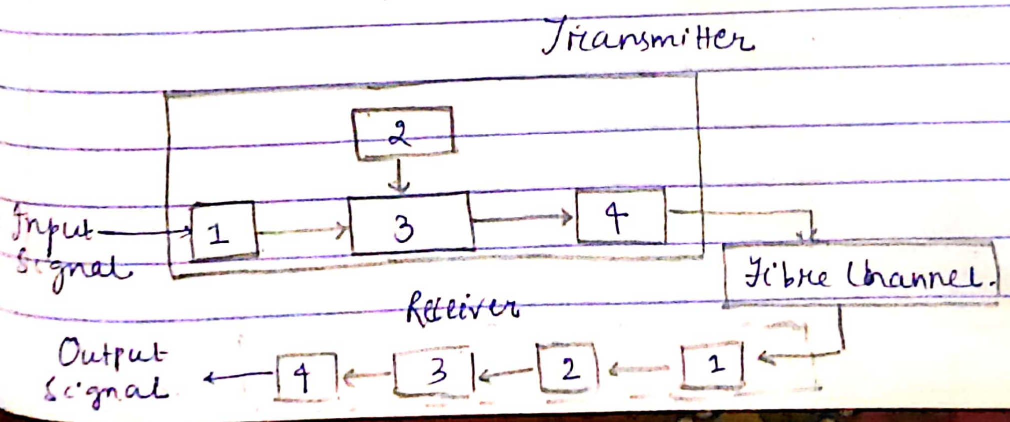
FOCL - Fibre Optics. Communication Link.

↳ This mainly consists of three section.

(i) Transmitter.

(ii) Optical fibre channel

(iii) Receiver.



Transmitter -

1 - Coder
2 - Carrier Source
3 - Intensity modulator
4 - Input Channel Coupler

Receiver -

1 - Output Channel Coupler
2 - Detector
3 - Signal Processor.
4 - Decoder.

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