Binomial Distribution / Bernoulli Distribution

It is a discrete probability distribution with Probability mass function $f(x) = \binom{n}{x} p^x q^{n-x}, \quad x = 0, 1, \dots n$

$= 0$, Other values of x

This is the probability that in 'n' independent trials an event A occurs precisely x times where p is the probability of A in a single trial and $q = 1 - p$.
The Occurrence of A is called success and the nonoccurrence is called failure. p is called the probability of success in a single trial.

The binomial distribution has mean

$$\boxed{\mu = E(x) = np}$$

Variance $\boxed{\sigma^2 = npq}$ where n: no. of trials.
p: probability of success.
q: probability of failure.

Q ① Four fair coins are tossed simultaneously. Find the probability function of the random variable X = number of heads and compute the probabilities of obtaining no heads, precisely 1 head, at least one head, not more than 3 heads.

Solution: Given $n = 4$

prob. mass function $f(x) = \binom{n}{x} p^x q^{n-x}$

Here $p = q = \frac{1}{2}$; $n = 4$

Scanned with CamScanner

$\therefore f(x) = \binom{4}{x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{4-x}$.

$P(\text{no heads}) = P(X=0) = \binom{4}{0}\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4$

$= \frac{4!}{0!\,4!}\,\frac{1}{2^4} = \frac{1}{16} = 0.0625$

$P(\text{precisely 1 head}) = P(X=1) = \binom{4}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^3$

$= \frac{4!}{1!\,3!}\,\frac{1}{2}\cdot\frac{1}{2^3} = \frac{1}{4} = 0.25$

$P(\text{at least one head}) = 1 - P(\text{no heads}) = 1 - P(X=0)$

$= 1 - 0.0625 = 0.9375$

$P(\text{not more than 3 heads}) = 1 - P(X=4)$

$= 1 - \binom{4}{4}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^0 = 1 - 0.0625 = 0.9375$

Q② If the probability of hitting a target is 10% and 10 shots are fired independently, what is the probability that the target will be hit at least once?

Ans: Given $n = 10$, $p = 10\% = \frac{10}{100}$

$q = 1 - p = 1 - \frac{10}{100} = \frac{90}{100}$

$\therefore f(x) = \binom{n}{x} p^x q^{n-x} = \binom{10}{x}\left(\frac{10}{100}\right)^x\left(\frac{90}{100}\right)^{10-x}$

$P(\text{the target will be hit at least once}) =$

$1 - P(\text{not hitting the target}) = 1 - P(X=0)$

$= 1 - \binom{10}{0}\left(\frac{10}{100}\right)^0\left(\frac{90}{100}\right)^{10} = 1 - \left(\frac{90}{100}\right)^{10}$

(3) Let $p = 1\%$ be the probability that a Certain type of light bulb will fail in the 24-hour test. Find the probability that a sign consisting of 10 such bulbs will burn 24 hrs with no bulb failures.

Ans: Given $n = 10$, $p = 1\%$, $q = 1 - p = 99\%$.

$$f(x) = \binom{n}{x} p^x q^{n-x} = \binom{10}{x}\left(\frac{1}{100}\right)^x \left(\frac{99}{100}\right)^{10-x}$$

$$P(X=0) = \binom{10}{0}\left(\frac{1}{100}\right)^0 \left(\frac{99}{100}\right)^{10} = (0.99)^{10}$$

Poisson Distribution

It is a discrete probability distribution with probability mass function $f(x) = \frac{e^{-\mu}\mu^{x}}{x!}$, $x = 0, 1, 2, \cdots$

It is a limiting case of binomial distribution.

For poison distribution mean = Variance = np.

$$\therefore \boxed{\mu = \sigma^2 = np}$$

Q. ① Let X be the number of Cars per minute passing a certain point of some road between 8 A.M and 10 A.M. On a Sunday. Assume that X has a poisson distribution with mean 5. Find the probability of observing 3 or fewer cars during any given minute.

Ans: Given mean $= \mu = 5$

Prob. mass function $f(x) = \frac{e^{-\mu}\mu^{x}}{x!} = \frac{e^{-5}5^{x}}{x!}$

P(3 or fewer Cars during any given minute)

$$= P(X=0) + P(X=1) + P(X=2) + P(X=3)$$

$$= \frac{e^{-5}5^{0}}{0!} + \frac{e^{-5}5^{1}}{1!} + \frac{e^{-5}5^{2}}{2!} + \frac{e^{-5}5^{3}}{3!}$$

$$= e^{-5}\left(1 + 5 + \frac{5^2}{2!} + \frac{5^3}{3!}\right)$$

$$= 0.2650$$

Q. Suppose that in the production of 50-ohm radio resistors, non defective items are those that have a resistance between 45 and 55 ohms and the probability of a resistor's being defective is 0.2%. The resistors are sold in lots of 100, with the guarantee that all resistors are non defective. What is the probability that a given lot will violate this guarantee?

Sol^n. Given $n=100$, $p = 0.2\%$

$$\text{mean} = \mu = np = 100 \times \frac{0.2}{100} = 0.2$$

Prob. mass function $f(x) = \dfrac{e^{-\mu}\mu^x}{x!} = \dfrac{e^{-0.2}(0.2)^x}{x!}$

$P(\text{lot will violate this guarantee}) = 1 - P(X=0)$

$$= 1 - \frac{e^{-0.2}(0.2)^0}{0!} = 1 - e^{-0.2} = 0.18126$$

Q. Suppose that 3% of bolts made by a machine are defective, the defective occuring at random during production. If the bolts are packaged 50 per box, what is the poisson approximation of the probability that a given box will contain x defectives?

Sol^n. Given $n=50$, $p=3\%$

$$\therefore \text{Mean} = \mu = np = 50 \times \frac{3}{100} = 1.5$$

$$P(\text{box contains } x \text{ defectives}) = \frac{e^{-1.5}(1.5)^x}{x!}$$