Wave Optics - (Module - I)

Wave is defined as a disturbance that moves through a medium.

$\psi \rightarrow$ Wave Function.

* $\psi(x,t) \rightarrow$ We imagine wave is travelling along x axis.
* $\psi(\vec{r},t) \rightarrow$ We imagine wave is travelling along x, y, z axis (in the space).

A wave can be represented by -

$$\boxed{\psi(x,t) = A\sin(kx - \omega t + \phi)}$$

$A \rightarrow$ Amplitude of wave

$k \rightarrow$ Wave Propagation vector. $(k = \frac{2\pi}{\lambda}) \rightarrow$ Wavelength

$\omega \rightarrow$ Angular frequency $(\omega = 2\pi f) \rightarrow$ Frequency

$\phi \rightarrow$ Initial phase.

Wave velocity, $\boxed{v = \frac{\omega}{K}}$

Relation betw^n wavelength (λ), frequency (f) and velocity (v).

$$\boxed{\lambda = \frac{v}{f}}$$

Differential Form Of Wave Equation -

$$\psi(x,t) = A\sin(kx - \omega t + \phi)$$

$$\frac{d\psi}{dt} = -\omega A\cos(kx - \omega t + \phi)$$

$$\frac{d^2\psi}{dt^2} = -\omega^2 \underbrace{A\sin(kx - \omega t + \phi)}_{\psi(x,t)} \Rightarrow \boxed{\frac{d^2\psi}{dt^2} = -\omega^2\psi(x,t)} \quad \text{— (1)}$$

$$\Rightarrow \boxed{\psi(x,t) = -\frac{1}{\omega^2}\frac{d^2\psi}{dt^2}} \quad \text{— (2)}$$

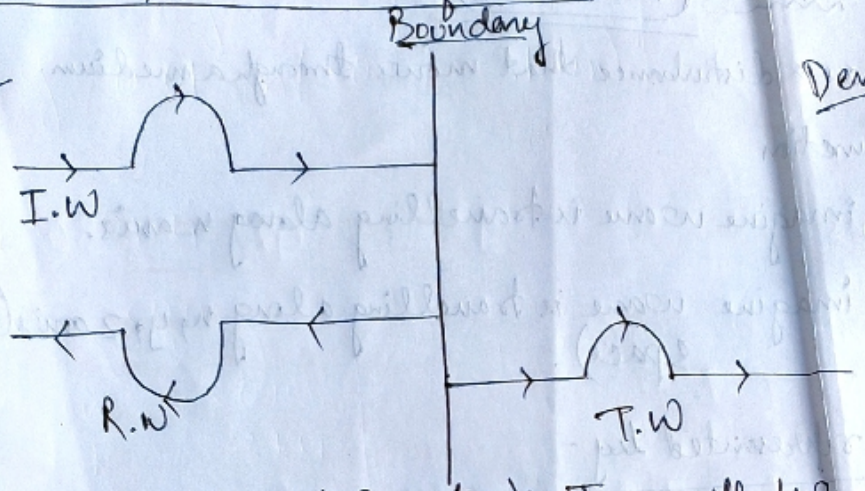
$$\frac{d\psi}{dx} = kA\cos(kx - \omega t + \phi)$$

$$\frac{d^2\psi}{dx^2} = -k^2 \underbrace{A\sin(kx - \omega t + \phi)}_{\psi(x,t)} \Rightarrow \boxed{\frac{d^2\psi}{dx^2} = -k^2\psi(x,t)} \quad \text{— (3)}$$

From (2) & (3) -

$$\frac{d^2\psi}{dx^2} = -k^2\left(-\frac{1}{\omega^2}\frac{d^2\psi}{dt^2}\right) \Rightarrow \frac{d^2\psi}{dx^2} = \underbrace{\frac{k^2}{\omega^2}}_{(1/v)^2}\frac{d^2\psi}{dt^2} \Rightarrow \boxed{\frac{d^2\psi}{dx^2} = \frac{1}{v^2}\frac{d^2\psi}{dt^2}}$$

Reflection and Transmission of Wave



	Reflected Wave (R.W)	Transmitted Wave (T.W)
Amplitude → (A)	less than I.W	Less than I.W
Speed → (v).	Speed is same in same medium.	Speed decreases as wave travelle from rarer to denser.
Frequency → (f)	Same	Same
Wavelength → (λ)	Same	Less than I.W
Phase →	Opposite.	Same Phase

$$\boxed{I \propto A^2}$$

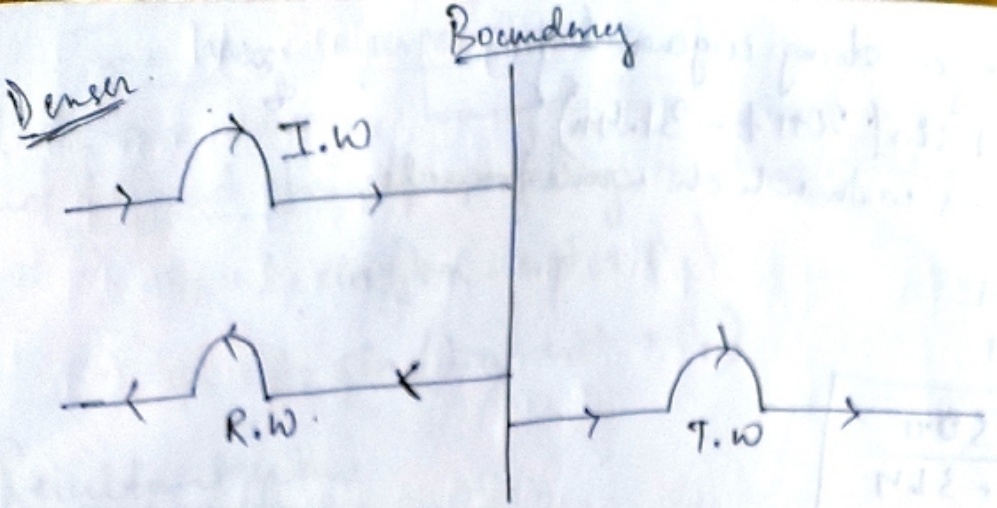
* Energy coming from the incident wave gets distributed with the I.W and R.W.
* Amplitude depends only on Intensity (light energy). Due to conservation of energy, the energy coming from the I.W gets distributed to the R.W and T.W.
* Speed remains same in the same medium and decreases when move from rarer to denser medium.
* Frequency of a wave is constant & doesn't change.

$$\text{Wavelength}(\lambda) = \frac{\text{Velocity}(v)}{\text{Frequency}(f)}$$

⇓

Constant for wave.

So, $\boxed{\text{for wave wavelength}(\lambda) \propto \text{Velocity}(v)}$



* When wave travels from rarer medium to denser medium the phase of transmitted wave remains same as incident wave but the phase of reflected wave becomes opposite.

* But when travels from denser medium to rarer medium, all the waves remain in the same phase.

	R.W	T.W
Amplitude (A) →	Decreases	Decreases
Speed (v) →	Same	Increases
Frequency (f) →	Same	Same
Wavelength (λ) →	Same	Increases
Phase (φ) →	Same	Same

Q) The progressive wave is represented by $\phi(x,t) = 4\sin(0.2x + 2t + \pi/6)$ Calculate (i) Amplitude (ii) Time (iii) Frequency (iv) Phase (v) Speed (vi) Wavelength.

Ans (i) Amplitude = 4m

(ii) time $= \frac{2\pi}{\omega} = \frac{10\pi}{0.2} = 10\pi$

(iii) Speed $(v) = \frac{\omega}{K}$

$= \frac{2}{0.2} = 10\,m/s$

(iv) Phase $\phi = \pi/6$

(v) Frequency (f)

$\omega = 2\pi f \Rightarrow 2 = 2\pi f \Rightarrow \boxed{f = \frac{1}{\pi}}$

(vi) Wavelength $(\lambda) = \frac{v}{f}$

$= \frac{10}{\frac{1}{\pi}} = 10\pi$

Q) A wave along a string is given by -

$$y = 0.01 \sin(50\pi t - 31.4x)$$

Calculate speed with which wave travels.

Ans. $k = -31.4$.

$\omega = 50\pi$

$$\boxed{V = \frac{\omega}{k} = \frac{50\pi}{-31.4}}$$

Superposition Of Wave -

* The principle of superposition states that when two or more waves act on a particle of a medium simultaneously then the resultant displacement of that ~~wave~~ particle is the algebric sum of the displacement of the same particle by the individual waves.

i.e. $\psi = \psi_1 + \psi_2$ (If both are in same direc^n)

$\psi = \psi_1 - \psi_2$ (If both are in opp. direc^n)

Types of Superposition -

(i) Coherent Superposition -

Constant phase difference betw^n the waves.

The resultant intensity differs from the sum of individual intensity i.e.

$$\boxed{I = A^2 \neq I_1 + I_2}$$

(ii) Incoherent Superposition -

* Phase difference changes continuously or randomly.
* Resultant Intensity = Sum of Individual Intensity

i.e $\boxed{I = A^2 = I_1 + I_2}$

Two beam superposition -

Let's take 2 waves having different amplitude, same frequency and different phases propagating in same direc^n.

$\Rightarrow \psi_1 = A_1 \sin(kx - \omega t + \phi_1)$

$\Rightarrow \psi_2 = A_2 \sin(kx - \omega t + \phi_2)$

Resultant wave -

$$\boxed{\psi = \psi_1 + \psi_2}$$

$$= A_1 \sin(kx - \omega t + \phi_1) + A_2 \sin(kx - \omega t + \phi_2)$$

$$= A_1 \left[\sin(kx - \omega t)\cos\phi_1 + \cos(kx - \omega t)\sin\phi_1\right]$$

$$+$$

$$A_2 \left[\sin(kx - \omega t)\cos\phi_2 + \cos(kx - \omega t)\sin\phi_2\right]$$

$$= A_1 \sin(kx - \omega t)\cos\phi_1 + A_1 \cos(kx - \omega t)\sin\phi_1 +$$

$$A_2 \sin(kx - \omega t)\cos\phi_2 + A_2 \cos(kx - \omega t)\sin\phi_2$$

$$= \sin(kx - \omega t)\left[A_1 \cos\phi_1 + A_2 \cos\phi_2\right] +$$

$$\cos(kx - \omega t)\left[A_1 \sin\phi_1 + A_2 \sin\phi_2\right]$$

Let $A_1 \cos\phi_1 + A_2 \cos\phi_2 = A\cos\phi$ —— (1)

and

$+$

$A_1 \sin\phi_1 + A_2 \sin\phi_2 = A\sin\phi$ —— (2)

$$= \sin(kx - \omega t) A\cos\phi + \cos(kx - \omega t) A\sin\phi.$$

$$= \boxed{A \sin(kx - \omega t + \phi)}$$

So, $\psi = \boxed{A \sin(kx - \omega t + \phi)}$

$\downarrow$

Amplitude of Resultant wave.

Now, $I = A^2 = (A\cos\phi)^2 + (A\sin\phi)^2$

$$= (A_1\cos\phi_1 + A_2\cos\phi_2)^2 + (A_1\sin\phi_1 + A_2\sin\phi_2)^2$$

$$= A_1^2\cos^2\phi_1 + A_2^2\cos^2\phi_2 + 2A_1\cos\phi_1 A_2\cos\phi_2$$

$$+$$

$$A_1^2\sin^2\phi_1 + A_2^2\sin^2\phi_2 + 2A_1\sin\phi_1 A_2\sin\phi_2$$

$$= A_1^2(\underbrace{\cos^2\phi_1 + \sin^2\phi_1}_{1}) + A_2^2(\underbrace{\cos^2\phi_2 + \sin^2\phi_2}_{1})$$

$$+ 2A_1A_2\sin\phi_1\sin\phi_2 + 2A_1A_2\cos\phi_1\cos\phi_2$$

$$= A_1^2 + A_2^2 + 2A_1A_2(\sin\phi_1\sin\phi_2 + \cos\phi_1\cos\phi_2)$$

$$\boxed{= A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_1 - \phi_2) = A^2 = I.}$$

$\Downarrow$

Resultant Intensity.

$(\phi_1 - \phi_2) \rightarrow$ Phase difference.

$[2A_1A_2\cos(\phi_1 - \phi_2)] \rightarrow$ Interference term.

Coherent Superposition -

Case-1 Let $(\phi_1 - \phi_2) = 0, \pm 2\pi, \pm 4\pi \ldots \pm 2n\pi$.

where $n \rightarrow 1, 2, 3, \ldots$

$\Rightarrow \cos(\phi_1 - \phi_2) = 1$.

$$\Rightarrow \boxed{I = I_{max} = A_1^2 + A_2^2 + 2A_1A_2 = (A_1 + A_2)^2}$$

SP case- If $A_1 = A_2 = a$.

$$\boxed{I = I_{max} = 4a^2.}$$ (Bright Fringe)

Case-2 Let $(\phi_1 - \phi_2) = \pm\pi, \pm 3\pi, \pm 5\pi, \ldots \pm(2n+1)\pi$

So, $\cos(\phi_1 - \phi_2) = -1$

So, $I = I_{min} = (A_1 - A_2)^2$

Sp case:- If $A_1 = A_2 = a$

$I = I_{min} = 0$ (Dark Fringe)

Incoherent Superposition -

* Phase difference changes frequently & randomly.
* Time Average value of $\langle \cos(\phi_1 - \phi_2) \rangle = 0$

$$\underset{(min)}{-1} \leftarrow 0 \rightarrow \underset{(max)}{1}$$

* $I = A^2 = A_1^2 + A_2^2$
$= I_1 + I_2$

* Resultant Intensity = Sum of Individual Intensity.

$A\sin\phi = A_1\sin\phi_1 + A_2\sin\phi_2$

$A\cos\phi = A_1\cos\phi_1 + A_2\cos\phi_2$

$\Rightarrow \tan\phi = \frac{A\sin\phi}{A\cos\phi} = \frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}$

$\Rightarrow \phi = \tan^{-1}\left(\frac{A_1\sin\phi_1 + A_2\sin\phi_2}{A_1\cos\phi_1 + A_2\cos\phi_2}\right)$

HW* Show that the resultant wave due to the superposition
L.Q of two harmonic waves of same frequency, different amplitude and different phases and travelling in same direction is also harmonic. Find amplitude & phase of resultant wave.

Ans. Lets take two harmonic waves ...

So, resultant wave is also harmonic ...

H.W. (1.Q) Coherent Superposition results in an interference pattern. Show it by taking two ~~means~~ beams of different amplitudes, different phases & same frequency & propagating in same direc^n in a med.

Ans. Write the superposition ---

Apply the coherent condn^s ---

Hence there is an interference pattern ---

Multiple Beam Superposition.

Let us consider n no. of waves of different amplitude, same frequency and different phase travelling in the same direction superpose.

$$\psi_1 = A_1 \sin(kx - \omega t + \phi_1)$$

$$\psi_2 = A_2 \sin(kx - \omega t + \phi_2)$$

$$\vdots$$

$$\psi_n = A_n \sin(kx - \omega t + \phi_n)$$

After Superposition -

The resultant wave,

$$\boxed{\psi = \psi_1 + \psi_2 + \psi_3 + \cdots \psi_n}$$

$$= \boxed{\sum_{i=1}^{n} \psi_i}$$

$$\Rightarrow \psi = A_1 \sin(kx - \omega t + \phi_1) + A_2 \sin(kx - \omega t + \phi_2) + \cdots + A_n \sin(kx - \omega t + \phi_n)$$

When $A \sin\phi = A_1 \sin\phi_1 + A_2 \sin\phi_2 + A_3 \sin\phi_3 + \cdots A_n \sin\phi_n$

$A \cos\phi = A_1 \cos\phi_1 + A_2 \cos\phi_2 + A_3 \cos\phi_3 + \cdots A_n \cos\phi_n$

We will get $\boxed{\psi = A \sin(kx - \omega t + \phi)}$

Resultant Intensity. *

$$\boxed{I = A^2 = \sum_{i=1}^{n} A_i^2 + 2 \sum_{\substack{i,j=1 \\ i \neq j}}^{n} A_i A_j \cos(\phi_i - \phi_j)}$$

For coherent Superposition

For max^m intensity (Let n=2).

$$I = A_1^2 + A_2^2 + 2A_1A_2 = (A_1 + A_2)^2$$

For $A_1 = A_2 = a$, $I = 4a^2 = (2a)^2$

For n = 3

$$A = A_1^2 + A_2^2 + A_3^2 + 2A_1A_2 \cos(\phi_1 - \phi_2) + 2A_2A_3 \cos(\phi_2 - \phi_3) + 2A_3A_1 \cos(\phi_3 - \phi_1)$$

Max.

$$\boxed{\begin{aligned} I_{coh} &= (na)^2 \\ I_{incoh} &= na^2 \end{aligned}}$$

$$\boxed{I_{coh} = n\, I_{incoh}}$$

* Coherent superposition with phases of successive waves suffering by a constant amount

$\psi_1 = A\sin(kx - \omega t) \Rightarrow \psi_1 = Ae^{i(kx-\omega t)}$

$\psi_2 = A\sin(kx - \omega t + \theta) \Rightarrow \psi_2 = Ae^{i(kx-\omega t+\theta)}$

$\psi_3 = A\sin(kx - \omega t + 2\theta) \Rightarrow \psi_3 = Ae^{i(kx-\omega t+2\theta)}$

$\vdots$

$\psi_n = A\sin(kx - \omega t + (n-1)\theta) \Rightarrow \psi_4 = Ae^{i(kx-\omega t+(n-1)\theta)}$

After superposition a resultant wave - (waves taken in exponential form)

$\psi = \psi_1 + \psi_2 + \psi_3 + \cdots \psi_n$

$= Ae^{i(kx-\omega t)} + Ae^{i(kx-\omega t+\theta)} + Ae^{i(kx-\omega t+2\theta)} + \cdots Ae^{i(kx-\omega t+(n-1)\theta)}$

$= Ae^{i(kx-\omega t)}\left[1 + e^{i\theta} + e^{2i\theta} + \cdots e^{i(n-1)\theta}\right]$

Formula-

$$1 + x^1 + x^2 + x^3 + \cdots + x^m = \frac{1 - x^{m+1}}{1 - x}.$$

$$= \boxed{Ae^{i(kx-\omega t)}\left[\frac{1 - e^{in\theta}}{1 - e^{i\theta}}\right]} = \psi$$

$$\Rightarrow \psi = A e^{i(kx-\omega t)} \left[\frac{e^{in\theta}-1}{e^{i\theta}-1}\right]$$

$$= A e^{i(kx-\omega t)} \frac{e^{in\theta/2}}{e^{i\theta/2}} \left[\frac{e^{in\theta/2}-e^{-in\theta/2}}{e^{i\theta/2}-e^{-i\theta/2}}\right]$$

$$\Rightarrow \psi = A e^{i(kx-\omega t)} e^{i\left(\frac{n-1}{2}\right)\theta} \left\{\left[\begin{array}{l} e^{i\theta} = \cos\theta + i\sin\theta \\ e^{-i\theta} = \cos\theta - i\sin\theta \end{array}\right]\right\}$$

$$\Rightarrow \boxed{\psi = \frac{A\sin n\theta/2}{\sin\theta/2} e^{i\left(kx-\omega t+\left(\frac{n-1}{2}\right)\theta\right)}} \rightarrow \text{(Rem)} \quad \left[\frac{2i\sin n\theta/2}{2i\sin\theta/2}\right]$$

$$\Rightarrow \boxed{\psi = R e^{i(kx-\omega t+\beta)}}$$

$\beta \rightarrow$ Phase of resultant wave.

(Rem).

$R \rightarrow$ Amplitude of resultant wave.

$$\boxed{\text{Resultant Amplitude}(R) = \frac{A\sin n\theta/2}{\sin\theta/2}} \quad \text{(Rem.)}$$

$$\boxed{\text{Resultant Phase}(\beta) = \left(\frac{n-1}{2}\right)\theta} \quad \text{(Rem).}$$

Interference -

The redistribution of intensities in the region of superposition is called as interference.

It is of two types -

(i) Constructive Interference

(ii) Destructive Interference.

Constructive Interference

The resultant amplitude is the sum of the individual amplitude.

$$\boxed{A = A_1 + A_2}$$

Destructive Interference -

Resultant Amplitude is the difference of individual amplitude.

$$\boxed{A = A_1 - A_2}$$

Methods of obtaining Interference Pattern (Coherent Sources) -

(i) Division of wave front. Eg - Young's Double Slit Experiment, Fresnel's Biprism

(ii) Division of amplitude

Eg - Newton's Rings, Michelson's Interferometer.

* When the incident wavefront is divided into two parts by reflection, refraction or diffraction and transverse through different optical path and reunited, then they produce an interference pattern which is called as the interference by division of wavefront.

Conditions for Interference -

① Conditions for sustained interference -

(i) Coherent Source.

(ii) Frequency must be equal or nearly equal, wavelength also equal or very nearly equal.

② Conditions for clear observation - (Imp for Short Ques^n)

(i) Background must be dark.

(ii) Distance betw^n slit and the screen is very large and distance betw^n the slits must be very very small.

③ Conditions for good contrast betw^n dark & bright fringes -

(i) Monochromatic source.

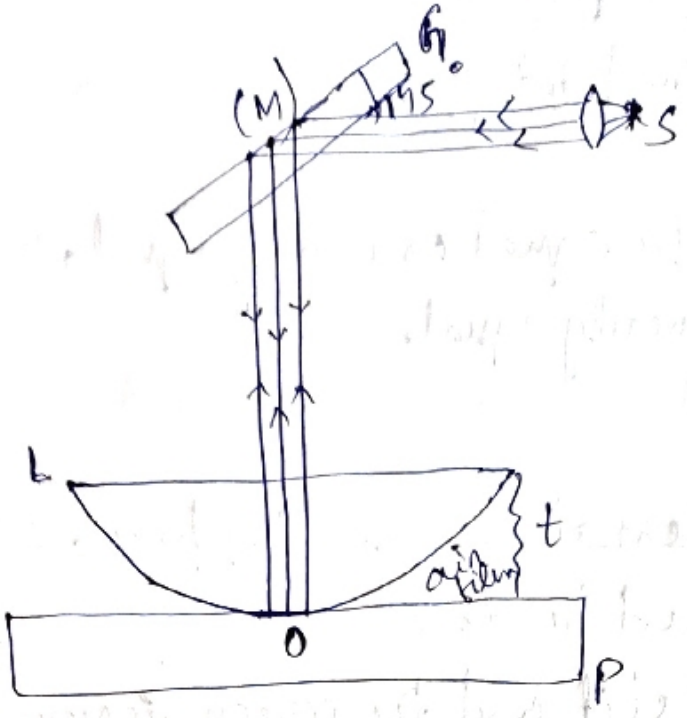
(ii) Amplitude must be equal or very nearly equal.

(iii) Slits must be narrow.

Newton's Ring - (Imp).

* When a plano convex lens is placed on a glass plate an air film is created with circular geometry, whose thickness gradually increases and become maximum at the edge of the lens. The thickness of the air film is zero at the center.

* Normal Incidence of monochromatic light on the plano convex lens will produce bright and dark concentric rings around the point of contact betw^n the lens and the glass plate. This phenomenon first explained by Newton, so called as Newton's ring.



S → Monochromatic source.

M → Travelling microscope.

G → Glass plate held at an angle of 45°.

t → thickness of air film.

P → Glass Plate

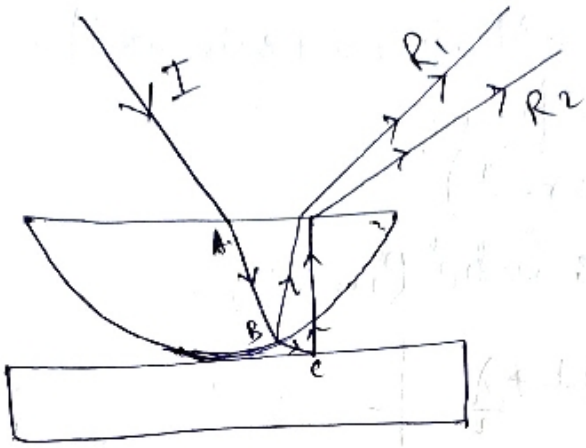
L → Plano convex lens having large radius of curvature.

(i) Let S be the monochromatic source from which light rendered parallel and fall on a glass plate G which is held at an angle of 45°, so that light incident on the plano convex lens normally.

(ii) The light passing through the plano convex lens and reflected from upper and lower surface of the air film formed in betw^n the plano convex lens and glass plate.

(iii) The reflected rays interfere and forms the fringe pattern which are circular in nature and viewed through the microscope.

Explanation of Fringe Formation - (Imp).



* Newton's ring are formed due to the interference of two reflected rays R_1 and R_2 coming from top and bottom surface of the air film formed between the lens and the glass plate.
* The incident ray I entered into the lens at the point A; then at point B, a part of it is reflected back in form of ray R_1 and rest part is transmitted through the air film & again reflected back to the same medium in form of ray R_2.
* So, R_1 and R_2 are coherent as produced from same incident ray I.

For reflected rays the path difference is -

$$\boxed{2\mu t \cos r + \frac{\lambda}{2}}$$

$r \rightarrow$ Angle of reflection.

In our case - (In Newton's ring experiment) -

$\mu = 1$ (air film)

$r = 0$ ($\cos r = 1$)

Path difference betn P_1 and P_2 -

$$\boxed{\Delta = 2t + \frac{\lambda}{2}} \quad \text{--- (1)}$$

At Centre ($t = 0$)

$$\Rightarrow \boxed{\Delta = \frac{\lambda}{2}}$$

Center is dark fringe.

Position of Bright fringe -

$\Delta = n\lambda$ [$n = 1, 2, 3 \ldots$]

$$\Rightarrow 2t + \frac{\lambda}{2} = n\lambda$$

$$\boxed{\Rightarrow 2t = (2n - 1)\frac{\lambda}{2}} \quad \text{--- (2)}$$

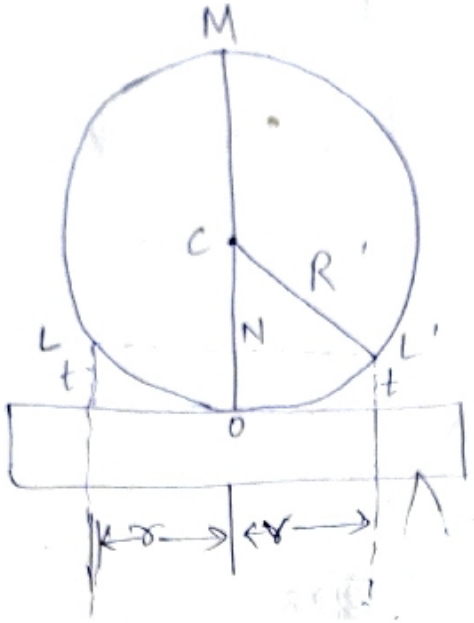
Position of dark fringe -

$\Delta = (2n+1)\lambda/2$ [$n = 1, 2, 3 \ldots$]

$$\Rightarrow 2t + \frac{\lambda}{2} = (2n+1)\lambda/2$$

$$\Rightarrow \boxed{2t = n\lambda} \quad \text{--- (3)}$$

Diameter of Newton's Ring -



Using Property of circle -

$$MN \times NO = LN \times NL'$$

$$\Rightarrow (2R - t) \times t = r \times r$$

$$\Rightarrow 2Rt - t^2 = r^2$$

$$t <<< R.$$

t^2 can be neglected.

$$\Rightarrow 2Rt = r^2$$

$$\Rightarrow \boxed{2t = \frac{r^2}{R}} \quad \text{--- (4)}$$

Comparing (2) & (4) - (Diameter of Bright fringe)

$$\Rightarrow \frac{r^2}{R} = (2n-1)\lambda/2$$

$$\Rightarrow \boxed{r^2 = \frac{R\lambda}{2}(2n-1)}$$

$$\Rightarrow \frac{D^2}{4} = \frac{R\lambda}{2}(2n-1)$$

$$\Rightarrow D^2 = 2R\lambda(2n-1)$$

$$\Rightarrow \boxed{D = \sqrt{2R\lambda(2n-1)}} \quad n = 1, 2, 3$$

$\Rightarrow D_n \propto \sqrt{2n-1}$

Diameter of nth Bright fringe.

Comparing (3) & (4) - (Diameter of dark fringe)

$\Rightarrow \frac{r^2}{R} = n\lambda$

$\Rightarrow r^2 = Rn\lambda$.

$\Rightarrow \frac{D^2}{4} = Rn\lambda$.

$\Rightarrow \boxed{D_n = \sqrt{4Rn\lambda}}$

Diameter of nth dark fringe.

* The fringes near to the center is more wider than the fringes away from the center as $D_n = \sqrt{4Rn\lambda}$.

$\Rightarrow \boxed{D_n \propto \sqrt{n}}$

~~$\Rightarrow D_1 = 1, D_2 = 1.414$~~

Newton's Rings formed by Transmitted light.

$\boxed{\text{Path difference}(\Delta) = 2t}$

Bright fringe.

$\frac{r^2}{R} = n\lambda$

$\boxed{2t = \frac{r^2}{R}}$

$\Rightarrow r^2 = Rn\lambda$

$\Rightarrow \boxed{D_n = \sqrt{4Rn\lambda}}$

$t = 0$ (center)

$\Delta = 0 \rightarrow$ Bright center

Dark fringe

$$2t = (2n-1)\lambda/2$$

$$\Rightarrow \frac{r^2}{R} = (2n-1)\frac{\lambda}{2}$$

$$\Rightarrow r^2 = \frac{R\lambda}{2}(2n-1)$$

$$\Rightarrow \boxed{D_n = \sqrt{2R\lambda(2n-1)}}$$ ⇒ Diameter of nth dark fringe.

* The fringe pattern formed in transmitted light and reflected light are complementary to each other.

Determination of Refractive Index of a liquid (N.R method)

$$2t = n\lambda \text{ (air)}$$

$$\Rightarrow 2\mu t = n\lambda \text{ (liq)}$$

$$\Rightarrow 2t = \frac{n\lambda}{\mu}$$

$$\Rightarrow \frac{r^2}{R} = \frac{n\lambda}{\mu} \qquad \Rightarrow r^2 = \frac{Rn\lambda}{\mu}$$

$$\Rightarrow \frac{D^2}{4} = \frac{Rn\lambda}{\mu}$$

$$\Rightarrow \boxed{D_{n_\ell}^2 = \frac{4Rn\lambda}{\mu}} \text{ (liq)}$$

$$\boxed{D_n^2 = 4Rn\lambda \text{ (air)}}$$

$$\Rightarrow \boxed{\mu = \frac{D_n^2}{D_{n_\ell}^2}}$$

Determination Of Wavelength Of Light By Newton's Ring Experiment.

$D_n^2 = 4Rn\lambda \rightarrow$ Dark fringe.

$D_n^2 = 2R\lambda(2n-1) \rightarrow$ Bright fringe } reflected

$D_{n+p}^2 = 4R(n+p)\lambda \longrightarrow$ Dark fringe.

$D_{n+p}^2 = 2R\lambda(2(n+p)-1) \rightarrow$ Bright fringe.

$D_{n+p}^2 - D_n^2 = \cancel{4(Rn+RP)} = 4RP\lambda$.

$\cancel{4Rn\lambda + P\lambda - 4Rn\lambda}$.

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4RP}}$$

$\Downarrow$

(Wavelength of light by Newton's ring.)

$\Rightarrow D_{n+p}^2 - D_n^2 = 4RP\lambda$ (for air medium).

$\Rightarrow D'^2_{n+p} - D'^2_n = \frac{4RP\lambda}{\mu}$ (for liq. med. of ref index μ)

$$\Rightarrow \boxed{\mu = \frac{D_{n+p}^2 - D_n^2}{D'^2_{n+p} - D'^2_n}}$$

2) Why the Newton's ring (fringe formation) is circular?

$2t = n\lambda$ (for dark fringe) $t \rightarrow$ thickness of air film

$n = 1, 2, 3 \ldots$

$\Rightarrow t = \text{const}$. $\lambda \rightarrow \text{const}$.

thickness of air film is const. for any particular order.

Locus of the pts for any constant value of t is circular.

So, newton's ring / fringe form^n is circular in nature with respect to the pt. of contact betw^n the plano convex lens and the glass plate.

(16) $\lambda = 589.3\,nm$ | $2t = n\lambda$.

$D_{10} = 0.005\,m$. | $t = ?$

$n = 10$.

$D_n^2 = 4Rn\lambda$.

$R = ?$

Bright

$\Delta = n\lambda$

Dark

$\Delta = (2n+1)\frac{\lambda}{2}$

(14) $R = 120\,cm$.

D_{15} (Bright) $\rightarrow 0.58\,cm$.

$D_{15} = \sqrt{2R\lambda(2n-1)} = \sqrt{2R\lambda \times 29}$.

$\Rightarrow R\lambda = ?$

$D_{25} = \sqrt{2R\lambda(2n-1)} = \sqrt{2R\lambda \times 49}$.

(13) $\lambda_1 = 600\,nm$, $\lambda_2 = 590\,nm$.

$D_n^2 = 4Rn\lambda_1$ & $D_{n+1}^2 = 4R(n+1)\lambda_2$

A.T.Q $\Rightarrow \cancel{4R}n\lambda_1 = \cancel{4R}(n+1)\lambda_2$

$\Rightarrow n = ?$ $n = 59$.

$\begin{array}{r} 349 \\ 4 \\ \hline 196 \end{array}$

Diffraction -

The bending of light at the corner of the aperture (may be an obstacle or a slit) is called as diffraction. When the dimension of the aperture is comparable to the wavelength of the light.

H.W (In class copy) Distinguish betn Interference and diffraction. (4 pts).

Interference	Diffraction
(i) It occurs due to the superposition of two light waves originating from the same source	(i) It occurs due to the superposition of ~~two light waves~~ many secondary wavelets that originates from the primary wave.
(ii) The region of minimum intensity (dark fringe) is completely dark.	(ii) The region of minimum intensity (minima) is not perfectly dark.
(iii) All bright fringes are of same intensity.	(iii) All bright fringes are not of same intensity.

Types -

(i) Fresnel's diffraction.

(ii) Fraunhofer's diffraction.

Fresnel's diffraction	Fraunhofer's diffraction
* The distance of the slit from the source and the screen is finite.	* The distance of the slit from the source and the screen is infinite.
* No lenses used.	* Lenses are used. One in between the source and the slit and another in between slit and screen
* The incident wavefronts are either spherical or cylindrical.	* The incident wavefront is plane wavefront.

Fraunhofer's Single Slit Diffraction-

Let AB be a single slit of width 'a'.

WW' be a plane wavefront originating from a monochromatic source of wavelength λ propagating normal to the slit be incident on it.

After diffraction, the diffracted rays are focused by using a convex lens and the screen is placed at the focal point of the convex lens.

When the wavefront reaches through the width of the slit, the secondary wavelets travelling normal to the slit are brought to focus by the lens on the screen at point P_0. So, P_0 be a point of maxima or central principal maxima.

In order to consider all the secondary wavelets originated and propagated in different directions, consider the secondary wavelets in the direction of θ are brought to focus by the lens at the point P_1 on the screen as

shown in figure.

To know the point P_1, a point of maxima/minima we have to calculate the path difference.

From figure the path difference is BM which is equal to $a \sin\theta$.

So, corresponding Phase difference $= \boxed{\frac{2\pi}{\lambda}(a\sin\theta)} = \phi$ (say).

To determine the resultant effect of all the secondary wavelets originating from different parts of the wavefront through the slit width, let it be divided into 'n' equal parts. (such that each part is just sufficient to produce one secondary wavelet).

The phase difference between any two secondary wavelets originated from successive parts of the wavefront in the direction of "θ" $= \frac{1}{n}\left(\frac{2\pi}{\lambda}(a\sin\theta)\right) = d$ (say)

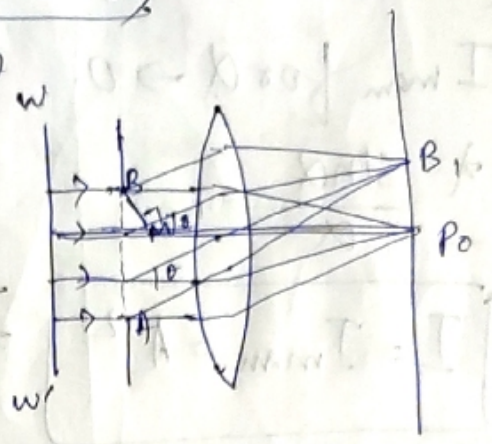
As all the secondary wavelets originated from the same wavefront, so all are having equal amplitude and let it be 'a'.

Here we are having 'n' number of secondary wavelets each having amplitude 'a' and successive phase difference 'd'.

So, their resultant amplitude at point P_1 is -

$$\boxed{R = \frac{a\sin\frac{nd}{2}}{\sin\frac{d}{2}}}$$

$$\Rightarrow R = \frac{a \sin \frac{\cancel{n}}{\cancel{2}}\left(\frac{1}{\cancel{n}} \times \frac{\cancel{2}\pi}{\lambda}(e\sin\theta)\right)}{\sin \frac{1}{\cancel{2}} \times \frac{\cancel{2}\pi}{n\lambda} e\sin\theta}$$



$$\Rightarrow R = \frac{a \sin \frac{\pi}{\lambda} e\sin\theta}{\sin \frac{\pi}{n\lambda} e\sin\theta}$$

Now, let $\frac{\pi}{\lambda} e\sin\theta = \alpha$

$$\Rightarrow R = \frac{a\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}$$

Here 'α' is very small but 'n' is very large.

$$\Rightarrow \sin\left(\frac{\alpha}{n}\right) = \frac{\alpha}{n}$$

$$\Rightarrow R = \frac{a\sin\alpha}{\frac{\alpha}{n}} = na\frac{\sin\alpha}{\alpha}$$

Let $na = A$.

$$\Rightarrow \boxed{R = \frac{A\sin\alpha}{\alpha}}$$

Resultant Intensity, $\boxed{I = A^2\frac{\sin^2\alpha}{\alpha^2}}$

↓

Complete Long Ques^n

$$I = \left(\frac{A\sin\alpha}{\alpha}\right)^2$$

$I = I_{max}$ for $\alpha \to 0$.

$\Rightarrow \frac{\sin\alpha}{\alpha} = 1$.

$\Rightarrow \boxed{I = I_{max} = A^2} \Rightarrow \boxed{I_{PM} = A^2}$

Posⁿ of Maxima

$\boxed{\alpha = 0} \longrightarrow$ (posⁿ of principal maxima)

$\Rightarrow \frac{\pi}{\lambda} e\sin\theta = 0$

$\Rightarrow \sin\theta = 0$

$\Rightarrow \boxed{\theta = 0} \longrightarrow$ P_0 (point of central/principal maxima)

$\boxed{\text{Intensity} = 0}$

Minima –

$I = I_{min} \Rightarrow 0$.

Posⁿ of minima $\Rightarrow \sin\alpha = 0$ But $\alpha \neq 0$. (posⁿ of minima)

$\Rightarrow \boxed{\alpha = \pm n\pi} \rightarrow$ Posⁿ of minima.

$n \neq 0$ and $n = 1, 2, 3$.

$\Rightarrow \frac{\pi}{\lambda} e\sin\theta = n\pi$

$\Rightarrow \boxed{e\sin\theta = n\lambda}$ $(n \neq 0,\ n = 1, 2, 3 \ldots)$

↳ Eqⁿ for minima in single slit.

$\Rightarrow \boxed{\theta = \sin^{-1}\left(\frac{n\lambda}{e}\right)}$

* With having 1 principal maxima at $\alpha = 0$ we have distinct minimas at $\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots$

So, the question of secondary maxima arise.

<u>The condition of secondary maxima can be obtained</u> -

$\Rightarrow \frac{dI}{d\alpha} = 0$

$$\Rightarrow \boxed{\frac{dI}{d\alpha} = A^2\, 2\left(\frac{\sin\alpha}{\alpha}\right)\left[\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right] = 0}$$

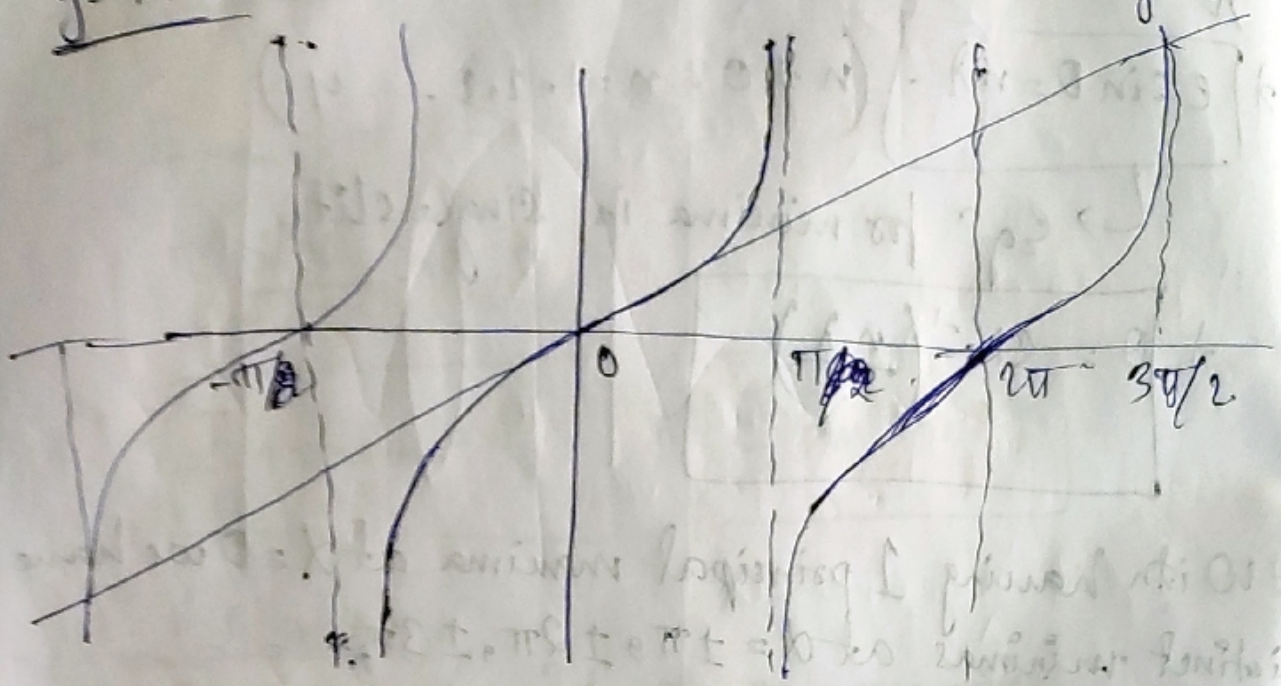
$\Rightarrow \frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$

$\Rightarrow \frac{\alpha\cancel{\cos\alpha}}{\cancel{\cos\alpha}} = \frac{\sin\alpha}{\cos\alpha}$

$\Rightarrow \boxed{\alpha = \tan\alpha}$ ← Condⁿ for secondary maxima.

* By drawing the curve after taking $y = \alpha$ and $y = \tan\alpha$ the point of intersection of both the curves will gi the posⁿ of secondary maxima.

$y = \tan\alpha$.



So, posⁿ of secondary maxima are

$$\boxed{\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} + \ldots}$$

* So, in betwⁿ any two minima there exists a secondary maxima.

$$\Rightarrow I = \left(\frac{A\sin\alpha}{\alpha}\right)^2$$

$\Rightarrow \alpha = 0 \rightarrow$ Principal maxima.

$\Rightarrow \alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \pm \ldots n\pi \rightarrow$ Minima

$\Rightarrow \alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} + \ldots \pm(2n+1)\frac{\pi}{2}$

$\rightarrow$ Secondary maxima

Intensity of Secondary Maxima -

Let I_1 be the intensity of first secondary maxima.

$$\Rightarrow I_1 = \frac{A^2 \sin^2\alpha}{\alpha^2} \qquad (\alpha = \pm 3\pi/2)$$

$$= \frac{A^2 \sin^2 \frac{3\pi}{2}}{\frac{9\pi^2}{4}}$$

$$\approx \boxed{\frac{A^2}{22}} = \boxed{\frac{I_{PM}}{22}}$$

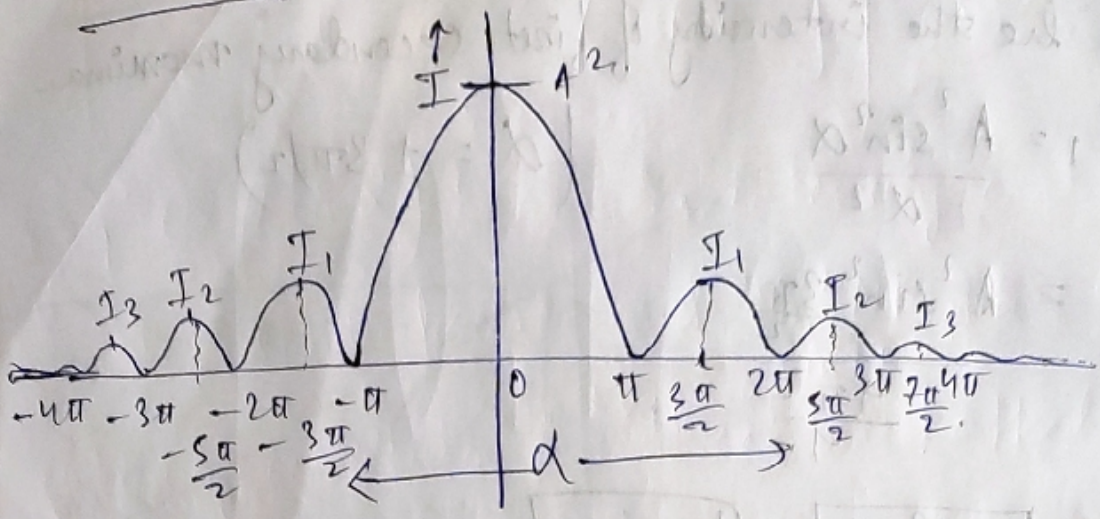
$$\Rightarrow I_2 = \frac{A^2 \sin^2\alpha}{\alpha^2} \qquad (\alpha = \pm 5\pi/2)$$

$$= \frac{A^2 \sin^2 5\pi/2}{\frac{25\pi^2}{4}}$$

$$= \boxed{\frac{A^2}{62}} \sim \boxed{\frac{I_{PM}}{62}}$$

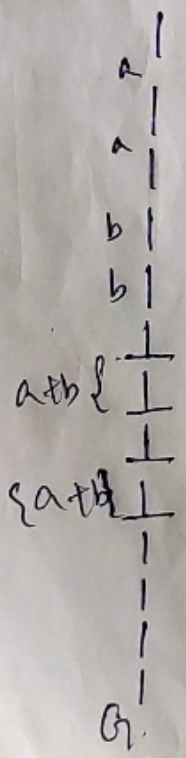
$$\Rightarrow I_3 = \frac{A^2}{128} \boxed{\frac{I_{PM}}{128}}$$

Intensity Distribution Curve -



Plane Diffraction Grating

* It is an arrangement of combination of n number of slits having equal width(a) and separated by equal opaque space(b)



a → Width of transparency.
b → width of opacity.
(a+b) → Grating element

* Grating element is defined as number of lines drawn in 1 cm.

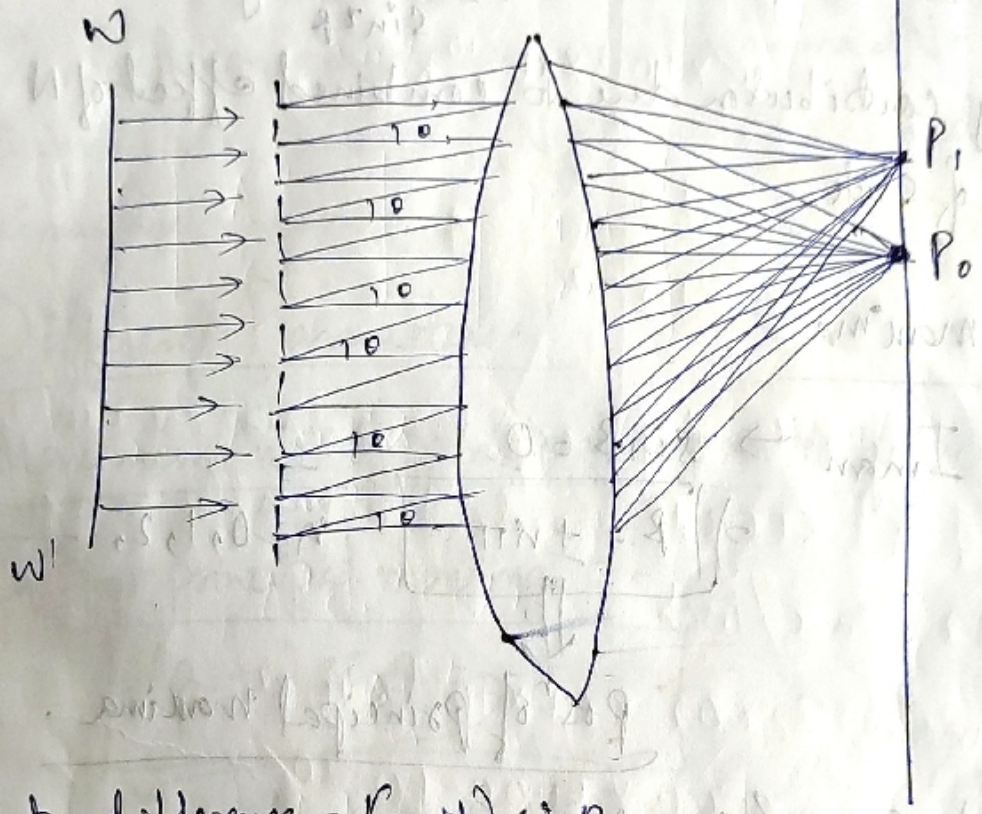
$N \longrightarrow$ No of lines drawn.

$L \longrightarrow$ Width of the grating in cm.

$L \longrightarrow$ in inches.

Grating element $= (L/N)$ in cm. $\quad L/N = G$.

$= (2.5/N)$ in inch. $\quad \Rightarrow N =$



Path difference $= (a+b)\sin\theta$

Corresponding phase difference $= \frac{2\pi}{\lambda}\left[(a+b)\sin\theta\right] = 2\beta$ (say)

<u>Resultant Amplitude</u> -

$$R = \frac{a\sin nd/2}{\sin d/2}$$

$\Rightarrow a = \left(\frac{A\sin\alpha}{\alpha}\right)$, $n = N$, $d = 2\beta$.

$$\Rightarrow R = \left(\frac{A\sin\alpha}{\alpha}\right)\frac{\sin N\cancel{2}\beta/\cancel{2}}{\sin \cancel{2}\beta/\cancel{2}}$$

$$\Rightarrow \boxed{R = \frac{A\sin\alpha}{\alpha}\,\frac{\sin N\beta}{\sin\beta}}$$

$$\Rightarrow \boxed{I = R^2 = \left(\frac{A\sin\alpha}{\alpha}\right)^2 \left[\frac{\sin^2 N\beta}{\sin^2\beta}\right]} \quad \text{--- (1)}$$

In Eq^n (1) $\left(\frac{A\sin\alpha}{\alpha}\right)^2$ gives the intensity contribution due to individual slits and $\frac{\sin^2 N\beta}{\sin^2\beta}$ gives the intensity contribution due to combined effect of N number of slits.

Pos^n of maxima

$I \rightarrow I_{max} \rightarrow \sin\beta = 0$

$$\Rightarrow \boxed{\beta = \pm n\pi} \quad n = 0, 1, 2, \ldots$$

$\Downarrow$

Pos^n of principal maxima.

$$\Rightarrow \beta = \frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi$$

$$\Rightarrow \boxed{(a+b)\sin\theta = \pm n\lambda}$$

$\Downarrow$

Eq^n of principal maxima in grating

$n = 0, 1, 2, 3, \ldots$

Minima

$I \rightarrow I_{min} \rightarrow \sin N\beta = 0$ but $\sin\beta \neq 0$.

$$\Rightarrow \boxed{N\beta = \pm m\pi}$$

* In between any two principal maxima, $(N-1)$ no. of minimas are there and $(N-2)$ no. of secondary maximas are there.

Different Parameters.

① Maximum no. of principal maxima

Eq^n for principal maxima is -

$$\boxed{(a+b)\sin\theta = \pm n\lambda}$$

$n \rightarrow$ order of principal maxima.

$n \rightarrow n_{max}$ when $\sin\theta = 1$.

Eq^n is $(a+b) = n_{max}\lambda$.

$$\Rightarrow \boxed{n_{max} = \frac{(a+b)}{\lambda}}$$

Ex- If a monochromatic light of wavelength 6000 A° incident normally on a plane diffraction grating having grating element 6×10^{-5} cm then find the maximum order of principal maxima observed.

Ans- $n_{max} = \frac{(a+b)}{\lambda}$

$= \frac{6\times10^{-5}\,cm}{6000\times10^{-10}\,m}$

$1m = 100\,cm$

$1cm = \frac{1}{100}\,m$

10^{-2}

$= \frac{6\times10^{-5}\times10^{-2}\,m}{6\times10^{3}\times10^{-10}\,m}$

$= \frac{10^{-7}}{10^{-7}} = 1.$

Q) How many order of principal maxima are visible if light of $\lambda = 5000A^{o}$ incident normally on a grating unit 2602 lines/inch.

Ans- $n_{max} = \frac{2602\times21\times10^{-1}\times10^{-2}}{5000\times10^{-10}}$

$= \frac{2602\times25\times10^{-3}}{5\times10^{3}\times10^{-10}}$

$2602 \times 25 = 65050$

$= \frac{5464\times10^{-3}}{5\times10^{-7}}$

$= 1928.5\times10^{4}$

$= 1.928.$

$$\cancel{n_{max} = \frac{2.5 \times}{2602 \times 5 \times 10^{3} \times 10^{-10}}}$$

Ans- $\lambda = 5000\ A^{\circ}$

$= 5000 \times 10^{-8}\ cm$

$$(a+b) = \frac{2.5}{2602}.$$

$$\Rightarrow n_{max} = \frac{2.5}{2602 \times 5000 \times 10^{-8}} = 19.2$$

Absente Spectra / Missing spectra -

$$\boxed{I = \left(\frac{A \sin\alpha}{\alpha}\right)^2 \frac{\sin^2 N\beta}{\sin^2 \beta}.}$$

$$\boxed{(a+b) \sin\theta = \pm n\lambda.}$$

The principal maxima occurs when it satisfies the maxima for single slit as well as for 'N' no. of slits.

When cond^n for principal maxima due to 'N' no. of slits and the cond^n for minima due to single slit are satisfied simultaneously for the same angle θ than the principal maxima due to grating for that particular order will be absent or missed which is called as absent spectra/missing spectra.

$(a+b)\sin\theta = n\lambda$ —— (1) $\rightarrow$ Eq^n for principal maxima due to grating

where $n = 0, 1, 2, 3, 4, \ldots$

$a\sin\theta = m\lambda$ —— (2) $\rightarrow$ Eq^n for minima in single slit diffraction

$n = 1, 2, 3, 4 \ldots$

Dividing both the eq^{n}s -

$$\Rightarrow \boxed{\frac{a+b}{a} = \frac{n}{m}}$$

Ex - If $a = b$, $n/m = 2$, $n = 2m$

$\Rightarrow 2, 4, 6, 8, \ldots$

These orders will be missed.

If $b = 2a$ or $a = b/2$.

$$\Rightarrow \frac{a+2a}{a} = \frac{n}{m} \Rightarrow 3 = \frac{n}{m} \Rightarrow n = 3m.$$

$\Rightarrow n = 3, 6, 9, 12 \ldots$

These orders will be missed.

Dispersive Power of Grating -

* It is defined as the ratio of variation of angle of diffraction with the variation of wavelength.

$\Rightarrow \frac{d\theta}{d\lambda} \rightarrow$ dispersive power.

$\Rightarrow (a+b)\sin\theta = n\lambda$.

$\Rightarrow \frac{d}{d\theta}\left((a+b)\sin\theta\right) = \frac{d}{d\lambda}(n\lambda)$

$\Rightarrow (a+b)\cos\theta \, d\theta = n \, d\lambda$.

$$\Rightarrow \boxed{\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}}$$

depends on -

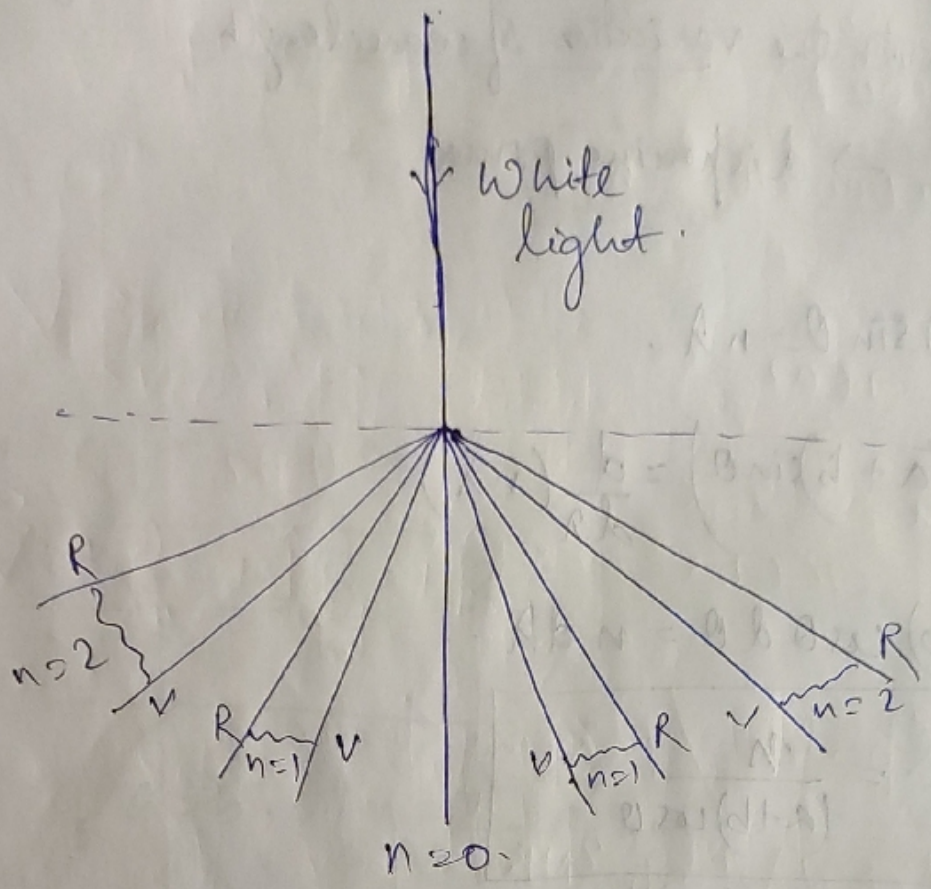
$\frac{d\theta}{d\lambda} \propto n$ (order of ~~principal~~ fringe)

and

$\frac{d\theta}{d\lambda} \propto \frac{1}{(a+b)}$ (grating element)

$\frac{d\theta}{d\lambda} \propto \frac{1}{\cos\theta}$ (angle of diffraction)

Formation Of Multiple Spectra -



* When a white light is passed through a grating then multiple spectra / colour spectra are formed. i.e from violet to red.

Characteristics Of Multiple Spectra -

* Most of the intensities pass through 0th order and rest will be distributed among other orders.
  So the spectra becomes less intense towards higher order.
* Spectra at different orders are situated simetrically on both the sides of 0th order image.

* Spectral colours are in the order from violet to red.
* Spectral lines are almost straight and sharp.
* For higher order, the spectral lines are more and more dispersed.

Module-2.

Electromagnetism

Vector Calculus-

$\vec{\nabla} \rightarrow$ $\boxed{\text{del operator}}$

$\rightarrow$ Vector differential operator.

$$\boxed{\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}}$$

1. Gradient of a scaler field $\Rightarrow$ grad$(\phi) = \vec{\nabla}\phi$ (out-vect)
2. Divergence of a vector field $\Rightarrow$ div$(\phi) = \vec{\nabla}\cdot\phi$ (out-scalar)
3. Curl of a ~~curl~~ vector field $\Rightarrow$ curl$(\phi) = \vec{\nabla}\times\phi$ (out-vector)

Let ϕ be a scaler field.

Then gradient of ϕ = grad $\phi = \vec{\nabla}\phi$.

$$\boxed{\vec{\nabla}\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}}$$

Ex $\Rightarrow$ $\vec{E} = -\vec{\nabla}V$

$\vec{E} \rightarrow$ Electric field

$V \rightarrow$ Electric Potential.

• Physical significance of gradient of scaler field - (1 mg)

* Gradiant of a scaler field is a vector quantity.
* It is the maximum space rate of change of the scaler field.
* The direction is along the direction of maximum increase.

Property -

* If ϕ and ψ are any two scaler field then.

① $\vec{\nabla}(\phi+\psi) = \vec{\nabla}\phi + \vec{\nabla}\psi$

② $\vec{\nabla}(\phi\psi) = \phi\vec{\nabla}\psi + \psi\vec{\nabla}\phi$

③ $\vec{\nabla}\left(\frac{\phi}{\psi}\right) = \frac{\psi\nabla\phi - \phi\nabla\psi}{\psi^2}$

Ex- Let $f = ax^2 + by^2 + cz$

where $a, b, c \rightarrow$ arbitary constant,

then find grad $f = \vec{\nabla} f$.

$\vec{\nabla} f = 2ax\hat{i} + 2by\hat{j} + c\hat{k}$

Ex- Find grad $|\vec{r}| = \vec{\nabla}|\vec{r}|$

$r \rightarrow$ pos^n vector.

$x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow \vec{\nabla}|\vec{r}| = \frac{\hat{i} + \hat{j} + \hat{k}}{\sqrt{x^2+y^2+z^2}}$

Ans - $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

$|\vec{r}| = \sqrt{x^2+y^2+z^2}$.

$\Rightarrow \vec{\nabla}|\vec{r}| = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)(x^2+y^2+z^2)^{1/2}$

$$= \frac{1}{\cancel{2}}\hat{i} \times \frac{\cancel{2}x}{(x^2+y^2+z^2)^{1/2}} + \frac{1}{\cancel{2}}\hat{j} \cdot \frac{\cancel{2}y}{(x^2+y^2+z^2)^{1/2}}$$

$$+ \frac{1}{\cancel{2}}\hat{k} \cdot \frac{\cancel{2}z}{(x^2+y^2+z^2)^{1/2}}$$

$$= \frac{x\hat{i} + y\hat{j} + z\hat{k}}{(x^2+y^2+z^2)^{1/2}}$$

$$= \frac{\vec{r}}{|\vec{r}|} = \boxed{\hat{r}}.$$

Divergence of a vector field

If $\vec{A}$ be a vector field -

$$\boxed{\text{divergence of } \vec{A} = \text{div}\,\vec{A} = \vec{\nabla}\cdot\vec{A}}$$

$A = \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$

$\Rightarrow \text{div}\,\vec{A} = \vec{\nabla}\cdot\vec{A}$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\hat{i}A_x + \hat{j}A_y + \hat{k}A_z\right)$$

$$= \boxed{\frac{\partial}{\partial x}A_x + \frac{\partial}{\partial y}A_y + \frac{\partial}{\partial z}A_z.}$$

$\Downarrow$

Scaler Quantity.

Q) Divergence of pos^n vector.

$$\text{div } \vec{r} = \vec{\nabla} \cdot \vec{r} \qquad \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Ans-

$$\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \cdot \left(x\hat{i} + y\hat{j} + z\hat{k}\right)$$

$$= \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 3.$$

Curl of A Vector field -

If $\vec{A}$ be a vector field

$$\boxed{\text{Curl of } \vec{A} = \text{curl } \vec{A} = \vec{\nabla} \times \vec{A}} \qquad A = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\Rightarrow \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{j}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$\Downarrow$

This is a vector quantity.

• Physical Significance of divergence of a vector field - (Imp)

* Divergence of a vector field is a scaler quantity
* It is the net flux coming out of the volume or going into the volume.
* Divergence of a vector field is positive if the flux is expanding and it is negative if the flux is contracting.
* If $\vec{\nabla}\cdot\vec{A} = 0$ then $\vec{A}$ is called as solenoidal.

• Physical significance of curl of a vector field (Imp)

* Curl of a vector field is a vector quantity.
* The curl of a vector field measures the rotation of the vector at the point of observation.
* If $\vec{\nabla}\times\vec{A} = 0$, then $\vec{A}$ is called irrotational.

Divergence Properties - (A & B are vector field and ϕ is any scaler field)

① $\vec{\nabla}\cdot(\vec{A}+\vec{B}) = \vec{\nabla}\cdot\vec{A} + \vec{\nabla}\cdot\vec{B}$

② $\vec{\nabla}\cdot(\phi\vec{A}) = (\vec{\nabla}\phi)\vec{A} + \phi(\vec{\nabla}\cdot\vec{A})$

Curl Properties -

(i) $\vec{\nabla}\times(\vec{A}+\vec{B}) = \vec{\nabla}\times\vec{A} + \vec{\nabla}\times\vec{B}$

(ii) $\vec{\nabla}\times(\phi\vec{A}) = \vec{\nabla}\phi\times\vec{A} + \phi(\vec{\nabla}\times\vec{A})$

Q) $\vec{F} = \hat{i}\,2xy + \hat{j}\,y^2 + \hat{k}\,3xz$

Find $\vec{\nabla}\cdot\vec{F}$ at $(1,1,0)$

$$\vec{\nabla}\cdot\vec{F} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(2xy\,\hat{i} + y^2\hat{j} + 3xz\,\hat{k}\right)$$

$$= \frac{\partial}{\partial x}(2xy) + \frac{\partial}{\partial y}(y^2) + \frac{\partial}{\partial z}(3xz)$$

$$= 2y + 2y + 3x$$

$$= 2 + 2 + 3 = 7$$

$3xz$

$2xy$

$2yx$

Q) $\vec{A} = 2xy\,\hat{i} + y^2\hat{j} - 2yz\,\hat{k}$

$\vec{\nabla}\cdot\vec{A}$ at $(1,0,2)$

$$\Rightarrow \vec{\nabla}\cdot\vec{A} = 2y + \cancel{2y} - \cancel{2y}$$

$$= 0.$$

Q) $\vec{V} = ax\,\hat{i} + 3y\,\hat{j} - z\,\hat{k}$

$\Rightarrow \vec{\nabla}\cdot\vec{V} = 0$

$\Rightarrow a + 3 - 1 = 0$

$\Rightarrow a + 2 = 0$

$\Rightarrow a = -2$

Q) $\vec{B} = 2x^2\hat{i} + 3yz\hat{j} + a\hat{k}$

$\vec{\nabla} \times \vec{B} = 0$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2 & 3yz & a \end{vmatrix}$$

$$\Rightarrow \hat{i}\left(\frac{\partial a}{\partial y} - \frac{\partial(3yz)}{\partial z}\right) + \hat{j}\left(\frac{\partial a}{\partial x} - \frac{\partial(2x^2)}{\partial z}\right) + \hat{k}\left(\frac{\partial}{\partial x}3yz - \frac{\partial}{\partial y}(2x^2)\right) = 0$$

$$\Rightarrow \hat{i}\left(\frac{\partial a}{\partial y} - 3y\right) + \hat{j}\left(0 - \frac{\partial a}{\partial x}\right) - \hat{k}(0 - 0) = 0$$

$\cancel{\Rightarrow (a - 3y)\hat{i}}$ $\Rightarrow \left(\frac{\partial a}{\partial y} - 3y\right)\hat{i} - \hat{j}\left(\frac{\partial a}{\partial x}\right) + 0\hat{k} = 0$

$= 0\hat{i} + 0\hat{j} + 0\hat{k}$

$$\Rightarrow \int \frac{\partial a}{\partial y} = \int 3y, \quad \int \frac{\partial a}{\partial x} = \int 0$$

$$\Rightarrow a = \frac{3y^2}{2}, \quad a = C$$

$$\Rightarrow \boxed{a = \frac{3y^2}{2} + C}$$

a) $\vec{A} = -\frac{2z^2y}{x^3}\hat{i} + \frac{z^2}{x^2}\hat{j} + \frac{2yz}{x^2}\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -\frac{2z^2y}{x^3} & \frac{z^2}{x^2} & \frac{2yz}{x^2} \end{vmatrix}$$

$$\hat{i}\left(\frac{\partial}{\partial y}\left(\frac{2yz}{x^2}\right) - \frac{\partial}{\partial z}\left(\frac{z^2}{x^2}\right)\right)$$

$$+\hat{j}\left(\frac{\partial}{\partial x}\left(\frac{2yz}{x^2}\right) - \frac{\partial}{\partial z}\left(-\frac{2z^2y}{x^3}\right)\right)$$

$$+\hat{k}\left(\frac{\partial}{\partial x}\left(\frac{z^2}{x^2}\right) - \frac{\partial}{\partial y}\left(-\frac{2z^2y}{x^3}\right)\right)$$

$$= \left(\cancel{\frac{2z}{x^2}} - \cancel{\frac{2z}{x^2}}\right)\hat{i} + \left(\cancel{2yz \times -\frac{2}{x^3}} + \cancel{\frac{2}{x^3} \times 2zy}\right)\hat{j}$$

$$+\hat{k}\left(\cancel{-\frac{2z^2}{x^3}} + \cancel{\frac{2z^2}{x^3}}\right)$$

$= 0.$

x^{-2}

x^3

$= 3$

$2yz \times$

$= 2yz \times -$

(6) $\vec{F} = 2xy\,\hat{i} + xy^2\,\hat{j} - e^{az}x^2\,\hat{k}$

$\vec{\nabla} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy & xy^2 & e^{az}x^2 \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}(e^{az}x^2) - \frac{\partial}{\partial z}(xy^2)\right)$$

$$+ \hat{j}\left(\frac{\partial}{\partial x}(e^{az}x^2) - \frac{\partial}{\partial z}(2xy)\right)$$

$$+ \hat{k}\left(\frac{\partial}{\partial x}(xy^2) - \frac{\partial}{\partial y}(2xy)\right)$$

$$= \cancel{\hat{i}\left(e^{az}x^2 - xy^2\right)} = \hat{i}(0-0)$$

$$+ \cdot \quad + \hat{j}\left(2xe^{az} - 0\right)$$

$$+ \hat{k}\left(y^2 - 2x\right)$$

$$= 0\hat{i} + 2xa^{az}\hat{j} + (y^2 - 2x)\hat{k}$$

Successive Operation.

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$$

$$\Rightarrow \vec{\nabla}\cdot\vec{\nabla} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\cdot\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$$

$$= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \nabla^2$$

$\downarrow$

Laplacian Operator.

$$\nabla^2 f = \vec{\nabla}\cdot\vec{\nabla} f$$

$$= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

① Curl of gradient of a scaler field is zero.

$$\boxed{\vec{\nabla}\times\vec{\nabla}\phi = 0}$$

if $\phi \rightarrow$ scaler field.

② Divergence of curl of a vector field is always equal to zero.

$$\boxed{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A}) = 0}$$

if $\vec{A} \rightarrow$ Vector field.

③ Curl of curl of a vector field is equal to

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}$$

i.e. $\boxed{\vec{\nabla}\times(\vec{\nabla}\times\vec{A}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{A}) - \nabla^2\vec{A}}$

④ $\boxed{\vec{\nabla}\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B})}$

$\vec{A} \rightarrow$ vector field
$\vec{B} \rightarrow$

① Proof

$$\boxed{\vec{\nabla}\times\vec{\nabla}\phi = 0}$$

$$\boxed{\vec{\nabla}\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}}$$

$\Rightarrow \vec{\nabla}\times\vec{\nabla}\phi$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial\phi}{\partial x} & \frac{\partial\phi}{\partial y} & \frac{\partial\phi}{\partial z} \end{vmatrix}$$

$$= \hat{i}\left(\frac{\partial}{\partial y}\left(\frac{\partial\phi}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial\phi}{\partial y}\right)\right)$$

$$+ \hat{j}\left(\frac{\partial}{\partial x}\left(\frac{\partial\phi}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial\phi}{\partial x}\right)\right)$$

$$+\hat{k}\left(\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial y}\right)-\frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial x}\right)\right)$$

$$=\hat{i}\left(\cancel{\frac{\partial^2 \phi}{\partial y \partial z}}-\cancel{\frac{\partial^2 \phi}{\partial y \partial z}}\right)+\hat{j}\left(\cancel{\frac{\partial^2 \phi}{\partial x \partial z}}-\cancel{\frac{\partial^2 \phi}{\partial x \partial z}}\right)$$

$$+\hat{k}\left(\cancel{\frac{\partial^2 \phi}{\partial x \partial y}}-\cancel{\frac{\partial^2 \phi}{\partial x \partial y}}\right)$$

$$=0\hat{i}+0\hat{j}+0\hat{k}=0.$$

Thus $\boxed{\vec{\nabla}\times\vec{\nabla}\phi=0}$

② <u>Proof</u>

$$\vec{\nabla}\times\vec{A}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A_x & A_y & A_z\end{vmatrix}$$

$$=\hat{i}\left(\frac{\partial}{\partial y}A_z-\frac{\partial}{\partial z}A_y\right)+\hat{j}\left(\frac{\partial}{\partial x}(A_z)-\frac{\partial}{\partial z}A_x\right)$$

$$+\hat{k}\left(\frac{\partial}{\partial x}A_y-\frac{\partial}{\partial y}A_x\right)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right).$$

$$+\hat{k}\left(\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial y}\right)-\frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial x}\right)\right)$$

$$=\hat{i}\left(\cancel{\frac{\partial^2 \phi}{\partial y \partial z}}-\cancel{\frac{\partial^2 \phi}{\partial y \partial z}}\right)+\hat{j}\left(\cancel{\frac{\partial^2 \phi}{\partial x \partial z}}-\cancel{\frac{\partial^2 \phi}{\partial x \partial z}}\right)$$

$$+\hat{k}\left(\cancel{\frac{\partial^2 \phi}{\partial x \partial y}}-\cancel{\frac{\partial^2 \phi}{\partial x \partial y}}\right)$$

$$=0\hat{i}+0\hat{j}+0\hat{k}=0.$$

Thus $\boxed{\vec{\nabla}\times\vec{\nabla}\phi=0}$

② <u>Proof</u>

$$\vec{\nabla}\times\vec{A}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ A_x & A_y & A_z\end{vmatrix}$$

$$=\hat{i}\left(\frac{\partial}{\partial y}A_z-\frac{\partial}{\partial z}A_y\right)+\hat{j}\left(\frac{\partial}{\partial x}(A_z)-\frac{\partial}{\partial z}A_x\right)$$

$$+\hat{k}\left(\frac{\partial}{\partial x}A_y-\frac{\partial}{\partial y}A_x\right)$$

$$\vec{\nabla}\cdot(\vec{\nabla}\times\vec{A})=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right).$$

$$= \cancel{\frac{\partial^2 A_z}{\partial x \partial y}} - \cancel{\frac{\partial^2 A_y}{\partial x \partial z}} + \cancel{\frac{\partial^2 A_x}{\partial y \partial z}} - \cancel{\frac{\partial^2 A_z}{\partial y \partial x}}$$

$$+ \cancel{\frac{\partial^2 A_y}{\partial z \partial x}} - \cancel{\frac{\partial^2 A_x}{\partial z \partial y}}$$

$$= 0.$$

Q) If $\vec{A}$ and $\vec{B}$ are irrotational then show that $\vec{A} \times \vec{B}$ is solenoidal.

Q) If a scaler field ϕ satisfies the relation $\nabla^2 \phi = 0$. then show that $\vec{\nabla} \phi$ is both irrotational and solenoidal.

Ans - $\vec{\nabla} \times \vec{A} = 0$ and $\vec{\nabla} \times \vec{B} = 0$

$$\Rightarrow \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right) \cdot (A$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = 0$$

$$\Rightarrow \hat{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{j}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)$$

$$+ \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) = 0.$$

$\Rightarrow \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = 0 \Rightarrow \frac{\partial A_z}{\partial y} = \frac{\partial A_y}{\partial z}$

$\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z} = 0 \Rightarrow \frac{\partial A_z}{\partial x} = \frac{\partial A_x}{\partial z}$

$\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = 0 \Rightarrow \frac{\partial A_y}{\partial x} = \frac{\partial A_x}{\partial y}$

$\vec{\nabla} \times \vec{B} = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix} = 0$$

$\vec{\nabla} \cdot (\vec{A} \times \vec{B}) = 0$

$\Rightarrow \frac{\partial B_z}{\partial y} = \frac{\partial B_y}{\partial z}, \quad \frac{\partial B_z}{\partial x} = \frac{\partial B_x}{\partial z}, \quad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y}$

$\vec{A} \times \vec{B} :$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ & & \\ & & \end{vmatrix}$$

Ans.

Since $\vec{A}$ and $\vec{B}$ are irrotational, $\vec{\nabla} \times \vec{A} = 0$ and $\vec{\nabla} \times \vec{B} = 0$.

$\Rightarrow \vec{\nabla} \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot \underbrace{(\vec{\nabla} \times \vec{A})}_{0} - \vec{A} \cdot \underbrace{(\vec{\nabla} \times \vec{B})}_{0}$

$\Rightarrow \vec{\nabla} \cdot (\vec{A} \times \vec{B}) = 0.$

So, $\vec{A} \times \vec{B}$ is solenoidal.

$\nabla^2 \phi = 0$

$\nabla^2 \phi = \vec{\nabla} \cdot (\vec{\nabla} \phi) = 0$

$\vec{\nabla} \phi$ is solenoidal.

Vector Integration -

① Line Integration $\int \vec{F} \cdot d\vec{r}$ $\oint \vec{F} \cdot d\vec{r}$

② Surface Integration. $\oiint_S \vec{F} \cdot d\vec{s}$

③ Volume Integration $\iiint_{z\,y\,x} A\,dx\,dy\,dz$ or $\int_V$

Gauss Divergence Theorem

* This relates the volume integration of a vector to the surface integration of that vector.

Mathematically -

$$\int_V (\vec{\nabla} \cdot \vec{A})\, dV = \oint_S \vec{A} \cdot d\vec{s}$$

* It states that the volume integration of divergence of a vector field $\vec{A}$ over any arbitary volume V is equal to the surface integration of that vector field $\vec{A}$ over a surface S which enclose the volume V.

Stokes Theorem.

* It relates the surface integration to line integration

Mathmetically -

$$\iint_S (\vec{\nabla} \times \vec{A}) \, d\vec{s} = \oint_C \vec{A} \cdot d\vec{r}.$$

* It states that the surface integration of curl of a vector field $\vec{A}$ over any arbitary surface S. is equal to line integration of that vector $\vec{A}$ along a close curve C which enclose the surface S.

Q) Using Gauss divergence theorem proof that the volume of a sphere is $\frac{4}{3}\pi r^3$. (Vimp)

Proof. let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

- According to Gauss divergence theorem -

$$\int_V (\vec{\nabla} \cdot \vec{A}) \, dv = \oint_S \vec{A} \cdot d\vec{s}$$

LHS -

$$(\vec{\nabla} \cdot \vec{r}) = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 3.$$

$$= \int_V 3 \cdot dv = \boxed{3v.}$$

RHS

$$\oiint_S \vec{r} \cdot \vec{dS}$$

$$= \oiint_S r\,ds\cos 0 = \oiint_S r\,ds$$

$$= r \oiint_S ds$$

$$= rS$$

Total Surface area of the sphere

$$= 4\pi r^2 \times r$$

$$= \boxed{4\pi r^3}$$

LHS = RHS

$$\Rightarrow 3V = 4\pi r^3$$

$$\Rightarrow \boxed{V = \frac{4\pi r^3}{3}}$$

Q) Using GDT ~~prove that~~ evaluate.

$$\oiint_S \vec{F}\cdot d\vec{S}$$

where $\vec{F} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$.

where S is surface of cube bounded by $x=0, x=1$, $y=0, y=1$, $z=0, z=1$

GDT

$$\iiint_V (\vec{\nabla}\cdot\vec{F})\cdot dv = \oiint_S \vec{F}\cdot d\vec{S}$$

LHS $\Rightarrow (\vec{\nabla}\cdot\vec{F})$

$$= \frac{\partial}{\partial x}(4xz) + \frac{\partial}{\partial y}(-y^2) + \frac{\partial}{\partial z}(yz)$$

$$= 4z - 2y + y = \boxed{4z - y}$$

$$= \iiint_{z\,y\,x} (\vec{\nabla}\cdot\vec{F})\,dx\,dy\,dz$$

$$= \int_{z=0}^{1}\int_{y=0}^{1}\int_{x=0}^{1} (4z - y)\,dx\,dy\,dz$$

$$= \int_{z=0}^{1}\int_{y=0}^{1} (4zx - yx)\Big|_0^1\,dy\,dz$$

$$= \int_{z=0}^{1}\int_{y=0}^{1} (4z - y)\,dy\,dz$$

$$= \int_{z=0}^{1} \left[y z y - \frac{y^2}{2} \right]_0^1 dz$$

$$= \int_0^1 \left(yz - \frac{1}{2} \right) dz$$

$$= \int_0^1 yz \, dz - \frac{1}{2} \int_0^1 dz$$

$$= \left[\frac{\cancel{y} z^2}{\cancel{2}} \right]_0^1 - \frac{1}{2} \left[z \right]_0^1$$

$$= 2 - \frac{1}{2} = \frac{3}{2}.$$

Q) $\vec{A} = x^2 \hat{i} + y^2 \hat{j} + z^2 \hat{k}$ taken over a cube bounded by the surface $0 \leq x, y, z \leq 1$, find $\oiint \vec{A} \cdot d\vec{S}$

$$\vec{\nabla} \cdot \vec{A} = 2x + 2y + 2z$$

$$LHS = \int_{z=0}^{1} \int_{y=0}^{1} \int_{x=0}^{1} (2x + 2y + 2z) \, dx \, dy \, dz.$$

$$= 2 \int_{z=0}^{1} \int_{y=0}^{1} \int_{x=0}^{1} (x + y + z) \, dx \, dy \, dz.$$

$$= 2 \int_{z=0}^{1} \int_{y=0}^{1} \left(\frac{x^2}{2} + yx + zx \right)_0^1 dy \, dz$$

$$= 2\int_{z=0}^{1}\int_{y=0}^{1}\left(\frac{1}{2}+y+z\right)dy\,dz$$

$$= 2\int_{z=0}^{1}\left(\frac{1}{2}y+\frac{y^2}{2}+yz\right)_0^1 dz$$

$$= 2\int_{z=0}^{1}\left(\frac{1}{2}+\frac{1}{2}+z\right)dz$$

$$= 2\int_{z=0}^{1}\left(\frac{1}{2}z+\frac{z^2}{2}+\frac{z^2}{2}\right)dz$$

$$= 2\left(\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)$$

$$= \cancel{2}\times\frac{3}{\cancel{2}} = 3.$$

Q) Using stokes theorem prove that the line integration of pos^n vector over a ~~cl~~ curve vanishes when the curve enclosed a surface S.

Ans- <u>Stokes theorem</u>-

$$\oiint_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{S} = \oint_C \vec{A} \cdot d\vec{r}$$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

<u>To prove</u>- $\oint_C \vec{r} \cdot d\vec{l} = 0$.

$\Rightarrow (\vec{\nabla} \times \vec{r}) \cdot d\vec{S}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$\Rightarrow (\vec{\nabla} \times \vec{r}) = 0$. & $\vec{\nabla} \cdot \vec{r} = 3$

$$\Rightarrow \oint_C \vec{r} \cdot d\vec{l} = \oiint_S (\vec{\nabla} \cdot \vec{r}) \, d\vec{S}$$

$$= 0$$

Derivation of Maxwell's Equation -

1. Gauss law of Electrostatics.
2. Gauss law of Magnetism.
3. Faraday's law of Electromagnetic Induction.
4. Modified Ampere's Circuital law.

① Gauss law of Electrostatics -

* The total electric flux ϕ_E over a closed surface is equal to $\frac{1}{\varepsilon_0}$ times total charge enclosed within the surface.

$$\phi_E = \oint_S \vec{E} \cdot \vec{dS} = \frac{q_{IN}}{\varepsilon_0} \quad \text{--- (1)}$$

Differential form of Gauss law of Electrostatics -

Consider a charge q is distributed uniformly in volume v and enclosed by the surface S.

(q). $\rho \rightarrow$ Charge density or charge per unit volume.

So, the amount of charge in a small volume element dv inside the surface is represented by $\rho\, dv$.

So, the total charge enclosed by the surface in the given volume is 'q' $= \int_v \rho\, dv$ --- (2)

Equating (1) and (2) -

$$\Rightarrow \oint_S E \cdot ds = \frac{1}{\varepsilon_0} \int_V \rho \, dv$$

Applying GDT to LHS -

$$\Rightarrow \int_V (\vec{\nabla} \cdot \vec{E}) \, dv = \frac{1}{\varepsilon_0} \int_V \rho \, dv$$

$$\Rightarrow \int_V \left((\vec{\nabla} \cdot \vec{E}) - \frac{\rho}{\varepsilon_0} \right) dv = 0$$

$$\Rightarrow \vec{\nabla} \cdot \vec{E} - \frac{\rho}{\varepsilon_0} = 0$$

$$\Rightarrow \boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}}$$

$\Downarrow$

Differential form of Gauss Law of Electrostatics.

or

1st Maxwell's Eqn

(2) Gauss law of Magnetism -

The total magnetic flux in a closed surface enclosing a volume V is equal to zero.

$$\oint_S \vec{B}\cdot\vec{ds} = 0$$

Apply GDT to LHS

$$\int_V (\vec{\nabla}\cdot\vec{B})\, dV = 0$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{B} = 0}$$

$\Downarrow$

Differential form of Gauss law of Magnetism or 2nd Maxwell's Eqn.

(3) Faraday's laws of EMI -

1st law: An emf is induced in a circuit whenever there is a change in magnetic flux linked through the circuit and induced current starts to flow if the circuit is closed.

2nd law: Magnitude of Induced Emf is equal to rate of change of magnetic flux linked through the circuit.

$$e = -\frac{d\phi}{dt} \quad \text{(1)}$$

$\phi \rightarrow$ Magnetic flux.

$e \rightarrow$ Induced Emf.

$$\phi = \int_S \vec{B}\cdot\vec{ds} \quad \text{--- (2)}$$

$$e = \oint \vec{E}\cdot\vec{dl} \quad \text{--- (3)}$$

* If $\vec{E}$ is the electric field produced in the conductor then an emf is induced and given by eq^n (3)

Substituting values from eq^n (2) and eq^n (3) in (1).

$$\Rightarrow \oint_C \vec{E}\cdot\vec{dl} = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot\vec{dS}$$

By Stokes Theorem

$$\Rightarrow \int_S (\vec{\nabla}\times\vec{E})\cdot\vec{dS} = \int -\frac{\partial\vec{B}}{\partial t}\cdot\vec{dS}$$

$$\Rightarrow \int_S \left\{(\vec{\nabla}\times\vec{E}) + \frac{\partial\vec{B}}{\partial t}\right\}\cdot dS = 0.$$

$$\Rightarrow \vec{\nabla}\times\vec{E} + \frac{\partial\vec{B}}{\partial t} = 0$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

$\Downarrow$

Differential form of Faraday's laws of EMI or Maxwell's 3rd Eq^n.

(4) Ampere's Circuital law -

* Ampere's circuital law relates the distribution of magnetic field / magnetic induction along a loop with the net electric current enclosed by the closed loop.

Statement -

* The line integral of magnetic induction $\vec{B}$ around the closed path is μ times the net current passing through the surface bounded by that closed path.

Mathematically -

$$\oint_C \vec{B}\cdot\vec{dl} = \mu_0 I \quad \text{--- (1)}$$

$$\oint_C \vec{H}\cdot\vec{dl} = I \quad \text{--- (2)}$$

$$I = \int_S \vec{J}\cdot\vec{dS}$$

$\vec{J} \rightarrow$ Current density.

$$\Rightarrow \oint_C \vec{B}\cdot\vec{dl} = \mu\int_S \vec{J}\cdot\vec{dS}$$

Applying Stokes Theorem $\Rightarrow \int_S (\vec{\nabla}\times\vec{B})\,\vec{dS} = \int_S \mu\vec{J}\cdot\vec{dS}$

$$\Rightarrow \int \{(\vec{\nabla}\times\vec{B}) - \mu\vec{J}\}\cdot\vec{dS} = 0$$

$$\Rightarrow \vec{\nabla}\times\vec{B} - \mu\vec{J} = 0$$

$$\Rightarrow \boxed{\vec{\nabla}\times\vec{B} = \mu\vec{J}} \text{ or } \quad \text{--- (3)}$$

$$\boxed{\vec{\nabla}\times\vec{H} = \vec{J}} \quad (\vec{B} = \mu\vec{H})$$

Differential form of Ampere's Circuital law.

Taking divergence of Eq^n (3) -

$$\Rightarrow \underbrace{\vec{\nabla}\cdot(\vec{\nabla}\times\vec{B})}_{0} = \vec{\nabla}\cdot(\mu\vec{J})$$

$$\Rightarrow 0 = \vec{\nabla}\cdot(\mu\vec{J})$$

$$\Rightarrow 0 = \mu(\vec{\nabla}\cdot\vec{J})$$

$$\Rightarrow \boxed{\vec{\nabla}\cdot\vec{J} = 0}$$

Eq^n of Continuity.

$$\boxed{\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0} \quad \text{--- (4)}$$

From (4) & above eq^n -

$$\frac{\partial \rho}{\partial t} + 0 = 0$$

$$\Rightarrow \boxed{\frac{\partial \rho}{\partial t} = 0}$$

$\Rightarrow \rho$ is static as derivative of constant is 0.

* So we conclude that ampere's circuital law is only valid for a static field and does not satisfy for a time varying field.

* So, ampere's circuital law needed modification and that was done by Maxwell by adding displacement current (I_d).

$$I_d = \varepsilon A \frac{\partial \vec{E}}{\partial t}$$

J_d = Displacement current density

$$= \varepsilon \frac{\partial E}{\partial t}.$$

Modified Ampere's Circuital law -

$$\boxed{\begin{aligned} &\oint \vec{B} \cdot \vec{dl} = \mu (I + I_d) \\ \Rightarrow\ &\oint \vec{H} \cdot \vec{dl} = I + I_d \end{aligned}}$$

Proceeding in the same way -

Eq^n (3) becomes -

$$\vec{\nabla} \times \vec{B} = \mu (\vec{J} + J_d)$$

$$\Rightarrow \boxed{\vec{\nabla} \times \vec{B} = \mu \vec{J} + \mu \varepsilon \frac{\partial \vec{E}}{\partial t}}$$

$\Downarrow$

Differential form of Modified Ampere's Circuital law or

4th Maxwell Eq^n.

Maxwell Equations -

① $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0}$ $\Rightarrow$ $\boxed{\vec{\nabla}\cdot\vec{D} = \rho}$

② $\vec{\nabla}\cdot\vec{B} = 0$ $\Rightarrow$ $\boxed{\vec{\nabla}\cdot\vec{H} = 0}$

③ $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ $\Rightarrow$ $\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$

④ $\vec{\nabla}\times\vec{B} = \mu\vec{J} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}$ $\Rightarrow$ $\boxed{\vec{\nabla}\times\vec{H} = J + \frac{\partial\vec{D}}{\partial t}}$

$\vec{D} = \varepsilon\vec{E}$

$\Rightarrow \frac{\partial\vec{D}}{\partial t} = \varepsilon\frac{\partial\vec{E}}{\partial t}$

Electric Displacement -

$\boxed{\vec{D} = \varepsilon\vec{E}}$, $\boxed{\vec{B} = \mu\vec{H}}$

Maxwell's Eq^n's in different medium -

① Free space - (source free region)
(Vaccum)

No charge source $\Rightarrow \rho = 0$
No current source $\Rightarrow J = 0$.
ε_0, μ_0

① $\vec{\nabla}\cdot\vec{E} = 0$.

② $\vec{\nabla}\cdot\vec{B} = 0$.

③ $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

④ $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$.

Charge free non-conducting / Dielectric medium.

$\rho = 0$, $\vec{J} = \sigma \vec{E}$ (σ = Conductivity of med).

$\sigma = 0$

$\vec{J} = 0$, ε, μ.

① $\vec{\nabla} \cdot \vec{E} = 0$

② $\vec{\nabla} \cdot \vec{B} = 0$

③ $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

④ $\vec{\nabla} \times \vec{B} = \mu \varepsilon \frac{\partial \vec{E}}{\partial t}$

Charge free Conducting Medium -

$\rho = 0$ (Charge free)

$J = \sigma \vec{E}$

μ, ε

① $\vec{\nabla} \cdot \vec{B} = 0$

② $\vec{\nabla} \cdot \vec{B} = 0$

③ $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

④ $\vec{\nabla} \times \vec{B} = \mu \sigma E + \mu \varepsilon \frac{\partial \vec{E}}{\partial t}$

Physical Significance of Maxwell's Eq^n:

① $\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}}$ or $\boxed{\vec{\nabla} \cdot \vec{D} = \rho}$

* It is a steady state eq^n independent of time.
* Electric flux depends on charge density.
* As $\vec{\nabla} \cdot \vec{D} = \rho$ so charge density is a scaler quantity.

② $\vec{\nabla} \cdot \vec{B} = 0$

* It is a steady state eq^n independent of time.

IMP * It signifies the non existence of magnetic monopoles. (single N or single S pole can't exist)

* Equal no. of magnetic lines of force enter and leave a given volume.

③ $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

* It is a time dependent eq^n.
* It relates the space variation of $\vec{E}$ with time variation of $\vec{B}$.
* It shows that the time variation of $\vec{B}$ generates electric field $\vec{E}$.

④ $\vec{\nabla} \times \vec{B} = \mu \varepsilon \frac{\partial \vec{E}}{\partial t} + \mu \vec{J}$

* It is a time dependent equation.
* It relates the space variation of magnetic induction $\vec{B}$ with time variation of $\vec{E}$.
* It shows a changing electric field produces a changing magnetic field.

Electromagnetic Wave Equation -

Maxwell Electromagnetic wave eq^n -

① Free space / vacuum $(\rho = 0, J = 0, \varepsilon_0, \mu_0)$

$$\vec{\nabla} \cdot \vec{E} = 0 \quad \text{--- (1)}$$

$$\vec{\nabla} \cdot \vec{B} = 0 \quad \text{--- (2)}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \text{--- (3)}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{--- (4)}$$

In terms of $\vec{E}$ -

Taking the curl of eq^n (3) -

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} \times \left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla} \cdot \vec{E}) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

Substituting values from (1) & (4) -

$$\Rightarrow \vec{\nabla}(0) - \nabla^2 \vec{E} = -\frac{\partial}{\partial t}\left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\Rightarrow \nabla^2 \vec{E} = \frac{\partial^2 \vec{E}}{\partial t^2} \mu_0 \varepsilon_0$$

Thus $\boxed{\nabla^2 \vec{E} - \frac{\partial^2 \vec{E}}{\partial t^2} \mu_0 \varepsilon_0 = 0}$

$\Downarrow$

Wave Eq^n in terms of $\vec{E}$ in free space.

In terms of $\vec{B}$.

Taking curl of (4) -

$$\Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \times \left(\mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$$

$$\Rightarrow \vec{\nabla} \cdot (\vec{\nabla} \times \vec{B}) - \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B})$$

Substituting values from (2) & (3) -

$$\Rightarrow -\nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial}{\partial t} \left(-\frac{\partial \vec{B}}{\partial t}\right)$$

$$\Rightarrow \nabla^2 \vec{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 \vec{B} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0.}$$

$\Downarrow$

Wave eq^n in terms of $\vec{B}$ in free space.

Derive the wave eq^n in terms of $\vec{E}$ and $\vec{B}$ in a charge free non conducting medium

$\rho = 0$, $J = 0$, μ, ε

① $\vec{\nabla}\cdot\vec{E} = 0$ —— ①

② $\vec{\nabla}\cdot\vec{B} = 0$ —— ②

③ $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ —— ③

④ $\vec{\nabla}\times\vec{B} = \mu\varepsilon\frac{\partial \vec{E}}{\partial t}$ —— ④

In terms of $\vec{E}$

Taking curl of ③

$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \nabla\times\left(-\frac{\partial \vec{B}}{\partial t}\right)$

$\Rightarrow \vec{\nabla}\cdot(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}(\nabla\times\vec{B})$

$\Rightarrow \cancel{-\nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial t}\right)}$

$\Rightarrow -\nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu\varepsilon\frac{\partial \vec{E}}{\partial t}\right)$

$\Rightarrow \nabla^2\vec{E} = \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$

$\Rightarrow \boxed{\nabla^2\vec{E} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0}$

$\Downarrow$

Wave eq^n in terms of $\vec{E}$ in charge free non conducting medium.

In terms of $\vec{B}$

Taking curl of (4) -

$$\Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{B}) = \vec{\nabla} \times \left(\mu \varepsilon \frac{\partial \vec{E}}{\partial t}\right)$$

$$\Rightarrow \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{B}) - \nabla^2 \vec{B} = \mu \varepsilon \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{E})$$

$$\Rightarrow -\nabla^2 \vec{B} = \mu \varepsilon \frac{\partial}{\partial t} \left(-\frac{\partial B}{\partial t}\right)$$

$$\Rightarrow \nabla^2 \vec{B} = \mu \varepsilon \frac{\partial^2 \vec{B}}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2 \vec{B} - \mu \varepsilon \frac{\partial^2 \vec{B}}{\partial t^2} = 0}$$

$\Downarrow$

Wave eq^n in terms of $\vec{B}$ in charge free, non-conducting medium

Derive the wave eq^n in terms of $\vec{E}$ and $\vec{B}$ in a charge free conducting medium.

$\rho = 0$, $J = \sigma\vec{E}$, μ, ε.

(1) $\vec{\nabla}\cdot\vec{E} = 0$ — (1)

(2) $\vec{\nabla}\cdot\vec{B} = 0$ — (2)

(3) $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$ — (3)

(4) $\vec{\nabla}\times\vec{B} = \mu\sigma\vec{E} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}$ — (4)

<u>In terms of $\vec{E}$</u>

<u>Taking curl of (3)</u> -

$$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{E}) = \vec{\nabla}\times\left(-\frac{\partial\vec{B}}{\partial t}\right)$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E} = -\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\Rightarrow -\nabla^2\vec{E} = -\frac{\partial}{\partial t}\left(\mu\sigma\vec{E} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}\right)$$

$$\Rightarrow \nabla^2\vec{E} = \mu\sigma\frac{\partial\vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2}$$

$$\Rightarrow \boxed{\nabla^2\vec{E} - \mu\sigma\frac{\partial\vec{E}}{\partial t} - \mu\varepsilon\frac{\partial^2\vec{E}}{\partial t^2} = 0}$$

In terms of $\vec{B}$

Taking curl of (4).

$$\Rightarrow \vec{\nabla}\times(\vec{\nabla}\times\vec{B}) = \vec{\nabla}\times\left(\mu\sigma\vec{E} + \mu\varepsilon\frac{\partial\vec{E}}{\partial t}\right)$$

$$\Rightarrow \vec{\nabla}(\vec{\nabla}\cdot\vec{B}) - \nabla^2\vec{B} = \vec{\nabla}\times(\mu\sigma\vec{E}) + \vec{\nabla}\times\left(\mu\varepsilon\frac{\partial\vec{E}}{\partial t}\right)$$

$$\Rightarrow -\nabla^2\vec{B} = \mu\sigma(\vec{\nabla}\times\vec{E}) + \mu\varepsilon\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{B})$$

$$\Rightarrow -\nabla^2\vec{B} = -\frac{\partial\vec{B}}{\partial t}\mu\sigma - \frac{\partial^2\vec{B}}{\partial t^2}\mu\varepsilon$$

$$\Rightarrow \boxed{\nabla^2\vec{B} - \frac{\partial\vec{B}}{\partial t}\mu\sigma - \frac{\partial^2\vec{B}}{\partial t^2}\mu\varepsilon = 0}$$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}}$$

$$\Rightarrow \boxed{\nabla^2\psi - \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} = 0} \quad \text{— (2)}$$

$\Rightarrow$ Mechanical wave eq^n

$$\boxed{\nabla^2\vec{B} - \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2} = 0} \quad \text{— (1)}$$

Comparing (1) and (2) -

$$\Rightarrow v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$\mu_0 = 4\pi \times 10^{-7}$

$\varepsilon_0 = 8.85 \times 10^{-12}$.

$$\Rightarrow \boxed{v = 3 \times 10^8 \, m/s.}$$

Wave eq^n in a charge free non-conducting medium

$$\Rightarrow \nabla^2 \vec{E} - \mu \varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \text{--- (3)}$$

$$\Rightarrow \boxed{\nabla^2 \psi - \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2} = 0.} \quad \text{--- (2)}$$

Comparing (3) & (2) -

$$\Rightarrow \frac{1}{v^2} = \mu \varepsilon$$

$$\Rightarrow v = \frac{1}{\sqrt{\mu \varepsilon}}.$$

$\mu < \mu_0$ & $\varepsilon < \varepsilon_0$

So,

$$\boxed{v = \frac{1}{\sqrt{\mu \varepsilon}} < v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} < V_{air/light}.}$$

Refractive Index of the Medium

$$\eta = \frac{c}{v} = \frac{\frac{1}{\sqrt{\mu_0 \varepsilon_0}}}{\frac{1}{\sqrt{\mu \varepsilon}}}$$

$$= \frac{\cancel{1}}{\sqrt{\mu_0 \varepsilon_0}} \times \frac{\sqrt{\mu \varepsilon}}{\cancel{1}}$$

$$= \frac{\sqrt{\mu \varepsilon}}{\sqrt{\mu_0 \varepsilon_0}}$$

$$\Rightarrow \frac{\cancel{\mu_0} \mu_r \cancel{\varepsilon_0} \varepsilon_r}{\cancel{\mu_0} \cancel{\varepsilon_0}}$$

$$\boxed{\eta = \sqrt{\mu_r \varepsilon_r}}$$

Characteristics of Electromagnetic Wave -

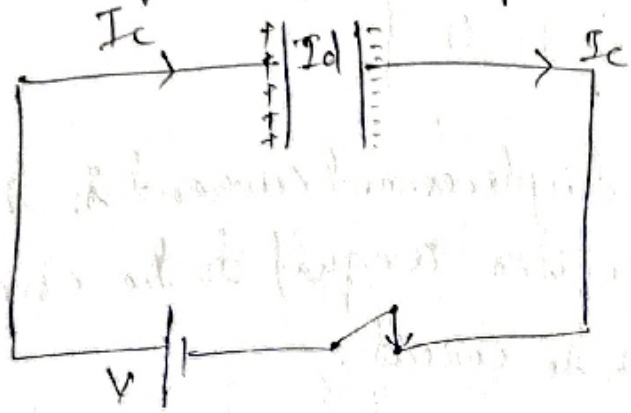
(i) Electromagnetic waves are transverse in nature.

(ii) They can propagate in free space or vaccum.

(iii) They propagate in free space with a velocity equal to velocity of light.

Displacement Current

Consider a parallel plate capacitor being charged by a cell. During the process of charging, the electric field between the plates of the capacitor changes with time.



It is observed that a magnetic field exists between the plates as long as the electric field changes. Even if there is no current between the plates. So, Maxwell introduced the concept of displacement current which is associated with the time varying electric field between the plates of capacitor.

If 'q' be the charge on the plate of the capacitor and 'A' is the area of each plate of capacitor then

$$\vec{E} = \frac{q}{\varepsilon A} \quad \left(C = \frac{q}{V}\right) \Rightarrow V = \frac{q}{C} = \frac{qd}{\varepsilon A}$$

$$\Rightarrow E = \frac{V}{d} = \frac{1}{d} \times \frac{qd}{\varepsilon A} = \boxed{\frac{q}{\varepsilon A}}$$

$C = \frac{\varepsilon A}{d}$

$E = \frac{q}{\varepsilon A}$

$V = \frac{qd}{\varepsilon A}$

$$\Rightarrow \frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon A} \frac{\partial q}{\partial t}$$

$$\Rightarrow \boxed{\frac{\partial \vec{E}}{\partial t} = \frac{1}{\varepsilon A} I_d}$$

$$\Rightarrow \boxed{I_d = \varepsilon A \frac{\partial \vec{E}}{\partial t}} \quad (1)$$

$\Downarrow$

Thus displacement current exists as long as change in electric field occurs.

$$\boxed{J_d = \varepsilon \frac{\partial \vec{E}}{\partial t}}$$

When capacitor is fully charged, electric field attains a constant value, So, $\frac{\partial \vec{E}}{\partial t} = 0$

So, $\boxed{I_d = 0}$

Q) Prove that the displacement current in the gap between the plates of a capacitor is equal to the conduction current flowing in the circuit.

Previous ans ↑

$C \rightarrow$ capacitance of capacitor

$V \rightarrow$ Voltage

$d \rightarrow$ distance betn the capacitor

$$E = \frac{V}{d}$$

$$\Rightarrow \frac{\partial E}{\partial t} = \frac{1}{d}\frac{\partial V}{\partial t}$$

Putting in previous eqn (2)

$$\Rightarrow I_d = \frac{\varepsilon A}{d}\frac{\partial V}{\partial t}$$

$$= C\frac{\partial V}{\partial t} = I_c$$

So, $\boxed{I_d = I_c}$

Conduction Current	Displacement Current
(i) Conduction current exists in a conducting medium due to the actual flow of charge carriers.	(i) Displacement current exists in vacuum or in any medium without the actual flow of charge carriers even if in the absence of charge carrier (due to magnetic field).
(ii) It depends on resistance and potential difference of the conductor.	(ii) It depends on electric permittivity of the medium and the rate of change of electric field.
(iii) $I_c = \frac{\partial q}{\partial t}$	(iii) $I_d = \varepsilon A \frac{\partial \vec{E}}{\partial t}$
(iv) It obeys Ohm's law.	(iv) It does not obey Ohm's law.
(v) It is a real current	(v) It is a fictious current.

Integral form of Maxwell Eq^n

① $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \oint \vec{E}\cdot d\vec{S} = \frac{1}{\varepsilon}\int \rho\, dv$ (Integral)

② $\vec{\nabla}\cdot\vec{B} = 0 \Rightarrow \oint \vec{B}\cdot d\vec{S} = 0$. (Integral form)

③ $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \oint \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\int \vec{B}\cdot d\vec{S}$.

④ $\vec{\nabla}\times\vec{B} = \mu\varepsilon\frac{\partial \vec{E}}{\partial t} + \mu\vec{J} \Rightarrow \oint \vec{H}\cdot d\vec{l} = \int\left(J + \frac{\partial \vec{D}}{\partial t}\right)\cdot d\vec{S}$

$$(a+b)\sin\theta = n\lambda \quad (1)$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} = \frac{n}{(a+b)\times\frac{\sqrt{(a+b)^2-n^2\lambda^2}}{(a+b)}}$$

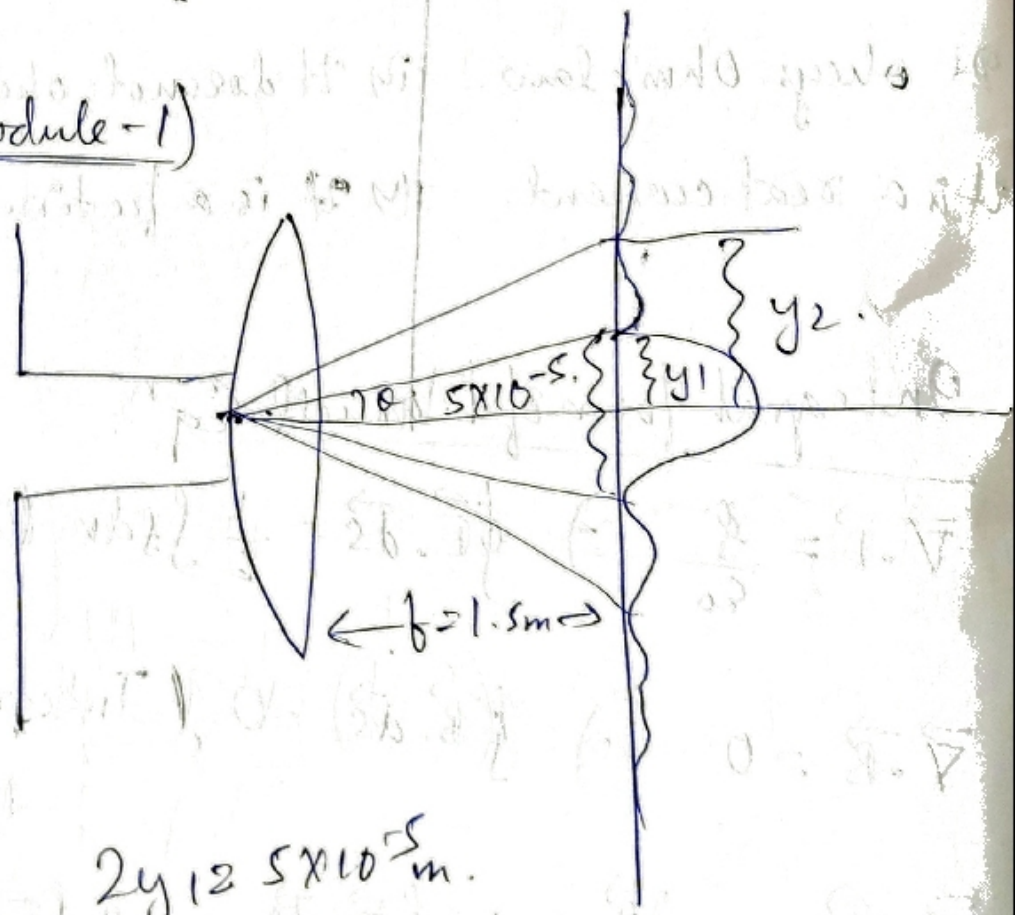
$$\sin\theta = \frac{n\lambda}{(a+b)} \qquad = \frac{n}{\sqrt{(a+b)^2-n^2\lambda^2}}$$

$$\cos\theta = \sqrt{1-\sin^2\theta}$$

$$= \sqrt{1-\frac{n^2\lambda^2}{(a+b)^2}} \qquad = \frac{\sqrt{(a+b)^2-n^2\lambda^2}}{(a+b)}$$

Med (Module-1)

(20)



$$2y_1 = 5\times10^{-5}\,m.$$

$$\tan\theta = \frac{y_1}{f} = \sin\theta$$

$$e\sin\theta = \frac{a\lambda}{590nm}$$

(22) $a+b = \frac{2.5}{15000}$

$(a+b)\sin\theta = n\lambda$.

$\Rightarrow \sin\theta = \frac{n\lambda}{(a+b)}$ — (1)

$\Rightarrow \boxed{\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}}$

$\Downarrow$

In given formula θ is calculated in above eqn (1).

On free space -

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{K} \times \vec{E} = \omega \vec{B}$$

$$= \omega \mu_0 \vec{H}$$

$$\Rightarrow KE \sin\theta = \omega \mu_0 \vec{H}$$

$$\theta = 90^\circ$$

$$\Rightarrow KE = \omega \mu_0 \vec{H}$$

$$\Rightarrow \cancel{\frac{E}{H}} = \Rightarrow H = \frac{K}{\omega} \cdot \frac{1}{\mu_0} E.$$

$$= \frac{1}{c} \frac{1}{\mu_0} E.$$

$$\Rightarrow \boxed{Z = \frac{E}{H} = \mu_0 c.}$$

$Z \rightarrow$ Wave Impedance.

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \Rightarrow \boxed{\frac{E}{H} = \frac{\mu_0}{\sqrt{\mu_0 \varepsilon_0}}} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \sqrt{\frac{4\pi \times 10^{-7}}{8.8 \times 10^{-12}}}$$

$$= 376.6\,\Omega.$$

* Wave Impedance Z in free space is $376.6\,\Omega$ which is positive and real. So, E and H are in same phase and have same relative magnitude at all time and at all point.

Poynting Theorem

Electromagnetic energy

The electric energy density = Electric Energy per unit volume

$$\boxed{u_E} = \frac{1}{2}\vec{E}\cdot\vec{D} = \frac{1}{2}\vec{E}\cdot(\varepsilon\vec{E})$$

$$= \boxed{\frac{1}{2}\varepsilon\vec{E}^2} \quad \text{(Rem.)}$$

Similarly Magnetic Energy density = Magnetic Energy per unit volume.

$$\boxed{u_M} = \frac{1}{2}\vec{H}\cdot\vec{B} = \boxed{\frac{1}{2}\mu H^2} \quad \text{(Rem.)}$$

The Electromagnetic Energy density-

~~Energy~~ Electromagnetic energy per unit volume.

$$u_{EM} = u_E + u_M$$

$$= \frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2$$

The total electromagnetic energy-

$$U_{EM} = \int_V u_{EM}\, dv = \int_V \left(\frac{1}{2}\varepsilon E^2 + \frac{1}{2}\mu H^2\right) dv. \quad \text{—— (1)}$$

As the electromagnetic waves propagate through space from their source to the distant recieving point, there is a transfer of energy from the source

to the reciever.

Now we establish a relation between the rate of energy transfer with the amplitudes of electric and magnetic field strengths ($\vec{E}$ and $\vec{H}$) so called poinding theorem.

Pointing Theorem-

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{E}\cdot(\vec{\nabla}\times\vec{H}).$$

$$= \vec{H}\left(-\frac{\partial B}{\partial t}\right) - \vec{E}\left(\vec{J} + \frac{\partial \vec{D}}{\partial t}\right)$$

$$= \vec{H}\left(-\frac{\partial}{\partial t}(\mu\vec{H})\right) - \vec{E}\cdot\vec{J} - \vec{E}\cdot\frac{\partial}{\partial t}(\varepsilon\vec{E})$$

$$= -\frac{\mu H}{2}\left(\frac{\partial}{\partial t}(2\vec{H})\right) - \frac{1}{2}\varepsilon\vec{E}\cdot\left(\frac{\partial}{\partial t}(2\vec{E})\right) - \vec{J}\cdot\vec{E}$$

$$= -\frac{1}{2}\mu\frac{\partial}{\partial t}(H^2) - \frac{1}{2}\varepsilon\frac{\partial}{\partial t}(E^2) - \vec{J}\cdot\vec{E}.$$

$$= -\frac{1}{2}\left[\frac{\partial}{\partial t}(\mu H^2) + \frac{\partial}{\partial t}(\varepsilon E^2) + \vec{J}\cdot\vec{E}\right]$$

$$= -\frac{\partial}{\partial t}\underbrace{\left[\frac{1}{2}\mu H^2 + \frac{1}{2}\varepsilon E^2\right]}_{u_{EM}} - \vec{J}\cdot\vec{E}.$$

$$= -\frac{\partial}{\partial t}(u_{EM}) - \vec{J}\cdot\vec{E}$$

Thus - $\boxed{\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\frac{\partial}{\partial t}(u_{EM}) - \vec{J}\cdot\vec{E}}$

Taking volume integration -

$$\Rightarrow -\vec{J}\cdot\vec{E} = \vec{\nabla}\cdot(\vec{E}\times\vec{H}) + \frac{\partial}{\partial t}(u_{EM}).$$

$$\Rightarrow -\int_V (\vec{J}\cdot\vec{E})\,dv = \int_V \frac{\partial}{\partial t}(u_{EM})\,dv + \int_V \vec{\nabla}\cdot(\vec{E}\times\vec{H})\,dv$$

Applying G.D.T to 3rd term

$$\Rightarrow \boxed{-\int_V (\vec{J}\cdot\vec{E})\,dv = \int_V \frac{\partial(u_{EM})}{\partial t}\,dv + \oint_S (\vec{E}\times\vec{H})\,d\vec{S}}$$

Interpretation of Each Term.

① $\boxed{-\int_V (\vec{J}\cdot\vec{E})\,dv}$

⇒ This represents the rate of energy transferred into the electromagnetic field through the motion of free charge in volume V.

② $\int_V \frac{\partial}{\partial t}(u_{EM})\,dv = \frac{\partial}{\partial t}\underbrace{\int_V (u_{EM})\,dv}_{\text{Total EME}}$

⇒ This represents the rate of electromagnetic energy stored in volume V.

③ $\oint_S (\vec{E} \times \vec{H}) \cdot d\vec{S}$

~~Electromagnetic Energy~~.

⇒ This represents the amount of Electromagnetic energy crossing the close surface per second which encloses the volume V.

$(\vec{E} \times \vec{H}) \Rightarrow$ Pointing vector $(\vec{S}$ or $\vec{P})$.

$$\Rightarrow -\int_V (\vec{J} \cdot \vec{E})\, dV = \int \frac{\partial}{\partial t}(u_{EM})\, dV + \oint_S \vec{S} \cdot d\vec{S} + \int_V (\vec{\nabla} \cdot \vec{S})\, dV.$$

$$\Rightarrow \boxed{-\vec{J} \cdot \vec{E} = \frac{\partial (u_{EM})}{dt} + (\vec{\nabla} \cdot \vec{S})}$$

$\Downarrow$

Pointing theorem /

Equation of conservation of energy in electromagnetism.

On the absence of source =

$J = 0$.

$$\Rightarrow \boxed{\frac{\partial (u_{EM})}{dt} + (\vec{\nabla} \cdot \vec{S}) = 0}$$

$\Downarrow$

(Eq^n of continuity in Electromagnetism

Poynting vector ($\vec{S}$) -

$$\vec{S} = \vec{E} \times \vec{H}$$

→ Power flux.

→ The power radiated per unit area placed perpendicular to the vector per unit time.

→ It is also the intensity of the wave.

$$|\vec{S}| = |\vec{E}||\vec{H}| \sin\theta \qquad \theta = 90^\circ$$

$$= |\vec{E}||\vec{H}|$$

But $\frac{|\vec{E}|}{|\vec{H}|} = \mu_0 c = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$$\Rightarrow \boxed{|\vec{S}| = \vec{E}\frac{|\vec{E}|}{\mu_0 c} = \frac{1}{\mu_0 c} E^2 = \sqrt{\frac{\varepsilon_0}{\mu_0}} E^2}$$

Also

$$\boxed{|\vec{S}| = \mu_0 c H^2 = \sqrt{\frac{\mu_0}{\varepsilon_0}} H^2}$$

Time Average value of $\vec{S}$ -

in terms of $\vec{E}$

$$\langle S \rangle = \sqrt{\frac{\varepsilon_0}{\mu_0}} \langle E_0^2 \sin^2 \omega t \rangle.$$

$E = E_0 \sin\omega t$

$$= \sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2 \langle \sin^2 \omega t \rangle \Rightarrow (0 \text{ to } 1) = \left(\frac{1}{2}\right)$$

$$= \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2$$

In terms of $\vec{H}$

$$H = H_0 \sin \omega t$$

$$\langle S \rangle = \sqrt{\frac{\mu_0}{\varepsilon_0}} \langle H_0^2 \sin^2 \omega t \rangle$$

$$\Rightarrow \boxed{\langle S \rangle = \frac{1}{2} \sqrt{\frac{\mu_0}{\varepsilon_0}} H_0^2}$$

$$B_{rms} = \frac{E_0}{\sqrt{2}} \quad \Big| \quad H_{rms} = \frac{H_0}{\sqrt{2}}$$

$$\Rightarrow \boxed{\begin{aligned} \langle S \rangle &= \sqrt{\frac{\varepsilon_0}{\mu_0}} E_{rms}^2 \\ \langle S \rangle &= \sqrt{\frac{\mu_0}{\varepsilon_0}} H_{rms}^2 \end{aligned}}$$

So, $\boxed{\frac{|\vec{E}|}{|\vec{H}|} = \mu_0 c = \frac{|E_0|}{|H_0|}}$

$\vec{S} = \vec{E} \times \vec{H}$

Unit = Watt/m^2

(Ques^n short)

Direction = Along $\vec{k}$ or along the direction of wave propagation.

(1) Electric field $\vec{E}$ of a plane electromagnetic wave propagating in a non magnetic medium ($\mu_r = 1$) is given by $\vec{E} = \hat{y}\, 50 \cos(10^8 t + 2z)$ V/m. Find -

(i) Direction of wave propagation: z axis

(ii) Frequency: $\omega = 10^8$

(iii) Wavelength.

(iv) ε_r

(v) The magnetic field vector ($\vec{H}$ or $\vec{B}$).

(2) An electromagnetic wave propagating in free space with magnetic vector $\vec{H} = \hat{i}\, e^{i(2\pi t + 3z)}$ Amp/m. Find the expression for $\vec{E}$.

(3) If $\vec{E} = \cos(y-t)\hat{k}$, $\vec{B} = \cos(y-t)\hat{i}$ then show whether they constitute a possible electromagnetic field or not.

(4) The maximum value of electric field in an electromagnetic wave in vacuum is 800 V/m. Find the maximum value of magnetic Intensity and the average value of poynting vector.

(5) A point source emits light with power 250W. Find the average value of poynting vector and Rms value of Electric and magnetic field at a distance 2m from the source. $\langle S \rangle = \frac{\text{Power}}{\text{Area}}$

(6) Calculate the value of pointing vector on the surface of the ~~[illegible]~~ sun if the power radiated by the sun is 4×10^{26} W while it's radius is 7×10^8 m.

(2) $\vec{E} = \hat{j}\, 376.6\, e^{i(2\pi t + 3z)}$ V/m

(3) It cannot constitute an electromagnetic wave because direction wise it is correct. but magnitude wise it is not correct because $\frac{E_0}{H_0} = 1$ but $\frac{B_0}{H_0} = \epsilon_0$

Q.10

Ionised medium

$\rho = 0, \mu_0, \epsilon_0, J = \sigma \vec{E}$

Find out wave equation in terms of $\vec{E}$ and $\vec{B}$ in an ionised medium.

Ans - $\vec{E} = \nabla^2 \vec{E} - \mu_0 \sigma \frac{d\vec{E}}{dt} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2}$.

$\vec{B} = \nabla^2 \vec{B} - \mu_0 \sigma \frac{\partial \vec{B}}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2}$.

Wave Equation in ionised medium and (skin depth)

$\Downarrow$

$\rho = 0, \epsilon_0, \mu_0, J = \sigma \vec{E}$.

$\sigma \rightarrow$ conductivity of the medium

$$\nabla^2 \vec{E} - \mu_0 \sigma \frac{\partial \vec{E}}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \quad \text{--- (1)}$$

$$\nabla^2 \vec{B} - \mu_0 \sigma \frac{\partial \vec{B}}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0 \quad \text{--- (2)}$$

$$\left(\nabla^2 \psi - \mu_0 \sigma \frac{\partial \psi}{\partial t} - \mu_0 \epsilon_0 \frac{\partial^2 \psi}{\partial t^2} = 0 \right) \quad \text{--- (3)}$$

General form.

The sol^n of the eq^n we can write -

$$\vec{E} = \vec{E_0} \cdot e^{i(\vec{K}\cdot\vec{r} - \omega t)} \quad \text{--- (4)}$$

$$\vec{B} = \vec{B_0} \cdot e^{i(\vec{K}\cdot\vec{r} - \omega t)} \quad \text{--- (5)}$$

$$\psi = \psi_0 e^{i(\vec{K}\cdot\vec{r} - \omega t)} \quad \text{--- (6)}$$

(General form).

$$- \nabla^2 \psi = -K^2 \psi$$ (Double derivative of ψ wrt space coordinate i.e $\vec{r}$)

$$- \frac{\partial \psi}{\partial t} = -i\omega\psi$$

$$- \frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi$$

Substituting above 3 values in (3) -

$$\Rightarrow -K^2\psi + i\omega\psi\mu_0\sigma + \omega^2\psi\mu_0\varepsilon_0 = 0$$

$$\Rightarrow -K^2 + i\omega\mu_0\sigma + \omega^2\mu_0\varepsilon_0 = 0.$$

$$\Rightarrow K^2 = \omega^2\mu_0\varepsilon_0 + i\omega\mu_0\sigma \quad \text{--- (7)}$$

So, K is a complex quantity

Let $K = \alpha + i\beta$

Substituting K in (7) -

$$\Rightarrow (\alpha + i\beta)^2 = \omega^2\mu_0\varepsilon_0 + i\omega\mu_0\sigma$$

$$\Rightarrow \alpha^2 - \beta^2 + 2i\alpha\beta = \omega^2\mu_0\varepsilon_0 + i\omega\mu_0\sigma$$

$$\Rightarrow \alpha^2 - \beta^2 = \omega^2\varepsilon_0\mu_0 \quad \text{--- (8)}$$

and $2\alpha\beta = \omega\mu_0\sigma$

$$\Rightarrow \boxed{\beta = \frac{\omega\mu_0\sigma}{2\alpha}} \quad \text{--- (9)}$$

Putting $K = \alpha + i\beta$ in (4) -

$$\vec{E} = E_0 e^{i\{(\alpha+i\beta)\cdot\vec{r}\}} \cdot e^{-i\omega t}$$

$$= E_0 e^{i(\alpha\cdot\vec{r})} \cdot e^{-\beta\cdot\vec{r}} e^{-i\omega t}$$

$$\Rightarrow \boxed{E = E_0 e^{i(\alpha\cdot\vec{r}-\omega t)} \cdot e^{-\beta\cdot\vec{r}}}$$

$$\boxed{E_n = E_{0n} e^{i(\alpha z-\omega t)} \cdot e^{-\beta z}}$$

* The electric field changes by a factor $e^{-\beta z}$ when wave propagating through the ionised medium. So, this factor is called as Attenuation factor

* This indicates that the electric field decays with the increase of distance z with the medium.

* The Attenuation factor $\boxed{e^{-\beta z} = e^{-\sqrt{\frac{\omega\mu\sigma}{2}}\, z}}$

* The Electric field reduces to $\frac{1}{e}$ times of it's original value after transversing the distance

$$\boxed{z = \left(\frac{2}{\omega\mu_0\sigma}\right)^{1/2}} \Rightarrow \text{Skin depth of ionised/conducting med}$$

* This distance is called as skindepth of the ionised or conducting medium.

* Thus the skin depth depends upon the frequency of the wave and the conductivity of the medium.
* It decreases with the increase of frequency of wave.
* It also decreases in a higher conducting medium.

i.e $\boxed{z \propto \frac{1}{\omega} \text{ and } z \propto \frac{1}{\sigma}}$

Q) Calculate the skin depth for aluminium at a frequency of 2×10^6 Hz if conductivity of the Aluminium is 38.2×10^6 mho/m. Assume $\mu_r = 1$. Calculate the attenuation constant, ~~and the~~ phase constant and speed of wave.

Ans - Attenuation const is α.

Phase constant is β

Skin depth is $z = \frac{1}{\alpha}$

Speed of wave $= \frac{\omega}{K}$

Module - 03

Quantum Mechanics -

Need of Quantum Mechanics:-

The quantum theories developed which couldn't explain by the help of classical theories -

(i) Plank's theory of Black Body Radiation in 1900

(ii) Einstein's theory of Photo Electric Effect in 1905.

(iii) Bohr's theory of Hydrogen spectrum in 1913. (Not in course)

(iv) Compton's theory of X ray scattering in 1922.

(v) De-broglie hypothesis in 1924.

(vi) Heisenberg's Uncertainity principle in 1927.

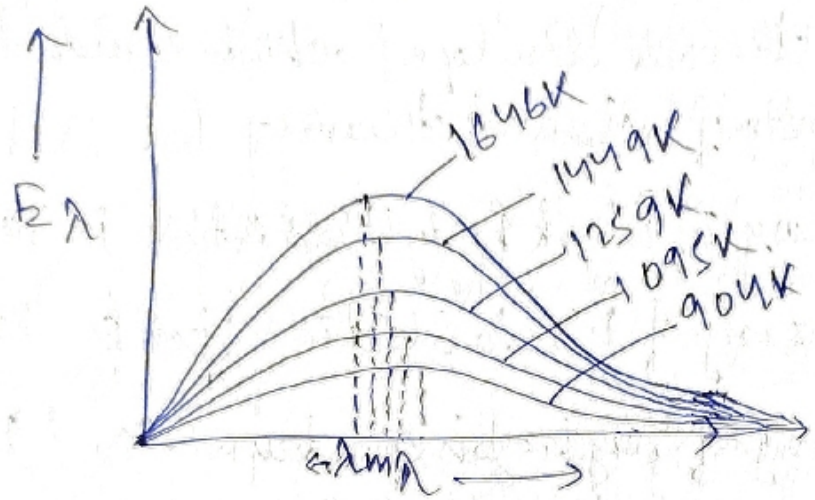
(vii) Schodinger's wave equation in 1928/29.

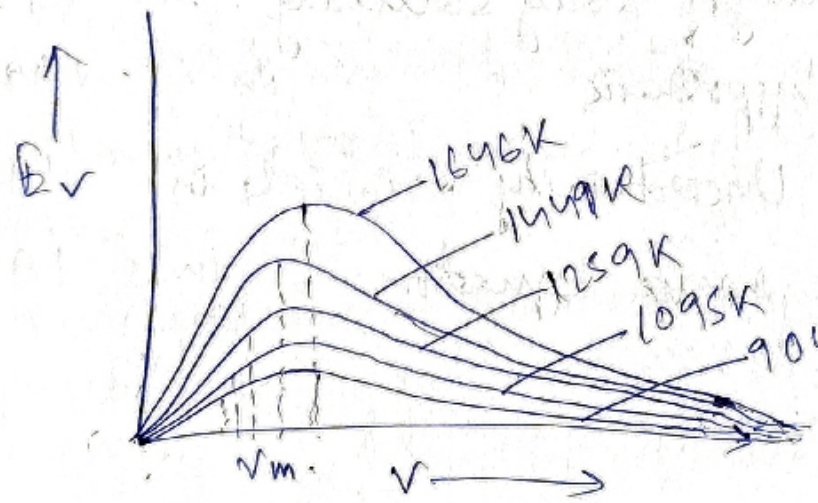
Module-4

Experimental Observation Of Black Body Radiation.

E_λ or E_ν → Radiant Energy density means Energy radiated per unit volume.

$\left\{ \nu = \frac{c}{\lambda} \right\}$



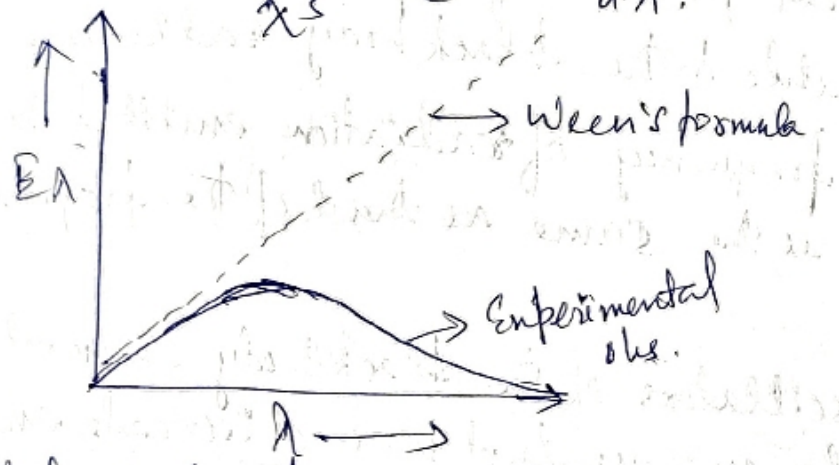


* At a given temperature, the energy density has maximum value corresponding to a value of frequency or wavelength.
* The frequency corresponding to the maximum energy density increase with increase of temperature.

* The energy density decreases to zero for both higher and lower values of frequency or wavelength.
* The energy density corresponding to a given frequency or wavelength increases with increase of temperature.
* The total energy radiated at any temperature is given by the area between the curve corresponding to that temperature and the horizontal axis.

Wien's formula for Black Body Radiation:-

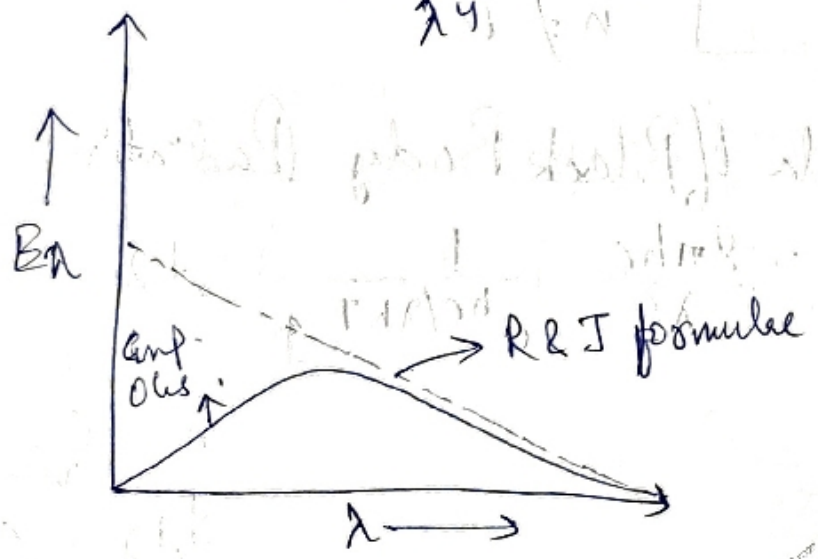
$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} e^{-hc/\lambda KT} d\lambda.$$



* Valid only for lower wavelength.

Rayleigh & Jean (1900)

$$E_\lambda d\lambda = \frac{8\pi KT}{\lambda^4} d\lambda.$$



* Valid only for higher wavelength.

Plank's Quantum Theory of Black Body Radiation (1900)

* In order to explain the experimental observe distribution of energy, Plank in 1900 suggested some assumptions which is called as Plank's Quantum Theory.

(i) A chamber containing black body radiations contain some simple harmonic oscillators of molecular dimensions which can vibrate with all possible frequencies.

(ii) Classical principle of Equipartition of energy is not applicable to the black body radiation.

(iii) The frequency of radiation emitted by an oscillator is the same as that of the frequency of vibration.

(iv) The oscillators of the black body cannot have all possible energy but has discreate energy which is integral multiple of some minimum amount of energy.

$$\boxed{E = nh\nu}$$

$n = 1, 2, 3, 4 \ldots$
$n \neq 0$

Plank's Formula of Black Body Radiation

$$\boxed{E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \frac{1}{e^{hc/\lambda KT} - 1} d\lambda} \quad \text{--- (1)}$$

$\lambda = \frac{c}{\nu}$

$\Rightarrow d\lambda = \frac{c}{\nu^2} d\nu$

$$\Rightarrow \boxed{E_\nu \, d\nu = \frac{8\pi h \nu^3}{c^3} \frac{1}{e^{h\nu/KT}-1} \, d\nu}$$

Eqn (1) reduce to ween's formula for low wavelength region.

for low wavelength λT is very small.

$$\Rightarrow e^{hc/\lambda KT} >>> 1.$$

$$\Rightarrow e^{hc/\lambda KT} - 1 \cong e^{hc/\lambda KT}$$

$$\Rightarrow E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda KT}} d\lambda.$$

$$= \frac{8\pi hc}{\lambda^5} e^{-hc/K\lambda T} d\lambda.$$

$$\Downarrow$$

Ween's formula.

So, plank's formula has a very good fitting to the experimental result at low wavelength region.

Plank's formula (Eq^n(1)) reduces to R&J formula for high wavelength region.

λT is very large

$$\Rightarrow e^{hc/K\lambda T} = 1 + \frac{hc}{\lambda KT} + \frac{1}{2!}\left(\frac{hc}{\lambda KT}\right)^2 + \cdots$$

$$= 1 + \frac{hc}{\lambda KT} \text{ (Neglecting higher order)}$$

$$\Rightarrow E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \times \frac{1}{e^{hc/\lambda KT} - 1} d\lambda$$

$$\Rightarrow E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \times \frac{1}{\cancel{1} + \frac{hc}{\lambda KT} - \cancel{1}} d\lambda$$

$$\Rightarrow E_\lambda d\lambda = \frac{8\pi \cancel{hc}}{\lambda^{\cancel{5}\,4}} \times \frac{\cancel{\lambda} KT}{\cancel{hc}} d\lambda$$

$$= \frac{8\pi KT}{\lambda^4} d\lambda \rightarrow \text{R\&J formula}$$

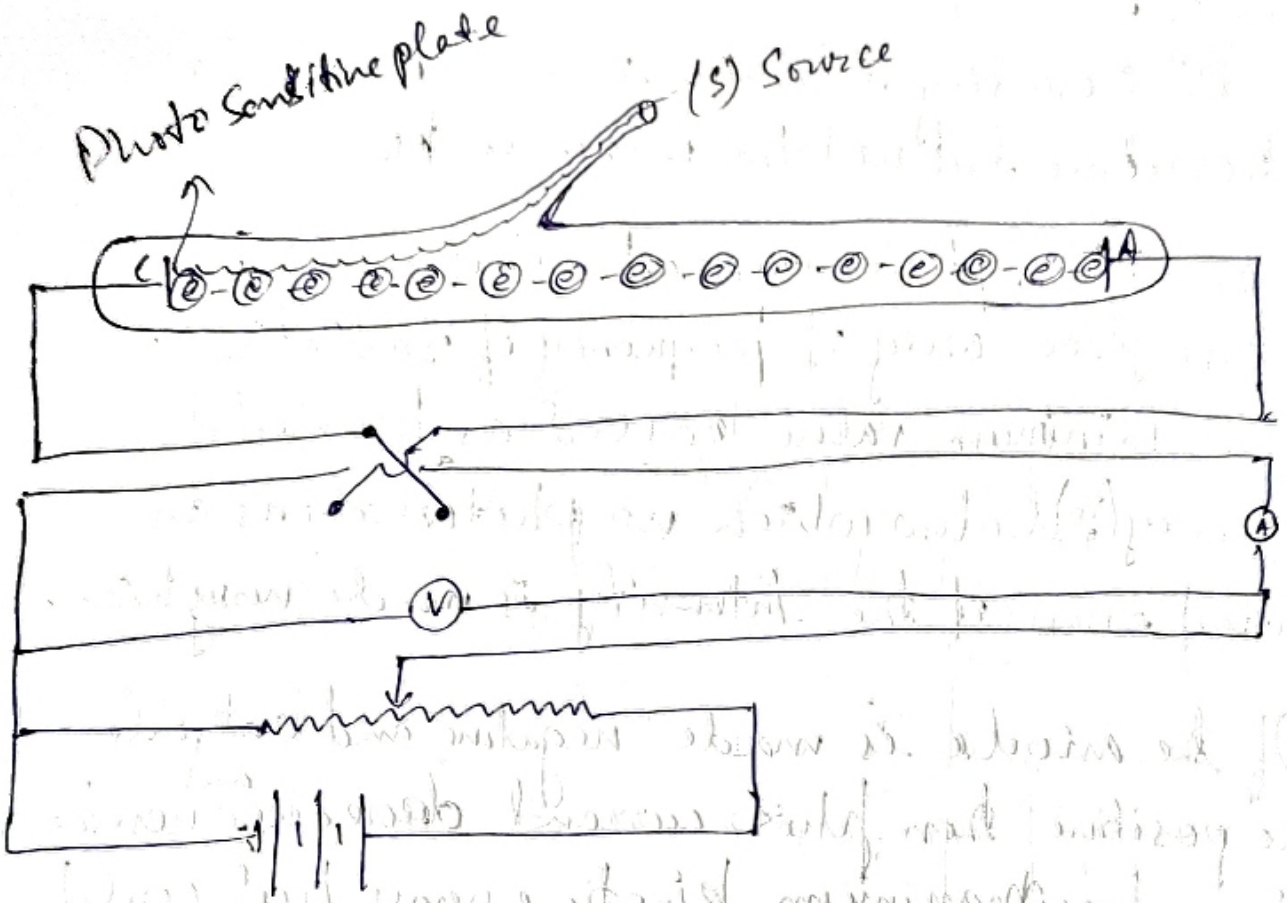
* So, plank's formula has a very good fitting to the experimental result for high wavelength region.

Thus we conclude Plank's formula agrees to the experimental result for all ~~values~~ ranges of wavelength

Photo Electric Effect -

* The emission of electrons from a metal plate when a light of suitable frequency is incident on the metal is called as Photoelectric effect.

* The emitted electrons are called as photoelectrons.



Experimental Observations -

(i) Photoelectric effect is an instantaneous process. As soon as the radiation is incident on the metal plate, the photoelectrons are emitted. The time lag between them is 10^{-8} sec.

(ii) For a given frequency and given potential difference between anode and photo cathode the photo current is directly proportional to intensity of incident radiation

iii) The emitted photoelectrons do not have same kinetic energy and varies from 0 to a maximum value.

For a given photo cathode and given potential difference the maximum kinetic energy depends upon the frequency of radiation but independent of intensity.

On increasing intensity increases the number of photoelectrons but not the maximum Kinetic Energy.

iv) For a given photo cathode, the photoelectric effect can take place only if frequency of radiation must have a minimum value called as threshold frequency (ν_0) below which no photoelectrons are emitted even if the intensity is made very high.

v) If the anode is made negative and cathode made positive then photo current decreases and even the electrons has maximum kinetic energy but could not reach the anode.

So, the minimum potential (negative potential) to get zero photocurrent is called as stopping potential.

vi) For a given metal surface, stopping potential is directly proportional to the frequency of incident radiation but independent of intensity.

Einstein's Explanation Of Photo Electric Effect.

* Einstein extended Plank's idea to explain photo-electric effect.

He assumed that Electromagnetic waves of frequency 'ν' can be regarded as a stream of particles of energy '$h\nu$' each and these particles are called as photons.

The energy of photons is used in two parts –

(i) A part of it is used to free the electrons from the atom and make them away from the metal surface. This energy is called as work function of the metal and denoted by (W_0).

(ii) The rest part of energy of photons is used in giving kinetic energy to the electrons.

$$\boxed{h\nu = W_0 + \frac{1}{2}mv^2}$$

$W_0 \rightarrow$ Work function of metal surface.

$v \rightarrow$ Velocity of the photoelectrons.

(Einstein's Photo Electric Equation)

$\boxed{W_0 = h\nu_0}$ → ν_0 → Minimum frequency required for which photoelectric effect takes place

$\nu_0 \rightarrow$ Threshold frequency

2) $$\boxed{h\nu = h\nu_0 + \frac{1}{2}mv_0^2}$$

$$\Rightarrow \boxed{\frac{1}{2}mv^2 = h\nu - h\nu_0}$$

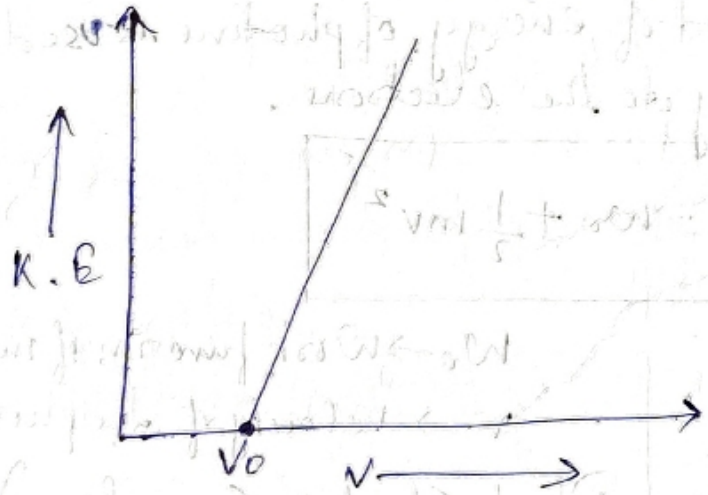
$h \rightarrow$ Plank's constant

$$\Rightarrow \boxed{\frac{1}{2}mv^2 = h(\nu - \nu_0)}$$

$\nu_0 \rightarrow$ constant

Thus $\boxed{\frac{1}{2}mv^2 \propto \nu}$ $\Rightarrow \boxed{v^2 \propto \nu}$

* K.E directly proportional to frequency of light.
* Velocity of electrons directly proportional to frequency.



Stopping Potential.

$$\frac{1}{2}mv^2 = qV$$

$q \rightarrow$ charge particle.

$V \rightarrow$ Potential given

Here $\boxed{\frac{1}{2}mv^2 = eV}$

$e \rightarrow$ charge of e^-

$V \rightarrow$ -ve potential between cathode & anode

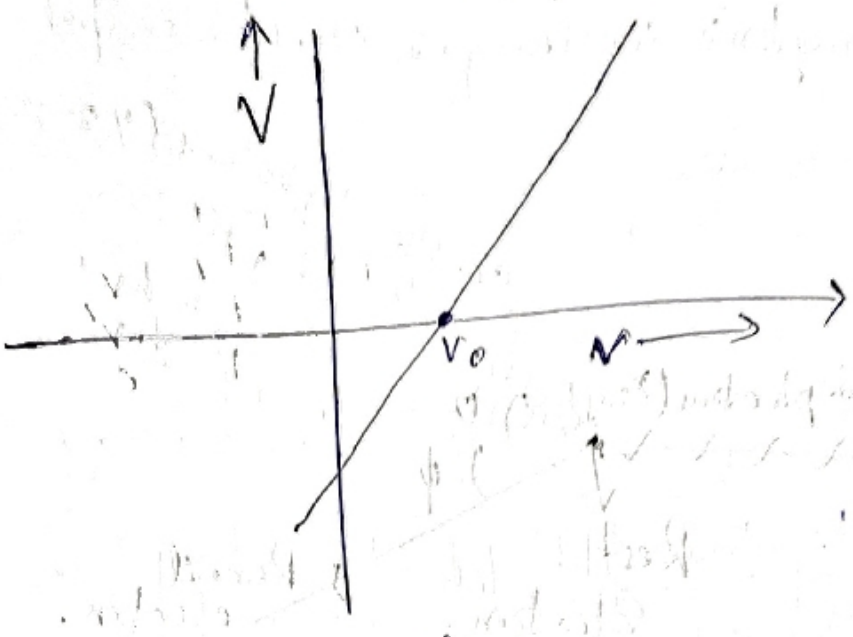
Stopping potential

$$eV = h\nu - h\nu_0$$

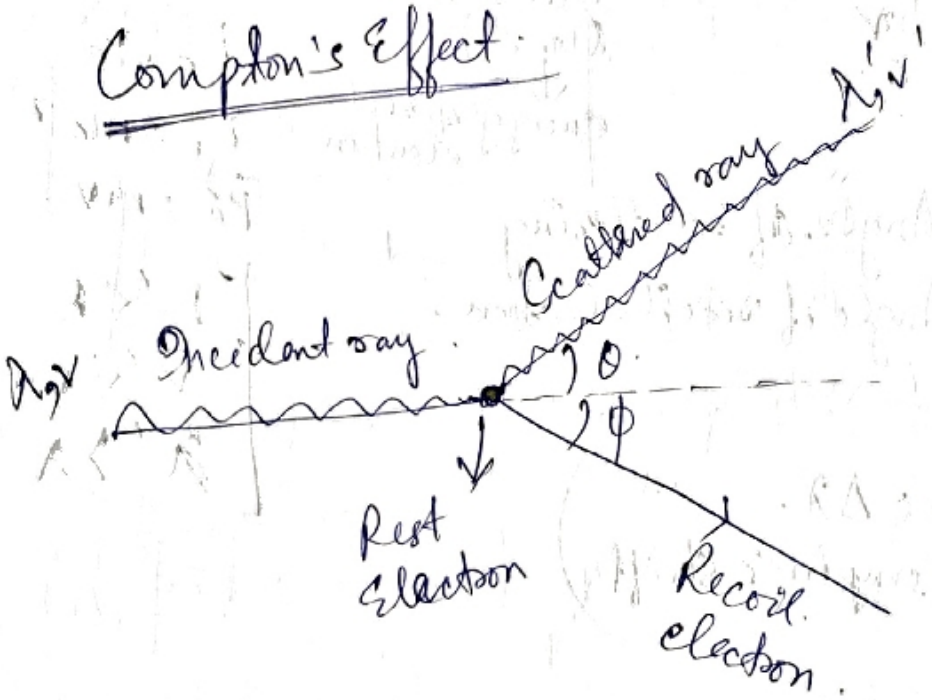
since $\frac{1}{2}mv^2 = eV$

$$\Rightarrow V \propto \nu$$

* Stopping potential (V) $\propto$ frequency (ν) since h, ν_0, e are constant.



Compton's Effect

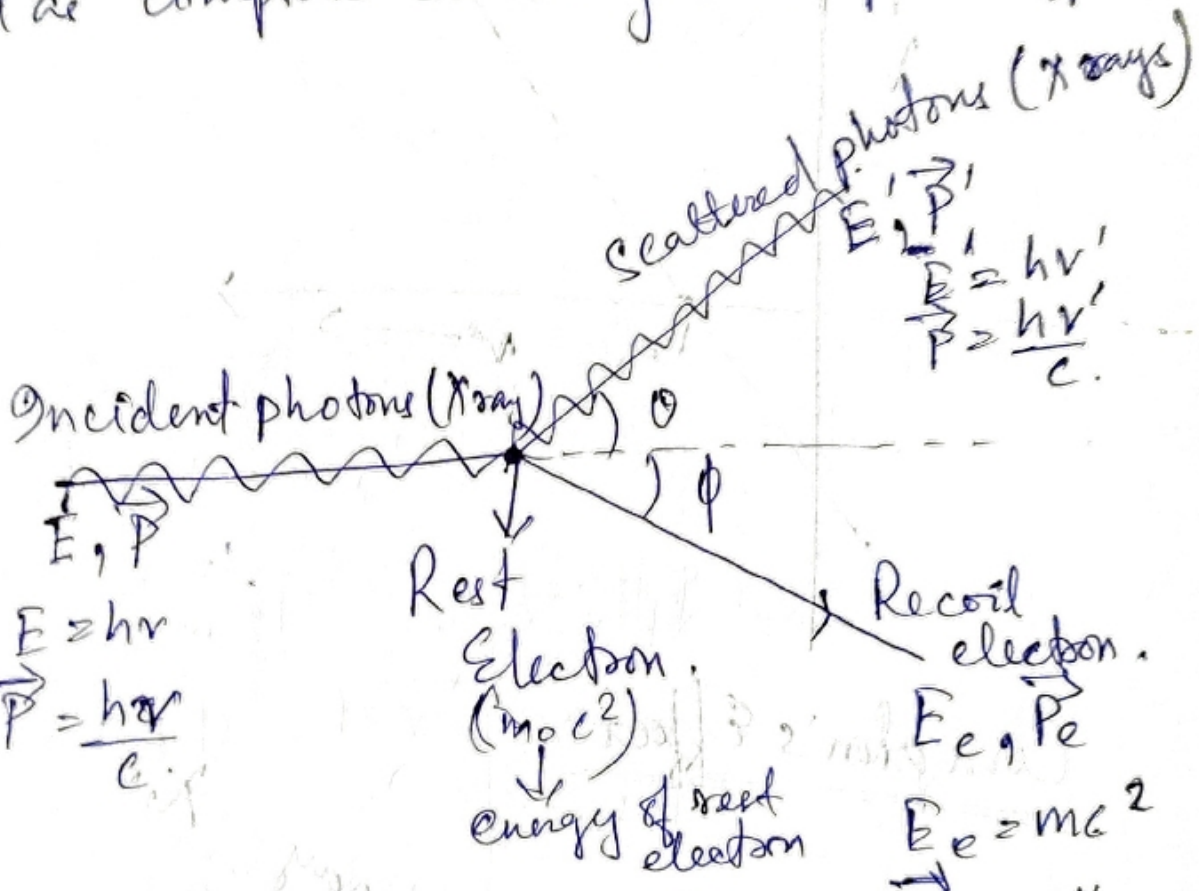


$$\boxed{\Delta\lambda = \lambda' - \lambda}$$

Compton's shift

Compton's Effect

* When a monochromatic beam of high frequency radiation such as X rays collides with the rest electron, a part of energy is given to that electron. Due to this electron will gain Kinetic Energy and scattered photon or (X rays) will have lower energy, so lower frequency or greater wavelength than the incident one which is called as compton's scattering or compton's effect.



$\theta \rightarrow$ Angle of Scattering.

$\phi \rightarrow$ Angle of recoil electron.

$$\left[\begin{array}{l} E' < E \\ \nu' < \nu \\ \lambda' > \lambda \end{array}\right]$$

$$\left(\begin{array}{l} \lambda' - \lambda = \Delta\lambda. \\ \Rightarrow \text{Compton's shift} \end{array}\right)$$

Experimental Observations -

* For a given angle of scattering the scattered x-rays have two components -
  (i) A component with wavelength same as that of the incident x-rays is called as unmodified component.
  (ii) Another component having lower frequency and greater wavelength is called as modified component or compton's component.

* The increase in wavelength in compton's component is called as compton's shift which depends upon the angle of scattering. (θ).
* The compton's shift is independent of the wavelength of the incident radiation.

Note: Classical theory failed to explain experimental observations by considering the radiations (x-rays) as wave nature.

Compton's Explanation -

An adequate explanation of this effect was provided by Compton in 1992 on the basis of plank's quantum theory of radiation. According to that quantum concept, the radiation is constituted by energy packets called 'Photons' having energy '$h\nu$'.

The incident x-rays with frequency 'ν' are regarded as a stream of particles with energy '$h\nu$'.

Before Collision -

(i) The energy of the incident photon -

$$E = h\nu$$

(ii) Momentum of incident photon -

$$\vec{P} = \frac{h\nu}{c}$$

(iii) Energy of the rest electron

$$= m_0 c^2$$

(iv) Momentum of the rest electron

$$= 0.$$

After Collision.

(i) Energy of scattered photon -

$$E' = h\nu'$$

(ii) Momentum of scattered photon -

$$\vec{P}' = \frac{h\nu'}{c}.$$

(iii) Energy of recoil electron -

$$E_e = mc^2$$

where $m = \frac{m_0}{1 - \frac{v^2}{c^2}}$

(iv) Momentum of recoil electron -

$$\vec{P}_e = m\vec{v}$$

$$\Rightarrow m^2 = \frac{m_0^2}{\frac{c^2 - v^2}{c^2}}$$

$$\Rightarrow m^2 = \frac{m_0^2 c^2}{c^2 - v^2}.$$

$$\Rightarrow m^2c^2 - m^2v^2 = m_0^2 c^2.$$

$$\Rightarrow c^2(m^2c^2 - m^2v^2) = m_0^2 c^2 \times c^2.$$

$\Rightarrow m^2c^4 - m^2c^2v^2 = m_0^2c^4$

$$\Rightarrow \boxed{E_e^2 = P_e^2c^2 + m_0^2c^4} \quad \text{--- (1)}$$

Applying law of conservation of energy -

$$\Rightarrow E + m_0c^2 = E' + E_e$$

$$\Rightarrow E + m_0c^2 = E' + \sqrt{P_e^2c^2 + m_0^2c^4}$$

$$\Rightarrow E - E' + m_0c^2 = \sqrt{P_e^2c^2 + m_0^2c^4} \quad \text{--- (2)}$$

Applying law of conservation of momentum -

$$\vec{P} = \vec{P'} + \vec{P_e}.$$

$$\Rightarrow \vec{P_e} = \vec{P} - \vec{P'}.$$

$$\Rightarrow P_e^2 = \vec{P_e}\cdot\vec{P_e} = (\vec{P} - \vec{P'})\cdot(\vec{P} - \vec{P'})$$

$$\Rightarrow P_e^2 = P^2 + P'^2 - 2PP'\cos\theta \quad \text{--- (3)}$$

Squaring equation (2) both the sides -

$$\Rightarrow (E - E')^2 + \cancel{m_0^2c^4} + 2(E - E')m_0c^2 = P_e^2c^2 + \cancel{m_0^2c^4}$$

$$\Rightarrow (E - E')^2 + 2(E - E')m_0c^2 = (P^2 + P'^2 - 2PP'\cos\theta)c^2.$$

Substituting value of P_e^2 from (3).

$$\Rightarrow (E - E')^2 + 2(E - E')m_0c^2 = P^2c^2 + P'^2c^2 - 2PP'c^2\cos\theta$$

$$= E^2 + E'^2 - 2EE'\cos\theta$$

Add and subtract $2EE'$ in right hand side-

$$\Rightarrow (E-E')^2 + 2(E-E')m_oc^2 = E^2 + E'^2 - 2EE' - 2EE'\cos\theta + 2EE'$$

$$\Rightarrow \cancel{(E-E')^2} + 2(E-E')m_oc^2 = \cancel{(E-E')^2} + 2EE'(1-\cos\theta)$$

$$\Rightarrow \cancel{2}(E-E')m_oc^2 = \cancel{2}EE'(1-\cos\theta)$$

$$\Rightarrow \frac{E-E'}{EE'} = \frac{1-\cos\theta}{m_oc^2}$$

$$\Rightarrow \frac{\cancel{E}}{\cancel{E}E'} - \frac{\cancel{E'}}{E\cancel{E'}} = \frac{1-\cos\theta}{m_oc^2}$$

$$\Rightarrow \frac{1}{E'} - \frac{1}{E} = \frac{1-\cos\theta}{m_oc^2}$$

$E = h\nu = \frac{hc}{\lambda}$

$E' = h\nu' = \frac{hc}{\lambda'}$

$$\Rightarrow \frac{\lambda'}{hc} - \frac{\lambda}{hc} = \frac{1-\cos\theta}{m_oc^2}$$

$$\Rightarrow \frac{1}{h\cancel{c}}(\lambda' - \lambda) = \frac{1-\cos\theta}{m_oc^{\cancel{2}}}$$

$$\Rightarrow \boxed{\lambda' - \lambda = \frac{h}{m_oc}(1-\cos\theta)}$$

$\Downarrow$ Compton's shift

So, $\boxed{\Delta\lambda = \lambda' - \lambda = \lambda_c(1-\cos\theta)}$

Compton's shift

$\lambda_c \rightarrow$ Compton's wavelength $= \frac{h}{m_oc}$

* Thus the Compton's shift $\Delta\lambda$ only depends upon the angle of scattering θ and independent of the wavelength of incident radiation.

Different cases -

Case-I $\theta = 0$

$$\Delta\lambda = \lambda_c(1-\cos\theta)$$

$$\Rightarrow \boxed{\Delta\lambda = 0.}$$

$$\Rightarrow \boxed{\lambda' = \lambda.}$$

There will be no change

Case-II $\theta = 90^\circ$

$$\Delta\lambda = \lambda_c(1-\cos 90^\circ)$$

$$\Rightarrow \Delta\lambda = \lambda_c.$$

$$\Rightarrow \lambda' - \lambda = \lambda_c.$$

$$\Rightarrow \boxed{\lambda' = \lambda + \lambda_c.}$$

Case-III $\theta = 180^\circ$

$$\boxed{\Delta\lambda = 2\lambda_c.}$$ ← Max. value of compton's shift

Q) Find compton's shift of x-rays of wavelength 1.5A° scattered by 60°. Also find wavelength of scattered x-rays.

Q) X rays of 1A° has scattered from a carbon block. Find wavelength of scattered beam in a direction making 90° with incident beam.
How much kinetic energy is imparted to the recoil electron.

Q) X rays of wavelength 1.2A° undergo compton's scattering due to electrons. What is maximum possible value of compton's shift if the compton's wavelength of the electron is $\boxed{\lambda_c = 0.0242 A^\circ}$

Photo Electric Effect

Q) Calculate velocity of photoelectron if work func^n of target material is 1.24eV & wavelength of incident rays is 4.36×10^{-7} m. What is the retarding potential necessary to stop the emission of these electrons.

Q) Electrons are emitted with zero velocity from a certain metal surface. When it is exposed to a radiation of 6800A°. Calculate the threshold frequency & work function of metal.

Q) Work function of aluminium is 4.2eV. Calculate the K.E of the fastest and slowest photoelectrons, the stopping potential and the cutoff wavelength. When

light of wavelength 2000 Å falls on the clean aluminium surface.

$\{1 eV = 1.62 \times 10^{-19} J\}$

$h = 6.626 \times 10^{-34} J/s$

$m_e \rightarrow 9.1 \times 10^{-31} kg$

$c \rightarrow 3 \times 10^{8} m/s$

$\boxed{\frac{h}{m_e c} = 2.42 \times 10^{-12} m.}$

Answer

(1)

Wavelength $\lambda = 1.5 Å$.

Compton's shift of x rays $\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta)$

$$= \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}}(1-\cos 60^\circ)$$

$$= \frac{6.626 \times 10^{-34}}{27.3 \times 10^{-23}}\left(1-\frac{1}{2}\right)$$

$$= \frac{6.626 \times 10^{-34} \times 5 \times 10^{-1}}{27.3 \times 10^{-23}}$$

$$= \frac{33.0 \times 10^{-35}}{27.3 \times 10^{-23}} = 1.2 \times 10^{-12} m.$$

Wavelength of scattered x rays = ?

$\Delta\lambda = \lambda' - \lambda$

$\Rightarrow 1.2 \times 10^{-12} = \lambda' - 1.5 \times 10^{-10}$

$\Rightarrow \lambda' = 1.2 \times 10^{-12} + 1.5 \times 10^{-10}$

$= \cancel{2.7 \times 10^{-22} m}\ 1.512 \times 10^{-10} m.$

(2) Wavelength $\lambda = 1 Å$.

$\theta = 90^\circ$

Compton's shift $\Delta\lambda = \frac{h}{m_0 c}(1-\cos 90^\circ)$

$\Rightarrow \Delta\lambda = 2.4 \times 10^{-12} m$

$\Rightarrow \lambda' - \lambda = \Delta\lambda \Rightarrow \lambda' = \Delta\lambda + \lambda$

$= 2.4 \times 10^{-12} + 1 \times 10^{-10} m.$

$= 1.02 \times 10^{-10} m$

Energy is conserved after collision.

$\Rightarrow h\nu = h\nu' + K.E$

$\Rightarrow K.E = h\nu - h\nu'$

$\Rightarrow K.E = \frac{hc}{\lambda} - \frac{hc}{\lambda'}$

$\Rightarrow K.E = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)$

$\Rightarrow K.E = 6.62\times10^{-34}\times3\times10^{8}\left(\frac{1}{1\times10^{-10}} - \frac{1}{1.02\times10^{-10}}\right)$

$\Rightarrow K.E = 19.86\times10^{-26}\left(\frac{10^{10}}{1} - \frac{10^{10}}{1.02}\right)$

$= 19.86\times10^{-26}\times10^{10}\left(1 - \frac{1}{1.02}\right)$

$= 19.86\times10^{-16}(0.02)$

$= 19.86\times10^{-16}\times\frac{2}{100}$

$= 39.72\times10^{-16}\times10^{-2}$

$= 39.72\times10^{-18}$ J

$= \boxed{247.85\,eV}$

(3) $\lambda = 1.2A^{\circ}$

$\lambda_c = 0.024A^{\circ}$

Maximum compton's shift is possible when $\theta = 180^{\circ}$ i.e $\Delta\lambda = hc(1-\cos 180^{\circ})$

$\Rightarrow \Delta\lambda = 2hc$

$= 2\times0.024A^{\circ}$

$= \boxed{0.048A^{\circ}}$

a) Work fun^n (w0) = 1.24 eV

$\lambda = 4.36 \times 10^{-7} m$.

According to Einstein's Photoelectric Eq^n-

$h\nu = \text{Work fun}^n + K.E$

$\Rightarrow \frac{hc}{\lambda} = 1.24 eV + \frac{1}{2} \times (9.1 \times 10^{-31}) \times v^2$

$\Rightarrow \frac{19.86 \times 10^{-26}}{4.36 \times 10^{-7}} = 1.24 + 4.55 \times 10^{-31} v^2$

$\Rightarrow 4.55 \times 10^{-19} = 1.24 + 4.55 \times 10^{-31} v^2$

$\Rightarrow 4.55 \times 10^{-19} = 1.24 \times 1.6 \times 10^{-19} + 4.55 \times 10^{-31} v^2$

$\Rightarrow 4.55 \times 10^{-19} = 1.98 \times 10^{-19} + 4.55 \times 10^{-31} v^2$

$\Rightarrow \frac{4.55 \times 10^{-19} - 1.98 \times 10^{-19}}{4.55 \times 10^{-31}} = v^2$

$\Rightarrow \frac{10^{-19}(2.57)}{4.55 \times 10^{-31}} = v^2$

$\Rightarrow \frac{2.57 \times 10^{-19}}{4.55 \times 10^{-31}} = v^2$

$\Rightarrow 0.56 \times 10^{12} = v^2$

$\Rightarrow v = \sqrt{0.56 \times 10^{12}}$

$= 748331.4 m/s$

$\frac{1}{2} m v^2 = e V_0$.

$\Rightarrow \frac{1}{2} \times 9.1 \times 10^{-31} \times 0.56 \times 10^{12} = 1.6 \times 10^{19} V_0$

$\Rightarrow \frac{4.55 \times 10^{-31} \times 0.56 \times 10^{12}}{1.6 \times 10^{19}} = V_0$.

$\Rightarrow \frac{2.54 \times 10^{-19}}{1.6 \times 10^{19}} = V_0$ $\boxed{1.58 \times 10^{-38} eV}$

(5) $v = 0$

$\Rightarrow$ K.E of electrons $= 0$.

$v = 6800 A^{\circ}$

$v_0 = ?$

$\cancel{h}v = \cancel{h}v_0 + \cancel{K.E}^{\,0}$

$\Rightarrow v_0 = v = 6800 A^{\circ}$

So, threshold frequency $= 6800 A^{\circ}$

Work fun^n $(W_0) = h v_0$

$= h \times 6800 A^{\circ}$

$= 6.62 \times 10^{-32} \times 6800 A^{\circ}$

$= 6.62 \times 6800 \times 10^{-10} \times 10^{-32}$

$= 45016 \times 10^{-42} J$

Q) Distinguish between the photoelectric effect and compton's scattering.

Dual Nature Of Radiation (De-Broglie Hypothesis)

$$\boxed{\lambda = \frac{h}{p}}$$

$\lambda \rightarrow$ Wavelength of a wave.

$p \rightarrow$ momentum of particle.

$= \boxed{mv}$

* Looking to the discussion given by photoelectric effect, compton's effect & pair production, Louis De Broglie proposed that all material particles are associated with waves called as <u>matter waves</u> or <u>de-Broglie's wave</u>.

According to his hypothesis the wavelength λ of matter wave associated with a moving particle of linear momentum 'p' and given by -

$$\boxed{\lambda = \frac{h}{p}}$$

where $h \rightarrow$ plank's const

$p \rightarrow$ linear momentum $= mv$.

Calculation Of Debroglie Wavelength -

(1) Free Particle -

No potential energy 0

So, (T) Total Energy $= \cancel{P.E} + K.E$

$\Rightarrow \boxed{T.E = K.E}$

$K.E = (E)$

$\Rightarrow K.E = \frac{p^2}{2m} \Rightarrow p^2 = 2mE$

$\Rightarrow p = \sqrt{2mE}$

So, $\lambda = \frac{h}{p} \Rightarrow \boxed{\lambda = \frac{h}{\sqrt{2mE}}}$

$E \rightarrow$ K.E of particle.

(2) For any particle -

Let the particle moves under potential 'V'.

$\Rightarrow T = E + V$ $\quad$ $T \rightarrow$ Total Eng.

$\Rightarrow E = T - V$ $\quad$ $E \rightarrow$ K.E

$V \rightarrow$ P.E

$$\Rightarrow \boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2m(T-V)}}}$$

(3) For a charged particle.

Let the charge particle having charge 'q' accelerated with potential difference 'V' of the charge

$K.E = qV$

$$\Rightarrow \boxed{\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}}$$

$q \rightarrow$ charge.

Sol case

For $q = e$

$$\Rightarrow \boxed{\lambda = \frac{h}{\sqrt{2meV}}} \qquad \Rightarrow \boxed{\lambda = \frac{12.27}{\sqrt{V}} \text{ Å}}$$

$V \rightarrow$ potential.

(iv) For thermal particle.

K.E of thermal particle = $\boxed{\frac{3}{2} KT}$

K → Boltz's const

T → Temp in Kelvin

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$= \frac{h}{\sqrt{\cancel{2} \times m \times \frac{3}{\cancel{2}} KT}}$$

$$= \frac{h}{\sqrt{3mKT}} \qquad \boxed{= \frac{h}{\sqrt{3mKT}}}$$

Q) Calculate debroglie wavelength associated with a electron subjected to a potential difference of 10V.

Ans- $\lambda = \frac{12.27}{\sqrt{10}} \text{ Å}$.

Q) A ball of mass 0.1 g has a speed of 300 m/s. Calculate the debroglie wavelength associated with it.

Heisenberg's Uncertainity Principle -

According to Heisenberg's uncertainity principle it is not possible to measure simultaneously the position and momentum of a particle with 100% accuracy.

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2} / \hbar / \hbar \qquad \hbar = \frac{h}{2\pi}$$

and.

$$\Delta t \cdot \Delta E \geq \frac{\hbar}{2} / \hbar / \hbar$$

① Non existance of $e^{\ominus}$ in the nucleous.

② Ground state energy of harmonic oscillator is non-zero.

③ Ground state energy of H-atom is non-zero.

Non existence of $e^{\ominus}$ in the nucleus.

① Let us assume the electron exists inside the nucleus.

dimension of nucleus $\approx 10^{-14}$ m.

$\Rightarrow \Delta x \approx 10^{-14}$ m.

Using Heisenberg's Uncertainity Principle -

$$\Delta x \cdot \Delta p \approx \frac{\hbar}{2}. \qquad \hbar = \frac{h}{2\pi} = 1.052 \times 10^{-34}$$

$$\Rightarrow \Delta p = \frac{\hbar}{2\Delta x}$$

$$= \frac{1.052 \times 10^{-34}}{2 \times 10^{-14}}$$

$$= 5.3 \times 10^{-21} \text{ kg m/s}$$

By Compton's theory -

$$E = \sqrt{m_0^2 c^4 + p^2 c^2}$$

$m_0 = 9.1 \times 10^{-31}$ kg

$c = 3 \times 10^8$ m/s.

Let $\Delta p = p$

$$= \sqrt{(9.1 \times 10^{-31})^2 \times (3 \times 10^8)^4 + (5.3 \times 10^{-21})^2 (3 \times 10^8)^2}$$

$$\Rightarrow E = 1.6 \times 10^{-12} \text{ J} \approx 10 \text{ MeV}$$

This means if the electron exists inside the nucleus then it's energy must be in the order of 10MeV.

* But the electrons emitted from the radioactive β decay has K.E of about 1MeV which is much smaller than the predicted value by Uncertainity principle.

*** Thus electrons do not exist inside the nucleus.

Q) Calculate the uncertainity in velocity from H.U.P.

Ans- $\Delta P = \frac{\hbar}{2\Delta r} = 5.3\times10^{-21}$ kg m/s.

$\Rightarrow m\Delta V = 5.3\times10^{-21}$

$\Rightarrow \Delta V = \frac{5.3\times10^{-21}}{9.1\times10^{-31}} =$

Uncertainity of velocity

(2) Ground State Energy of Harmonic Oscillator is non-zero:-

The energy of the harmonic oscillator is -

$$E = \frac{P^2}{2m} + \frac{1}{2}m\omega^2 x^2 \quad \text{--- (1)}$$

Using Heisenberg's Uncertainity principle -

$$\Delta x \cdot \Delta p \approx \frac{\hbar}{2}$$

Let $\Delta x \approx x$ and $\Delta p \approx P$.

$\Rightarrow x \cdot p = \frac{\hbar}{2}$

$\Rightarrow \boxed{p = \frac{\hbar}{2x}}$ $\Rightarrow p^2 = \frac{\hbar^2}{4x^2}$

Substituting p in ①.

$\Rightarrow \cancel{p = \frac{\hbar}{2x}}$ $\Rightarrow E = \frac{\hbar^2}{8mx^2} + \frac{1}{2}m\omega^2 x^2$ ②

for $x = x_0$

As the ground state energy is minimum, so the 1st order derivative of energy at that pt is 0.

$$\Rightarrow \left.\frac{\partial E}{\partial x}\right]_{x=x_0} = 0$$

$$\Rightarrow \frac{\hbar^2}{8m}\frac{\partial}{\partial x}(x^{-2}) + \frac{1}{2}m\omega^2\frac{\partial}{\partial x}(x^2) = 0$$

$$\Rightarrow \frac{\hbar^2}{\cancel{8}_4 m} \times -\cancel{2}x^{-3} + \frac{1}{\cancel{2}}m\omega^2 \cancel{2}x = 0$$

$$\Rightarrow -\frac{\hbar^2}{4mx^3} + m\omega^2 x = 0.$$

$$\Rightarrow \frac{-\hbar^2}{4mx^3} = m\omega^2 x$$

$$\Rightarrow \frac{\hbar^2}{4m^2\omega^2} = x^4$$

$$\Rightarrow \boxed{x_0^2 = \frac{\hbar}{2m\omega}}$$

$$E_0 = \frac{\hbar^2}{4\cancel{8}\cancel{m} \cdot \frac{\hbar}{\cancel{2}\cancel{m}\omega}} + \frac{1}{2}\cancel{m}\omega^2 \cdot \frac{\hbar}{2\cancel{m}\omega} = \frac{\cancel{\hbar^2}\omega}{4\cancel{\hbar}} + \frac{\hbar\omega}{4}$$

$$= \boxed{\frac{\hbar\omega}{2}}$$

Thus ground state energy of harmonic oscillator is non zero.

Ground state energy of Hydrogen atom is non-zero

The energy of H atom -

$$E = \frac{p^2}{2m} - \frac{e^2}{4\pi\varepsilon_0 r} \quad \text{--- (1)}$$

$r \rightarrow$ radius of orbit of atom.

$p \rightarrow$ Momentum.

Using Heisenberg's Uncertainity Principle -

Here $\Delta r . \Delta p \approx \hbar$

Let $\Delta r = r$ & $\Delta p = p$

$\Rightarrow r_0 p = \hbar \quad \Rightarrow \boxed{p = \frac{\hbar}{r}}$

Thus energy expression is -

$$E = \frac{\hbar^2}{2mr^2} - \frac{e^2}{4\pi\varepsilon_0 r} \quad \text{--- (2)}$$

r^{-1}
$= -1r$

$$\left.\frac{\partial E}{\partial r}\right|_{r=r_0} = 0$$

$$\Rightarrow \frac{\hbar^2}{\cancel{2}m} \times \frac{-\cancel{2}}{r^3} - \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{-1}{r^2} = 0$$

$$\Rightarrow \frac{-\hbar^2}{mr_0^3} + \frac{e^2}{4\pi\varepsilon_0 r_0^2} = 0$$

$$\Rightarrow \frac{\hbar^2}{mr_0^3} = \frac{e^2}{4\pi\varepsilon_0 r_0^2}.$$

$$\Rightarrow \frac{\cancel{r_0^2}\hbar^2}{mr_0^{\cancel{3}}} = \frac{e^2}{4\pi\varepsilon_0}.$$

$$\Rightarrow \frac{\hbar^2}{mr_0} = \frac{e^2}{4\pi\varepsilon_0}.$$

$$\Rightarrow \frac{4\pi\varepsilon_0 \hbar^2}{me^2} = r_0$$

$$\Rightarrow E_0 = \frac{\hbar^2}{2m \times \frac{16\pi^2\varepsilon_0^2\hbar^4}{m^2e^4}} - \frac{e^2}{4\pi\varepsilon_0 \times \frac{4\pi\varepsilon_0\hbar^2}{me^2}}$$

$$\Rightarrow E_0 = \frac{\cancel{\hbar^2} e^4 m}{32\pi^2\varepsilon_0^2 \cancel{\hbar^4}} - \frac{me^4}{16\pi^2\varepsilon_0^2\hbar^2}$$

$$\Rightarrow E_0 = \frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2} - \frac{me^4}{16\pi^2\varepsilon_0^2\hbar^2}.$$

$$= \frac{me^4}{\pi^2\varepsilon_0^2\hbar^2}\left(\frac{1}{32} - \frac{1}{16}\right)$$

$$= \frac{me^4}{\pi^2\varepsilon_0^2\hbar^2}\left(\frac{1-2}{32}\right)$$

$$= \boxed{\frac{-me^4}{32\pi^2\varepsilon_0^2\hbar^2}}$$

Thus ground state energy of Hydrogen atom is non-zero.

Q) An electron and a proton are accelerated at a same potential. If their masses are m_e and m_p respectively then calculate the ratio of their de-broglie wavelength.

Q) Estimate the de broglie wavelength for a beam of electron whose energy is 45 eV.

Q) An electron is accelerated by potential of V volt has a debroglie wavelength λ. If the $e^{\ominus}$ is accelerated again by the potential difference of 4V find it's de broglie wavelength.

Q) A proton and a α-particle are accelerated through the same potential difference. Find the ratio of their de-broglie wavelength.

Ans.

(1) m_e, m_p.

electron & proton are accelerated at same potential

$\Rightarrow$ K.E of electron = eV.

K.E of proton = pV.

$$\frac{\lambda_e}{\lambda_p} = \frac{\frac{h}{\sqrt{2m_e eV}}}{\frac{h}{\sqrt{2m_p pV}}} = \frac{\cancel{h}}{\sqrt{\cancel{2}m_e e\cancel{V}}} \times \frac{\sqrt{\cancel{2}m_p p\cancel{V}}}{\cancel{h}}$$

$$= \frac{m_p \times \cancel{p}}{m_e \times \cancel{e}} = \frac{1.6\times10^{-27}}{9.1\times10^{-31}}$$

$$= 0.1836\times10^{4}$$

$$= \boxed{1836}$$

(2) Energy = 45 eV.

$= 45\times1.6\times10^{-19}$ J.

1.3104×10^{-4}

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.626\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times45\times1.6\times10^{-19}}}$$

$$= \frac{6.626\times10^{-34}}{3.61\times10^{-24}} = 1.83\times10^{-10}\,m.$$

$= 1.83\,\text{Å}$.

$V \rightarrow \lambda$.

$4V \rightarrow \lambda = ?$

$$\lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda' = \frac{h}{\sqrt{2me \cdot 4V}} = \frac{h}{\sqrt{4}\sqrt{2meV}} = 2\frac{h}{\sqrt{2meV}} \lambda.$$

$$\Rightarrow \boxed{\lambda' = \frac{\lambda}{2}.}$$

Thus the debroglie wavelength becomes half the original value.

4) $m_p = 1.6 \times 10^{-27}$ kg. charge of α particle $= 2e$

$m_\alpha = 6.64 \times 10^{-27}$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{\frac{h}{\sqrt{2m_p PV}}}{\frac{h}{\sqrt{2m_\alpha \cdot 2eV}}}$$

$$= \frac{\cancel{h}}{\sqrt{\cancel{2}m_p P \cancel{V}}} \times \frac{\sqrt{\cancel{2}m_\alpha 2e \cancel{V}}}{\cancel{h}}$$

$$= \frac{m_\alpha \times \cancel{2e}}{m_p \times \cancel{P}} = 2\frac{m_\alpha}{m_p}$$

Module - 4

Transition from Deterministic to Probabilistic

In quantum mechanics, every physical system is characterised by a wave function which contains all informations for probabilistic description of a system.

Any physical property of a system instead of having exact value, can take a number of allowed values with different probabilities.

So this probabilistic description is the basic characteristic of quantum physics and achieved by the wave function.

Q) What is a wave function?

Ans: Wave functions are the mathematical representation of particles which contain all information required for probabilistic description of the particle.

Characteristics of the Wave Function -

① Wave function is in general a mathematical function of space and time such that -

$$\boxed{\psi(\vec{r},t) = \psi(x,y,z,t) = Ae^{i(\vec{k}\cdot\vec{r}-\omega t)}}$$

② Wave function is in general a complex quantity such that

$$\psi\psi^* = \psi^*\psi = |\psi|^2 \Rightarrow \text{Probability density}$$

$\psi^* \rightarrow$ Complex conjugate of wave fun^n

which determine the probability density and it's a real quantity. $(|\psi|^2)$

(3) ψ must be finite everywhere.

(4) It is a single value function. That means it has one particular value at a point $\vec{r}$ and at any time t. It vanishes at infinity.

(5) The wave function ψ and it's first derivative is continuous everywhere including boundaries.

Application of Schodinger's wave eqn.

(6) The wave func^n ψ satisfies schodinger's wave equation.

(7) ψ must be normalizable or ψ is a normalized wave function.

The condition for normalization is -

$$\int_0^V |\psi|^2 dV = 1$$

i.e the total probality $\left(\int_V |\psi|^2 dV\right)$ is always $= 1$.

Probability Density ($|\psi|^2$)

* It is the probability of finding the particle per unit volume and is given by -

$$\boxed{\psi\psi^* = \psi^*\psi = |\psi|^2}.$$

So, Total probability is $\boxed{\left(\int_0^V |\psi|^2 dV\right) = 1.}$

* Principle of Superposition -

⇒ The wave function representing the actual state of a system is a linear superposition (combination) of different possible allowed states in which the system can exist.

Let $\psi_1, \psi_2, \psi_3, \ldots, \psi_n$ ~~are the dif~~ represent the different allowed states in which the system can exist.

Then - $\boxed{\psi = c_1\psi_1 + c_2\psi_2 + c_3\psi_3 + \cdots + c_n\psi_n}$

$$\Rightarrow \boxed{\psi = \sum_{i=1}^{n} c_i \psi_i}$$

where $c_1, c_2, c_3, \ldots, c_n$ are the coefficients called as the probability amplitude and $\boxed{c_i^2}$ is the probability of the system being in the state ψ_i.

Thus $\boxed{c_1^2 + c_2^2 + c_3^2 + \cdots + c_n^2 = 1}$

Operators, eigen values, eigen functions and Expectation Values.

Operators -

* Any dynamical quantity like position, momentum, energy etc. can be observed / measured and known as observables.

To every observable quantity there exist an operator. So, every physical quantity

is associated with a quantum mechanical operator.

Observables (Physical Quantity)	Operators. 3D	1D
Total Energy (E)	$i\hbar \frac{\partial}{\partial t}$	$i\hbar \frac{\partial}{\partial t}$
Linear Momentum (P)	$-i\hbar \nabla$	$i\hbar \frac{\partial}{\partial x}, -i\hbar \frac{\partial}{\partial y}, -i\hbar \frac{\partial}{\partial z}$
Kinetic energy $\left(\frac{P^2}{2m}\right)$	$\frac{-\hbar^2}{2m} \nabla^2$	$\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2}, \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial y^2}, \frac{-\hbar^2}{2m} \frac{\partial^2}{\partial z^2}$
Potential Energy (V)	$V(r)$	$V(x), V(y), V(z)$
Hamiltonian (H)	$\frac{-\hbar^2}{2m} \nabla^2 + V(r)$	$\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V(x)$, $\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial y^2} + V(y)$, $\frac{-\hbar^2}{2m} \frac{\partial^2}{\partial z^2} + V(z)$

* Operators are written as $\hat{E}, \hat{P}, \ldots$ etc.

Eigen Functions -

* The allowed quantum states of a physical system are called as eigen states and represented by a set of functions called eigen function.

The actual wave function of the system is the linear combination of eigen functions.

If $\psi_1, \psi_2, \psi_3, \ldots, \psi_n$ are the eigen functions of a system with coefficient $C_1, C_2, C_3, \ldots, C_n$; then the wave func^n $\psi = C_1\psi_1 + C_2\psi_2 + C_3\psi_3 + \ldots + C_n\psi_n$

Eigen Values -

* The set of allowed values of a physical quantity for a given system is called eigen values.

Expectation Values -

* In quantum mechanics, every system is described in a probabilistic manner. For a given state of a system, the different eigen values of a quantity occur with different relative probabilities.

If $q_1, q_2, q_3, \ldots, q_n$ are the eigen values of a physical quantity Q and they occur with probabilities $P_1, P_2, P_3, \ldots, P_n$ then the expectation values of Q (weighted average value) represented by

$$\langle Q \rangle = \frac{P_1q_1 + P_2q_2 + P_3q_3 + \ldots + P_nq_n}{P_1 + P_2 + P_3 + \ldots + P_n}$$

$\Rightarrow \langle Q \rangle = p_1 q_1 + p_2 q_2 + p_3 q_3 + \dots + p_n q_n$

$$= \sum_{i=1}^{n} p_i q_i$$

By Integration -

If the system is represented by ψ, then it's complex conjugate is ψ^*.

Ex- let us find the momentum expectation value

$$\Rightarrow \langle p \rangle = \frac{\int_{-\infty}^{\infty} \psi^* \hat{p} \psi \, dx}{\cancel{\int_{-\infty}^{\infty} \psi^* \psi \, dx} \to 1} = \int_{-\infty}^{\infty} \psi^* \hat{p} \psi \, dx$$

Q) (1) The probability that a system can be represented by eigen functions ψ_1, ψ_2, ψ_3 are $\frac{1}{2}, \frac{1}{3}$ & $\frac{1}{6}$ respectively (i) Write the wave func^n for the system.

(ii) If the energy eigen values for the above states are 4 eV, 6 eV and 9 eV then find energy expectation value.

(2) The wave function ψ of a system is a linear combination of the eigen function $\phi_1, \phi_2, \phi_3, \phi_4$ and ϕ_5 and given by $\psi = \frac{1}{\sqrt{3}} \phi_1 + \frac{1}{\sqrt{3}} \phi_2 + \frac{1}{\sqrt{6}} \phi_3 + \frac{1}{\sqrt{24}} \phi_4 + \frac{1}{\sqrt{8}} \phi_5$.

What is the probability of the system being in the state ϕ_3.

③ A particle is observed to have 5 quantum states $\psi_1, \psi_2, \psi_3, \psi_4$ & ψ_5 with relative probabilities 0.2, 0.1, 0.3, 0.2, 0.2 respectively. If corresponding energy eigen values for these states are 2eV, 3eV, 3eV, 1eV and 1eV. Calculate the energy expectation values. Write the wave fun^n

Ans:- Answers.

$P_1 = \frac{1}{2}, P_2 = \frac{1}{3}, P_3 = \frac{1}{6}$

$C_1 = \frac{1}{\sqrt{2}}, C_2 = \frac{1}{\sqrt{3}}, C_3 = \frac{1}{\sqrt{6}}$

$\psi = C_1\psi_1 + C_2\psi_2 + C_3\psi_3$

① $(C)^2 = (P)$ ⇒ $C = \sqrt{P}$

⇒ $\boxed{\psi = \frac{1}{\sqrt{2}}\psi_1 + \frac{1}{\sqrt{3}}\psi_2 + \frac{1}{\sqrt{6}}\psi_3}$

Energy eigen values q_1, q_2, q_3 are 4eV, 6eV & 9eV

$E_1 = 4eV, E_2 = 6eV, E_3 = 9eV$

⇒ $\langle E \rangle = P_1 E_1 + P_2 E_2 + P_3 E_3$

$= \frac{1}{2} \cdot 4 + \frac{1}{3} \cdot 6 + \frac{1}{6} \times 9$

$= 2 + 2 + \frac{3}{2} = \frac{4+4+3}{2} = \frac{11}{2} = \boxed{5.5\ eV}$

② Probability of system being in state ϕ_3

$= (C_3)^2 = \left(\frac{1}{\sqrt{6}}\right)^2 = \boxed{\frac{1}{6}}$

③ Wave fun^n $\psi = C_1\psi_1 + C_2\psi_2 + C_3\psi_3 + C_4\psi_4 + C_5\psi_5$

⇒ $\boxed{\psi = \frac{1}{\sqrt{5}}\psi_1 + \frac{1}{\sqrt{10}}\psi_2 + \sqrt{\frac{3}{10}}\psi_3 + \frac{1}{\sqrt{5}}\psi_4 + \frac{1}{\sqrt{5}}\psi_5}$

$$\langle E \rangle = p_1q_1 + p_2q_2 + p_3q_3 + p_4q_4 + p_5q_5$$

$$= \frac{2}{10} \times 2 + \frac{1}{10} \times 3 + \frac{3}{10} \times 3 + \frac{2}{10} \times 1 + \frac{2}{10} \times 8$$

$$= \frac{4}{10} + \frac{3}{10} + \frac{9}{10} + \frac{2}{10} + \frac{2}{10}$$

$$= \frac{20}{10} = \boxed{2eV}$$

Integration Type Queen.

(4) The wave function $\psi(x)$ is given by -

$$\psi(x) = A_n \sin \frac{2n\pi x}{L}$$ in the region

$0 <= x <= L$. Find the normalisation constant and normalised wave func^n.

Ans- The normalisable cond^n is-

$$\int_0^L \psi^* \psi \, dx = 1$$

$$\psi = A_n \sin \frac{2n\pi x}{L}.$$

$$\psi^* = A_n \sin \frac{2n\pi x}{L}.$$

$$\Rightarrow \int_0^L A_n^2 \sin^2 \frac{2n\pi x}{L} dx = 1.$$

Ans $\rightarrow A_n = \sqrt{\frac{2}{L}}$

(1) The wave function for a quantum mechanical particle constrained to move in the region $-\frac{\pi}{2} \leq x < \frac{\pi}{2}$ is given by $\psi = A\sin^2 x$. Normalise the wave function and obtain the normalisation constant.

(2) Calculate the probability of finding the particle in the region $2 < x < 4$ if wave fun^n of particle is given by $\psi = 0.25 e^{2ix}$.

(3) The normalised wave function for certain particle is $\psi(x) = \sqrt{\frac{3}{\pi}} \cos x$ in the region $\frac{-\pi}{2} < x < \frac{\pi}{2}$. Calculate probability of finding the particle between $0 < x < \pi/4$.

(4) Calculate the expectation value of linear momentum for the wave function $\psi(x) = \sqrt{\frac{2}{L}} \frac{\sin \pi x}{L}$ in the region $0 < x < L$ and $\psi(x) = 0$ for $|x| > L$.

(5) Normalise the wave function in one dimension for $\psi(x) = Ae^{-\alpha x}$ for $x > 0$ and $\psi(x) = Ae^{\alpha x}$ for $x < 0$. Here α is a positive constant.

Answers-

(1) $\psi = A\sin^2 x$, $\psi^* = A\sin^2 x$.

Condition for normalisation-

$$\int_{-\pi/2}^{\pi/2} \psi\psi^* \, dx = 1$$

$$\Rightarrow \int_{-\pi/2}^{\pi/2} A\sin^2 x \cdot A\sin^2 x \, dx = 1$$

$$\Rightarrow A^2 \int_{-\pi/2}^{\pi/2} \sin^4 x \, dx = 1$$

$$\Rightarrow A^2 \int_{-\pi/2}^{\pi/2} (\sin^2 x)^2 \, dx = 1.$$

$$\frac{1-\cos 2x}{2}$$

$$\Rightarrow A^2 \int_{-\pi/2}^{\pi/2} \left(\frac{1-\cos 2x}{2}\right)^2 dx = 1$$

$\sin^2 x + \cos^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \cos^2$

$\sin 2x = 2\sin$

$$\Rightarrow A^2 \int_{-\pi/2}^{\pi/2} \frac{1}{2} \, dx - A^2 \int_{-\pi/2}^{\pi/2} \frac{\cos 2x}{2} \, dx = 1$$

$$\Rightarrow \frac{A^2}{2} \int_{-\pi/2}^{\pi/2} dx - \frac{A^2}{2} \int_{-\pi/2}^{\pi/2} \cos 2x \, dx = 1$$

$$\Rightarrow \frac{A^2}{2} [\pi] - \frac{A^2}{2}$$

$$\Rightarrow A^2 \int_{-\pi/2}^{\pi/2} \frac{(1-\cos 2x)^2}{4} \, dx = 1.$$

$$\Rightarrow \frac{A^2}{4} \int_{-\pi/2}^{\pi/2} (1 + \cos^2 2x - 2\cos x) \, dx = 1.$$

$$\Rightarrow \frac{A^2}{4} \int_{-\pi/2}^{\pi/2} 1 \, dx + \frac{A^2}{4} \int_{-\pi/2}^{\pi/2} \cos^2 2x \, dx - \cancel{2} \cdot \frac{A^2}{\cancel{4}\,2} \int_{-\pi/2}^{\pi/2} \cos x \, dx = 1$$

$\frac{1+\cos 4x}{2}$

$$\Rightarrow \frac{A^2}{4} \times \pi + \frac{A^2}{4} \int_{-\pi/2}^{\pi/2} \left(\frac{1+\cos 4x}{2}\right) dx - \frac{A^2}{2} \cdot [\sin x]_{-\pi/2}^{\pi/2} = 1$$

$$\Rightarrow \frac{A^2 \pi}{4} + \frac{A^2}{4 \times 2} \int_{-\pi/2}^{\pi/2} dx + \frac{A^2}{4 \cdot 2} \int_{-\pi/2}^{\pi/2} \cos 4x \, dx - \frac{A^2}{\cancel{2}} \cdot \cancel{2} = 1$$

$$\Rightarrow \frac{A^2 \pi}{4} + \frac{A^2}{8} \times \pi + \frac{A^2}{8} [\qquad = 1$$

(2) $\psi = 0.25 e^{2in}$ & $\psi^* = 0.25 e^{-2in}$.

$$P = \int_2^4 \psi^* \psi \, dn$$

$$= \int_2^4 0.25 \cancel{e^{2in}} \cdot 0.25 \cancel{e^{-2in}} \, dn$$

$$= \int_2^4 \frac{25}{100} \cdot \frac{25}{100} \, dn$$

$$= \left(\frac{25}{100}\right)^2 \int_2^4 dn$$

$$= \left(\frac{25}{100}\right)^2 \times (4-2)$$

$$= \left(\frac{25}{100}\right)^2 \times 2$$

$$= \frac{625}{10000} \times 2$$

$$= 0.0625 \times 2$$

$$= 0.1250$$

So, probability of finding the particle in the above region is 12.5%

8) $\psi(x) = \sqrt{\frac{3}{\pi}} \cos x \qquad \psi^*(x) = \sqrt{\frac{3}{\pi}} \cos x$

$$P = \int_0^{\pi/4} \psi \psi^* dx = \int_0^{\pi/4} \sqrt{\frac{3}{\pi}} \cos x \cdot \sqrt{\frac{3}{\pi}} \cos x \, dx$$

$$= \frac{3}{\pi} \int_0^{\pi/4} \cos^2 x \, dx = \frac{3}{\pi} \int_0^{\pi/4} \frac{1+\cos 2x}{2} dx$$

$$\boxed{P \simeq 61.3\%}$$

$$\cancel{= \frac{3}{\pi} \int_0^{\pi/4} \frac{1-\cos 2x}{2} dx}$$

$$\cancel{= \frac{3}{\pi} = \frac{3}{\pi} \int_0^{\pi/4} 1 - \cos^2 x \, dx}$$

$$\cancel{= \frac{3}{\pi} \int_0^{\pi/4} dx - \int_0^{\pi/4} \frac{1-\cos 2x}{2} dx}$$

$$\cancel{= \frac{3}{\pi} \times \frac{\pi}{4} - \left[\frac{1}{2} \int_0^{\pi/4} dx - \frac{1}{2} \int_0^{\pi/4} \cos 2x \, dx \right]}$$

$$\cancel{= \frac{3}{4} - \left[\frac{1}{2} \right.}$$

9) $\psi(x) = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L} \qquad \psi^* = \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L} \qquad -i\hbar \frac{\partial}{\partial x}$

$$\langle P \rangle = \int_0^L \psi^* \hat{P} \psi \, dx$$

$$= \int_0^L \sqrt{\frac{2}{L}} \sin \frac{\pi x}{L} \left\{ -i\hbar \frac{\partial}{\partial x} \left(\sqrt{\frac{2}{L}} \sin \frac{\pi x}{L} \right) \right\} dx$$

$$= -i\hbar \frac{2}{L} \int_0^L \sin \frac{\pi x}{L} \cos \frac{\pi x}{L} \cdot \frac{\pi}{L} \, dx$$

$$= -i\hbar \frac{2}{L} \cdot \frac{\pi}{L} \int_0^L \sin \frac{\pi x}{L} \cos \frac{\pi x}{L} \, dx$$

$$= -\frac{i\hbar 2\pi}{L^2}\int_0^L \sin\frac{\pi x}{L}\cos\frac{\pi x}{L}\,dx$$

$$= -\frac{\cancel{2}i\hbar\pi}{L\times\cancel{L}}\int_0^L 2\sin\frac{\pi x}{L}\cdot\cos\frac{\pi x}{L}\,dx$$

$\sin 4x = -\frac{\cos 4x}{4}$

$$= -\frac{i\hbar\pi}{L}\int_0^L \sin 2\frac{\pi x}{L}\,dx \qquad = \frac{i\hbar\cancel{\pi}}{\cancel{L}}\cdot\frac{\cancel{L}}{2\cancel{\pi}}\left(\cos\frac{2\pi x}{L}\right)_0^L$$

$$= -\frac{i\hbar\pi}{L}\cdot -\left(\cos\frac{2\pi x}{\cancel{L}}\cdot\frac{\cancel{L}}{2\pi}\right)_0^L = \frac{i\hbar}{2}\cancel{\cos(2\pi-0)}$$

$$= -\frac{i\hbar\pi}{L}\cdot -(\cos 2x)_0^L \qquad = \boxed{0}$$

$$= \frac{i\hbar\pi}{L}\cos 2 - \cos 0$$

(5) H.W

Expression for Momentum Operator and Energy Operator.

Consider the wave propagate along x axis within 1 dimen

$$P = \frac{h}{\lambda} = \frac{h}{2\pi}\times\frac{2\pi}{\lambda} = \hbar k. \quad \text{—(1)}$$

$$E = h\nu = \frac{h}{2\pi}\times 2\pi\nu = \hbar\omega. \quad \text{—(2)}$$

$$\psi = Ae^{i(kx-\omega t)} \quad \text{—(3)}$$

$$\text{i)}\ \frac{\partial\psi}{dx} = (ik)\underbrace{Ae^{i(kx-\omega t)}}_{\psi} = ik\psi$$

$$= i\frac{P}{\hbar}\psi$$

$$\Rightarrow p\psi = \frac{\hbar}{i}\frac{\partial\psi}{\partial x}$$

$$\Rightarrow \frac{i\hbar}{i^2}\frac{\partial\psi}{\partial x} = \boxed{-i\hbar\frac{\partial\psi}{\partial x}}$$

So, $\boxed{\hat{p} = -i\hbar\frac{\partial\psi}{\partial x}}$

<u>Energy operator.</u>

$$\psi = Ae^{i(kx-\omega t)}$$

$$\Rightarrow \frac{\partial\psi}{\partial t} = (-i\omega)\underbrace{Ae^{i(kx-\omega t)}}_{\psi}$$

$$\Rightarrow \frac{\partial\psi}{\partial t} = -i\omega\psi.$$

$$\Rightarrow \frac{\partial\psi}{\partial t} = -i\frac{E}{\hbar}\psi$$

$$\Rightarrow E\psi = \frac{-\hbar\times i}{i\times i}\frac{\partial\psi}{\partial t}.$$

$$\Rightarrow E\psi = \frac{-\hbar i}{i^2}\frac{\partial\psi}{\partial t}.$$

$$\Rightarrow \boxed{\hat{E} = i\hbar\frac{\partial\psi}{\partial t}} \quad (1\text{-}D)$$

① Schodinger's Wave Equation

1-D Time dependent Schondinger's wave eqn for free particle along X axis.

Let the wave func^n associated with the particle be represented by $\psi(x,t) = Ae^{i(kx-\omega t)}$

$$\frac{\partial \psi}{\partial x} = ikAe^{i(kx-\omega t)} = ik\psi \quad \text{--- (1)}$$

$$\frac{\partial \psi}{\partial t} = -i\omega Ae^{i(kx-\omega t)} = -i\omega\psi \quad \text{--- (2)}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{i^2}{-1}k^2 Ae^{i(kx-\omega t)} = -k^2\psi \quad \text{--- (3)}$$

$$P = \frac{h}{\lambda} = \hbar k$$

$$E = h\nu = \hbar\omega.$$

From (2) -

$$\boxed{\psi = -\frac{1}{i\omega}\frac{\partial \psi}{\partial t}}$$

Substituting in (3) -

$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} = \frac{+k^2}{+i\omega}\frac{\partial \psi}{\partial t}$$

$$\Rightarrow -\frac{\omega}{k^2}\frac{\partial^2 \psi}{\partial x^2} = -\frac{1}{i}\frac{\partial \psi}{\partial t}$$

$$\Rightarrow -\frac{\omega}{k^2}\frac{\partial^2 \psi}{\partial x^2} = \frac{i^2}{i}\frac{\partial \psi}{\partial t} \qquad (i^2 = -1)$$

$$\Rightarrow -\frac{\omega}{k^2}\frac{\partial^2 \psi}{\partial x^2} = i\frac{\partial \psi}{\partial t}$$

Multiplying by $\hbar$ -

$$\Rightarrow -\frac{\hbar\omega}{k^2}\frac{\partial^2 \psi}{\partial x^2} = i\hbar\frac{\partial \psi}{\partial t} \quad \text{--- (4)}$$

$E = \frac{P^2}{2m}$

$$\Rightarrow \boxed{\hbar\omega = \frac{\hbar^2 k^2}{2m}}$$

Substituting $\hbar\omega$ in (4) -

$$\Rightarrow -\frac{\hbar^2 \cancel{k^2}}{2m\cancel{k^2}} \frac{\partial^2 \psi}{\partial x^2} = i\hbar \frac{\partial \psi}{\partial t}.$$

$$\Rightarrow \boxed{i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2}} \quad \text{—(5)}$$

(1D Time dependent Schodinger's wave eq^n for free particle)

(2) For any particle -

Let the particle moves under a potential $V(x)$.

$$\Rightarrow E = \frac{P^2}{2m} + V(x). \quad \text{—(6)}$$

$$\Rightarrow \boxed{i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi(x)}$$

(1-D Time dependent Schodinger's wave eq^n for any particle)

In 3-D -

$$\boxed{i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi} \quad \text{—(7)}$$

(3-D time dependent Schodinger's wave eq^n for free particle).

In 3D for any particle -

$$\Rightarrow \boxed{i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \nabla^2 \psi + V\psi(\vec{r})} \quad \text{—(8)}$$

(3-D time dependent Schodinger's wave eq^n for any particle).

1-D Time Independent Schodinger's Wave Eq^n for free particle :-

As we know-

$$\boxed{\hat{E} = i\hbar \frac{\partial}{\partial t}}$$

So,

$$\boxed{E\psi = i\hbar \frac{\partial \psi}{\partial t}}$$

So, substituting this in eq^n (5) -

$$\Rightarrow E\psi = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} \quad \Rightarrow \frac{2mE\psi}{\hbar^2} = -\frac{\partial^2 \psi}{\partial x^2}$$

$$\Rightarrow \boxed{\frac{2mE\psi}{\hbar^2} + \frac{\partial^2 \psi}{\partial x^2} = 0}$$

(1-D time independent S.W.E for a free particle along x axis)

For any particle -

$$\Rightarrow E\psi = \frac{-\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(\psi)$$

$$\Rightarrow (E-V)\psi = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2}$$

$$\Rightarrow \frac{2m}{\hbar^2}(E-V)\psi = -\frac{\partial^2 \psi}{\partial x^2}$$

$$\Rightarrow \boxed{\frac{2m}{\hbar^2}(E-V)\psi + \frac{\partial^2 \psi}{\partial x^2} = 0}$$

(1-D time independent S.W.E for any particle along x axis)

~~* Any quantum~~

For 3D -

(Time Independent)

$$\cancel{\nabla^2 E\psi = -\frac{\hbar^2}{2m}}$$

$$\boxed{\nabla^2\psi + \frac{2mE\psi}{\hbar^2} = 0}$$

$\Downarrow$ ~~any particle~~ free particle

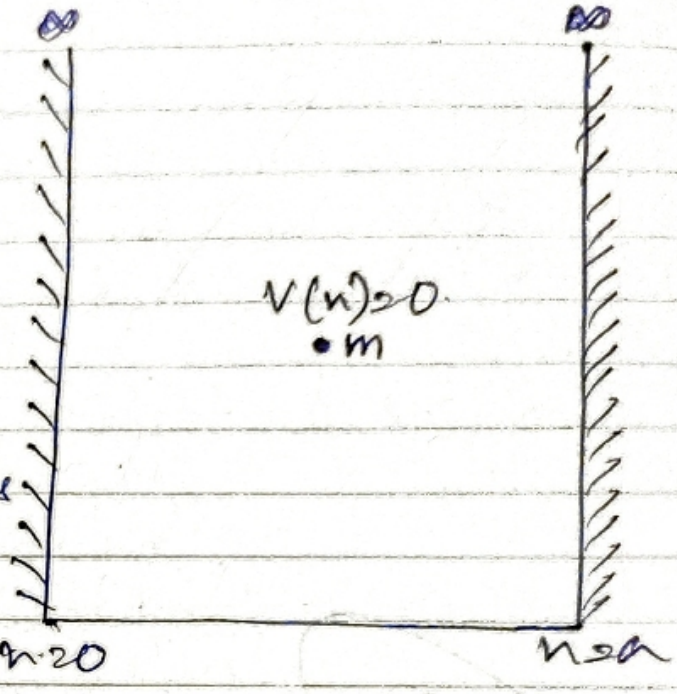
$$\boxed{\nabla^2\psi + \frac{2m(E-V)\psi}{\hbar^2} = 0}$$

$\Rightarrow$ any particle

Application of Schodinger's Wave Equation.

① Potential Well Box (Particle in a well dimensional well / box of infinite height).

Let us consider the case of a particle of mass m moving along x axis between the two walls at $x=0$ and at $x=a$.



The particle is free to move between the walls i.e $V(x)=0$.

When the particle strikes any of the walls, it is reflected back immediately as the walls are perfectly rigid.

The potential func^n is defined as -

Imp

$$\begin{cases} V(x) = 0 & 0 < x < a \text{ (within the box)} \\ V(x) = \infty & x \leq 0 \text{ and } x \geq a \text{ (outside the box)} \end{cases}$$

* Since the height of the potential well is infinite, so the probability of finding the particles at the rigid walls and outside the rigid wall vanishes.

Thus, inside the walls, the Schodinger's wave eq^n can be written as -

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2mE\psi}{\hbar^2} = 0 \quad \text{— ①} \quad (\text{as } V(x) = 0)$$

$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + k^2 \psi = 0$ — (2) where $k = \sqrt{\frac{2mE}{\hbar^2}} = \frac{\sqrt{2mE}}{\hbar}$;

The solution of eq^n (2) can be written as $\boxed{\psi(x) = A \sin kx + B \cos kx}$ where A and B are arbitary constant and can be found out by using boundary conditions

$$\Rightarrow \boxed{\psi(x) = A \sin kx + B \cos kx} \quad \text{— (3)}$$

$\psi(x) = \begin{cases} 0 \\ \text{when} \\ x = 0 \end{cases}$ and $\psi(x) = \begin{cases} 0 \\ \text{when} \\ x = a \end{cases}$

$\Downarrow$ $\Downarrow$

$\Rightarrow \boxed{B = 0}$ $\qquad 0 = A \sin ka + \cancel{B \cos ka}^{\,0}$

$\Rightarrow A \sin ka = 0$

$\sin ka = 0$ (as $A \neq 0$)

$\Rightarrow \boxed{ka = \pm n\pi}$

$n \neq 0, n = 1, 2, 3, 4$

Energy Eigen Values:

$ka = \pm n\pi \Rightarrow k = \frac{n\pi}{a} \Rightarrow k^2 = \frac{n^2\pi^2}{a^2}$. But $k = \frac{\sqrt{2mE}}{\hbar}$

$\Rightarrow k^2 = \frac{2mE}{\hbar^2}$

$\Rightarrow \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{a^2} \Rightarrow E = \frac{\hbar^2}{2m} \cdot \frac{n^2\pi^2}{a^2}$

$\Rightarrow E = \left(\frac{h}{2\pi}\right)^2 \times \frac{1}{2m} \times \frac{n^2\pi^2}{a^2}$

$\Rightarrow E = \frac{h^2}{4\pi^2} \cdot \frac{n^2\pi^2}{2ma^2} \Rightarrow \boxed{E_n = \frac{h^2}{8ma^2} \cdot n^2}$

$n = 1, 2, 3, 4$

$E = \frac{p^2}{2m} = \frac{h^2}{8 \times 4 a^2} \cdot n^2$ $\quad p^2 = \frac{h^2}{4a^2} \cdot n^2$ $\quad \boxed{p = \frac{h}{2a} \cdot n}$ (Momentum is quantised)

$\Rightarrow E_1 = \frac{h^2}{8ma^2}$ (n = 1, ground state)

$\Rightarrow E_2 = \frac{h^2}{8ma^2} \cdot 4$ (n = 2, 1st excited state)

$\Rightarrow E_3 = \frac{h^2}{8ma^2} \cdot 9$ (n = 3, 2nd excited state)

So, Energy is quantised.

Q) Show that momentum is quantised.

(2) Debroglie's Wave length -

$P = h/\lambda$

$\Rightarrow P = \hbar k = \hbar \frac{n\pi}{a}$

$= \frac{h}{2\pi} \times \frac{n\pi}{a} = \frac{h}{2} \cdot \frac{n}{a}$

$\Rightarrow \frac{h}{\lambda} = \frac{h}{2} \cdot \frac{n}{a}$

$\Rightarrow \boxed{\lambda = \frac{2a}{n}}$

$\Rightarrow \boxed{a = \frac{n\lambda}{2}}$ $\quad \Rightarrow \boxed{a = \frac{n}{2} \cdot \lambda}$ (Rem).

Width

Wavelength

Thus, (Width of the box is half integral multiple of debroglie wavelength.)

So, the allowed ~~level~~ momentum states are possible for the energy, for which the width of the

potential well is equal to half integral multiple of wavelength.

Eigen Function -

The normalization cond^n is -

$$\int |\psi|^2 dx = 1.$$

$$\psi(x) = A \sin kx \quad (as\ B = 0)$$

$$\Rightarrow \int_0^a A^2 \sin^2 kx\, dx = 1.$$

$$\Rightarrow A^2 \int_0^a \left(\frac{1-\cos 2kx}{2}\right) dx = 1.$$

$$\Rightarrow \frac{A^2}{2}\left[\int_0^a dx - \int_0^a \cos 2kx\, dx\right] = 1$$

$$\Rightarrow \boxed{A = \frac{\sqrt{2}}{a}} \quad (Ans)$$

Now, the normalised wave function is -

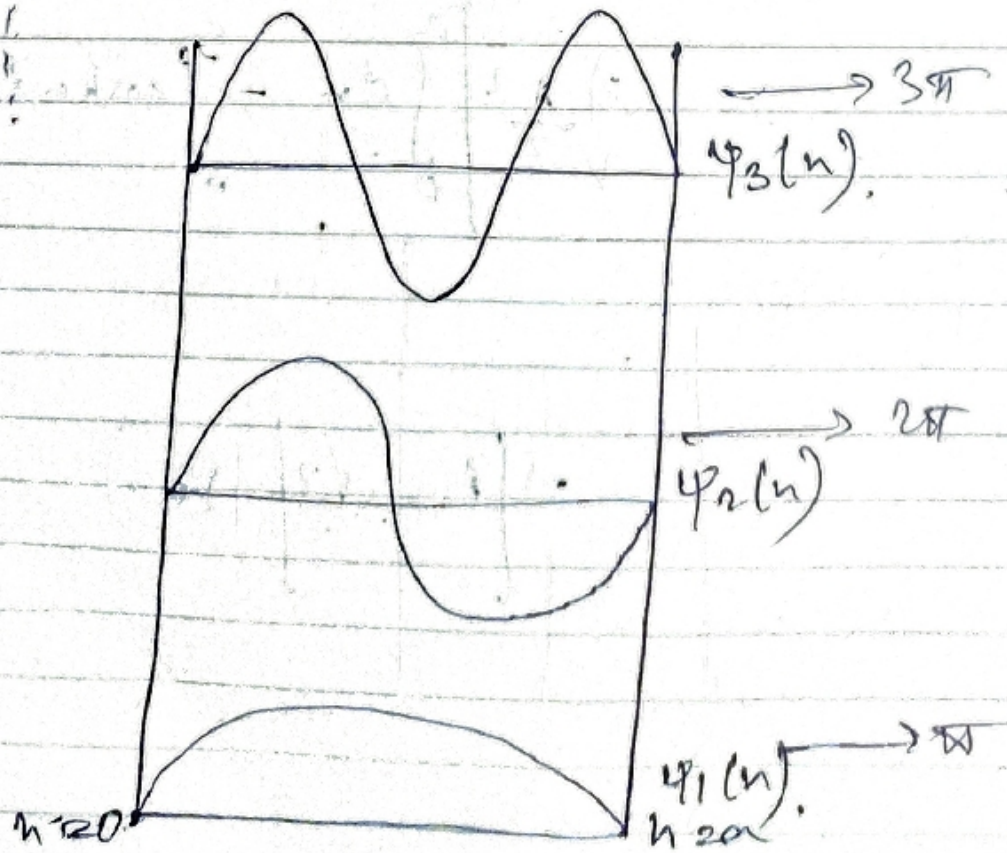
$$\boxed{\psi(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x}$$ $k = \frac{n\pi}{a}$ & $A = \sqrt{\frac{2}{a}}$

$$\boxed{\psi_n(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi}{a} x}$$

$\psi_1(x) = \sqrt{\frac{2}{a}} \sin \frac{\pi x}{a}$ (Eigen func^n for ground state)

$\psi_2(x) = \sqrt{\frac{2}{a}} \sin \frac{2\pi}{a} x$ (Eigen func^n for 1st excited state)

$\psi_3(x) = \sqrt{\frac{2}{a}} \sin \frac{3\pi}{a} x$ (Eigen func^n for 2nd excited state)

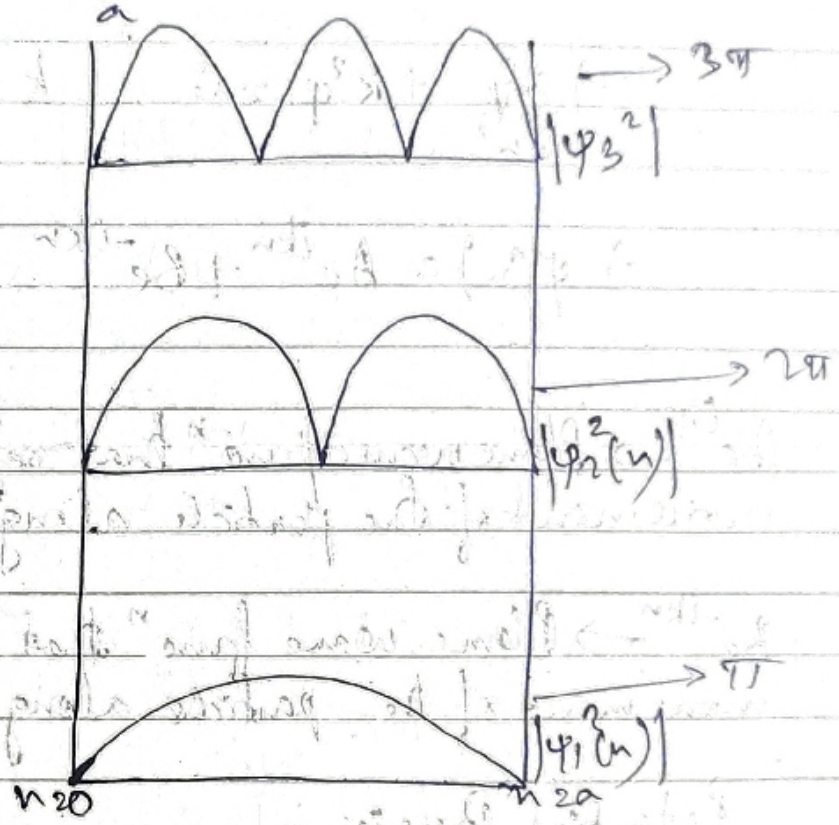


$$|\psi_1^2| = \frac{2}{a}\sin^2\frac{\pi x}{a}$$

$$|\psi_2^2| = \frac{2}{a}\sin^2\frac{2\pi x}{a}$$

$$|\psi_3^2| = \frac{2}{a}\sin^2\frac{3\pi x}{a}$$

$$\Rightarrow |\psi_n^2| = \frac{2}{a}\sin\frac{n\pi x}{a}$$



Discussion – (Short Questions)

(i) The ground state energy $E_1 = \frac{h^2}{8ma^2}$ which is the minimum energy of particle & nonzero.

(ii) $E_n = E_1 \cdot n^2$, So the energy of the higher allowed levels are the multiple of E_1 and $\boxed{E_n \propto n^2}$. So, the values of the energy lying between the allowed states are forbidden.

(iii) The energy levels are not equispaced.

(iv) The spacing between two successive levels increases with increase of n.

Force free particle.

* Let the particle be freely move along x axis
So, Schrodinger's Eq^n is-

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2mE\psi}{\hbar^2} = 0$$

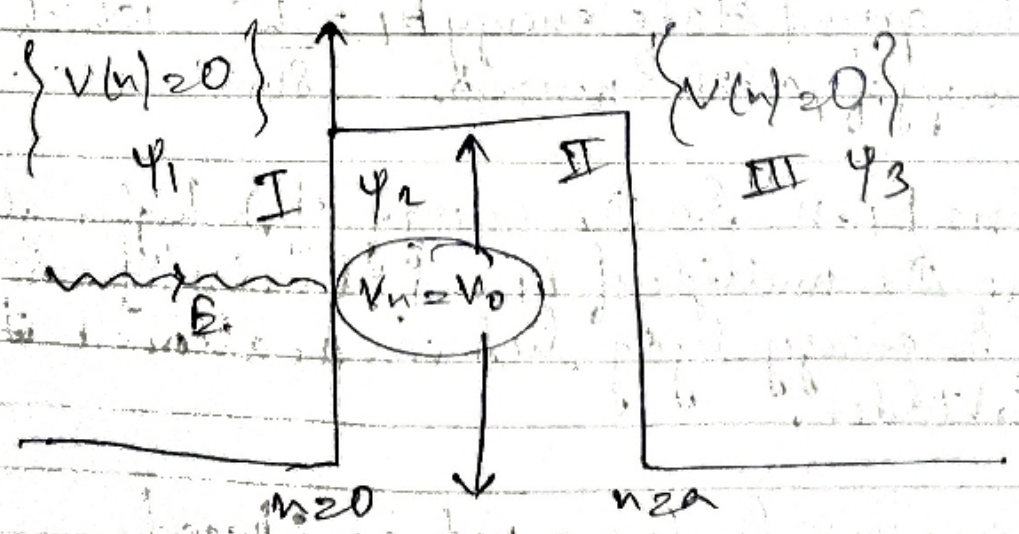
$$\Rightarrow \frac{\partial^2 \psi}{\partial x^2} + k^2\psi = 0 \qquad k = \frac{\sqrt{2mE}}{\hbar}$$

$$\Rightarrow \psi(x) = Ae^{ikx} + Be^{-ikx}$$

$Ae^{ikx} \rightarrow$ Plane wave func^n that represents the movement of the particle along positive x axis.

$Be^{-ikx} \rightarrow$ Plane wave func^n that represents the movement of the particle along negative x axis.

Potential Barrier -



* It is a region of finite height and finite width in which the potential is significantly higher than the points either side of it, so that the particle requires energy to pass through it.

Imp

The potential function is defined as

$$V(x) = 0 \quad \text{for } x < 0 \text{ and } x > a$$
$$V(x) = V_0 \quad \text{for } 0 \leq x \leq a$$

* Classical physics tells particle cannot cross through the barrier and total reflection occurs as $E < V_0$. but quantum mechanics predicts that, the particle have a non-zero probability of crossing the barrier and will reach at region (III) even if it's energy is less than the height of the barrier which is called as quantum mechanical tunneling.

For $E < V_0$

For region-I :-

The Schrodinger's wave eq^n (Time Independent) is -

$$\frac{\partial^2 \psi_1}{\partial x^2} + \frac{2mE\psi_1}{\hbar^2} = 0 \quad \text{--- (1)}$$

$$\Rightarrow \frac{\partial^2 \psi_1}{\partial x^2} + k^2\psi = 0 \quad \text{--- (2)} \qquad \text{where } k = \frac{\sqrt{2mE}}{\hbar}$$

The sol^n of eq^n (2) is -

$$\boxed{\psi_1(x) = Ae^{ikx} + Be^{-ikx}}$$

$Ae^{ikx} \rightarrow$ Incident wave in region I.

$Be^{-ikx} \rightarrow$ Reflected wave in region II.

A and B are the arbitrary constants and can be found out by boundary conditions.

For region II -

Time independent Schrodinger's Wave eq^n is -

$$\frac{\partial^2 \psi_2}{\partial x^2} - \frac{2m(V_0 - E)}{\hbar^2}\psi_2 = 0$$

$$\Rightarrow \frac{\partial^2 \psi_2}{\partial x^2} - \alpha^2 \psi_2 = 0 \quad \text{--- (3)} \qquad \text{where } \left(\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}\right)$$

Solution of eq^n (3) -

$$\psi_2(x) = Ce^{\alpha x} + De^{-\alpha x}$$

$Ce^{\alpha x} \to \infty$ when $x \to \infty$

$\Rightarrow$ Doesn't exist (as wave fun^n vanishes at ∞)

$$\Rightarrow \boxed{\psi_2 = De^{-\alpha x}}$$

for region - III -

$$\Rightarrow \frac{\partial^2 \psi_3}{\partial x^2} + \frac{2mE}{\hbar^2}\psi_3 = 0$$

$$\Rightarrow \frac{\partial^2 \psi_3}{\partial x^2} + k_1 \psi_3 = 0 \quad \text{— (4)} \qquad k_1 = \frac{\sqrt{2mE}}{\hbar}$$

The sol of (4) -

$$\psi_3(x) = Fe^{ikx} + Ge^{-ikx}$$

Not exist (as there is no reflected rays due to the absence of any boundary)

$$\Rightarrow \boxed{\psi_3(x) = Fe^{ikx}}$$

Transmitted wave.

Using the boundary condition the arbitary constant A, B, D and F can be found out. Then the transmission coefficient

$$\boxed{T = 16\frac{E}{V_0}\left(1-\frac{E}{V_0}\right)e^{-2\alpha a} \simeq \boxed{e^{-2\alpha a}} \text{ (For solving numericals)}}$$

where $\left(\alpha = \frac{\sqrt{2m(V_0-E)}}{\hbar}\right)$

where $a \rightarrow$ dimensions / width of the barrier.

The transmission probability increases with the decrease in height V_0 and the width 'a' of the potential barrier.

Quantum Mechanical Tunneling / Tunneling Effect -

* It is the penetration of the particle of lower energy to a region of higher potential with finite probability and escaping them from the potential region is called as tunneling.

* The phenomenon of barrier penetration or quantum mechanical tunneling suggest that microscopic particle trapped in a deep potential have a non zero probability of escaping them from the potential well even if they do not have enough energy to climb over the wave.

Eg of tunneling -

1) Alpha particle scattering
2) Nuclear fusion.

Numericals -

① Electrons with energy 3eV are incident on a potential barrier of 10eV hight and 4A° wide. Find the transmission probability. $(T = e^{-2\alpha a})$. $\hbar = 1.054 \times 10^{-34}$

② If 20×10^{-6} electrons with energy 2eV are incident on a potential barrier of 8eV high and 0.5 nm width then calculate how many electrons will tunnel through the barrier.

Answers -

①

$E = 3eV \times 1.6 \times 10^{-19}$ J
$V_0 = 10eV \times 1.6 \times 10^{-19}$ J
$a = 4A° = 4 \times 10^{-10}$ m

$\hbar = 1.054 \times 10^{-34}$
$m = 9.1 \times 10^{-31}$

$(V_0 - E) = 7 \times 1.6 \times 10^{-19}$ J

$\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar} = 1.354 \times 10^{10}$

$T = e^{-2\alpha a} = e^{-2 \times 1.354 \times 10^{10} \times 4 \times 10^{-10}} = \boxed{1.97 \times 10^{-5}}$

② $E = 2eV$ $= \boxed{\cancel{3.8 \times 10^{-3}}}$

$V_0 = 8eV$

$a = 0.5nm$

$m = 9.1 \times 10^{-31}$ kg

$(V_0 - E) = 6 \times 1.6 \times 10^{-19}$ J

$\hbar = 1.054 \times 10^{-34}$

$\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar} = \frac{\sqrt{18.2 \times 10^{-31} \times 9.6 \times 10^{-19}}}{1.054 \times 10^{-34}}$

$\alpha = 12.54 \times 10^{9}$ $= \cancel{\frac{\sqrt{3.56 \times 10^{-48}}}{1.054 \times 10^{-34}}}$

$a = 0.5nm$ $= \cancel{\frac{1.88 \times 10^{-24}}{1.054 \times 10^{-34}}}$

$T = 3.58 \times 10^{-6}$

$n_{inc} = 20 \times 10^{-6}$

$n_{Transm} = n_{inc} \times T$

$= 20 \times 10^{-6} \times 3.58 \times 10^{-6}$

$= \boxed{71.6}$

Module - 5

LASER AND FIBER OPTICS -

LASER -> Light Amplification By Stimulated Emission Of Radiation.

Laser ->

* A laser is a device which amplifies electromagnetic radiation and generates extreame intense, coherent, monochromatic and directional radiations.

Q. Characteristics Of the laser light -

(i) They are highly directional

(ii) They are monochromatic

(iii) The laser beam is spatially and temporarily coherent to an extraordinary degree. So interference and diffraction effect can be observed by taking two independent laser beams.

(iv) The laser beams are highly intense as compared to ordinary light.

Q. Difference between Ordinary light and laser light.

=> Spontaneous Emission

=> Stimulated Emission

=> Meta Stable State

=> Population Inversion

=> Pumping

=> ~~Spontaneous Emission~~

Atomic transition -

There are three kinds of atomic transition.

(i) Spontaneous Emission.
(ii) Stimulated absorption
(iii) Stimulated Emission.

Difference between Spontaneous emission and stimulated emission

Ans-

Spontaneous Emission	Stimulated Emission
(i) The atoms in the excited state at higher energy level may drop to lower energy level by emitting photon without any external provocation.	(i) The atoms in the excited state at higher energy level may force to drop to lower energy level by emitting photons with external provocation (by incidence photons).
(ii) The atom emits the electromagnetic waves has no definite phase or directional relation with photons emitted by another atom.	(ii) The emitted photon is in phase with that of the incident photon and also in the same direction with incident photons.
(iii) No coherent sources can be produced by this method.	(iii) Coherent sources are produced by this method.

Meta Stable State -

* Meta stable state is an intermediate state in between excited state and ground state and it is more closer to the excited state.
* The lifetime of atoms in this state are from 10^{-5} sec to 10^{-3} sec.
* In excited state the lifetime of atoms is 10^{-8} sec. So it is insufficient for the interaction of incident photons with the atoms before stimulated emission.
* So for production of laser meta-stable state is required.

Population Inversion -

* In a normal condition or in thermal equilibrium the density of atoms or population of atoms decreases if we go from lower to higher energy levels.
* But when the population of higher energy level is more than that of lower energy level then it is called as population inversion for the system or it is called as an active system.

Probability of Spontaneous emission rate Vs Stimulated Emission rate & Einstein's coefficients -

Let us consider a system or assembly of atoms in thermal equilibrium i.e at temperature T (in Kelvin) with radiation frequency ν and energy density $\rho(\nu)$

Under this condition let all the transition process occur.

1. Stimulated Absorption (Upward direction) ↑ high
2. Spontaneous Emission (Downward direction) ↓
3. Stimulated Emission. (Downward direction) ↓

Let -

$N_1 \rightarrow$ Number of Atoms per unit volume in lower energy level.

$E_1 \rightarrow$ Energy of atoms in lower energy level.

$N_2 \rightarrow$ Number of Atoms per unit volume in higher energy level.

$E_2 \rightarrow$ Energy of atoms in higher energy level.

If $f(\nu) \rightarrow$ Energy density of interacting photons.

$n \rightarrow$ Number of interacting photons per unit volume

$f(\nu) = n h\nu$ and $(E_2 - E_1) = h\nu$.

Case-I

Stimulated Absorption

Stimulated absorption rate $\propto N_1$

$\propto f(\nu)$

$= B_{12} N_1 f(\nu)$

$B_{12} \rightarrow$ Proportionality constant and known as Einstein's stimulated absorption coefficient.

Case-II Spontaneous Emission.

Spontaneous Emission rate $\propto N_2$

$= A_{21} N_2$

$A_{21} \rightarrow$ Proportionality constant and known as Einstein's Spontaneous Emission Coefficient.

Case-III Stimulated Emission:

Stimulated Emission rate $\propto N_2$

$\propto f(\nu)$

$= B_{21} N_2 f(\nu)$

(2) $B_{21} \rightarrow$ Proportionality constant or Einstein's Stimulated Emission coefficient.

Under Thermal Equilibrium total Upward transition = Total Downward transition.

$$\Rightarrow B_{12} N_1 f(\nu) = A_{21} N_2 + B_{21} N_2 f(\nu)$$

$$\Rightarrow B_{12} N_1 f(\nu) - B_{21} N_2 f(\nu) = A_{21} N_2$$

$$\Rightarrow f(\nu)\left[B_{12} N_1 - B_{21} N_2\right] = A_{21} N_2$$

$$\Rightarrow f(\nu) = \frac{A_{21} N_2}{B_{12} N_1 - B_{21} N_2}$$

$$\Rightarrow f(\nu) = \frac{\frac{A_{21} N_2}{B_{21} N_2}}{\frac{B_{12} N_1}{B_{21} N_2} - \cancel{\frac{B_{21} N_2}{B_{21} N_2}}}$$

$$\Rightarrow f(\nu) = \frac{\frac{A_{21} \cancel{N_2}}{B_{21} \cancel{N_2}}}{\frac{B_{12}}{B_{21}}\left(\frac{N_1}{N_2}\right) - 1}$$

$$\Rightarrow \boxed{f(\nu) = \frac{A_{21}/B_{21}}{\frac{B_{12}}{B_{21}}\left(\frac{N_1}{N_2}\right) - 1}} \quad \text{--- (1)}$$

By Boltzmann Distribution law -

The population density at different energy levels.

$$\Rightarrow N_i = N_0 e^{-E_i/KT}$$

$N_i \rightarrow$ Population density in the ith energy level.
$N_0 \rightarrow$ Population density at the ground state.

$$\Rightarrow N_1 = N_0 e^{-E_1/KT}$$
$$\Rightarrow N_2 = N_0 e^{-E_2/KT}$$

$$\Rightarrow \frac{N_1}{N_2} = e^{(E_2-E_1)/KT}$$

$$= \boxed{e^{h\nu/KT}}$$

From Eq^n ① -

$$\Rightarrow f(\nu) = \frac{A_{21}/B_{21}}{\left[\frac{B_{12}}{B_{21}} e^{h\nu/KT} - 1\right]}$$

From Plank's law of Blackbody radiation the Energy density $f(\nu)$

$$\rightarrow f(\nu) = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/KT} - 1}$$

Comparing both eq^{n}s -

$$\Rightarrow \frac{B_{12}}{B_{21}} = 1 \quad \Rightarrow B_{12} = B_{21} \text{ and } \frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$$

Let $R \rightarrow$ Ratio of Spontaneous Emission Rate to Stimulated Emission rate.

$$R = \frac{N_2 A_{21}}{N_2 B_{21} f(\nu)} = \frac{A_{21}}{B_{21}} \left(\frac{1}{f(\nu)} \right)$$

Substituting $\frac{A_{21}}{B_{21}}$ and $f(\nu)$ from previous page.

$$\Rightarrow R = \frac{8\pi h \nu^3}{c^3} \times \frac{c^3}{8\pi h \nu^3} \left(e^{h\nu/KT} - 1 \right)$$

$$\Rightarrow R = e^{h\nu/KT} - 1$$

$$\Rightarrow \boxed{R = e^{(E_2 - E_1)/KT} - 1}$$

As $(E_2 - E_1) = h\nu$ is always positive the probability of Spontaneous emission increase rapidly with the energy difference between the two states. So under thermal equilibrium, Spontaneous emission is a dominant process.

Population Inversion $(N_2 > N_1)$

$$\frac{N_2}{N_1} = e^{-(E_2 - E_1)/KT}$$

$$\Rightarrow \frac{N_2}{N_1} - 1 = e^{-(E_2 - E_1)/KT} - 1$$

$$\Rightarrow \boxed{\underbrace{N_2 - N_1}_{\Delta N\ (+ve)\ as\ N_2 > N_1} = N_1 \left(e^{-(E_2 - E_1)/KT} - 1 \right)}$$

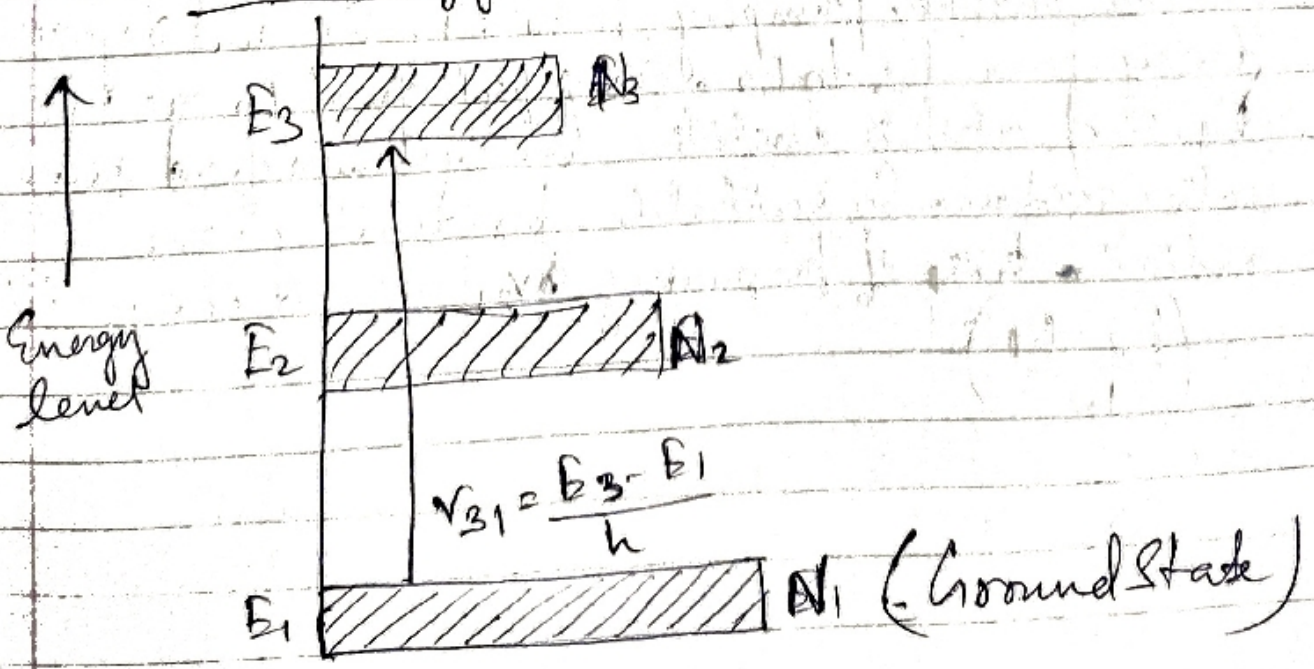
* RHS can be +ve only when T is -ve (not possible)
* Population Inversion is called as a negative temperature state

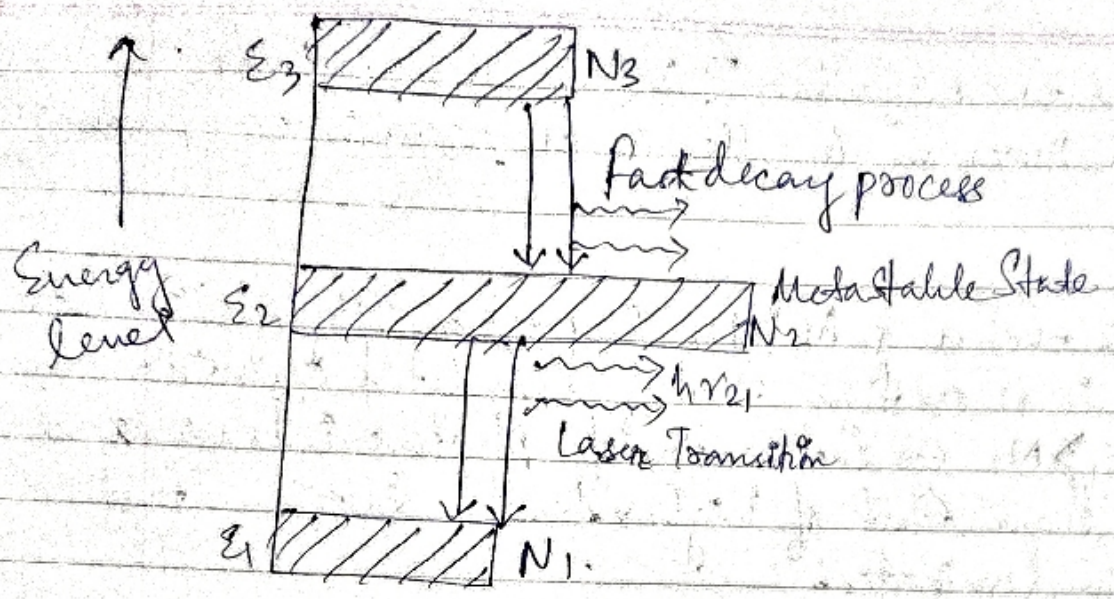
* As RHS is -ve and $N_1 > N_2$ so probability of stimulated absorption is more than that of stimulated emission.
  As $N_1 > N_2$ so interaction of photons in lower level is more than that of higher level. So stimulated absorption dominates.

* So, the conclusion drawn is that the higher energy state can never be more populated than the lower energy state or ground state. So population inversion is not possible by direct excitation from a lower energy level to a higher energy level.

This is only possible if we consider atleast a three energy level state.

Three Energy level State -





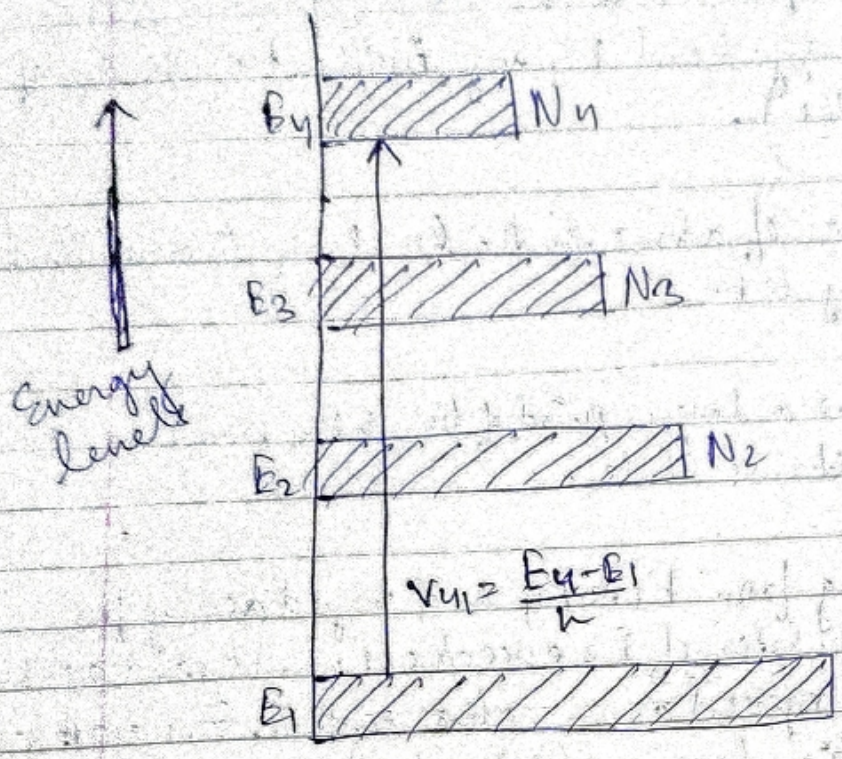
* The stimulated absorption excites the atoms from ground state E_1 to the highest level E_3 by incident radiation of frequency $\left(\nu_{31} = \frac{E_3 - E_1}{h}\right)$.

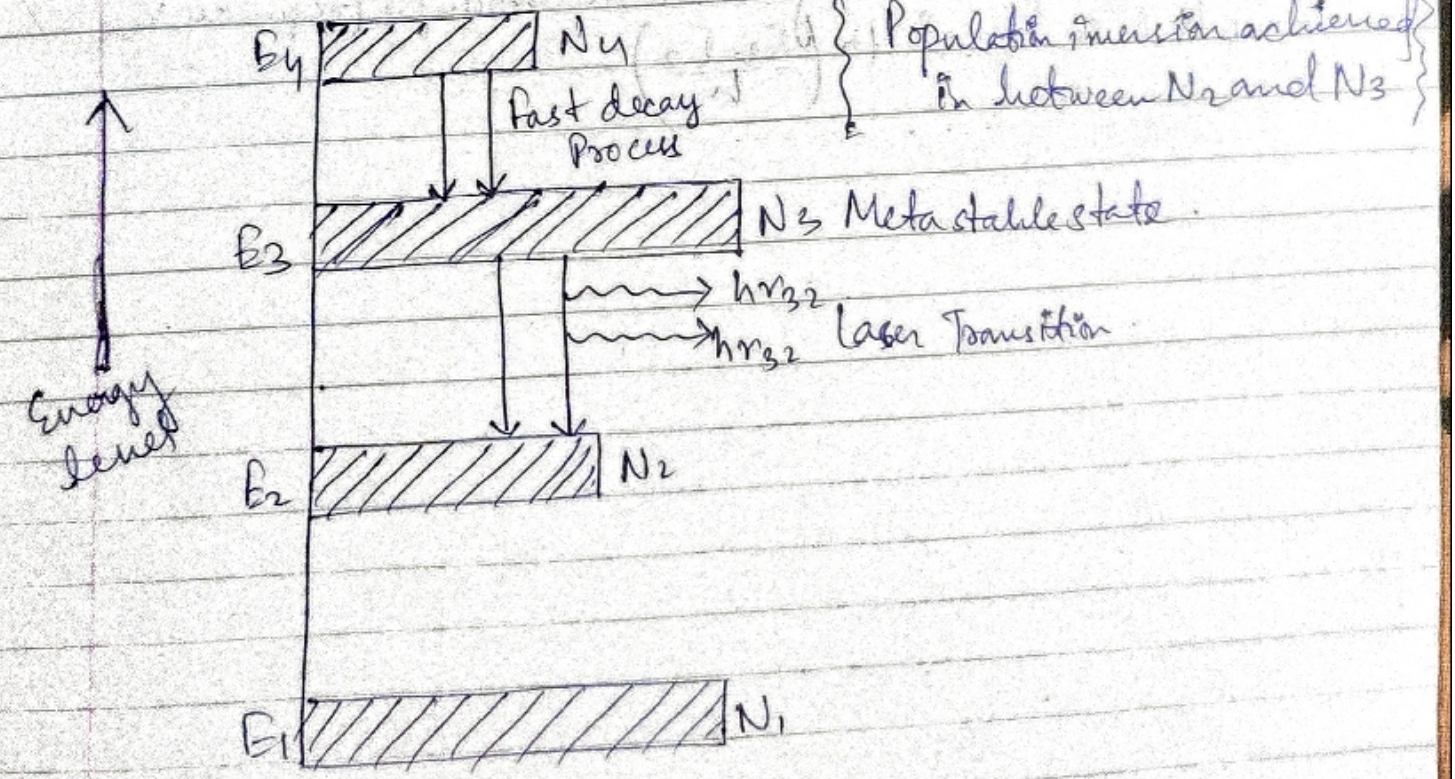
* But as the lifetime of the excited atoms in the level E_3 is very short so they decay quickly to E_2.

* It will remain for a longer period in E_2 than E_3. So, E_2 is called as a meta stable state.

* With intense pumping from E_1 to E_3 and rapid decay from E_3 to E_2, the population of E_2 exceeds E_1 and at this condition the system refered as an <u>active system</u>. So suitable for emitting laser radiation of frequency ν_{21} which is equal to $\left(\frac{E_2 - E_1}{h}\right)$.

Four Energy Level State:





* The stimulated absorption excites the atoms from ground state E_1 to the highest energy level E_4 by incident radiation of frequency $\left(\nu_{41} = \frac{E_4 - E_1}{h}\right)$.

* But as the lifetime of atoms in the level E_4 is very short so they decay quickly to E_3.

* It will remain for a longer period of time in E_3 than E_4. So E_3 is called meta stable state.

* With intense pumping from E_1 to E_4 and rapid decay from E_4 to E_3, the population of E_3 exceeds E_2 and E_1 and at this condition, system is referred as an active system. So, now it is suitable for emitting laser radiation for frequency ν_{32} which is equal to $\left(\frac{E_3 - E_2}{h}\right)$.

Advantages / Merits of Four level State wrt three level State.
(Demerits of Three Energy level).

(i) As the ground state level is not the lower level for laser transition so there is no need to pump more than one-half of the population from the ground level. So, less energy is required for pumping.

(ii) As E_2 is the lower laser level so it is easy to maintain the population inversion in between E_3 and E_2 because the atoms does not have a tendency to move to E_1 from E_3.

(iii) The laser transition is a continuous emission in this case.

Pumping (Short Ques^n)

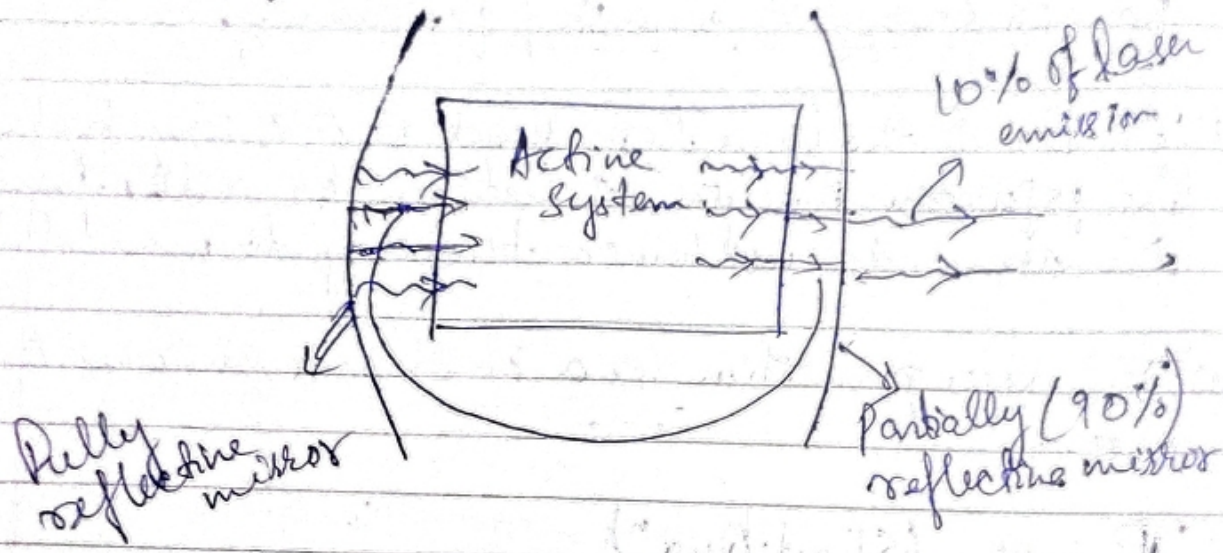
* Pumping is a method of achieving Population inversion externally (by some external agency).

* There are various types of pumping.
  (i) Optical Pumping like light radiation / lamp etc.
  (ii) Electric field pumping.
  (iii) Electron beam pumping
  (iv) Chemical pumping.

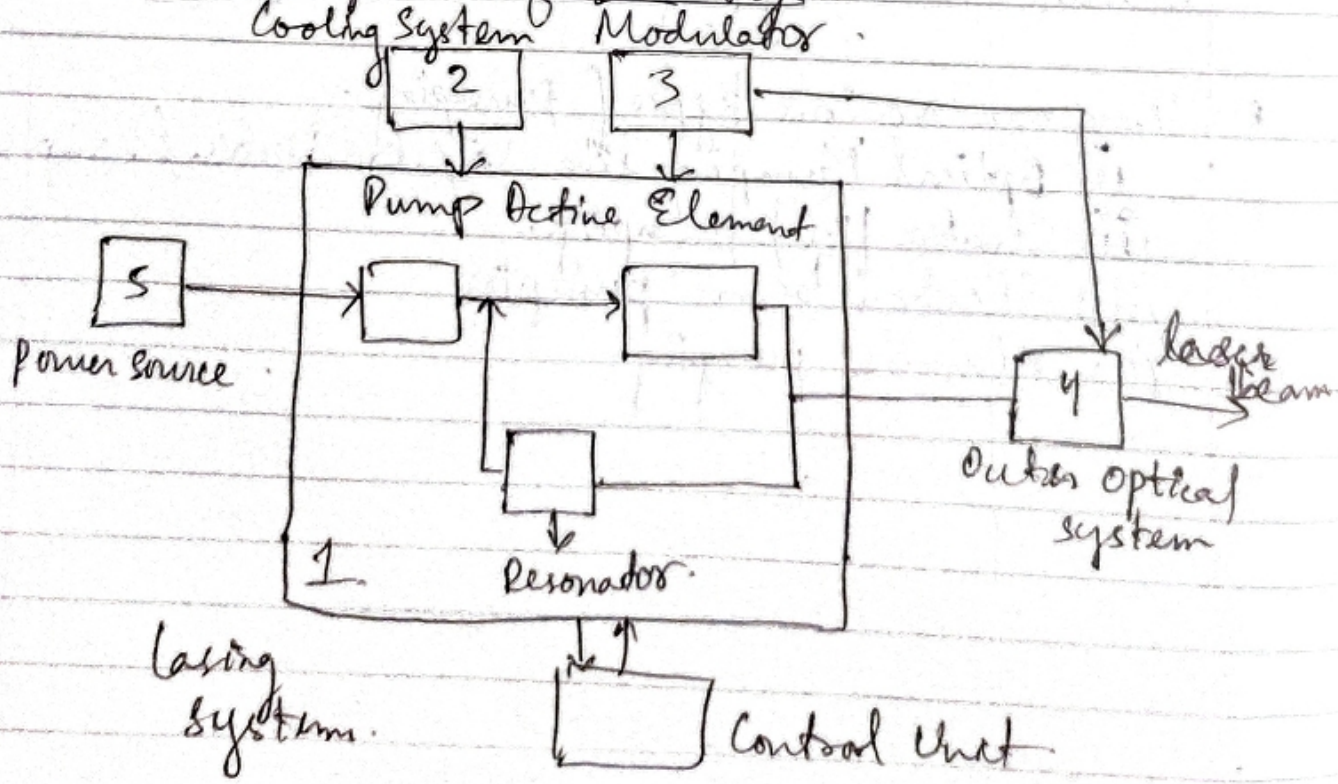
Components of Laser -

* Energy source (pumping) (which will excite the atom for population inversion)

* Optical feedback.



Block Diagram of laser light -



* Lasing system - It consists of -
(i) Active Element
(ii) Pumping system.
(iii) Optical Resonator (which provides the positive feedback)

* Cooling system - It is required to cool the system and maintain the temperature when heated.

* Light Modulator - It is an additional feature which enables emission control.

* Outer Optical system - It serves to ~~come~~ send the laser beam to a desired pattern.

* Power Source - It supplies required power necessary for pumping.

Module-3

monday.

① a) State Heisenberg's Uncertainity principle. Using this prove that
(i) Ground state energy of harmonic oscillator is non zero.
(ii) Non existence of electrons inside the nucleus.
(iii) Ground state energy of hydrogen atom is non-zero.
(iv) Calculate the minimum uncertainity in the velocity of an electron confined to a box of 10^{-8} length.

② Write Plank's formula for black body radiation. Discuss how could it explain the black body energy spectrum completely.

③ What is Compton's effect? Show that compton's shift is independent of the wavelength of incident photons.

④ Write the difference between photoelectric effect and compton's scattering. Establish Einstein's photoelectric equation and discuss Einstein's theory of photoelectric effect.

⑤ (i) Write the laws of photoelectric effect.
(ii) Define stopping potential. Discuss on which factor it depends. Show it graphically.

Module-5

① What is population inversion? Explain why population inversion cannot be achieved in two energy level system.

② Show that population inversion is reffered as a negative temperature state.

③ Explain the construction and working of a ruby laser with neat diagram.

④ Explain the construction and working of helium-neon laser with neat diagram.

Module-4

① Define a wavefunction. Write it's characteristics.

① Q) A ruby laser has it's meta stable state at 1.79eV from which stimulated emission produces laser light. Calculate the wavelength of the light. At room temperature when population inversion is achieved, calculate the ratio of the population of atoms in the meta stable state to the ground state.

② Q) A ruby laser emits light of 693.95 nm wavelength. If 1 mole of Chromium ions are involved in population inversion process in a pulse, calculate the pulse energy in eV.

③ Q) In a material, transition occurs between a metastable state and an energy level of 0.5eV. The wavelength of the emitted radiation is 2200 nm. Calculate the energy of the meta stable state.

(4) Q) If the wavelength of the laser is 7000A°, find the intensity of laser if power delivered is 10^3 W. $\left(I = \frac{P}{\lambda^2}\right)$

(5) Q) The laser light emitted in ruby laser has a wavelength of 7000A° and total number of Cr^{3+} ions is 2.8×10^{19}. Calculate the energy of a photon and total energy available per laser pulse.

(6) Q) Calculate the population inversion ratio for the two states involved in lasing action in a ruby laser at temp 27°C. Consider the emission wavelength is 7000A°.

Answers -

(4) $E = 1.79$ eV $\qquad KT = 0.025$ eV

$= 1.79 \times 1.6 \times 10^{-19}$ J

$E = h\nu$

$\Rightarrow \cancel{\nu = \frac{6.626 \times 10^{-34}}{1.7 \times 1.6 \times 10^{-19}}}$ $\Rightarrow \nu = \frac{1.79 \times 1.6 \times 10^{-19}}{6.626 \times 10^{-34}}$

$= 0.45 \times 10^{15}$

$= 4.5 \times 10^{14}$ Hz

$\Rightarrow \lambda = \frac{c}{\nu} \Rightarrow \lambda = 6.6 \times 10^{-7}$ m

Now $\frac{N_2}{N_1} = e^{-(E_2 - E_1)/KT}$

$= e^{-1.79/0.025}$

$= 8.02 \times 10^{-32}$

(2) $\lambda = 693.95\,nm$

$$\Delta E = E_2 - E_1 = \frac{hc}{\lambda} = \frac{6.626\times10^{-34}\times3\times10^{8}}{693.9\times10^{-9}} = \frac{19.87\times10^{-26}}{693.9\times10^{-9}}$$

$$= 0.028\times10^{-17}$$
$$= 2.8\times10^{-19}\,J$$
$$= 1.79\,eV$$

1 mole of $Cr^{3+} = 1\times6.022\times10^{23}$ ions are involved in 1 pulse

So, pulse energy $= 6.022\times10^{23}\times1.79\,eV$

(3) $E_1 = 0.5\,eV = 0.5\times1.6\times10^{-19} = 0.8\times10^{-19}\,J$

$\lambda = 2200\,nm$

$$E_2 - E_1 = \frac{hc}{\lambda}$$

(Ans- 1.06 eV)

$$\Rightarrow E_2 - 0.8\times10^{-19} = \frac{6.626\times10^{-34}\times3\times10^{8}}{2200\times10^{-9}}$$

$$\Rightarrow E_2 = \frac{19.87\times10^{-26}}{2200\times10^{-9}} + 0.8\times10^{-19}$$

$$\Rightarrow E_2 = \frac{9.03\times10^{-3}\times10^{-26}}{10^{-9}} + 0.8\times10^{-19}$$

$$\Rightarrow E_2 = 9.03\times10^{-29+9} + 0.8\times10^{-19}$$

$$\Rightarrow E_2 = 9.03\times10^{-20} + 0.8\times10^{-19}$$

$$\Rightarrow E_2 = 1.703\times10^{-19}\,J$$

$$= 1.06\,eV$$

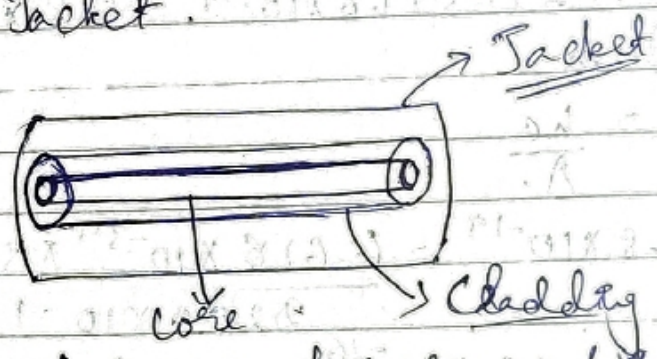
Quiz. Advantages of Optical fibre lines over conventional line (wire)

1. long distance signal transmission.
2. large bandwidth, light weight and small diameter.
3. Easy installation and easy transportation.
4. Non-conductivity.
5. It is dielectric in nature
6. Used for security concerns.
7. It is also cheaper.

Structure.

The optical fibre mainly consists of three sections.

(i) Core
(ii) Cladding
(iii) Outer core / Jacket.



* Core.

It's the most innermost section and is made up of glass / plastic. It's diameter is about 5μm to 100μm. It has a remarkable property of conducting an optical beam.

* Cladding.

Core is surrounded by cladding which is made up of plastic or glass and it's diameter is about 125μm.

It has the different optical property than that of the core such that refractive index of core is

greater than refractive index of cladding.

* Jacket -
It is the outermost section made up of Plastic, polymer or some suitable material and mainly used for the protection against moisture, crushing or any type of environmental hazards and it's diameter is about 250 μm

Classification of Optical fibre -

There are two methods of classification of optical fibre -

⇒ (i) One method is based on the variation of refractive index of the core.

a) Step index fibre.
b) Graded index fibre.

⇒ (ii) ~~Depend~~ The other method is based on the core diameter or mode of transmission of light. These are -

a) Single mode fibre.
b) Multi mode fibre.

Principle of Optical fibre -

i → angle of incidence with core
r → angle of reflection with axis.
θ → angle of incidence at core & cladding interface

* The total Internal Reflection of optical fibre communication. The light signal entering at one end of the fibre has to travel through the entire length and emerge at the other end without much loss.

Derivation for Numerical Aperture and Acceptance angle

The condition for propagation of light signal through the optical fibre -

(i) Core must have higher refractive index than that of cladding

$\mu_1 \rightarrow$ Refractive index of core

$\mu_2 \rightarrow$ Refractive index of cladding

$$\boxed{\mu_1 > \mu_2}$$

(ii) The angle of incidence at the cladding (θ) should be greater than that of the critical angle. i.e

$$\boxed{\theta \geq \theta_c}$$

$\theta_c \rightarrow$ Critical angle.

$$\Rightarrow \sin\theta \geq \sin\theta_c \quad \text{--- (1)}$$

$$\sin\theta_c = \frac{\mu_2}{\mu_1} \quad \text{--- (2)}$$

But $\theta = (90 - r)$

$$\Rightarrow \sin\theta = \sin(90 - r) = \cos r \quad \text{--- (3)}$$

From (1), (2) and (3) -

$$\Rightarrow \cos r \geq \frac{\mu_2}{\mu_1} \quad \text{--- (4)}$$

By <u>Snell's law</u> -

$$\frac{\sin i}{\sin r} = \mu_1$$

$$\Rightarrow \sin r = \frac{\sin i}{\mu_1}$$

$$\Rightarrow \cos r = \sqrt{1-\sin^2 r} = \sqrt{1-\frac{\sin^2 i}{\mu_1^2}} \quad \text{--- (5)}$$

From (4) and (5) -

$$\Rightarrow \sqrt{1-\frac{\sin^2 i}{\mu_1^2}} \geq \frac{\mu_2}{\mu_1}$$

$$\Rightarrow 1-\frac{\sin^2 i}{\mu_1^2} \geq \frac{\mu_2^2}{\mu_1^2}$$

$$\Rightarrow \mu_1^2 - \sin^2 i \geq \mu_2^2$$

$$\Rightarrow \mu_1^2 - \mu_2^2 \geq \sin^2 i$$

$$\Rightarrow \sin i \leq \sqrt{\mu_1^2 - \mu_2^2}$$

Take the maximum value of i. Let it be i_m

$$\Rightarrow \boxed{\sin i_m = \sqrt{\mu_1^2 - \mu_2^2}} \Rightarrow \text{Numerical aperture}$$

N.A.

$\Rightarrow N.A = \sqrt{(\mu_1+\mu_2)(\mu_1-\mu_2)}$

$\cong \sqrt{2\mu_1 \Delta\mu}$

<u>limiting case</u> ($i_m = 90^\circ$)

$\Rightarrow 1 = \mu_1^2 - \mu_2^2$

$\Rightarrow \mu_1^2 = 1 + \mu_2^2$

This is not a practically feasible optical fibre through which optical fibre can take place.

In all the case :-

$\mu_1^2 < 1 + \mu_2^2$

<u>Lower limit</u>

$\theta = 0^\circ$

$\Rightarrow \mu_1 = \mu_2$.

Not possible

<u>Acceptance angle</u> -

* Acceptance angle is the twice of the critical angle and the critical angle is the maximum value of the incidence angle or is

$\Rightarrow \boxed{i_c = 2 i_m} \rightarrow$ Max. value of incidence angle.

$\downarrow$

Acceptance angle.

Q) The refractive index of core and cladding for a step index fibre are 1.52 and 1.41 respectively. Calculate the numerical aperture of the fibre.

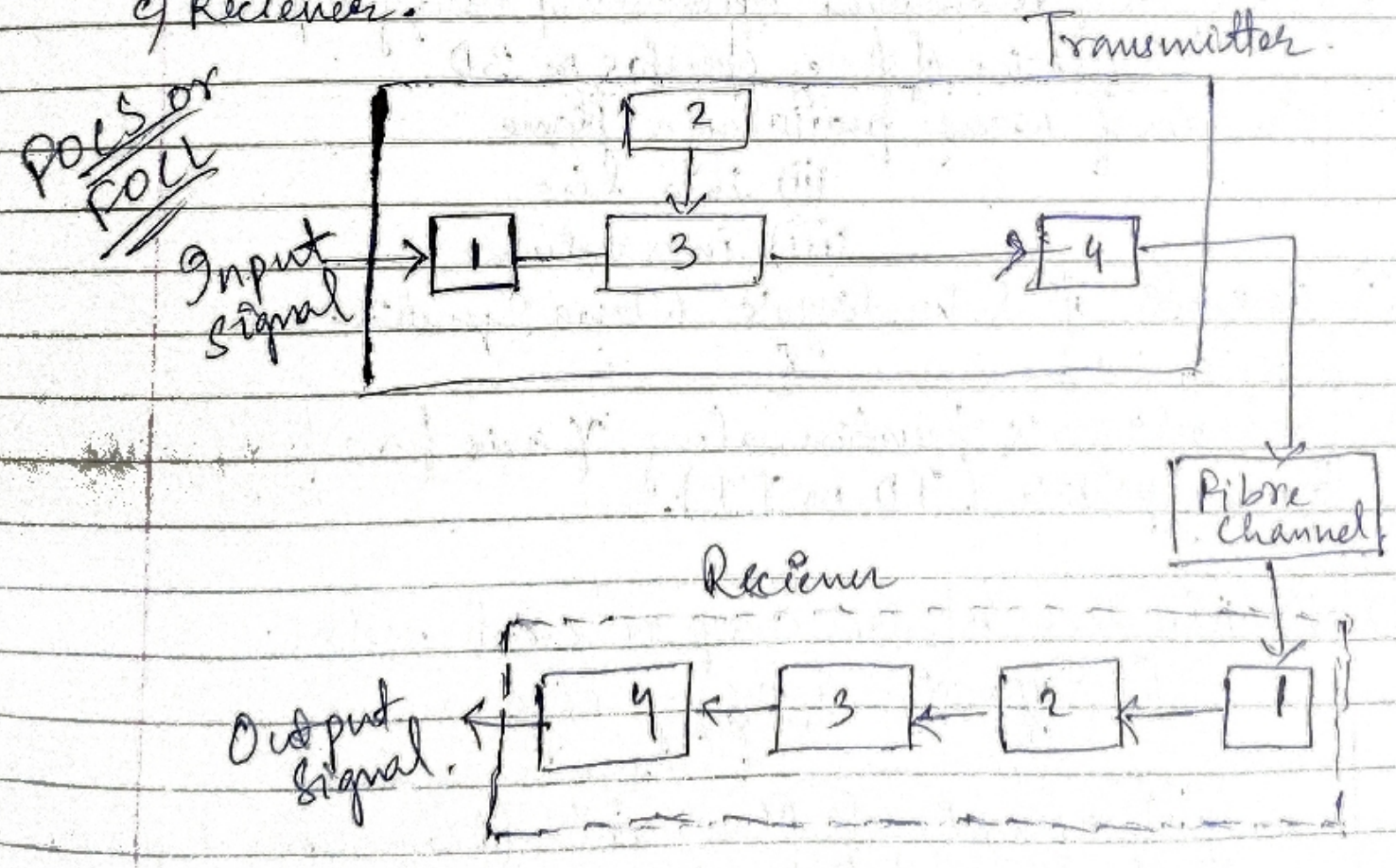
Advantages of optical fibre communication -

* FOCL or FOCS -

Fibre Optics Communication Link or
Fibre Optics Communication System

This mainly consist of three section -

a) Transmitter.
b) Optical fibre channel.
c) Reciever.



Transmitter-

1 ⇒ Coder.
2 ⇒ Carrier source.
3 ⇒ Intensity Modulator
4 ⇒ Input channel Coupler

Reciever-

1 ⇒ Output channel coupler
2 ⇒ Detector
3 ⇒ Signal Processor.
4 ⇒ Decoder.

Module-4 (Outline) -

1. Wave fun^n and the characteristics of wave fun^n.
2. Derivation of Momentum Operator and Energy Operator.
   ↳ Expression of these operators in 3D.
3. Dimension of wave fun^n (i) along a plane
   (ii) in a line
   (iii) in a volume
4. Derivation of Schrodinger's Wave Equation.
5. Write down the Schrodinger's Equation along x axis for a free particle or any particle (TD or TI).
6. Particle in a infinite deep potential well.
   (i) Expression of energy eigen value. (Energy is quantised)
   (ii) Expression of Momentum (Momentum is quantised)
   (iii) Expression of Debroglie wavelength.
   (iv) Expression of Eigen fun^n.

(7) Define potential barrier. Setup potential fun^n for it and setup Schrödinger's equation in three region.

(8) How classical physics differs from Quantum Physics

(9) Explain Tunneling Effect with examples.

(10) Write the expression for transmission probability.

Q) Find the dimension of the wave function for the moving particle (i) along an axis (1D)
(ii) along a plane (2D)
(iii) In a volume (3D)

Ans- <u>For 1-D</u>

<u>The normalization condⁿ -</u>

$$\int_0^L |\psi(x)|^2 dx = 1$$

$$\Rightarrow |\psi(x)|^2 \int_0^L dx = 1$$

$$\Rightarrow |\psi(x)|^2 [L] = 1$$

$$\Rightarrow \psi(x)^2 = \frac{1}{L}$$

$$\Rightarrow \psi(x) = L^{-1/2}$$

Dimension $= M^0 L^{-1/2} T^0$

2D (along a plane)

$$\int_0^L \int_0^L |\psi(x)|^2 \, dx\, dy = 1$$

$$\Rightarrow |\psi(x)|^2 \int_0^L \int_0^L dx\, dy = 1$$

$$\Rightarrow |\psi(x)|^2 \int_0^L L \, dy = 1$$

$$\Rightarrow |\psi(x)^2| \times L \int_0^L dx = 1$$

$$\Rightarrow \psi(x)^2 \times L^2 = 1$$

$$\Rightarrow \psi(x)^2 = \frac{1}{L^2}$$

$$\Rightarrow \psi(x) = \frac{1}{L} = L^{-1}$$

Dimension = $M^0 L^{-1} T^0$

3D (along a volume)

$$\int_0^L\int_0^L\int_0^L |\psi(x)|^2 \, dx\,dy\,dz = 1$$

Dimension - $M^0 L^{-3/2} T^0$

Module-4

(6) $V_0 = 6\,eV = 6 \times 1.6 \times 10^{-19}\,J$

$a = 0.2\,nm = 0.2 \times 10^{-9}\,m$.

$T = e^{-2\alpha a}$.

$T = 1\% = \frac{1}{100}$

$\cancel{\Rightarrow \frac{1}{100} = e^{-2 \times 10^{-9}}}$

$\Rightarrow \frac{1}{100} = e^{-2 \times 0.2 \times 10^{-9} \times \alpha}$.

$\cancel{\Rightarrow 0.01 = e}$

$\alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar}}$

$\Rightarrow \alpha = \sqrt{\frac{2 \times 9.1 \times 10^{-31}(9.6 \times 10^{-19} - E)}{1.05 \times 10^{-34}}}$.

$\Rightarrow \alpha = \sqrt{\frac{18.2 \times 10^{-31}(9.6 \times 10^{-19} - E)}{1.05 \times 10^{-34}}}$

$\Rightarrow \alpha = \frac{1.34 \times 10^{-15}}{1.05 \times 10^{-34}} \sqrt{9.6 \times 10^{-19} - E}$

$\Rightarrow T = e^{-2\alpha a}$

$\Rightarrow 0.01 = e^{-2\alpha a}$

$\Rightarrow \ln(0.01) = \ln(e^{-2\alpha a})$

$\Rightarrow \cancel{-}4.605 = \cancel{-}2\alpha a$

$\Rightarrow 4.605 = 2\dfrac{\sqrt{2m(V_0 - E)}}{\hbar} a$

$\Rightarrow (4.605)^2 = \dfrac{8m(V_0 - E)a^2}{\hbar^2}$

$\Rightarrow (4.605)^2 = \dfrac{8ma^2}{\hbar^2}(V_0 - E)$

$\Rightarrow (V_0 - E) = \dfrac{\hbar^2}{8ma^2}\dfrac{(4.605)^2}{1.6\times10^{-19}}\,eV$

$\Rightarrow E = V_0 - \dfrac{\hbar^2}{8ma^2}\dfrac{(4.605)^2}{1.6\times10^{-19}}$

$\Rightarrow E = V_0 - \dfrac{(1.05)^2\times10^{-68}\times21.206}{8\times9.1\times10^{-31}\times(0.02\times10^{-9})^2\times1.6\times10^{-19}}$

$\Rightarrow E = 6 - 5.01$

$= 1\,eV.$

Module - 3 (Outline)

- Plank's Quantum Theory
- Plank's Formula for Black Body Radiation
  From it derive Wien's formula & R&J formula.
- Einstein's Theory of Photoelectric Effect
- Compton's effect
  - Defination & Explanation
  - Show that compton's shift depends on angle of scattering.
- Heisenberg's Uncertainity Principle
- Debroglie Hypothesis:
  - Formula & Numerical

Photo
compton
Heisenberg

Q) ~~Show that~~

~~For proto~~

Module - 1

- Interference (Newton's Ring)
- Superposition

} LQ (7)

Module - 2

- Maxwell's Eqn
- Physical Significance
- Different States
- Poynting theorem & Poynting Vector

Q) What do you mean by poynting vector? Write it's SI unit.
Proove that $\langle S \rangle = \frac{1}{2}\sqrt{\frac{\varepsilon_0}{\mu_0}}\, E_0^2$ Start from Poynting Vector $\vec{S} = \vec{E} \times \vec{H}$
$\Rightarrow S = EH \sin\theta$

$$\langle S \rangle \frac{1}{2}\sqrt{\frac{\mu_0}{\varepsilon_0}}\, H_0^2$$

$$\langle S \rangle = \frac{\varepsilon_0}{\mu_0} E_{rms}^2$$

- Derivation of Poynting Theorem ✗
- Derivation of Transverse nature ✗
- " " Skin Depth ✗

- Define Poynting Theorem ✓
- Poynting Theorem Numericals ✓