モノイダル圏における左右単位子の一致の証明

西坂 毅

2022年9月11日

Proof.

以下、 λ_I に恒等射を入れて行きます。

```
\lambda_I = \lambda_I
     \circ (\lambda_I \otimes Id_I) \circ (\lambda_I^{-1} \otimes Id_I)
       =\lambda_I
       \circ (\lambda_I \otimes Id_I)
     \circ (\rho_I^{-1} \otimes Id_I) \circ (\rho_I \otimes Id_I)
      \circ (\lambda_I^{-1} \otimes Id_I)
       =\lambda_I
       \circ (\lambda_I \otimes Id_I)
      \circ \left( \rho_I^{-1} \otimes Id_I \right)
     \circ \rho_I^{-1} \circ \rho_I
       \circ \left( \rho_{I}\otimes Id_{I}\right)
      \circ (\lambda_I^{-1} \otimes Id_I)
       =\lambda_I
       \circ (\lambda_I \otimes Id_I)
      \circ (\rho_I^{-1} \otimes Id_I) \circ \rho_I^{-1} \circ \rho_I \circ (\rho_I \otimes Id_I)
     \circ \alpha_{I,I,I}^{-1} \circ \alpha_{I,I,I}
      \circ (\lambda_I^{-1} \otimes Id_I)
       =\lambda_I
       \circ (\lambda_I \otimes Id_I)
       \circ (\rho_I^{-1} \otimes Id_I) \circ \rho_I^{-1} \circ \rho_I \circ (\rho_I \otimes Id_I)
      \circ \alpha_{I,I,I}^{-1} \circ (Id_I \otimes \lambda_I^{-1}) \circ (Id_I \otimes \lambda_I) \circ \alpha_{I,I,I}
      \circ (\lambda_I^{-1} \otimes Id_I)
```

$$= \lambda_{I}
\circ (\lambda_{I} \otimes Id_{I})
\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})
\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1})
\circ (\lambda_{I} \otimes Id_{I}) \circ (\lambda_{I}^{-1} \otimes Id_{I}) \circ (Id_{I} \otimes \lambda_{I}) \circ \alpha_{I,I,I}
\circ (\lambda_{I}^{-1} \otimes Id_{I})
= \lambda_{I}
\circ (\lambda_{I} \otimes Id_{I})
\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})
\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I})
\circ \alpha_{I,I,I}^{-1} \circ \alpha_{I,I,I}
\circ (\lambda_{I}^{-1} \otimes Id_{I}) \circ (Id_{I} \otimes \lambda_{I}) \circ \alpha_{I,I,I}
\circ (\lambda_{I}^{-1} \otimes Id_{I})
= \lambda_{I}
\circ (\lambda_{I} \otimes Id_{I})
\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})
\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1}
\circ \lambda_{I \otimes I}^{-1} \circ \lambda_{I \otimes I}
\circ \alpha_{I,I,I} \circ (\lambda_{I}^{-1} \otimes Id_{I}) \circ (Id_{I} \otimes \lambda_{I}) \circ \alpha_{I,I,I}
\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

```
= \lambda_{I}
\circ (\lambda_{I} \otimes Id_{I})
\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})
\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1})
\circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1} \circ \lambda_{I \otimes I}^{-1}
\circ \lambda_{I}^{-1} \circ \lambda_{I}
\circ \lambda_{I \otimes I} \circ \alpha_{I,I,I} \circ (\lambda_{I}^{-1} \otimes Id_{I}) \circ (Id_{I} \otimes \lambda_{I}) \circ \alpha_{I,I,I}
\circ (\lambda_{I}^{-1} \otimes Id_{I})
= \lambda_{I}
\circ (\lambda_{I} \otimes Id_{I})
\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})
\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I})
\circ \alpha_{I,I,I}^{-1} \circ \lambda_{I \otimes I}^{-1} \circ \lambda_{I}^{-1} \circ \lambda_{I} \circ \lambda_{I \otimes I} \circ \alpha_{I,I,I} \circ (\lambda_{I}^{-1} \otimes Id_{I}) \circ (Id_{I} \otimes \lambda_{I}) \circ \alpha_{I,I,I}
\circ ((\lambda_{I} \otimes Id_{I}) \otimes Id_{I}) \circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})
\circ (\lambda_{I}^{-1} \otimes Id_{I})
```

α が自然変換であることから以下の図式が可換。

$$((I \otimes I) \otimes I) \otimes I \xrightarrow{\alpha_{I \otimes I, I, I}} (I \otimes I) \otimes (I \otimes I)$$

$$(\lambda_{I} \otimes Id_{I}) \otimes Id_{I} \qquad (I \otimes I) \otimes I \xrightarrow{\alpha_{I, I, I}} I \otimes (I \otimes I)$$

$$(I \otimes I) \otimes I \xrightarrow{\alpha_{I, I, I}} I \otimes (I \otimes I)$$

従って以下の等式が成り立つ

$$\lambda_I \otimes (Id_I \otimes Id_I) \circ \alpha_{I,I,I \otimes I} = \alpha_{I,I,I} \circ (\lambda_I \otimes Id_I) \otimes Id_I$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1} \circ \lambda_{I}^{-1} \circ \lambda_{I}^{-1}$$

$$\circ \lambda_{I} \circ \lambda_{I \otimes I} \circ \alpha_{I,I,I} \circ (\lambda_{I}^{-1} \otimes Id_{I}) \circ (Id_{I} \otimes \lambda_{I})$$

$$\circ (\lambda_{I} \otimes (Id_{I} \otimes Id_{I})) \circ \alpha_{I,I,I \otimes I}$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

以下の等式が成り立つ $%\lambda_{I}$ を当てる順番を入れ替えただけ

$$(\lambda_I \otimes Id_I) \circ ((Id_I \otimes Id_I) \otimes \lambda_I) = \lambda_I \otimes \lambda_I = (Id_I \otimes \lambda_I) \circ (\lambda_I \otimes (Id_I \otimes Id_I))$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1} \circ \lambda_{I \otimes I}^{-1}$$

$$\circ \lambda_{I}^{-1} \circ \lambda_{I} \circ \lambda_{I \otimes I} \circ \alpha_{I,I,I} \circ (\lambda_{I}^{-1} \otimes Id_{I})$$

$$\circ (\lambda_{I} \otimes Id_{I}) \circ ((Id_{I} \otimes Id_{I}) \otimes \lambda_{I})$$

$$\circ \alpha_{I,I,I \otimes I}$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

 $=\lambda_I$

以下は恒等射なので削除

$$(\lambda_I^{-1} \otimes Id_I) \circ (\lambda_I \otimes Id_I)$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1} \circ \lambda_{I \otimes I}^{-1}$$

$$\circ \lambda_{I}^{-1} \circ \lambda_{I} \circ \lambda_{I \otimes I} \circ \alpha_{I,I,I}$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I}) \circ (\lambda_{I} \otimes Id_{I})$$

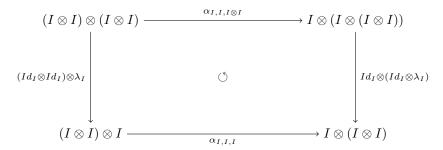
$$\circ ((Id_{I} \otimes Id_{I}) \otimes \lambda_{I})$$

$$\circ \alpha_{I,I,I \otimes I}$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

α が自然変換であることから以下の図式が可換。



従って以下の等式が成り立つ

$$(Id_I \otimes (Id_I \otimes \lambda_I)) \circ \alpha_{I,I,I \otimes I} = \alpha_{I,I,I} \circ ((Id_I \otimes Id_I) \otimes \lambda_I)$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1} \circ \lambda_{I}^{-1} \circ \lambda_{I} \circ$$

五角等式より

$$(I \otimes (I \otimes I)) \otimes I \xrightarrow{\alpha_{I,I \otimes I,I}} I \otimes ((I \otimes I) \otimes I)$$

$$\alpha_{I,I,I} \otimes I \qquad \circlearrowleft \qquad I \otimes (I \otimes I)$$

$$((I \otimes I) \otimes I) \otimes I \qquad \circlearrowleft \qquad I \otimes (I \otimes (I \otimes I))$$

$$\alpha_{I,I,I \otimes I} \qquad \circlearrowleft \qquad (I \otimes I) \otimes (I \otimes I)$$

従って以下の等式が成り立つ

$$(Id_I \otimes \alpha_{I,I,I}) \circ \alpha_{I,I \otimes I,I} \circ (\alpha_{I,I,I} \otimes Id_I) = \alpha_{I,I,I \otimes I} \circ \alpha_{I,I,I \otimes I}$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1} \circ \lambda_{I}^{-1} \circ \lambda_{I} \circ \lambda_{I} \circ \lambda_{I} \otimes Id_{I})$$

$$\circ (Id_{I} \otimes (Id_{I} \otimes \lambda_{I}))$$

$$\circ (Id_{I} \otimes \alpha_{I,I,I}) \circ \alpha_{I,I \otimes I,I} \circ (\alpha_{I,I,I} \otimes Id_{I})$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

λ が自然変換であることから以下の図式が可換。

$$I \otimes (I \otimes (I \otimes I)) \xrightarrow{\lambda_{I \otimes (I \otimes I)}} I \otimes (I \otimes I)$$

$$Id_{I} \otimes (Id_{I} \otimes \lambda_{I}) \downarrow \qquad \qquad \downarrow Id_{I} \otimes \lambda_{I}$$

$$I \otimes (I \otimes I) \xrightarrow{\lambda_{I \otimes I}} I \otimes I$$

従って以下の等式が成り立つ

$$(Id_I \otimes \lambda_I) \circ \lambda_{I \otimes (I \otimes I)} = \lambda_{I \otimes I} \circ (Id_I \otimes (Id_I \otimes \lambda_I))$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

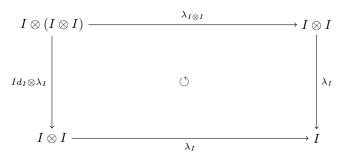
$$\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1} \circ \lambda_{I}^{-1} \circ \lambda_{I}$$

$$\circ (Id_{I} \otimes \lambda_{I}) \circ \lambda_{I \otimes (I \otimes I)}$$

$$\circ (Id_{I} \otimes \alpha_{I,I}) \circ \alpha_{I,I} \circ \alpha_{I,I} \circ \alpha_{I,I} \circ \alpha_{I} \circ \alpha_{I}$$

- $\circ (Id_I \otimes \alpha_{I,I,I}) \circ \alpha_{I,I \otimes I,I} \circ (\alpha_{I,I,I} \otimes Id_I)$
- $\circ ((\lambda_I^{-1} \otimes Id_I) \otimes Id_I)$
- $\circ (\lambda_I^{-1} \otimes Id_I)$

λが自然変換であることから以下の図式が可換。



従って以下の等式が成り立つ

$$\lambda_I \circ \lambda_{I \otimes I} = \lambda_I \circ (Id_I \otimes \lambda_I)$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1} \circ \lambda_{I \otimes I}^{-1} \circ \lambda_{I}^{-1}$$

$$\circ \lambda_{I} \circ \lambda_{I \otimes I}$$

$$\circ \lambda_{I \otimes (I \otimes I)}$$

$$\circ (Id_{I} \otimes \alpha_{I,I,I}) \circ \alpha_{I,I \otimes I,I} \circ (\alpha_{I,I,I} \otimes Id_{I})$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

$$\lambda_{I\otimes I}^{-1}\circ\lambda_{I}^{-1}\circ\lambda_{I}\circ\lambda_{I\otimes I}$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1}$$

$$\circ \lambda_{I \otimes I}^{-1} \circ \lambda_{I}^{-1} \circ \lambda_{I} \circ \lambda_{I \otimes I}$$

$$\circ \lambda_{I \otimes (I \otimes I)}$$

$$\circ (Id_{I} \otimes \alpha_{I,I,I}) \circ \alpha_{I,I \otimes I,I} \circ (\alpha_{I,I,I} \otimes Id_{I})$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

λが自然変換であることから以下の図式が可換。

$$I \otimes ((I \otimes (I \otimes I) \xrightarrow{\lambda_{(I \otimes I) \otimes I}} (I \otimes I) \otimes I$$

$$Id_{I} \otimes \alpha_{I,I,I} \qquad \circlearrowleft \qquad \qquad \downarrow \alpha_{I,I,I}$$

$$I \otimes (I \otimes (I \otimes I)) \xrightarrow{\lambda_{I \otimes (I \otimes I)}} I \otimes (I \otimes I)$$

従って以下の等式が成り立つ

$$\alpha_{I,I,I} \circ \lambda_{(I \otimes I) \otimes I} = \lambda_{I \otimes (I \otimes I)} \circ (Id_{I} \otimes \alpha_{I,I,I})$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1}$$

$$\circ \alpha_{I,I,I} \circ \lambda_{(I \otimes I) \otimes I}$$

$$\circ \alpha_{I,I \otimes I,I} \circ (\alpha_{I,I,I} \otimes Id_{I})$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

$$(\lambda_I^{-1} \otimes I) \circ \lambda_I^{-1} \circ \lambda_I \circ (\lambda_I \otimes I)$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1}$$

$$\circ \alpha_{I,I,I}$$

$$\circ (\lambda_{I}^{-1} \otimes I) \circ \lambda_{I}^{-1} \circ \lambda_{I} \circ (\lambda_{I} \otimes I)$$

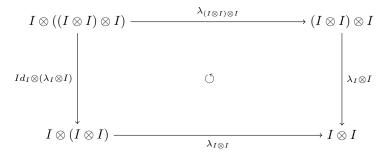
$$\circ \lambda_{(I \otimes I) \otimes I}$$

$$\circ \alpha_{I,I \otimes I,I} \circ (\alpha_{I,I,I} \otimes Id_{I})$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

λ が自然変換であることから以下の図式が可換。



従って以下の等式が成り立つ

 $\lambda_{I\otimes I}\circ (Id_I\otimes (\lambda_I\otimes I))=(\lambda_I\otimes I)\circ \lambda_{(I\otimes I)\otimes I}$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1}$$

$$\circ \alpha_{I,I,I}$$

$$\circ (\lambda_{I}^{-1} \otimes I) \circ \lambda_{I}^{-1} \circ \lambda_{I}$$

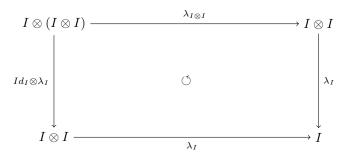
$$\circ \lambda_{I \otimes I} \circ (Id_{I} \otimes (\lambda_{I} \otimes I))$$

$$\circ \alpha_{I,I \otimes I,I} \circ (\alpha_{I,I,I} \otimes Id_{I})$$

 $\circ ((\lambda_I^{-1} \otimes Id_I) \otimes Id_I)$

 $\circ (\lambda_I^{-1} \otimes Id_I)$

λ が自然変換であることから以下の図式が可換。



従って以下の等式が成り立つ

$$\lambda_I \circ (Id_I \otimes \lambda_I) = \lambda_I \circ \lambda_{I \otimes I}$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1}$$

$$\circ \alpha_{I,I,I}$$

$$\circ (\lambda_{I}^{-1} \otimes I) \circ \lambda_{I}^{-1}$$

$$\circ \lambda_{I} \circ (Id_{I} \otimes \lambda_{I})$$

$$\circ (Id_{I} \otimes (\lambda_{I} \otimes I))$$

$$\circ \alpha_{I,I \otimes I,I} \circ (\alpha_{I,I,I} \otimes Id_{I})$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

以下は恒等射なので削除

$$\lambda_I^{-1} \circ \lambda_I$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \alpha_{I,I,I}^{-1} \circ (Id_{I} \otimes \lambda_{I}^{-1}) \circ (\lambda_{I} \otimes Id_{I}) \circ \alpha_{I,I,I}^{-1}$$

$$\circ \alpha_{I,I,I}$$

$$\circ (\lambda_{I}^{-1} \otimes I)$$

$$\circ \lambda_{I}^{-1} \circ \lambda_{I}$$

$$\circ (Id_{I} \otimes \lambda_{I})$$

$$\circ (Id_{I} \otimes (\lambda_{I} \otimes I))$$

$$\circ \alpha_{I,I \otimes I,I} \circ (\alpha_{I,I,I} \otimes Id_{I})$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

$$(Id_I \otimes \lambda_I^{-1}) \circ (\lambda_I \otimes Id_I) \circ \alpha_{I,I,I}^{-1} \circ \alpha_{I,I,I} \circ (\lambda_I^{-1} \otimes Id_I) \circ (Id_I \otimes \lambda_I)$$

$$=\lambda_I$$

$$\circ (\lambda_I \otimes Id_I)$$

$$\circ (\rho_I^{-1} \otimes Id_I) \circ \rho_I^{-1} \circ \rho_I \circ (\rho_I \otimes Id_I)$$

$$\circ \alpha_{I,I,I}^{-1}$$

$$\circ (Id_I \otimes \lambda_I^{-1}) \circ (\lambda_I \otimes Id_I) \circ \alpha_{I,I,I}^{-1} \circ \alpha_{I,I,I} \circ (\lambda_I^{-1} \otimes Id_I) \circ (Id_I \otimes \lambda_I)$$

$$\circ \left(Id_I \otimes (\lambda_I \otimes Id_I)\right)$$

$$\circ \alpha_{I,I\otimes I,I} \circ (\alpha_{I,I,I} \otimes Id_I)$$

$$\circ ((\lambda_I^{-1} \otimes Id_I) \otimes Id_I)$$

$$\circ (\lambda_I^{-1} \otimes Id_I)$$

α が自然変換であることから以下の図式が可換。

$$(I \otimes (I \otimes I)) \otimes I \xrightarrow{\alpha_{I,I \otimes I,I}} I \otimes ((I \otimes I) \otimes I)$$

$$(Id_{I} \otimes \lambda_{I}) \otimes Id_{I} \qquad \circlearrowleft \qquad \downarrow Id_{I} \otimes (\lambda_{I} \otimes Id_{I})$$

$$(I \otimes I) \otimes I \xrightarrow{\alpha_{I,I,I}} I \otimes (I \otimes I)$$

従って以下の等式が成り立つ

$$\alpha_{I,I,I} \circ ((Id_I \otimes \lambda_I) \otimes Id_I) = (Id_I \otimes (\lambda_I \otimes Id_I)) \circ \alpha_{I,I \otimes I,I}$$

$$= \lambda_I$$

$$\circ (\lambda_I \otimes Id_I)$$

$$\circ (\rho_I^{-1} \otimes Id_I) \circ \rho_I^{-1} \circ \rho_I \circ (\rho_I \otimes Id_I)$$

$$\circ \alpha_{I,I,I}^{-1}$$

$$\circ \alpha_{I,I,I} \circ ((Id_I \otimes \lambda_I) \otimes Id_I)$$

$$\circ (\alpha_{I,I,I} \otimes Id_I)$$

$$\circ ((\lambda_I^{-1} \otimes Id_I) \otimes Id_I)$$

$$\circ (\lambda_I^{-1} \otimes Id_I)$$

$$\alpha_{I,I,I}^{-1} \circ \alpha_{I,I,I}$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \alpha_{I,I,I}^{-1} \circ \alpha_{I,I,I}$$

$$\circ ((Id_{I} \otimes \lambda_{I}) \otimes Id_{I})$$

$$\circ (\alpha_{I,I,I} \otimes Id_{I})$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

三角等式より $((I\otimes I)\otimes I)\otimes I \xrightarrow{\alpha_{I,I,I}\otimes Id_I} (I\otimes (I\otimes I))\otimes I$ $(\rho_I\otimes Id_I)\otimes Id_I \qquad (Id_I\otimes \lambda_I)\otimes Id_I$ $(I\otimes I)\otimes I$

従って以下の等式が成り立つ

$$(\rho_{I} \otimes Id_{I}) \otimes Id_{I} = ((Id_{I} \otimes \lambda_{I}) \otimes Id_{I}) \circ (\alpha_{I,I,I} \otimes Id_{I})$$

$$= \lambda_{I}$$

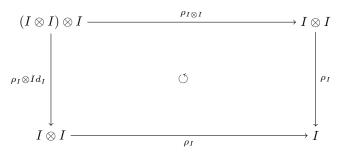
$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ ((\rho_{I} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$



従って以下の等式が成り立つ

$$\rho_I \circ \rho_{I \otimes I} = \rho_I \circ (\rho_I \otimes Id_I)$$

$$=\lambda_I$$

$$\circ (\lambda_I \otimes Id_I)$$

$$\circ (\rho_I^{-1} \otimes Id_I) \circ \rho_I^{-1}$$

$\circ \rho_I \circ \rho_{I \otimes I}$

$$\circ \left(\left(\rho_{I} \otimes Id_{I} \right) \otimes Id_{I} \right)$$

$$\circ ((\lambda_I^{-1} \otimes Id_I) \otimes Id_I)$$

$$\circ \left(\lambda_{I}^{-1}\otimes Id_{I}\right)$$

$$((I \otimes I) \otimes I) \otimes I \xrightarrow{\rho_{(I \otimes I) \otimes I}} (I \otimes I) \otimes I$$

$$\downarrow^{(\rho_{I} \otimes Id_{I}) \otimes Id_{I}} \qquad \circlearrowleft \qquad \downarrow^{\rho_{I} \otimes Id_{I}}$$

$$(I \otimes I) \otimes I \xrightarrow{\rho_{I \otimes I}} I \otimes I$$

従って以下の等式が成り立つ

$$(\rho_{I} \otimes Id_{I}) \circ \rho_{(I \otimes I) \otimes I} = \rho_{I \otimes I} \circ ((\rho_{I} \otimes Id_{I}) \otimes Id_{I})$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I})$$

$$\circ \rho_{I}^{-1}$$

$$\circ \rho_{I}$$

$$\circ (\rho_{I} \otimes Id_{I}) \circ \rho_{(I \otimes I) \otimes I}$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

$$(\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$= \lambda_{I}$$

$$\circ (\lambda_{I} \otimes Id_{I})$$

$$\circ (\rho_{I}^{-1} \otimes Id_{I}) \circ \rho_{I}^{-1} \circ \rho_{I} \circ (\rho_{I} \otimes Id_{I})$$

$$\circ \rho_{(I \otimes I) \otimes I}$$

$$\circ ((\lambda_{I}^{-1} \otimes Id_{I}) \otimes Id_{I})$$

$$\circ (\lambda_{I}^{-1} \otimes Id_{I})$$

$$((I \otimes I) \otimes I) \otimes I \xrightarrow{\rho_{(I \otimes I) \otimes I}} (I \otimes I) \otimes I$$

$$(\lambda_{I} \otimes Id_{I}) \otimes Id_{I} \qquad (I \otimes I) \otimes I \xrightarrow{\rho_{I \otimes I}} I \otimes I$$

従って以下の等式が成り立つ

$$\rho_{I\otimes I} \circ ((\lambda_I \otimes Id_I) \otimes Id_I) = (\lambda_I \otimes Id_I) \circ \rho_{(I\otimes I)\otimes I}$$

$$= \lambda_I$$

$$\circ \rho_{I\otimes I} \circ ((\lambda_I \otimes Id_I) \otimes Id_I)$$

$$\circ ((\lambda_I^{-1} \otimes Id_I) \otimes Id_I)$$

$$\circ (\lambda_I^{-1} \otimes Id_I)$$

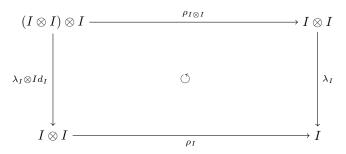
以下は恒等射なので削除 $((\lambda_I\otimes Id_I)\otimes Id_I)\circ ((\lambda_I^{-1}\otimes Id_I)\otimes Id_I)$

$$= \lambda_I$$

$$\circ \rho_{I \otimes I}$$

$$\circ ((\lambda_I \otimes Id_I) \otimes Id_I) \circ ((\lambda_I^{-1} \otimes Id_I) \otimes Id_I)$$

$$\circ (\lambda_I^{-1} \otimes Id_I)$$



従って以下の等式が成り立つ

$$\rho_I \circ (\lambda_I \otimes Id_I) = \lambda_I \circ \rho_{I \otimes I}$$
$$= \rho_I \circ (\lambda_I \otimes Id_I)$$
$$\circ (\lambda_I^{-1} \otimes Id_I)$$

以下は恒等射なので削除

$$(\lambda_I \otimes Id_I) \circ (\lambda_I^{-1} \otimes Id_I)$$

$$= \rho_I \circ (\lambda_I \otimes Id_I) \circ (\lambda_I^{-1} \otimes Id_I)$$

= ρ_I