

問図より、

$$r = 90^\circ - r' \quad \cdots \textcircled{1}$$

$$\frac{\sin i}{\sin r} = \frac{\sin i'}{\sin r'} = n \quad \cdots \textcircled{2}$$

$$\begin{aligned} \sin(90^\circ - r) &= \sin 90^\circ \cos r - \cos 90^\circ \sin r \\ &= \cos r \quad \cdots \textcircled{3} \quad (\text{加法定理}) \end{aligned}$$

②, ③式より、

$$\frac{\sin i}{\sin r} = n, \frac{\sin i'}{\sin r'} = n$$

$$\sin r = \frac{\sin i}{n} \quad \cdots \textcircled{4}$$

$$\cos r = \frac{\sin i'}{n} \quad \cdots \textcircled{5}$$

④, ⑤式より、

$$\sin r + \cos r = \frac{\sin i}{n} + \frac{\sin i'}{n}$$

両辺を二乗すると、

$$\sin^2 r + \cos^2 r = \frac{\sin^2 i + \sin^2 i'}{n^2}$$

$$n^2 = \sin^2 i + \sin^2 i'$$

$$\therefore n = \sqrt{\sin^2 i + \sin^2 i'}$$