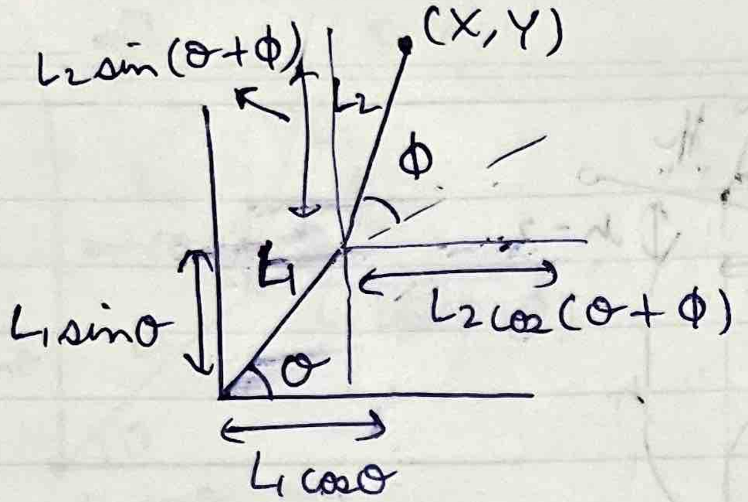
INVERSE KINEMATICS



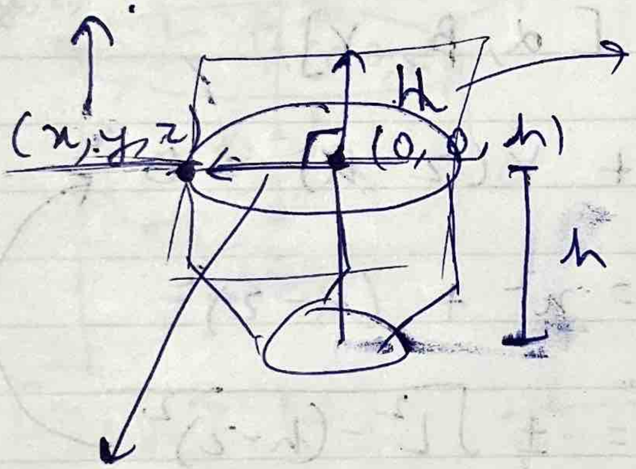
$$X = L_1 \cos\theta + L_2 \cos(\theta+\phi)$$

$$Y = L_1 \sin\theta + L_2 \sin(\theta+\phi)$$

?

F.K. (Direct)

?



x-z plane

$$\mathcal{H} = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$

normal vector

(Dot product = 0)

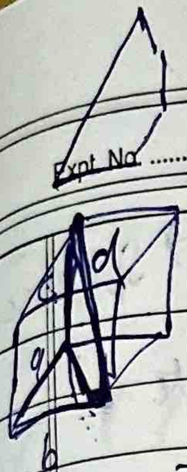
[∵ 90 degree]

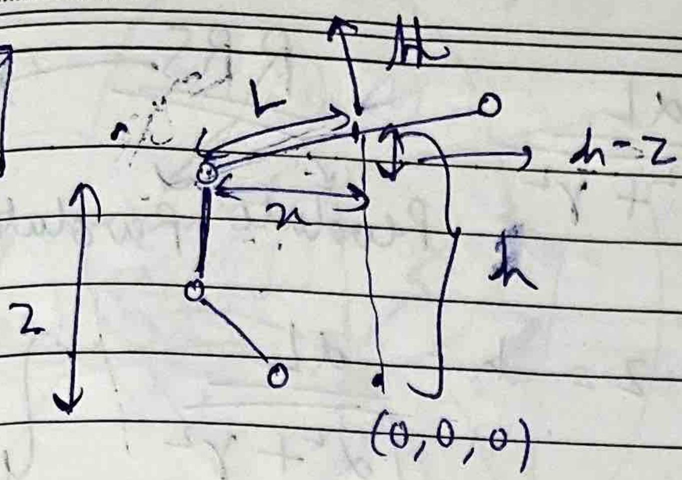
$$(x, y, z-h) \cdot \mathcal{H} = 0$$

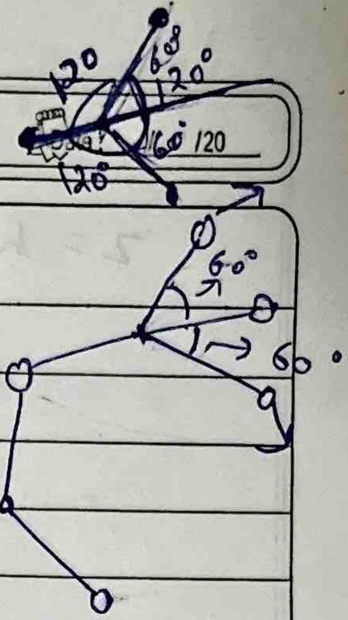
$$\alpha x + \beta y + \gamma(z-h) = 0$$

$(x-z)$ plane $\Longrightarrow$ $y = 0$

$$\alpha x + \gamma(z-h) = 0 \quad -(1)$$







Pythagoras thm

$$L^2 = x^2 + (h-z)^2$$

$$x = \pm\sqrt{L^2 - (h-z)^2} \quad — ②$$

Put ② in ①

$$\alpha\sqrt{L^2 - (h-z)^2} + \gamma(z-h) = 0$$

$$L^2 - (h-z)^2 = \frac{\gamma^2 (z-h)^2}{\alpha^2} \quad \cancel{③}$$

$$(a-b)^2 = (b-a)^2$$

$$L^2 = \frac{\gamma^2 (z-h)^2}{\alpha^2} + (z-h)^2$$

$$L^2 = \frac{\alpha^2+\gamma^2}{\alpha^2}(z-h)^2$$

$$\cancel{z-h = \pm L\sqrt{\frac{\alpha^2+\gamma^2}{\alpha^2}}}$$

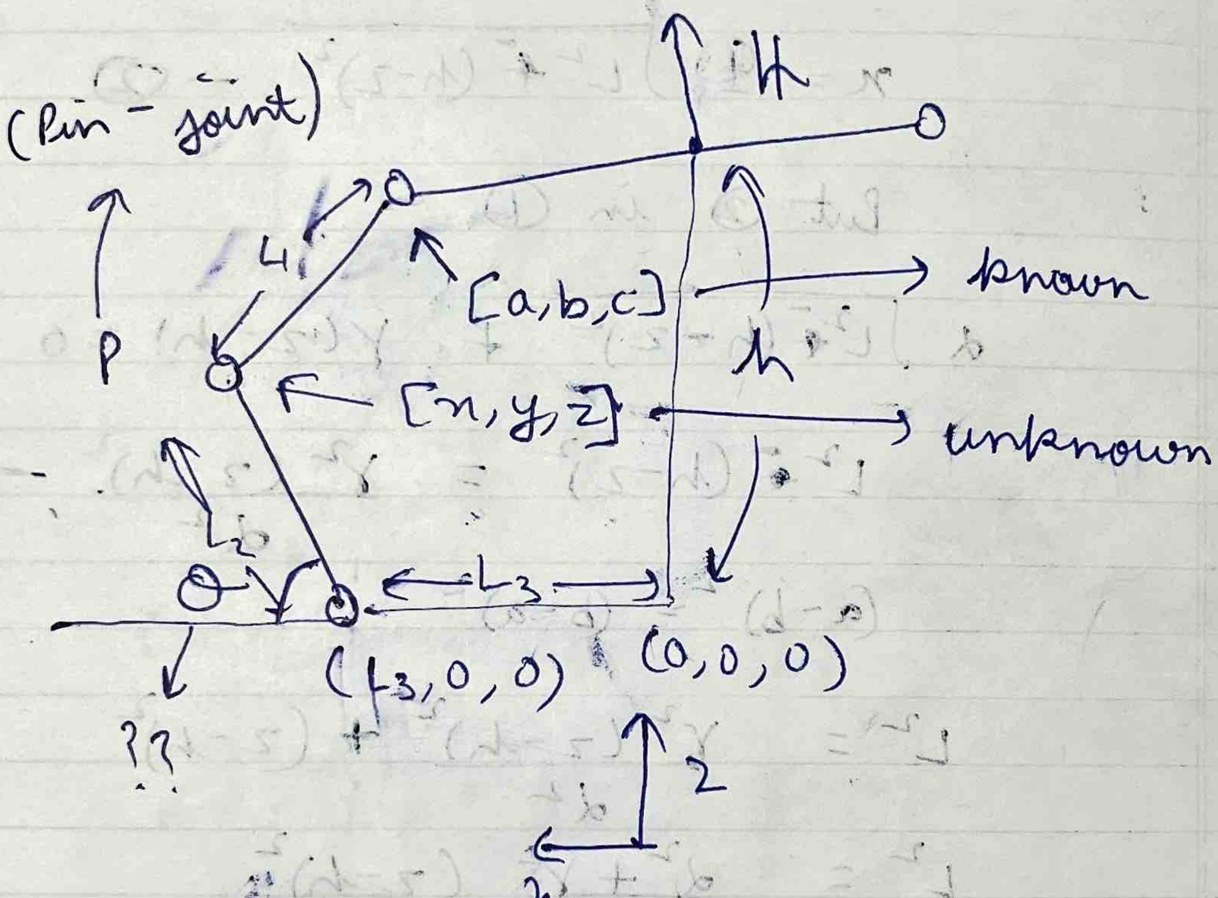
$$z-h = \pm\frac{\alpha L}{\sqrt{\alpha^2+\gamma^2}}$$

$$z = h \pm \frac{\alpha L}{\sqrt{\alpha^2 + \gamma^2}}$$

RRS → Spherical

Revolute, Revolute

$$\begin{cases} z = h - \dfrac{\alpha L}{\sqrt{\alpha^2 + \gamma^2}} \\ x = \sqrt{L^2 - (h - z)^2} \end{cases}$$



Circle (formula)

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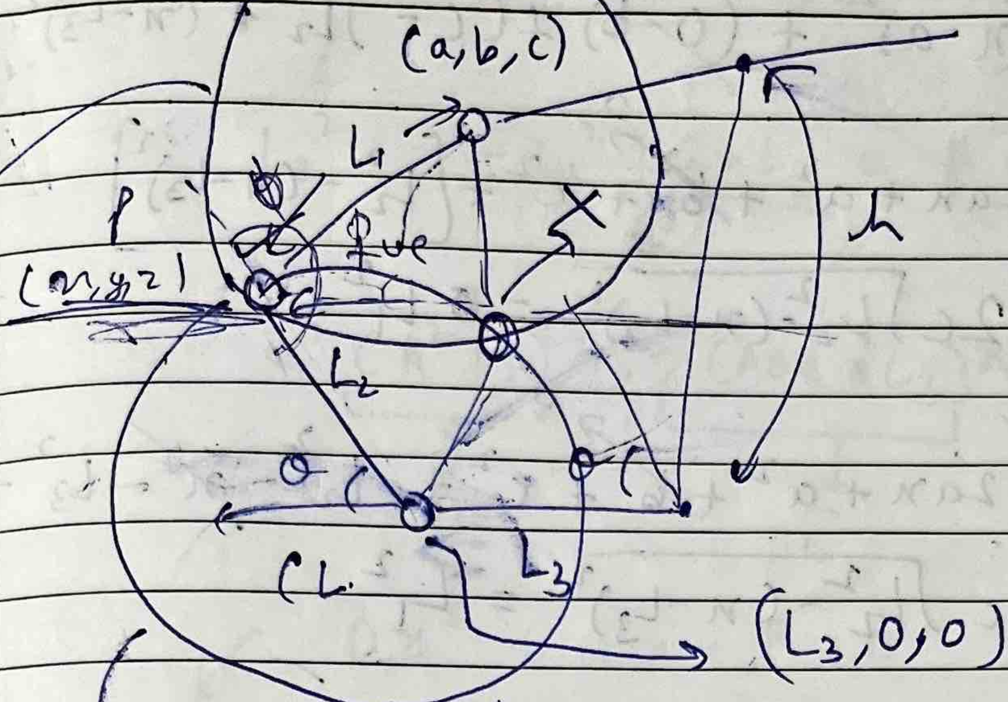


$$x = L_3 + L_2 \cos\theta \qquad y = 0$$

$$z = L_2 \sin\theta$$

G

$$(x-a)^2 + \cancel{(y-b)}^2 + (c-z)^2 = L_1^2$$

0

$$(x-L_3)^2 + (y-0)^2 + (z-0)^2 = L_2^2$$

$$(x-z) \Rightarrow y = 0$$

$$(x-L_3)^2 + z^2 = L_2^2$$

$$z = \pm\sqrt{L_2^2 - (x-L_3)^2}$$

$$(x-a)^2 + \cancel{(0-b)^2} + \left(c - \sqrt{L_2^2 - (x-L_3)^2}\right)^2 = L_1^2$$

$$x^2 - 2ax + a^2 + \cancel{b^2} + c^2 + \left[L_2^2 - (x-L_3)^2\right]$$
$$- 2c\sqrt{L_2^2 - (x-L_3)^2} = L_1^2$$

$$\cancel{x^2} - 2ax + a^2 + \cancel{b^2} + c^2 + L_2^2 - \cancel{x^2} - L_3^2 + 2xL_3$$
$$- 2c\sqrt{L_2^2 - (x-L_3)^2} = L_1^2$$

$$\sqrt{L_2^2 - (x-L_3)^2} = \frac{2(L_3-a)x + a^2 + c^2 + L_2^2 - L_3^2 - L_1^2}{2c}$$

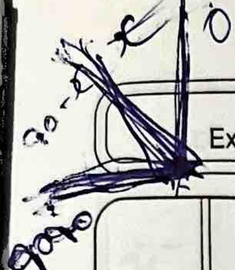
$$= \frac{(L_3-a)x}{c} + \frac{a^2+c^2+L_2^2-L_3^2-L_1^2}{2c}$$

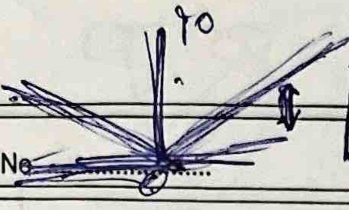
$$= -\left[\underbrace{\boxed{\frac{(a-L_3)}{c}}}_{A}x \underbrace{\boxed{- \frac{a^2+c^2+L_2^2-L_3^2-L_1^2}{2c}}}_{B}\right]$$

$$\sqrt{L_2^2 - (x-L_3)^2} = Ax + B$$

$$\downarrow$$

$$-(x^2 + L_3^2 - 2xL_3)$$





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$$\Rightarrow \quad L_2^2 - x^2 - L_3^2 + 2xL_3 = A^2x^2 + B^2 + 2ABx$$

$$\Rightarrow \quad x^2\underbrace{(A^2+1)}_{D} + \underbrace{2(AB - L_3)}_{E}x + \overbrace{B^2 - L_2^2 + L_3^2}^{F} = 0$$

$$Dx^2 + Ex + F = 0$$

Quad eqn

$$\left\{ x = \frac{-E \pm \sqrt{E^2 - 4DF}}{2D} \right\}$$

$$x = L_3 + L_2 \cos\theta \qquad [y=0]$$
$$z = L_2 \sin\theta$$

$P(d, e, f)$

$$d = L_3 + L_2 \cos\theta$$
$$e = 0, \quad f = L_2 \sin\theta$$

$$\theta = \cos^{-1}\left(\frac{d - L_3}{L_2}\right)$$

$$\text{②} \quad \cos\theta = \frac{d - L_3}{L_2}, \quad \sin\theta = \frac{f}{L_2} \quad \text{①}$$

motor angle

$$\frac{\text{①}}{\text{②}} \Rightarrow \tan\theta = \frac{f}{d - L_3} \Rightarrow \theta = \tan^{-1}\left(\frac{f}{d - L_3}\right)$$

$$\theta = \operatorname{atan2}(\overset{\text{①}}{\sin\theta}, \overset{\text{②}}{\cos\theta})$$

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$$z = h - \frac{\alpha L}{\sqrt{\alpha^2+\gamma^2}}$$

$$x = \sqrt{L^2 - (h-z)^2}$$

20 15 10 5 0

$$x = \sqrt{L^2 - \left(\cancel{h} - \left[\cancel{h} - \frac{\alpha L}{\sqrt{(\alpha^2+\gamma^2)}}\right]\right)^2}$$

100

$$x = \sqrt{L^2 - \left(\frac{\alpha L}{\sqrt{\alpha^2+\gamma^2}}\right)^2}$$

$$= \sqrt{L^2 - \frac{\alpha^2 L^2}{\alpha^2+\gamma^2}}$$

$$= L\sqrt{1 - \frac{\alpha^2}{\alpha^2+\gamma^2}}$$

$$= L\sqrt{\frac{\cancel{\alpha^2+\gamma^2}\,\gamma^2}{\alpha^2+\gamma^2}} = \frac{L\gamma}{\sqrt{\alpha^2+\gamma^2}}$$

$$x = \frac{L\gamma}{\sqrt{\alpha^2+\gamma^2}}$$

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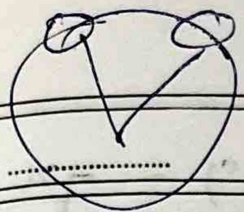


$x \rightarrow$ a_m_x = ✓ $\qquad$ A_m = [a_m_x, a_m_y, a_m_z]

$y \rightarrow$ a_m_y = ✓

$z \rightarrow$ a_m_z = ✓

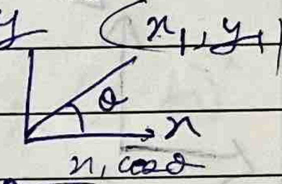
$$x = \frac{L r}{\sqrt{d^2 + r^2}}, \quad y = 0, \quad z = h - \frac{dL}{\sqrt{d^2 + r^2}}$$

$$A = \frac{(L_3 - a)\,x}{c} \quad ✓$$

$$B = \frac{a^2 + c^2 + L_2^2 - L_3^2 - L_1^2}{2c}$$

$$C = A^2 + 1 \qquad D = 2(AB - L_3)$$

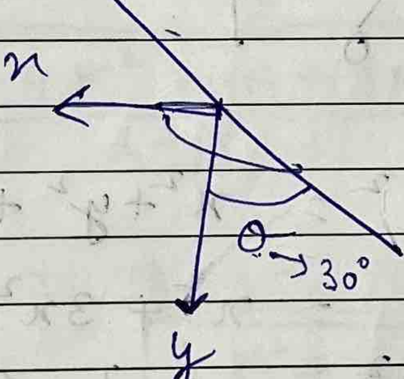
$$E = B^2 - L_2^2 + L_3^2$$

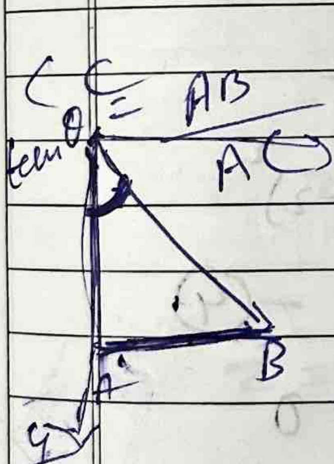


~~tanθ =~~

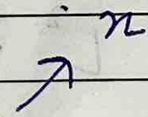
theta_a = 60 — ?

$$\boxed{b^2 = ?}$$





$\tan\theta = \frac{AB}{AC}$ → x



$$\tan\theta = \frac{AB}{AC} \rightarrow y$$

$$\frac{1}{\sqrt{3}}$$

$y = \sqrt{3}$

$AC = \sqrt{3}\,AB$

$+y = \sqrt{3}\,AB$

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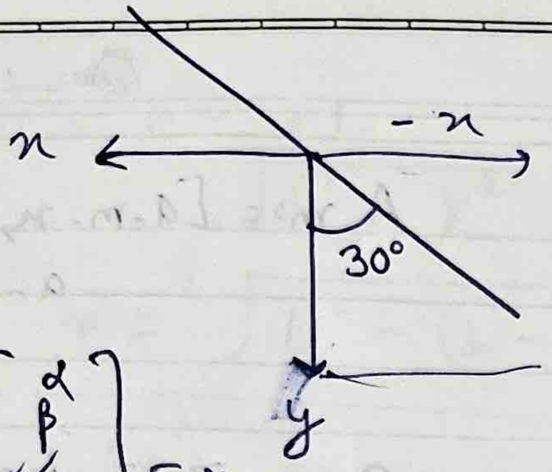


$$\tan(30^\circ) = \frac{-x}{y}$$

$$\frac{1}{\sqrt{3}} = \frac{-x}{y}$$

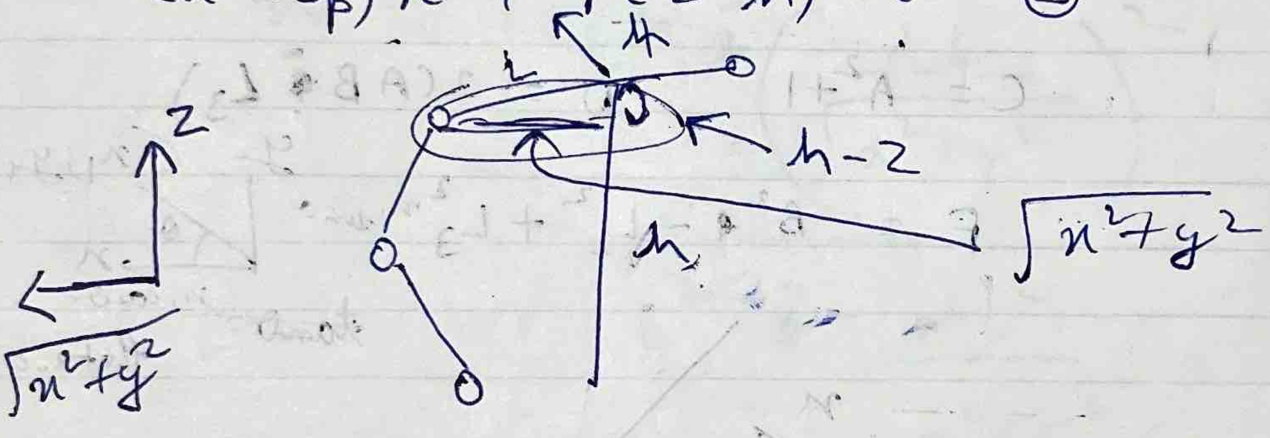
$$y = -\sqrt{3}x \quad -①$$

$$n = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}$$ Eq^n

$$\alpha x + \beta y + \gamma(z-h) = 0$$

$$\alpha x + (-\sqrt{3}\beta x) + \gamma(z-h) = 0 \quad -②$$

$$(\alpha - \sqrt{3}\beta)x + \gamma(z-h) = 0 \quad -③$$



$$L^2 = x^2 + y^2 + (h-z)^2$$

$$= x^2 + 3x^2 + (h-z)^2$$

$$L^2 = 4x^2 + (h-z)^2 \quad -④$$

$$(\alpha - \sqrt{3}\beta)x + \gamma(z-h) = 0$$

$$x = -\frac{\gamma(z-h)}{(\alpha - \sqrt{3}\beta)}$$

$$x^2 = \frac{L^2 - (h-z)^2}{4}$$

$$x = \pm \frac{1}{2}\sqrt{L^2 - (h-z)^2}$$

$$\cancel{\pm}\frac{\gamma(z-h)}{(\alpha-\sqrt{3}\beta)} = \cancel{\pm}\frac{1}{2}\sqrt{L^2-(h-z)^2}$$

$$L^2 - (h-z)^2 = \frac{4\gamma^2(z-h)^2}{(\alpha-\sqrt{3}\beta)^2}$$

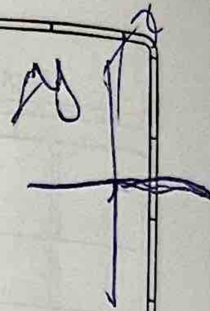
$$L^2 = \frac{4\gamma^2(z-h)^2}{(\alpha-\sqrt{3}\beta)^2} + (z-h)^2$$

$$L^2 = \left[\frac{4\gamma^2 + (\alpha-\sqrt{3}\beta)^2}{(\alpha-\sqrt{3}\beta)^2}\right] \times (z-h)^2$$

$$L = \pm \frac{\sqrt{4\gamma^2 + (\alpha-\sqrt{3}\beta)^2}}{\alpha-\sqrt{3}\beta} \times (z-h)$$

$$z-h = \pm \frac{(\alpha-\sqrt{3}\beta)L}{\sqrt{4\gamma^2+(\alpha-\sqrt{3}\beta)^2}}$$

$$z = h \pm \frac{(\alpha-\sqrt{3}\beta)L}{\sqrt{4\gamma^2+(\alpha-\sqrt{3}\beta)^2}}$$



~~$z = h \pm$~~

$$\left\{\begin{aligned} z &= h + \frac{(\alpha - \sqrt{3}\beta)L}{\sqrt{4\gamma^2 + (\alpha - \sqrt{3}\beta)^2}} \\ x &= -\frac{1}{2}\sqrt{L^2 - (h-z)^2} \\ y &= -\sqrt{3}\,x \end{aligned}\right.$$

$$z = h + \frac{(\alpha - \sqrt{3}\beta)L}{\sqrt{4\gamma^2 + \alpha^2 + 3\beta^2 - 2\sqrt{3}\alpha\beta}}$$

$$x = -\frac{1}{2}\sqrt{L^2 - \left[\cancel{h} - \left[\cancel{h} + \frac{(\alpha - \sqrt{3}\beta)L}{\sqrt{4\gamma^2 - \alpha^2 - 3\beta^2 + 2\sqrt{3}\alpha\beta}}\right]\right]^2}$$

$$= -\frac{1}{2}\sqrt{L^2 + \left[\frac{(\alpha - \sqrt{3}\beta)L}{\sqrt{4\gamma^2 - \alpha^2 - 3\beta^2 + 2\sqrt{3}\alpha\beta}}\right]^2}$$

$$= -\frac{1}{2}L\left[\sqrt{1 + \frac{(\alpha - \sqrt{3}\beta)^2}{4\gamma^2 - \alpha^2 - 3\beta^2 + 2\sqrt{3}\alpha\beta}}\right]$$

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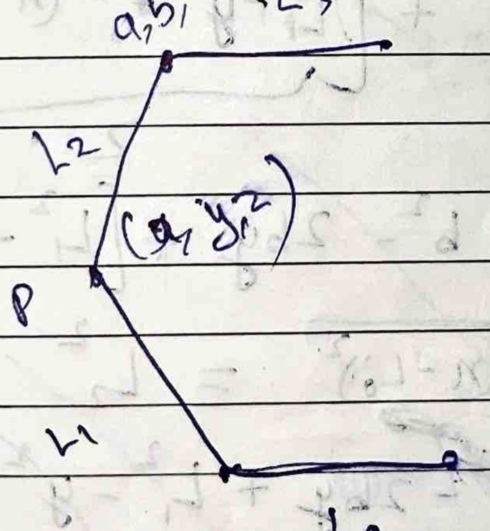
$$-\frac{1}{2}L\left[\sqrt{\frac{4r^2 - \cancel{\alpha^2} - \cancel{3\beta^2} + \cancel{2\sqrt{3}\alpha\beta} + \cancel{\alpha^2} + \cancel{3\beta^2} - \cancel{2\sqrt{3}\alpha\beta}}{4r^2 - \alpha^2 - 3\beta^2 + 2\sqrt{3}\,\alpha\beta}}\right.$$

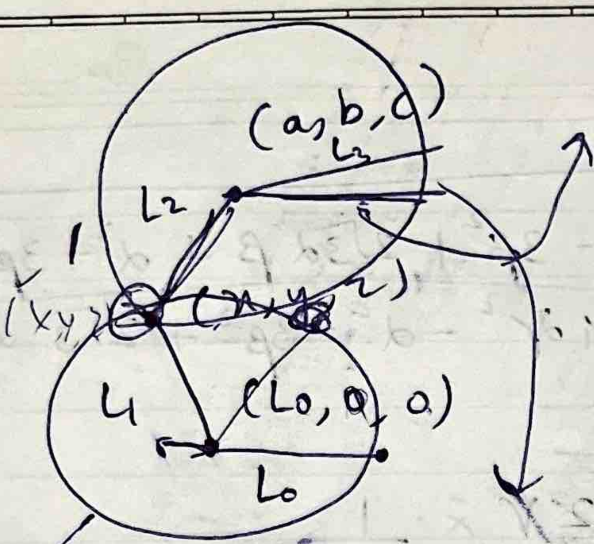
$$= -\frac{1}{\cancel{2}}L \times \cancel{2}r \times \frac{1}{\sqrt{4r^2 - \alpha^2 - 3\beta^2 + 2\sqrt{3}\,\alpha\beta}}$$

$$x = \frac{-Lr}{\sqrt{4r^2 + \alpha^2 + 3\beta^2 \pm 2\sqrt{3}\,\alpha\beta}}$$

$$y = \frac{-\sqrt{3}\,x + Lr}{\sqrt{4r^2 + \alpha^2 + 3\beta^2 \pm 2\sqrt{3}\,\alpha\beta}}$$



$A\underline{x}+b$

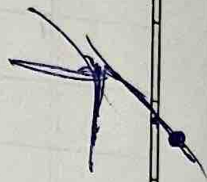


$\sqrt{x^2+y^2}$ Sphere

$L_0 \cos 60$

~~Circle~~ eqⁿ

$\frac{L_0}{2}$ $\frac{\sqrt{3}L_0}{2}$



$$(x-a)^2+(y-b)^2+(z-c)^2=L_2^2$$

$$(x-L_0)^2+y^2+z^2=L_1^2$$

$$z^2=L_1^2-y^2-(x-L_0)^2$$

$$z=\pm\sqrt{L_1^2-y^2-(x-L_0)^2}$$

$$(x-a)^2+(y-b)^2+\left[\sqrt{L_1^2-y^2-(x-L_0)^2}-c\right]^2=L_2^2$$

$$x^2+a^2-2ax+y^2+b^2-2by+\left[L_1^2-y^2-(x-L_0)^2\right]+c^2-2c\sqrt{L_1^2-y^2-(x-L_0)^2}=L_2^2$$

$$\cancel{x^2}+a^2-2ax+\cancel{y^2}+b^2-2by+L_1^2-\cancel{y^2}-\cancel{x^2}-L_0^2+2xL_0+c^2-2c\sqrt{L_1^2-y^2-(x-L_0)^2}=L_2^2$$

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$$\cancel{a^2 - 2}\; a^2 + L_1^2 - L_0^2 + c^2 - 2ax - 2by + 2xL_0 - L_2^2 = 2c\sqrt{L_1^2 - y^2 - (x-L_0)^2}$$

$Ax + B$

$$\sqrt{L_1^2 - y^2 - (x-L_0)^2} = \frac{\{a^2 + L_1^2 - L_0^2 - L_2^2 + c^2 + 2x(L_0 - a) - 2by\}}{2c}$$

$$= \frac{x(\cancel{2}L_0 - \cancel{2}a)}{\cancel{2}c} + \frac{a^2 + L_1^2 + c^2 - L_0^2 - L_2^2 - 2by}{2c}$$

A B

$y = -\sqrt{3}x$

$-2by = 2\sqrt{3}bx$

$$\frac{L_0 + \sqrt{3}b - a}{c} \rightarrow A$$

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$$\left[(-\sqrt{3}x) - b\right]^2 \quad \left[(-a)-b\right]^2 \quad y = -\sqrt{3}x$$

$$-\sqrt{3}x^2 + b^2 \quad x^2 + b^2 - 2bx$$

$$a^2 + b^2 + 2bx$$

$$(x-a)^2 + (-\sqrt{3}x - b)^2 + (z-c)^2 = L_2^2$$

$$\underline{(x-L_0)^2 + (-\sqrt{3}x)^2 + z^2 = L_1^2}$$

$$x^2 + L_0^2 - 2xL_0 + 3x^2 + z^2 = L_1^2$$

$$\therefore z^2 = L_1^2 - L_0^2 - 4x^2 + 2xL_0$$

$$z = \pm\sqrt{L_1^2 - L_0^2 - 4x^2 + 2xL_0}$$

code

OPEN

$$\cancel{x^2} + a^2 - 2ax + 3\cancel{x^2} + b^2 + 2\sqrt{3}bx + L_1^2 - L_0^2 - \cancel{4x^2} + 2xL_0$$
$$+ c^2 - 2c\sqrt{L_1^2 - L_0^2 - 4x^2 + 2xL_0} = L_2^2$$

$$a^2 + b^2 + c^2 + L_1^2 - L_0^2 - L_2^2 - 2ax + 2\sqrt{3}bx + 2xL_0$$
$$= 2c\sqrt{L_1^2 - L_0^2 - 4x^2 + 2xL_0}$$

$$\Rightarrow 0 = \frac{\cancel{2}x\left[-a + \sqrt{3}b + 2L_0\right]}{\cancel{2}c} + \underbrace{\frac{a^2 + b^2 + c^2 + L_1^2 - L_0^2 - L_2^2}{2c}}_{\beta}$$

$$\xrightarrow{} A$$

$$\underline{-\left[a - \sqrt{3}b - 2L_0\right]x}$$

$$c$$

$$\left(x + \frac{L_0}{2}\right)^2 + \left(-\sqrt{3}x - \frac{\sqrt{3}L_0}{2}\right)^2 + z^2 = L_1^2$$

x $\sqrt{x^2+y^2}$

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$$x^2 + \frac{L_0^2}{4} + xL_0 + 3x^2 + \frac{3L_0^2}{4} + 3xL_0 + z^2 = L_1^2$$

$$4x^2 + L_0^2 + 4xL_0 + z^2 = L_1^2 \rightarrow z^2 = \pm\sqrt{L_1^2 - L_0^2 - 4x^2 - 4xL_0}$$

$$\sqrt{L_1^2 - L_0^2 - 4x^2 + 2xL_0} = Ax + B$$

$$L_1^2 - L_0^2 - 4x^2 + 2xL_0 = A^2x^2 + B^2 + 2ABx$$

$$x^2\underbrace{(A^2+4)}_{C} + x\underbrace{(2AB - 2L_0)}_{D} + \underbrace{B^2 + L_0^2 - L_1^2}_{E} = 0$$

$$\cancel{x =} \quad CX^2 + DX + E = 0$$

Quad. Eq^n

$$X = \frac{-D \pm \sqrt{D^2 - 4CE}}{2C}$$

L $x-z$ $\sqrt{x^2+y^2}$

~~$L_0 \cos\theta$~~

d $\quad x = L_0 + L_1 \cos\theta$

e $\quad y = \cancel{L_1 \sin\theta}\ \ -\sqrt{3}x$

f $\quad z = L_1 \sin\theta$

d $\quad \sqrt{d^2+e^2}$

$$\cos\theta = \frac{d - L_0}{L_1} \qquad \sin\theta = \frac{f}{L_1}$$

$$\sqrt{L_1^2 - L_0^2 - 4x^2 - 4xL_0} \rightarrow 3x^2$$

$$\cancel{L_1^2 - L_0^2 - 4x^2 - 4xL_0}$$

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