Tutorial - 3

(1) Evaluate:

(a) $L^{-1}\left\{\frac{1-s}{s^2+4}\right\}$

Ans - (a) $L^{-1}\left\{\frac{1}{s^2+4}\right\} - L^{-1}\left\{\frac{s}{s^2+4}\right\}$

$\frac{\sin 2t}{2} - \cos 2t$

(b) $L^{-1}\left\{\frac{2-s+3s^3}{s5}\right\}$

Ans - (b) $L^{-1}\left\{\frac{2}{s^5}\right\} - L^{-1}\left\{\frac{s}{s^5}\right\} + 3L^{-1}\left\{\frac{s^3}{s^5}\right\}$

$= 2L^{-1}\left\{\frac{1}{s^5}\right\} - L^{-1}\left\{\frac{1}{s^4}\right\} + 3L^{-1}\left\{\frac{1}{s^2}\right\}$

$= \frac{2t^4}{5!} - \frac{t^3}{4!} + \frac{3t}{2!}$

$= \frac{t^4}{60} - \frac{t^3}{24} - \frac{3t}{2}$

(c) $L^{-1}\left\{\frac{(3+\sqrt{s})^2}{7s^3}\right\}$

Ans - (c) $L^{-1}\left\{\frac{9+s+6s^{1/2}}{7s^3}\right\}$

$= L^{-1}\left\{\frac{9}{7s^3}\right\} + L^{-1}\left\{\frac{s}{7s^3}\right\} + L^{-1}\left\{\frac{6s^{1/2}}{7s^3}\right\}$

$= \frac{9t^2}{7\cdot\lfloor 3!} + \cancel{6}\frac{\cancel{t}}{7\cdot\lfloor 2} + L^{-1}\left\{\frac{6}{7}s^{\frac{1}{2}-3}\right\}$

$= \frac{9\cancel{8}t^2}{14} + \frac{t}{\cancel{14}7} + \frac{6\cdot t^{-\frac{5}{2}-1}}{7\cdot\left\lceil\left(\frac{-5}{2}\right)!\right.}$

$= \frac{9t^2}{14} + \frac{t}{7} + \frac{6t^{-7/2}}{7\left\lceil\frac{-5}{2}\right.}$

(d) $L^{-1}\left\{\frac{2(s-3)^2}{s^4}\right\}$

Ans - (d) $2L^{-1}\left\{\frac{s^2+9-6s}{s^4}\right\}$

$= 2\left[L^{-1}\left\{\frac{s^2}{s^4}\right\} + L^{-1}\left\{\frac{9}{s^4}\right\} - L^{-1}\left\{\frac{6s}{s^4}\right\}\right]$

$= 2\left[t + \frac{9t^3}{6} - \frac{t^2}{2}\right]$ //

(2) Evaluate :

(a) $L^{-1}\left\{\frac{s+10}{s^2-s-2}\right\}$

Ans - (a) $L^{-1}\left\{\frac{s+10}{s^2-2s+s-2}\right\}$

$= L^{-1}\left\{\frac{s+10}{(s+1)(s-2)}\right\}$

$\frac{s+10}{(s+1)(s-2)} = \frac{A}{(s+1)} + \frac{B}{(s-2)}$ (∴ By using Partial fraction).

$s+10 = A(s-2) + B(s+1)$

$s = -1$, $-1+10 = A(-3) + 0$

$9 = A(-3)$

$A = -3$

$s = 2$, $2+10 = 0 + B(3)$

$B = 4$

$L^{-1}\left\{\frac{s+10}{(s+1)(s-2)}\right\} = L^{-1}\left(\frac{(-3)}{s+1} + \frac{4}{(s-2)}\right)$

$= -3L^{-1}\left(\frac{1}{s+1}\right) + 4L^{-1}\left(\frac{1}{s-2}\right)$

$= -3 \cdot e^{-t} + 4e^{2t}$

(b) $L^{-1}\left\{ \dfrac{6+s}{s^2+6s+13} \right\}$

Ans - (b) $L^{-1}\left\{ \dfrac{6+s}{s^2+6s+9+4} \right\}$

$= L^{-1}\left\{ \dfrac{6+s}{s^2+3s+3s+9+4} \right\}$

$= L^{-1}\left\{ \dfrac{s+6}{(s+3)^2+4} \right\}$

$= L^{-1}\left\{ \dfrac{s}{(s+3)^2+2^2} \right\} + L^{-1}\left\{ \dfrac{6}{(s+3)^2+4} \right\}$

$= e^{-3t} L^{-1}\left\{ \dfrac{s}{s^2+2^2} \right\} + 6e^{-3t} L^{-1}\left\{ \dfrac{1}{s^2+2^2} \right\}$

$= e^{-3t} \left(\cos 2t + 3 \sin 2t \right)$

(c) $L^{-1}\left\{ \dfrac{10}{(s-2)^4} \right\}$

Ans - (c) $10\, L^{-1}\left\{ \dfrac{1}{(s-2)^4} \right\}$ $= 10 e^{2t} \dfrac{t^3}{6}$

$= 10\, e^{2t} L^{-1}\left\{ \dfrac{1}{s^4} \right\}$ $= \dfrac{2e^{2t} \cdot t^3}{3}$ //

(d) $L^{-1}\left\{\frac{1}{(s+a)^2}\right\}$

Ans - (d) $L^{-1}\left\{\frac{1}{(s+a)^2}\right\}$

$= e^{-at}\, L^{-1}\left\{\frac{1}{s^2}\right\}$

$= e^{-at} \cdot t$

(3) Using Partial fraction method, Evaluate:

(a) $L^{-1}\left\{\frac{1}{(s+\sqrt{2})(s-\sqrt{3})}\right\}$

Ans - (a) $\frac{1}{(s+\sqrt{2})(s-\sqrt{3})} = \frac{A}{(s+\sqrt{2})} + \frac{B}{(s-\sqrt{3})}$

$1 = A(s-\sqrt{3}) + B(s+\sqrt{2})$

$s = \sqrt{3}, \quad 1 = 0 + B(\sqrt{3}+\sqrt{2})$

$B = \frac{1}{\sqrt{3}+\sqrt{2}}$

$s = -\sqrt{2}, \quad 1 = A(-\sqrt{2}-\sqrt{3})$

$A = -\frac{1}{\sqrt{3}+\sqrt{2}}$

$L^{-1}\left\{\frac{1}{(s+\sqrt{2})(s-\sqrt{3})}\right\} = L^{-1}\left\{\frac{-1/\sqrt{3}+\sqrt{2}}{s+\sqrt{2}}\right\} + L^{-1}\left\{\frac{1/\sqrt{3}+\sqrt{2}}{s-\sqrt{3}}\right\}$

$$= \frac{-1}{\sqrt{3}+\sqrt{2}} e^{-\sqrt{2}t} + \frac{1}{\sqrt{3}+\sqrt{2}} e^{\sqrt{3}t}$$

(b) $L^{-1}\left\{\frac{1}{s(s^2-3s+3)}\right\}$

Ans-(b) $L^{-1}\left\{\frac{1}{s(s^2-3s+9-6)}\right\}$

$$= L^{-1}\left\{\frac{1}{s((s-3)^2-6)}\right\}$$

$$\frac{1}{s((s-3)^2-6)} = \frac{A}{s} + \frac{Bs+C}{(s-3)^2-6}$$

$$1 = ((s-3)^2-6)A + (Bs+C)s$$

$s=0$, $1 = (9-6)A + 0$

$A = 1/3$

$s=3$, $1 = -6A + (3B+C)3$

$1 = -6 \cdot \frac{1}{3} + (3B+C)3$

$3 = (3B+C)3$

$3B + C = 1$

$B = \frac{1-C}{3}$

$s=1, \quad 1 = (4-6)A + (B+C)$

$1 = -2 \cdot \frac{1}{3} + B + C$

$B + C = \frac{5}{3}$ $\quad \therefore \quad B = \frac{1-2}{3}$

$\frac{1-C}{3} + C = \frac{5}{3}$ $\quad B = \frac{-1}{3}$

$\frac{1-C+3C}{\cancel{3}} = \frac{5}{\cancel{3}}$

$1 + 2C = 5$

$C = 2$

$$L^{-1}\left\{\frac{1}{s((s-3)^2-6)}\right\} = L^{-1}\left\{\frac{1/3}{s} + \frac{-\frac{1}{3}s + 2}{(s-3)^2-6}\right\}$$

$$= \frac{1}{3} L^{-1}\frac{1}{s} + L^{-1}\left(\frac{-\frac{1}{3}s + 2}{(s-3)^2-6}\right)$$

$$= \frac{1}{3} + \frac{1}{3} L^{-1}\left(\frac{-s+6}{(s-3)^2-6}\right)$$

$$= \frac{1}{3} - \frac{1}{3} L^{-1}\left(\frac{(s-3)-3}{(s-3)^2-6}\right)$$

$$= \frac{1}{3} - \frac{1}{3} e^{3t} L^{-1}\left(\frac{s-3}{s^2-6}\right)$$

$$= \frac{1}{3}\left[1 - e^{3t}\left(L^{-1}\left(\frac{s}{s^2-6}\right) - L^{-1}\left(\frac{3}{s^2-6}\right)\right)\right]$$

$$= \frac{1}{3}\left[1 - e^{3t}\left(\cancel{\cosh t}\ \cosh\sqrt{6}t - \frac{3}{\sqrt{6}} \sinh\sqrt{6}t \right)\right]$$

(c) $L^{-1}\left\{ \frac{4s+5}{(s-1)^2(s+2)} \right\}$

Ans - (c) $L^{-1}\left\{ \frac{4s+5}{(s-1)^2(s+2)} \right\}$

$$\frac{4s+5}{(s-1)^2(s+2)} = \frac{A}{(s-1)} + \frac{B}{(s-1)^2} + \frac{C}{s+2}$$

$$4s+5 = A(s-1)(s+2) + B(s+2) + C(s-1)^2$$

$s=1$, $4+5 = 0 + B(3) + 0$

$B = 3$

$s=-2$, $-8+5 = 0 + 0 + C(-2-1)^2$

$-3 = C(-3)^2$

$C = -3$

$s=0$, $5 = A(-1)(2) + B(2) + C(-1)^2$

$5 = -2A + 2B + C$

$5 = -2A + 6 - 3$

$2 = -2A$

$A = -1$

$$L^{-1}\left\{\frac{4s+5}{(s-1)^2(s+2)}\right\} = L^{-1}\left(\frac{-1}{(s-1)}\right) + L^{-1}\left\{\frac{3}{(s-1)^2}\right\} + L^{-1}\left\{\frac{-3}{s+2}\right\}$$

$$= -e^{t} + 3e^{t} L^{-1}\left\{\frac{1}{s^2}\right\} - 3e^{-2t}$$

$$= -e^{t} + 3e^{t}\cdot t - 3e^{-2t}$$

(d) $L^{-1}\left\{\dfrac{s+3}{(s^2+1)(s^2+9)}\right\}$

Ans - (d) $L^{-1}\left\{\dfrac{s+3}{(s^2+1)(s^2+9)}\right\}$

$$\frac{s+3}{(s^2+1)(s^2+9)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+9}$$

$$s+3 = (As+B)(s^2+9) + (Cs+D)(s^2+1)$$

$$s+3 = As^3 + 9As + Bs^2 + 9B + Cs^3 + Cs + Ds^2 + D$$

$$s+3 = s^3(A+C) + (B+D)s^2 + s(9A+C) + 9B + D$$

By Comparing Coefficient

$A+C=0$ $B+D=0$ $9A+C=1$ $9B+D=3$

$A=-C$ $B=-D$ $9(-C)+C=1$ $9(-D)+D=3$

$A=\frac{1}{8}$ $B=\frac{3}{8}$ $-8C=1$ $-8D=3$

$C=-\frac{1}{8}$ $D=\frac{-3}{8}$

$$= L^{-1}\left\{\frac{s+3}{(s^2+1)(s^2+9)}\right\} = L^{-1}\left\{\frac{\frac{1}{8}(s+3)}{s^2+1}\right\} - L^{-1}\left\{\frac{\frac{1}{8}(s+3)}{s^2+9}\right\}$$

$$= \frac{1}{8}\left[\left(L^{-1}\frac{s}{s^2+1} + 3L^{-1}\frac{1}{s^2+1}\right) - \left(L^{-1}\frac{s}{s^2+9} + 3L^{-1}\left(\frac{1}{s^2+9}\right)\right)\right]$$

$$= \frac{1}{8}\left[(\cos t + 3\sin t) - \cos 3t - \cancel{3}\sin 3t\right]$$

$$= \frac{1}{8}\left[\cos t + 3\sin t - \cos 3t - \sin 3t\right]$$

(4) Using convolution theorem, Evaluate :

(a) $L^{-1}\left\{\frac{a}{s^2(s^2+a^2)}\right\}$

Ans - (a) $F(s) = \frac{a}{s^2+a^2} \Rightarrow f(t) = \sin at \Rightarrow f(u) = \sin au$

$G(s) = \frac{a}{s^2} \Rightarrow g(t) = a.t \Rightarrow g(t-u) = a(t-u)$

$$L^{-1} F(s)\, G(s) = \int_0^t f(u)\cdot g(t-u)\, du$$

$$= \int_0^t \sin au \,.\, a(t-u)\, du$$

$$= \int_0^t \sin au \cdot at - \sin au \cdot au \, du$$

$$= at \int_0^t \sin au - a\int_0^t u \cdot \sin au \, du$$

$$= at \left[\frac{-\cos au}{a} \right]_0^t - a\left[\frac{-u \cos au}{a} - \frac{\sin au}{a^2} \right]_0^t$$

$$= t\left[-\cos at - 1 \right] - a\left[\left(\frac{-t \cdot \cos at}{a} - \frac{\sin at}{a^2} \right) - 0 \right]$$

$$= -t \cdot \cancel{\cos at} - t + t \cdot \cancel{\cos at} + \frac{\sin at}{a}$$

$$= \frac{\sin at}{a} + t$$

(b) $L^{-1}\left\{ \frac{1}{(s^2+a^2)^2} \right\}$

Ans (b) $F(s) = \frac{1}{s^2+a^2} \Rightarrow f(t) = \frac{\sin at}{a} \Rightarrow f(u) = \frac{\sin au}{a}$

$G(s) = \frac{1}{s^2+a^2} \Rightarrow g(t) = \frac{\sin at}{a} \Rightarrow g(t-u) = \frac{\sin a(t-u)}{a}$

$$L^{-1} F(s) \cdot G(s) = \frac{1}{a}\int_0^t \sin au \cdot \sin a(t-u) \, du$$

$$= \frac{1}{a}\int_0^t \sin au \left(\sin at \cdot \cos au - \cos at \cdot \sin au \right)$$

$$= \frac{1}{a}\int_0^t \left(\sin au \cdot \sin at \cdot \cos au - \sin au \cdot \cos at \cdot \sin au \right) du$$

$$= \frac{1}{a}\int_0^t \frac{\sin at}{2}\left(2\sin au\cdot\cos au\right) - \cos at\left(\sin^2 au\right)$$

$$= \frac{1}{a}\int_0^t \frac{\sin at}{2}\sin 2au - \cos at\left(\frac{1-\cos 2au}{2}\right)$$

$$= \frac{\sin at}{2a}\int_0^t \sin 2au - \frac{\cos at}{2a}\int_0^t 1-\cos 2au \, du$$

$$= \frac{\sin at}{2a}\left[\frac{-\cos 2au}{2a}\right]_0^t - \frac{\cos at}{2a}\left[u - \frac{\sin 2au}{2a}\right]_0^t$$

$$= -\frac{\sin at}{2a}\left[\left(\frac{\cos 2at}{2a} - \frac{1}{2a}\right)\right] - \frac{\cos at}{2a}\left[\left(t - \frac{\sin 2at}{2a}\right) - 0\right]$$

$$= -\frac{\sin at\cos 2at}{4a^2} + \frac{\sin at}{4a^2} - \frac{t\cos at}{4a^2} + \frac{\cos at\cdot\sin 2at}{4a^2}$$

$$= \frac{1}{4a^2}\left[\sin at - t\cos at + \sin(2at - at)\right]$$

$$= \frac{1}{4a^2}\left[\sin at - t\cos at + \sin at\right]$$

$$= \frac{1}{4a^2}\left[2\sin at - t\cos at\right]$$

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(c) $L^{-1}\left\{\frac{1}{(s+1)(s+3)}\right\}$

Ans - (c) $F(s) = \frac{1}{s+1} \Rightarrow f(t) = e^{-t} \Rightarrow f(u) = e^{-u}$

$G(s) = \frac{1}{s+3} \Rightarrow g(t) = e^{-3t} \Rightarrow g(t-u) = e^{-3(t-u)}$

$$L^{-1} F(s) \cdot G(s) = \int_0^t e^{-u} \cdot e^{-3t} \cdot e^{3u} \, du$$

$$= \int_0^t e^{2u} \cdot e^{-3t} \, du$$

$$= e^{-3t} \int_0^t e^{2u} \, du$$

$$= e^{-3t} \left[\frac{e^{2u}}{2}\right]_0^t$$

$$= e^{-3t} \left[\frac{e^{2t}}{2} - \frac{1}{2}\right]$$

$$= \frac{e^{-t}}{2} - \frac{e^{-3t}}{2}$$

(5) (a) Solve by Laplace Transform : $y'' + 6y = 1$, $y(0) = 2$, $y'(0) = 0$

Ans - (a) $L\, y'' + L\, 6y = L\, 1 \Rightarrow (s^2 Y(s) + s\, y(0) + y'(0)) + 6(Y(s)) = L$

$\Rightarrow s^2 Y(s) + 2s + 0 + 6Y(s) = \frac{1}{s}$

$$Y(s)(s^2+6) = \frac{1}{s} - 2s$$

$$Y(s) = \frac{1-2s^2}{s(s^2+6)}$$

Taking inverse Laplace both side.

$$L^{-1}Y(s) = L^{-1}\left(\frac{1-2s^2}{s(s^2+6)}\right)$$

$$Y(t) = L^{-1}\left(\frac{1-2s^2}{s(s^2+6)}\right)$$

$$= L^{-1}\left(\frac{1}{s(s^2+6)}\right) - 2L^{-1}\left(\frac{s}{s^2+6}\right)$$

$$Y(t) = \left[1 - 13\cos\sqrt{6}\,t\right]\frac{1}{6}$$

(b) $\frac{dy}{dt} - 2y = 4$; $t=0$, $y=1$

Ans - (b) $L(y' - 2y) = L4$

$$Ly' - 2Ly = 4L1$$

$$(sY(s) - y(0)) - 2Y(s) = \frac{4}{s}$$

$$sY(s) - 1 - 2Y(s) = \frac{4}{s}$$

$$Y(s)(s-2) = \frac{4}{s} + 1$$

$$Y(s) = \frac{4+s}{s(s-2)}$$

Taking Inverse Laplace both side

$$L^{-1} Y_{(s)} = L^{-1} \frac{s+4}{s(s-2)}$$

$$Y_{(t)} = L^{-1} \frac{s}{s(s-2)} + L^{-1} \frac{4}{s(s-2)}$$

$$= L^{-1} \frac{1}{s-2} + \frac{4}{2}\left[L^{-1}\frac{1}{s} - L^{-1}\frac{1}{s-2}\right]$$

$$= e^{2t} + 2(1) - 2e^{2t}$$

$$= 2 - e^{2t}$$

(c) Solve the IVP using the Laplace Transform :
$y'' + 4y = 0$, $y(0) = 1$, $y'(0) = 6$.

Ans - (c)

$$L y'' + 4 L y = 0$$

$$(s^2 Y_{(s)} - s y_{(0)} - y'_{(0)}) + 4 Y_{(s)} = 0$$

$$s^2 Y_{(s)} - s - 6 + 4 Y_{(s)} = 0$$

$$Y_{(s)}(s^2+4) = s+6$$

$$Y_{(s)} = \frac{s+6}{s^2+4}$$

Taking Inverse Laplace both side.

$$L^{-1} Y_{(s)} = L^{-1}\left(\frac{s+6}{s^2+4}\right)$$

$$Y_{(t)} = L^{-1}\frac{s}{s^2+4} + 6 L^{-1}\frac{1}{s^2+4}$$

$$Y_{(t)} = \cos 2t + 3 \sin 2t$$

(d) Solve the initial Value problem $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} = e^t \sin t$, $y(0) = y'(0) = 0$ using Laplace Transforms.

Ans. (d) $y'' - 2y' = e^t \sin t$

$Ly'' - 2Ly' = Le^t \sin t$

$(s^2 Y_{(s)} - s\,y(0) - y'(0)) - 2(sY_{(s)} - y_{(0)}) = L\,e^t \sin t$

$s^2 Y_{(s)} - 0 - 0 - 2sY_{(s)} - 0 = \frac{1}{(s-1)^2+1}$

$Y_{(s)}(s^2 - 2s) = \frac{1}{(s-1)^2+1}$

$Y_{(s)} = \frac{1}{s(s-2)((s-1)^2+1)}$

Taking Inverse Laplace both side

$L^{-1} Y_{(s)} = L^{-1}\left(\frac{1}{s(s-2)((s-1)^2+1)}\right)$

$Y_{(t)} = L^{-1}\left(\frac{1}{s(s-2)((s-1)^2+1)}\right)$

$\frac{1}{s(s-2)((s-1)^2+1)} = \frac{A}{s} + \frac{B}{(s-2)} + \frac{Cs+D}{(s-2)^2+1}$

$1 = A((s-2)^2+1)(s-2) + B((s-2)^2+1)s + (Cs+D)(s-2)s$

$s = 0, \quad 1 = A(4+1)(-2)$

$A = \frac{-1}{10}$

$s=2$, $\quad 1 = 0 + B(1)(2) + 0$

$B = \frac{1}{2}$

$s=1$, $\quad 1 = -\frac{1}{10}(1+1)(-1) + \frac{1}{2}(1+1)(1) + (C+D)(-1)1$

$1 = \frac{1}{5} + 1 - (C+D)$

$C+D = \frac{1}{5} \rightarrow (1)$

$s=3$, $\quad 1 = 2A + 6B + 3(3C+D)$

$1 = -\frac{1}{5} + 3 + 3(3C+D)$

$3C+D = -\frac{3}{5} \rightarrow (2)$

From (1) & (2)

$C = -\frac{2}{5}$ and $D = \frac{3}{5}$

$$y(t) = L^{-1}\left(\frac{-1/10}{s} + \frac{1/2}{(s-2)} + \frac{-\frac{2}{5}s + \frac{3}{5}}{(s-2)^2+1}\right)$$

$$= -\frac{1}{10} + \frac{1}{2}e^{2t} + \frac{1}{5}L^{-1}\frac{-2s+3}{(s-2)^2+1}$$

$$= -\frac{1}{10} + \frac{e^{2t}}{2} - \frac{1}{5}L^{-1}\frac{2s-3}{(s-2)^2+1}$$

$$= -\frac{1}{10} + \frac{e^{2t}}{2} - \frac{1}{5}\left[2L^{-1}\frac{s-2+2}{(s-2)^2+1} - \frac{3}{(s-2)^2+1}\right]$$

$$= -\frac{1}{10} + \frac{e^{2t}}{2} - \frac{1}{5}\left[2\left[L^{-1}\frac{s-2}{(s-2)^2+1} + L^{-1}\frac{2}{(s-2)^2+1}\right] - L^{-1}\frac{3}{(s-2)^2+1}\right]$$

$$= \frac{-1}{10} + \frac{e^{2t}}{2} - \frac{1}{5}\left[2\left[e^{2t}\sin t\right] - e^{2t}\sin t\right]$$

$$= -\frac{1}{10} + \frac{e^{2t}}{2} - \frac{1}{5}e^{2t}\sin t$$

(6) Evaluate:

(a) $L^{-1}\left\{\frac{e^{-5s}}{s}\right\}$

Ans - (a) $L^{-1}\, e^{-5s} \cdot \frac{1}{s} = U_{(t-5)} \cdot f(t-5)$

$F(s) = \frac{1}{s} \Rightarrow f(t) = 1 \Rightarrow f(t-5) = 1$

$L^{-1}\, \frac{e^{-5s}}{s} = U_{(t-5)} \cdot \cancel{f(t-5)}$

(b) $L^{-1}\left\{\frac{e^{-3s}}{s^3}\right\}$

Ans (b) $L^{-1}\, e^{-3s}\, \frac{1}{s^3} = U_{(t-3)} \cdot f(t-3)$

$F(s) = \frac{1}{s^3} \Rightarrow f(t) = \frac{t^2}{2} \Rightarrow f(t-3) = \frac{(t-3)^2}{2}$

$L^{-1}\, \frac{e^{-3s}}{s^3} = U_{(t-3)} \cdot \frac{(t-3)^2}{2}$

(c) $L^{-1}\left\{\frac{s e^{-2s}}{s^2+\pi^2}\right\}$

Ans - (c) $L^{-1}\, e^{-2s} \frac{s}{s^2+\pi^2} = U_{(t-2)}\, f(t-2)$

$F(s) = \frac{s}{s^2+\pi^2} \Rightarrow f(t) = \cos \pi t \Rightarrow f(t-2) = \cos\pi(t-2)$

$L^{-1} \frac{e^{-2s} \cdot s}{s^2+\pi^2} = U_{(t-2)} \cos\pi(t-2)$

(d) $L^{-1}\left\{\frac{e^{-4s}(s+2)}{s^2+4s+5}\right\}$

Ans - (d) $L^{-1}\, e^{-4s} \cdot \frac{s+2}{s^2+4s+5} = U_{(t-4)}\, f(t-4)$

$F(s) = \frac{s+2}{s^2+4s+5} \Rightarrow f(t) = L^{-1} \frac{s+2}{(s+2)^2+1} = e^{-2t} \cdot L^{-1} \frac{s}{s^2+1}$

$= e^{-2t} \cdot \cos t$

$f(t-4) = e^{-2(t-4)} \cos(t-4)$

$L^{-1} \frac{e^{-4s}(s+2)}{s^2+4s+5} = U_{(t-4)} \cdot e^{-2(t-4)} \cos(t-4)$

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(7) Evaluate:

(a) $L^{-1}\left\{ \cot^{-1}(s-a) \right\}$

Ans-(a) $F(s) = \cot^{-1}(s-a)$

$= \dfrac{-1}{(s-a)^2+1}$

$F'(s) = \dfrac{-1}{(s-a)^2+1}$

$L^{-1} F'(s) = -t \cdot f(t)$

$-L^{-1} \dfrac{1}{(s-a)^2+1} = -t \cdot f(t)$

$e^{+at} \cdot \sin t = t \cdot f(t)$

$f(t) = \dfrac{e^{at} \cdot \sin t}{t}$

(b) $L^{-1}\left\{ \log\left(\dfrac{s+a}{s+b} \right) \right\}$

Ans-(b) $F(s) = \log(s+a) - \log(s+b)$

$F'(s) = \dfrac{1}{s+a} - \dfrac{1}{s+b}$

$L^{-1} F'(s) = -t \cdot f(t)$

$L^{-1} \dfrac{1}{s+a} - L^{-1} \dfrac{1}{s+b} = -t\, f(t) \Rightarrow e^{-at} - e^{-bt} = -t\, f(t)$

$f(t) = \dfrac{-e^{-at} + e^{-bt}}{t}$