Solution:

let, $Y_i := (a_i, b_i)$

let, $L = L(Y_1, Y_2, \dots Y_n)$

↳ L.C.S. of A, B

let $X_i := E(L \mid Y_1, Y_2 \dots Y_i)$

X_i is a doob martigale of L w.r.t Y_i

Proof: → ① X_i is function of $Y_1, Y_2 \dots Y_i$

② → $E[|X_i|] < E[|L|]$

$< n$

$\therefore E[|X_i|] < \infty$

③ → $E[X_{i+1} \mid Y_0, Y_1, \dots Y_i]$

$= E[E[L \mid Y_0, \dots Y_{i+1}] \mid Y_0, Y_1, \dots Y_i]$

$= E[L \mid Y_0, Y_1 \dots Y_i]$

$= X_i$

Also, L is 2-Lipschitz since length of subsequence can change by 2 at max, by adding /removing one (a_i, b_i) pair.

Given a_i, b_i are chosen indepently, so Y_i are independent.

→ Applying Azuma inequality,

$$\Pr(|X_n - \mu_n| > \lambda) \leq \frac{2}{e^{\frac{\lambda^2}{2(4n)}}}$$

$$\boxed{\Pr(|X_n - \mu_n| > \lambda) \leq 2e^{-\frac{\lambda^2}{8n}}}$$

(Here each $c_i = 2 \Rightarrow \sum_{i=1}^{n} c_i^2 = 4n$)

* Hence the probability decreases exponentially as λ increase.

* So LCS is concentrated around mean.