CHAPTER 5

Basics of Options

In our study of derivative pricing, we have thus far examined forwards, futures, and swaps, which constitute noncontingent claims. These contracts entail both the writer and the holder having the right and obligation to execute the contract at the expiration date. In this chapter, let us shift our focus to another crucial aspect of derivative markets: *options*. Options are contingent claims that offer a distinct level of flexibility. Buyers have the right, but not the obligation, to buy or sell an underlying asset at a predetermined price within a specified time frame. The buyer enjoys the contingent claim by paying a *premium* to the contract, which is the price paid by the buyer to the seller in order to own the option contract. This unique characteristic of options serves as a valuable instrument for investors seeking strategic opportunities and risk management solutions in financial markets.

In options, there are two primary types, namely, call options and put options. A call option gives the holder the right, but not the obligation, to buy an underlying asset at a predetermined price, known as the *strike price*, within a specified period. On the other hand, a put option grants the holder the right, but not the obligation, to sell an underlying asset at the strike price within a specified timeframe.

A crucial part of options theory is the concept of option premiums and how they are determined. Option premiums are influenced by various factors, including the current market price of the underlying asset, the time remaining until the option expires, and the level of volatility in the market. Factors such as interest rates and dividend payments may also affect option premiums. Understanding these factors is essential for investors as they evaluate potential option trades and assess the risk-reward profile of their strategies. Moreover, comprehending option pricing mechanics allows traders to make informed decisions and effectively manage their portfolios in dynamic market environments.

In this chapter, we discuss some fundamental properties of option premiums. In Section 5.1, we start with an overview of the types of options, including call and put options, as well as the distinctions between American and European options. We also introduce the concepts of option payoffs and gains, providing insight into the relationship between option payoffs and underlying asset prices. An option price comprises intrinsic value and time value. We discuss these two components of option pricing and provide formulas for various cases. Further, in Section 5.2, we examine various bounds for option premiums in a no-arbitrage market environment, including the derivation and application of put-call parity estimates. The discussion of basic concepts and principles of option premiums in this chapter sets the stage for further exploration of option pricing models and advanced trading strategies in subsequent chapters.

5.1 Types of Options

As with forwards and futures, options contracts involve two parties. The party initiating the contract is known as the *writer* or *seller* of the option contract, while the other party is the *holder* or *buyer*. A buyer is said to hold a *long position*, whereas, a seller is said to hold a *short position* in an option contract.

Two types of options exist depending on the trade set in the contract. These are *call options* and *put options*.

Call Option: A *call option* grants the holder the right but no obligation to **buy** the underlying asset for a specified strike price, denoted by K, by (or within) a predetermined time, denoted by T.

Put Option: A *put option* grants the holder the right but no obligation to **sell** the underlying asset for a specified strike price K by (or within) a predetermined time T.

To acquire the contingent position in the contract, the holder must pay a token amount upfront to the writer at the contract's acceptance time, set as $t = 0$. This token amount is known as the *premium* or *price* of the option.

Note

The price of an option (*i.e.* the premium) should not be confused with the future or forward) price in a future (or forward) contract. A future price in a future contract is equivalent to the strike price in an option. Whereas the price of an option (often referred to as option price) is the amount paid by the holder to the writer in order to gain only the right but not the obligation to exercise the option contract (*i.e.* the contingent position in the contract).

Remark 5.1.1.

The following is the list of all the basic parameters needed to define an option:

- Contract issue time, generally taken as $t = 0$;
- Expiration or maturity (date or time or both), denoted by $t = T$;
- Strike price or exercise price, denoted by K;
- *Contract size*, also called a lot, which is the number of units of the underlying assets exercised in a contract; and
- Option premium, denoted by X in general. When it comes to call option premium, we use the notation C and for the put option premium, we use P.

An option (a call or a put option) can further be categorized into two types depending on the time of execution of the contract. These are the *American option* and the *European option*.

American Option: In an *American option*, the contract can be exercised at any time up to the expiration time.

European Option: In a *European option*, the contract is allowed to be exercised only on the expiration time.

Note

In both the NSE and BSE, index options are typically European style, while other stock options are typically American style.

The underlying asset of an option can include stocks, commodities, or foreign currencies, which are settled by physical delivery. However, there are also underlying assets that cannot be physically delivered at expiration, such as stock indices (like Nifty and Sensex) and interest rates. In these cases, settlements are made in cash by paying the difference between the strike price and the spot price.

5.1.1 Payoff and Gain

Let $S(t)$ (also denoted by S_t) be the spot market price of the underlying asset at any time t and K be the strike price of the option. Let the period of the option be $[0, T]$, where T is the expiration time or maturity. We proceed to find the payoff of an option at maturity T.

Observe first that, since the holder of an option has no obligation to exercise the option, the holder will exercise it if and only if there is a positive return. This means that

- the holder of a **call** option will exercise it if and only if $K < S_t$ ($t = T$ if it is an European option);
- the holder of a **put** option will exercise it if and only if $K > S_t$ ($t = T$ if it is an European option).

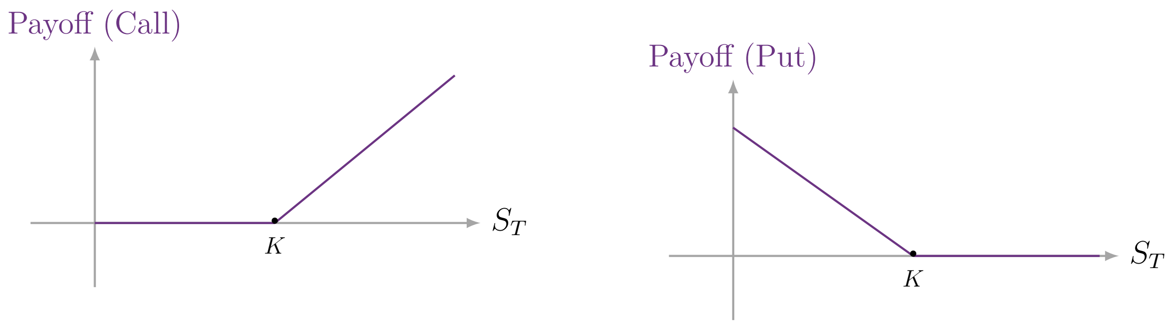

Figure 5.1: Buyer's payoff graphs.

Definition 5.1.2 [Payoff].

A call option is said to be

1. *in-the-money* if $K < S_t$;
2. *at-the-money* if $K = S_t$; and
3. *out-of-the-money* if $K > S_t$,

at any time $t \in [0, T]$. The *payoff* of a call option at the expiration T is defined as

$$C_T = \begin{cases} \max\big(0, S_T - K\big), & \text{for buyer (long position)} \\ \min\big(0, K - S_T\big), & \text{for seller (short position)} \end{cases}$$

A put option is said to be

1. *in-the-money* if $K > S_t$;
2. *at-the-money* if $K = S_t$; and
3. *out-of-the-money* if $K < S_t$,

at any time $t \in [0, T]$. The *payoff* of a put option at the expiration T is defined as

$$P_T = \begin{cases} \max\big(0, K - S_T\big), & \text{for buyer (long position)} \\ \min\big(0, S_T - K\big), & \text{for seller (short position)} \end{cases}$$

Graphical illustrations of the buyer's payoff s are shown in Figure 5.1.

Problem 5.1.

Investor-C bought a 50-strike call option on a stock, and Investor-P bought a 112-strike put option on the same stock with the same expiration. Let both the options be European, and there are no transaction costs (frictionless). If the investor-C's payoff is 10, then find the payoff of Investor-P. **Answer:** 52

Remark 5.1.3.

The payoff of an option is the value of the option at the expiration date. We can also define the value of an option at any time $t \in [0, T]$.

The *intrinsic value* of a call option is defined as

$$V_c(t) = \max\big(0, S(t) - K\big), \quad t \in [0, T]$$

and the *intrinsic value* of a put option is defined as

$$V_p(t) = \max\big(0, K - S(t)\big), \quad t \in [0, T].$$

Observe that we have not incorporated the option premium while defining the payoff in the above definition. The gain (or loss) of an option comprises the payoff in addition to the future value of the premium.

Let X be the premium paid for an option contract, and r be the prevailing interest rate continuously compounded. Then the gain (or loss) in a call option is defined as

$$G_T = \begin{cases} C_T - Xe^{rT}, & \text{for long call} \\ Xe^{rT} - C_T, & \text{for short call} \end{cases}$$

Similarly, we can define the gain (or loss) in a put option. Illustrations of the gain (depicted by dashed lines) along with the payoff (represented by solid lines) in four possible scenarios are depicted in Figure 5.2.

Problem 5.2.

An investor buys a 14900-strike put on Nifty when the Nifty is trading in the spot market at 14875 points. The premium for the put is ₹27. Assume that the lot size is 100 units and the expiration is 91 days. Considering the prevailing interest rate as 6%, calculate the investor's gain if the index declines by 75 points from the strike price at expiration. **Answer:** 4759.31

Hints:
Since the interest rate scheme is not specified, one has to take it as continuous compounding. Also, consider 365 days a year.

Problem 5.3.

Consider a European call option with a 90-strike and 6 months expiration. Given that the underlying stock takes the price ₹87, ₹92, or ₹97 with probability 1/3 each on the expiration date. If the option is bought for ₹8 and the prevailing interest rate is 9% continuously compounded, then find the expected gain (or loss) for the long call. **Answer:** -5.3682

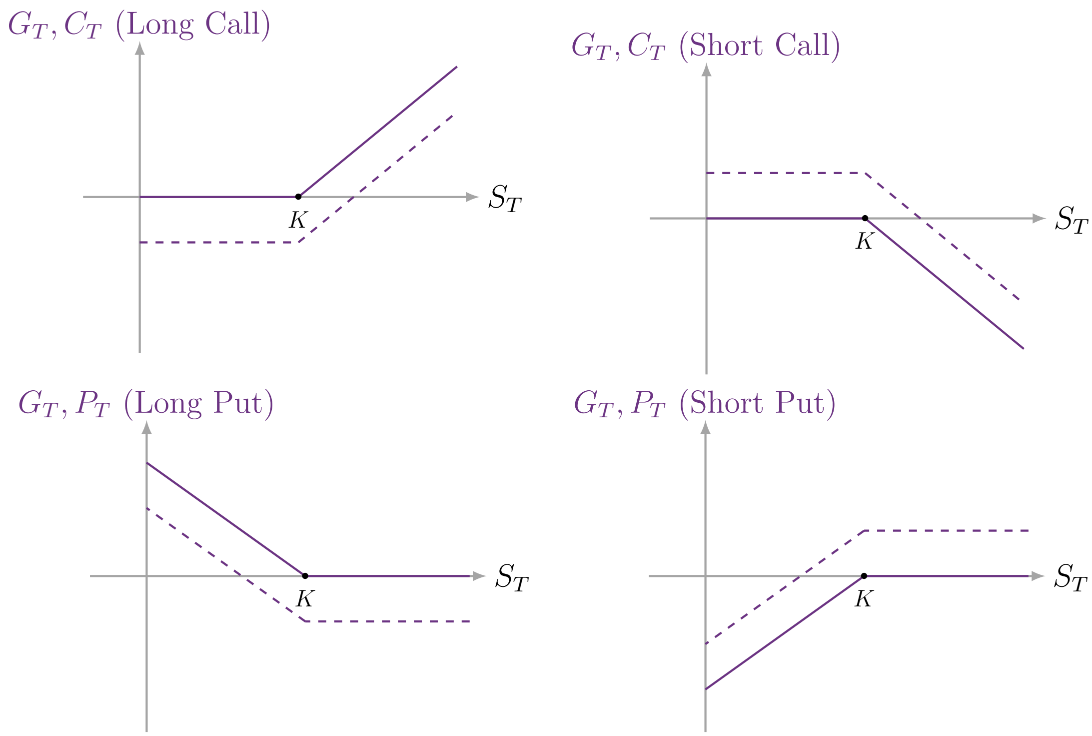

Figure 5.2: Gain (dashed lines) and Payoff (solid lines) graphs.

Remark 5.1.4.

Let us give the formula for the gain in an option using the discrete compounding scheme with frequency m.

The future value of the premium X at time T with discrete compounding scheme with frequency m is given by

$$\mathrm{FV}(X) = X\left(1+\frac{r}{m}\right)^{mT}.$$

Therefore, the required gain is given by

$$G_T = \begin{cases} C_T - X\left(1+\frac{r}{m}\right)^{mT}, & \text{for long call} \\ X\left(1+\frac{r}{m}\right)^{mT} - C_T, & \text{for short call} \\ P_T - X\left(1+\frac{r}{m}\right)^{mT}, & \text{for long put} \\ X\left(1+\frac{r}{m}\right)^{mT} - P_T, & \text{for short put.} \end{cases}$$

Option Type	Payoff Type	Strike	Jan′21	Feb′21	Mar′21
Call	In-the-money	1240	63.60	70.00	114.05
		1260	53.65	48.75	104.40
	Out-of-the-money	1280	44.80	43.85	95.40
		1300	37.05	57.10	87.00
Put	Out-of-the-money	1240	40.75	60.00	87.80
		1260	50.60	68.00	98.00
	In-the-money	1280	61.55	199.00	108.85
		1300	74.10	214.30	120.25

Table 5.1: Option data of a stock as of January 1st, 2021, for various expirations when the spot price of the stock was ₹1260.45 per share.

5.1.2 Time Value in an Option Price

One may argue for considering the intrinsic value of an option as the price of the option. However, the option price is positive at any time $t \in [0, T)$, whereas the option value, can be zero if the option is out-of-the-money. This shows that an option price includes more than just the intrinsic value.

The option price can be decomposed into (at least) two parts, namely, the intrinsic value and the time value, and is written as

$$\text{Option Price} = \text{Intrinsic Value} + \text{Time Value}.$$

We use the notations $C^e(t)$ (or C^e_t) and $C^a(t)$ (or C^a_t), $t \in [0, T]$, to represent European and American call option prices, respectively, and similar notations for other cases.

From the above decomposition, the *time value* can be written as

$$\text{TV}(t) = \begin{cases} C^e(t) - \max\big(0, S(t) - K\big), & \text{for European call} \\ P^e(t) - \max\big(0, K - S(t)\big), & \text{for European put} \\ C^a(t) - \max\big(0, S(t) - K\big), & \text{for American call} \\ P^a(t) - \max\big(0, K - S(t)\big), & \text{for American put.} \end{cases}$$

Example 5.1.5.

Consider a stock on 1st January 2021 (taken as $t = 0$) which was trading at ₹1260.45 per share. Table 5.1 shows the option price for different expirations and different strike prices.

Our interest is to obtain the time value of all the options at $t = 0$ for all the data shown in Table 5.1.

Let us take the 1240-strike call option with Jan′21 expiration, where the option price was $C^a(0) = 63.6$ and the intrinsic value of the option was

$$\max\big(0, S(0) - K\big) = \max\big(0, 1260.45 - 1240\big) = 20.45.$$

Therefore, the time value of the option is

$$\mathrm{TV}_c^a(0) = 63.6 - 20.45 = 43.15.$$

The same 1240-strike put option was out-of-the-money and therefore, the intrinsic value is given by

$$\max\big(0, K - S(t)\big) = \max\big(0, 1240 - 1260.45\big) = 0.$$

Hence, the time value of the option as on 1st January is

$$\mathrm{TV}_p^a(0) = 40.75.$$

Similarly, we can find the time value for other strike prices and for other expirations.

5.2 Bounds for Premium

The primary focus of options theory is to determine an appropriate price (premium) for a given option. As we progress into the theory, we will see that this is a difficult task. However, obtaining bounds for the price of an option in a no-arbitrage market is relatively straightforward. Throughout this section, we assume that the market allows short selling and that lending and borrowing occur at the same prevailing interest rate r, with continuous compounding.

5.2.1 For European Options

Let us first explore the simplest case of a European option, where the underlying stock does not pay dividends during the option period.

Theorem 5.2.1 [European Call on Non-Dividend-Paying Stock].

Consider a European call option with

- K-strike;
- period $[0, T]$; and
- premium C_0^e,

where the underlying stock with spot price S_0 pays no dividend during the option period. If the market does not allow arbitrage, then

$$\max\big(0, S_0 - Ke^{-rT}\big) \le C_0^e \le S_0,$$

where r is the prevailing annual interest rate continuously compounded.

Proof.

First, let us prove the upper bound.

Assume the contrary that there is a K-strike call option with premium $C_0^e > S_0$. Then, construct a portfolio using the following trades:

- short K-strike call option;
- buy the stock in the spot market;
- invest $C_0^e - S_0$ in a risk-free interest rate instrument up to the expiration date of the call option.

The portfolio is given by

$$\Pi_0 = (\boldsymbol{b}, \boldsymbol{s}, \boldsymbol{c}); \quad \boldsymbol{b} = (1), \boldsymbol{s} = (1), \boldsymbol{c} = (-1).$$

Here, we assumed that one unit of the risk-free instrument costs $C_0^e - S_0$ (one may simply assume the risk-free instrument as the savings bank account). The value of the portfolio at $t = 0$ is

$$V(\Pi_0)(0) = 1 \times (C_0^e - S_0) + 1 \times S_0 + (-1) \times C_0^e = 0.$$

At time $t = T$, the value of the portfolio Π_0 is given by

$$V(\Pi_0)(T) = (C_0^e - S_0)e^{rT} + \min\left\{K, S_T\right\}.$$

Since $K > 0$, $S_T > 0$, and by our assumption $C_0^e > S_0$, we see that $V(\Pi_0)(T) > 0$ and this happens with probability 1. This shows that Π_0 is an arbitrage portfolio. This contradicts the no-arbitrage market assumption.

Let us now prove the lower bound.

It is clear that $C_0^e \geq 0$. Therefore, we assume that $S_0 - Ke^{-rT} > 0$ and prove that $S_0 - Ke^{-rT} \leq C_0^e$.

Assume the contrary $S_0 - Ke^{-rT} > C_0^e$. Then, we construct a portfolio by making the following trades:

- short a stock in the spot market at the price of S_0 per share;
- buy one K-strike call option by paying the premium C_0^e; and
- invest $S_0 - C_0^e$ in a risk-free interest rate instrument up to the expiration date of the call option.

The portfolio is given by

$$\Pi_0 = (\boldsymbol{b}, \boldsymbol{s}, \boldsymbol{c}); \quad \boldsymbol{b} = (1), \boldsymbol{s} = (-1), \boldsymbol{c} = (1),$$

where we have assumed that one unit of the risk-free asset is $S_0 - C_0^e$. The value of the portfolio at $t = 0$ is

$$V(\Pi_0)(0) = 1 \times (S_0 - C_0^e) + (-1) \times S_0 + 1 \times C_0^e = 0.$$

At time $t = T$, make the following trades:

- close the risk-free investment;
- if $S_T > K$, then exercise the call option;
  otherwise, buy one stock at the spot market; and
- close the short position at the spot market.

The payoff of Π_0 is

$$V(\Pi_0)(T) = (S_0 - C_0^e)e^{rT} - \min\left\{K, S_T\right\}.$$

By our assumption, we get (how?)

$$V(\Pi_0)(T) > 0$$

with probability 1. Thus, Π_0 is an arbitrage portfolio.

Note

While Theorem 5.2.1 is proven for $t = 0$, the result can be extended to any $t \in [0, T]$, and the estimate is given by:

$$\max\left(0, S_t - Ke^{-r(T-t)}\right) \leq C_t^e \leq S_t.$$

Similar modifications in the estimates apply to all theorems proved in this section.

Theorem 5.2.2 [European Put on Non-Dividend-Paying Stock].

Consider the European put option

- K-strike;
- period $[0, T]$;
- premium P_0^e,

where the underlying stock with spot price S_0 pays no dividend during the option period. If the market does not allow arbitrage, then

$$\max\left(0, Ke^{-rT} - S_0\right) \leq P_0^e \leq Ke^{-rT},$$

where r is the prevailing annual interest rate continuously compounded.

Proof of this theorem is left as an exercise.

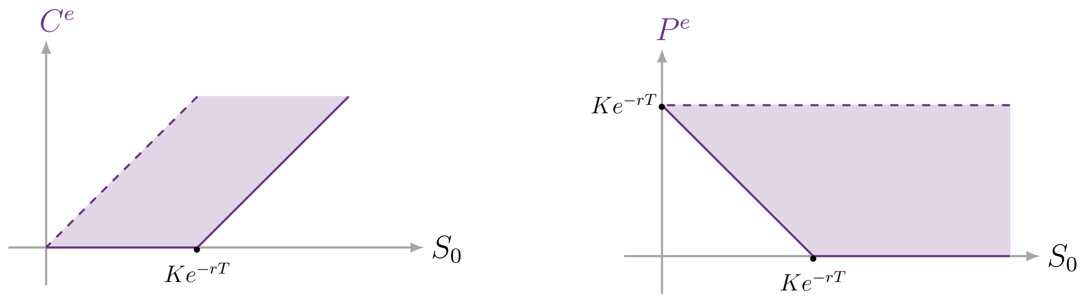

Figure 5.3: An illustration of the bounds for European call (left) and put (right) options for no-dividend-paying stocks.

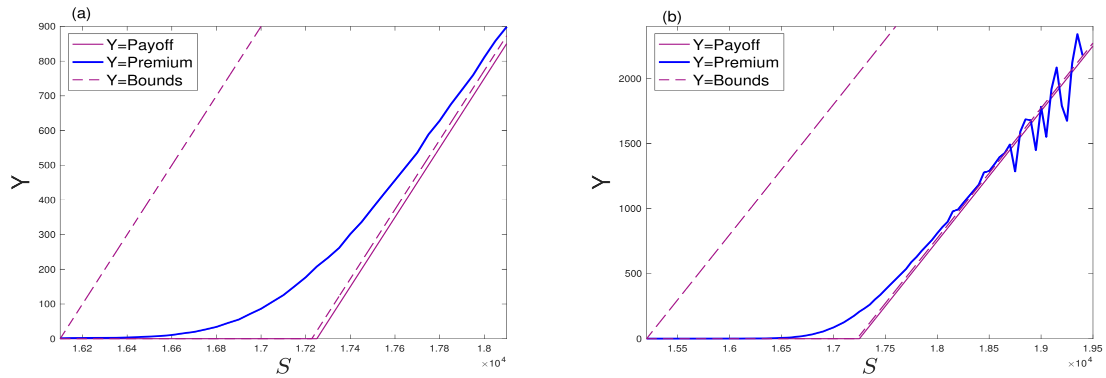

Figure 5.4: A real-time illustration of the bounds of a European call option premium proved in Theorem 5.2.1. Details are in Example 5.2.3.

The bounds obtained in the above theorem are graphically illustrated in Figure 5.3.

Example 5.2.3 [A real-time illustration].

Let us illustrate the bounds proved in Theorem 5.2.1 using real-time data. To this end, we have obtained the option chain data for NIFTY with an expiration date of February 24, 2022, from the NSE website

`https://www.nseindia.com/option-chain`

on February 18, 2022, after the market closed. On that day, NIFTY closed at approximately $17,276$ points. The last traded price (LTP) of the call options with various strike prices were extracted from the downloaded option chain table. These premiums are depicted in Figure 5.4 by a solid blue line, where the roles of the strike price and spot price are interchanged. The dashed lines represent the corresponding bounds obtained from Theorem 5.2.1, while the payoff graph is illustrated by a solid line.

The lower bound is calculated using $K = 17250$ and $r = 0.0675$, which is approximately the 10-year government bond yield as of the last week of February 2022. In Figure 5.4(a) a zoomed view around K is provided, representing the nearly at-the-money options.

From this figure, it is evident that there are no arbitrage opportunities according to the considered interest rate r. This can be attributed to the substantial liquidity present in these options. However, as we move away from the at-the-money option, liquidity diminishes, and sometimes visible arbitrage opportunities emerge, particularly at the market's closing time, due to speculators. This phenomenon is clearly observed in Figure 5.4(b) around strikes greater than 18700, which correspond to far in-the-money options. It is worth noting that at the next day's market opening, such arbitrage gaps either cease to exist or are swiftly filled by arbitrageurs.

Problem 5.4 [European Options on Dividend-Paying Stock].

Consider the European (call or put) option

- K-strike;
- period $[0, T]$; and
- premium C_0^e for call and P_0^e for put,

where the underlying stock with spot price S_0 pays a dividend D_0 at some time during the option period. If the market does not allow arbitrage, then show that

$$\begin{aligned} \max\left(0, S_0 - D_0 - Ke^{-rT}\right) &\leq C_0^e \leq S_0 - D_0, && \text{for call option;} \\ \max\left(0, Ke^{-rT} + D_0 - S_0\right) &\leq P_0^e \leq Ke^{-rT}, && \text{for put option,} \end{aligned}$$

where r is the prevailing annual interest rate continuously compounded.

Project 5.1.

Write a Python code that takes the following inputs:

1. The name of the CSV file containing an option chain for NIFTY (download the CSV file from the website provided in Example 5.2.3);
2. The spot market value of NIFTY corresponding to the option chain given in the CSV file;
3. The prevailing interest rate;
4. The expiration time T.

Furthermore, the Python code should perform the following:

1. Read the CSV file;
2. Identify all arbitrage opportunities in the option chain based on the estimates provided in Theorem 5.2.1 and Theorem 5.2.2;
3. Display the identified opportunities along with the arbitrage gain.