CHAPTER 8

Trading Stratergies and Sensitivity

This chapter is divided into two parts. Section 8.1 explores popular trading strategies using options, while Section 8.2 covers the topic of Greeks where we study the sensitivity of option price by analysing the effect of different parameters involved in the Black-Scholes formula.

8.1 Option Trading Strategies

There are many strategies to make portfolios by combining options and the underlying assets. One can also build portfolios by combining several options. We discuss here a few which are more popular.

Options are primarily used for hedging. However, speculators come into the market as a consequence of high liquidity. Speculators are also attracted to a great advantage of *leverage* in trading options rather than trading directly in the underlying assets. The idea of leverage is well understood through simple examples.

Example 8.1.1 [Leverage].

Assume that a stock traded at the spot market for ₹1950 per share on 1$^{\text{st}}$ February and the same stock was trading at ₹2150 on 25$^{\text{th}}$ February. Mr. Megh bought 100 shares of the stock on 1$^{\text{st}}$ February by paying ₹1,95,000 and sold it on 25$^{\text{th}}$ February and obtained ₹20,000, which gave him 10.26% on his initial investment.

On the other hand, Mrs. Sahana bought a call option (with contract size 100 shares) for 1950-strike, expiration 25th February, and the premium of ₹50 per share. Hence, she paid ₹5000 ($= X$) as the premium to the call option. On the expiration date, the spot price was $S_T = 2150$. Since the strike price was less than the spot price of the stock at expiration, Mrs. Sahana's call option was in-the-money. So, she would certainly exercise the option on the expiration and gone for the cash settlement of

$$C_T = (2150 - 1950) \times 100 = 20,000 \text{ (payoff)}.$$

Since she paid 5000 as premium, the gain in the trade (with $r = 0$) was

$$G_T = 20000 - 5000 = 15,000.$$

Mrs. Sahana obtained ₹5000 less than Mr. Megh's return in the spot market trade. However, Mrs. Sahana's gain was 300% of her initial investment, whereas Mr. Megh obtained only 10.26%.

Obtaining a higher percentage return from an option is called *leverage*. In other words, the call option that Mrs. Sahana opted for had provided a leveraged return to her.

Caution: On the contrary to the scenario considered above, if the spot price of the stock on 25th February was at ₹1949, say, just ₹1 less than the strike price. Then, compute the percentage loss for Mrs. Sahana and Mr. Megh.

In the above example, the options trader is a *speculator*. This is because the trader's intention was not to reduce the uncertainty in the price fluctuation of a stock in the portfolio. Instead, the trader somehow predicted that the stock price would go up (or merely guessed it) and took a risk in the trade.

Hedgers are just opposite to speculators. They are more worried about the risk involved in their investment, and they do not mind compromising some portion of their return to reduce the risk. This is something similar to insurance. A hedge can be understood through simple examples.

Example 8.1.2 [Hedge using protective put].

Assume that Mr. Megh bought 100 shares of a stock at the rate of ₹1950 per share on 1st February. He anticipated an up move in the stock price by the end of February, and hence he had the intention of selling it by the month-end. However, he was worried about the possibility of the stock price going downwards. He decided to reduce his risk by looking for an option.

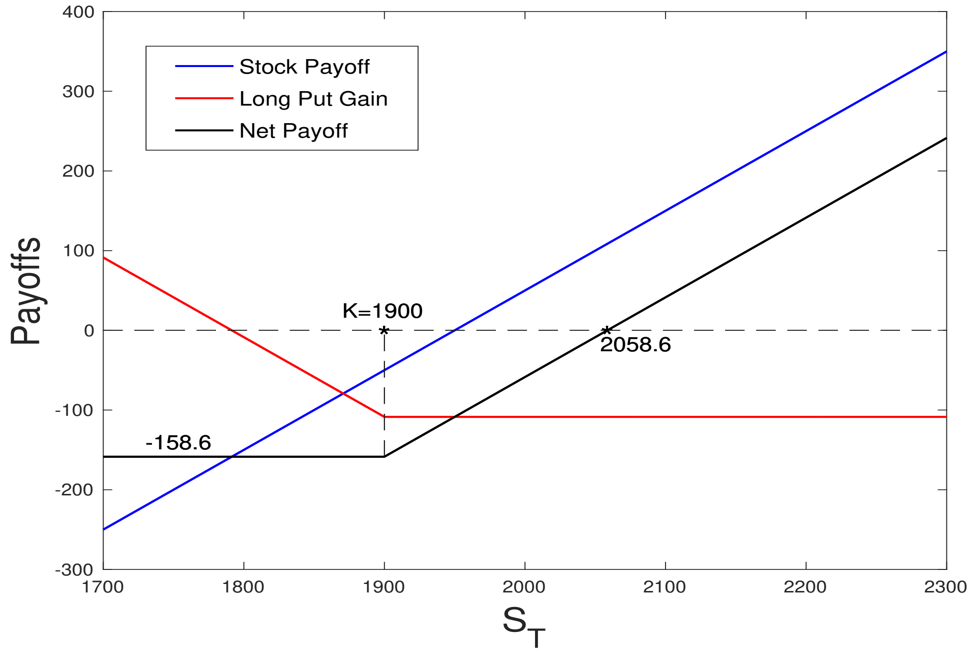

Figure 8.1: Protective put graph for Example 8.1.2.

Mr. Megh noticed a put option with 1900-strike, expiration 25th February, and the premium of ₹108.6 per share. He took a long position in one put option, whose contract size (*i.e.* lot) was 100 shares.

Mr. Megh's long put trade gave him a downward protection, which can be seen from the *net gain* given by

$$\begin{aligned}\mathcal{N}(S_T) &= \text{Stock payoff} + \text{Long put gain/loss} \\ &= (S_T - S_0) + G_T,\end{aligned}$$

where $S_0 = 1950$ and G_T is calculated with $r = 0$.

Figure 8.1 depicts the payoff graph. One can see that the long put option position limited Mr. Megh's loss (which we call *limited loss*) in the spot market trade to -158.6 per share, and the profit is unbounded. One can also see that Mr. Megh makes a profit if $S_T > 2058.6 =: S_*$.

Protecting a risky investment from losing too much using some counter trades (in the present case, it is the long put) is a *hedge*. In other words, the put option Mr. Megh opted for had provided a hedge for his exposure to the stock that he traded in the spot market.

Problem 8.1 [Hedge using covered call].

Obtain a hedge for Mr. Megh in the above example using the following call option:

- 1990-strike;
- expiration 25$^{\text{th}}$ February;
- contract size 100; and
- premium ₹91 per share.

Obtain the net gain function, limited loss, and the minimum stock price S_* such that $S_T > S_*$ gives profit to Mr. Megh. Also, draw the graphs of stock payoff, gain/loss of the call, and net gain.

Hints:
See whether you have to enter into the call option trade as a writer or a holder. In other words, see whether you have to take a short or a long position in the call option in order to hedge the spot market trade. For the net gain function, give the complete expression, including the expression for gain/loss; just writing G_T is not enough.

Remark 8.1.3 [Covered put].

A *covered put* is a portfolio involving a short position in the spot market and a short position in the corresponding option.

8.1.1 Spreads

A *spread* is a portfolio consisting of options of the same type (either all calls or all puts). There are three basic kinds of spreads:

1. A *vertical spread* (or *price spread*) is a portfolio in which the options have the same expiration date but different strike prices.
2. An *horizontal spread* (or *calendar spread*) is a portfolio in which the options have the same strike price but different expiration dates.
3. A *diagonal spread* is a portfolio in which the options have different strike prices and expiration dates.

We restrict to only vertical spreads and discuss some popular vertical spreads among traders.

Bull Spread

Bull spread is a portfolio created when a trader anticipates an upward trend in the underlying asset price.

Definition 8.1.4.

For given $0 < K_1 < K_2$, the portfolio consisting of a long position in a K_1-strike call (put) option and a short position in a K_2-strike call (put) option with the same expiration and the same underlying asset is called a ***bull spread***.

A bull spread is an example of a vertical spread. The general form of the *net gain function* for bull spread is given by

$$\begin{aligned}
\mathcal{N}(S_T) &= \text{Gain in Long call} + \text{Gain in Short call} \\
&= \big(\max(0, S_T - K_1) - X_1\big) + \big(X_2 - \max(0, S_T - K_2)\big) \\
&= \begin{cases} X_2 - X_1, & \text{if } S_T \le K_1 \\ S_T - K_1 + X_2 - X_1, & \text{if } K_1 \le S_T \le K_2 \\ K_2 - K_1 + X_2 - X_1, & \text{if } K_2 \le S_T. \end{cases}
\end{aligned}$$

Observe that in order to maximize the profit, we have to look for maximizing $K_2 - K_1$ and minimize $X_2 - X_1$, wherever the later is negative. Note that $X_2 - X_1$ is typically negative in practice. Opting for out-of-the-money calls, which generally trade with lower premiums, can be advantageous in this regard.

Example 8.1.5.

Mrs. Sahana believes that the share price of a stock will increase to ₹2150 in a month from its current price of ₹1950. In other words, Mrs. Sahana is *bullish* on the stock. She can perform one of the following trades:

- she can buy the stock at the spot market; or
- she can buy a future in the futures market; or
- she can buy a 1950-strike call option; or
- she can write a 1950-strike put option.

The first choice is expensive, and all the choices are highly risky. However, if her prediction goes correct, they all will give a considerable profit.

Alternatively, Mrs. Sahana can go for a bull spread if she is ready to give up a part of the predicted profit.

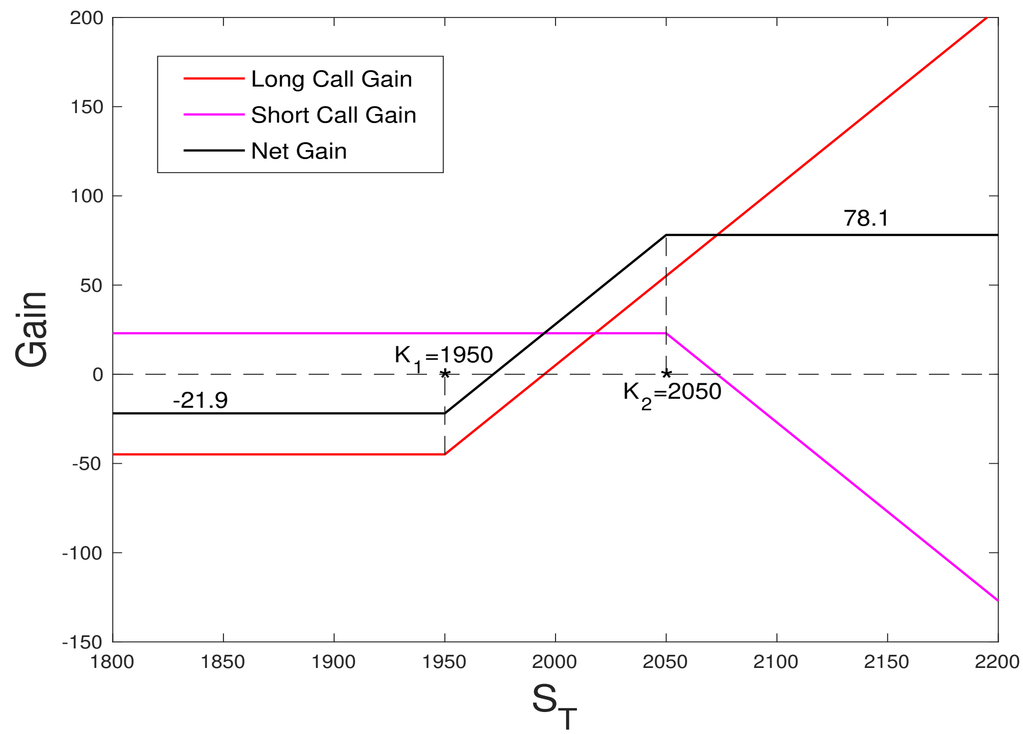

Figure 8.2: Gain graphs for Example 8.1.5.

Mrs. Sahana made the following portfolio 1st February:

- She bought the 1950-strike European call option for the premium ₹44.90 per share; and
- sold the 2050-strike European call option for the premium ₹23 per share,

both have expiration 25th February. From the above description, we can see that the above trades give rise to a portfolio, which is a bull spread.

The net gain of the bull spread is given by

$$\mathcal{N}(S_T) = \begin{cases} -21.9, & \text{if } S_T \leq 1950 \\ S_T - 1971.9, & \text{if } 1950 \leq S_T \leq 2050 \\ 78.1, & \text{if } 2050 \leq S_T. \end{cases}$$

The gain graphs are depicted in Figure 8.2. Observe that Mrs. Sahana had invested ₹21.9 per share. The maximum loss that she can take in this portfolio is -21.9, and the maximum profit she can achieve is 78.1.

Problem 8.2.

Consider the following active call options trading at the market and all these options have the same expiration:

Strike	Premium
1980	40.25
2040	25.1

Construct a bull spread out of these call options. Obtain the general form of the net gain function and also obtain the net gain function for this problem. Draw the gain graphs for all the traded options and also draw the graph of the net gain function.

Note

One can also create a bull spread with put options.

Problem 8.3.

Consider two active European put options

Strike	Premium
K_1	X_1
K_2	X_2

trading at the market with same underlying and expiration with $K_1 < K_2$. Construct a bull spread out of these put options and obtain the general form of the net gain function. In particular, if $K_1 = 1500$, $X_1 = 30.05$, $K_2 = 1520$, and $X_2 = 40.45$, then obtain the net gain function for the bull spread. Draw the gain graphs for all the traded options and also draw the graph of the net gain function.

Bear Spread

Bear spread is a portfolio formed by a trading having a bearish view on the underlying asset price.

Definition 8.1.6 [Bear Spread].

For given $0 < K_1 < K_2$, a ***bear spread*** is a portfolio can be created by buying a K_2-strike call (put) option and write a K_1-strike call (put) option with the same underlying asset and a same expiration date.

Problem 8.4.

Obtain the general form of the net gain function of the following:

1. a bear call spread
2. a bear put spread

Example 8.1.7.

On 2nd March 2020, Mrs. Sahana felt that the entire market would crash in a month. In particular, she felt that HDFC Bank stock, which was trading at ₹1200 per share on the day, would crash up to ₹1100 per share. In other words, Mrs. Sahana was *bearish* on the HDFC Bank's stock.

Mrs. Sahana constructed a bear spread out of the stock options as follows:

- she bought a European put option of 1180-strike with premium ₹28; and
- she wrote another European put option of 1140-strike with premium ₹20,

both had expiration 28th March 2020.

The net gain of the bear spread is given by

$$\mathcal{N}(S_T) = \begin{cases} 32, & \text{if } S_T \leq 1140 \\ 1172 - S_T, & \text{if } 1140 \leq S_T \leq 1180 \\ -8, & \text{if } 1180 \leq S_T. \end{cases}$$

The gain graphs are depicted in Figure 8.3. Observe that Mrs. Sahana had invested ₹8 per share to build the portfilio. The maximum loss that she can take in this portfolio is -8, and the maximum profit she can achieve is 32.

Note

One can also create a bear spread with call options.

Problem 8.5.

The following are the active put options trading at the market on 2nd March 2020:

Strike	**Premium**
1120	21.65
1140	20
1160	29.25
1180	28.00
1200	29.30

Mrs. Sahana (in Example 8.1.7) is looking for another bear spread. All the options were with expiration 28th March 2020. Suggest another bear spread than the one opted in Example 8.1.7. Obtain the net gain per share and draw the gain graphs.

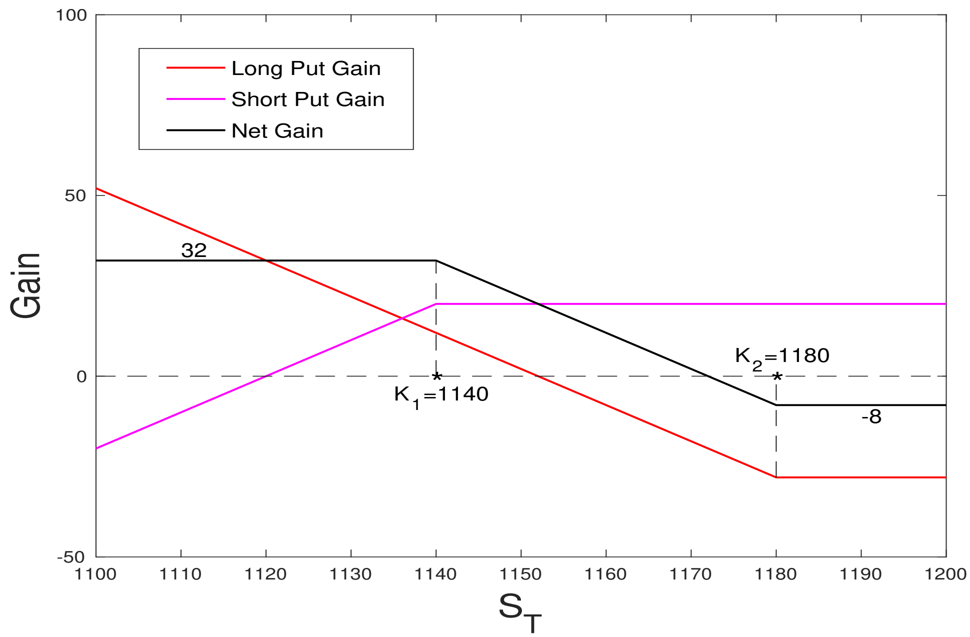

Figure 8.3: Gain graphs for Example 8.1.7.

Butterfly Spread

Definition 8.1.8.

Let $0 < K_1 < K_2 < K_3$, and $K_2 \in [S_0 - \epsilon, S_0 + \epsilon]$ for some sufficiently small $\epsilon > 0$. Buy two call options, one in each of K_1-strike and K_3-strike. Write two call options with K_2-strike. All options have the same underlying and expiration. The resulting portfolio is a *butterfly spread*.

Note

The role of ϵ in the above definition is to indicate that K_2-strike option is very near to the in-the-money option. It is not always necessary that $\epsilon \ll 1$. Practically, K_2 can be $\pm 1\%$ of the spot price or even more depending on the situations.

The general form of the net gain function for butterfly spread is given by

$$\begin{aligned}
\mathcal{N}(S_T) &= \text{Gain in Long call-1} + 2 \times \text{Gain in Short call-2} + \text{Gain in Long call-3} \\
&= \big(\max(0, S_T - K_1) - X_1\big) + 2\big(X_2 - \max(0, S_T - K_2)\big) \\
&\qquad\qquad + \big(\max(0, S_T - K_3) - X_3\big) \\
&= \begin{cases}
-X_1 + 2X_2 - X_3, & \text{if } S_T \le K_1 \\
S_T - K_1 - X_1 + 2X_2 - X_3, & \text{if } K_1 \le S_T \le K_2 \\
-S_T - K_1 + 2(K_2 + X_2) - X_1 - X_3, & \text{if } K_2 \le S_T \le K_3 \\
-K_1 + 2K_2 - K_3 - X_1 + 2X_2 - X_3, & \text{if } K_3 \le S_T.
\end{cases}
\end{aligned}$$

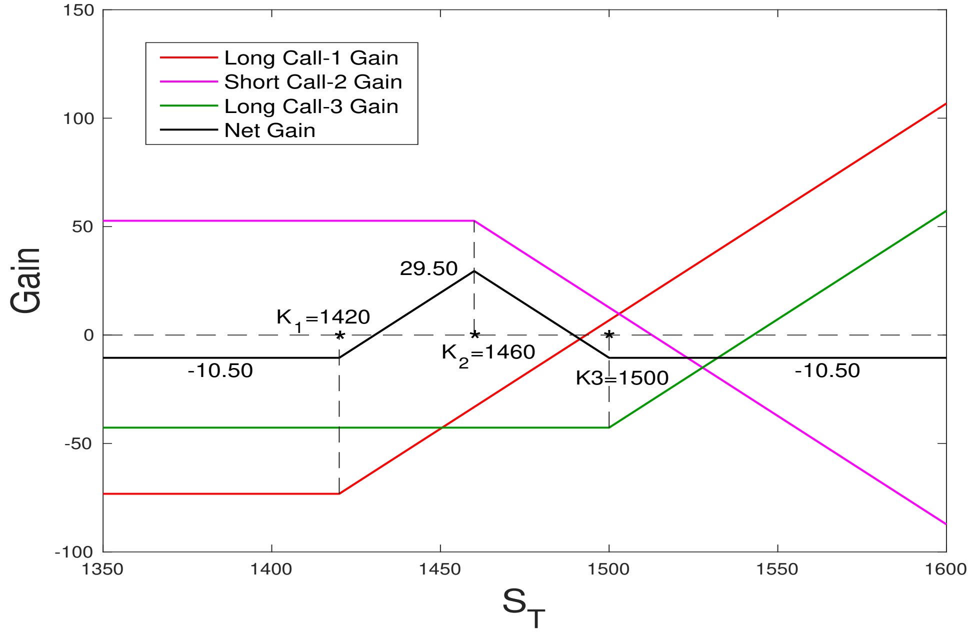

Figure 8.4: Gain graphs for Example 8.1.9.

Example 8.1.9.

Assume that Titan was trading at ₹1457 ($= S_0$) per share on 1st February 2021. Mrs. Sahana felt that the share price would not vary significantly during the month. Based on her prediction, she made the following trades:

- she wrote two call options with 1460-strike at the premium of ₹52.7 per share;
- she bought one call option with 1420-strike for ₹73.2 per share; and
- she also bought one call option with 1500-strike for ₹42.7 per share.

With these trades, she created a portfolio which is a butterfly spread.

The net gain of the butterfly spread is given by

$$\mathcal{N}(S_T) = \begin{cases} -10.5, & \text{if } S_T \leq 1420 \\ S_T - 1430.5, & \text{if } 1420 \leq S_T \leq 1460 \\ 1489.5 - S_T, & \text{if } 1460 \leq S_T \leq 1500 \\ -10.5, & \text{if } 1500 \leq S_T. \end{cases}$$

The gain graphs are depicted in Figure 8.4.

Note
One can also create a butterfly spread with put options.

Problem 8.6.

The following are the active call options trading at the market on 1^{st} February 2021, and Mrs. Sahana (in Example 8.1.9) is looking for a better butterfly spread:

Strike	Premium
1400	89.35
1420	73.20
1440	54.25
1460	52.70
1480	51.80
1500	42.70
1520	26.60

All the options are with expiration 25^{th} February 2020. Suggest a butterfly spread different from the one opted in Example 8.1.9. Obtain the net gain per share and draw the gain graphs. Also, write the portfolio and find its initial value using the mathematical notations introduced in the course.

Problem 8.7.

Consider the following active put options trading at the market and all these options have the same expiration :

Strike	Premium
140	7.60
160	9.25
180	8.00

The current spot market price of the underlying stock is ₹160.2. Construct a butterfly spread out of these put options. Obtain the general form of the net gain function and also obtain the net gain function for this problem. Draw the gain graphs for all the traded options and also draw the graph of the net gain function.

8.1.2 Volatility Speculation

The bull and bear spreads are directional speculations, whereas the butterfly spread is a volatility speculation. In this section, we introduce two more strategies used as volatility speculations.

Straddle

A *straddle* is a portfolio made by taking long positions in a call and a put simultaneously with the same strike price, underlying asset, and the expiration.

For a given $K > 0$, the general form of the net gain function of a straddle is given by

$$\begin{aligned}
\mathcal{N}(S_T) &= \text{Gain in Long call} + \text{Gain in Long put} \\
&= \big(\max(0, S_T - K) - C\big) + \big(\max(0, K - S_T) - P\big) \\
&= \begin{cases} -S_T + K - C - P, & \text{if } S_T \le K \\ S_T - K - C - P, & \text{if } K \le S_T. \end{cases}
\end{aligned}$$

A trader anticipating a less volatility in the underlying asset can take a *short straddle*, means that the trader should take short positions in both options. On the other hand, if a trader thinks that the stock price will be volatile, but not clear about which direction it will move, then a (long) straddle may be preferred.

Strangle

The disadvantage of straddle is that both the options have the same strike price. Thus, we have to go for either both at-the-money options or one in-the-money and another out-of-the-money options. In any case, we can see that the total price of the portfolio will be high. Generally, out-of-the-money options are cheaper than in-the-money options. Therefore, if we relax the condition of having the same strike price, we can building portfolio with two long positions, one in call and another in put, but now with different strike prices. Such a portfolio is called a *strangle*.

For given real numbers $0 < K_1 < K_2$, a portfolio consisting of a long K_1-strike put option and a long K_2 strike call option on the same underlying and with same expiration is called a *strangle* (also called *long strangle*). A *short strangle* is a short position in a strangle, which means selling a K_1-strike put option and a K_2 strike call option on the same underlying and with same expiration.

Problem 8.8.

Determine the net gain function for both a long strangle and a short strangle.

Given a pair of real numbers $0 < K_1 < K_2$, find the positive real numbers S_* and S^* such that the net gain function in the corresponding (long) strangle satisfies:

$$\mathcal{N}(S_T) > 0, \text{ for all } S_T < S_* \text{ and } S_T > S^*.$$