

# Introduction to AI ML

EE1390

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# Problem

Find the equation of the tangent to the circle at the point

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

whose center is the point of intersection of the straight lines

$$\begin{pmatrix} 2 & 1 \end{pmatrix} x = 3$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix} x = 1$$

Let  $O$  be the center of the circle  $C$  given by the intersection of the straight lines

$$\begin{pmatrix} 2 & 1 \end{pmatrix} x = 3$$

$$\begin{pmatrix} 1 & -1 \end{pmatrix} x = 1$$

The intersection point of two lines given by

$$n_1^T x = p_1$$

$$n_2^T x = p_2$$

can be given by

$$\begin{pmatrix} n_1^T \\ n_2^T \end{pmatrix} x = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

let

$$N^T = \begin{pmatrix} n_1^T \\ n_2^T \end{pmatrix} \text{ and } p = \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

Thus,

$$N^T x = p$$

$$N = \begin{pmatrix} n_1 & n_2 \end{pmatrix}$$

$$x = N^{-T} p$$

Since O is the point of intersection

$$O = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{-2}{3} \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$O = \begin{pmatrix} \frac{4}{3} \\ \frac{1}{3} \end{pmatrix}$$

$$\text{Let } k = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$t = k * OA$$

$$= \begin{pmatrix} \frac{4}{3} \\ \frac{-1}{3} \end{pmatrix}$$

$$Q = t^T$$

$$= \begin{pmatrix} \frac{4}{3} & \frac{-1}{3} \end{pmatrix}$$

Equation of the tangent

$$Qx = \text{constant}$$

A is the point  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Since  $x = A$  satisfies the above equation

We get

$$(4 \ -1)x = 5$$

which is the equation of the tangent to the circle C at the point

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

