A Comparative Analysis on Optimizing the Ideal Point of Linear Programming Problem under Uncertainty with Left and Right Reference Function PMA21P01L Review-2 Presentation

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INTRODUCTION

- Linear Programming Problem is a mathematical technique which is used to optimize the objective function with the limited resources.
- Linear Programming Problem applications may include production schedul ing, inventory policies, investment portfolio, allocation of advertising budget, construction of warehouses, etc.
- Conventional linear programming is not suitable for many real time problems which involve data with inherent vagueness or impreciseness.
 The ambiguity or uncertainty in reality is efficiently handled by fuzzy set theory.
- This manuscript includes comparative analysis of lexicographic approach which is applied in multi objective linear programming problem and the revised simplex algorithm utilizing gaussian elimination to linear programming problem under uncertainty with left and right reference function.

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PRELIMINARIES

Definition 1.1 [1]

Let X be an universe of discourse. A fuzzy set A is defined as,

 $\tilde{\mathcal{A}}=\{(x,\,\mu\;(x)),\,x\in X:\,\mu\in[0,1]\;\}$ where $\mu(x)$ is a membership function.

Definition 1.2 [1]

A fuzzy set is a fuzzy number \tilde{A} : $F(R) \rightarrow [0\ 1]$ which has characteristics such as,

- $ilde{m{A}}$ is normal, there exist an ${\sf x}\in{\sf X}$ such that $ilde{A}({\sf x})=1$
- $ilde{m{ heta}}$ $ilde{A}$ is convex, $ilde{A}\{\lambda x_1+(1-\lambda)x_2\}\geq \min~\{ ilde{A}(x_1), ilde{A}(x_2)\},~orall~x_1,x_2\in \mathsf{R}~\mathrm{and}~\lambda\in [0,1]$
- \mathbf{m} \tilde{A} is piecewise continuous.

Definition 1.3 [2]

A fuzzy number \tilde{A} is of LR-type if there exist shape functions L (for left) and R (for right), and scalers $\alpha>0$, $\beta>0$ with membership function

$$\mu = \left\{ \begin{array}{l} L\left(\frac{m-x}{\alpha}\right), x \leq m \\ R\left(\frac{x-m}{\beta}\right), x \geq m \end{array} \right.$$

where the real number m is called the mean value or peak of \tilde{A} , and α and β are called the left and right spreads, respectively. Symbolically, \tilde{A} is denoted by $(m, \alpha, \beta)_{LR}$.

Definition 1.4- Arithematic operations

Suppose two fuzzy numbers \tilde{M} and \tilde{N} , represented as L-R fuzzy number of the form $\tilde{M}=(m_1,\alpha_1,\beta_1)_{LR}$ and $\tilde{N}=(m_2,\alpha_2,\beta_2)_{LR}$, We define

ADDITION

$$\tilde{M} + \tilde{N} = (m_1 + m_2, \alpha_1 + \alpha_2, \beta_1 + \beta_2)_{LR}$$

SUBTRACTION

$$\tilde{M} - \tilde{N} = (m_1 - m_2, \alpha_1 + \beta_2, \beta_1 + \alpha_2)_{LR}$$

MULTIPLICATION

- For $\tilde{M} = (m_1, \alpha_1, \beta_1)_{LR}$, $\tilde{N} = (m_2, \alpha_2, \beta_2)_{LR}$ positive, $\tilde{M} \times \tilde{N} = (m_1, m_2, m_1, \alpha_2 + m_2, \alpha_1, m_1, \beta_2 + m_2, \beta_1)_{LR}$
- For $\tilde{N}=(m_2,\alpha_2,\beta_2)_{LR}$ is positive, $\tilde{M}=(m_1,\alpha_1,\beta_1)_{LR}$ is negative $\tilde{M}\times \tilde{N}=(m_1.m_2,m_2.\alpha_1-m_1.\alpha_2,m_2.\beta_1-m_1.\beta_2)_{LR}$
- For $\tilde{M} = (m_1, \alpha_1, \beta_1)_{LR}$, $\tilde{N} = (m_2, \alpha_2, \beta_2)_{LR}$ negative, $\tilde{M} \times \tilde{N} = (m_1.m_2, -m_2.\beta_1 m_1.\alpha_2, -m_2.\alpha_1 m_1.\beta_2)_{LR}$

DIVISION

$$\tilde{M}/\tilde{N} = (m_1/m_2, (m_1\beta_2 + m_2\alpha_1)/m_2^2, (m_1\alpha_2 + m_2\beta_1)/m_2^2)_{LR},$$
 Where $\tilde{N} \neq (0, 0, 0)$

Definition 1.5 [2]

Two LR fuzzy numbers $\tilde{M}=(m_1,\alpha_1,\beta_1)_{LR}$ and $\tilde{N}=(m_2,\alpha_2,\beta_2)_{LR}$ are said to be equal if and only if $m_1=m_2$ and $\alpha_1=\alpha_2$ and $\beta_1=\beta_2$.

Definition 1.6 [4]

Let \tilde{M} is denoted by $(m_1, \alpha_1, \beta_1)_{LR}$ and \tilde{N} is denoted by $(m_2, \alpha_2, \beta_2)_{LR}$ be two fuzzy numbers of LR-type. Then $\tilde{M} \leq \tilde{N}$ if and only if

- $1.m_1 \leq m_2$,
- $2.m_1 = m_2 \text{ and } \alpha_1 + \beta_1 \ge \alpha_2 + \beta_2$,
- $3.m_1 = m_2$ and $\alpha_1 + \beta_1 = \alpha_2 + \beta_2$ and $2m_1 \alpha_1 + \beta_1 \le 2m_2 \alpha_2 + \beta_2$.

Definition 1.7 [4]

A ranking function is a function R: $F(R) \to R$, where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into a real line.Let $\tilde{M}=(m,\alpha,\beta)_{LR}$ be a LR- type fuzzy number; then $R(\tilde{M})=m+((\beta-\alpha)\div 4)$.

The ranking function is used to compare the fuzzy numbers. For any two fuzzy numbers \tilde{A} , \tilde{B} in F(R), we have the following comparison:

- $\tilde{A} \geq \tilde{B}$ if and only if $R(\tilde{A}) \geq R(\tilde{B})$
- $extbf{@}$ $\tilde{A} \leq \tilde{B}$ if and only if $R(\tilde{A}) \leq R(\tilde{B})$
- $\tilde{A} = \tilde{B}$ if and only if $R(\tilde{A}) = R(\tilde{B})$

LITERATURE REVIEW

| Year | Author | Layout | |
|------|-------------------|--|--|
| 2013 | Haifang cheng, | Compromise programming approach. | |
| | Weilai Huang | | |
| 2016 | A.Hosseinzadeh, | Lexicography method. | |
| | S.A.Edalatpanah | | |
| 2013 | Izaz Ullah Khan, | Gaussian elimination and revised simplex | |
| | Tahir Ahmad | method. | |
| 2021 | Saeid Jafarzadeh, | Modified triangular fuzzy numbers, With mid- | |
| | Ghoushchi Elnaz | dle as objective and upper and lower as con- | |
| | | straints. | |
| 2014 | M. Otadi | Two phase method. | |

FUZZY LINEAR PROGRAMMING PROBLEM

- Linear programming is a mathematical technique used to optimize a linear objective function subject to linear constraints.
- The observed values of the data in real-life problems are incomplete or non-obtainable information. In such situations, fuzzy sets theory is an appropriate approach to handle imprecise data.
- Linear Programming Problem with uncertainty, impreciseness, or incomplete parameter termed as fuzzy linear programming problem. The objective function and constraints of the Fuzzy Linear Programming problem are linear expressions involving the fuzzy variables.
- The goal is to optimizes the membership value of the fuzzy objective function with fuzzy constraints.

MATHEMATICAL FORMULATION OF FUZZY LPP

 Fuzzy linear programming problems with m fuzzy constraints and n fuzzy variables may be formulated as follows:

$$\begin{array}{ll} \text{Maximize (Minimize)} & ((c^Tx)^m, (c^Tx)^I, (c^Tx)^u)_{LR} \\ \text{subject to} & ((Ax)^m, (Ax)^I, (Ax)^u)_{LR} \leq ((b)^m, (b)^I, (b)^u)_{LR} \\ & ((Ax)^m, (Ax)^I, (Ax)^u)_{LR} \geq ((b)^m, (b)^I, (b)^u)_{LR} \\ & ((Ax)^m, (Ax)^I, (Ax)^u)_{LR} = ((b)^m, (b)^I, (b)^u)_{LR} \\ & (x)^m \geq 0 \\ & (x)^m - (x)^I \geq 0 \\ & (x)^m + (x)^u \geq 0 \end{array}$$

(1)

ALGORITHM OF LEXICOGRAPHIC METHOD

STEP-1

• Change the inequality constraints of problem(1) into equality constraints by adding slack variable. Then convert the constraints by using the definition in equality of LR variable.

$$\label{eq:maximize} \begin{array}{ll} \text{Maximize (minimize)} & ((c^Tx)^m,(c^Tx)^I,(c^Tx)^u)_{LR} \\ \text{subject to} & (Ax)^m = (b)^m \\ & (Ax)^I = (b)^I \\ & (Ax)^u = (b)^u \\ & (x)^m \geq 0 \\ & (x)^m - (x)^I \geq 0 \\ & (x)^m + (x)^u \geq 0 \end{array} \tag{2}$$

Convert above problem into the following multiobjective Linear Programming problem:

$$\begin{array}{ll} \text{Maximize (minimize)} & (\mathbf{c^Tx})^m \\ \text{Minimize (Maximize)} & ((\mathbf{c^Tx})^l + (\mathbf{c^Tx})^u) \\ \text{Maximize (minimize)} & (2(\mathbf{c^Tx})^m - (\mathbf{c^Tx})^l + (\mathbf{c^Tx})^u) \\ \text{subject to} & (\mathbf{Ax})^m = (\mathbf{b})^m \\ & (\mathbf{Ax})^l = (\mathbf{b})^l \\ & (\mathbf{Ax})^u = (\mathbf{b})^u \\ & (\mathbf{x})^m \geq \mathbf{0} \\ & (\mathbf{x})^m - (\mathbf{x})^l \geq \mathbf{0} \\ & (\mathbf{x})^m + (\mathbf{x})^u \geq \mathbf{0} \end{array}$$

 By lexicogrophic method, in terms of the preference of objective functions, take first objective functions with constraints

$$\label{eq:maximize} \begin{array}{ll} \text{Maximize (minimize)} & (\mathbf{c^Tx})^m \\ \text{subject to} & (\mathbf{Ax})^m = (\mathbf{b})^m \\ & (\mathbf{Ax})^l = (\mathbf{b})^l \\ & (\mathbf{Ax})^u = (\mathbf{b})^u \\ & (\mathbf{x})^m \geq 0 \\ & (\mathbf{x})^m - (\mathbf{x})^l \geq 0 \\ & (\mathbf{x})^m + (\mathbf{x})^u \geq 0 \end{array}$$

• find the solution of above linear programming problem.

 Then they considered second objective function, where first objective function is included as a constraints along with the other constraints as shown below and solved.

Minimize (Maximize)
$$((\mathbf{c}^\mathsf{T}\mathbf{x})^\mathsf{I} + (\mathbf{c}^\mathsf{T}\mathbf{x})^\mathsf{u})$$
 subject to
$$((\mathbf{c}^\mathsf{T}\mathbf{x})^\mathsf{m}) = \mathbf{s}^*$$

$$(\mathbf{A}\mathbf{x})^\mathsf{m} = (\mathbf{b})^\mathsf{m}$$

$$(\mathbf{A}\mathbf{x})^\mathsf{I} = (\mathbf{b})^\mathsf{I}$$

$$(\mathbf{A}\mathbf{x})^\mathsf{u} = (\mathbf{b})^\mathsf{u}$$

$$(\mathbf{x})^\mathsf{m} \geq \mathbf{0}$$

$$(\mathbf{x})^\mathsf{m} - (\mathbf{x})^\mathsf{I} \geq \mathbf{0}$$

$$(\mathbf{x})^\mathsf{m} + (\mathbf{x})^\mathsf{u} > \mathbf{0}$$

 Then they considered third objective function, where first objective function and second objective function are included as a constraints along with the other constraints as shown below and solved.

$$\label{eq:maximize} \begin{array}{ll} \text{Maximize (minimize)} & (2(c^Tx)^m - (c^Tx)^I + (c^Tx)^u) \\ \text{subject to} & ((c^Tx)^I + (c^Tx)^u) = n^* \\ & ((c^Tx)^m) = s^* \\ & (Ax)^m = (b)^m \\ & (Ax)^I = (b)^I \\ & (Ax)^u = (b)^u \\ & (x)^m \geq 0, (x)^m - (x)^I \geq 0 \\ & (x)^m + (x)^u > 0 \end{array} \tag{6}$$

• The solution obtained by above linear programming problem gives the optimum objective value for the original problem.

ALGORITHM OF GAUSSIAN ELIMINATION PROCESS

STEP-1

- Convert all the inequality constraints of problem (1) into equation by introducing fuzzy slack variables. Put the coefficients of these fuzzy slack variables equal to zero in the objective function.
- Let us \tilde{x}_{n+i} , i = 1, 2, 3,...,m and \tilde{x}_i , i = 1, 2, 3,...,n represent the slack and decision variables, respectively and initial basic feasible solution is $[0,0,...,0,...,0,b_1,b_2,...,b_r,...,b_m]^T$.
- The constraints matrix can be augmented as

$$[\tilde{A}\tilde{I}] \begin{bmatrix} \tilde{X} \\ \tilde{X_s} \end{bmatrix} = \tilde{b}$$

• Let us $\tilde{c}_1, \tilde{c}_2, \ldots, \tilde{c}_n$ are the coefficients of $\tilde{x}_1, \ldots, \tilde{x}_n$.

 A Simplex tableau is used to perform row operations on the linear programming model as well as to check a solution for optimality. The table consists of the coefficient corresponding to the linear constraints variables and the coefficients of the objective function. Then construct the fuzzy initial simplex table in the following format:

Table: Initial Table:1

| | $\left[\tilde{x_j}\right]_{1\times n}$ | $[\tilde{x_s}]_{1\times(n+m)}$ | \tilde{b} |
|--|--|--------------------------------|---------------------------|
| $\left[\tilde{x}_{s}\right]_{n+1}^{n+m}$ | $	ilde{[aij]}_{m 	imes n}$ | $[\tilde{I}]_{n+1 \times n+m}$ | $[ilde{b_i}]_{m	imes 1}$ |
| Ĩ | $\tilde{[-Cj]}_{1\times n}$ | $[0]_{1\times n+m}$ | |

- For maximization problems, find the value of \tilde{c}_j with the most negative ranking value. Let it be \tilde{c}_k .
- For minimization problems, search for the value of \tilde{c}_j with the most positive ranking value. Let it be \tilde{c}_k .

STEP-3

• Find the rank values $R(\tilde{b_i}), R(\tilde{a_{ij}}), i = 1, 2, 3,...,m$ and the ranking ratio

$$\frac{R(\tilde{b_i})}{R(\tilde{a_i}j)}$$
; $R(\tilde{a_i}j) \neq 0$

where i = 1, 2, 3, ..., m

- Find the least ranking ratio from $\frac{R(\tilde{b}_i)}{R(\tilde{a}_{ij})}$, i = 1, 2, 3,...,m. Let it be related to the basic row variable \tilde{x}_I .
- Furthermore, the basic variable $\tilde{x_l}$ leaves the basis and $\tilde{x_k}$ enters the basis.

STEP-5

- Let us consider pivot element is \tilde{a}_{lk} and R_l is the pivot row.
- Identify a fuzzy number \tilde{w} by using arithmetic operation of LR such that:

$$\tilde{a_{lk}} \cdot \tilde{w} = (1,0,0)$$

After identifying \tilde{w} , with row operation $R'_{l} - > R_{l} \cdot \tilde{w}$, the pivot element \tilde{a}_{lk} become (1,0,0).

• Identify a fuzzy number \tilde{v} by using arithmetic operation of LR such that:

$$\tilde{a_{ik}} \cdot v + (1,0,0,) = (0,0,0)$$

After identifying \tilde{v} , with row operation $R_i' - > R_i \cdot v + R_I'$, the elementat at the position of above and below of pivotal element $\tilde{a_{ik}}$

STEP-6

become (0,0,0).

- if there is no number in the \tilde{c}_j equation with negative ranking value, so the obtained solution is optimal.
- If atleast \tilde{c}_j has negative ranking value go to step 2 and repeat the computational procedure until either an optimum solution is obtained.

NUMERICAL ILLUSTRATION

- In this section, we will demonstrate the efficiency and superiority of the methods using numerical example.
- Here we considered fuzzy linear programming problem from the literature [6].
- M. Otadi et al [6] is solving this problem by two phase method. This
 method can be applied only if the elements of the coefficient matrix
 are triangular fuzzy numbers.
- Lexicographic method and gaussian elimination with revised simplex method is used to solve fuzzy linear programming problem that it can be applied for without any restriction on the elements of coefficient matrix.

Example 1.1

$$\begin{array}{l} ((1,1,1)_{LR}) \cdot \tilde{X}_1 + ((2,1,2)_{LR}) \cdot \tilde{X}_2 \leq ((4,3,4)_{LR}) \\ ((-2,1,1)_{LR}) \cdot \tilde{X}_1 + ((3,1,2)_{LR}) \cdot \tilde{X}_2 \leq ((5,3,2)_{LR}) \\ ((3,2,1)_{LR}) \cdot \tilde{X}_1 + ((-3,2,1)_{LR}) \cdot \tilde{X}_2 \leq ((5,4,3)_{LR}) \end{array}$$

$$((3,2,1)_{LR}) \cdot X_1 + ((-3,2,1)_{LR}) \cdot X_2 \le ((5,4,3)_{LR})$$

and $\tilde{X}_1, \tilde{X}_2 > 0$.

SOLUTIONS

SOLUTION BY LEXICOGRAPHIC METHOD

- According to the algorithm, the inequality constraints are changed into equality constraints.
- Then FLPP is converted into multi objective linear programming problem[MOLPP].
- By lexicographic method the MOLPP is solved. The obtained solution provides the optimum solution for the LR fuzzy linear programming problem.
- The optimum objective value for LR-fuzzy linear programming problem by lexicographic method is $(3.3333, 0.3332, 1.3332)_{LR}$.

SOLUTION BY GAUSSIAN ELIMINATION METHOD

- According to the algorithm, the initial table is created and found the most negative value of rank function in the \tilde{z} equation.
- Found the least rank ratio and pivot element.
- Create (1, 0, 0) at the position of pivot element and (0, 0, 0) at the positions above and below of pivot element by the row operations.
- The first iteration is complete. For the second iteration, there is no number in the \tilde{z} equation with a negative ranking value, so the solution is optimal.
- The optimum objective value for LR-fuzzy linear programming problem by Gaussian elimination method is $(3.333, 2.111, 2.555)_{LR}$.

Result and Discussion

- The optimum objective value for LR-fuzzy linear programming problem by lexicographic method is $(3.3333, 0.3332, 1.3332)_{LR}$ and using the ranking function the objective value is 3.583.
- The optimum objective value for LR-fuzzy linear programming problem by Gaussian elimination method is $(3.3333, 2.11, 2.555)_{LR}$ and using the ranking function the objective value is 3.444.
- The optimum objective value for LR-fuzzy linear programming problem from the literature[6] by two phase method is $(3.3333, 1.6666, 2.5555)_{LR}$ and using the ranking function the objective value is 3.555.
- Comparatively lexicographic method gives relatively better result in maximisation problem which is shown in the following graphical representation.

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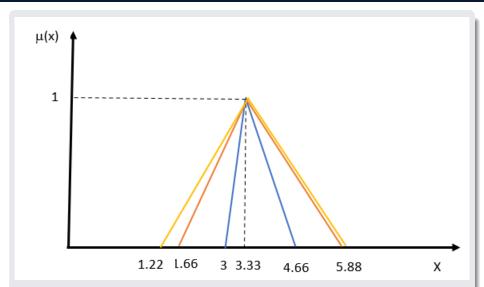


Figure: Comparative result of the optimum objective value for LR-fuzzy linear programming problem.

CONCLUSION

- Fuzzy linear programming is a valuable tool for modeling uncertain and ambiguous data in decision-making..
- This manuscript includes comparative analysis of different algorithm to optimize Linear Programming Problem under uncertainty with left and right reference function.
- The algorithms discussed are the lexicographic approach which is applied in multi objective linear programming problem and the revised simplex algorithm utilizing gaussian elimination.
- Comparatively lexicographic method gives relatively better result than revised simplex algorithm, which is illustrated numerically.
- The outcomes of this result will be useful for researchers and practitioners working in the field of decision-making and optimization, particularly for those dealing with problems where the input parameters are vague or imprecise.

SUSTAINABLE DEVELOPMENT GOALS

- Fuzzy Linear Programming problem can be used to optimize agricultural systems to meet the goals of SDG 2 (Zero Hunger) by finding the best solution that maximizes food production while minimizing environmental impact.
- Fuzzy Linear Programming problem can be used to optimize health care systems that meet the goals of SDG 3 (Good Health and Well-being) by finding the best solution that balances cost, quality of care, and access to health services.
- Fuzzy Linear Programming problem can be used to optimize the problems in business and industry that meet the goals of SDG 9 (Industry, Innovation and Infrastructure) by finding the best solution to maximize profit or minimize costs based upon the resources available in the company.

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THANK YOU