

The Grimoire of Programming

Chapter 1

The Trial of Number Theory

```
/* Number theory template :
   Deals with various aspects of integer, division, modulo, etc.
*/
#include <cmath>
#include <complex>
#include <vector>
namespace number {
    /* Basic operation :
       long long inverse (const long long &x, const long long &mod) :
       returns the inverse of x modulo mod.
       i.e. x * inv (x) % mod = 1.
       int fpm (int x, int n, int mod) :
       returns x^n % mod. i.e. Fast Power with Modulo.
    */
    long long inverse (const long long &x, const long long &mod) {
        if (x == 1) return 1;
        return (mod - mod / x) * inverse (mod % x, mod) % mod;
    }
    int fpm (int x, int n, int mod) {
        register int ans = 1, mul = x;
        while (n) {
            if (n & 1) ans = int (1ll * ans * mul % mod);
            mul = int (1ll * mul * mul % mod);
            n >>= 1;
        }
        return ans;
    }
    /* Discrete Fourier transform :
       int dft::prepare (int n) : readys the transformation with dimension n.
       void dft::main (complex *a, int n, int f) :
       transforms array a with dimension n to its frequency representation.
       transforms back when f = 1.
    */
    namespace dft {
        const int MAXN = 1E6;
        const double PI = acos (-1);
        typedef std::complex <double> complex;
        complex e[2][MAXN];
        int prepare (int n) {
            int len = 1;
            for (; len <= 2 * n; len <= 1);
            for (int i = 0; i < len; i++) {
                e[0][i] = complex (cos (2 * PI * i / len), sin (2 * PI * i / len));
                e[1][i] = complex (cos (2 * PI * i / len), -sin (2 * PI * i / len));
            }
            return len;
        }
        void main (complex *a, int n, int f) {
            for (int i = 0, j = 0; i < n; i++) {
                if (i > j) std::swap (a[i], a[j]);
                for (int t = n >> 1; (j ^= t) < t; t >>= 1);
            }
            for (int i = 2; i <= n; i <= 1)
                for (int j = 0; j < n; j += i)
                    for (int k = 0; k < (i >> 1); k++) {
                        complex A = a[j + k];
                        complex B = e[f][n / i * k] * a[j + k + (i >> 1)];
                        a[j + k] = A + B;
                        a[j + k + (i >> 1)] = A - B;
                    }
            if (f == 1) {
                for (int i = 0; i < n; i++)
                    a[i] = complex (a[i].real () / n, a[i].imag ());
            }
        }
    }
    /* Number-theoretic transform :
       void ntt::main (int *a, int n, int f, int mod, int prt) :
       converts polynomial f (x) = a[0] * x^0 + a[1] * x^1 + ... + a[n - 1] * x^(n - 1)
       to a vector (f (prt^0), f (prt^1), f (prt^2), ..., f (prt^(n - 1))). (module mod)
       Converts back if f = 1.
       Requires specific mod and corresponding prt to work. (given in MOD and PRT)
       int ntt::crt (int *a, int mod) :
       makes up the results a from module 3 primes to a certain module mod.
    */
    namespace ntt {
        const int MAXN = 1E6;
```

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void main (int *a, int n, int f, int mod, int prt) {
    for (register int i = 0, j = 0; i < n; i++) {
        if (i > j) std::swap (a[i], a[j]);
        for (register int t = n >> 1; (j ^ t) < t; t >>= 1);
    }
    for (register int i = 2; i <= n; i <= 1) {
        static int exp[MAXN];
        exp[0] = 1;
        exp[1] = fpm (prt, (mod - 1) / i, mod);
        if (f == 1) exp[1] = fpm (exp[1], mod - 2, mod);
        for (register int k = 2; k < (i >> 1); k++) {
            exp[k] = int (1ll * exp[k - 1] * exp[1] % mod);
        }
        for (register int j = 0; j < n; j += i) {
            for (register int k = 0; k < (i >> 1); k++) {
                register int &pA = a[j + k], &pB = a[j + k + (i >> 1)];
                register int A = pA, B = int (1ll * pB * exp[k] % mod);
                pA = (A + B) % mod;
                pB = (A - B + mod) % mod;
            }
        }
    }
    if (f == 1) {
        register int rev = fpm (n, mod - 2, mod);
        for (register int i = 0; i < n; i++) {
            a[i] = int (1ll * a[i] * rev % mod);
        }
    }
}

const int MOD[3] = {1045430273, 1051721729, 1053818881}, PRT[3] = {3, 6, 7};
int crt (int *a, int mod) {
    static int inv[3][3];
    for (int i = 0; i < 3; i++)
        for (int j = 0; j < 3; j++)
            inv[i][j] = (int) inverse (MOD[i], MOD[j]);
    static int x[3];
    for (int i = 0; i < 3; i++) {
        x[i] = a[i];
        for (int j = 0; j < i; j++) {
            int t = (x[i] - x[j] + MOD[i]) % MOD[i];
            if (t < 0) t += MOD[i];
            x[i] = int (1LL * t * inv[j][i] % MOD[i]);
        }
    }
    int sum = 1, ret = x[0] % mod;
    for (int i = 1; i < 3; i++) {
        sum = int (1LL * sum * MOD[i - 1] % mod);
        ret += int (1LL * x[i] * sum % mod);
        if (ret >= mod) ret -= mod;
    }
    return ret;
}

/* Chinese Remainder Theroem :
   bool crt::main (const std::vector <std::pair<long long, long long> > &input,
                  std::pair<long long, long long> &output) :
       solves for an integer set x = output.first + k * output.second
       that satisfies x % input[i].second = input[i].first.
       Returns whether a solution exists.
*/
namespace crt {
    void euclid (const long long &a, const long long &b,
                 long long &x, long long &y) {
        if (b == 0) x = 1, y = 0;
        else euclid (b, a % b, y, x), y -= a / b * x;
    }
    long long gcd (const long long &a, const long long &b) {
        if (b == 0) return a;
        return gcd (b, a % b);
    }
    long long fix (const long long &a, const long long &b) {
        return (a % b + b) % b;
    }
    bool solve (const std::vector <std::pair<long long, long long> > &input,
                std::pair<long long, long long> &output) {
        output = std::make_pair (1, 1);
        for (int i = 0; i < (int) input.size (); ++i) {
            long long number, useless;
            euclid (output.second, input[i].second, number, useless);
            long long divisor = gcd (output.second, input[i].second);
            if ((input[i].first - output.first) % divisor) {
                return false;
            }
            number *= (input[i].first - output.first) / divisor;
            number = fix (number, input[i].second);
            output.first += output.second * number;
            output.second *= input[i].second / divisor;
            output.first = fix (output.first, output.second);
        }
        return true;
    }
}

#include <cstdio>
int main () {
    return 0;
}

```

Chapter 2

The Trial of Geometry

```
/* Geometry template :
   Most templates of points, lines and circles on a 2D plane.
*/
#include <cmath>
#include <vector>
#include <algorithm>
namespace geometry {
    /* Basic constant & function :
       EPS : fixes the possible error of data.
           i.e.  $x == y$  iff  $|x - y| < EPS$ .
       PI : the value of PI.
       int sgn (const double &x) : returns the sign of x.
       int cmp (const double &x, const double &y) : returns the sign of  $x - y$ .
       double sqr (const double &x) : returns  $x * x$ .
    */
    const double EPS = 1E-8;
    const double PI = acos (-1);
    int sgn (const double &x) { return x < -EPS ? -1 : x > EPS; }
    int cmp (const double &x, const double &y) { return sgn (x - y); }
    double sqr (const double &x) { return x * x; }
    /* struct point : defines a point and its various utility.
       point (const double &x, const double &y) gives a point at (x, y).
       It also represents a vector on a 2D plane.
       point unit () const : returns the unit vector of (x, y).
       point rot90 () const :
           returns a point rotated 90 degrees counter-clockwise with respect to the origin.
       point _rot90 () const : same as above except clockwise.
       point rotate (const double &t) const : returns a point rotated t radian(s) counter-clockwise.
       Operators are mostly vector operations. i.e. vector +, -, *, / and dot/det product.
    */
    struct point {
        double x, y;
        point (const double &x = 0, const double &y = 0) : x (x), y (y) {}
        double norm () const { return sqrt (x * x + y * y); }
        double norm2 () const { return x * x + y * y; }
        point unit () const {
            double l = norm ();
            return point (x / l, y / l);
        }
        point rot90 () const { return point (-y, x); }
        point _rot90 () const { return point (y, -x); }
        point rotate (const double &t) const {
            double c = cos (t), s = sin (t);
            return point (x * c - y * s, x * s + y * c);
        }
    };
    bool operator == (const point &a, const point &b) {
        return cmp (a.x, b.x) == 0 && cmp (a.y, b.y) == 0;
    }
    bool operator != (const point &a, const point &b) {
        return ! (a == b);
    }
    bool operator < (const point &a, const point &b) {
        if (cmp (a.x, b.x) == 0) return cmp (a.y, b.y) < 0;
        return cmp (a.x, b.x) < 0;
    }
    point operator - (const point &a) { return point (-a.x, -a.y); }
    point operator + (const point &a, const point &b) {
        return point (a.x + b.x, a.y + b.y);
    }
    point operator - (const point &a, const point &b) {
        return point (a.x - b.x, a.y - b.y);
    }
    point operator * (const point &a, const double &b) {
        return point (a.x * b, a.y * b);
    }
    point operator / (const point &a, const double &b) {
        return point (a.x / b, a.y / b);
    }
    double dot (const point &a, const point &b) {
        return a.x * b.x + a.y * b.y;
    }
    double det (const point &a, const point &b) {
        return a.x * b.y - a.y * b.x;
    }
}
```

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double dis (const point &a, const point &b) {
    return sqrt (sqr (a.x - b.x) + sqr (a.y - b.y));
}
/* struct line : defines a line (segment) based on two points, s and t.
   line (const point &s, const point &t) gives a basic line from s to t.
   double length () const : returns the length of the segment.
*/
struct line {
    point s, t;
    line (const point &s = point (), const point &t = point ()) : s (s), t (t) {}
    double length () const { return dis (s, t); }
};
/* Point & line interaction :
   bool point_on_line (const point &a, const line &b) : checks if a is on b.
   bool intersect_judgement (const line &a, const line &b) : checks if segment a and b intersect.
   point line_intersect (const line &a, const line &b) : returns the intersection of a and b.
       Fails on colinear or parallel situations.
   double point_to_line (const point &a, const line &b) : returns the distance from a to b.
   double point_to_segment (const point &a, const line &b) : returns the distance from a to b.
       i.e. the minimized length from a to segment b.
   bool in_polygon (const point &p, const std::vector <point> &po) :
       checks if a is in a polygon with vetices po (clockwise or counter-clockwise order).
   double polygon_area (const std::vector <point> &a) :
       returns the signed area of polygon a (positive for counter-clockwise order, and vise-versa).
   point project_to_line (const point &a, const line &b) :
       returns the projection of a on b,
*/
bool point_on_line (const point &a, const line &b) {
    return sgn (det (a - b.s, b.t - b.s)) == 0 && sgn (dot (b.s - a, b.t - a)) <= 0;
}
bool two_side (const point &a, const point &b, const line &c) {
    return sgn (det (a - c.s, c.t - c.s)) * sgn (det (b - c.s, c.t - c.s)) < 0;
}
bool intersect_judgement (const line &a, const line &b) {
    if (point_on_line (b.s, a) || point_on_line (b.t, a)) return true;
    if (point_on_line (a.s, b) || point_on_line (a.t, b)) return true;
    return two_side (a.s, a.t, b) && two_side (b.s, b.t, a);
}
point line_intersect (const line &a, const line &b) {
    double s1 = det (a.t - a.s, b.s - a.s);
    double s2 = det (a.t - a.s, b.t - a.s);
    return (b.s * s2 - b.t * s1) / (s2 - s1);
}
double point_to_line (const point &a, const line &b) {
    return fabs (det (b.t - b.s, a - b.s)) / dis (b.s, b.t);
}
double point_to_segment (const point &a, const line &b) {
    if (sgn (dot (b.s - a, b.t - b.s)) * dot (b.t - a, b.t - b.s)) <= 0)
        return fabs (det (b.t - b.s, a - b.s)) / dis (b.s, b.t);
    return std::min (dis (a, b.s), dis (a, b.t));
}
bool in_polygon (const point &p, const std::vector <point> &po) {
    int n = (int) po.size ();
    int counter = 0;
    for (int i = 0; i < n; ++i) {
        point a = po[i], b = po[(i + 1) % n];
        /*
           The following statement checks if p is on the border of the polygon.
           The boolean returned may be changed if necessary.
           i.e. the algorithm may check if p is strictly in the polygon.
        */
        if (point_on_line (p, line (a, b))) return true;
        int x = sgn (det (p - a, b - a)), y = sgn (a.y - p.y), z = sgn (b.y - p.y);
        if (x > 0 && y <= 0 && z > 0) counter++;
        if (x < 0 && z <= 0 && y > 0) counter--;
    }
    return counter != 0;
}
double polygon_area (const std::vector <point> &a) {
    double ans = 0.0;
    for (int i = 0; i < (int) a.size (); ++i)
        ans += det (a[i], a[(i + 1) % a.size ()]) / 2.0;
    return ans;
}
point project_to_line (const point &a, const line &b) {
    return b.s + (b.t - b.s) * (dot (a - b.s, b.t - b.s) / (b.t - b.s).norm2 ());
}
/* Centers of a triangle :
   returns various centers of a triangle with vertices (a, b, c).
*/
point incenter (const point &a, const point &b, const point &c) {
    double p = dis (a, b) + dis (b, c) + dis (c, a);
    return (a * dis (b, c) + b * dis (c, a) + c * dis (a, b)) / p;
}
point circumcenter (const point &a, const point &b, const point &c) {
    point p = b - a, q = c - a, s (dot (p, p) / 2, dot (q, q) / 2);
    double d = det (p, q);
    return a + point (det (s, point (p.y, q.y)), det (point (p.x, q.x), s)) / d;
}
point orthocenter (const point &a, const point &b, const point &c) {
    return a + b + c - circumcenter (a, b, c) * 2.0;
}
/* Fermat point :
   point fermat_point (const point &a, const point &b, const point &c) :
       returns a point p that minimizes |pa| + |pb| + |pc|.
*/
point fermat_point (const point &a, const point &b, const point &c) {
    if (a == b) return a;
    if (b == c) return b;
}

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    if (c == a) return c;
    double ab = dis (a, b), bc = dis (b, c), ca = dis (c, a);
    double cosa = dot (b - a, c - a) / ab / ca;
    double cosb = dot (a - b, c - b) / ab / bc;
    double cosc = dot (b - c, a - c) / ca / bc;
    double sq3 = PI / 3.0;
    point mid;
    if (sgn (det (b - a, c - a)) < 0) swap (b, c);
    if (sgn (cosa + 0.5) < 0) mid = a;
    else if (sgn (cosb + 0.5) < 0) mid = b;
    else if (sgn (cosc + 0.5) < 0) mid = c;
    else mid = line_intersect (line (a, c + (b - c).rotate (sq3)), line (c, b + (a - b).rotate (sq3)));
    return mid;
}
/* struct circle defines a circle.
   circle (point c, double r) gives a circle with center c and radius r.
*/
struct circle {
    point c;
    double r;
    circle (point c = point (), double r = 0) : c (c), r (r) {}
};
bool operator == (const circle &a, const circle &b) {
    return a.c == b.c && cmp (a.r, b.r) == 0;
}
bool operator != (const circle &a, const circle &b) {
    return ! (a == b);
}
/* Circle interaction :
   bool in_circle (const point &a, const circle &b) : checks if a is in or on b.
   circle make_circle (const point &a, const point &b) :
       generates a circle with diameter ab.
   circle make_circle (const point &a, const point &b, const point &c) :
       generates a circle passing a, b and c.
   std::pair<point, point> line_circle_intersect (const line &a, const circle &b) :
       returns the intersections of a and b.
       Fails if a and b do not intersect.
   std::pair<point, point> circle_circle_intersect (const circle &a, const circle &b) :
       returns the intersections of a and b.
       Fails if a and b do not intersect.
   std::pair<line, line> tangent (const point &a, const circle &b) :
       returns the tangent lines of b passing through a.
       Fails if a is in b.
*/
bool in_circle (const point &a, const circle &b) {
    return cmp (dis (a, b.c), b.r) <= 0;
}
circle make_circle (const point &a, const point &b) {
    return circle ((a + b) / 2, dis (a, b) / 2);
}
circle make_circle (const point &a, const point &b, const point &c) {
    point p = circumcenter (a, b, c);
    return circle (p, dis (p, a));
}
std::pair<point, point> line_circle_intersect (const line &a, const circle &b) {
    double x = sqrt (sqr (b.r) - sqr (point_to_line (b.c, a)));
    return std::make_pair (project_to_line (b.c, a) + (a.s - a.t).unit () * x,
        project_to_line (b.c, a) - (a.s - a.t).unit () * x);
}
point __circle_intersect (const circle &a, const circle &b) {
    point r = (b.c - a.c).unit ();
    double d = dis (a.c, b.c);
    double x = .5 * ((sqr (a.r) - sqr (b.r)) / d + d);
    double h = sqrt (sqr (a.r) - sqr (x));
    return a.c + r * x + r.rot90 () * h;
}
std::pair<point, point> circle_intersect (const circle &a, const circle &b) {
    return std::make_pair (__circle_intersect (a, b), __circle_intersect (b, a));
}
std::pair<line, line> tangent (const point &a, const circle &b) {
    circle p = make_circle (a, b.c);
    return circle_intersect (p, b);
}
/* Convex hull :
   std::vector<point> convex_hull (std::vector<point> a) :
       returns the convex hull of point set a (counter-clockwise).
*/
bool turn_left (const point &a, const point &b, const point &c) {
    return sgn (det (b - a, c - a)) >= 0;
}
bool turn_right (const point &a, const point &b, const point &c) {
    return sgn (det (b - a, c - a)) <= 0;
}
std::vector<point> convex_hull (std::vector<point> a) {
    int n = (int) a.size (), cnt = 0;
    std::sort (a.begin (), a.end ());
    std::vector<point> ret;
    for (int i = 0; i < n; ++i) {
        while (cnt > 1 && turn_left (ret[cnt - 2], a[i], ret[cnt - 1])) {
            --cnt;
            ret.pop_back ();
        }
        ret.push_back (a[i]);
        ++cnt;
    }
    int fixed = cnt;
    for (int i = n - 1; i >= 0; --i) {
        while (cnt > fixed && turn_right (ret[cnt - 2], a[i], ret[cnt - 1])) {
            --cnt;
            ret.pop_back ();
        }
    }
}

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        ret.push_back (a[i]);
        ++cnt;
    }
    ret.pop_back ();
    return ret;
}
/* Minimum circle of a point set :
   circle minimum_circle (std::vector<point> p) : returns the minimum circle of point set p.
*/
circle minimum_circle (std::vector<point> p) {
    circle ret;
    std::random_shuffle (p.begin (), p.end ());
    for (int i = 0; i < (int) p.size (); ++i)
        if (!in_circle (p[i], ret)) {
            ret = circle (p[i], 0);
            for (int j = 0; j < i; ++j)
                if (!in_circle (p[j], ret)) {
                    ret = make_circle (p[j], p[i]);
                    for (int k = 0; k < j; ++k)
                        if (!in_circle (p[k], ret)) ret = make_circle (p[i], p[j], p[k]);
                }
        }
    return ret;
}
/* Online half plane intersection (complexity = O(c.size ())) :
   std::vector<point> cut (const std::vector<point> &c, line p) :
   returns the convex polygon cutting convex polygon c with half plane p.
   (left hand with respect to vector p)
   If such polygon does not exist, returns an empty set.
   e.g.
       static const double BOUND = 1e5;
       convex.clear ();
       convex.push_back (point (-BOUND, -BOUND));
       convex.push_back (point (BOUND, -BOUND));
       convex.push_back (point (BOUND, BOUND));
       convex.push_back (point (-BOUND, BOUND));
       convex = cut (convex, line(point, point));
       if (convex.empty ()) { ... }
*/
std::vector<point> cut (const std::vector<point> &c, line p) {
    std::vector<point> ret;
    if (c.empty ()) return ret;
    for (int i = 0; i < (int) c.size (); ++i) {
        int j = (i + 1) % (int) c.size ();
        if (!turn_right (p.s, p.t, c[i])) ret.push_back (c[i]);
        if (two_side (c[i], c[j], p))
            ret.push_back (line_intersect (p, line (c[i], c[j])));
    }
    return ret;
}
/* Offline half plane intersection (complexity = O(nlogn), n = h.size ()) :
   std::vector<point> half_plane_intersect (std::vector<line> h) :
   returns the intersection of half planes h.
   (left hand with respect to the vector)
   If such polygon does not exist, returns an empty set.
*/
bool turn_left (const line &l, const point &p) {
    return turn_left (l.s, l.t, p);
}
std::vector<point> half_plane_intersect (std::vector<line> h) {
    typedef std::pair<double, line> polar;
    std::vector<polar> g;
    g.resize (h.size ());
    for (int i = 0; i < (int) h.size (); ++i) {
        point v = h[i].t - h[i].s;
        g[i] = std::make_pair (atan2 (v.y, v.x), h[i]);
    }
    sort (g.begin (), g.end (), [] (const polar &a, const polar &b) {
        if (cmp (a.first, b.first) == 0)
            return sgn (det (a.second.t - a.second.s, b.second.t - a.second.s)) < 0;
        else
            return cmp (a.first, b.first) < 0;
    });
    h.resize (std::unique (g.begin (), g.end (), [] (const polar &a, const polar &b) {
        return cmp (a.first, b.first) == 0;
    }) - g.begin ());
    for (int i = 0; i < (int) h.size (); ++i)
        h[i] = g[i].second;
    int fore = 0, rear = -1;
    std::vector<line> ret;
    for (int i = 0; i < (int) h.size (); ++i) {
        while (fore < rear && !turn_left (h[i], line_intersect (ret[rear - 1], ret[rear]))) {
            --rear;
            ret.pop_back ();
        }
        while (fore < rear && !turn_left (h[i], line_intersect (ret[fore], ret[fore + 1])))
            ++fore;
        ret.push_back (h[i]);
    }
    while (rear - fore > 1 && !turn_left (ret[fore], line_intersect (ret[rear - 1], ret[rear]))) {
        --rear;
        ret.pop_back ();
    }
    while (rear - fore > 1 && !turn_left (ret[rear], line_intersect (ret[fore], ret[fore + 1])))
        ++fore;
    if (rear - fore < 2) return std::vector<point> ();
    std::vector<point> ans;
    ans.resize (ret.size ());
    for (int i = 0; i < (int) ret.size (); ++i)
        ans[i] = line_intersect (ret[i], ret[(i + 1) % ret.size ()]);
    return ans;
}
/* Intersection of a polygon and a circle :

```

```

double polygon_circle_intersect::main (const std::vector<point> &p, const circle &c) :
    returns the area of intersection of polygon p (vertices in either order) and c.
*/
namespace polygon_circle_intersect {
// The area of the sector with center (0, 0), radius r and segment ab.
double sector_area (const point &a, const point &b, const double &r) {
    double c = (2.0 * r * r - (a - b).norm2 ()) / (2.0 * r * r);
    double al = acos (c);
    return r * r * al / 2.0;
}
// The area of triangle (a, b, (0, 0)) intersecting circle (point (), r).
double area (const point &a, const point &b, const double &r) {
    double dA = dot (a, a), dB = dot (b, b), dC = point_to_segment (point (), line (a, b)), ans = 0.0;
    if (sgn (dA - r * r) <= 0 && sgn (dB - r * r) <= 0) return det (a, b) / 2.0;
    point tA = a.unit () * r;
    point tB = b.unit () * r;
    if (sgn (dC - r) > 0) return sector_area (tA, tB, r);
    std::pair<point, point> ret = line_circle_intersect (line (a, b), circle (point (), r));
    if (sgn (dA - r * r) > 0 && sgn (dB - r * r) > 0) {
        ans += sector_area (tA, ret.first, r);
        ans += det (ret.first, ret.second) / 2.0;
        ans += sector_area (ret.second, tB, r);
        return ans;
    }
    if (sgn (dA - r * r) > 0)
        return det (ret.first, b) / 2.0 + sector_area (tA, ret.first, r);
    else
        return det (a, ret.second) / 2.0 + sector_area (ret.second, tB, r);
}
// Main process.
double main (const std::vector<point> &p, const circle &c) {
    double ret = 0.0;
    for (int i = 0; i < (int) p.size (); ++i) {
        int s = sgn (det (p[i] - c.c, p[ (i + 1) % p.size ()] - c.c));
        if (s > 0)
            ret += area (p[i] - c.c, p[ (i + 1) % p.size ()] - c.c, c.r);
        else
            ret -= area (p[ (i + 1) % p.size ()] - c.c, p[i] - c.c, c.r);
    }
    return fabs (ret);
}
}
/* Union of circles :
std::vector<double> union_circle::main (const std::vector<circle> &c) :
    returns the union of circle set c.
    The i-th element is the area covered with at least i circles.
*/
namespace union_circle {
struct cp {
    double x, y, angle;
    int d;
    double r;
    cp (const double &x = 0, const double &y = 0, const double &angle = 0,
        int d = 0, const double &r = 0) : x (x), y (y), angle (angle), d (d), r (r) {}
};
double dis (const cp &a, const cp &b) {
    return sqrt (sqr (a.x - b.x) + sqr (a.y - b.y));
}
double cross (const cp &p0, const cp &p1, const cp &p2) {
    return (p1.x - p0.x) * (p2.y - p0.y) - (p1.y - p0.y) * (p2.x - p0.x);
}
int cir_cross (cp p1, double r1, cp p2, double r2, cp &cp1, cp &cp2) {
    double mx = p2.x - p1.x, sx = p2.x + p1.x, mx2 = mx * mx;
    double my = p2.y - p1.y, sy = p2.y + p1.y, my2 = my * my;
    double sq = mx2 + my2, d = - (sq - sqr (r1 - r2)) * (sq - sqr (r1 + r2));
    if (sgn (d) < 0) return 0;
    if (sgn (d) <= 0) d = 0;
    else d = sqrt (d);
    double x = mx * ((r1 + r2) * (r1 - r2) + mx * sx) + sx * my2;
    double y = my * ((r1 + r2) * (r1 - r2) + my * sy) + sy * mx2;
    double dx = mx * d, dy = my * d;
    sq *= 2;
    cp1.x = (x - dy) / sq;
    cp1.y = (y + dx) / sq;
    cp2.x = (x + dy) / sq;
    cp2.y = (y - dx) / sq;
    if (sgn (d) > 0) return 2;
    else return 1;
}
bool circmp (const cp &u, const cp &v) {
    return sgn (u.r - v.r) < 0;
}
bool cmp (const cp &u, const cp &v) {
    if (sgn (u.angle - v.angle)) return u.angle < v.angle;
    return u.d > v.d;
}
double calc (cp cir, cp cp1, cp cp2) {
    double ans = (cp2.angle - cp1.angle) * sqr (cir.r)
        - cross (cir, cp1, cp2) + cross (cp (0, 0), cp1, cp2);
    return ans / 2;
}
std::vector<double> main (const std::vector<circle> &c) {
    int n = c.size ();
    std::vector<cp> cir, tp;
    std::vector<double> area;
    cir.resize (n);
    tp.resize (2 * n);
    area.resize (n + 1);
    for (int i = 0; i < n; i++)
        cir[i] = cp (c[i].c.x, c[i].c.y, 0, 1, c[i].r);
    cp cp1, cp2;
}

```



```

std::sort (cir.begin (), cir.end (), circmp);
for (int i = 0; i < n; ++i)
    for (int j = i + 1; j < n; ++j)
        if (sgn (dis (cir[i], cir[j]) + cir[i].r - cir[j].r) <= 0)
            cir[i].d++;
for (int i = 0; i < n; ++i) {
    int tn = 0, cnt = 0;
    for (int j = 0; j < n; ++j) {
        if (i == j) continue;
        if (cir_cross (cir[i], cir[i].r, cir[j], cir[j].r, cp2, cp1) < 2) continue;
        cp1.angle = atan2 (cp1.y - cir[i].y, cp1.x - cir[i].x);
        cp2.angle = atan2 (cp2.y - cir[i].y, cp2.x - cir[i].x);
        cp1.d = 1;
        tp[tn++] = cp1;
        cp2.d = -1;
        tp[tn++] = cp2;
        if (sgn (cp1.angle - cp2.angle) > 0) cnt++;
    }
    tp[tn++] = cp (cir[i].x - cir[i].r, cir[i].y, PI, -cnt);
    tp[tn++] = cp (cir[i].x - cir[i].r, cir[i].y, -PI, cnt);
    std::sort (tp.begin (), tp.begin () + tn, cmp);
    int p, s = cir[i].d + tp[0].d;
    for (int j = 1; j < tn; ++j) {
        p = s;
        s += tp[j].d;
        area[p] += calc (cir[i], tp[j - 1], tp[j]);
    }
}
return area;
}
}
}
#include <cstdio>
using namespace geometry;
int main() {
    return 0;
}

```

Chapter 3

The Trial of Graph

```
namespace graph {
    const int MAXN = 1E5, MAXM = 1E5;
    struct edge_list {
        int size;
        int begin[MAXN], dest[MAXM], next[MAXM];
        void clear (int n) {
            size = 0;
            for (int i = 0; i < n; i++)
                begin[i] = -1;
        }
        edge_list (int n = MAXN) {
            clear (n);
        }
        void add_edge (int u, int v) {
            dest[size] = v; next[size] = begin[u]; begin[u] = size++;
        }
    };
}
#include <cstdio>
int main () {
    return 0;
}
```

Chapter 4

The Trial of String

```
//string.cpp
#include <string>
#include <vector>
#include <map>
namespace string {
    struct suffix_automaton {
        std::vector<std::map<char, int>> edges; // edges[i] : the labeled edges from node i
        std::vector<int> link; // link[i] : the parent of i
        std::vector<int> length; // length[i] : the length of the longest string in the ith class
        std::vector<int> terminals; // terminals : the terminal state of the automaton
        int last; // the index of the equivalence class of the whole string
        suffix_automaton (std::string s) {
            edges.push_back (std::map<char,int> ());
            link.push_back (-1);
            length.push_back (0);
            last = 0;
            for (int i = 0; i < (int) s.size(); ++i) {
                edges.push_back (std::map<char,int> ());
                length.push_back (i + 1);
                link.push_back (0);
                int r = edges.size() - 1;
                int p = last;
                while (p >= 0 && edges[p].find (s[i]) == edges[p].end ()) {
                    edges[p][s[i]] = r;
                    p = link[p];
                }
                if (p != -1) {
                    int q = edges[p][s[i]];
                    if (length[p] + 1 == length[q]) {
                        link[r] = q;
                    } else {
                        edges.push_back (edges[q]);
                        length.push_back (length[p] + 1);
                        link.push_back (link[q]);
                        int qq = edges.size() - 1;
                        link[q] = qq;
                        link[r] = qq;
                        while (p >= 0 && edges[p][s[i]] == q) {
                            edges[p][s[i]] = qq;
                            p = link[p];
                        }
                    }
                }
                last = r;
            }
            int p = last;
            while (p > 0) {
                terminals.push_back (p);
                p = link[p];
            }
        }
    };
};
#include <cstdio>
int main () {
    return 0;
}
```

Chapter 5

Reference

5.1 vimrc

```
set ruler
set number
set tabstop=4
set softtabstop=4
set shiftwidth=4
set smartindent
set showmatch
set hlsearch
set incsearch
set autoread
set backspace=2
set mouse=a
syntax on
nmap <C-A> ggVG
vmap <C-C> "+y
nmap <C-P> "+p
autocmd FileType cpp set cindent
autocmd FileType cpp map <F3> :vsplit %<.in <CR>
autocmd FileType cpp map <F5> :!time ./%.exe <CR>
autocmd FileType cpp map <F7> :!gdb ./%.exe <CR>
autocmd FileType cpp map <F8> :!time ./%.exe < %<.in <CR>
autocmd FileType cpp map <F9> :!g++ % -o %< -g -std=c++11 -Wall -Wextra -Wconversion && size %<.exe <CR>
autocmd FileType cpp map <F10> :!astyle -A2 -t -C -S -N -O -xd % <CR>
autocmd FileType java map <F3> :vsplit %<.in <CR>
autocmd FileType java map <F5> :!time java %< <CR>
autocmd FileType java map <F8> :!time java %< < %<.in <CR>
autocmd FileType java map <F9> :!javac % <CR>
autocmd FileType java map <F10> :!astyle -A2 -t -C -S -N -O -xd % <CR>
```

5.2 Java Reference

```
/* Java reference :
   References on Java IO, Structures, etc.
*/
import java.io.*;
import java.lang.*;
import java.math.*;
import java.util.*;
/* Regular usage:
   Slower IO :
   Scanner in = new Scanner (System.in));
   Scanner in = new Scanner (new BufferedInputStream (System.in));
   Input :
       in.nextInt () / in.nextBigInteger () / in.nextBigDecimal () / in.nextDouble ()
       in.nextLine () / in.hasNext ()
   Output :
       System.out.print (...);
       System.out.println (...);
       System.out.printf (...);
   Faster IO :
   Shown below.
   BigInteger :
       BigInteger.valueOf (int) : convert to BigInteger.
       abs / negate () / max / min / add / subtract / multiply /
       divide / remainder (BigInteger) : BigInteger algebraic.
       gcd (BigInteger) / modInverse (BigInteger mod) /
       modPow (BigInteger ex, BigInteger mod) / pow (int ex) : Number Theory.
       not () / and / or / xor (BigInteger) / shiftLeft / shiftRight (int) : Bit operation.
       compareTo (BigInteger) : comparison.
       intValue () / longValue () / toString (int radix) : converts to other types.
       isProbablePrime (int certainty) / nextProbablePrime () : checks primitive.
   BigDecimal :
       consists of a BigInteger value and a scale.
       The scale is the number of digits to the right of the decimal point.
       divide (BigDecimal) : exact divide.
       divide (BigDecimal, int scale, RoundingMode roundingMode) :
           divide with roundingMode, which may be:
               CEILING / DOWN / FLOOR / HALF_DOWN / HALF_EVEN / HALF_UP / UNNECESSARY / UP.
       BigDecimal setScale (int newScale, RoundingMode roundingMode) :
           returns a BigDecimal with newScale.
       doubleValue () / toPlainString () : converts to other types.
   Arrays :
       Arrays.sort (T [] a);
```

```

        Arrays.sort (T [] a, int fromIndex, int toIndex);
        Arrays.sort (T [] a, int fromIndex, int toIndex, Comparator <? super T> comparator);
LinkedList <E> :
    addFirst / addLast (E) / getFirst / getLast / removeFirst / removeLast () :
        deque implementation.
    clear () / add (int, E) / remove (int) : clear, add & remove.
    size () / contains / removeFirstOccurrence / removeLastOccurrence (E) :
        deque methods.
    ListIterator <E> listIterator (int index) : returns an iterator :
        E next / previous () : accesses and iterates.
        hasNext / hasPrevious () : checks availability.
        nextIndex / previousIndex () : returns the index of a subsequent call.
        add / set (E) / remove () : changes element.
PriorityQueue <E> (int initcap, Comparator <? super E> comparator) :
    add (E) / clear () / iterator () / peek () / poll () / size () :
        priority queue implementations.
TreeMap <K, V> (Comparator <? super K> comparator) :
    Map.Entry <K, V> ceilingEntry / floorEntry / higherEntry / lowerEntry (K) :
        getKey / getValue () / setValue (V) : entries.
    clear () / put (K, V) / get (K) / remove (K) : basic operation.
    size () : size.
StringBuilder :
    Mutable string.
    StringBuilder (string) : generates a builder.
    append (int, string, ...) / insert (int offset, ...) : adds objects.
    charAt (int) / setCharAt (int, char) : accesses a char.
    delete (int, int) : removes a substring.
    reverse () : reverses itself.
    length () : returns the length.
    toString () : converts to string.
String :
    Immutable string.
    String.format (String, ...) : formats a string. i.e. sprintf.
    toLowerCase / toUpperCase () : changes the case of letters.
*/
/* Examples on Comparator :
public class Main {
    public static class Point {
        public int x;
        public int y;
        public Point () {
            x = 0;
            y = 0;
        }
        public Point (int xx, int yy) {
            x = xx;
            y = yy;
        }
    }
    public static class Cmp implements Comparator <Point> {
        public int compare (Point a, Point b) {
            if (a.x < b.x) return -1;
            if (a.x == b.x) {
                if (a.y < b.y) return -1;
                if (a.y == b.y) return 0;
            }
            return 1;
        }
    }
    public static void main (String [] args) {
        Cmp c = new Cmp ();
        TreeMap <Point, Point> t = new TreeMap <Point, Point> (c);
        return;
    }
};
*/
/* Another way to implement is to use Comparable.
However, equalTo and hashCode must be rewritten.
Otherwise, containers may fail and give strange answers.
Example :
public static class Point implements Comparable <Point> {
    public int x;
    public int y;
    public Point () {
        x = 0;
        y = 0;
    }
    public Point (int xx, int yy) {
        x = xx;
        y = yy;
    }
    public int compareTo (Point p) {
        if (x < p.x) return -1;
        if (x == p.x) {
            if (y < p.y) return -1;
            if (y == p.y) return 0;
        }
        return 1;
    }
    public boolean equalTo (Point p) {
        return (x == p.x && y == p.y);
    }
    public int hashCode () {
        return x + y;
    }
}
};
*/
//Faster IO :
public class Main {
    static class InputReader {
        public BufferedReader reader;
        public StringTokenizer tokenizer;
        public InputReader (InputStream stream) {

```

```

        reader = new BufferedReader (new InputStreamReader (stream), 32768);
        tokenizer = null;
    }
    public String next() {
        while (tokenizer == null || !tokenizer.hasMoreTokens()) {
            try {
                String line = reader.readLine();
                tokenizer = new StringTokenizer (line);
            } catch (IOException e) {
                throw new RuntimeException (e);
            }
        }
        return tokenizer.nextToken();
    }
    public BigInteger nextBigInteger() {
        return new BigInteger (next (), 10);    //  customize the radix here.
    }
    public int nextInt() {
        return Integer.parseInt (next());
    }
    public double nextDouble() {
        return Double.parseDouble (next());
    }
}
public static void main (String[] args) {
    InputReader in = new InputReader (System.in);
    //  Put your code here.
}
}

```