

The Trial of Number Theory

```
Number theory template :
      Deals with various aspects of integer, division, modulo, etc.
#include <cmath>
#include <complex>
#include <vector>
namespace number {
       /* Basic operation :
                    long long inverse (const long long &x, const long long &mod) :
                    iong long inverse (const long long &x, const long long &mod) {
   if (x == 1) return 1;
   return (mod - mod / x) * inverse (mod % x, mod) % mod;
      int fpm (int x, int n, int mod) {
    register int ans = 1, mul = x;
    while (n) {
        if (n & 1) ans = int (111 * ans * mul * mod);
            mul = int (111 * mul * mul * mod);
            n >>= 1;
}
             }
return ans;
      long long gcd (const long long &a, const long long &b) {
   if (b == 0) return a;
   return gcd (b, a % b);
      /* Discrete Fourier transform :
    int dft::prepare (int n) : readys the transformation with dimension n.
    void dft::main (complex *a, int n, int f) :
        transforms array a with dimension n to its frequency representation.
        transforms back when f = 1.
       struct dft {
             constexpr static int MAXN = 1E6;
              constexpr static double PI = acos (-1);
             typedef std::complex <double> complex;
             complex e[2][MAXN];
             int prepare (int n) {
   int len = 1;
   for (; len <= 2 * n; len <<= 1);
   for (int i = 0; i < len; i++) {
       e[0][i] = complex (cos (2 * PI * i / len), sin (2 * PI * i / len));
       e[1][i] = complex (cos (2 * PI * i / len), -sin (2 * PI * i / len));
}</pre>
                    }
return len;
             void main (complex *a, int n, int f) {
    for (int i = 0, j = 0; i < n; i++) {
        if (i > j) std::swap (a[i], a[j]);
        for (int t = n >> 1; (j ^= t) < t; t >>= 1);
                    a[j + k] = A + B;

a[j + k + (i >> 1)] = A - B;
                    if (f == 1) {
   for (int i = 0; i < n; i++)
       a[i] = complex (a[i].real () / n, a[i].imag ());</pre>
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}
};
/* Number-theoretic transform :
    void ntt::main (int *a, int n, int f, int mod, int prt) :
        converts polynominal f (x) = a[0] * x^0 + a[1] * x^1 + ... + a[n - 1] * x^(n - 1)
            to a vector (f (prt^0), f (prt^1), f (prt^2), ..., f (prt^(n - 1))). (module mod)
        Converts back if f = 1.
        Requries specific mod and corresponding prt to work. (given in MOD and PRT)
    int ntt::crt (int *a, int mod) :
        makes up the results a from module 3 primes to a certain module mod.
*/
struct ntt {
        constexpr static int MAXN = 1E6;
        void main (int *a, int n, int f, int mod, int prt) {
    for (register int i = 0, j = 0; i < n; i++) {
        if (i > j) std::swap (a[i], a[j]);
        for (register int t = n >> 1; (j ^= t) < t; t >>= 1);
                 for (register int i = 2; i <= n; i <<= 1) {
                         (register int 1 = 2; 1 <= n; 1 <<= 1) {
    static int exp[MAXN];
    exp[0] = 1;
    exp[1] = fpm (prt, (mod - 1) / i, mod);
    if (f == 1) exp[1] = fpm (exp[1], mod - 2, mod);
    for (register int k = 2; k < (i >> 1); k++) {
        exp[k] = int (111 * exp[k - 1] * exp[1] % mod);
}
                         for (register int j = 0; j < n; j += i) {
   for (register int k = 0; k < (i >> 1); k++) {
      register int &pA = a[j + k], &pB = a[j + k + (i >> 1)];
      register int A = pA, B = int (111 * pB * exp[k] % mod);
      pA = (A + B) % mod;
      pB = (A - B + mod) % mod;
}
                         }
                }
if (f == 1) {
    register int rev = fpm (n, mod - 2, mod);
    for (register int i = 0; i < n; i++) {
        a[i] = int (111 * a[i] * rev % mod);
}</pre>
        constexpr static int MOD[3] = {1045430273, 1051721729, 1053818881}, PRT[3] = {3, 6, 7};
       int sum = 1, ret = x[0] % mod;
for (int i = 1; i < 3; i ++) {
    sum = int (1LL * sum * MOD[i - 1] % mod);
    ret += int (1LL * x[i] * sum % mod);
    if (ret >= mod) ret -= mod;
                 return ret;
        }
};
       struct crt {
        long long fix (const long long &a, const long long &b) {
   return (a % b + b) % b;
       number *= (input[i].first - output.first) / divisor;
                         number = fix (number, input[i].second);
output.first += output.second * number;
output.second *= input[i].second / divisor;
                         output.first = fix (output.first, output.second);
                 return true;
        }
        Miller Rabin :
    Checks whether a certain integer is prime.
    Usage : bool miller_rabin::solve (long long).
*/
struct miller_rabin {
```

```
const static int BASE[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
                                   const static int BASE[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
bool check (const long long &prime, const long long &base) {
   long long number = prime - 1;
   for (; "number & 1; number >>= 1);
   long long result = power_mod (base, number, prime);
   for (; number != prime - 1 && result != 1 && result != prime - 1; number <<= 1) {
      result = multiply_mod (result, result, prime);
   }</pre>
                                                        return result == prime - 1 || (number & 1) == 1;
                                    bool solve (const long long Enumber) {
   if (number < 2) {
      return false;
   }</pre>
                                                       if (number < 4) {
    return true;</pre>
                                                        if (~number & 1) {
    return false;
                                                        for (int i = 0; i < 12 && BASE[i] < number; ++i) {
    if (!check (number, BASE[i])) {
        return false;
    }
}</pre>
                                                                           1
                                                        return true;
                                     }
                   /* Pollard Rho :
                                                        Factorizes an integer.
                                                        Usage : void pollard_rho::solve (long long, std::vector <long long> &).
                   struct pollard_rho {
                                   long long pollard_rho (const long long &number, const long long &seed) {
  long long x = rand() % (number - 1) + 1, y = x;
  for (int head = 1, tail = 2; ; ) {
    x = multiply_mod (x, x, number);
    x = add_mod (x, seed, number);
    if (x == y) {
        return number;
    }
}
                                                                           } long long answer = std::__gcd (abs (x - y), number);
if (answer > 1 && answer < number) {
    return answer;
}</pre>
                                                                           if (++head == tail) {
    y = x;
    tail <<= 1;
                                                                         }
                                                       }
                                    void solve (const long long &number, std::vector<long long> &divisor) {
   if (number > 1) {
      if (miller_rabin (number)) {
            divisor.push_back (number);
      } eleminor |
            | column |
            | 
                                                                          divisor.pusn_back (number,)
} else {
   long long factor = number;
   for (; factor >= number;
        factor = pollard_rho (number, rand() % (number - 1) + 1));
   factorize (number / factor, divisor);
   factorize (factor, divisor);
}
                                                     }
                                   }
                };
 #include <cstdio>
int main () {
    return 0;
```

The Trial of Geometry

```
#include <cmath>
#include <vector>
#include <algorithm>
namespace geometry {
         /* Basic constant & function
                          EPS : fixes the possible error of data
                                 i.e. x == y \text{ iff } |x - y| < EPS.
                          PI : the value of PI.
int sgn (const double &x) : returns the sign of x.
int cmp (const double &x, const double &y) : returns the sign of x - y.
double sqr (const double &x) : returns x * x.
         const double EPS = 1E-8;
const double PI = acos (-1);
        int sgn (const double &x) { return x < -EPS ? -1 : x > EPS; } int cmp (const double &x, const double &y) { return sgn (x - y); } double sqr (const double &x) { return x * x; }
         /* struct point : defines a point and its various utility.
    point (const double &x, const double &y) gives a point at (x, y).
    It also represents a vector on a 2D plane.
    point unit () const : returns the unit vector of (x, y).
    point rot90 () const :
                          returns a point rotated 90 degrees counter-clockwise with respect to the origin.

point _rot () const : same as above except clockwise.

point rotate (const double &t) const : returns a point rotated t radian(s) counter-clockwise.

Operators are mostly vector operations. i.e. vector +, -, *, / and dot/det product.
       */
struct point {
    double x, y;
    point (const double &x = 0, const double &y = 0) : x (x), y (y) {}
    double norm () const { return sqrt (x * x + y * y); }
    double norm2 () const { return x * x + y * y; }
    point unit () const {
        double 1 = norm ();
        return point (x / 1, y / 1);
}
                 point rot90 () const {return point (-y, x); }
point _rot90 () const {return point (y, -x); }
point rotate (const double &t) const {
    double c = cos (t), s = sin (t);
    return point (x * c - y * s, x * s + y * c);
}
        bool operator == (const point &a, const point &b) {
   return cmp (a.x, b.x) == 0 && cmp (a.y, b.y) == 0;
        bool operator != (const point &a, const point &b) {
    return ! (a == b);
        bool operator < (const point &a, const point &b) {
   if (cmp (a.x, b.x) == 0) return cmp (a.y, b.y) < 0;
   return cmp (a.x, b.x) < 0;</pre>
         point operator - (const point &a) { return point (-a.x, -a.y); }
        point operator + (const point &a, const point &b) {
   return point (a.x + b.x, a.y + b.y);
        point operator - (const point &a, const point &b) {
   return point (a.x - b.x, a.y - b.y);
        point operator * (const point &a, const double &b) {
   return point (a.x * b, a.y * b);
        point operator / (const point &a, const double &b) {
   return point (a.x / b, a.y / b);
         double dot (const point &a, const point &b) {
    return a.x * b.x + a.y * b.y;
        double det (const point &a, const point &b) {
   return a.x * b.y - a.y * b.x;
```

```
double dis (const point &a, const point &b) {
   return sqrt (sqr (a.x - b.x) + sqr (a.y - b.y));
         struct line : defines a line (segment) based on two points, s and t.
    line (const point &s, const point &t) gives a basic line from s to t.
    double length () const : returns the length of the segment.
struct line {
         point s, t;
line (const point &s = point (), const point &t = point ()) : s (s), t (t) {}
double length () const { return dis (s, t); }
/* Point & line interaction :
    bool point_on_line (const point &a, const line &b) : checks if a is on b.
    bool intersect_judgement (const line &a, const line &b) : checks if segment a and b intersect.
    point line_intersect (const line &a, const line &b) : returns the intersection of a and b.
        Fails on colinear or parallel situations.
    double point_to_line (const point &a, const line &b) : returns the distance from a to b.
    double point_to_segment (const point &a, const lint &b) : returns the distance from a to b.
    i.e. the minimized length from a to segment b.
    bool in polvgon (const point &p, const std::vector <point> &po) :
                   1.e. the minimized length from a to segment b.
bool in_polygon (const point &p, const std::vector <point> &po):
    checks if a is in a polygon with vetices po (clockwise or counter-clockwise order).
double polygon_area (const std::vector <point> &a):
    returns the signed area of polygon a (positive for counter-clockwise order, and vise-versa).
point project_to_line (const point &a, const line &b):
    returns the projection of a on b,
bool point_on_line (const point &a, const line &b) {
   return sgn (det (a - b.s, b.t - b.s)) == 0 && sgn (dot (b.s - a, b.t - a)) <= 0;</pre>
bool two_side (const point &a, const point &b, const line &c) {
   return sgn (det (a - c.s, c.t - c.s)) * sgn (det (b - c.s, c.t - c.s)) < 0;</pre>
bool intersect_judgement (const line &a, const line &b) {
         if (point_on_line (b.s, a) || point_on_line (b.t, a)) return true;
if (point_on_line (a.s, b) || point_on_line (a.t, b)) return true;
return two_side (a.s, a.t, b) && two_side (b.s, b.t, a);
point line_intersect (const line &a, const line &b) {
    double s1 = det (a.t - a.s, b.s - a.s);
    double s2 = det (a.t - a.s, b.t - a.s);
    return (b.s * s2 - b.t * s1) / (s2 - s1);
double point_to_line (const point &a, const line &b) {
   return fabs (det (b.t - b.s, a - b.s)) / dis (b.s, b.t);
double point_to_segment (const point &a, const line &b) {
   if (sgn (dot (b.s - a, b.t - b.s) * dot (b.t - a, b.t - b.s)) <= 0)
      return fabs (det (b.t - b.s, a - b.s)) / dis (b.s, b.t);
   return std::min (dis (a, b.s), dis (a, b.t));</pre>
bool in_polygon (const point &p, const std::vector <point> & po) {
         #/
if (point_on_line (p, line (a, b))) return true;
int x = sgn (det (p - a, b - a)), y = sgn (a.y - p.y), z = sgn (b.y - p.y);
if (x > 0 && y <= 0 && z > 0) counter++;
if (x < 0 && z <= 0 && y > 0) counter--;
          return counter != 0;
double polygon_area (const std::vector <point> &a) {
    double ans = 0.0;
    for (int i = 0; i < (int) a.size (); ++i)
        ans += det (a[i], a[ (i + 1) % a.size ()]) / 2.0;
    return ans;</pre>
point project_to_line (const point &a, const line &b) {
    return b.s + (b.t - b.s) * (dot (a - b.s, b.t - b.s) / (b.t - b.s).norm2 ());
/*
         Centers of a triangle :
    returns various centers of a triangle with vertices (a, b, c).
point incenter (const point &a, const point &b, const point &c) {
   double p = dis (a, b) + dis (b, c) + dis (c, a);
   return (a * dis (b, c) + b * dis (c, a) + c * dis (a, b)) / p;
point circumcenter (const point &a, const point &b, const point &c) {
   point p = b - a, q = c - a, s (dot (p, p) / 2, dot (q, q) / 2);
   double d = det (p, q);
   return a + point (det (s, point (p.y, q.y)), det (point (p.x, q.x), s)) / d;
point orthocenter (const point &a, const point &b, const point &c) {
   return a + b + c - circumcenter (a, b, c) * 2.0;
/* Fermat point :
                 point fermat_point (const point &a, const point &b, const point &c) :
    returns a point p that minimizes |pa| + |pb| + |pc|.
point fermat_point (const point &a, const point &b, const point &c) {
   if (a == b) return a;
   if (b == c) return b;
```

```
if (c == a) return c;
double ab = dis (a, b), bc = dis (b, c), ca = dis (c, a);
double cosa = dot (b - a, c - a) / ab / ca;
double cosb = dot (a - b, c - b) / ab / bc;
double cosc = dot (b - c, a - c) / ca / bc;
double sq3 = PI / 3.0;
         double sq3 = F1 / 3.0;
point mid;
if (sgn (det (b - a, c - a)) < 0) swap (b, c);
if (sgn (cosa + 0.5) < 0) mid = a;
else if (sgn (cosb + 0.5) < 0) mid = b;
else if (sgn (cosc + 0.5) < 0) mid = c;
else mid = line_intersect (line (a, c + (b - c).rotate (sq3)), line (c, b + (a - b).rotate (sq3)));</pre>
          return mid;
 /* struct circle defines a circle.
circle (point c, double r) gives a circle with center c and radius r.
 struct circle
         point c;
double r;
circle (point c = point (), double r = 0) : c (c), r (r) {}
bool operator == (const circle &a, const circle &b) {
   return a.c == b.c && cmp (a.r, b.r) == 0;
bool operator != (const circle &a, const circle &b) {
   return ! (a == b);
/* Circle interaction :
    bool in_circle (const point &a, const circle &b) : checks if a is in or on b.
    circle make_circle (const point &a, const point &b) :
        generates a circle with diameter ab.
    circle make_circle (const point &a, const point &b, const point &c) :
        generates a circle passing a, b and c.
    std::pair <point, point> line_circle_intersect (const line &a, const circle &b) :
        returns the intersections of a and b.
        Fails if a and b do not intersect.
    std::pair <point, point> circle_intersect (const circle &a, const circle &b):
        returns the intersections of a and b.
        Fails if a and b do not intersect.
    std::pair std::pair sine> tangent (const point &a, const circle &b) :
        returns the tangent lines of b passing through a.
        Fails if a is in b.
bool in_circle (const point &a, const circle &b) {
   return cmp (dis (a, b.c), b.r) <= 0;</pre>
 circle make_circle (const point &a, const point &b) {
   return circle ((a + b) / 2, dis (a, b) / 2);
 circle make_circle (const point &a, const point &b, const point &c) {
   point p = circumcenter (a, b, c);
   return circle (p, dis (p, a));
 nt __circle_intersect (const circle &a, const circle &b) {
point r = (b.c - a.c).unit ();
double d = dis (a.c, b.c);
double x = .5 * ((sqr (a.r) - sqr (b.r)) / d + d);
double h = sqrt (sqr (a.r) - sqr (x));
return a.c + r * x + r.rot90 () * h;
point
 std::pair <point, point> circle_intersect (const circle &a, const circle &b) {
    return std::make_pair (__circle_intersect (a, b), __circle_intersect (b, a));
 std::pair <line, line> tangent (const point &a, const circle &b) {
    circle p = make_circle (a, b.c);
    return circle_intersect (p, b);
                  std::vector <point> convex_hull (std::vector <point> a) :
    returns the convex hull of point set a (counter-clockwise).
bool turn_left (const point &a, const point &b, const point &c) {
    return sgn (det (b - a, c - a)) >= 0;
bool turn_right (const point &a, const point &b, const point &c) {
    return sgn (det (b - a, c - a)) <= 0;</pre>
ret.pop_back ();
                   ret.push_back (a[i]); ++cnt;
```

```
ret.push_back (a[i]);
++cnt;
       ret.pop_back ();
return ret;
       Minimum circle of a point set : circle minimum_circle (std::vector <point> p) : returns the minimum circle of point set p.
circle minimum_circle (std::vector <point> p) {
       circle ret;
       }
return ret;
      Online half plane intersection (complexity = O(c.size ())) :
    std::vector <point> cut (const std::vector<point> &c, line p) :
        returns the convex polygon cutting convex polygon c with half plane p.
        (left hand with respect to vector p)
    If such polygon does not exist, returns an empty set.
                      If such polygon goes not end;
e.g.
static const double BOUND = 1e5;
convex.clear ();
convex.push_back (point (-BOUND, -BOUND));
convex.push_back (point (BOUND, -BOUND));
convex.push_back (point (BOUND, BOUND));
convex.push_back (point (-BOUND, BOUND));
convex = cut (convex, line(point, point));
if (convex.empty ()) { ... }
return ret;
}
       Offline half plane intersection (complexity = O(nlogn), n = h.size ()) : std::vector <point> half_plane_intersect (std::vector <line> h) : returns the intersection of half planes h. (left hand with respect to the vector)
/*
                       If such polygon does not exist, returns an empty set.
bool turn_left (const line &1, const point &p) {
    return turn_left (1.s, 1.t, p);
std::vector <point> half_plane_intersect (std::vector <line> h) {
    typedef std::pair <double, line> polar;
    std::vector <polar> g;
    g.resize (h.size ());
    for (int i = 0; i < (int) h.size (); ++i) {
        point v = h[i].t - h[i].s;
        g[i] = std::make_pair (atan2 (v.y, v.x), h[i]);
}</pre>
        sort (g.begin (), g.end (), [] (const polar &a, const polar &b) {
   if (cmp (a.first, b.first) == 0)
      return sgn (det (a.second.t - a.second.s, b.second.t - a.second.s)) < 0;</pre>
                       return cmp (a.first, b.first) < 0;
       });
h.resize (std::unique (g.begin (), g.end (), [] (const polar &a, const polar &b) {
    return cmp (a.first, b.first) == 0;
}) - g.begin ());
for (int i = 0; i < (int) h.size (); ++i)
    h[i] = g[i].second;
int fore = 0, rear = -1;
std::vector <line> ret;
for (int i = 0; i < (int) h.size (); ++i) {
    while (fore < rear && !turn_left (h[i], line_intersect (ret[rear - 1], ret[rear]))) {
        --rear;
        ret.pop back ();
}</pre>
                      ret.pop_back ();
               ret.push_back (h[i]);
       ret.pop_back ();
                   (rear - fore > 1 && !turn_left (ret[rear], line_intersect (ret[fore], ret[fore + 1])))
        ++fore;
if (rear - fore < 2) return std::vector <point> ();
       ir (rear - fore < 2) return std::vector <point> ();
std::vector <point> ans;
ans.resize (ret.size ());
for (int i = 0; i < (int) ret.size (); ++i)
    ans[i] = line_intersect (ret[i], ret[ (i + 1) % ret.size ()]);
return ans;</pre>
      Intersection of a polygon and a circle :
```

```
double polygon_circle_intersect::solve (const std::vector <point> &p, const circle &c) :
    returns the area of intersection of polygon p (vertices in either order) and c.
struct polygon_circle_intersect {
         // The area of the sector with center (0, 0), radius r and segment ab.
         double sector_area (const point &a, const point &b, const double &r) {
   double c = (2.0 * r * r - (a - b).norm2 ()) / (2.0 * r * r);
   double al = acos (c);
   return r * r * al / 2.0;
         // The area of triangle (a, b, (0, 0)) intersecting circle (point (), r).
        // The area of triangle (a, b, (0, 0)) intersecting circle (point (), r).
double area (const point &a, const point &b, const double &r) {
   double dA = dot (a, a), dB = dot (b, b), dC = point_to_segment (point (), line (a, b)), ans = 0.0;
   if (sgn (dA - r * r) <= 0 && sgn (dB - r * r) <= 0) return det (a, b) / 2.0;
   point tA = a.unit () * r;
   point tB = b.unit () * r;
   if (sgn (dC - r) > 0) return sector_area (tA, tB, r);
   std::pair <point, point> ret = line_circle_intersect (line (a, b), circle (point (), r));
   if (sgn (dA - r * r) > 0 && sgn (dB - r * r) > 0) {
      ans += sector_area (tA, ret.first, r);
      ans += det (ret.first, ret.second) / 2.0;
      ans += sector_area (ret.second, tB, r);
      return ans;
}
                   if (sgn (dA - r * r) > 0)
    return det (ret.first, b) / 2.0 + sector_area (tA, ret.first, r);
                   else return det (a, ret.second) / 2.0 + sector_area (ret.second, tB, r);
         // Main process.
        double solve (const std::vector <point> &p, const circle &c) {
   double ret = 0.0;
   for (int i = 0; i < (int) p.size (); ++i) {
      int s = sgn (det (p[i] - c.c, p[ (i + 1) % p.size ()] - c.c));
      if (s > 0)
                                       ret += area (p[i] - c.c, p[ (i + 1) % p.size ()] - c.c, c.r);
                                      ret -= area (p[ (i + 1) % p.size ()] - c.c, p[i] - c.c, c.r);
                   return fabs (ret);
         }
};
        Union of circles:
    std::vector <double> union_circle::solve (const std::vector <circle> &c) :
        returns the union of circle set c.
        The i-th element is the area covered with at least i circles.
struct union_circle {
         struct cp {
    double x, y, angle;
                  double x, y, angle,
int d;
double r;
cp (const double &x = 0, const double &y = 0, const double &angle = 0,
    int d = 0, const double &r = 0) : x (x), y (y), angle (angle), d (d), r (r) {}
         double dis (const cp &a, const cp &b) {
   return sqrt (sqr (a.x - b.x) + sqr (a.y - b.y));
         double cross (const cp &p0, const cp &p1, const cp &p2) {
    return (p1.x - p0.x) * (p2.y - p0.y) - (p1.y - p0.y) * (p2.x - p0.x);
                 cir_cross (cp p1, double r1, cp p2, double r2, cp &cp1, cp &cp2) {
    double mx = p2.x - p1.x, sx = p2.x + p1.x, mx2 = mx * mx;
    double my = p2.y - p1.y, sy = p2.y + p1.y, my2 = my * my;
    double sq = mx2 + my2, d = - (sq - sqr (r1 - r2)) * (sq - sqr (r1 + r2));
    if (sgn (d) < 0) return 0;
    if (sgn (d) <= 0) d = 0;
    else d = sqrt (d);
    double x = mx * ((r1 + r2) * (r1 - r2) + mx * sx) + sx * my2;
    double y = my * ((r1 + r2) * (r1 - r2) + my * sy) + sy * mx2;
    double dx = mx * d, dy = my * d;
    sq *= 2;
    cp1.x = (x - dy) / sq;
    cp2.x = (x + dy) / sq;
    cp2.y = (y - dx) / sq;
    if (sgn (d) > 0) return 2;
    else return 1;
        bool circmp (const cp &u, const cp &v) {
   return sgn (u.r - v.r) < 0;</pre>
         bool cmp (const cp &u, const cp &v) {
    if (sgn (u.angle - v.angle)) return u.angle < v.angle;
    return u.d > v.d;
         std::vector <double> solve (const std::vector <circle> &c) {
  int n = c.size ();
  std::vector <cp> cir, tp;
  std::vector <double> area;
                  std::vector <double> area;
cir.resize (n);
tp.resize (2 * n);
area.resize (n + 1);
for (int i = 0; i < n; i++)
    cir[i] = cp (c[i].c.x, c[i].c.y, 0, 1, c[i].r);</pre>
                   cp cp1, cp2;
```

The Trial of Graph

```
#include <algorithm>
#include <array>
#include <vector>
#include <queue>
namespace graph {
      /* Edge list:
Various kinds of edge list.
       const int MAXN = 1E5, MAXM = 1E5;
       const int MAXN = IE5, MAAR - IE5,
struct edge_list {
   int size;
   int begin[MAXN], dest[MAXM], next[MAXM];
   void clear (int n) {
       size = 0;
       for (int i = 0; i < n; i++)
            begin[i] = -1;
}</pre>
               }
edge_list (int n = MAXN) {
    clear (n);
               }
void add_edge (int u, int v) {
    dest[size] = v;
    next[size] = begin[u];
    begin[u] = size++;
               }
        struct flow_edge_list {
               int size;
int size;
int begin[MAXN], dest[MAXM], next[MAXM], flow[MAXM], inv[MAXM];
void clear (int n) {
    size = 0;
    for (int i = 0; i < n; i++)
        begin[i] = -1;
}</pre>
                }
flow_edge_list (int n = MAXN) {
    clear (n);
               clear (n);
}
void add_edge (int u, int v, int f) {
    dest[size] = v;
    next[size] = begin[u];
    flow[size] = f;
    inv[size] = size + 1;
    begin[u] = size++;
    dest[size] = u;
    next[size] = begin[v];
    flow[size] = 0;
    inv[size] = size - 1;
    begin[v] = size++;
}
              Hopcoft-Carp algorithm :
                       maximum matching with complexity O (m * n^0.5).
                       Initialize: pass n, m as the size of both vertex sets, e as the reference of the edge_list.

Usage: solve() for maximum matching. The matching is in matchx and matchy.
        struct hopcoft_carp {
               static const int MAXN = 1E5;
               int n, m;
edge_list &e;
               int matchx[MAXN], matchy[MAXN], level[MAXN];
               int w = matchy[y];
if (w == -1 || (level[x] + 1 == level[w] && dfs (w))) {
    matchx[x] = y;
    matchy[y] = x;
    return true;
                               }
                       level[x] = -1;
return false;
```

```
for (int head = 0; head < (int) queue.size(); ++head) {</pre>
                      int x = queue[head];
for (int i = e.begin[x]; i != -1; i = e.next[i]) {
                           (Int 1 = e.begin[x]; 1 != -1; .
int y = e.dest[i];
int w = matchy[y];
if (w != -1 && level[w] < 0) {
    level[w] = level[x] + 1;
    queue.push_back (w);
}</pre>
                      }
                int delta = 0;
for (int i = 0; i < n; ++i)
    if (matchx[i] == -1 && dfs (i)) delta++;
if (delta == 0) return answer;
else answer += delta;</pre>
     }
     Kuhn Munkres algorithm :
           weighted maximum matching algorithm. Complexity O (N^3).
           struct kuhn_munkres :
                Initialize: pass nx, ny as the size of both sets, w as the weight matrix.

Usage: solve() for the minimum matching. The matching is in link.
struct kuhn_munkres {
     static const int MAXN = 500;
static const int INF = 1E9;
     int nx, ny;
int w[MAXN][MAXN];
     int lx[MAXN], ly[MAXN], visx[MAXN], visy[MAXN], slack[MAXN], link[MAXN];
    } else slack[y] = std::max (slack[y], t);
           }
return 0;
         int solve () {
           }
int res = 0;
for (i = 0; i < ny; i ++)
    if (link[i] > -1) res += w[link[i]][i];
    }
};
     Weighted matching algorithm : maximum match for graphs. Not stable.
           Usage: Set k to the size of vertices, w to the weight matrix.

Note that k has to be even for the algorithm to work.
struct weighted_match {
     static const int MAXN = 500;
     int k;
long long w[MAXN][MAXN];
int match[MAXN], path[MAXN], p[MAXN], len;
long long d[MAXN];
```

```
bool v[MAXN];
        bool dfs (int i) {
  path[len++] = i;
  if (v[i]) return true;
  v[i] = true;
                path:
if (v[i]) return true,
v[i] = true;
for (int j = 0; j < k; ++j) {
   if (i != j && match[i] != j && !v[j]) {
     int kok = match[j];
     if (d[kok] < d[i] + w[i][j] - w[j][kok]) {
        d[kok] = d[i] + w[i][j] - w[j][kok];
        if (dfs (kok)) return true;
}</pre>
                  }
--len;
v[i] = false;
return false;
        long long solve () {
   if (k & 1) ++k;
   for (int i = 0; i < k; ++i) p[i] = i, match[i] = i ^ 1;
   int cnt = 0;
   for (:) {</pre>
                           (;;) {
len = 0;
                           flag = true;
int t = match[path[len - 1]], j = len - 2;
while (path[j] != path[len - 1]) {
   match[t] = path[j];
   std::swap (t, match[path[j]]);
                                               match[t] = path[j];
match[path[j]] = t;
                                     }
                           }
if (!flag) {
    if (++cnt >= 1) break;
    std::random_shuffle (p, p + k);
                  long long ans = 0;
for (int i = 0; i < k; ++i)
    ans += w[i][match[i]];
return ans / 2;</pre>
       }
        Weighted blossom algorithm (vfleaking ver.) :
    maximum match for graphs. Complexity O (n^3).
    Note that the base index is 1.
    struct weighted_blossom :
                  struct weig
Usage :
                                     ge:
Set n to the size of the vertices.
Run init ().
Set g[][].w to the weight of the edge.
Run solve ().
The first result is the answer, the second one is the number of matching pairs.
Obtain the matching with match[].
struct weighted_blossom {
        const static int INF = INT_MAX;
const static int MAXN = 500;
         struct edge {
                  int u, v, w;
edge (int u = 0, int v = 0, int w = 0): u (u), v (v), w (w) {}
        int n, n_x;
edge g[MAXN * 2 + 1] [MAXN * 2 + 1];
int lab[MAXN * 2 + 1];
int match[MAXN * 2 + 1], slack[MAXN * 2 + 1], st[MAXN * 2 + 1], pa[MAXN * 2 + 1];
int flower_from[MAXN * 2 + 1] [MAXN + 1], S[MAXN * 2 + 1], vis[MAXN * 2 + 1];
std::vector<int> flower[MAXN * 2 + 1];
         std::queue<int> q;
int e_delta (const edge &e) {
   return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2;
         }
void update_slack (int u, int x) {
   if (!slack[x] || e_delta (g[u][x]) < e_delta (g[slack[x]][x]))slack[x] = u;</pre>
         void set_slack (int x) {
    slack[x] = 0;
    for (int u = 1; u <= n; ++u)
        if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)update_slack (u, x);
         void q_push (int x) {
    if (x <= n)q.push (x);
    else for (size_t i = 0; i < flower[x].size(); i++)q_push (flower[x][i]);</pre>
        }
void set_st (int x, int b) {
    st[x] = b;
    if (x > n) for (size_t i = 0; i < flower[x].size(); ++i)
        set_st (flower[x][i], b);</pre>
        }
int get_pr (int b, int xr) {
    int pr = find (flower[b].begin(), flower[b].end(), xr) - flower[b].begin();
    if (pr % 2 == 1) {
        reverse (flower[b].begin() + 1, flower[b].end());
        return (int) flower[b].size() - pr;
```

```
} else return pr;

} else lettan r.,

void set_match (int u, int v) {
    match[u] = g[u][v].v;
    if (u > n) {
        edge e = g[u][v];
        int xr = flower_from[u][e.u], pr = get_pr (u, xr);
        for (int i = 0; i < pr; ++i)set_match (flower[u][i], flower[u][i ^ 1]);
        set_match (xr, v);
        rotate (flower[u].begin(), flower[u].begin() + pr, flower[u].end());
}
</pre>

}
void augment (int u, int v) {
    for (;;) {
        int xnv = st[match[u]];
        set_match (u, v);
        if (!xnv)return;
        set_match (xnv, st[pa[xnv]]);
        u = st[pa[xnv]], v = xnv;
}

  fint get_lca (int u, int v) {
    static int t = 0;
    for (++t; u || v; std::swap (u, v)) {
        if (u == 0) continue;
        if (vis[u] == t) return u;
        vis[u] = t;
        u = st[match[u]];
        if (u) u = st[pa[u]];
}
 return 0;
                 set_slack (b);
    }
void expand_blossom (int b) {
    for (size_t i = 0; i < flower[b].size(); ++i)
        set_st (flower[b][i], flower[b][i]);
    int xr = flower_from[b][g[b][pa[b]].u], pr = get_pr (b, xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flower[b][i], xns = flower[b][i + 1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set_slack (xns);
        q push (xns);
}</pre>
                              q_push (xns);
                }
s[xr] = 1, pa[xr] = pa[b];
for (size_t i = pr + 1; i < flower[b].size(); ++i) {
   int xs = flower[b][i];
   S[xs] = -1, set_slack (xs);
}</pre>
                 st[b] = 0;
  st[b] = v;
}
bool on_found_edge (const edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, q push (nu);
} else if (S[v] == 0) {
        int lca = get_lca (u, v);
        if (!lca)return augment (u, v), augment (v, u), true;
        else add_blossom (u, lca, v);
}
                 return false;
```

```
if (on_found_edge (g[u][v]))return true;
} else update_slack (u, st[v]);
                                  int d = INF;
for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b && S[b] == 1)d = std::min (d, lab[b] / 2);
for (int x = 1; x <= n_x; ++x)
    if (st[x] == x && slack[x]) {
        if (S[x] == -1)d = std::min (d, e_delta (g[slack[x]][x]));
        else if (S[x] == 0)d = std::min (d, e_delta (g[slack[x]][x]) / 2);
}</pre>
                                  for (int u = 1; u <= n; ++u) {
   if (S[st[u]] == 0) {
      if (lab[u] <= d)return 0;
      lab[u] -= d;
   } else if (S[st[u]] == 1)lab[u] += d;</pre>
                                  for (int b = n + 1; b <= n_x; ++b)
  if (st[b] == b) {
    if (S[st[b]] == 0)lab[b] += d * 2;
    else if (S[st[b]] == 1)lab[b] -= d * 2;</pre>
                                  q = std::queue<int>();
for (int x = 1; x <= n_x; ++x)
    if (st[x] == x && slack[x] && st[slack[x]] != x && e_delta (g[slack[x]][x]) == 0)
        if (on_found_edge (g[slack[x]][x]))return true;
for (int b = n + 1; b <= n_x; ++b)
    if (st[b] == b && S[b] == 1 && lab[b] == 0)expand_blossom (b);</pre>
                       return false;
          }
std::pair<long long, int> solve() {
    std::fill (match + 1, match + n + 1, 0);
    n_x = n;
    int n_matches = 0;
    lang tot weight = 0;
                      int n_matches = 0;
long long tot_weight = 0;
for (int u = 0; u <= n; ++u)st[u] = u, flower[u].clear();
int w_max = 0;
for (int u = 1; u <= n; ++u)
    for (int v = 1; v <= n; ++v) {
        flower_from[u][v] = (u == v ? u : 0);
        w_max = std::max (w_max, g[u][v].w);
}</pre>
                      for (int u = 1; u <= n; ++u)lab[u] = w_max;
while (matching())++n_matches;
for (int u = 1; u <= n; ++u)
    if (match[u] && match[u] < u)
        tot_weight += g[u][match[u]].w;
return std::make_pair (tot_weight, n_matches);</pre>
          }
void init () {
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v)
        g[u][v] = edge (u, v, 0);</pre>
};
           Sparse graph maximum flow :
    int isap::solve (const flow_edge_list &e, int n, int s, int t) :
                                 e : edge list.
n : vertex size
s : source.
t : sink.
struct isap {
           const static int MAXN = 1E3;
const static int INF = 1E9;
           int pre[MAXN], d[MAXN], gap[MAXN], cur[MAXN];
           int solve (flow_edge_list &e, int n, int s, int t) {
   std::fill (pre, pre + n + 1, 0);
   std::fill (d, d + n + 1, 0);
   std::fill (gap, gap + n + 1, 0);
   for (int i = 0; i < n; i++) cur[i] = e.begin[i];
   gap[0] = n;
   int u = pre[s] = s v maxflow = 0;</pre>
                       int u = pre[s] = s, v, maxflow = 0;
while (d[s] < n) {
    v = n:
                                  v = n;
for (int i = cur[u]; i != -1; i = e.next[i])
    if (e.flow[i] && d[u] == d[e.dest[i]] + 1) {
        v = e.dest[i];
        cur[u] = i;
        becal
                                break;
                                                          u = s;
while (p != s) {
   p = pre[p];
   dflow = std::min (dflow, e.flow[cur[p]]);
                                                          maxflow += dflow;
                                                          maxrlow += drlow;
p = t;
while (p != s) {
    p = pre[p];
    e.flow[cur[p]] -= dflow;
    e.flow[e.inv[cur[p]]] += dflow;
                                                          }
                                  } else {
   int mindist = n + 1;
   for (int i = e.begin[u]; i != -1; i = e.next[i])
      if (e.flow[i] && mindist > d[e.dest[i]]) {
```

```
mindist = d[e.dest[i]];
cur[u] = i;
                                   if (!--gap[d[u]]) return maxflow;
gap[d[u] = mindist + 1]++;
u = pre[u];
                            }
                     }
return maxflow;
             }
       };
             Dense graph maximum flow :
                     int dinic::solve (const flow_edge_list &e, int n, int s, int t) :
                            e : edge list
                            n : vertex size.
s : source.
t : sink.
       struct dinic {
              const static int MAXN = 1E3;
const static int INF = 1E9;
              flow_edge_list &e;
              int n, s, t;
              int d[MAXN], w[MAXN], q[MAXN];
             int d[MAXN], w[MAXN], q[MAXN],
int bfs() {
   for (int i = 0; i < n; i ++) d[i] = -1;
   int l, r;
   q[l = r = 0] = s, d[s] = 0;
   for (; l <= r; l ++)
        for (int k = e.begin[q[l]]; k > -1; k = e.next[k])
        if (d[e.dest[k]] == -1 && e.flow[k] > 0) d[e.dest[k]] = d[q[l]] + 1, q[++r] = e.dest[k];
   return d[t] > -1 ? 1 : 0;
}
             }
                     }
if (k == -1) d[u] = -1;
return ret;
             void solve (flow_edge_list &e, int n, int s, int t) {
    dinic::e = e; dinic::n = n; dinic::s = s; dinic::t = t;
    while (bfs ()) {
        for (int i = 0; i < n; i ++) w[i] = e.begin[i];
        dfs (s, INF);
    }</pre>
             }
      };
#include <cstdio>
using namespace graph;
int main () {
    return 0;
```

The Trial of String

```
//string.cpp
#include <string>
#include <vector>
#include <map>
namespace string {
        struct suffix_automaton {
   std::vector <std::map <char, int> > edges; // edges[i] : the labeled edges from node i
   std::vector <int> link; // link[i] : the parent of i
   std::vector <int> length; // length[i] : the length of the longest string in the ith class
   std::vector <int> terminals; // terminals : the terminal state of the automaton
   int last; // the index of the equivalence class of the whole string
                  p = link[p];
                                     lse {
  edges.push_back (edges[q]);
  length.push_back (length[p] + 1);
  link.push_back (link[q]);
  int qq = edges.size() - 1;
  link[q] = qq;
  link[r] = qq;
  while (p >= 0 && edges[p][s[i]] == q) {
     edges[p][s[i]] = qq;
     p = link[p];
  }
                                               }
                                      }
last = r;
                           int p = last;
while (p > 0) {
    terminals.push_back (p);
    p = link[p];
        };
#include <cstdio>
int main () {
    return 0;
```

Reference

5.1 vimrc

```
set ruler
set tabstop=4
set softtabstop=4
set softtabstop=4
set siftwidth=4
set smartindent
set showmatch
set hlsearch
set autoread
set backspace=2
set set autoread
set backspace=2
syntax on
nmap <C-A> ggVG
vmap <C-P> "+p
autocmd FileType cpp map <F3> :vsplit %<.in <CR>
autocmd FileType cpp map <F3> :!time ./%<.exe <CR>
autocmd FileType cpp map <F>:!time ./%<.exe <<CR>
autocmd FileType cpp map <F>:!time ./%<.exe <<CR>
autocmd FileType cpp map <F>:!time ./%<.exe <<.in <CR>
autocmd FileType pdp map <F>:!style -A2 -t -C -S -N -O -xd -p -H % <CR>
autocmd FileType java map <F8> :!time java %< <CR>
autocmd FileType java map <F8> :!time java %< <CR>
autocmd FileType java map <F9> :!java %<.cR>
autocmd FileType tex map <F9> :!java %<.cR>
autocmd FileType tex map <F9> :!java istyle -A2 -t -C -S -N -O -xd -p -H %</cR>
autocmd FileType tex map <F9> :!java %<.cR>
autocmd FileType tex map <F9> :!pdflatex % && rm %<.aux && rm %<.log <CR>
autocmd FileType tex map <F9> :!pdflatex % && rm %<.aux && rm %<.log <CR>
autocmd FileType tex map <F9> :!pdflatex % && rm %<.aux && rm %<.log <CR>
autocmd FileType tex map <F9> :!pdflatex % && rm %<.aux && rm %<.log <CR>
autocmd FileType tex map <F9> :!pdflatex % && rm %<.aux && rm %<.log <CR>
autocmd FileType tex map <F8> :!chrome %:p:h/%<.pdf <CR>
```

5.2 Java Reference

```
doubleValue () / toPlainString () : converts to other types.
                       Arrays:
Arrays.sort (T [] a);
Arrays.sort (T [] a, int fromIndex, int toIndex);
Arrays.sort (T [] a, int fromIndex, int toIndex, Comperator <? super T> comperator);

**TabedTiet <E>:

**TabedTiet <E>:

**TabedTiet / GetLast / removeFirst / removeLast () :
                        LinkedList <E>:

addFirst / addLast (E) / getFirst / getLast / removeFirst / removeLast ():

deque implementation.

clear () / add (int, E) / remove (int): clear, add & remove.

size () / contains / removeFirstOccurrence / removeLastOccurrence (E):

deque methods.
                      deque methods.

ListIterator <E> listIterator (int index) : returns an iterator :
    E next / previous () : accesses and iterates.
    hasNext / hasPrevious () : checks availablity.
    nextIndex / previousIndex () : returns the index of a subsequent call.
    add / set (E) / remove () : changes element.

PriorityQueue <E> (int initcap, Comparator <? super E> comparator) :
    add (E) / clear () / iterator () / peek () / poll () / size () :
        priority queue implementations.

TreeMap <K, V> (Comparator <? super K> comparator) :
    Map.Entry <K, V> ceilingEntry / floorEntry / higherEntry / lowerEntry (K):
        getKey / getValue () / setValue (V) : entries.
    clear () / put (K, V) / get (K) / remove (K) : basic operation.
    size () : size.

StringBuilder :
                       clear () / put (K, V) / get (K) / remove (K) : basic operation.
    size () : size.
StringBuilder :
    Mutable string.
    StringBuilder (string) : generates a builder.
    append (int, string, ...) / insert (int offset, ...) : adds objects.
    charAt (int) / setCharAt (int, char) : accesses a char.
    delete (int, int) : removes a substring.
    reverse () : reverses itself.
    length () : returns the length.
    toString () : converts to string.
                         String :
                                    Immutable string.

String.format (String, ...) : formats a string. i.e. sprintf. toLowerCase / toUpperCase () : changes the case of letters.
/* Examples on Comparator :
public class Main {
           public static class Point {
                        public int x;
public int y;
                        public Point () {
                                   x = 0;

y = 0;
                        public Point (int xx, int yy) {
    x = xx;
    y = yy;
            1;
           public static class Cmp implements Comparator <Point> {
   public int compare (Point a, Point b) {
      if (a.x < b.x) return -1;
      if (a.x == b.x) {
        if (a.y < b.y) return -1;
        if (a.y == b.y) return 0;
   }</pre>
                                    return 1:
                        }
            1;
           public static void main (String [] args) {
   Cmp c = new Cmp ();
   TreeMap <Point, Point> t = new TreeMap <Point, Point> (c);
            1
};
            Another way to implement is to use Comparable. However, equalTo and hashCode must be rewritten. Otherwise, containers may fail and give strange answers.
            Example :
           public static class Point implements Comparable <Point> {
                        public int x;
public int y;
public Point () {
                                   x = 0;

y = 0;
                        public Point (int xx, int yy) {
    x = xx;
    y = yy;
                        public int compareTo (Point p) {
    if (x < p.x) return -1;
    if (x == p.x) {
        if (y < p.y) return -1;
        if (y == p.y) return 0;
    }
}</pre>
                                    return 1;
                        public boolean equalTo (Point p) {
    return (x == p.x && y == p.y);
                        public int hashCode () {
    return x + y;
           };
//Faster IO :
public class Main {
            static class InputReader {
```

5.3 Hacks