

# The Trial of Number Theory

```
//number.cpp
#include <cmath>
#include <complex>
namespace number {
           iong long inverse (const long long &x, const long long &mod) {
   if (x == 1) return 1;
   return (mod - mod / x) * inverse (mod % x, mod) % mod;
          int fpm (int x, int n, int mod) {
   register int ans = 1, mul = x;
   while (n) {
      if (n & 1) ans = int (111 * ans * mul * mod);
      mul = int (111 * mul * mul * mod);
      n >>= 1;
}
                      return ans;
          namespace FFT {
                      const int MAXN = 1E6;
const double PI = acos (-1);
                      typedef std::complex <double> Complex;
                      Complex e[2][MAXN];
                      int prepare (int n) {
   int len = 1;
   for (; len <= 2 * n; len <<= 1);
   for (int i = 0; i < len; i++) {
       e[0][i] = Complex (cos (2 * PI * i / len), sin (2 * PI * i / len));
       e[1][i] = Complex (cos (2 * PI * i / len), -sin (2 * PI * i / len));
}</pre>
                                 }
return len;
                     void DFT (Complex *a, int n, int f) {
    for (int i = 0, j = 0; i < n; i++) {
        if (i > j) std::swap (a[i], a[j]);
        for (int t = n >> 1; (j ^= t) < t; t >>= 1);
                                 for (int i = 2; i <= n; i <<= 1)
    for (int j = 0; j < n; j += i)
        for (int k = 0; k < (i >> 1); k++) {
            Complex A = a[j + k];
            Complex B = e[f][n / i * k] * a[j + k + (i >> 1)];
            a[i + k] = A + B:
                                                                   a[j + k] = A + B;

a[j + k + (i >> 1)] = A - B;
                                 if (f == 1) {
   for (int i = 0; i < n; i++)
        a[i] = Complex (a[i].real () / n, a[i].imag ());</pre>
           namespace NTT {
                      const int MAXN = 1E6;
                      void DFT (int *a, int n, int f, int mod, int prt) {
    for (register int i = 0, j = 0; i < n; i++) {
        if (i > j) std::swap (a[i], a[j]);
        for (register int t = n >> 1; (j ^= t) < t; t >>= 1);
                                 for (register int i = 2; i <= n; i <<= 1) {
    static int exp[MAXN];
    exp[0] = 1;
    exp[1] = fpm (prt, (mod - 1) / i, mod);
    if (f == 1) exp[1] = fpm (exp[1], mod - 2, mod);
    for (register int k = 2; k < (i >> 1); k++) {
        exp[k] = int (111 * exp[k - 1] * exp[1] % mod);
}
                                            for (register int j = 0; j < n; j += i) {
    for (register int k = 0; k < (i >> 1); k++) {
        register int &pA = a[j + k], &pB = a[j + k + (i >> 1)];
        register int A = pA, B = int (111 * pB * exp[k] % mod);
        pA = (A + B) % mod;
        pB = (A - B + mod) % mod;
}
                                                       }
                                            }
                                 }
if (f == 1) {
    register int rev = fpm (n, mod - 2, mod);
    for (register int i = 0; i < n; i++) {</pre>
```

## The Trial of Geometry

```
#include <cmath>
#include <vector>
#include <algorithm>
namespace geometry {
         /* Basic constant & function
                          EPS : fixes the possible error of data
                                 i.e. x == y \text{ iff } |x - y| < EPS.
                          PI : the value of PI.
int sgn (const double &x) : returns the sign of x.
int cmp (const double &x, const double &y) : returns the sign of x - y.
double sqr (const double &x) : returns x * x.
         const double EPS = 1E-8;
const double PI = acos (-1);
        int sgn (const double &x) { return x < -EPS ? -1 : x > EPS; } int cmp (const double &x, const double &y) { return sgn (x - y); } double sqr (const double &x) { return x * x; }
         /* struct point : defines a point and its various utility.
    point (const double &x, const double &y) gives a point at (x, y).
    It also represents a vector on a 2D plane.
    point unit () const : returns the unit vector of (x, y).
    point rot90 () const :
                          returns a point rotated 90 degrees counter-clockwise with respect to the origin.

point _rot () const : same as above except clockwise.

point rotate (const double &t) const : returns a point rotated t radian(s) counter-clockwise.

Operators are mostly vector operations. i.e. vector +, -, *, / and dot/det product.
       */
struct point {
    double x, y;
    point (const double &x = 0, const double &y = 0) : x (x), y (y) {}
    double norm () const { return sqrt (x * x + y * y); }
    double norm2 () const { return x * x + y * y; }
    point unit () const {
        double 1 = norm ();
        return point (x / 1, y / 1);
}
                 point rot90 () const {return point (-y, x); }
point _rot90 () const {return point (y, -x); }
point rotate (const double &t) const {
    double c = cos (t), s = sin (t);
    return point (x * c - y * s, x * s + y * c);
}
        bool operator == (const point &a, const point &b) {
   return cmp (a.x, b.x) == 0 && cmp (a.y, b.y) == 0;
        bool operator != (const point &a, const point &b) {
    return ! (a == b);
        bool operator < (const point &a, const point &b) {
   if (cmp (a.x, b.x) == 0) return cmp (a.y, b.y) < 0;
   return cmp (a.x, b.x) < 0;</pre>
         point operator - (const point &a) { return point (-a.x, -a.y); }
        point operator + (const point &a, const point &b) {
   return point (a.x + b.x, a.y + b.y);
        point operator - (const point &a, const point &b) {
   return point (a.x - b.x, a.y - b.y);
        point operator * (const point &a, const double &b) {
   return point (a.x * b, a.y * b);
        point operator / (const point &a, const double &b) {
   return point (a.x / b, a.y / b);
         double dot (const point &a, const point &b) {
    return a.x * b.x + a.y * b.y;
        double det (const point &a, const point &b) {
   return a.x * b.y - a.y * b.x;
```

```
double dis (const point &a, const point &b) {
   return sqrt (sqr (a.x - b.x) + sqr (a.y - b.y));
         struct line : defines a line (segment) based on two points, s and t.
    line (const point &s, const point &t) gives a basic line from s to t.
    double length () const : returns the length of the segment.
 struct line {
         point s, t;
line (const point &s = point (), const point &t = point ()) : s (s), t (t) {}
double length () const { return dis (s, t); }
/* Point & line interaction :
    bool point_on_line (const point &a, const line &b) : checks if a is on b.
    bool intersect_judgement (const line &a, const line &b) : checks if segment a and b intersect.
    point line_intersect (const line &a, const line &b) : returns the intersection of a and b.
        Fails on colinear or parallel situations.
    double point_to_line (const point &a, const line &b) : returns the distance from a to b.
    double point_to_segment (const point &a, const lint &b) : returns the distance from a to b.
    i.e. the minimized length from a to segment b.
    bool in polvgon (const point &p, const std::vector <point> &po) :
                  bool in_polygon (const point &p, const std::vector <point> &po) :
    checks if a is in a polygon with vetices po (clockwise or counter-clockwise order).
point project_to_line (const point &a, const line &b) :
    returns the projection of a on b,
bool point_on_line (const point &a, const line &b) {
   return sgn (det (a - b.s, b.t - b.s)) == 0 && sgn (dot (b.s - a, b.t - a)) <= 0;</pre>
bool two_side (const point &a, const point &b, const line &c) {
   return sgn (det (a - c.s, c.t - c.s)) * sgn (det (b - c.s, c.t - c.s)) < 0;</pre>
bool intersect_judgement (const line &a, const line &b) {
         if (point_on_line (b.s, a) || point_on_line (b.t, a)) return true;
if (point_on_line (a.s, b) || point_on_line (a.t, b)) return true;
return two_side (a.s, a.t, b) && two_side (b.s, b.t, a);
 }
point line_intersect (const line &a, const line &b) {
   double s1 = det (a.t - a.s, b.s - a.s);
   double s2 = det (a.t - a.s, b.t - a.s);
   return (b.s * s2 - b.t * s1) / (s2 - s1);
double point_to_line (const point &a, const line &b) {
   return fabs (det (b.t - b.s, a - b.s)) / dis (b.s, b.t);
double point_to_segment (const point &a, const line &b) {
   if (sgn (dot (b.s - a, b.t - b.s) * dot (b.t - a, b.t - b.s)) <= 0)
      return fabs (det (b.t - b.s, a - b.s)) / dis (b.s, b.t);
   return std::min (dis (a, b.s), dis (a, b.t));
}</pre>
*/
if (point_on_line (p, line (a, b))) return true;
int x = sgn (det (p - a, b - a)), y = sgn (a.y - p.y), z = sgn (b.y - p.y);
if (x > 0 && y <= 0 && z > 0) counter++;
if (x < 0 && z <= 0 && y > 0) counter--;
         return counter != 0;
point project_to_line (const point &a, const line &b) {
   return b.s + (b.t - b.s) * (dot (a - b.s, b.t - b.s) / (b.t - b.s).norm2 ());
      Centers of a triangle : returns various centers of a triangle with vertices (a, b, c).
point incenter (const point &a, const point &b, const point &c) {
    double p = (a - b).norm () + (b - c).norm () + (c - a).norm ();
    return (a * (b - c).norm () + b * (c - a).norm () + c * (a - b).norm ()) / p;
point circumcenter (const point &a, const point &b, const point &c) {
   point p = b - a, q = c - a, s (dot (p, p) / 2, dot (q, q) / 2);
   double d = det (p, q);
   return a + point (det (s, point (p.y, q.y)), det (point (p.x, q.x), s)) / d;
point orthocenter (const point &a, const point &b, const point &c) {
    return a + b + c - circumcenter (a, b, c) * 2.0;
 /*
         struct circle defines a circle. circle (point c, double r) gives a circle with center c and radius r.
 struct circle {
         point c;
double r;
circle (point c = point (), double r = 0) : c (c), r (r) {}
bool operator == (const circle &a, const circle &b) {
   return a.c == b.c && cmp (a.r, b.r) == 0;
bool operator != (const circle &a, const circle &b) {
   return ! (a == b);
 /* Circle interaction :
```

```
bool in_circle (const point &a, const circle &b) : checks if a is in or on b.
circle make_circle (const point &a, const point &b) :
    generates a circle with diameter ab.
circle make_circle (const point &a, const point &b, const point &c) :
    generates a circle passing a, b and c.
std::pair <point, point> line_circle_intersect (const line &a, const circle &b) :
    returns the intersections of a and b.
    Fails if a and b do not intersect.
std::pair <point, point> circle_intersect (const circle &a, const circle &b):
    returns the intersections of a and b.
    Fails if a and b do not intersect.
std::pair std::pair intersections of a and b.
    Fails if a and b do not intersect.
std::pair intersections of a and b.
    Fails if a intersections 
                                   Fails if a is in b.
bool in_circle (const point &a, const circle &b) {
   return cmp (dis (a, b.c), b.r) <= 0;</pre>
circle make_circle (const point &a, const point &b) {
   return circle ((a + b) / 2, dis (a, b) / 2);
circle make_circle (const point &a, const point &b, const point &c) {
           point p = circumcenter (a, b, c);
return circle (p, dis (p, a));
nt __circle_intersect (const circle &a, const circle &b) {
point r = (b.c - a.c).unit ();
double d = dis (a.c, b.c);
double x = .5 * ((sqr (a.r) - sqr (b.r)) / d + d);
double h = sqrt (sqr (a.r) - sqr (x));
return a.c + r * x + r.rot90 () * h;
point
std::pair <point, point> circle_intersect (const circle &a, const circle &b) {
    return std::make_pair (__circle_intersect (a, b), __circle_intersect (b, a));
std::pair <line, line> tangent (const point &a, const circle &b) {
    circle p = make_circle (a, b.c);
    return circle_intersect (p, b);
 /* Convex hull
                      std::vector <point> convex_hull (std::vector <point> a) :
    returns the convex hull of point set a (counter-clockwise).
bool turn_left (const point &a, const point &b, const point &c) {
    return sgn (det (b - a, c - a)) >= 0;
bool turn_right (const point &a, const point &b, const point &c) {
   return sgn (det (b - a, c - a)) <= 0;</pre>
}
ret.pop_back ();
                       ret.push_back (a[i]);
++cnt;
           ret.push_back (a[i]); ++cnt;
           ret.pop_back ();
return ret;
          Minimum circle of a point set : circle minimum_circle (std::vector <point> p) : returns the minimum circle of point set p.
circle minimum_circle (std::vector <point> p) {
            circle ret:
           if (!in_circle (p[k], ret)) ret = make_circle (p[i], p[j], p[k]);
            return ret;
/* Online half plane intersection (complexity = O(c.size ())) :
    std::vector <point> cut (const std::vector<point> &c, line p) :
    returns the convex polygon cutting convex polygon c with half plane p.
        (left hand with respect to vector p)
        If such polygon does not exist, returns an empty set.
        e.g.
```

```
static const double BOUND = 1e5;
convex.clear ();
convex.push_back (point (-BOUND, -BOUND));
convex.push_back (point (BOUND, -BOUND));
convex.push_back (point (BOUND, BOUND));
convex.push_back (point (-BOUND, BOUND));
convex = cut (convex, line(point, point));
if (convex.empty ()) { . . . }
std::vector <point> cut (const std::vector<point> &c, line p) {
    std::vector <point> ret;
    if (c.empty ()) return ret;
    for (int i = 0; i < (int) c.size (); ++i) {
        int j = (i + 1) % (int) c.size ();
        if (!turn_right (p.s, p.t, c[i])) ret.push_back (c[i]);
        if (two_side (c[i], c[j], p))
            ret.push_back (line_intersect (p, line (c[i], c[j])));
}</pre>
        return ret;
}
/* Offline half plane intersection (complexity = O(nlogn), n = h.size ()) :
    std::vector <point> half_plane_intersect (std::vector <line> h) :
        returns the intersection of half planes h.
        (left hand with respect to the vector)
        If such polygon does not exist, returns an empty set.
bool turn_left (const line &1, const point &p) {
    return turn_left (1.s, 1.t, p);
std::vector <point> half_plane_intersect (std::vector <line> h) {
    typedef std::pair <double, line> polar;
    std::vector <polar> g;
    g.resize (h.size ());
    for (int i = 0; i < (int) h.size (); ++i) {
        point v = h[i].t - h[i].s;
        g[i] = std::make_pair (atan2 (v.y, v.x), h[i]);
}</pre>
        if (g.begin (), g.end (), [] (const polar &a, const polar &b) {
   if (cmp (a.first, b.first) == 0)
     return sgn (det (a.second.t - a.second.s, b.second.t - a.second.s)) < 0;</pre>
                        return cmp (a.first, b.first) < 0;
       ret.pop_back ();
                ++rore;
++rear;
ret.push_back (h[i]);
        ret.pop_back ();
        while (rear - fore > 1 && !turn_left (ret[rear], line_intersect (ret[fore], ret[fore + 1])))
        ++fore;
if (rear - fore < 2) return std::vector <point> ();
std::vector <point> ans;
        ans.resize (ret.size ());
for (int i = 0; i < (int) ret.size (); ++i)
    ans[i] = line_intersect (ret[i], ret[ (i + 1) % ret.size ()]);</pre>
namespace polygon_circle_intersect {
        // The area of the sector with center (0, 0), radius r and segment ab.
       double sector_area (const point &a, const point &b, const double &r) {
   double c = (2.0 * r * r - (a - b).norm2 ()) / (2.0 * r * r);
   double al = acos (c);
   return r * r * al / 2.0;
        . The area of triangle (a, b, (0, 0)) intersecting circle (point (), r).
        double area (const point &a, const point &b, const double &r) {
  double dA = dot (a, a), dB = dot (b, b), dC = point_to_segment (point (), line (a, b)), ans = 0.0;
  if (sgn (dA - r * r) <= 0 && sgn (dB - r * r) <= 0) return det (a, b) / 2.0;
  point tA = a.unit () * r;
  point tB = b.unit () * r;
  if (sgn (dC - r) > 0) return sector_area (tA, tB, r);
  std:rei* (point reint) ret = line sirele intersect (line (a, b) sirele (point (), r));
                if (sgn (uc - r) > 0) return sector_area (tA, tB, r);
std::pair <point, point> ret = line_circle_intersect (line (a, b), circle (point (), r));
if (sgn (dA - r * r) > 0 && sgn (dB - r * r) > 0) {
    ans += sector_area (tA, ret.first, r);
    ans += det (ret.first, ret.second) / 2.0;
    ans += sector_area (ret.second, tB, r);
    return ans;
}
                if (sgn (dA - r * r) > 0)
   return det (ret.first, b) / 2.0 + sector_area (tA, ret.first, r);
                else
  return det (a, ret.second) / 2.0 + sector_area (ret.second, tB, r);
        }
```

```
// Main process.
double main (const std::vector <point> &p, const circle &c) {
    double ret = 0.0;
    for (int i = 0; i < (int) p.size (); ++i) {
        int s = sgn (det (p[i] - c.c, p[ (i + 1) % p.size ()] - c.c));
        if (s > 0)
        ret += area (p[i] - c.c, p[ (i + 1) % p.size ()] - c.c, c.r);
    else
                  ret -= area (p[ (i + 1) % p.size ()] - c.c, p[i] - c.c, c.r);
      return fabs (ret);
}
Union of circles:
    std::vector <double> union_circle::main (std::vector <circle> &cir) :
        returns the union of circle set cir.
        The i-th element is the area covered by i circles.
double cub (const double &x) { return x * x * x; }
struct arc {
      double t:
      point p;
int d;
arc() {};
      arc (const double &t, const point &p, int d) : t (t), p (p), d (d) {}
std::vector <arc> vec:
std::vector <double> ans;
std::vector <point> c;
int cnt = 0;
bool operator < (const arc &a, const arc &b) {
   return cmp (a.t, b.t) < 0;</pre>
void psh (const double t1, const point p1, const double t2, const point p2) {
   if (cmp (t2, t1) < 0) cnt++;
   vec.push_back (arc (t1, p1, 1));
   vec.push_back (arc (t2, p2, -1));
}</pre>
void combine (int d, const double &ar, const point &o) {
   if (sgn (ar) == 0) return;
   c[d] = (c[d] * ans[d] + o * ar) * (1.0 / (ans[d] + ar));
   ans[d] += ar;
f[i] = false;
break;
      c.clear ();
ans.resize (n + 1);
c.resize (n + 1);
      point dvd;
for (int i = 0; i < n; ++i) {
    dvd = cir[i].c - point (cir[i].r, 0);
```

# The Trial of Graph

# The Trial of String

```
//string.cpp
#include <string>
#include <vector>
#include <map>
namespace string {
        struct suffix_automaton {
   std::vector <std::map <char, int> > edges; // edges[i] : the labeled edges from node i
   std::vector <int> link; // link[i] : the parent of i
   std::vector <int> length; // length[i] : the length of the longest string in the ith class
   std::vector <int> terminals; // terminals : the terminal state of the automaton
   int last; // the index of the equivalence class of the whole string
                  p = link[p];
                                     lse {
  edges.push_back (edges[q]);
  length.push_back (length[p] + 1);
  link.push_back (link[q]);
  int qq = edges.size() - 1;
  link[q] = qq;
  link[r] = qq;
  while (p >= 0 && edges[p][s[i]] == q) {
     edges[p][s[i]] = qq;
     p = link[p];
  }
                                               }
                                      }
last = r;
                           }
int p = last;
while (p > 0) {
    terminals.push_back (p);
    p = link[p];
        };
#include <cstdio>
int main () {
    return 0;
```

#### Reference

#### 5.1 vimrc

```
set ruler
set number
set tabstop=4
set softtabstop=4
set siftwidth=4
set smartindent
set showmatch
set incsearch
set autoread
set autoread
set backspace=2
set mouse=a
syntax on
nmap <C-A> ggVG
vmap <C-C> "+y
nmap <C-P> "+p
autocmd FileType cpp set cindent
autocmd FileType cpp map <F3> :vsplit %<.in <CR>
autocmd FileType cpp map <F5> :!time ./%<.exe < CR>
autocmd FileType cpp map <F5> :!time ./%<.exe < CR>
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autocmd FileType cpp map <F10> :!astyle -A2 -t -C -S -N -O -xd % <CR>
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autocmd FileType cpp map <F10> :!astyle -A2 -t -C -S -N -O -xd % <CR>
autocmd FileType cpp map <F10> :!astyle -A2 -t -C -S -N -O -xd % <CR>
autocmd FileType cpp map <F10> :!astyle -A2 -t -C -S -N -O -xd % <CR>
autocmd FileType cpp map <F10> :!astyle -A2 -t -C -S -N -O -xd % <CR>
autocmd FileType cpp map <F10> :!astyle -A2 -t -C -S -N -O -xd % <CR>
autocmd FileType cpp map <F10> :!astyle -A2 -t -C -S -N -O -xd % <CR>
autocmd FileType cpp map <F10> :!astyle -A2 -t -C -S -N -O -xd % <CR>
autocmd FileType cpp map <F10> :!astyle -A2
```