The Grimoire of Programming

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Contents

Chapter 1

Trial of number theory

1.1 Constants and basic functions

```
const double PI = acos (-1);
/*  Basic constants & functions :
        long long inverse (const long long &x, const long long &mod) :
            returns the inverse of x modulo mod.
            i.e. x * inv (x) % mod = 1.
        int fpm (int x, int n, int mod) :
            returns x^n % mod. i.e. Fast Power with Modulo.
        void euclid (const long long &a, const long long &b,
                     long long &x, long long &y) :
            solves for ax + by = gcd (a, b).
        long long gcd (const long long &a, const long long &b) :
            solves for the greatest common divisor of a and b.
        long long mul_mod (const long long &a, const long long &b, const long long &mod) :
            returns a * b % mod.
        long long llfpm (const long long &x, const long long &n, const long long &mod) :
            returns x^n % mod.
*/
long long abs (const long long &x) { return x > 0 ? x : -x; }
long long inverse (const long long &x, const long long &mod) {
    if (x == 1) return 1;
    return (mod - mod / x) * inverse (mod % x, mod) % mod;
}
int fpm (int x, int n, int mod) {
    register int ans = 1, mul = x;
    while (n) {
        if (n & 1) ans = int (1ll * ans * mul % mod);
        mul = int (1ll * mul * mul % mod);
        n >>= 1;
    }
    return ans;
}
void euclid (const long long &a, const long long &b,
             long long &x, long long &y) {
    if (b == 0) x = 1, y = 0;
    else euclid (b, a % b, y, x), y -= a / b * x;
}
long long gcd (const long long &a, const long long &b) {
    if (!b) return a;
    long long x = a, y = b;
    while (x > y ? (x = x % y) : (y = y % x));
    return x + y;
}
long long mul_mod (const long long &a, const long long &b, const long long &mod) {
    long long ans = 0, add = a, k = b;
    while (k) {
        if (k & 1) ans = (ans + add) % mod;
        add = (add + add) % mod;
        k >>= 1;
    }
    return ans;
}
long long llfpm (const long long &x, const long long &n, const long long &mod) {
    long long ans = 1, mul = x, k = n;
    while (k) {
        if (k & 1) ans = mul_mod (ans, mul, mod);
        mul = mul_mod (mul, mul, mod);
        k >>= 1;
    }
    return ans;
}
```

1.2 Discrete Fourier transform

```
/*  Discrete Fourier transform :
        int dft::init (int n) : initializes the transformation with dimension n.
        void dft::main (complex *a, int n, int f) :
            transforms array a with dimension n to its frequency representation.
            Transforms back when f = 1.
*/
template <int MAXN = 1E6>
struct dft {
    typedef std::complex <double> complex;
    complex e[2][MAXN];
```

```
    int init (int n) {
        int len = 1;
        for (; len <= 2 * n; len <<= 1);
        for (int i = 0; i < len; i++) {
            e[0][i] = complex (cos (2 * PI * i / len), sin (2 * PI * i / len));
            e[1][i] = complex (cos (2 * PI * i / len), -sin (2 * PI * i / len));
        }
        return len;
    }
    void main (complex *a, int n, int f) {
        for (int i = 0, j = 0; i < n; i++) {
            if (i > j) std::swap (a[i], a[j]);
            for (int t = n >> 1; (j ^= t) < t; t >>= 1);
        }
        for (int i = 2; i <= n; i <<= 1)
            for (int j = 0; j < n; j += i)
                for (int k = 0; k < (i >> 1); k++) {
                    complex A = a[j + k];
                    complex B = e[f][n / i * k] * a[j + k + (i >> 1)];
                    a[j + k] = A + B;
                    a[j + k + (i >> 1)] = A - B;
                }
        if (f == 1) {
            for (int i = 0; i < n; i++)
                a[i] = complex (a[i].real () / n, a[i].imag ());
        }
    }
};
```

1.3 Number-theoretic transform

```
/* Number-theoretic transform :
    void ntt::main (int *a, int n, int f, int mod, int prt) :
        converts polynominal f (x) = a[0] * x^0 + a[1] * x^1 + ... + a[n - 1] * x^(n - 1)
            to a vector (f (prt^0), f (prt^1), f (prt^2), ..., f (prt^(n - 1))). (module mod)
        Converts back if f = 1.
        Requries specific mod and corresponding prt to work. (given in MOD and PRT)
    int ntt::crt (int *a, int mod) :
        fixes the results a from module 3 primes to a certain module mod.
*/
template <int MAXN = 1E6>
struct ntt {
    void main (int *a, int n, int f, int mod, int prt) {
        for (register int i = 0, j = 0; i < n; i++) {
            if (i > j) std::swap (a[i], a[j]);
            for (register int t = n >> 1; (j ^= t) < t; t >>= 1);
        }
        for (register int i = 2; i <= n; i <<= 1) {
            static int exp[MAXN];
            exp[0] = 1;
            exp[1] = fpm (prt, (mod - 1) / i, mod);
            if (f == 1) exp[1] = fpm (exp[1], mod - 2, mod);
            for (register int k = 2; k < (i >> 1); k++) {
                exp[k] = int (1ll * exp[k - 1] * exp[1] % mod);
            }
            for (register int j = 0; j < n; j += i) {
                for (register int k = 0; k < (i >> 1); k++) {
                    register int &pA = a[j + k], &pB = a[j + k + (i >> 1)];
                    register int A = pA, B = int (1ll * pB * exp[k] % mod);
                    pA = (A + B) % mod;
                    pB = (A - B + mod) % mod;
                }
            }
        }
        if (f == 1) {
            register int rev = fpm (n, mod - 2, mod);
            for (register int i = 0; i < n; i++) {
                a[i] = int (1ll * a[i] * rev % mod);
            }
        }
    }
    int MOD[3] = {1045430273, 1051721729, 1053818881}, PRT[3] = {3, 6, 7};
    int crt (int *a, int mod) {
        static int inv[3][3];
        for (int i = 0; i < 3; i++)
            for (int j = 0; j < 3; j++)
                inv[i][j] = (int) inverse (MOD[i], MOD[j]);
        static int x[3];
        for (int i = 0; i < 3; i++) {
            x[i] = a[i];
            for (int j = 0; j < i; j++) {
                int t = (x[i] - x[j] + MOD[i]) % MOD[i];
                if (t < 0) t += MOD[i];
                x[i] = int (1LL * t * inv[j][i] % MOD[i]);
            }
        }
        int sum = 1, ret = x[0] % mod;
        for (int i = 1; i < 3; i ++) {
            sum = int (1LL * sum * MOD[i - 1] % mod);
            ret += int (1LL * x[i] * sum % mod);
            if (ret >= mod) ret -= mod;
        }
        return ret;
    }
};
```

1.4 Chinese remainder theorem

```
/* Chinese remainder theroem :
```

```
        bool crt::solve (const std::vector <std::pair<long long, long long> > &input,
                        std::pair<long long, long long> &output) :
            solves for an integer set x = output.first + k * output.second
                that satisfies x % input[i].second = input[i].first.
            Returns whether a solution exists.
*/
struct crt {
    long long fix (const long long &a, const long long &b) {
        return (a % b + b) % b;
    }
    bool solve (const std::vector <std::pair <long long, long long> > &input,
                std::pair <long long, long long> &output) {
        output = std::make_pair (1, 1);
        for (int i = 0; i < (int) input.size (); ++i) {
            long long number, useless;
            euclid (output.second, input[i].second, number, useless);
            long long divisor = gcd (output.second, input[i].second);
            if ((input[i].first - output.first) % divisor) {
                return false;
            }
            number *= (input[i].first - output.first) / divisor;
            number = fix (number, input[i].second);
            output.first += output.second * number;
            output.second *= input[i].second / divisor;
            output.first = fix (output.first, output.second);
        }
        return true;
    }
};
```

1.5 Miller Rabin primality test

```
/*  Miller Rabin :
        bool miller_rabin::solve (const long long &) :
            tests whether a certain integer is prime.
*/
struct miller_rabin {
    int BASE[12] = {2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37};
    bool check (const long long &prime, const long long &base) {
        long long number = prime - 1;
        for (; ~number & 1; number >>= 1);
        long long result = llfpm (base, number, prime);
        for (; number != prime - 1 && result != 1 && result != prime - 1; number <<= 1)
            result = mul_mod (result, result, prime);
        return result == prime - 1 || (number & 1) == 1;
    }
    bool solve (const long long &number) {
        if (number < 2) return false;
        if (number < 4) return true;
        if (~number & 1) return false;
        for (int i = 0; i < 12 && BASE[i] < number; ++i)
            if (!check (number, BASE[i]))
                return false;
        return true;
    }
};
```

1.6 Pollard's Rho algorithm

```
/*  Pollard Rho :
        std::vector <long long> pollard_rho::solve (const long long &) :
            factorizes an integer.
*/
struct pollard_rho {
    miller_rabin is_prime;
    const long long threshold = 13E9;
    long long factorize (const long long &number, const long long &seed) {
        long long x = rand() % (number - 1) + 1, y = x;
        for (int head = 1, tail = 2; ; ) {
            x = mul_mod (x, x, number);
            x = (x + seed) % number;
            if (x == y)
                return number;
            long long answer = gcd (abs (x - y), number);
            if (answer > 1 && answer < number)
                return answer;
            if (++head == tail) {
                y = x;
                tail <<= 1;
            }
        }
    }
    void search (const long long &number, std::vector<long long> &divisor) {
        if (number > 1)  {
            if (is_prime.solve (number))
                divisor.push_back (number);
            else {
                long long factor = number;
                for (; factor >= number;
                       factor = factorize (number, rand () % (number - 1) + 1));
                search (number / factor, divisor);
                search (factor, divisor);
            }
        }
    }
    std::vector <long long> solve (const long long &number) {
        std::vector <long long> ans;
```

```
        if (number > threshold)
            search (number, ans);
        else {
            long long rem = number;
            for (long long i = 2; i * i <= rem; ++i)
                if (number % i)
                    ++i;
                else {
                    ans.push_back (i);
                    rem %= i;
                }
        }
        return ans;
    }
};
```

1.7 Adaptive Simpson's method

```
/*  Adaptive Simpson's method :
        double simpson::solve (double (*f) (double), double l, double r, double eps) :
            integrates f over (l, r) with error eps.
*/
struct simpson {
    double area (double (*f) (double), double l, double r) {
        double m = l + (r - l) / 2;
        return (f (l) + 4 * f (m) + f (r)) * (r - l) / 6;
    }
    double solve (double (*f) (double), double l, double r, double eps, double a) {
        double m = l + (r - l) / 2;
        double left = area (f, l, m), right = area (f, m, r);
        if (fabs (left + right - a) <= 15 * eps) return left + right + (left + right - a) / 15.0;
        return solve (f, l, m, eps / 2, left) + solve (f, m, r, eps / 2, right);
    }
    double solve (double (*f) (double), double l, double r, double eps) {
        return solve (f, l, r, eps, area (f, l, r));
    }
};
```

Chapter 2

Trial of geometry

2.1 Constants and basic functions

```
/*  Constants & basic functions :
        EPS : fixes the possible error of data.
            i.e. x == y iff |x - y| < EPS.
        PI : the value of PI.
        int sgn (const double &x) : returns the sign of x.
        int cmp (const double &x, const double &y) : returns the sign of x - y.
        double sqr (const double &x) : returns x * x.
*/
const double EPS = 1E-8;
const double PI = acos (-1);
int sgn (const double &x) { return x < -EPS ? -1 : x > EPS; }
int cmp (const double &x, const double &y) { return sgn (x - y); }
double sqr (const double &x) { return x * x; }
```

2.2 Point class

```
/*  struct point : defines a point and its various utility.
        point (const double &x, const double &y) gives a point at (x, y).
        It also represents a vector on a 2D plane.
        point unit () const : returns the unit vector of (x, y).
        point rot90 () const :
            returns a point rotated 90 degrees counter-clockwise with respect to the origin.
        point _rot () const : same as above except clockwise.
        point rotate (const double &t) const : returns a point rotated t radian(s) counter-clockwise.
        Operators are mostly vector operations. i.e. vector +, -, *, / and dot/det product.
*/
struct point {
    double x, y;
    point (const double &x = 0, const double &y = 0) : x (x), y (y) {}
    double norm () const { return sqrt (x * x + y * y); }
    double norm2 () const { return x * x + y * y; }
    point unit () const {
        double l = norm ();
        return point (x / l, y / l);
    }
    point rot90 () const {return point (-y, x); }
    point _rot90 () const {return point (y, -x); }
    point rotate (const double &t) const {
        double c = cos (t), s = sin (t);
        return point (x * c - y * s, x * s + y * c);
    }
};
bool operator == (const point &a, const point &b) {
    return cmp (a.x, b.x) == 0 && cmp (a.y, b.y) == 0;
}
bool operator != (const point &a, const point &b) {
    return ! (a == b);
}
bool operator < (const point &a, const point &b) {
    if (cmp (a.x, b.x) == 0) return cmp (a.y, b.y) < 0;
    return cmp (a.x, b.x) < 0;
}
point operator - (const point &a) { return point (-a.x, -a.y); }
point operator + (const point &a, const point &b) {
    return point (a.x + b.x, a.y + b.y);
}
point operator - (const point &a, const point &b) {
    return point (a.x - b.x, a.y - b.y);
}
point operator * (const point &a, const double &b) {
    return point (a.x * b, a.y * b);
}
point operator / (const point &a, const double &b) {
    return point (a.x / b, a.y / b);
}
double dot (const point &a, const point &b) {
    return a.x * b.x + a.y * b.y;
}
double det (const point &a, const point &b) {
    return a.x * b.y - a.y * b.x;
```

```
}
double dis (const point &a, const point &b) {
    return sqrt (sqr (a.x - b.x) + sqr (a.y - b.y));
}
```

2.3 Line class

```
/*  struct line : defines a line (segment) based on two points, s and t.
        line (const point &s, const point &t) gives a basic line from s to t.
        double length () const : returns the length of the segment.
*/
struct line {
    point s, t;
    line (const point &s = point (), const point &t = point ()) : s (s), t (t) {}
    double length () const { return dis (s, t); }
};
```

2.4 Interactions between points and lines

```
/*  Point & line interactions :
        bool point_on_line (const point &a, const line &b) : checks if a is on b.
        bool intersect_judgement (const line &a, const line &b) : checks if segment a and b intersect.
        point line_intersect (const line &a, const line &b) : returns the intersection of a and b.
            Fails on colinear or parallel situations.
        double point_to_line (const point &a, const line &b) : returns the distance from a to b.
        double point_to_segment (const point &a, const lint &b) : returns the distance from a to b.
            i.e. the minimized length from a to segment b.
        bool in_polygon (const point &p, const std::vector <point> &po) :
            checks if a is in a polygon with vetices po (clockwise or counter-clockwise order).
        double polygon_area (const std::vector <point> &a) :
            returns the signed area of polygon a (positive for counter-clockwise order, and vise-versa).
        point project_to_line (const point &a, const line &b) :
            returns the projection of a on b,
*/
bool point_on_line (const point &a, const line &b) {
    return sgn (det (a - b.s, b.t - b.s)) == 0 && sgn (dot (b.s - a, b.t - a)) <= 0;
}
bool two_side (const point &a, const point &b, const line &c) {
    return sgn (det (a - c.s, c.t - c.s)) * sgn (det (b - c.s, c.t - c.s)) < 0;
}
bool intersect_judgement (const line &a, const line &b) {
    if (point_on_line (b.s, a) || point_on_line (b.t, a)) return true;
    if (point_on_line (a.s, b) || point_on_line (a.t, b)) return true;
    return two_side (a.s, a.t, b) && two_side (b.s, b.t, a);
}
point line_intersect (const line &a, const line &b) {
    double s1 = det (a.t - a.s, b.s - a.s);
    double s2 = det (a.t - a.s, b.t - a.s);
    return (b.s * s2 - b.t * s1) / (s2 - s1);
}
double point_to_line (const point &a, const line &b) {
    return fabs (det (b.t - b.s, a - b.s)) / dis (b.s, b.t);
}
double point_to_segment (const point &a, const line &b) {
    if (sgn (dot (b.s - a, b.t - b.s) * dot (b.t - a, b.t - b.s)) <= 0)
        return fabs (det (b.t - b.s, a - b.s)) / dis (b.s, b.t);
    return std::min (dis (a, b.s), dis (a, b.t));
}
bool in_polygon (const point &p, const std::vector <point> & po) {
    int n = (int) po.size ();
    int counter = 0;
    for (int i = 0; i < n; ++i) {
        point a = po[i], b = po[ (i + 1) % n];
        /*          The following statement checks is p is on the border of the polygon.
                    The boolean returned may be changed if necessary.
                        i.e. the algorithm may check if p is strictly in the polygon.
        */
        if (point_on_line (p, line (a, b))) return true;
        int x = sgn (det (p - a, b - a)), y = sgn (a.y - p.y), z = sgn (b.y - p.y);
        if (x > 0 && y <= 0 && z > 0) counter++;
        if (x < 0 && z <= 0 && y > 0) counter--;
    }
    return counter != 0;
}
double polygon_area (const std::vector <point> &a) {
    double ans = 0.0;
    for (int i = 0; i < (int) a.size (); ++i)
        ans += det (a[i], a[ (i + 1) % a.size ()]) / 2.0;
    return ans;
}
point project_to_line (const point &a, const line &b) {
    return b.s + (b.t - b.s) * (dot (a - b.s, b.t - b.s) / (b.t - b.s).norm2 ());
}
```

2.5 Centers of a triangle

```
/*  Centers of a triangle :
        returns various centers of a triangle with vertices (a, b, c).
*/
point incenter (const point &a, const point &b, const point &c) {
    double p = dis (a, b) + dis (b, c) + dis (c, a);
    return (a * dis (b, c) + b * dis (c, a) + c * dis (a, b)) / p;
}
```

```
point circumcenter (const point &a, const point &b, const point &c) {
    point p = b - a, q = c - a, s (dot (p, p) / 2, dot (q, q) / 2);
    double d = det (p, q);
    return a + point (det (s, point (p.y, q.y)), det (point (p.x, q.x), s)) / d;
}
point orthocenter (const point &a, const point &b, const point &c) {
    return a + b + c - circumcenter (a, b, c) * 2.0;
}
```

2.6 Fermat point

```
/*  Fermat point :
        point fermat_point (const point &a, const point &b, const point &c) :
            returns a point p that minimizes |pa| + |pb| + |pc|.
*/
point fermat_point (const point &a, const point &b, const point &c) {
    if (a == b) return a;
    if (b == c) return b;
    if (c == a) return c;
    double ab = dis (a, b), bc = dis (b, c), ca = dis (c, a);
    double cosa = dot (b - a, c - a) / ab / ca;
    double cosb = dot (a - b, c - b) / ab / bc;
    double cosc = dot (b - c, a - c) / ca / bc;
    double sq3 = PI / 3.0;
    point mid;
    if (sgn (cosa + 0.5) < 0) mid = a;
    else if (sgn (cosb + 0.5) < 0) mid = b;
    else if (sgn (cosc + 0.5) < 0) mid = c;
    else if (sgn (det (b - a, c - a)) < 0)
        mid = line_intersect (line (a, b + (c - b).rotate (sq3)), line (b, c + (a - c).rotate (sq3)));
    else
        mid = line_intersect (line (a, c + (b - c).rotate (sq3)), line (c, b + (a - b).rotate (sq3)));
    return mid;
}
```

2.7 Circle class

```
/*  struct circle defines a circle.
    circle (point c, double r) gives a circle with center c and radius r.
*/
struct circle {
    point c;
    double r;
    circle (point c = point (), double r = 0) : c (c), r (r) {}
};
bool operator == (const circle &a, const circle &b) {
    return a.c == b.c && cmp (a.r, b.r) == 0;
}
bool operator != (const circle &a, const circle &b) {
    return ! (a == b);
}
```

2.8 Interactions of circles

```
/*  Circle interaction :
        bool in_circle (const point &a, const circle &b) : checks if a is in or on b.
        circle make_circle (const point &a, const point &b) :
            generates a circle with diameter ab.
        circle make_circle (const point &a, const point &b, const point &c) :
            generates a circle passing a, b and c.
        std::pair <point, point> line_circle_intersect (const line &a, const circle &b) :
            returns the intersections of a and b.
            Fails if a and b do not intersect.
        std::pair <point, point> circle_intersect (const circle &a, const circle &b):
            returns the intersections of a and b.
            Fails if a and b do not intersect.
        std::pair <line, line> tangent (const point &a, const circle &b) :
            returns the tangent lines of b passing through a.
            Fails if a is in b.
*/
bool in_circle (const point &a, const circle &b) {
    return cmp (dis (a, b.c), b.r) <= 0;
}
circle make_circle (const point &a, const point &b) {
    return circle ((a + b) / 2, dis (a, b) / 2);
}
circle make_circle (const point &a, const point &b, const point &c) {
    point p = circumcenter (a, b, c);
    return circle (p, dis (p, a));
}
std::pair <point, point> line_circle_intersect (const line &a, const circle &b) {
    double x = sqrt (sqr (b.r) - sqr (point_to_line (b.c, a)));
    return std::make_pair (project_to_line (b.c, a) + (a.s - a.t).unit () * x,
                           project_to_line (b.c, a) - (a.s - a.t).unit () * x);
}
point __circle_intersect (const circle &a, const circle &b) {
    point r = (b.c - a.c).unit ();
    double d = dis (a.c, b.c);
    double x = .5 * ((sqr (a.r) - sqr (b.r)) / d + d);
    double h = sqrt (sqr (a.r) - sqr (x));
    return a.c + r * x + r.rot90 () * h;
}
std::pair <point, point> circle_intersect (const circle &a, const circle &b) {
    return std::make_pair (__circle_intersect (a, b), __circle_intersect (b, a));
}
```

```
std::pair <line, line> tangent (const point &a, const circle &b) {
    circle p = make_circle (a, b.c);
    return circle_intersect (p, b);
}
```

2.9 Convex hull

```
/*  Convex hull :
        std::vector <point> convex_hull (std::vector <point> a) :
            returns the convex hull of point set a (counter-clockwise).
*/
bool turn_left (const point &a, const point &b, const point &c) {
    return sgn (det (b - a, c - a)) >= 0;
}
bool turn_right (const point &a, const point &b, const point &c) {
    return sgn (det (b - a, c - a)) <= 0;
}
std::vector <point> convex_hull (std::vector <point> a) {
    int n = (int) a.size (), cnt = 0;
    std::sort (a.begin (), a.end ());
    std::vector <point> ret;
    for (int i = 0; i < n; ++i) {
        while (cnt > 1 && turn_left (ret[cnt - 2], a[i], ret[cnt - 1])) {
            --cnt;
            ret.pop_back ();
        }
        ret.push_back (a[i]);
        ++cnt;
    }
    int fixed = cnt;
    for (int i = n - 1; i >= 0; --i) {
        while (cnt > fixed && turn_left (ret[cnt - 2], a[i], ret[cnt - 1])) {
            --cnt;
            ret.pop_back ();
        }
        ret.push_back (a[i]);
        ++cnt;
    }
    ret.pop_back ();
    return ret;
}
```

2.10 Minimum circle

```
/*  Minimum circle of a point set :
    circle minimum_circle (std::vector <point> p) : returns the minimum circle of point set p.
*/
circle minimum_circle (std::vector <point> p) {
    circle ret;
    std::random_shuffle (p.begin (), p.end ());
    for (int i = 0; i < (int) p.size (); ++i)
        if (!in_circle (p[i], ret)) {
            ret = circle (p[i], 0);
            for (int j = 0; j < i; ++j)
                if (!in_circle (p[j], ret)) {
                    ret = make_circle (p[j], p[i]);
                    for (int k = 0; k < j; ++k)
                        if (!in_circle (p[k], ret)) ret = make_circle (p[i], p[j], p[k]);
                }
        }
    return ret;
}
```

2.11 Half plane intersection

```
/*  Online half plane intersection (complexity = O(c.size ())) :
        std::vector <point> cut (const std::vector<point> &c, line p) :
            returns the convex polygon cutting convex polygon c with half plane p.
                (left hand with respect to vector p)
            If such polygon does not exist, returns an empty set.
            e.g.
                static const double BOUND = 1e5;
                convex.clear ();
                convex.push_back (point (-BOUND, -BOUND));
                convex.push_back (point (BOUND, -BOUND));
                convex.push_back (point (BOUND, BOUND));
                convex.push_back (point (-BOUND, BOUND));
                convex = cut (convex, line(point, point));
                if (convex.empty ()) { ... }
*/
std::vector <point> cut (const std::vector<point> &c, line p) {
    std::vector <point> ret;
    if (c.empty ()) return ret;
    for (int i = 0; i < (int) c.size (); ++i) {
        int j = (i + 1) % (int) c.size ();
        if (!turn_right (p.s, p.t, c[i])) ret.push_back (c[i]);
        if (two_side (c[i], c[j], p))
            ret.push_back (line_intersect (p, line (c[i], c[j])));
    }
    return ret;
}
/*  Offline half plane intersection (complexity = O(nlogn), n = h.size ()) :
        std::vector <point> half_plane_intersect (std::vector <line> h) :
            returns the intersection of half planes h.
                (left hand with respect to the vector)
            If such polygon does not exist, returns an empty set.
```

```
*/
bool turn_left (const line &l, const point &p) {
    return turn_left (l.s, l.t, p);
}
std::vector <point> half_plane_intersect (std::vector <line> h) {
    typedef std::pair <double, line> polar;
    std::vector <polar> g;
    g.resize (h.size ());
    for (int i = 0; i < (int) h.size (); ++i) {
        point v = h[i].t - h[i].s;
        g[i] = std::make_pair (atan2 (v.y, v.x), h[i]);
    }
    sort (g.begin (), g.end (), [] (const polar &a, const polar &b) {
        if (cmp (a.first, b.first) == 0)
            return sgn (det (a.second.t - a.second.s, b.second.t - a.second.s)) < 0;
        else
            return cmp (a.first, b.first) < 0;
    });
    h.resize (std::unique (g.begin (), g.end (), [] (const polar &a, const polar &b) {
        return cmp (a.first, b.first) == 0;
    }) - g.begin ());
    for (int i = 0; i < (int) h.size (); ++i)
        h[i] = g[i].second;
    int fore = 0, rear = -1;
    std::vector <line> ret;
    for (int i = 0; i < (int) h.size (); ++i) {
        while (fore < rear && !turn_left (h[i], line_intersect (ret[rear - 1], ret[rear]))) {
            --rear;
            ret.pop_back ();
        }
        while (fore < rear && !turn_left (h[i], line_intersect (ret[fore], ret[fore + 1])))
            ++fore;
        ++rear;
        ret.push_back (h[i]);
    }
    while (rear - fore > 1 && !turn_left (ret[fore], line_intersect (ret[rear - 1], ret[rear]))) {
        --rear;
        ret.pop_back ();
    }
    while (rear - fore > 1 && !turn_left (ret[rear], line_intersect (ret[fore], ret[fore + 1])))
        ++fore;
    if (rear - fore < 2) return std::vector <point> ();
    std::vector <point> ans;
    ans.resize (ret.size ());
    for (int i = 0; i < (int) ret.size (); ++i)
        ans[i] = line_intersect (ret[i], ret[ (i + 1) % ret.size ()]);
    return ans;
}
```

2.12 Intersection of a polygon and a circle

```
/*  Intersection of a polygon and a circle :
        double polygon_circle_intersect::solve (const std::vector <point> &p, const circle &c) :
            returns the area of intersection of polygon p (vertices in either order) and c.
*/
struct polygon_circle_intersect {
    //  The area of the sector with center (0, 0), radius r and segment ab.
    double sector_area (const point &a, const point &b, const double &r) {
        double c = (2.0 * r * r - (a - b).norm2 ()) / (2.0 * r * r);
        double al = acos (c);
        return r * r * al / 2.0;
    }
    //  The area of triangle (a, b, (0, 0)) intersecting circle (point (), r).
    double area (const point &a, const point &b, const double &r) {
        double dA = dot (a, a), dB = dot (b, b), dC = point_to_segment (point (), line (a, b)), ans = 0.0;
        if (sgn (dA - r * r) <= 0 && sgn (dB - r * r) <= 0) return det (a, b) / 2.0;
        point tA = a.unit () * r;
        point tB = b.unit () * r;
        if (sgn (dC - r) > 0) return sector_area (tA, tB, r);
        std::pair <point, point> ret = line_circle_intersect (line (a, b), circle (point (), r));
        if (sgn (dA - r * r) > 0 && sgn (dB - r * r) > 0) {
            ans += sector_area (tA, ret.first, r);
            ans += det (ret.first, ret.second) / 2.0;
            ans += sector_area (ret.second, tB, r);
            return ans;
        }
        if (sgn (dA - r * r) > 0)
            return det (ret.first, b) / 2.0 + sector_area (tA, ret.first, r);
        else
            return det (a, ret.second) / 2.0 + sector_area (ret.second, tB, r);
    }
    //  Main procedure.
    double solve (const std::vector <point> &p, const circle &c) {
        double ret = 0.0;
        for (int i = 0; i < (int) p.size (); ++i) {
            int s = sgn (det (p[i] - c.c, p[ (i + 1) % p.size ()] - c.c));
            if (s > 0)
                ret += area (p[i] - c.c, p[ (i + 1) % p.size ()] - c.c, c.r);
            else
                ret -= area (p[ (i + 1) % p.size ()] - c.c, p[i] - c.c, c.r);
        }
        return fabs (ret);
    }
};
```

2.13 Union of circles

```
/*  Union of circles :
```

```
        std::vector <double> union_circle::solve (const std::vector <circle> &c) :
            returns the union of circle set c.
            The i-th element is the area covered with at least i circles.
*/
struct union_circle {
    struct cp {
        double x, y, angle;
        int d;
        double r;
        cp (const double &x = 0, const double &y = 0, const double &angle = 0,
            int d = 0, const double &r = 0) : x (x), y (y), angle (angle), d (d), r (r) {}
    };
    double dis (const cp &a, const cp &b) {
        return sqrt (sqr (a.x - b.x) + sqr (a.y - b.y));
    }
    double cross (const cp &p0, const cp &p1, const cp &p2) {
        return (p1.x - p0.x) * (p2.y - p0.y) - (p1.y - p0.y) * (p2.x - p0.x);
    }
    int cir_cross (cp p1, double r1, cp p2, double r2, cp &cp1, cp &cp2) {
        double mx = p2.x - p1.x, sx = p2.x + p1.x, mx2 = mx * mx;
        double my = p2.y - p1.y, sy = p2.y + p1.y, my2 = my * my;
        double sq = mx2 + my2, d = - (sq - sqr (r1 - r2)) * (sq - sqr (r1 + r2));
        if (sgn (d) < 0) return 0;
        if (sgn (d) <= 0) d = 0;
        else d = sqrt (d);
        double x = mx * ((r1 + r2) * (r1 - r2) + mx * sx) + sx * my2;
        double y = my * ((r1 + r2) * (r1 - r2) + my * sy) + sy * mx2;
        double dx = mx * d, dy = my * d;
        sq *= 2;
        cp1.x = (x - dy) / sq;
        cp1.y = (y + dx) / sq;
        cp2.x = (x + dy) / sq;
        cp2.y = (y - dx) / sq;
        if (sgn (d) > 0) return 2;
        else return 1;
    }
    static bool circmp (const cp &u, const cp &v) {
        return sgn (u.r - v.r) < 0;
    }
    static bool cmp (const cp &u, const cp &v) {
        if (sgn (u.angle - v.angle)) return u.angle < v.angle;
        return u.d > v.d;
    }
    double calc (cp cir, cp cp1, cp cp2) {
        double ans = (cp2.angle - cp1.angle) * sqr (cir.r)
                   - cross (cir, cp1, cp2) + cross (cp (0, 0), cp1, cp2);
        return ans / 2;
    }
    std::vector <double> solve (const std::vector <circle> &c) {
        int n = c.size ();
        std::vector <cp> cir, tp;
        std::vector <double> area;
        cir.resize (n);
        tp.resize (2 * n);
        area.resize (n + 1);
        for (int i = 0; i < n; i++)
            cir[i] = cp (c[i].c.x, c[i].c.y, 0, 1, c[i].r);
        cp cp1, cp2;
        std::sort (cir.begin (), cir.end (), circmp);
        for (int i = 0; i < n; ++i)
            for (int j = i + 1; j < n; ++j)
                if (sgn (dis (cir[i], cir[j]) + cir[i].r - cir[j].r) <= 0)
                    cir[i].d++;
        for (int i = 0; i < n; ++i) {
            int tn = 0, cnt = 0;
            for (int j = 0; j < n; ++j) {
                if (i == j) continue;
                if (cir_cross (cir[i], cir[i].r, cir[j], cir[j].r, cp2, cp1) < 2) continue;
                cp1.angle = atan2 (cp1.y - cir[i].y, cp1.x - cir[i].x);
                cp2.angle = atan2 (cp2.y - cir[i].y, cp2.x - cir[i].x);
                cp1.d = 1;
                tp[tn++] = cp1;
                cp2.d = -1;
                tp[tn++] = cp2;
                if (sgn (cp1.angle - cp2.angle) > 0) cnt++;
            }
            tp[tn++] = cp (cir[i].x - cir[i].r, cir[i].y, PI, -cnt);
            tp[tn++] = cp (cir[i].x - cir[i].r, cir[i].y, -PI, cnt);
            std::sort (tp.begin (), tp.begin () + tn, cmp);
            int p, s = cir[i].d + tp[0].d;
            for (int j = 1; j < tn; ++j) {
                p = s;
                s += tp[j].d;
                area[p] += calc (cir[i], tp[j - 1], tp[j]);
            }
        }
        return area;
    }
};
```

Chapter 3

Trial of graph

3.1 Constants and edge lists

```
const int INF = 1E9;
/*  Edge list:
        Various kinds of edge list.
*/
template <int MAXN = 1E5, int MAXM = 1E5>
struct edge_list {
    int size;
    int begin[MAXN], dest[MAXM], next[MAXM];
    void clear (int n) {
        size = 0;
        std::fill (begin, begin + n, -1);
    }
    edge_list (int n = MAXN) {
        clear (n);
    }
    void add_edge (int u, int v) {
        dest[size] = v; next[size] = begin[u]; begin[u] = size++;
    }
};
template <int MAXN = 1E5, int MAXM = 1E5>
struct cost_edge_list {
    int size;
    int begin[MAXN], dest[MAXM], next[MAXM], cost[MAXM];
    void clear (int n) {
        size = 0;
        std::fill (begin, begin + n, -1);
    }
    cost_edge_list (int n = MAXN) {
        clear (n);
    }
    void add_edge (int u, int v, int c) {
        dest[size] = v; next[size] = begin[u]; cost[size] = c; begin[u] = size++;
    }
};
template <int MAXN = 1E5, int MAXM = 1E5>
struct flow_edge_list {
    int size;
    int begin[MAXN], dest[MAXM], next[MAXM], flow[MAXM], inv[MAXM];
    void clear (int n) {
        size = 0;
        std::fill (begin, begin + n, -1);
    }
    flow_edge_list (int n = MAXN) {
        clear (n);
    }
    void add_edge (int u, int v, int f) {
        dest[size] = v; next[size] = begin[u]; flow[size] = f; inv[size] = size + 1; begin[u] = size++;
        dest[size] = u; next[size] = begin[v]; flow[size] = 0; inv[size] = size - 1; begin[v] = size++;
    }
};
template <int MAXN = 1E5, int MAXM = 1E5>
struct cost_flow_edge_list {
    int size;
    int begin[MAXN], dest[MAXM], next[MAXM], cost[MAXM], flow[MAXM], inv[MAXM];
    void clear (int n) {
        size = 0;
        std::fill (begin, begin + n, -1);
    }
    cost_flow_edge_list (int n = MAXN) {
        clear (n);
    }
    void add_edge (int u, int v, int c, int f) {
        dest[size] = v; next[size] = begin[u]; cost[size] = c;
        flow[size] = f; inv[size] = size + 1; begin[u] = size++;
        dest[size] = u; next[size] = begin[v]; cost[size] = c;
        flow[size] = 0; inv[size] = size - 1; begin[v] = size++;
    }
};
```

3.2 SPFA improved

```
/*  SPFA :
```

```
        Shortest path fast algorithm. (with SLF and LLL)
        bool spfa::solve (const cost_edge_list &e, int n, int s) :
            dist[] gives the distance from s.
            last[] gives the previous vertex.
*/
template <int MAXN = 1E5, int MAXM = 1E5>
struct spfa {
    int dist[MAXN], last[MAXN];
    int queue[MAXN], cnt[MAXN];
    bool inq[MAXN];
    bool solve (const cost_edge_list <MAXN, MAXM> &e, int n, int s) {
        std::fill (dist, dist + MAXN, INF);
        std::fill (last, last + MAXN, -1);
        std::fill (cnt, cnt + MAXN, 0);
        std::fill (inq, inq + MAXN, false);
        int p = 0, q = 1, size = 1;
        long long avg = 0;
        dist[s] = 0; queue[0] = s; inq[s] = true;
        while (p != q) {
            int n = queue[p]; p = (p + 1) % MAXN;
            if (1LL * dist[n] * size > avg) {
                queue[q] = n;
                q = (q + 1) % MAXN;
                continue;
            }
            inq[n] = false; avg -= dist[n]; --size;
            for (int i = e.begin[n]; ~i; i = e.next[i]) {
                int v = e.dest[i];
                if (dist[v] > dist[n] + e.cost[i]) {
                    dist[v] = dist[n] + e.cost[i]; last[v] = n;
                    if (!inq[v]) {
                        if (++cnt[v] > n) return false;
                        inq[v] = true; avg += dist[v]; --size;
                        if (dist[v] < dist[queue[p]])
                            queue[p = (p + MAXN - 1) % MAXN] = v;
                        else {
                            queue[q] = v;
                            q = (q + 1) % MAXN;
                        }
                    }
                }
            }
        }
        return true;
    }
};
```

3.3 Dijkstra's shortest path algorithm

```
/*  Dijkstra :
        Shortest path algorithm.
*/
template <int MAXN = 1E5, int MAXM = 1E5>
struct dijkstra {
    int dist[MAXN], last[MAXN];
    bool vis[MAXN];
    void solve (const cost_edge_list <MAXN, MAXM> &e, int s) {
        std::priority_queue <std::pair <int, int>, std::vector <std::pair <int, int> >,
            std::greater <std::pair <int, int> > > queue;
        std::fill (dist, dist + MAXN, INF);
        std::fill (last, last + MAXN, -1);
        std::fill (vis, vis + MAXN, false);
        dist[s] = 0;
        queue.push (std::make_pair (0, s));
        while (!queue.empty ()) {
            int n = queue.top ().second; queue.pop (); vis[n] = true;
            for (int i = e.begin[n]; ~i; i = e.next[i]) {
                int v = e.dest[i];
                if (dist[v] > dist[n] + e.cost[i]) {
                    dist[v] = dist[n] + e.cost[i]; last[v] = n;
                    queue.push (std::make_pair (dist[v], v));
                }
            }
            while (!queue.empty () && vis[queue.top ().second]) queue.pop ();
        }
    }
};
```

3.4 Tarjan

```
/*  Tarjan :
        returns strongly connected components.
        void tarjan::solve (const edge_list &) :
            component[] gives which component a vertex belongs to.
*/
template <int MAXN = 1E5, int MAXM = 1E5>
struct tarjan {
    int component[MAXN], component_size;
    int dfn[MAXN], low[MAXN], vis[MAXN], s[MAXN], s_s, ins[MAXN], ind;
    void dfs (const edge_list <MAXN, MAXM> &e, int u) {
        dfn[u] = low[u] = ind++;
        vis[u] = ins[u] = 1; s[s_s++] = u;
        for (int i = e.begin[u]; ~i; i = e.next[i]) {
            if (!vis[e.dest[i]]) {
                dfs (e, e.dest[i]);
                low[u] = std::min (low[u], low[e.dest[i]]);
```

```
            } else if (ins[e.dest[i]])
                low[u] = std::min (low[u], dfn[e.dest[i]]);
        }
        if (dfn[u] == low[u]) {
            do {
                component[s[--s_s]] = component_size;
                ins[s[s_s]] = 0;
            } while (s[s_s] != u);
            component_size++;
        }
    }
    void solve (const edge_list <MAXN, MAXM> &e) {
        std::fill (vis, vis + MAXN, 0);
        std::fill (ins, ins + MAXN, 0);
        s_s = ind = 0;
        dfs (e, 0);
    }
};
```

3.5 Hopcoft-Carp

```
/*  Hopcoft-Carp algorithm :
        maximum matching with complexity O (m * n^0.5).
        struct hopcoft_carp :
            Usage : solve () for maximum matching. The matching is in matchx and matchy.
*/
template <int MAXN = 1E5, int MAXM = 1E5>
struct hopcoft_carp {
    int n, m;
    int matchx[MAXN], matchy[MAXN], level[MAXN];
    bool dfs (edge_list <MAXN, MAXM> &e, int x) {
        for (int i = e.begin[x]; ~i; i = e.next[i]) {
            int y = e.dest[i];
            int w = matchy[y];
            if (w == -1 || (level[x] + 1 == level[w] && dfs (e, w))) {
                matchx[x] = y;
                matchy[y] = x;
                return true;
            }
        }
        level[x] = -1;
        return false;
    }
    int solve (edge_list <MAXN, MAXM> &e, int n, int m) {
        std::fill (matchx, matchx + n, -1);
        std::fill (matchy, matchy + m, -1);
        for (int answer = 0; ; ) {
            std::vector <int> queue;
            for (int i = 0; i < n; ++i) {
                if (matchx[i] == -1) {
                    level[i] = 0;
                    queue.push_back (i);
                } else {
                    level[i] = -1;
                }
            }
            for (int head = 0; head < (int) queue.size(); ++head) {
                int x = queue[head];
                for (int i = e.begin[x]; ~i; i = e.next[i]) {
                    int y = e.dest[i];
                    int w = matchy[y];
                    if (w != -1 && level[w] < 0) {
                        level[w] = level[x] + 1;
                        queue.push_back (w);
                    }
                }
            }
            int delta = 0;
            for (int i = 0; i < n; ++i)
                if (matchx[i] == -1 && dfs (e, i)) delta++;
            if (delta == 0) return answer;
            else answer += delta;
        }
    }
};
```

3.6 Kuhn-Munkres

```
/*  Kuhn Munkres algorithm :
        weighted maximum matching algorithm. Complexity O (N^3).
        struct kuhn_munkres :
            Initialize : pass nx, ny as the size of both sets, w as the weight matrix.
            Usage : solve () for the minimum matching. The exact matching is in link[].
*/
template <int MAXN = 500>
struct kuhn_munkres {
    int nx, ny;
    int w[MAXN][MAXN];
    int lx[MAXN], ly[MAXN], visx[MAXN], visy[MAXN], slack[MAXN], link[MAXN];
    int dfs (int x) {
        visx[x] = 1;
        for (int y = 0; y < ny; y ++) {
            if (visy[y]) continue;
            int t = lx[x] + ly[y] - w[x][y];
            if (t == 0) {
                visy[y] = 1;
                if (link[y] == -1 || dfs (link[y])) {
```

```
                link[y] = x;
                return 1;
            }
        } else slack[y] = std::max (slack[y], t);
    }
    return 0;
}
int solve () {
    int i, j;
    std::fill (link, link + ny, -1);
    std::fill (ly, ly + ny, 0);
    for (i = 0; i < nx; i++)
        for (j = 0, lx[i] = INF; j < ny; j++)
            lx[i] = std::min (lx[i], w[i][j]);
    for (int x = 0; x < nx; x++) {
        for (i = 0; i < ny; i++) slack[i] = -INF;
        while (true) {
            std::fill (visx, visx + nx, 0);
            std::fill (visy, visy + ny, 0);
            if (dfs (x)) break;
            int d = INF;
            for (i = 0; i < ny; i++)
                if (!visy[i] && d < slack[i]) d = slack[i];
            for (i = 0; i < nx; i++)
                if (visx[i]) lx[i] -= d;
            for (i = 0; i < ny; i++)
                if (visy[i]) ly[i] += d;
                else slack[i] -= d;
        }
    }
    int res = 0;
    for (i = 0; i < ny; i ++)
        if (link[i] > -1) res += w[link[i]][i];
    return res;
}
};
```

3.7 Stochastic weighted maximum matching

```
/*  Weighted matching algorithm :
        maximum match for graphs. Not stable.
        struct weighted_match :
            Usage : Set k to the size of vertices, w to the weight matrix.
            Note that k has to be even for the algorithm to work.
*/
template <int MAXN = 500>
struct weighted_match {
    int k;
    long long w[MAXN][MAXN];
    int match[MAXN], path[MAXN], p[MAXN], len;
    long long d[MAXN];
    bool v[MAXN];
    bool dfs (int i) {
        path[len++] = i;
        if (v[i]) return true;
        v[i] = true;
        for (int j = 0; j < k; ++j) {
            if (i != j && match[i] != j && !v[j]) {
                int kok = match[j];
                if (d[kok] < d[i] + w[i][j] - w[j][kok]) {
                    d[kok] = d[i] + w[i][j] - w[j][kok];
                    if (dfs (kok)) return true;
                }
            }
        }
        --len;
        v[i] = false;
        return false;
    }
    long long solve () {
        if (k & 1) ++k;
        for (int i = 0; i < k; ++i) p[i] = i, match[i] = i ^ 1;
        int cnt = 0;
        for (;;) {
            len = 0;
            bool flag = false;
            std::fill (d, d + k, 0);
            std::fill (v, v + k, 0);
            for (int i = 0; i < k; ++i) {
                if (dfs (p[i])) {
                    flag = true;
                    int t = match[path[len - 1]], j = len - 2;
                    while (path[j] != path[len - 1]) {
                        match[t] = path[j];
                        std::swap (t, match[path[j]]);
                        --j;
                    }
                    match[t] = path[j];
                    match[path[j]] = t;
                    break;
                }
            }
            if (!flag) {
                if (++cnt >= 2) break;
                std::random_shuffle (p, p + k);
            }
        }
        long long ans = 0;
        for (int i = 0; i < k; ++i)
            ans += w[i][match[i]];
        return ans / 2;
```

```
        }
    };
```

3.8 Weighted blossom (vfleaking ver.)

```
/* Weighted blossom algorithm (vfleaking ver.) :
        maximum match for graphs. Complexity O (n^3).
        Note that the base index is 1.
        struct weighted_blossom :
            Usage :
                Set n to the size of the vertices.
                Run init ().
                Set g[][].w to the weight of the edge.
                Run solve ().
                The first result is the answer, the second one is the number of matching pairs.
                Obtain the matching with match[].
*/
template <int MAXN = 500>
struct weighted_blossom {
    struct edge {
        int u, v, w;
        edge (int u = 0, int v = 0, int w = 0): u (u), v (v), w (w) {}
    };
    int n, n_x;
    edge g[MAXN * 2 + 1][MAXN * 2 + 1];
    int lab[MAXN * 2 + 1];
    int match[MAXN * 2 + 1], slack[MAXN * 2 + 1], st[MAXN * 2 + 1], pa[MAXN * 2 + 1];
    int flower_from[MAXN * 2 + 1][MAXN + 1], S[MAXN * 2 + 1], vis[MAXN * 2 + 1];
    std::vector<int> flower[MAXN * 2 + 1];
    std::queue<int> q;
    int e_delta (const edge &e) {
        return lab[e.u] + lab[e.v] - g[e.u][e.v].w * 2;
    }
    void update_slack (int u, int x) {
        if (!slack[x] || e_delta (g[u][x]) < e_delta (g[slack[x]][x]))slack[x] = u;
    }
    void set_slack (int x) {
        slack[x] = 0;
        for (int u = 1; u <= n; ++u)
            if (g[u][x].w > 0 && st[u] != x && S[st[u]] == 0)update_slack (u, x);
    }
    void q_push (int x) {
        if (x <= n)q.push (x);
        else for (size_t i = 0; i < flower[x].size(); i++)q_push (flower[x][i]);
    }
    void set_st (int x, int b) {
        st[x] = b;
        if (x > n)for (size_t i = 0; i < flower[x].size(); ++i)
                set_st (flower[x][i], b);
    }
    int get_pr (int b, int xr) {
        int pr = find (flower[b].begin(), flower[b].end(), xr) - flower[b].begin();
        if (pr % 2 == 1) {
            reverse (flower[b].begin() + 1, flower[b].end());
            return (int)flower[b].size() - pr;
        } else return pr;
    }
    void set_match (int u, int v) {
        match[u] = g[u][v].v;
        if (u > n) {
            edge e = g[u][v];
            int xr = flower_from[u][e.u], pr = get_pr (u, xr);
            for (int i = 0; i < pr; ++i)set_match (flower[u][i], flower[u][i ^ 1]);
            set_match (xr, v);
            rotate (flower[u].begin(), flower[u].begin() + pr, flower[u].end());
        }
    }
    void augment (int u, int v) {
        for (;;) {
            int xnv = st[match[u]];
            set_match (u, v);
            if (!xnv)return;
            set_match (xnv, st[pa[xnv]]);
            u = st[pa[xnv]], v = xnv;
        }
    }
    int get_lca (int u, int v) {
        static int t = 0;
        for (++t; u || v; std::swap (u, v)) {
            if (u == 0)continue;
            if (vis[u] == t)return u;
            vis[u] = t;
            u = st[match[u]];
            if (u)u = st[pa[u]];
        }
        return 0;
    }
    void add_blossom (int u, int lca, int v) {
        int b = n + 1;
        while (b <= n_x && st[b])++b;
        if (b > n_x)++n_x;
        lab[b] = 0, S[b] = 0;
        match[b] = match[lca];
        flower[b].clear();
        flower[b].push_back (lca);
        for (int x = u, y; x != lca; x = st[pa[y]])
            flower[b].push_back (x), flower[b].push_back (y = st[match[x]]), q_push (y);
        reverse (flower[b].begin() + 1, flower[b].end());
        for (int x = v, y; x != lca; x = st[pa[y]])
            flower[b].push_back (x), flower[b].push_back (y = st[match[x]]), q_push (y);
        set_st (b, b);
        for (int x = 1; x <= n_x; ++x)g[b][x].w = g[x][b].w = 0;
```

```
    for (int x = 1; x <= n; ++x)flower_from[b][x] = 0;
    for (size_t i = 0; i < flower[b].size(); ++i) {
        int xs = flower[b][i];
        for (int x = 1; x <= n_x; ++x)
            if (g[b][x].w == 0 || e_delta (g[xs][x]) < e_delta (g[b][x]))
                g[b][x] = g[xs][x], g[x][b] = g[x][xs];
        for (int x = 1; x <= n; ++x)
            if (flower_from[xs][x])flower_from[b][x] = xs;
    }
    set_slack (b);
}
void expand_blossom (int b) {
    for (size_t i = 0; i < flower[b].size(); ++i)
        set_st (flower[b][i], flower[b][i]);
    int xr = flower_from[b][g[b][pa[b]].u], pr = get_pr (b, xr);
    for (int i = 0; i < pr; i += 2) {
        int xs = flower[b][i], xns = flower[b][i + 1];
        pa[xs] = g[xns][xs].u;
        S[xs] = 1, S[xns] = 0;
        slack[xs] = 0, set_slack (xns);
        q_push (xns);
    }
    S[xr] = 1, pa[xr] = pa[b];
    for (size_t i = pr + 1; i < flower[b].size(); ++i) {
        int xs = flower[b][i];
        S[xs] = -1, set_slack (xs);
    }
    st[b] = 0;
}
bool on_found_edge (const edge &e) {
    int u = st[e.u], v = st[e.v];
    if (S[v] == -1) {
        pa[v] = e.u, S[v] = 1;
        int nu = st[match[v]];
        slack[v] = slack[nu] = 0;
        S[nu] = 0, q_push (nu);
    } else if (S[v] == 0) {
        int lca = get_lca (u, v);
        if (!lca)return augment (u, v), augment (v, u), true;
        else add_blossom (u, lca, v);
    }
    return false;
}
bool matching() {
    std::fill (S + 1, S + 1 + n_x, -1);
    std::fill (slack + 1, slack + 1 + n_x, -1);
    q = std::queue<int>();
    for (int x = 1; x <= n_x; ++x)
        if (st[x] == x && !match[x])pa[x] = 0, S[x] = 0, q_push (x);
    if (q.empty())return false;
    for (;;) {
        while (q.size()) {
            int u = q.front();
            q.pop();
            if (S[st[u]] == 1)continue;
            for (int v = 1; v <= n; ++v)
                if (g[u][v].w > 0 && st[u] != st[v]) {
                    if (e_delta (g[u][v]) == 0) {
                        if (on_found_edge (g[u][v]))return true;
                    } else update_slack (u, st[v]);
                }
        }
        int d = INF;
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b && S[b] == 1)d = std::min (d, lab[b] / 2);
        for (int x = 1; x <= n_x; ++x)
            if (st[x] == x && slack[x]) {
                if (S[x] == -1)d = std::min (d, e_delta (g[slack[x]][x]));
                else if (S[x] == 0)d = std::min (d, e_delta (g[slack[x]][x]) / 2);
            }
        for (int u = 1; u <= n; ++u) {
            if (S[st[u]] == 0) {
                if (lab[u] <= d)return 0;
                lab[u] -= d;
            } else if (S[st[u]] == 1)lab[u] += d;
        }
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b) {
                if (S[st[b]] == 0)lab[b] += d * 2;
                else if (S[st[b]] == 1)lab[b] -= d * 2;
            }
        q = std::queue<int>();
        for (int x = 1; x <= n_x; ++x)
            if (st[x] == x && slack[x] && st[slack[x]] != x && e_delta (g[slack[x]][x]) == 0)
                if (on_found_edge (g[slack[x]][x]))return true;
        for (int b = n + 1; b <= n_x; ++b)
            if (st[b] == b && S[b] == 1 && lab[b] == 0)expand_blossom (b);
    }
    return false;
}
std::pair <long long, int> solve () {
    std::fill (match + 1, match + n + 1, 0);
    n_x = n;
    int n_matches = 0;
    long long tot_weight = 0;
    for (int u = 0; u <= n; ++u)st[u] = u, flower[u].clear();
    int w_max = 0;
    for (int u = 1; u <= n; ++u)
        for (int v = 1; v <= n; ++v) {
            flower_from[u][v] = (u == v ? u : 0);
            w_max = std::max (w_max, g[u][v].w);
        }
    for (int u = 1; u <= n; ++u)lab[u] = w_max;
    while (matching())++n_matches;
    for (int u = 1; u <= n; ++u)
```

```
            if (match[u] && match[u] < u)
                tot_weight += g[u][match[u]].w;
        return std::make_pair (tot_weight, n_matches);
    }
    void init () {
        for (int u = 1; u <= n; ++u)
            for (int v = 1; v <= n; ++v)
                g[u][v] = edge (u, v, 0);
    }
};
```

3.9 Maximum flow

```
/*  Sparse graph maximum flow :
        int isap::solve (flow_edge_list &e, int n, int s, int t) :
            e : edge list.
            n : vertex size.
            s : source.
            t : sink.
*/
template <int MAXN = 1E3, int MAXM = 1E5>
struct isap {
    int pre[MAXN], d[MAXN], gap[MAXN], cur[MAXN];
    int solve (flow_edge_list <MAXN, MAXM> &e, int n, int s, int t) {
        std::fill (pre, pre + n + 1, 0);
        std::fill (d, d + n + 1, 0);
        std::fill (gap, gap + n + 1, 0);
        for (int i = 0; i < n; i++) cur[i] = e.begin[i];
        gap[0] = n;
        int u = pre[s] = s, v, maxflow = 0;
        while (d[s] < n) {
            v = n;
            for (int i = cur[u]; ~i; i = e.next[i])
                if (e.flow[i] && d[u] == d[e.dest[i]] + 1) {
                    v = e.dest[i];
                    cur[u] = i;
                    break;
                }
            if (v < n) {
                pre[v] = u;
                u = v;
                if (v == t) {
                    int dflow = INF, p = t;
                    u = s;
                    while (p != s) {
                        p = pre[p];
                        dflow = std::min (dflow, e.flow[cur[p]]);
                    }
                    maxflow += dflow;
                    p = t;
                    while (p != s) {
                        p = pre[p];
                        e.flow[cur[p]] -= dflow;
                        e.flow[e.inv[cur[p]]] += dflow;
                    }
                }
            } else {
                int mindist = n + 1;
                for (int i = e.begin[u]; ~i; i = e.next[i])
                    if (e.flow[i] && mindist > d[e.dest[i]]) {
                        mindist = d[e.dest[i]];
                        cur[u] = i;
                    }
                if (!--gap[d[u]]) return maxflow;
                gap[d[u] = mindist + 1]++;
                u = pre[u];
            }
        }
        return maxflow;
    }
};
/*  Dense graph maximum flow :
        int dinic::solve (flow_edge_list &e, int n, int s, int t) :
            e : edge list.
            n : vertex size.
            s : source.
            t : sink.
*/
template <int MAXN = 1E3, int MAXM = 1E5>
struct dinic {
    int n, s, t;
    int d[MAXN], w[MAXN], q[MAXN];
    int bfs (flow_edge_list <MAXN, MAXM> &e) {
        for (int i = 0; i < n; i ++) d[i] = -1;
        int l, r;
        q[l = r = 0] = s, d[s] = 0;
        for (; l <= r; l ++)
            for (int k = e.begin[q[l]]; k > -1; k = e.next[k])
                if (d[e.dest[k]] == -1 && e.flow[k] > 0) d[e.dest[k]] = d[q[l]] + 1, q[++r] = e.dest[k];
        return d[t] > -1 ? 1 : 0;
    }
    int dfs (flow_edge_list <MAXN, MAXM> &e, int u, int ext) {
        if (u == t) return ext;
        int k = w[u], ret = 0;
        for (; k > -1; k = e.next[k], w[u] = k) {
            if (ext == 0) break;
            if (d[e.dest[k]] == d[u] + 1 && e.flow[k] > 0) {
                int flow = dfs (e, e.dest[k], std::min (e.flow[k], ext));
                if (flow > 0) {
                    e.flow[k] -= flow, e.flow[e.inv[k]] += flow;
                    ret += flow, ext -= flow;
                }
```

```
            }
        }
        if (k == -1) d[u] = -1;
        return ret;
    }
    void solve (flow_edge_list <MAXN, MAXM> &e, int n, int s, int t) {
        dinic::n = n; dinic::s = s; dinic::t = t;
        while (bfs (e)) {
            for (int i = 0; i < n; i ++) w[i] = e.begin[i];
            dfs (e, s, INF);
        }
    }
};
```

3.10 Minimum cost flow

```
/*  Sparse graph minimum cost flow :
        std::pair <int, int> minimum_cost_flow::solve (cost_flow_edge_list &e,
                                                        int n, int s, int t) :
            e : edge list.
            n : vertex size.
            s : source.
            t : sink.
            returns the flow and the cost respectively.
*/
template <int MAXN = 1E3, int MAXM = 1E5>
struct minimum_cost_flow {
    int n, source, target;
    int prev[MAXN];
    int dist[MAXN], occur[MAXN];
    bool augment (cost_flow_edge_list <MAXN, MAXM> &e) {
        std::vector <int> queue;
        std::fill (dist, dist + n, INF);
        std::fill (occur, occur + n, 0);
        dist[source] = 0;
        occur[source] = true;
        queue.push_back (source);
        for (int head = 0; head < (int)queue.size(); ++head) {
            int x = queue[head];
            for (int i = e.begin[x]; ~i; i = e.next[i]) {
                int y = e.dest[i];
                if (e.flow[i] && dist[y] > dist[x] + e.cost[i]) {
                    dist[y] = dist[x] + e.cost[i];
                    prev[y] = i;
                    if (!occur[y]) {
                        occur[y] = true;
                        queue.push_back (y);
                    }
                }
            }
            occur[x] = false;
        }
        return dist[target] < INF;
    }
    std::pair <int, int> solve (cost_flow_edge_list <MAXN, MAXM> &e, int n, int s, int t) {
        minimum_cost_flow::n = n;
        source = s; target = t;
        std::pair <int, int> answer = std::make_pair (0, 0);
        while (augment (e)) {
            int number = INF;
            for (int i = target; i != source; i = e.dest[e.inv[prev[i]]]) {
                number = std::min (number, e.flow[prev[i]]);
            }
            answer.first += number;
            for (int i = target; i != source; i = e.dest[e.inv[prev[i]]]) {
                e.flow[prev[i]] -= number;
                e.flow[e.inv[prev[i]]] += number;
                answer.second += number * e.cost[prev[i]];
            }
        }
        return answer;
    }
};
/*  Dense graph minimum cost flow :
        std::pair <int, int> zkw_flow::solve (cost_flow_edge_list &e,
                                              int n, int s, int t) :
            e : edge list.
            n : vertex size.
            s : source.
            t : sink.
            returns the flow and the cost respectively.
*/
template <int MAXN = 1E3, int MAXM = 1E5>
struct zkw_flow {
    int n, s, t, totFlow, totCost;
    int dis[MAXN], slack[MAXN], visit[MAXN];
    int modlable() {
        int delta = INF;
        for (int i = 0; i < n; i++) {
            if (!visit[i] && slack[i] < delta) delta = slack[i];
            slack[i] = INF;
        }
        if (delta == INF) return 1;
        for (int i = 0; i < n; i++) if (visit[i]) dis[i] += delta;
        return 0;
    }
    int dfs (cost_flow_edge_list <MAXN, MAXM> &e, int x, int flow) {
        if (x == t) {
            totFlow += flow;
            totCost += flow * (dis[s] - dis[t]);
            return flow;
```

```
        }
        visit[x] = 1;
        int left = flow;
        for (int i = e.begin[x]; ~i; i = e.next[i])
            if (e.flow[i] > 0 && !visit[e.dest[i]]) {
                int y = e.dest[i];
                if (dis[y] + e.cost[i] == dis[x]) {
                    int delta = dfs (e, y, std::min (left, e.flow[i]));
                    e.flow[i] -= delta;
                    e.flow[e.inv[i]] += delta;
                    left -= delta;
                    if (!left) { visit[x] = false; return flow; }
                } else
                    slack[y] = std::min (slack[y], dis[y] + e.cost[i] - dis[x]);
            }
        return flow - left;
    }
    std::pair <int, int> solve (cost_flow_edge_list <MAXN, MAXM> &e, int n, int s, int t) {
        zkw_flow::n = n; zkw_flow::s = s; zkw_flow::t = t;
        totFlow = 0; totCost = 0;
        std::fill (dis + 1, dis + t + 1, 0);
        do {
            do {
                std::fill (visit + 1, visit + t + 1, 0);
            } while (dfs (e, s, INF));
        } while (!modlable ());
        return std::make_pair (totFlow, totCost);
    }
};
```

Chapter 4

Trial of string

4.1 KMP

```
/*  KMP algorithm :
        void kmp::build (const std::string & str) :
            initializes and builds the failure array. Complexity O (n).
        int kmp::find (const std::string &str) :
            finds the first occurence of match in str. Complexity O (n).
        Note : match is cylic when L % (L - 1 - fail[L - 1]) == 0 &&
            L / (L - 1 - fail[L - 1]) > 1, where L = match.size ().
*/
template <int MAXN = 1E6>
struct kmp {
    std::string match;
    int fail[MAXN];
    void build (const std::string &str) {
        match = str; fail[0] = -1;
        for (int i = 1; i < (int) str.size (); ++i) {
            int j = fail[i - 1];
            while (~j && str[i] != str[j + 1]) j = fail[j];
            fail[i] = str[i] == str[j + 1] ? j + 1 : -1;
        }
    }
    int find (const std::string &str) {
        for (int i = 0, j = 0; i < (int) str.size (); ++i, ++j) {
            if (j == match.size ()) return i - match.size ();
            while (~j && str[i] != match[j]) j = fail[j];
        }
        return str.size ();
    }
};
```

4.2 Suffix automaton

```
/*  Suffix automaton :
        void suffix_automaton::init () :
            initializes the automaton with an empty string.
        void suffix_automaton::extend (int token) :
            extends the string with token. Complexity O (1).
        head : the first state.
        tail : the last state.
            Terminating states can be reached via visiting the ancestors of tail.
        state::len : the longest length of the string in the state.
        state::parent : the parent link.
        state::dest : the automaton link.
*/
template <int MAXN = 1E6, int MAXC = 26>
struct suffix_automaton {
    state *head, *tail;
    struct state {
        int len;
        state *parent, *dest[MAXC];
        state (int len = 0) : len (len), parent (NULL) {
            memset (dest, 0, sizeof (dest));
        }
        state *extend (state *, int token);
    } node_pool[MAXN * 2], *tot_node, *null = new state();
    state *state::extend (state *start, int token) {
        state *p = this;
        state *np = this -> dest[token] ? null : new (tot_node++) state (this -> len + 1);
        while (p && !p -> dest[token])
            p -> dest[token] = np, p = p -> parent;
        if (!p) np -> parent = start;
        else {
            state *q = p -> dest[token];
            if (p -> len + 1 == q -> len) {
                np -> parent = q;
            } else {
                state *nq = new (tot_node++) state (*q);
                nq -> len = p -> len + 1;
                np -> parent = q -> parent = nq;
                while (p && p -> dest[token] == q) {
                    p -> dest[token] = nq, p = p -> parent;
                }
```

```
            }
        }
        return np == null ? np -> parent : np;
    }
    void init () {
        tot_node = node_pool;
        head = tail = new (tot_node++) state();
    }
    suffix_automaton () {
        init ();
    }
    void extend (int token) {
        tail = tail -> extend (head, token);
    }
};
```

4.3 Palindromic tree

```
/*  Palindromic tree :
        void palindromic_tree::init () : initializes the tree.
        bool palindromic_tree::extend (int) : extends the string with token.
            returns whether the tree has generated a new node.
            Complexity O (log MAXC).
        odd, even : the root of two trees.
        last : the node representing the last char.
        node::len : the palindromic string length of the node.
*/
template <int MAXN = 1E6, int MAXC = 26>
struct palindromic_tree {
    struct node {
        node *child[MAXC], *fail;
        int len;
        node (int len) : fail (NULL), len (len) {
            memset (child, NULL, sizeof (child));
        }
    } node_pool[MAXN * 2], *tot_node;
    int size, text[MAXN];
    node *odd, *even, *last;
    node *match (node *now) {
        for (; text[size - now -> len - 1] != text[size]; now = now -> fail);
        return now;
    }
    bool extend (int token) {
        text[++size] = token;
        node *now = match (last);
        if (now -> child[token])
            return last = now -> child[token], false;
        last = now -> child[token] = new (tot_node++) node (now -> len + 2);
        if (now == odd) last -> fail = even;
        else {
            now = match (now -> fail);
            last -> fail = now -> child[token];
        }
        return true;
    }
    void init() {
        text[size = 0] = -1;
        tot_node = node_pool;
        last = even = new (tot_node++) node (0); odd = new (tot_node++) node (-1);
        even -> fail = odd;
    }
    palindromic_tree () {
        init ();
    }
};
```

Chapter 5

Reference

5.1 Vimrc

```
set ruler
set number
set tabstop=4
set softtabstop=4
set shiftwidth=4
set smartindent
set showmatch
set hlsearch
set incsearch
set autoread
set backspace=2
set mouse=a
syntax on
nmap <C-A> ggVG
vmap <C-C> "+y
nmap <C-P> "+p
autocmd FileType cpp set cindent
autocmd FileType cpp map <F3> :vsplit %<.in <CR>
autocmd FileType cpp map <F5> :!time ./%<.exe <CR>
autocmd FileType cpp map <F7> :!gdb ./%<.exe <CR>
autocmd FileType cpp map <F8> :!time ./%<.exe < %<.in <CR>
autocmd FileType cpp map <F9> :!g++ % -o %< -g -std=c++11 -Wall -Wextra -Wconversion && size %<.exe <CR>
autocmd FileType java map <F3> :vsplit %<.in <CR>
autocmd FileType java map <F5> :!time java %< <CR>
autocmd FileType java map <F8> :!time java %< < %<.in <CR>
autocmd FileType java map <F9> :!javac % <CR>
```

5.2 Java reference

```java
/*  Java reference :
        References on Java IO, structures, etc.
*/
import java.io.*;
import java.lang.*;
import java.math.*;
import java.util.*;
/*  Regular usage:
        Slower IO :
            Scanner in = new Scanner (System.in));
            Scanner in = new Scanner (new BufferedInputStream (System.in));
            Input :
                in.nextInt () / in.nextBigInteger () / in.nextBigDecimal () / in.nextDouble ()
                in.nextLine () / in.hasNext ()
            Output :
                System.out.print (...);
                System.out.println (...);
                System.out.printf (...);
        Faster IO :
            Shown below.
        BigInteger :
            BigInteger.valueOf (int) : convert to BigInteger.
            abs / negate () / max / min / add / subtract / multiply /
                divide / remainder (BigInteger) : BigInteger algebraic.
            gcd (BigInteger) / modInverse (BigInteger mod) /
                modPow (BigInteger ex, BigInteger mod) / pow (int ex) : Number Theory.
            not () / and / or / xor (BigInteger) / shiftLeft / shiftRight (int) : Bit operation.
            compareTo (BigInteger) : comparation.
            intValue () / longValue () / toString (int radix) : converts to other types.
            isProbablePrime (int certainty) / nextProbablePrime () : checks primitive.
        BigDecimal :
            consists of a BigInteger value and a scale.
            The scale is the number of digits to the right of the decimal point.
            divide (BigDecimal) : exact divide.
            divide (BigDecimal, int scale, RoundingMode roundingMode) :
                divide with roundingMode, which may be:
                    CEILING / DOWN / FLOOR / HALF_DOWN / HALF_EVEN / HALF_UP / UNNECESSARY / UP.
            BigDecimal setScale (int newScale, RoundingMode roundingMode) :
                returns a BigDecimal with newScale.
            doubleValue () / toPlainString () : converts to other types.
        Arrays :
            Arrays.sort (T [] a);
            Arrays.sort (T [] a, int fromIndex, int toIndex);
            Arrays.sort (T [] a, int fromIndex, int toIndex, Comperator <? super T> comperator);
```

```
        LinkedList <E> :
            addFirst / addLast (E) / getFirst / getLast / removeFirst / removeLast () :
                deque implementation.
            clear () / add (int, E) / remove (int) : clear, add & remove.
            size () / contains / removeFirstOccurrence / removeLastOccurrence (E) :
                deque methods.
            ListIterator <E> listIterator (int index) : returns an iterator :
                E next / previous () : accesses and iterates.
                hasNext / hasPrevious () : checks availablity.
                nextIndex / previousIndex () : returns the index of a subsequent call.
                add / set (E) / remove () : changes element.
        PriorityQueue <E> (int initcap, Comparator <? super E> comparator) :
            add (E) / clear () / iterator () / peek () / poll () / size () :
                priority queue implementations.
        TreeMap <K, V> (Comparator <? super K> comparator) :
            Map.Entry <K, V> ceilingEntry / floorEntry / higherEntry / lowerEntry (K):
                getKey / getValue () / setValue (V) : entries.
            clear () / put (K, V) / get (K) / remove (K) : basic operation.
            size () : size.
        StringBuilder :
            Mutable string.
            StringBuilder (string) : generates a builder.
            append (int, string, ...) / insert (int offset, ...) : adds objects.
            charAt (int) / setCharAt (int, char) : accesses a char.
            delete (int, int) : removes a substring.
            reverse () : reverses itself.
            length () : returns the length.
            toString () : converts to string.
        String :
            Immutable string.
            String.format (String, ...) : formats a string. i.e. sprintf.
            toLowerCase / toUpperCase () : changes the case of letters.
*/
/* Examples on Comparator :
public class Main {
    public static class Point {
        public int x;
        public int y;
        public Point () {
            x = 0;
            y = 0;
        }
        public Point (int xx, int yy) {
            x = xx;
            y = yy;
        }
    };
    public static class Cmp implements Comparator <Point> {
        public int compare (Point a, Point b) {
            if (a.x < b.x) return -1;
            if (a.x == b.x) {
                if (a.y < b.y) return -1;
                if (a.y == b.y) return 0;
            }
            return 1;
        }
    };
    public static void main (String [] args) {
        Cmp c = new Cmp ();
        TreeMap <Point, Point> t = new TreeMap <Point, Point> (c);
        return;
    }
};
*/
/*  Another way to implement is to use Comparable.
    However, equalTo and hashCode must be rewritten.
    Otherwise, containers may fail.
    Example :
    public static class Point implements Comparable <Point> {
        public int x;
        public int y;
        public Point () {
            x = 0;
            y = 0;
        }
        public Point (int xx, int yy) {
            x = xx;
            y = yy;
        }
        public int compareTo (Point p) {
            if (x < p.x) return -1;
            if (x == p.x) {
                if (y < p.y) return -1;
                if (y == p.y) return 0;
            }
            return 1;
        }
        public boolean equalTo (Point p) {
            return (x == p.x && y == p.y);
        }
        public int hashCode () {
            return x + y;
        }
    };
*/
//Faster IO :
public class Main {
    static class InputReader {
        public BufferedReader reader;
        public StringTokenizer tokenizer;
        public InputReader (InputStream stream) {
            reader = new BufferedReader (new InputStreamReader (stream), 32768);
            tokenizer = null;
```

```
        }
        public String next() {
            while (tokenizer == null || !tokenizer.hasMoreTokens()) {
                try {
                    String line = reader.readLine();
                    tokenizer = new StringTokenizer (line);
                } catch (IOException e) {
                    throw new RuntimeException (e);
                }
            }
            return tokenizer.nextToken();
        }
        public BigInteger nextBigInteger() {
            return new BigInteger (next (), 10);    //  customize the radix here.
        }
        public int nextInt() {
            return Integer.parseInt (next());
        }
        public double nextDouble() {
            return Double.parseDouble (next());
        }
    }
    public static void main (String[] args) {
        InputReader in = new InputReader (System.in);
        //  Put your code here.
    }
}
```

5.3 Operator precedence

Precedence	Operator	Description	Associativity
1	`::`	Scope resolution	Left-to-right

2	`a++ a--`
	`type() type{}`
	`a()`
	`a[]`
	`. ->`
		Suffix/postfix increment and decrement
		Functional cast
		Function call
		Subscript
		Member access	

3	`++a --a`
	`+a -a`
	`! ~`
	`(type)`
	`*a`
	`&a`
	`sizeof`
	`new new[]`
	`delete delete[]`
		Prefix increment and decrement
		Unary plus and minus
		Logical NOT and bitwise NOT
		C-style cast
		Indirection (dereference)
		Address-of
		Size-of
		Dynamic memory allocation
		Dynamic memory deallocation
			Right-to-left

4	`.* ->*`	Pointer-to-member	Left-to-right
5	`a*b a/b a%b`	Multiplication, division, and remainder	
6	`a+b a-b`	Addition and subtraction	
7	`<< >>`	Bitwise left shift and right shift	
8	`< <=` `> >=`	For relational operators $<$ and $\leq$ respectively For relational operators $>$ and $\geq$ respectively	
9	`== !=`	For relational operators $=$ and $\neq$ respectively	
10	`a&b`	Bitwise AND	
11	`^`	Bitwise XOR (exclusive or)	
12	`|`	Bitwise OR (inclusive or)	
13	`&&`	Logical AND	
14	`||`	Logical OR	

15	`a?b:c`
	`throw`
	`=`
	`+= -= *= /= %=`
	`<<= >>=`
	`&= ^= |=`
		Ternary conditional
		throw operator
		Direct assignment
		Compound assignment by arithmetic operation
		Compound assignment by bitwise shift
		Compound assignment by bitwise AND, XOR, and OR
			Right-to-left

16	`,`	Comma	Left-to-right

5.4 Hacks

5.4.1 Formating long long in scanf & printf

```
#ifdef WIN32
    #define LL "%I64d"
#else
    #define LL "%lld"
#endif
```

5.4.2 Optimizing

```
#pragma GCC optimize ("O3")
#pragma GCC optimize ("whole-program")
```

5.4.3 Larger stack

5.4.3.1 C++

```
#pragma comment(linker, "/STACK:36777216")
```

5.4.3.2 G++

```
int __size__ = 256 << 20;   // 256MB
char *__p__ = (char*)malloc(__size__) + __size__;
__asm__("movl %0, %%esp\n" :: "r"(__p__));
```

5.5 Math reference

5.5.1 Integration table

5.5.1.1 $ax^2 + bx + c(a > 0)$

$$\int \frac{\mathrm{d}x}{ax^2+bx+c} = \begin{cases} \frac{2}{\sqrt{4ac-b^2}} \arctan \frac{2ax+b}{\sqrt{4ac-b^2}} + C & b^2 < 4ac \\ \frac{1}{\sqrt{b^2-4ac}} \ln \left| \frac{2ax+b-\sqrt{b^2-4ac}}{2ax+b+\sqrt{b^2-4ac}} \right| + C & b^2 > 4ac \end{cases}$$

$$\int \frac{x}{ax^2+bx+c} \mathrm{d}x = \frac{1}{2a} \ln \left| ax^2+bx+c \right| - \frac{b}{2a} \int \frac{\mathrm{d}x}{ax^2+bx+c}$$

5.5.1.2 $\sqrt{\pm ax^2 + bx + c}(a > 0)$

$$\int \frac{\mathrm{d}x}{ax^2+bx+c} = \frac{1}{\sqrt{a}} \ln \left| 2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c} \right| + C$$

$$\int \sqrt{ax^2+bx+c}\, \mathrm{d}x = \frac{2ax+b}{4a} \sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8\sqrt{a^3}} \ln \left| 2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c} \right| + C$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}} \mathrm{d}x = \frac{1}{a} \sqrt{ax^2+bx+c} - \frac{b}{2\sqrt{a^3}} \ln \left| 2ax+b+2\sqrt{a}\sqrt{ax^2+bx+c} \right| + C$$

$$\int \frac{\mathrm{d}x}{\sqrt{-ax^2+bx+c}} = -\frac{1}{\sqrt{a}} \arcsin \frac{2ax-b}{\sqrt{b^2+4ac}} + C$$

$$\int \sqrt{-ax^2+bx+c}\, \mathrm{d}x = \frac{2ax-b}{4a} \sqrt{-ax^2+bx+c} + \frac{b^2+4ac}{8\sqrt{a^3}} \arcsin \frac{2ax-b}{\sqrt{b^2+4ac}} + C$$

$$\int \frac{x}{\sqrt{-ax^2+bx+c}} \mathrm{d}x = -\frac{1}{a} \sqrt{-ax^2+bx+c} + \frac{b}{2\sqrt{a^3}} \arcsin \frac{2ax-b}{\sqrt{b^2+4ac}} + C$$

5.5.1.3 Triangular

$$\int \tan x \,\mathrm{d}x = -\ln|\cos x| + C$$

$$\int \cot x \,\mathrm{d}x = \ln|\sin x| + C$$

$$\int \sec x \,\mathrm{d}x = \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right| + C = \ln|\sec x + \tan x| + C$$

$$\int \csc x \,\mathrm{d}x = \ln\left|\tan \frac{x}{2}\right| + C = \ln|\csc x - \cot x| + C$$

5.5.2 Prefix sum of multiplicative functions

Define the Dirichlet convolution $f * g(n)$ as:

$$f * g(n) = \sum_{d=1}^{n}[d|n]f(n)g(\frac{n}{d})$$

Assume we are going to calculate some function $S(n) = \sum_{i=1}^{n} f(i)$, where $f(n)$ is a multiplicative function. Say we find some $g(n)$ that is simple to calculate, and $\sum_{i=1}^{n} f * g(i)$ can be figured out in $O(1)$ complexity. Then we have

$$\begin{aligned}
\sum_{i=1}^{n} f * g(i) &= \sum_{i=1}^{n}\sum_{d}[d|i]g(\frac{i}{d})f(d) \\
&= \sum_{\frac{i}{d}=1}^{n}\sum_{d=1}^{\left\lfloor \frac{n}{\frac{i}{d}} \right\rfloor} g(\frac{i}{d})f(d) \\
&= \sum_{i=1}^{n}\sum_{d=1}^{\left\lfloor \frac{n}{i} \right\rfloor} g(i)f(d) \\
&= g(1)S(n) + \sum_{i=2}^{n} g(i)S(\left\lfloor \frac{n}{i} \right\rfloor) \\
S(n) &= \frac{\sum_{i=1}^{n} f * g(i) - \sum_{i=2}^{n} g(i)S(\left\lfloor \frac{n}{i} \right\rfloor)}{g(1)}
\end{aligned}$$

It can be proven that $\left\lfloor \frac{n}{i} \right\rfloor$ has at most $O(\sqrt{n})$ possible values. Therefore, the calculation of $S(n)$ can be reduced to $O(\sqrt{n})$ calculations of $S(\left\lfloor \frac{n}{i} \right\rfloor)$. By applying the master theorem, it can be shown that the complexity of such method is $O(n^{\frac{3}{4}})$.

Moreover, since $f(n)$ is multiplicative, we can process the first $n^{\frac{2}{3}}$ elements via linear sieve, and for the rest of the elements, we apply the method shown above. The complexity can thus be enhaced to $O(n^{\frac{2}{3}})$.

For the prefix sum of Euler's function $S(n) = \sum_{i=1}^{n} \varphi(i)$, notice that $\sum_{d|n} \varphi(d) = n$. Hence $\varphi * I(n) = id(n)$.($I(n) = 1, id(n) = n$) Now let $g(n) = I(n)$, and we have $S(n) = \sum_{i=1}^{n} i - \sum_{i=2}^{n} S(\left\lfloor \frac{n}{i} \right\rfloor)$.

For the prefix sum of Mobius function $S(n) = \sum_{i=1}^{n} \mu(i)$, notice that $\mu * I(n) = [n = 1]$. Hence $S(n) = 1 - \sum_{i=2}^{n} S(\left\lfloor \frac{n}{i} \right\rfloor)$.