ESO 207A Programming Assignment 2: Due on 09/02/2019

January 23, 2019

1 Dominating Set in 2-D

Given two points p, q in 2-D space, p is said to *dominate* q if $p_x \geq q_x$ and $p_y \geq q_y$. Define a partial ordering on the points of the 2-D space, $\preceq$, as follows: $q \preceq p$ if p dominates q.

Let S be a set of points in a 2-D space. Then $Dom(S)$ denotes those members of S which are not dominated by any other point in S, i.e., $Dom(S) = \{q \in S | q \not\preceq p \ \forall \ p \in \mathbb{R}^2 \setminus \{q\}\}$.

Example $Dom(\{(2,1),(3,2),(2,3),(1,2)\}) = \{(3,2),(2,3)\}$.

2 Problem

Given a set S of n points in a 2-D plane, write a C program to compute $Dom(S)$. Observe that a trivial algorithm can be designed which has $O(n^2)$ time complexity. Your task is to use *divide and conquer* and design an algorithm with $O(n. \log \ n)$ time complexity.

3 Details

Save all input points in an array $A[.]$ at locations 0 to $n-1$ and return the set $Dom(S)$ in array $B[.]$ in consecutive locations, starting from 0. For simplicity, assume that all points have integer coordinates.

Initially store all points in a 2-D array $Point$. If the i-th point is (x_i, y_i), then $Point[0, i] = x_i$ and $Point[1, i] = y_i$. So index i uniquely identifies the point.

Array A and B are integer arrays and they only store the indices of the points. That is, if $A[j] = i$, then it is referring to the point $(Point[0, i], Points[1, i])$. Same is true for B.

Hint1: Devise a subroutine $Dom(i, j)$, $i \leq j$, which computes the $Dom(\{A[i], \ldots, A[j]\})$ and returns it in array B in the consecutive locations starting from $B[i]$.

Hint2: Initially sort the points in A in increasing order of their x-values.

4 I/O Format

- The first line of the input consists of an integer n denoting the number of points in set S.

- Next n lines will contain n points with each line containing two integers separated by a **single space**. First number being the x-coordinate and the second the y-coordinate.

- The output must have integer m, the number of points in $Dom(S)$, in the first line. Subsequently m lines must have x and y coordinates of one point each, separated by a single space. The points must be sorted in increasing order of x-values of the points.

Sample Input:

```
4
3  2
1  2  |
2  3
2  1
```

Corresponding Output:

```
2
2   3
3   2
```

Constraints:

$1 \leq n \leq 10^5$

$0 \leq x \leq 10^5$

$0 \leq y \leq 10^5$