PROJECT 1

General information

- The project work may be done in groups of at most three people – you are encouraged to collaborate, since the projects will be rather time consuming.
- The results of your analyses, including answers to all questions stated below, shall be summarised in a report which is no longer than 10 pages (A4, 12 pt font, standard spacing). No derivations are needed in the report unless you use this explicitly as a part of your argumentation.
- Please make sure that all graphs and tables used are easy to read and clearly referenced from the main text in the report.
- The report must be submitted using the online submission system due to that the department is using software for detecting plagiarism. How this can be done will be announced via the course forum.
- The deadline for submitting project work is 2024-01-12 (12th of January). Revisions of project work may be handed in until the day of the first re-exam 2024. Note that the maximum score for a revised project work is the lowest pass score. This applies separately to both projects
- **N.B.** There are technical tips available at the course homepage.

Project description

- Consider a health insurance with the following dynamics: There are the states healthy = 2, sick = 1, and dead = 0, with the following transitions being allowed

$$\begin{aligned} 2 &\leftrightarrow 1 \\ 1 &\rightarrow 0 \\ 2 &\rightarrow 0 \end{aligned}$$

- The benefit payment structure is as follows: at time 0, corresponding to "today", an annual sickness benefit of 15 % of a monthly salary of 30 000 is paid for a person being sick for one year. These payments are assumed to occur continuously in time, and there is an indexation corresponding to an increase of 1.5 % per annum (deterministic). That is,

$$b_1(t) = 12 \cdot 30\ 000 \cdot 0.15 \cdot 1.015^t.$$

- Assume that the mortality is described by the cohort mortality given by the Makeham-function

$$\mu(t) = a + b\exp\{ct\},$$

with parameters $a = 3.5 \cdot 10^{-4}, b = 7 \cdot 10^{-8}$, and $c = 0.157$ (where t here should be thought of as age in years), and use the interest rates found on in the file

`interest_rates.txt`

on the course homepage. The interest rates corresponds to continuously compounded spot rates r_t.

Questions to be answered:

(A.1) The spot rates r_t produces *yearly* discount factors given by $D(0,t) = \exp\{-tr_t\}$, which means that the rates $r(t)$ used in the modelling will be a function of r_t. One suggestion is to express $r(t)$ as a function of r_t assuming $r(t)$ is constant over calendar years. If you do not succeed with this, clearly state this in the report and use the constant interest rate $r(t) = 0.04$.

(A.2) Use the data in the file

`transition_data.RData`

consisting of i.i.d. 50 year old individuals to estimate the transition rates for transitions between the states 2 and 1, as functions of age (this only contains very few observed deaths). Each row in the file corresponds to a contract and each column corresponds to a specific day, and all values corresponds to the states $0, 1$, and 2. Is it reasonable to assume that the rates are approximately the same over different calendar years? if not, argue for a parametric approximation that can be used for extrapolation.

If you are unable to solve this, clearly state this in the report and use the rates $\mu_{21}(t) = 2$ and $\mu_{12}(t) = 50$.

(A.3) Determine the single premium to be paid from the time point when a healthy individual turns 50, in order for this to be a fair premium, if the last time point when benefits can be received corresponds to the individuals 65 birthday. Do these calculations using a discretisation corresponding to the time units 1 day, 1 month, and 1 year. Report how the results are affected by the discretisation, and illustrate how the reserves evolve as a function of time.

N.B. You may use the example `R`-code for solving differential equations that is available on the course homepage!

(A.4) Redo (A.2) when using a premium paid continuously in time as long as an individual is healthy, which is increased with 1.5 % per annum. Only do this for one suitably chosen time discretisation. Try what happens if you only allow a non-zero premium for the first 10 years in order to illustrate the effect of premium payment structure.

(A.5) Do a number of bootstrap simulations of the dataset

`transition_data.RData`

by sampling individual policies with replacement, and for each of the bootstrap simulations, re-estimate all parameters, and calculate the fair single premium. Discuss the results, including comments on estimation.