PROJECT 2

General information

- The project work may be done in groups of at most three people – you are encouraged to collaborate, since the projects may be time consuming.
- The results of your analyses, including answers to all questions stated below, shall be summarised in a report which is no longer than 10 pages (A4, 12 pt font, standard spacing). If you use results available in the literature, just state the result together with a precise reference (no need to repeat lengthy derivations).
- Please make sure that graphs and tables used are easy to read and clearly referenced from the main text in the report.
- The report must be submitted using the online submission system. The department is using software for detecting plagiarism.
- The deadline for submitting project work is 2024-01-12 (12th of January). Revisions of project work may be handed in until the day of the first re-exam 2024. Note that the maximum score for a revised project work is the lowest pass score. This applies separately to both projects.

Project description

- Valuation of the policy holders benefits and the insurance company's gains is done by computing discounted expected values, where expectation is with respect to risk-neutral probabilities according to a probability measure Q.
- Consider a simplified unit-linked insurance only consisting of two states, alive and dead, where the mortality rates (with respect to Q) are deterministic and given by the Makeham-function

$$\mu(t) = a + b\exp\{ct\},$$

  with parameters $a = 3.5 \cdot 10^{-4}, b = 7 \cdot 10^{-8}$, and $c = 0.157$ (where t here should be thought of as age in years).
- The payment structure is as follows: at time 0, corresponding to "today", a single payment is made by the insured, which corresponds to 1 SEK. This payment, after possibly withdrawing a fee (see below), is then invested in a risky asset $X(t)$. The market only consists of this risky asset and a deterministic risk-free account which grows according to e^{rt}, where $r = 0.015$ is the risk-free interest rate. The risk-neutral dynamics of the risky asset is given by

$$X^Q(t) := \exp\{(r - \frac{1}{2}\sigma^2)t + \sigma W^Q(t)\},$$

  where $\sigma = 0.06$ is a constant and where $W^Q(t)$ corresponds to a standard Brownian motion, i.e. with respect to Q, $W^Q(t) - W^Q(s), t > s$, is distributed according to a normal random variable with mean 0 and variance

$t - s$, and where $W^Q(t) - W^Q(s)$ is independent of $W^Q(v) - W^Q(u)$, for all $s < t \leq u < v$.

- Death events for different individuals are independent, and independent of how the price of the risky asset evolves.
- Given that the insured individual is alive at some future, pre-determined, time point τ, the insured will receive the invested amount in accordance with the investment gain / loss produced by the risky asset dynamics. Moreover, the insured will also receive a possible inheritance gain (sv. "arvsvinst") due to that other individuals in the insured collective may have died. That is, when an individual dies before time τ, its invested capital will be distributed equally amongst the remaining active contracts (the survivors), which will be re-invested in the risky asset. This re-distribution of capital due to inheritance gains will be done at the end of each calendar year.

Questions to be answered:

(A.1) Consider a portfolio consisting of 10 000 contracts with 50 year old individuals, all of which just turned 50 before the beginning of the current year. A policy holder will receive the benefits the day she turns 70, and nothing if she dies before the age of 70. Assume that the insurer does not charge any fee on the initial 1 SEK investment, but will collect an annual fee of 0.1 % of the invested capital at the end of each calendar year (including at the 70th birthday), which is re-invested in the risk-free asset. Calculate the risk-neutral value of the benefits for one such contract and calculate the risk-neutral value of the total gain for the insurer.

(A.2) Redo (A.1), but replace the annual fee with a single fee taken from the initially invested 1 SEK. Determine this amount so that the risk-neutral value of the benefits for one contract is equal to that in (A.1). Calculate the risk-neutral value of the total gain for the insurer.

(A.3) Redo (A.1), but replace the annual fee with taking 20 % of the inheritance gain. Calculate the risk-neutral value of the benefits for one contract and calculate the risk-neutral value of the total gain for the insurer.

(A.4) Plot 10 trajectories of (1) the evolution of the number of alive policy holders, (2) the evolution of the price of the risky asset. Determine the distribution of (3) the number of alive policy holders at age 70, (4) the price of the risky asset at that time.

You may calculate expectations analytically or approximate expectations by simulating asset price dynamics and death events and use empirical averages as proxies for expected values. Simulated sample sizes should be sufficiently large to obtain reasonable accuracy.

The above description does not say what happens if no policy holders survive until the age of 70. But this event can be safely ignored. Why?