G+ আরো পরবর্তী ব্লগ» nitaibanik29@gmail.c

I, ME AND MYSELF !!!

SATURDAY, NOVEMBER 20, 2010

Matrix Exponentiation

Introduction:

Don't be confused with the title, this article has nothing to do with "how to calculate an exponent of a given matrix", rather it will discuss on how to use this technique to solve a specific class of problems.

Sometimes we face some problems, where, we can easily derive a recursive relation (mostly suitable for dynamic programming approach), but the given constraints make us about to cry, there comes the matrix exponentiation idea. The situation can be made more clear with the following example:

Let, a problem says: *find f(n) : n'th Fibonacci number*. When n is sufficiently small, we can solve the problem with a simple recursion, *f(n) = f(n-1) + f(n-2)*, or, we can follow dynamic programming approach to avoid the calculation of same function over and over again. But, what will you do if the problem says, *given 0 < n < 1000000000, find f(n) % 999983* ? No doubt dynamic programming will fail!

We'll develop the idea on how and why these types of problems could be solved by matrix exponentiation later, first lets see how matrix exponentiation can help is to represent recurrence relations.

Prerequisite:

Before continuing, you need to know:

- Given two matrices, how to find their product, or given the product matrix of two matrices, and one of them, how to find the other matrix.
- Given a matrix of size d x d, how to find its n'th power in O(d^3log(n)).

Patterns:

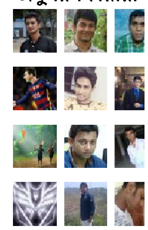
What we need first, is a recursive relation, and what we want to do, is to find a matrix *M* which can lead us to the desired state from a set of already known states. Let, we know *k* states of a given recurrence relation, and want to find the *(k+1)*th state. Let *M* be a *k x k* matrix, and we build a matrix *A*:[*k x 1*] matrix from the known states of the recurrence relation, now we want to get a matrix *B*:[*k x 1*] which will represent the set of next states, i.e. *M x A = B*, as shown below:

```
    | f(n)   |   | f(n+1) |
    | f(n-1) |   |  f(n)  |
M x | f(n-2) | = | f(n-1) |
    | ...... |   | ...... |
    | f(n-k) |   |f(n-k+1)|
```

So, if we can design *M* accordingly, job's done!, the matrix will then be used to represent the recurrence relation.

- **Type 1:**

  Lets start by the simplest one, *f(n) = f(n-1) + f(n-2)*.
  So, *f(n+1) = f(n) + f(n-1)*
  Let, we know, *f(n)* and *f(n-1)*; we want to get *f(n+1)*
  From the above situation, matrix *A* and *B* can be formed as shown below:

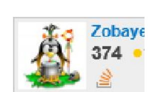
অনুসরণকারীরা

অনুসরণ

SUBSCRIBE
Posts
Comments

BLOG HITS


BLOG ARCHIV
► 2015 (4)
► 2014 (6)
► 2013 (19)
► 2012 (14)
► 2011 (15)
▼ 2010 (33)
► Decembe
▼ Novembe
CRC - 32
Matrix Exp
Merge Sor
► Septembe
► August (3
► June (2)
► May (5)
► March (5)
► February
► January (
► 2009 (27)

ABOUT ME


STACK OVERF


Matrix A Matrix B

```
| f(n)   |        | f(n+1) |
| f(n-1) |        |  f(n)  |
```

[Note: matrix *A* will be always designed in such a way that, every state on which *f(n+1)* depends, will be present]
So, now, we need to design a 2x2 matrix *M* such that, it satisfies *M x A = B* as stated above.
The first element of *B* is *f(n+1)* which is actually *f(n) + f(n-1)*. To get this, from matrix *A*, we need, 1 *f(n)* and 1 *f(n-1)*. So, the 1st row of *M* will be *[1 1]*.

```
| 1   1 | x |  f(n)  | = | f(n+1) |
| ----- |   | f(n-1) |   | ------ |
```

[Note: ----- means, we are not concerned about this value]
Similarly, 2nd item of *B* is *f(n)* which we can get by simply taking 1 *f(n)* from *A*. So, the 2nd row of *M* is *[1 0]*.

```
| ----- | x |  f(n)  | = | ------ |
| 1   0 |   | f(n-1) |   |  f(n)  |
```

[I hope you know how a matrix multiplication is done and how the values ar assigned!]
Thus we get the desired 2 x 2 matrix *M*:

```
| 1   1 | x |  f(n)  | = | f(n+1) |
| 1   0 |   | f(n-1) |   |  f(n)  |
```

If you are confused about how the above matrix is calculated, you might try doing it this way:

We know, the multiplication of an *n x n* matrix *M* with an *n x 1* matrix *A* will generate an *n x 1* matrix *B*, i.e. *M x A = B*. The *k'th* element in the product matrix *B* is the product of k'th row of the *n x n* matrix *M* with the *n x 1* matrix *A* in the left side.
In the above example, the 1st element in *B* is *f(n+1) = f(n) + f(n-1)*. So, it's the product of 1st row of matrix *M* and matrix *B*. Let, the first row of *M* is *[x y]*. So, according to matrix multiplication,

```
 x * f(n) + y * f(n-1) = f(n+1) = f(n) + f(n-1)
⇒ x = 1, y = 1
```

Thus we can find the first row of matrix *M* is *[1 1]*. Similarly, let, the 2nd row of matrix *M* is *[x y]*, and according to matrix multiplication:

```
 x * f(n) + y * f(n-1) = f(n)
⇒ x = 1, y = 0
```

Thus we get the second row of *M* is *[1 0]*.

- **Type 2:**

  Now, we make it a bit complex: find *f(n) = a * f(n-1) + b * f(n-2)*, where *a, b* are some constants.
  This tells us, *f(n+1) = a * f(n) + b * f(n-1)*.
  By this far, this should be clear that the dimension of the matrices will be equal to the number of dependencies, i.e. in this particular example, again 2. So, for *A* and *B*, we can build two matrices of size 2 x 1:

  Matrix A Matrix B

```
| f(n)   |
| f(n-1) |                 | f(n+1) |
                           |  f(n)  |
```

Now for *f(n+1) = a * f(n) + b * f(n-1)*, we need *[a b]* in the first row of objective matrix *M* instead of *[1 1]* from the previous example. Because, now we need *a* of *f(n)*'s and *b* of *f(n-1)*'s.

```
| a   b | x |  f(n)  | = | f(n+1) |
| ----- |   | f(n-1) |   | ------ |
```

And, for the 2nd item in *B* i.e. *f(n)*, we already have that in matrix *A*, so we just take that, which leads, the 2nd row of the matrix *M* will be *[1 0]* as the previous one.

```
| ----- | x |  f(n)  | = | ------ |
| 1   0 |   | f(n-1) |   |  f(n)  |
```

So, this time we get:

```
| a   b | x |  f(n)  | = | f(n+1) |
| 1   0 |   | f(n-1) |   |  f(n)  |
```

Pretty simple as the previous one...

- **Type 3:**

  We've already grown much older, now lets face a bit complex relation: find *f(n) = a * f(n-1) + c * f(n-3)*.
  Ooops! a few minutes ago, all we saw were contiguous states, but here, the state *f(n-2)* is missing. Now? what to do?

  Actually, this is not a problem anymore, we can convert the relation as follows: *f(n) = a * f(n-1) + 0 * f(n-2) + c * f(n-3)*, deducing *f(n+1) = a * f(n) + 0 * f(n-1) + c * f(n-2)*. Now, we see that, this is actually a form described in Type 2. So, here, the objective matrix *M* will be 3 x 3, and the elements are:

```
| a  0  c |   |  f(n)  |   | f(n+1) |
| 1  0  0 | x | f(n-1) | = |  f(n)  |
| 0  1  0 |   | f(n-2) |   | f(n-1) |
```

  These are calculated in the same way as Type 2. [Note, if you find it difficult, try on pen and paper!]

- **Type 4:**

  Life is getting complex as hell, and Mr. problem now asks you to find *f(n) = f(n-1) + f(n-2) + c* where *c* is any constant.

  Now, this is a new one and all we have seen in past, after the multiplication, each state in *A* transforms to its next state in *B*.
  f(n) = f(n-1) + f(n-2) + c
  f(n+1) = f(n) + f(n-1) + c
  f(n+2) = f(n+1) + f(n) + c
  .... so on
  So, normally we can't get it through the previous fashions. But, how about now we add *c* as a state?

```
    |  f(n)  |   | f(n+1) |
M x | f(n-1) | = |  f(n)  |
    |   c    |   |   c    |
```

  Now, its not much hard to design *M* according to the previous fashion. Here it is done, but don't forget to verify on yours:

```
| 1  1  1 |   |  f(n)   |   | f(n+1) |
| 1  0  0 | x | f(n-1)  | = |  f(n)  |
| 0  0  1 |   |    c    |   |    c   |
```

- **Type 5:**

  Lets put it altogether: find matrix suitable for *f(n) = a * f(n-1) + c * f(n-3) + d * f(n-4) + e*.
  I would leave it as an exercise to reader. The final matrix is given here, check if it matches with your solution. Also find matrix *A* and *B*.

```
| a  0  c  d  1 |
| 1  0  0  0  0 |
| 0  1  0  0  0 |
| 0  0  1  0  0 |
| 0  0  0  0  1 |
```

  [Note: you may take a look back to Type 3 and 4]

- **Type 6:**

  Sometimes, a recurrence is given like this:

```
f(n) = if n is odd, f(n-1) else, f(n-2)
In short:
f(n) = (n&1) * f(n-1) + (!(n&1)) * f(n-2)
```

  Here, we can just split the functions in the basis of odd even and keep 2 different matrix for both of them and calculate separately.
  Actually, there might appear many different patterns, but these are the basic patterns.

- **Type 7:**

  Sometimes we may need to maintain more than one recurrence, where they are interrelated. For example, let a recurrence relation be:
  g(n) = 2g(n-1) + 2g(n-2) + f(n), where, *f(n) = 2f(n-1) + 2f(n-2)*. Here, recurrence *g(n)* is dependent upon *f(n)* and the can be calculated in the same matrix but of increased dimensions. Lets design the matrices *A, B* then we'll try to find matrix *M*.

```
   Matrix A          Matrix B

|   g(n)  |       | g(n+1) |
|  g(n-1) |       |  g(n)  |
|  f(n+1) |       | f(n+2) |
|   f(n)  |       | f(n+1) |
```

  Here, *g(n+1) = 2g(n) + 2g(n-1) + f(n+1)* and *f(n+2) = 2f(n+1) + 2f(n)*.
  Now, using the above process, we can generate the objective matrix *M* as follows:

```
| 2  2  1  0 |
| 1  0  0  0 |
| 0  0  2  2 |
| 0  0  1  0 |
```

So, these are the basic categories of recurrence relations which are used to be solved by this simple technique.

Analysis:

Now that we have seen how matrix multiplication can be used to maintain recurrence relations, we are back to out first question, how this helps us in solving recurrences on a huge range.

Recall the recurrence *f(n) = f(n-1) + f(n-2)*.
We already know that:

```
M x |  f(n)   | = | f(n+1) | ............(1)
    | f(n-1)  |   |  f(n)  |
```

How about we multiply *M* multiple times? Like this:

```
M x M x |  f(n)  | = | f(n+1) |
        | f(n-1) |   |  f(n)  |
```

Replacing from (1):

```
M x M x |  f(n)  | = M x | f(n+1) | = | f(n+2) |
        | f(n-1) |       |  f(n)  |   | f(n+1) |
```

So, we finally get:

```
M^2 x |  f(n)  | = | f(n+2) |
      | f(n-1) |   | f(n+1) |
```

Similarly we can show:

```
M^3 x |  f(n)  | = | f(n+3) |
      | f(n-1) |   | f(n+2) |


M^4 x |  f(n)  | = | f(n+4) |
      | f(n-1) |   | f(n+3) |


..............................
..............................
..............................


M^k x |  f(n)  | = | f(n+k) |
      | f(n-1) |   |f(n+k-1)|
```

Thus we can get any state *f(n)* by simply raising the power of objective matrix *M* to n-1 in O($d^3\log(n)$), where d is the dimension of square matrix *M*. So, even if n = 1000000000, still this can be calculated pretty easily as long as d^3 is sufficiently small.

Related problems:

UVa 10229 : Modular Fibonacci
UVa 10870 : Recurrences
UVa 11651 : Krypton Number System
UVa 10754 : Fantastic Sequence
UVa 11551 : Experienced Endeavour

Regards, Zobayer Hasan.

Posted by Zobayer Hasan at 2:12 PM

95 comments:

Anonymous November 20, 2010 at 3:17 PM

Thank you, Zobayer.
It's worth reading :)

Reply

Anonymous November 26, 2010 at 9:45 AM

Hey Zobayer. Do have any idea to solve that:
we consider F(n) function determined by following term
if n is quadratic value then F(n)=0
otherwise f(n) = [1 / {sqrt(n)}]
So what is S = F(1)+F(2)+...+F(N^2).
I think there is no formula exists to solve it. But How do i solve it easier??

Reply

Zobayer Hasan November 30, 2010 at 3:59 AM

I don't think I can solve this type using matrix exponentiation, I can go only for linear ones. I don't think quadratic equations can be solved.

But, still this could be solved, like, sometimes approximation formulas can be found, or, you can also try divide and conquer / dp techniques. Might help.

Reply

Sabbir Yousuf Sanny December 21, 2010 at 4:45 AM

Really nice tutorial! Helped me to solve this problem on latest Codechef - http://www.codechef.com/COOK05/problems/SEEDS/

Keep it up!

Reply

Sankalp January 9, 2011 at 5:18 PM

Hey Zobayer!
I have a problem where the recursive relation is like this:
f(n,h,k) = f(n-1,h-1,k+1) + f(n-1,h,k-1) + f(n-1,h+1,k)+1

do you have any idea how to solve this by matrix exponentiation..?

Reply

Zobayer Hasan January 9, 2011 at 10:25 PM

Is it not possible with dp? I don't think it's possible with matrix exponentiation as all of the parameters are not shifting at a same order. I am sorry if I am wrong, actually I don't know the answer of your question :(

Reply

Aakash Johari (IIITA) February 2, 2011 at 6:25 PM

Hi, Zobayar. Can you explain type-7 matrix A and B. According to me it should be
| g(n) |
A = | g(n-1) |
| f(n) |
| f(n-1) |

B = | g(n+1) |
| g(n) |
| f(n+1) |
| f(n) |

Reply

Zobayer Hasan February 2, 2011 at 10:13 PM

@Aakash, if you want to find out g(n+1) then you will be needing:

g(n+1) = 2g(n) + 2g(n-1) + f(n+1)

So, you must have f(n+1) on the left side. Hope you get it.

Reply

Radhakrishnan February 16, 2011 at 1:25 AM

This comment has been removed by the author.

Reply

Zobayer Hasan February 16, 2011 at 1:55 AM

nice question. look carefully, you can always do something like this, say for n = 3
1 + A + A^2 + A^3
= (1 + A) + A^2(1 + A)
so divide and conquer is possible.
I have a post related to this, which u may wish to see.
http://zobayer.blogspot.com/2009/12/power-series-evaluation.html
thanks for commenting.

Reply

Anonymous April 16, 2011 at 7:31 PM

Thanks for the informative post!
It would be more helpful, if you could point out/give links to resources about finding n'th power of matrix in O(d^3 * log(n)).

Reply

Zobayer Hasan April 16, 2011 at 10:05 PM

I think, you can find b^n in log(n) right?
a recursive algorithm might look like this:
f(b, n):
----If n == 0 return 1
----If n is odd return f(b, n-1) * b;
----else return square(f(b, n/2))

See, you are multiplying b here which is definitely O(1), however, for matrix, b will be a dxd matrix, and to multiply two dxd matrix, you need O(d^3). So the actual complexity is O(d^3 lg(n))

Reply

Anonymous May 27, 2011 at 2:23 AM

hey can you tell about uva 11651 I am not able to figure it out

Reply

Anonymous May 27, 2011 at 6:13 PM

excellent one............worth reading

Reply

Voltorb October 8, 2011 at 3:24 PM

I tried the HDU 2802 problem using matrix exponentiation. I got TLE, so tried storing the matrices for A, A^2, A^4,... to quicken the calculation of A^n. But still got TLE!

Can this be solved using matrix exponentiation for sure? Because there is this Chinese blog, which tells there is some cycle form - http://hi.baidu.com/xiinho/blog/item/1307e6156a9be10d4b90a741.html

Reply

Zobayer Hasan October 9, 2011 at 10:11 PM

I could not do it with matrix expo either, but I have listed it here because I have found this problem in the list of matrix expo problems somewhere, can't remember now where, many days passed. Sorry!

Reply

Anonymous December 22, 2011 at 6:16 PM

f(n) = n*f(n-1) + f(n-2)

Can this be solved using matrix exponentiation ?

Reply

Anonymous March 12, 2012 at 5:15 PM

Hi Zobayer,

Can you show how to find the result for

f(n) = a* f(n-1) + b?

Thanks :)

Reply

Replies

Zobayer Hasan March 12, 2012 at 9:04 PM

This is quite simple, I think you can try the following relation
{ { a, 1 }, { 0, 1 } } * { { f(n), b } } = { f(n+1), b }

Reply

Anonymous June 4, 2012 at 2:25 PM

Hmm...the new theme is cool ;-), nice article btw. Although I knew about matrix exponentiation before but learned a couple of new things here. Very well explained too.

Reply

Replies

Zobayer Hasan June 4, 2012 at 9:51 PM

Thank you :)

Reply

Anonymous June 5, 2012 at 11:28 AM

Can we do it for recurrence having constant as a function of n.
Like for example f(n) = f(n-1) + f(n-2) + n ?

Reply

Replies

Zobayer Hasan June 5, 2012 at 5:50 PM

I don't think so, or to be precise, I can't.

Nobody November 9, 2013 at 4:07 PM

Yes, we can do it. Just define g(n) = n => g(n) = g(n-1) + 1, g(1) = 1.
Then, f(n+1) = f(n) + f(n-1) + g(n+1) = f(n) + f(n-1) + (n+1).
This if of type 7 as given in the article. We can solve it as:

f(n+1)		1 1 1 0		f(n)
f(n)	=	1 0 0 0		f(n-1)
g(n+2)		0 0 1 1		g(n+1)
1		0 0 0 1		1

We need to find out { { f(n+1) }, { f(n) }, { g(n+2) }, { 1 } }.
We know { { f(n) }, { f(n-1) }, { g(n+1) }, { 1 } }.
The matrix is
1 1 1 0
1 0 0 0
0 0 1 1
0 0 0 1

And thank you Zobayer for this wonderful article. It helped me a lot.

Zobayer Hasan April 23, 2015 at 10:10 AM

@Nobody, clever thinking, would you mind if I add this to the article? I wish I knew your email address / website or something.

Reply

Muhammed Hedayet June 7, 2012 at 1:28 AM

Cool. I know Matrix exponentiation technique and have solved some problems using this technique, but nice organized article. This article let me think in a new fashion. Thanks. Want more. . . :-)

Reply

Replies

Zobayer Hasan June 7, 2012 at 2:03 AM

Thanks :)

Reply

Anonymous July 3, 2012 at 12:18 PM

Very nice post, keep up the great work.

Reply

Shahin ShamS July 3, 2012 at 6:31 PM

I didn't know about the topic before read the article. Many many thanks for this.

Reply

Shahin ShamS July 3, 2012 at 6:33 PM

Many many thanks to your for this article. Because the topic is new for me.... ans thanks again.

Reply

Shahin ShamS July 3, 2012 at 6:36 PM

Many many thanks to you for this article. Because the topic is new for me........ thanks again.

Reply

sahil sharma July 7, 2012 at 11:07 AM

good work dude..really nice

Reply

riazur July 8, 2012 at 4:07 AM

Thanks Vaia.It's really helpful.Thanks a lot :)

Reply

sameer July 17, 2012 at 12:42 PM

Thanks for sharing.
Keep up the good work !!!

Reply

Anonymous September 5, 2012 at 5:37 AM

Can we do it if we have a n on the right side?

For example:

f(n) = a*f(n-1) - n?

Reply

Replies

Nobody November 9, 2013 at 4:11 PM

Yes we can. Just define g(n) = n => g(n) = g(n-1) + 1, g(1) = 1.
Then f(n) = a*f(n-1) - g(n). This is of type 7 as given in the article.
We need to find out the 1 column matrix:
f(n+1)
g(n+2)
1
We know:
f(n)
g(n+1)
1

Thanks a lot bro. I wanted to learn about this and at the right time found this blog.

Reply

Replies

Zobayer Hasan February 28, 2013 at 8:55 PM

Thanks for visiting, I'm glad that it helped :)

Reply

Salman Estyak March 1, 2013 at 12:10 AM

while(1)
printf("ধন্যবাদ,ভাইয়া :)");

Reply

Anonymous March 3, 2013 at 9:45 PM

Hi , Thanks for writing this beautiful note :)

Can you please explain, how can we build the M for the following type of function? Thanks in advance.

g(n)=g(n-1)+g(n-2)+f(n-3)
where
f(n)=f(n-1)+f(n-2)+g(n-3)

Reply

Replies

Zobayer Hasan March 3, 2013 at 9:59 PM

you can do it using a 6x6 matrix.
all you need to do is derive this matrix
f(n+1)
f(n)
f(n-1)
g(n+1)
g(n)
g(n-1)
and what you know is this matrix
f(n)
f(n-1)
f(n-2)
g(n)
g(n-1)
g(n-2)
so the matrix will be
1 1 0 0 0 1
1 0 0 0 0 0
0 1 0 0 0 0
0 0 1 1 1 0
0 0 0 1 0 0
0 0 0 0 1 0

Reply

Anonymous March 4, 2013 at 2:32 PM

I'm having trouble understanding the Formation of this matrix ...
f(n+1)
f(n)
f(n-1)
g(n+1)
g(n)
g(n-1)
I'm writing my reasoning , please correct it ......

I need to find , f(n+1) ...
f(n+1)=f(n)+f(n-1)+g(n-2)

Here , f(n+1) function's dependencies are f(n) , f(n-1) and g(n-2) ... now , I need to find out the the dependencies of g(n-2) to , get the value of g(n-2) , g(n-2)=g(n-3)+g(n-4)+f(n-5) //g() again is related to f() , I'm finding this confusing//
then , f(n+1) total dependencies are f(n),f(n-1),g(n-3),g(n-4),f(n-5) ....so , A becomes
a matrix will all the up stated values , and B will be a matrix with same functions but '1' added to the arguments . Please , Explain the fault in the reasoning a give a bit detail on what is really going on there .

Thanks .

Reply

Replies

Zobayer Hasan March 4, 2013 at 8:32 PM

Yes, there are some mistakes in your reasoning, I mean, you are correct about the dependencies, but the fact is, if you already know what g(n-2) is, you wont need to expand it further. In matrix exponentiation algorithm, with M*A = B form, the 1 column matrix you showed is B here, and A is what you already have. No need to follow recursions any more. Just try to test if recurrences hold. As I have written above, just multiply it and see for yourself you u can get the right side or not.
first row is 1 1 0 0 0 1, and the 1 column A matrix is
f(n)
f(n-1)
f(n-2)
g(n)
g(n-1)
g(n-2)
so, multiplying, we get the first row of B which is
f(n)*1 + f(n-1)*1 + f(n-2)*0 + g(n)*0 + g(n-1)*0 + g(n-2)*1
=> f(n) + f(n-1) + g(n-2)
=> f(n+1).
similarly, second row is 1 0 0 0 0 0, this is f(n)
from third row, 0 1 0 0 0 0, we get f(n-1)
from fourth row 0 0 1 1 1 0, we get f(n-2) + g(n) + g(n-1) => g(n+1)
from fifth row 0 0 0 1 0 0, we get g(n)
from sixth row, 0 0 0 0 1 0, we get g(n-1)
so, multiplying the matrix M, we get
f(n+1)
f(n)
f(n-1)
g(n+1)
g(n)
g(n-1)
Now, imagine, if we already know
f(n)
f(n-1)
f(n-2)
g(n)
g(n-1)
g(n-2)
we can then find M^n to get f(n+1) and g(n+1) at the same time, don't follow typical recurrence reasoning here. I think the later part of the post describes how this is done actually. I don't know how can I make it more clear.

Zobayer Hasan March 4, 2013 at 8:35 PM

Correction, I wanted to write this in the last:
Now, imagine, if we already know
f(2)
f(1)
f(0)
g(2)
g(1)
g(0),
then we can find M^n to get f(2+n), g(2+n) at the same time. the other values are just carried with the process.

Anonymous March 4, 2013 at 11:46 PM

Thanks , I think I got it finally .
:) I was taking the whole thing as the formal recursion and was trying to trace back and ...from f() I was trying to trace back through g() .....again f() ...all that made me confused ...

sorry but I will nag you one more time :) I'm writing another recurrence function , please say whether it's okay or not :)

Let ,
f(n)=f(n-1)+g(n-2)+h(n-3)
g(n)=g(n-1)+h(n-2)+f(n-3)
h(n)=h(n-1)+f(n-2)+g(n-3)

then ,

B will be
|f(n+1)|
|f(n) |
|f(n-1)|
|g(n+1)|
|g(n) |
|g(n-1)|
|h(n+1)|

|h(n) |
|h(n-1)|

A will be :
|f(n) |
|f(n-1)|
|f(n-2)|
|g(n) |
|g(n-1)|
|g(n-2)|
|h(n) |
|h(n-1)|
|h(n-2)|

So , M will be

| 1 0 0 0 1 0 0 0 1 |
| 1 0 0 0 0 0 0 0 0 |
| 0 1 0 0 0 0 0 0 0 |
| 0 0 1 1 0 0 0 1 0 |
| 0 0 0 1 0 0 0 0 0 |
| 0 0 0 0 1 0 0 0 0 |
| 0 1 0 0 0 1 1 0 0 |
| 0 0 0 0 0 0 1 0 0 |
| 0 0 0 0 0 0 0 1 0 |

:) Thanks

Zobayer Hasan March 5, 2013 at 12:26 AM

Yeah, its correct!

Reply

Anonymous March 30, 2013 at 11:52 AM

it realy helped me to get thrgh the concept so easily..thank u...

Reply

Ravi Shankar October 15, 2013 at 2:45 AM

Thanks Sir for this post.

Reply

Anonymous October 29, 2013 at 11:52 PM

দারুণ আইডিয়া...

Reply

Anonymous December 23, 2013 at 11:54 AM

please give some idea about the prerequisite

Reply

Swagnik Dutta January 14, 2014 at 5:30 PM

That was an awesome article. However i failed to understand a certain part in type 7. I was hoping if you could explain how to arrive at this equation g(n+1) = 2g(n) + 2g(n-1) + f(n+1). from the main equation :g(n) = 2g(n-1) + 2g(n-2) + f(n), where, f(n) = 2f(n-1) + 2f(n-2).

I could form the equations for all the other cases/types ,but I don't know how to work with equations with more than one recurrence. It would be very helpful if you could explain a bit briefly.

Reply

Replies

Zobayer Hasan January 14, 2014 at 10:20 PM

Hello there, about the recursion, isn't that putting n+1 instead of n ?

Reply

Swagnik Dutta January 15, 2014 at 1:29 AM

Sorry my previous question was rather silly. I don't understand why there are 4 rows in matrix A and B. I will try to explain what i have understood. Please rectify me if I'm wrong anywhere.

Given:
g(n) = 2g(n-1) + 2g(n-2) + f(n) --->(1)
f(n) = 2f(n-1) + 2f(n-2)---->(2)

replacing n by (n+1) in equation (1).
g(n+1) = 2g(n) + 2g(n-1) + f(n+1)
RHS of the equation is the present set of states. so they should be in matrix A.
hence matrix A should be:
| g(n) |
| g(n-1) |
| f(n+1) |
and matrix B should be containing the next set of states:
| g(n+1) |
| g(n) |
| f(n+2) |

All the types from 1 to 6 were solved this way,but here in case 7 the fourth row was introduced.
Can you tell me the reason behind that or why its necessary?

Reply

Replies

Swagnik Dutta January 17, 2014 at 11:46 PM

I was hoping if you could clear my doubt mentioned above. :)

Zobayer Hasan January 18, 2014 at 12:19 AM

Sorry for late response, got carried away by the chores of daily life.
Anyway, its nothing different from the other types. Look closely, to know f(n+1), you need to know both f(n) and f(n-2). So, if you only use three rows, how are you going to get the value of f(n-2)? So, you will need two g()s and two f()s in the matrix.
Hope it is clear now. The thing is, you need to have all the dependencies on the LHS matrix.

Zobayer Hasan January 18, 2014 at 12:20 AM

Oops! sorry about the type, its not f(n-2) its f(n-1). I think you got that as well,

Reply

TUSHAR PODDAR January 18, 2014 at 5:22 PM

What about f(x,y)=f(x,y-1)+f(x-1,y) ?
It can be done using DP, but if x and y are as large as 10^6 then one has to look around!
Matrix Expo possible?

Reply

TUSHAR PODDAR January 18, 2014 at 5:23 PM

What about f(x,y)=f(x,y-1)+f(x-1,y) ?
It can be done using DP, but if x and y are as large as 10^6 then one has to look around!
Matrix Expo possible?

Reply

Replies

Zobayer Hasan January 24, 2014 at 3:37 AM

No, I don't think it's possible, or at least not in my knowledge.

Reply

TUSHAR PODDAR January 23, 2014 at 2:40 PM

Any hope with Matrix Expo? Another property of the function f(x,y) was f(x,y)=f(y,x)

Reply

Replies

Zobayer Hasan January 24, 2014 at 3:36 AM

I don't think that can be done using matrix exponentiation. If you can express one variable in terms of the other one, then some thoughts can be made, but I have no clue how to handle function on more than one variable recurrences using matrix exponentiation.

Reply

Anchit Jain April 2, 2014 at 8:45 PM

Please explain Type 7 . It's given that g(n) = 2g(n-1) + 2g(n-2) + f(n) and f(n) = 2f(n-1) + 2f(n-2).
So g(n+1)=2g(n)+2g(n-1)+f(n+1) and f(n+1)=2f(n)+2f(n-1)
so shouldn't matrix A be [g(n) ,g(n-1) , f(n) , f(n-1)] .
Please explain how did you got | g(n) |
| g(n-1) |
| f(n+1) |
| f(n) |
Someone also has asked this doubt but I could not understand your explanation for that.Please explain asap.

Reply

$ATYA October 14, 2014 at 11:11 PM

Nice Article!

Can u please tell how to make approach for 11651?

Thanks

Reply

Shivam October 28, 2014 at 2:39 AM

Nice article...can u please share ur email address!!

Reply

Mahesh Dhoni November 10, 2014 at 9:46 AM

zobayer why are we considering f(n)&f(n-1)in matrix exponentation after all we want to find f(n+1)

Reply

Replies

Zobayer Hasan November 10, 2014 at 11:33 AM

do you mean why we are keeping them on right hand side? they are generated as partial result, but we are not worried about them, as long as we find f(n+1).

but on the left side matrix, we need them both because f(n+1) depends on both.

Enter your reply...

Reply as: Nitai Chandra E ▼ Sign out

Publish Preview Notify me

Anonymous January 25, 2015 at 12:27 PM

T(n)= max (a+ T(n-b), c+ T(n-d)).
how can we solve this with matrix exponentiation ?

Reply

Replies

Zobayer Hasan April 23, 2015 at 10:14 AM

Seems more like dynamic programming to me. I have never tried anything of this sort, lets see if someone else can help you.

Reply

Ishu Garg June 13, 2015 at 10:31 PM