Default propagation on Financial Networks

Group 11
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Motivation

- **Applying network to Financial systematic risk**

  Research in financial contagion analysis using social contagion network model (The Eisenberg–Noe model) is completely deterministic, triggered only by an actual insolvency. External assets should also be introduced in Network models to account for the uncertainty on value. [1]

- **Reproduction of cascading effects by studying SOC**

  In Drupsteens slow-driven Network ABM of the IRS market, agents have stochastic exposure and attempt to hedge risk by creating links with other agents. Financial market behaviours such as power law in default cascade sizes are frequently attributed to SOC [2]. Can we see signs of self organized criticality in Drupsteens model?

- **Extension to drupsteens model: Considering volatility of external assets and preferential attachment**

  In reality, external volatility of assets will also influence the behaviour of default cascade sizes. So we add exposure to an external price time series (interest rate). This allows heterogeneous effects of an assets price swing different balance sheets as is found in reality. And instead of linking randomly with other agents with opposite exposure, agents prefer to link with “optimal” partner to swap most exposure, so we add preferential attachment mechanism to our model.

Research Question

How do defaults propagate across a financial network?

The Model: Financial Network

1. Nodes exposure is a brownian motion
2. Nodes can form links (contracts) to reduce exposure
3. Nodes can become bankrupt (default) when exposure exceeds threshold
4. Bankrupt nodes are removed and transfer their exposure to linked nodes
5. New node is added to the system for every bankrupt node

Comparison to Drupsteens Model

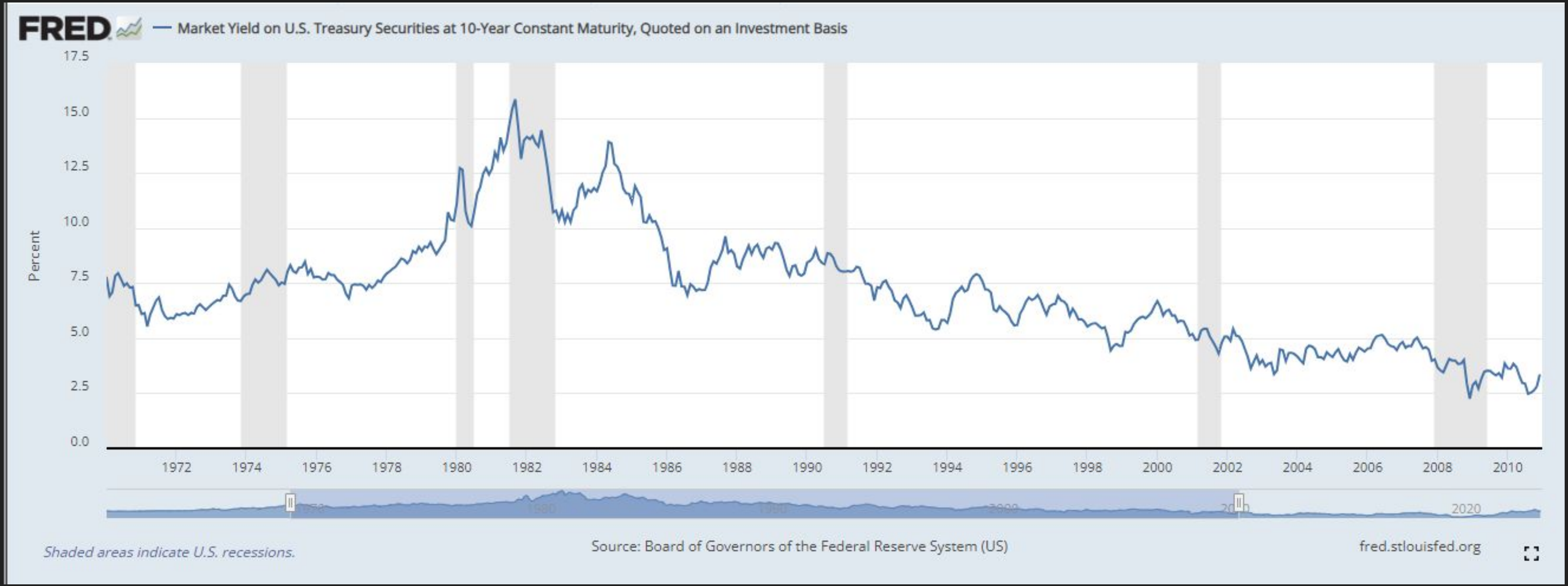
Drupsteen model:

- **Linking:** Random - node links randomly with any other node with opposite sign exposure
- **Bankruptcy:** Internal exposure - only consider node's own exposure changing over time

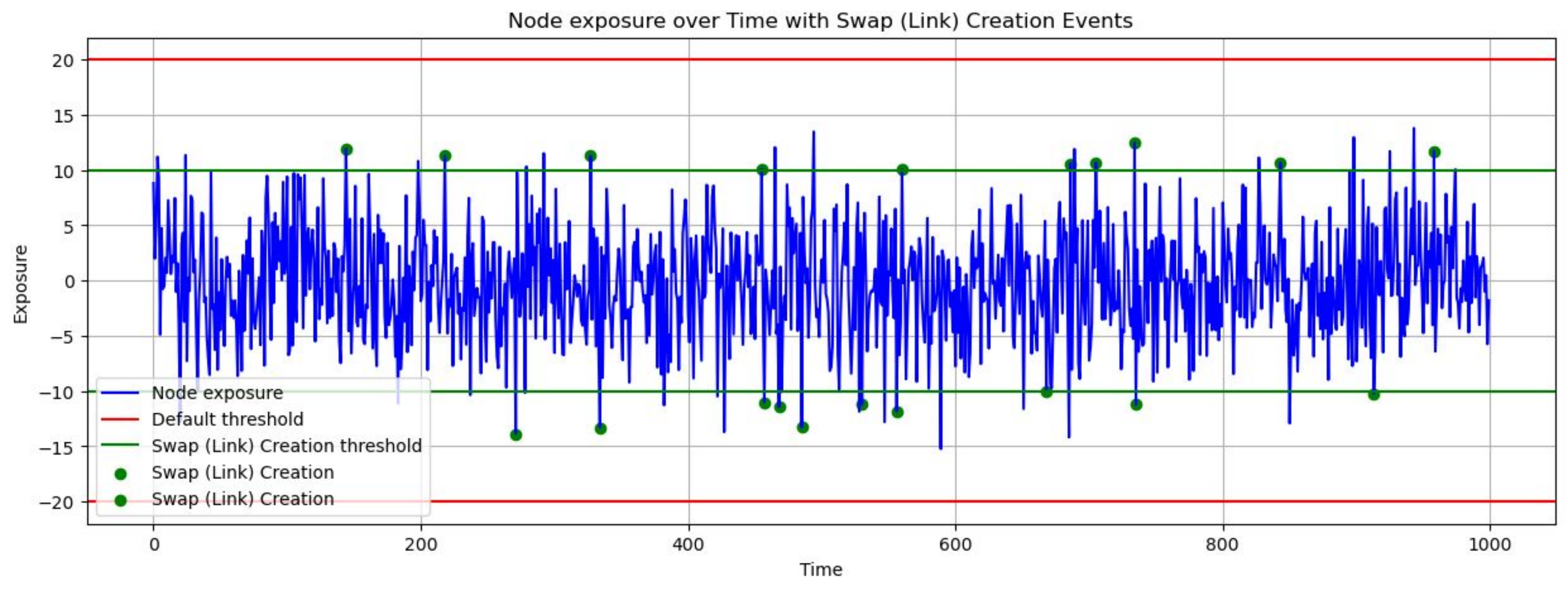
Our model:

- **Linking:** Preferential attachment - node traverses all other opposite sign exposure nodes, for every pair calculate sum of exposure, choose the smallest sum pair as optimal linking node
- **Bankruptcy**: Consider both internal exposure and external price (interest rate) changing over time

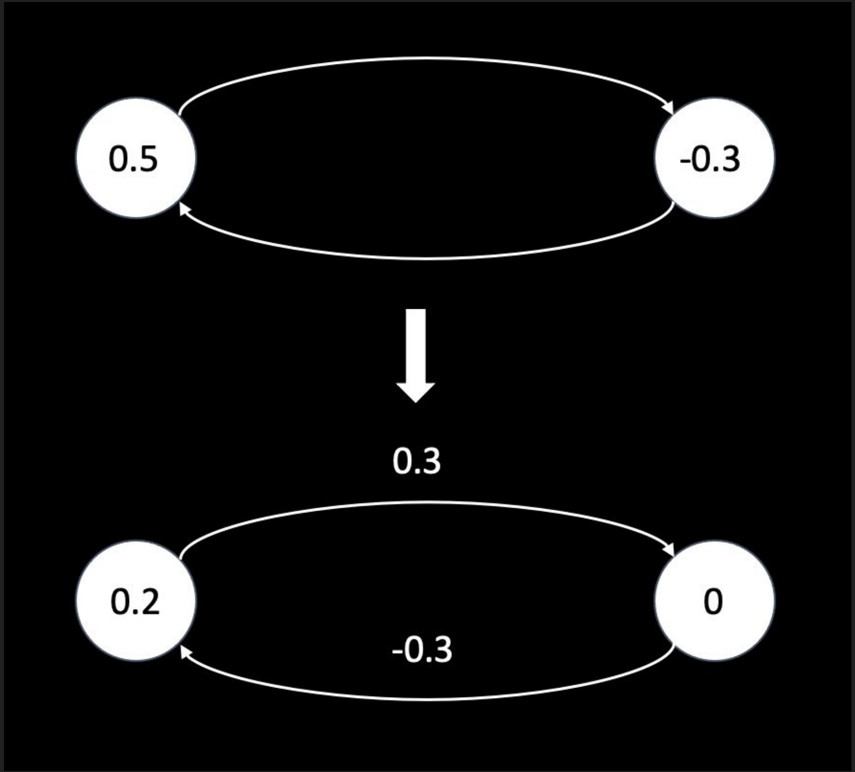
Model Behavior: Interest Rate Change



Model Behavior: Node Exposure (slow drive)



Model Behavior: Form Link (store exposure)

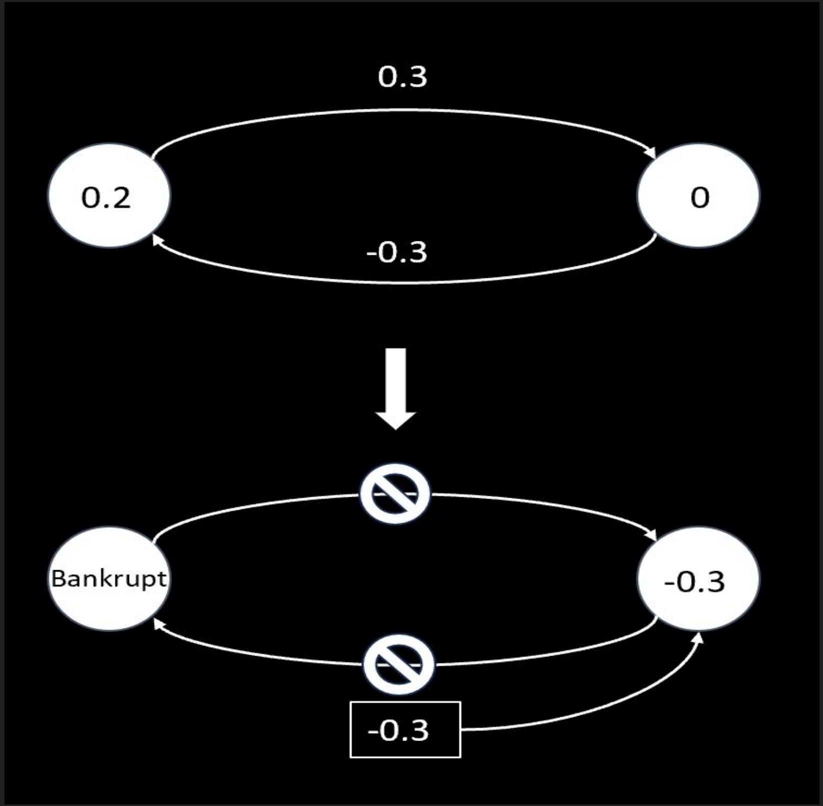


Pseudocode

For each node in Network
    Find closest_node with exposure sum closest to 0/ find any other opposite sign node

If closest_node exist
    Choose the smaller absolute exposure of 2 nodes as abs value of link weight
    Determine link weights based on node's exposure sign
    Update 2 node’s exposures based on link weights

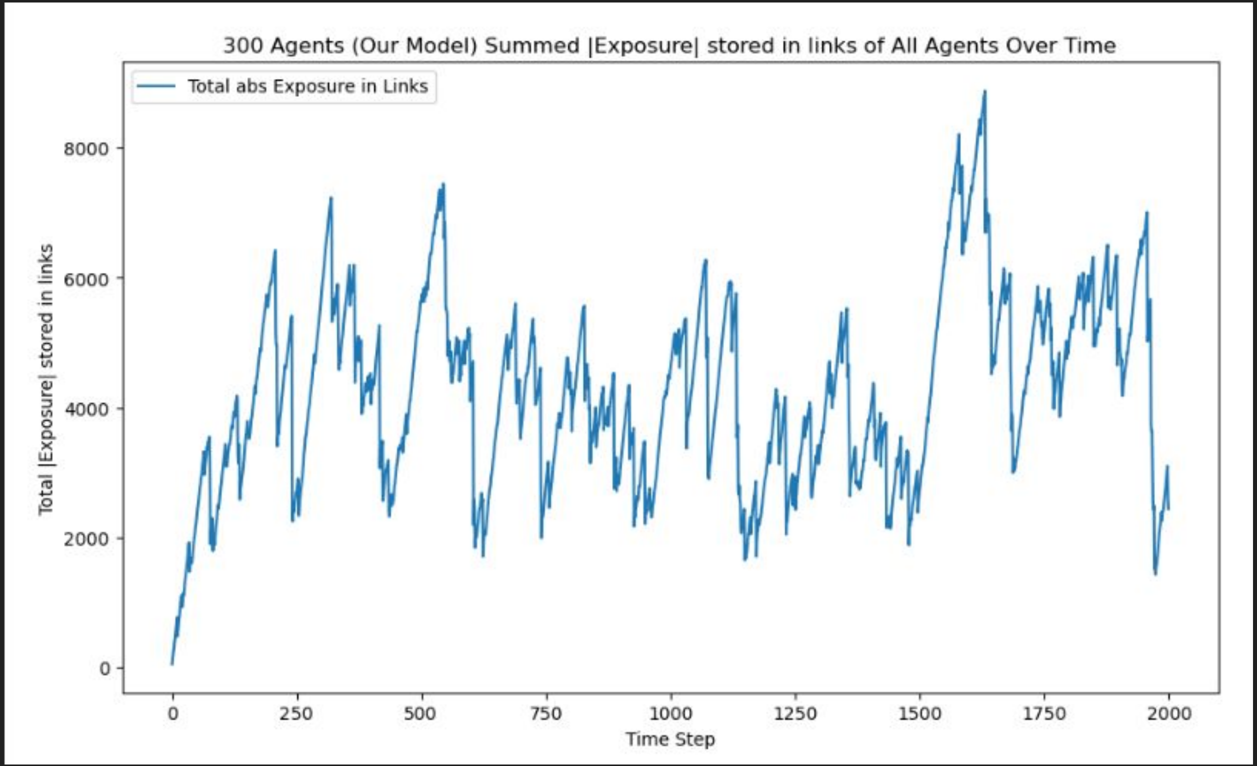
Model Behavior: Nodes Bankruptcy (Release/Avalanche)



Pseudocode

If abs. node’s exposure > threshold
 node in bankrupt_nodes
    for neighbors not in bankrupt_nodes
        Adjust neighbor's exposure by their link weight to self

Model Behavior: Self organization towards critical point?



Self organized criticality?

1. Nodes add exposure each timestep (slow drive)
2. Nodes can form links (store stress)
3. Nodes can become bankrupt (release)
4. Bankrupt node can cause other to default (avalanche)

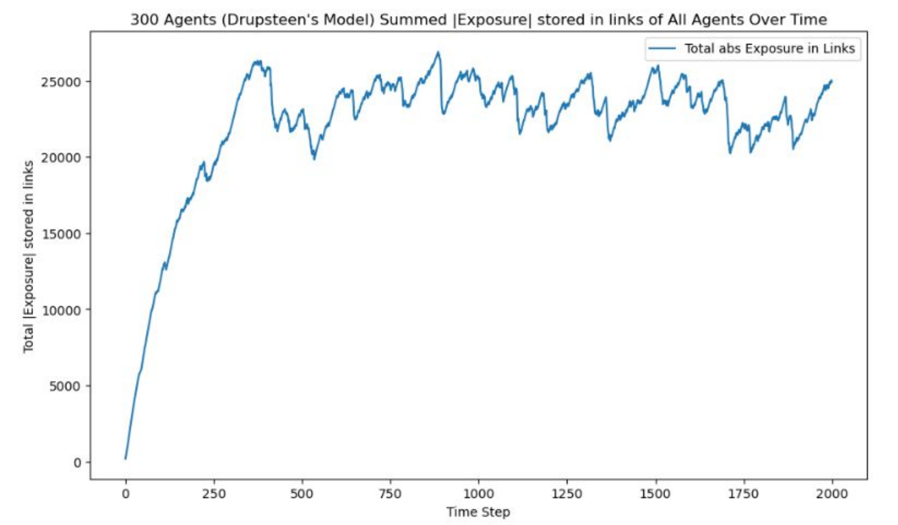
Hypotheses

- Hypothesis 1a: We expect powerlaw distribution in the PDF of absolute number of agents dying per timestep.

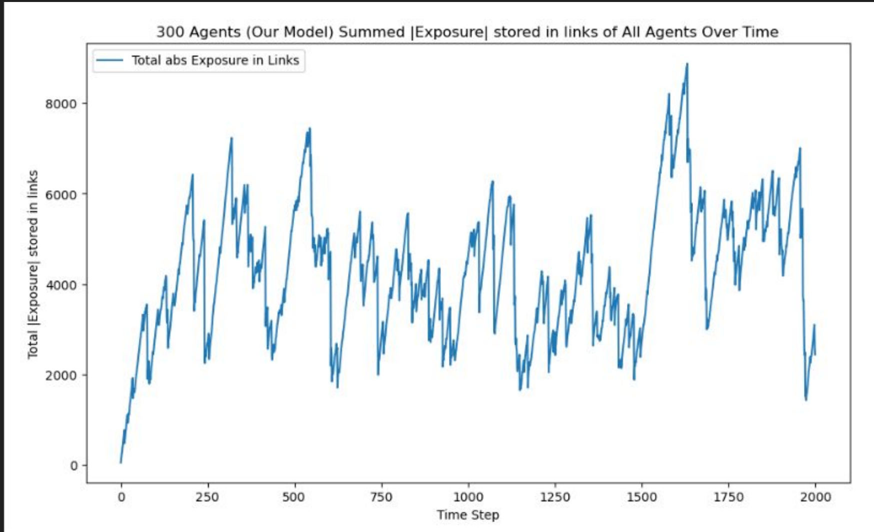
- Hypothesis 2a: We expect average default size to increase with system size.

- Hypothesis 3a: We expect increased volatility in stock prices leads to a higher probability of an agent dying per timestep when the network has more exposure stored in links.

Comparing Model behavior: Drup steen vs. our model



Parameters: runs=10, time_steps=2000, delta_t=1, sigma_exposure_node=1, sigma_intrestrate=1, linking_threshold=1, threshold_v=5, time_to_maturity=400, link_threshold_mode=hard cutoff, link_formation_mode=_random link logic, bankruptcy_mode=link logic_exposure



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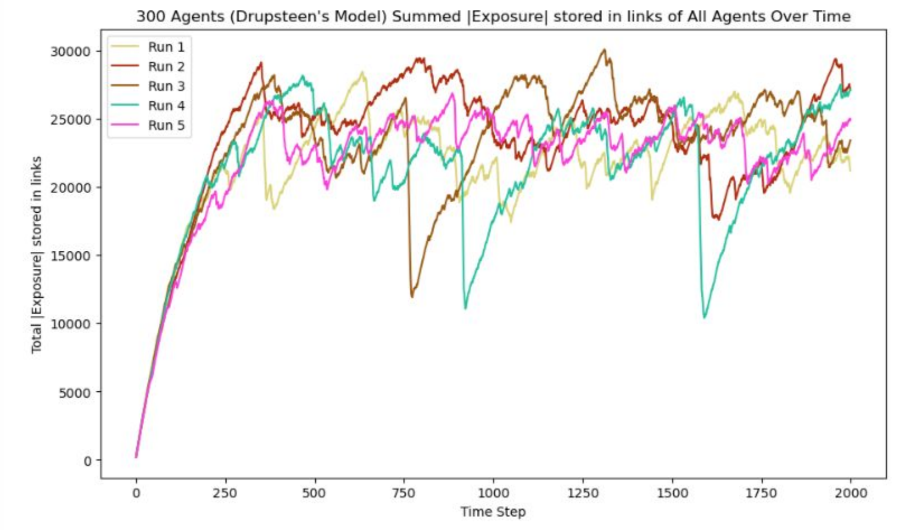

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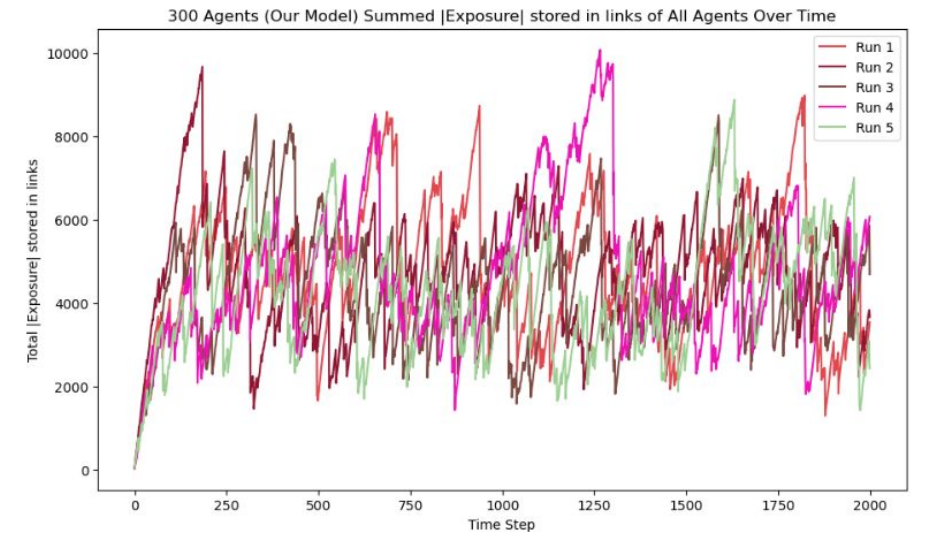

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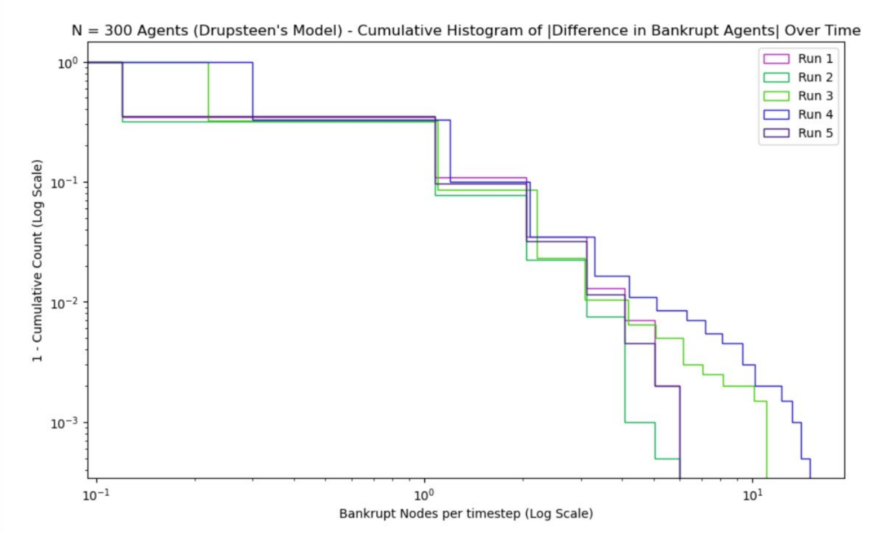

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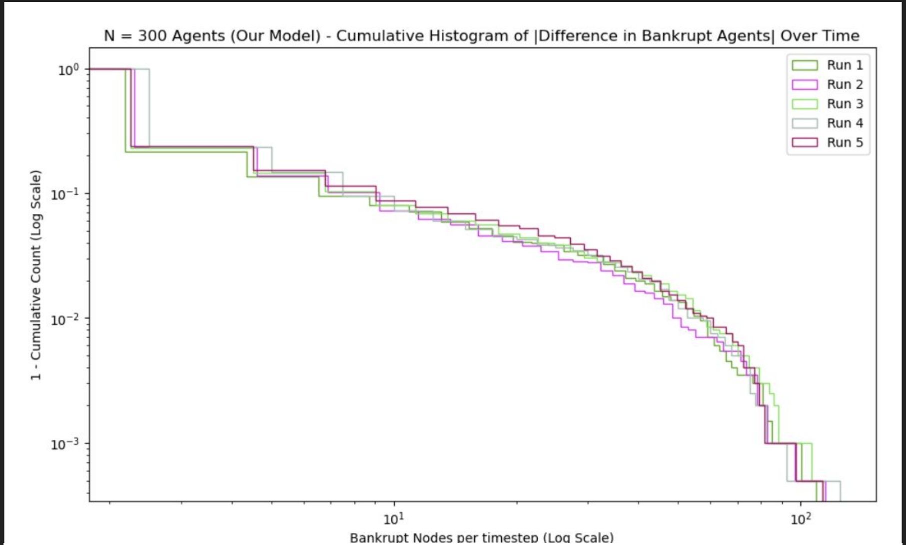

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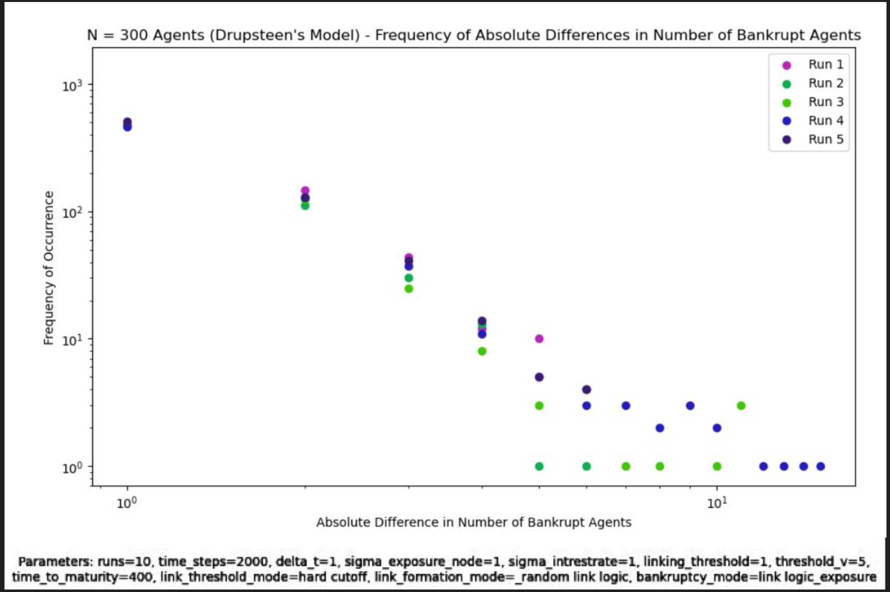
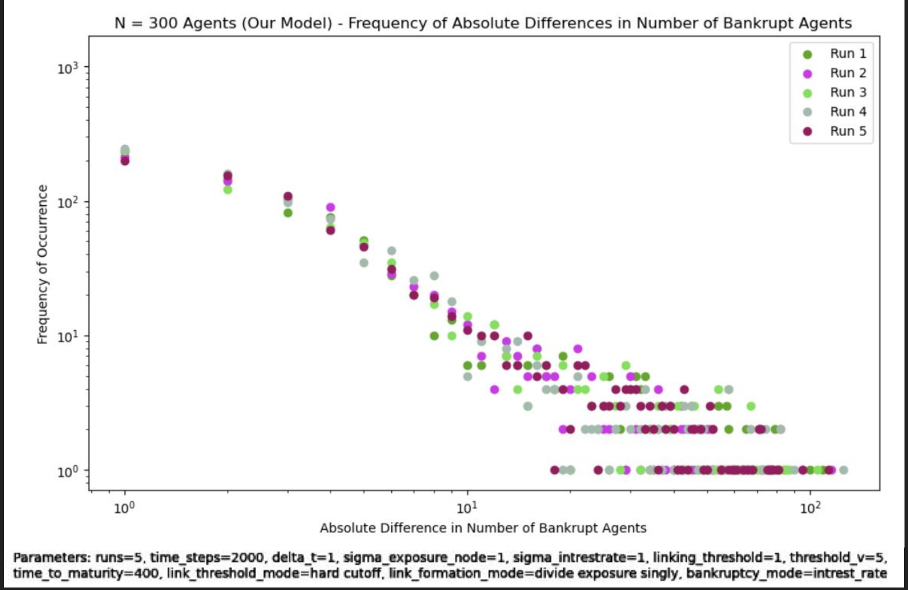
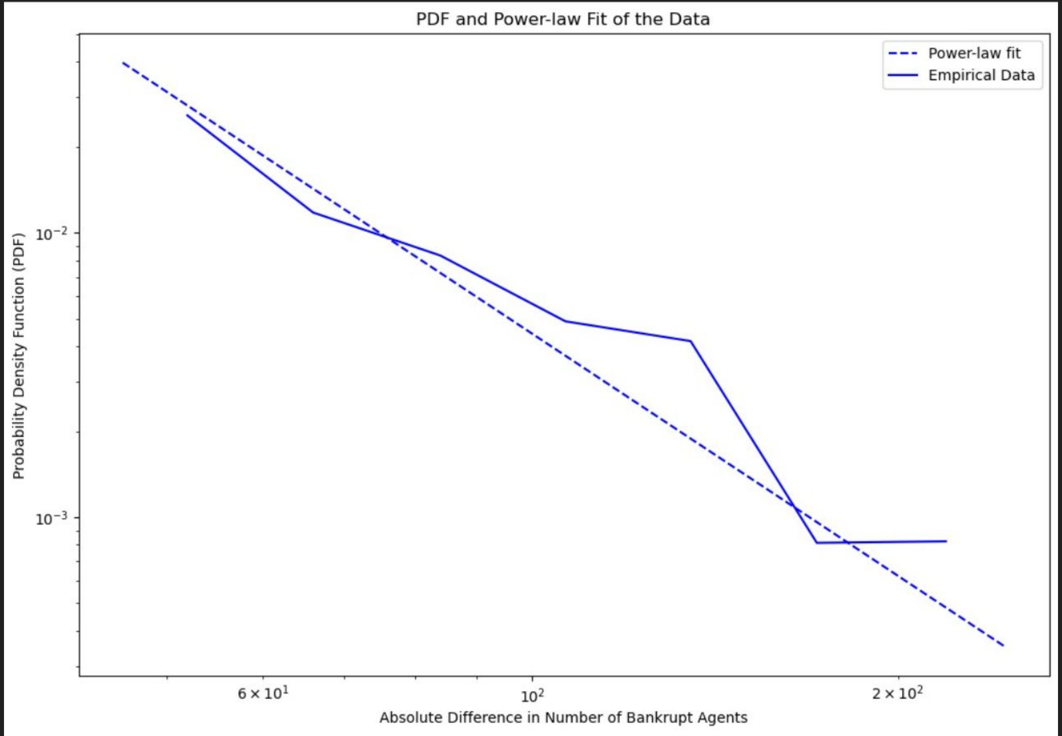
Comparing Model behavior: Drup steen vs. our model

PDF





Powerlaw fitting



Results: H1a

H1a: PDF of defaults follows a power law distribution

H0a: states that the PDF follows an exponential distribution

Drupsteens model:

Estimated Powerlaw Alpha exponents: ranging from 7-55 (!!!)

<u>For system sizes N = 100, 200 and 300,</u>

<u>H0a is rejected in favor of H1a rejected alpha = 0.05,</u>

<u>given this sample HOWEVER powerlaw exponents look spurrious</u>

	N agents (Nodes)	run	alpha exponent	likelihood ratio	p-value
0	100 Agents (Drupsteen's Model)	1	9.135399	13.067375	0.0***
1	100 Agents (Drupsteen's Model)	2	10.928717	2.703785	0.0069**
2	100 Agents (Drupsteen's Model)	3	12.691453	4.436069	0.0***
3	100 Agents (Drupsteen's Model)	4	32.284535	3.826657	0.0001***
4	100 Agents (Drupsteen's Model)	5	55.258676	15.012201	0.0***
5	Stouffer's p-value				0.0***
6	200 Agents (Drupsteen's Model)	1	6.509864	14.617672	0.0***
7	200 Agents (Drupsteen's Model)	2	11.684209	4.424535	0.0***
8	200 Agents (Drupsteen's Model)	3	8.665191	1.506681	0.1319
9	200 Agents (Drupsteen's Model)	4	13.377589	4.573385	0.0***
10	200 Agents (Drupsteen's Model)	5	5.202756	2.916121	0.0035**
11	Stouffer's p-value				0.0***
12	300 Agents (Drupsteen's Model)	1	7.176636	3.661485	0.0003***
13	300 Agents (Drupsteen's Model)	2	24.862223	3.815034	0.0001***
14	300 Agents (Drupsteen's Model)	3	4.398227	3.521408	0.0004***
15	300 Agents (Drupsteen's Model)	4	3.915704	4.201539	0.0***
16	300 Agents (Drupsteen's Model)	5	7.841798	3.875167	0.0001***
17	Stouffer's p-value				0.0***

Results: H1a

H1a: PDF of defaults follows a power law distribution

H0a: states that the PDF follows an exponential distribution

Our model:

Estimated Power law Alpha exponents: mostly between 2 and 3

For system sizes N = 100, 200 and 300, H0a is rejected in favor of H1a at the alpha = 0.05 level, given this sample.

	N agents (Nodes)	run	alpha exponent	likelihood ratio	p-value
0	100 Agents	0	2.582411	2.253132	0.0243*
1	100 Agents	1	2.495865	4.384189	0.0***
2	100 Agents	2	4.170634	-0.14068	0.8881
3	100 Agents	3	2.824595	-0.817323	0.4137
4	100 Agents	4	3.504318	0.322458	0.7471
5		Stouffer's p-value			0.0212*
6	200 Agents	0	2.229182	7.322882	0.0***
7	200 Agents	1	2.215366	5.920257	0.0***
8	200 Agents	2	2.239663	5.87563	0.0***
9	200 Agents	3	2.25435	5.218964	0.0***
10	200 Agents	4	2.230617	5.521491	0.0***
11		Stouffer's p-value			0.0***
12	300 Agents	0	2.012058	7.440271	0.0***
13	300 Agents	1	2.111148	8.193237	0.0***
14	300 Agents	2	2.025498	8.218183	0.0***
15	300 Agents	3	2.080259	6.048644	0.0***
16	300 Agents	4	1.933861	9.986647	0.0***
17		Stouffer's p-value			0.0***

Results: H2a

Hypothesis 2a - We expect average default size to increase with system size

- Linear regression: Our model

  Coefficients

  Intercept : ß_0 = 0.9897, p-value = 0.0069

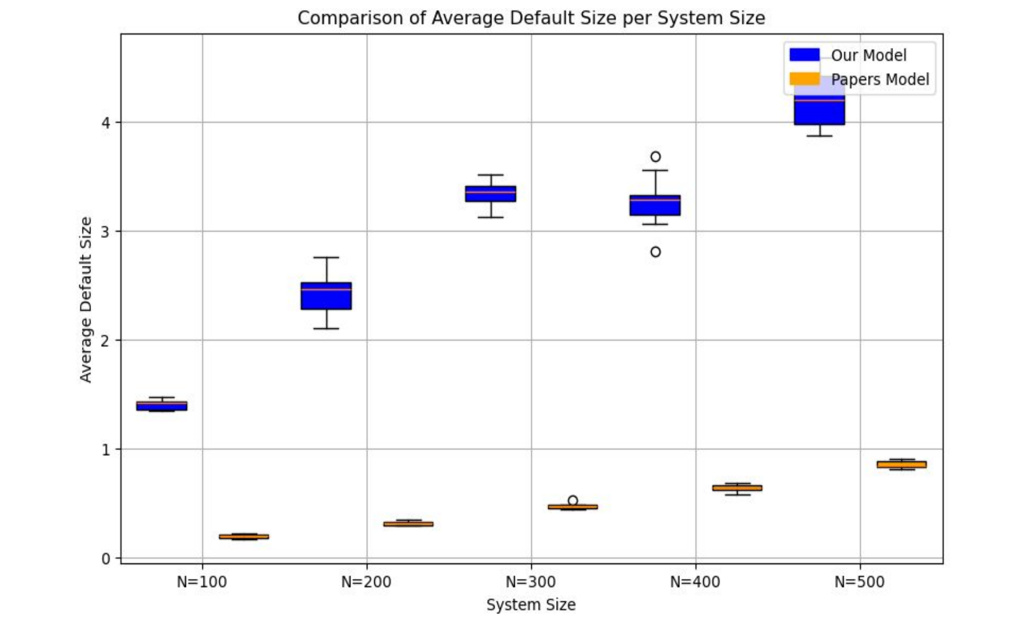
  System size: ß_1 = 0.0065, p-value = 0.0016

- Linear regression: Drupsteens model

  Coefficients

  System size: ß_1 = 0.0016 p-value = 9.74e-05

<u>Null hypothesis of ß_1 = 0 rejected at alpha = 0.05 level, thus H2a is supported, given this sample. But interpret with caution as coefficients are tiny</u>

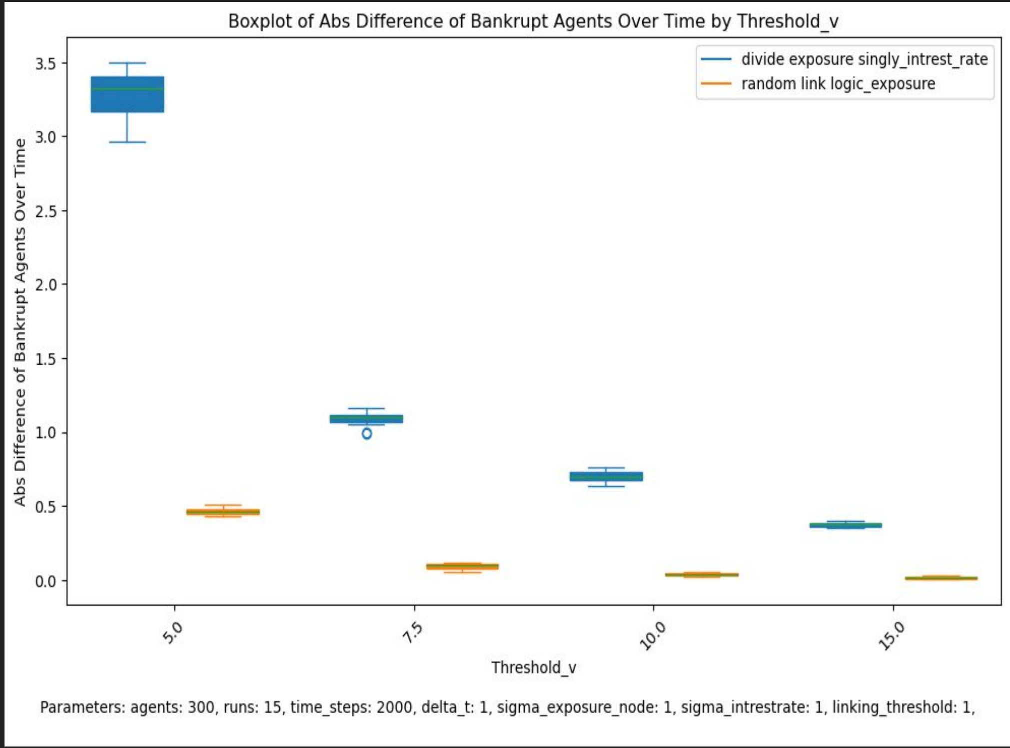


Parameters: runs: 10, time_steps: 2000, delta_t: 1, sigma_exposure_node: 1, sigma_intrestrate: 1, linking_threshold: 1, threshold_v: 5, time_to_maturity: 400, link_threshold_mode: hard cutoff

Results: Sensitivity analysis

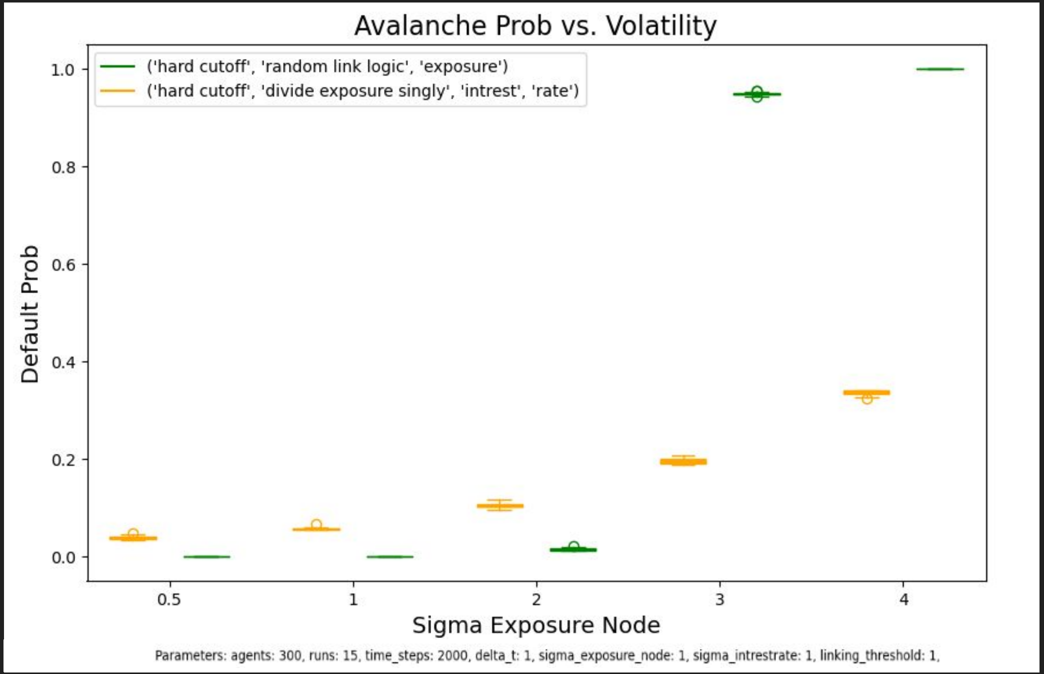
When the value of the threshold of bankrupt increases:

- The number of bankrupt nodes per timestep will decrease.
- The distribution of the number of bankrupt agencies per timestep will have a tendency to get narrow.
- For every threshold, our mode (blue) has a larger number of bankrupt agencies per timestep than the paper mode (orange).



Results: Sensitivity analysis

- We treat bankruptcies exceeding 5% of Node population as a cascade default (avalanche)
- Default probability for our model slowly increasing with node exposure standard deviation
- Default probability for Drupsteens model slowly, then suddenly increasing and exceeding our models default probability as node exposure standard deviation increases



Results H3a

H3a: volatility in stock prices leads to a higher probability of an agent dying per timestep when the network has more exposure stored in links.

- We treat bankruptcies exceeding 5% of Node population as a cascade default (avalanche)
- Linear regression:

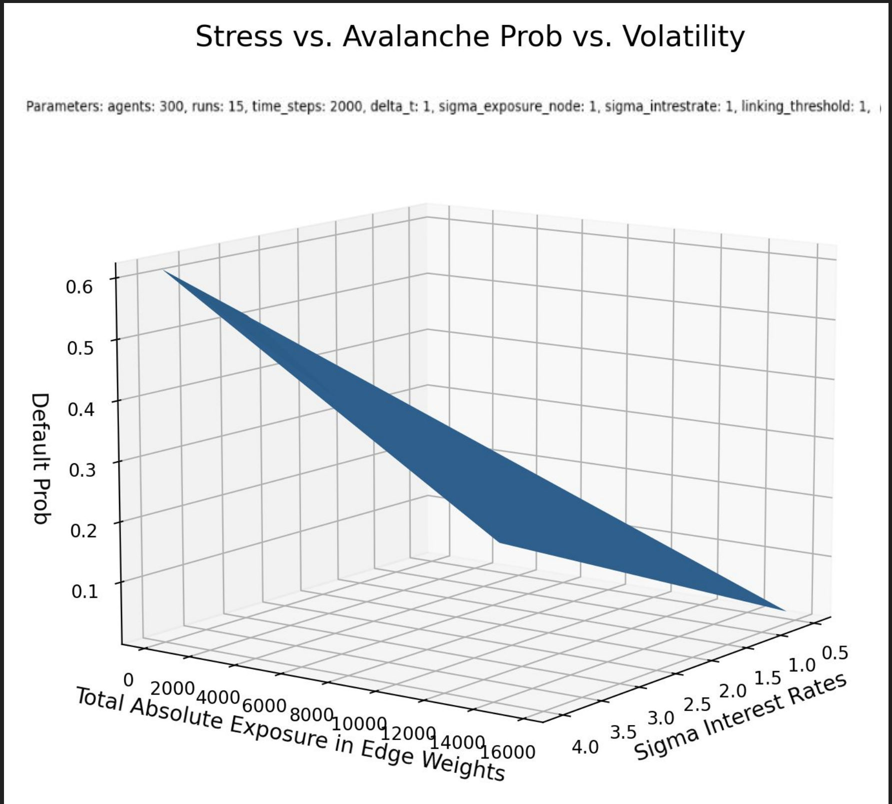
  Coefficients

  Sigma interest rate: ß_1 = 0.1141, p - value = 0.092

  Total link exposure: ß_2 = 1.8e-05, p - value = 0.209

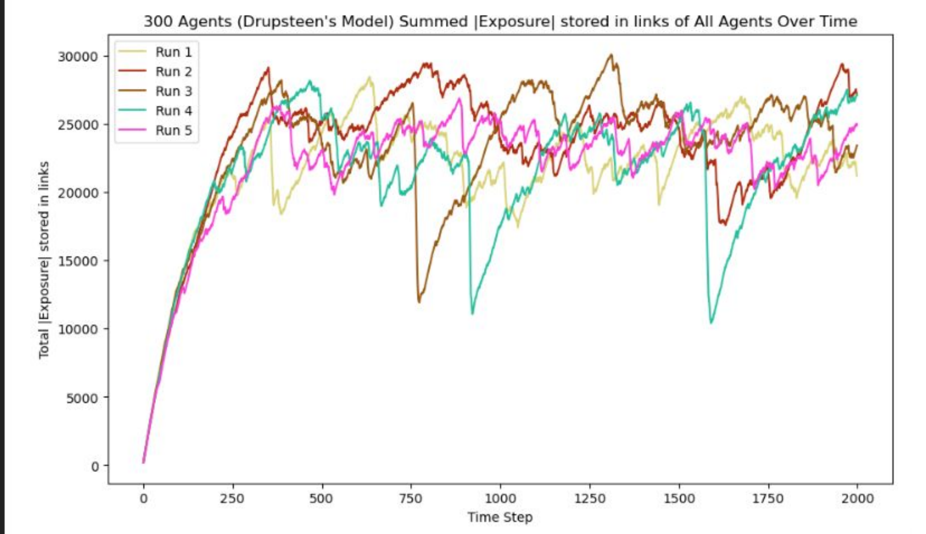
  Interaction Sigma interest rate * Total link exposure: ß_interaction = 0.159, p - value = 0.159

<u>Null hypothesis of ß_interaction = 0 not rejected at alpha = 0.05 level, given this sample: H3a is not supported</u>



Time series analysis

Can we time dependent relationship among the chaotic pattern?



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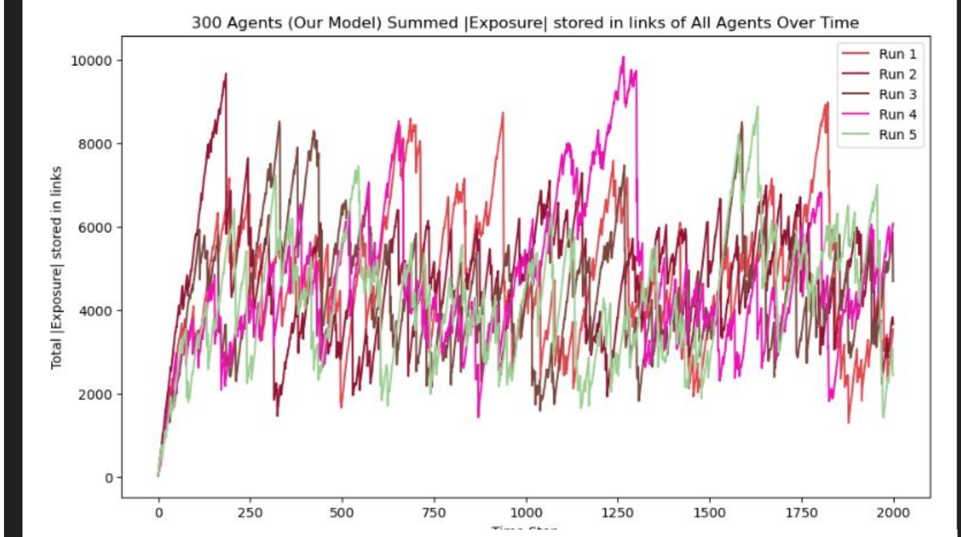

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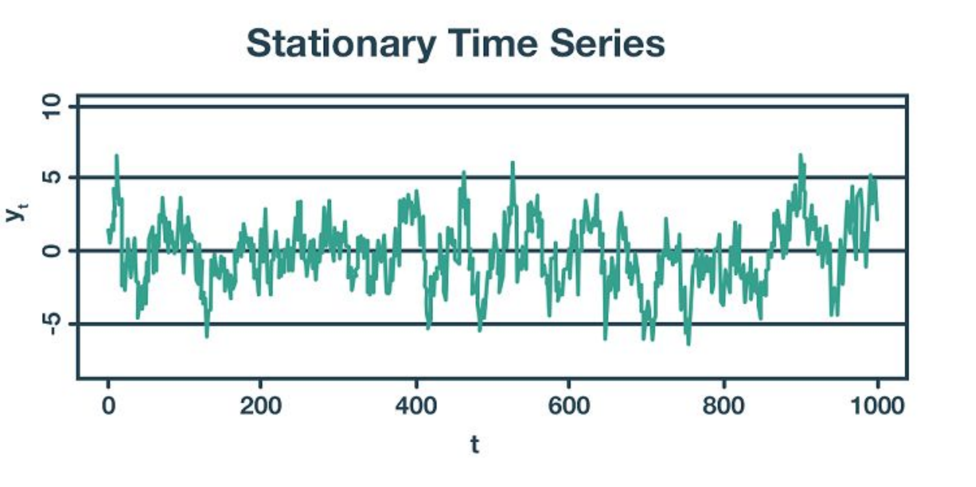
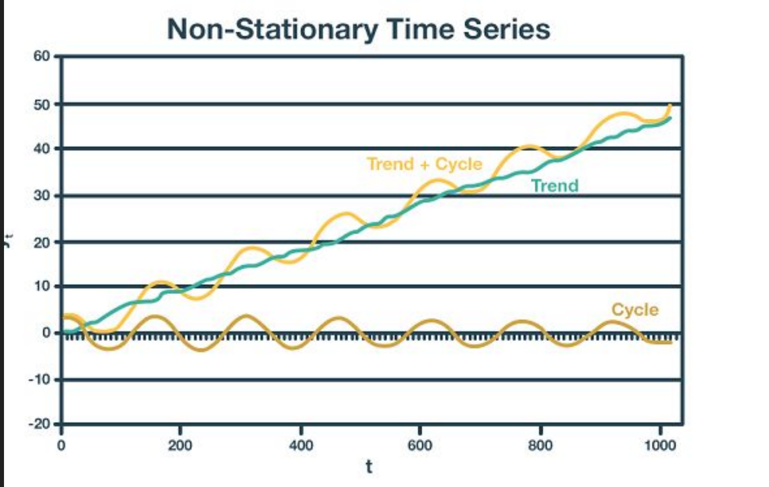
Time series analysis

Stationarity: Mean and Variance do not change over time

Example:

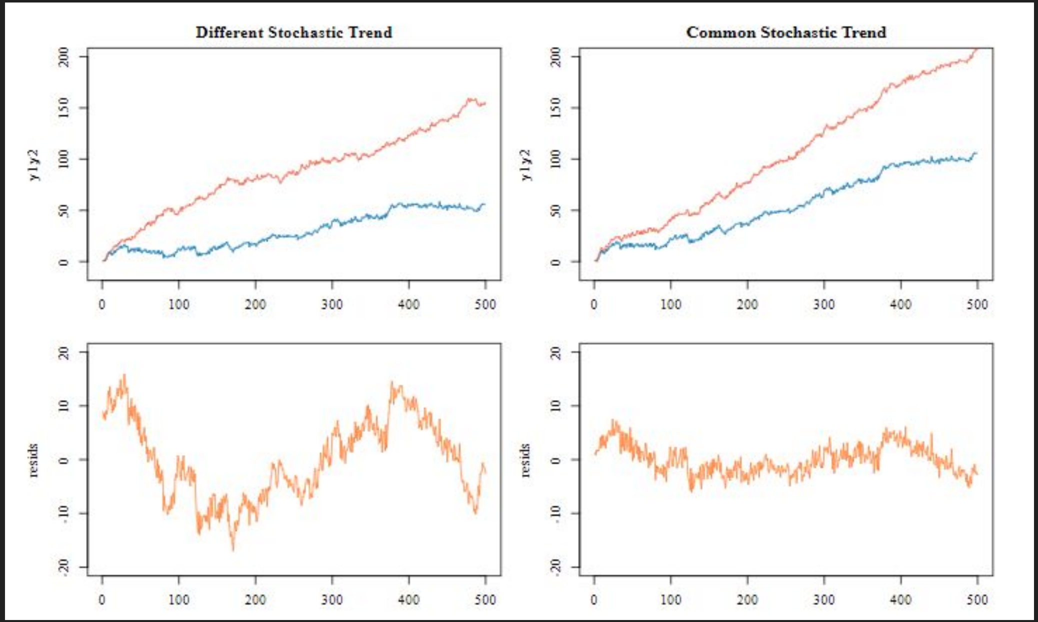




Time series analysis

Cointegration is when a linear combination of time series integrated of order 1 is stationary, meaning have the time series have a long run equilibrium relationship

Example:



Time series analysis

- ADF test showed both Bankruptcy per timestep and Total node exposure time series stationary, thus we fit a VAR model

VAR model:

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} e_{1,t} \\ e_{2,t} \end{bmatrix}$$

Time series analysis

VAR results (test on our model):

Run	Coefficient of y_2 on y_1	p-value (y_2 on y_1)	Coefficient of y_1 on y_2	p-value (y_1 on y_2)
1	0.000069	0.615	1.521808*	0.025*
2	0.000251	0.097	1.703884*	0.016*
3	0.000167	0.299	1.509921*	0.023*
4	-0.000127	0.329	2.528099*	0.000*
5	-0.004640* (L1), 0.004553* (L2)	0.040* (L1), 0.041* (L2)	0.801291 (L1), -0.370538 (L2)	0.616 (L1), 0.559 (L2)

Table 1: Comparison of Coefficients and Their Significance Across Runs, with Significant Lags Marked

Where y1 is |bankruptcies| per timestep, and y2 is Total exposure stored in the links

Conclusion: Bankruptcy is significantly positively influences Total exposure stored in links in 4/5 runs, vice versa in the last run.

Comparing to Real world data

- Real Data: numbers of bankruptcies in 1970-2010 from IMF [3]
- Fitting our & Drupsteems model using standard deviation of real treasury 10 year interest rate of 3.56990 from FED [4]

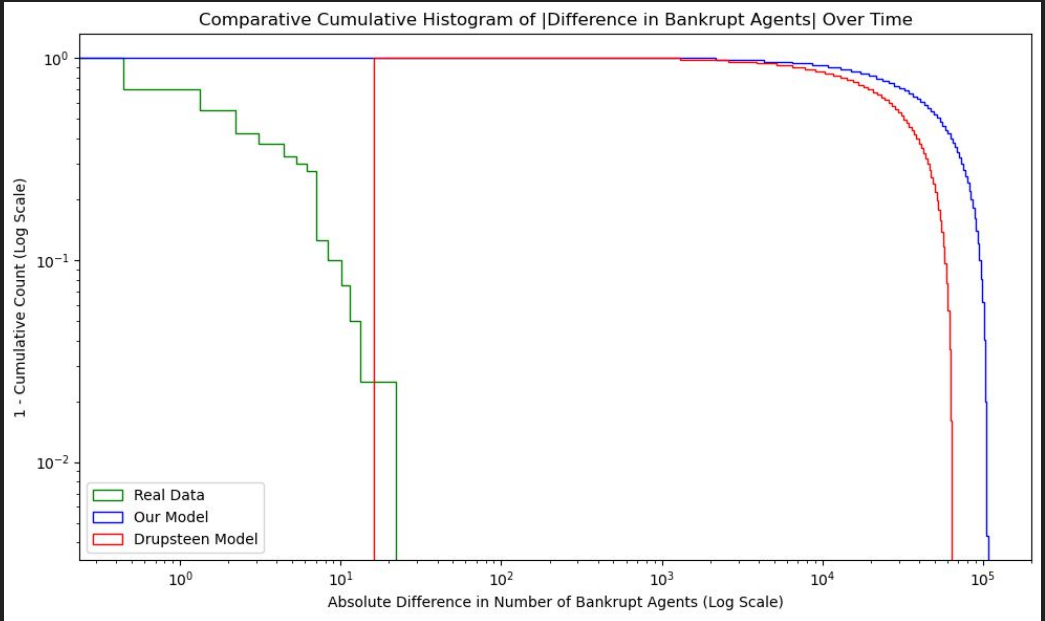
As expected from the plot, Kolmogrov Smirnov test

for similarity of distributions rejects H0:

Sample come from same distribution at alpha = 0.05 given this sample

Our model vs. real data: p - value = 1.6510558917300213e-144

Drupsteen model vs. real data: p - value = 1.815355151065231e-132



Discussion

- In our implementation, modeling exposure to an external asset and using preferential linking produces power law bankruptcy PDF
- We were unable to reproduce power law exponents with Drupsteens model
- Real world data not fit by either model
- Limited accuracy of statistical testing for H1, H2 and H3, only 5 samples due to runtime constraints
- Limited accuracy of transactional modeling by linking mechanism (max 2 links between nodes)
- Exposure to a single asset instead of multiple
- Exposure to external assets and preferential exposure linking are recommended topic for future research in Financial Network models

References

- [1] Caccioli, F., Barucca, P. & Kobayashi, T. Network models of financial systemic risk: a review. J Comput Soc Sc 1, 81–114 (2018). https://doi.org/10.1007/s42001-017-0008-3
- [2] Drupsteen D. (2019) *Self-organized criticality and synchronisation in an interest rate swap market model* [Unpublished MSc Thesis]. University of Amsterdam
- [3] Laeven, L., & Valencia, F. (2013). Systemic banking crises database. IMF Economic Review, 61(2), 225-270.
- [4] Federal Reserve Bank of St. Louis, 10-Year Treasury Constant Maturity Minus 2-Year Treasury Constant Maturity [T10Y2Y], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/T10Y2Y, January 31, 2024.