KYBER

By Pakin Panawattanakul

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PRESENTATION OUTLINE

1. Introduction
   a) Objective
   b) What is FPGA
2. Kyber PKE
3. Kyber KEM

INTRODUCTION

OBJECTIVE

Our group previous presentation

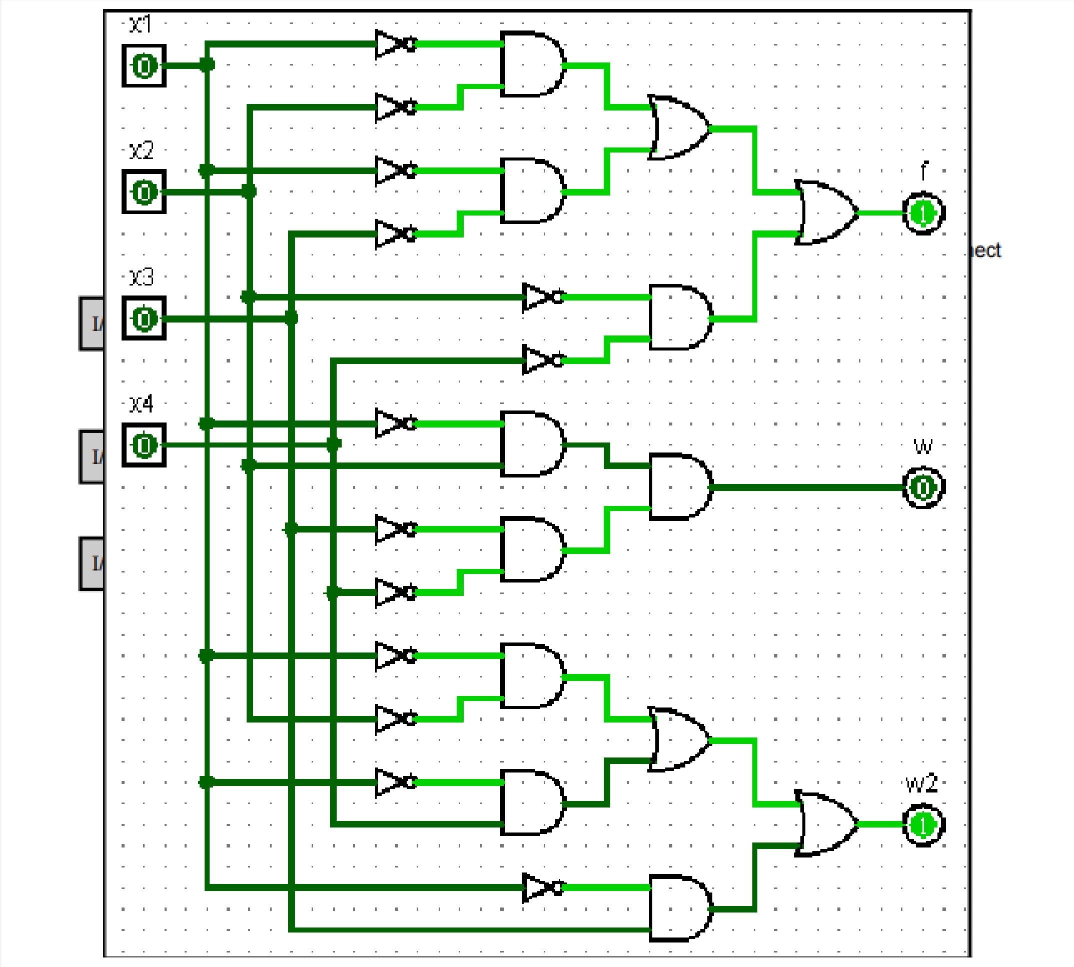
- Post quantum cryptography : variations, math problem, performance → Lattice based
- Frodo KEM : Lattice based algorithm that haven't been implement → It is suck !

Today Objective

- AJ. Vasin : Find Lattice based cryptography implementation methods on FPGA
- Understanding basic idea of Kyber-PKE & Kyber-KEM

FIELD PROGRAMMABLE GATES ARRAY

- No instruction set like traditional CPU
- Reconfigurable Logic Block
- Using Hardware description language
  - E.g. Verilog and VHDL
- Parallel execution
- Flexibility and Customizable



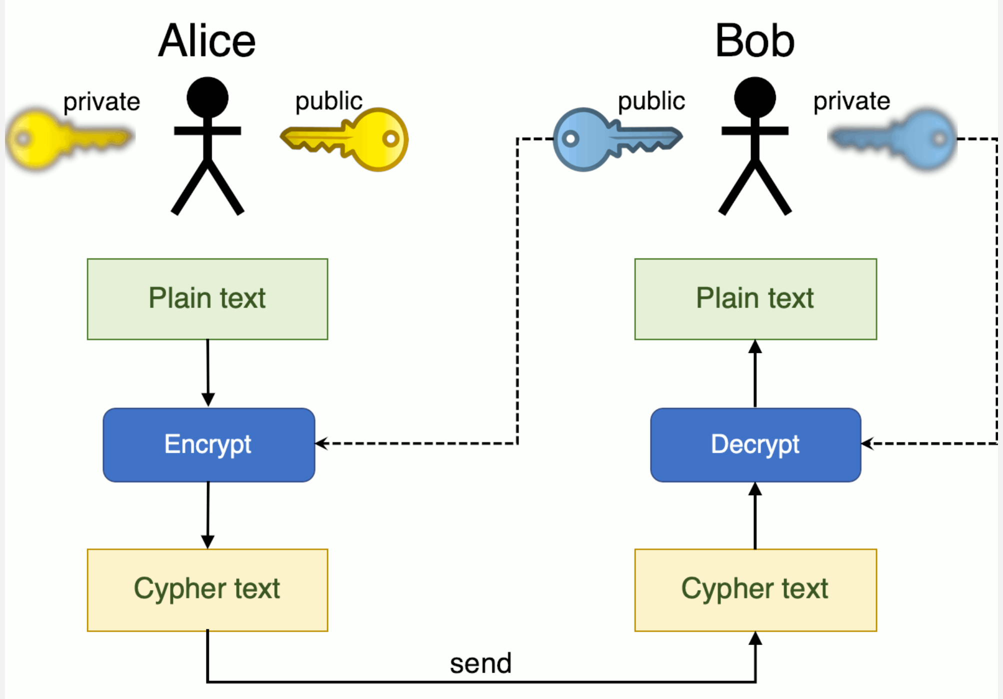
https://eee.poriyaan.in/topic/fpga--field-programmable-gate-arrays--11689/
https://www.researchgate.net/figure/Digital-circuit-with-two-inputs-OR-and-AND-gates_fig11_309907692

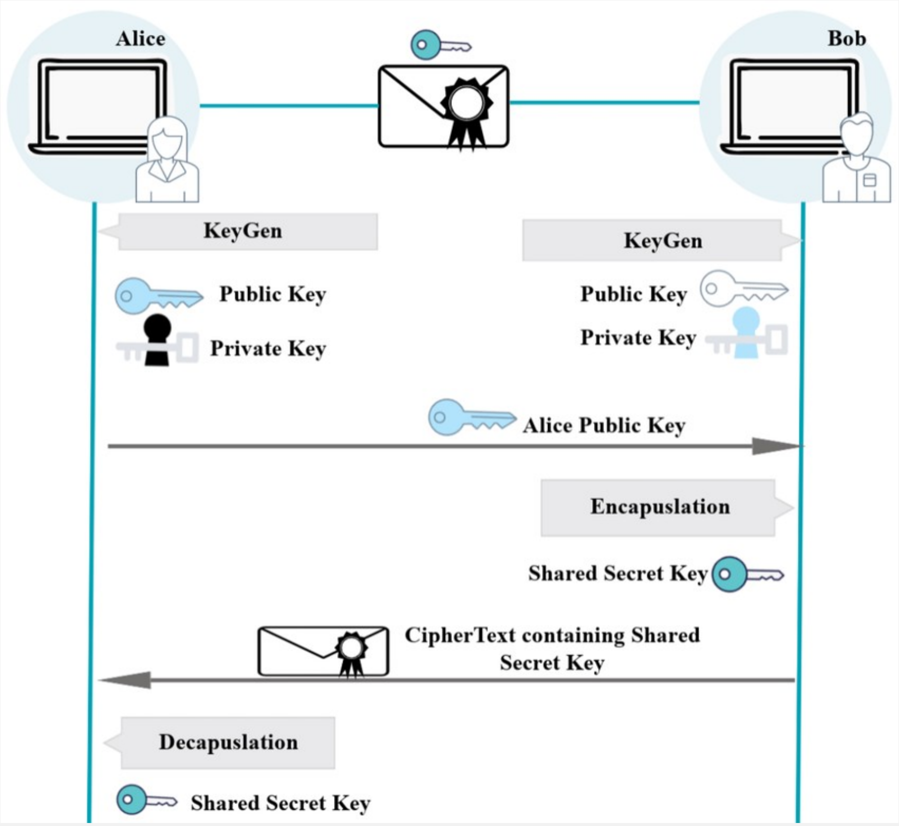
KYBER-PKE & KYBER-KEM

- Base on Module Learning with Error

  (Tony's presentation)
- Kyber-KEM chosen to be standard Key encapsulation algorithm by NIST

NIST (National Institute of Standards and Technology) is a U.S. agency that develops and promotes standards for technology, including cybersecurity and cryptography.





- Public key encryption(PKE)
- Encrypted message

- Key Encapsulation Mechanism (KEM)
- Established shared secret key (symmetric key)

https://www.coengoedegebure.com/surviving-an-infosec-job-interview-cryptography/

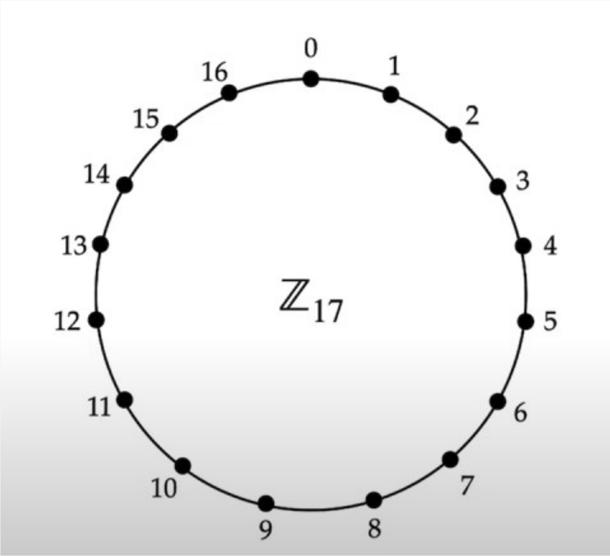
https://www.researchgate.net/publication/371388643_Resilience_Optimization_of_Post-Quantum_Cryptography_Key_Encapsulation_Algorithms

IMPORTANT MATH NOTATION

- Integer modulo $\mathbb{Z}_q$
- Polynomial ring : $R_q = \mathbb{Z}_q[x]/(x^n + 1)$
- Polynomial vector R_q^k
- “ Small” Polynomial : S_η
- bit string $\{0,1\}^n$

- Integer modulo : $\mathbb{Z}_q$
  - Integer 0,1,2,…, q-1
  - After math operation → mod q

e.g. 15+3 = 18%17 = 1



- Polynomial ring : $R_q = \mathbb{Z}_q[x]/(x^n + 1)$
  - q , n
  - Coefficient in $\mathbb{Z}_q \in \{0,1,2, \dots, q-1\}$
  - Degree = $n-1$ → $a_0 + a_1 x + \cdots + a_{n-1} x^{n-1}$
  - After multiplying apply modular reduction $/(x^n + 1)$

e.g. q = 17, n = 4

f(x) = $2 + 16x + 3x^2 + 5x^3$

g(x) = $9 + x + 14x^3$

$$f(x)g(x) = 18 + 146x + 43x^2 + 76x^3 + 229x^4 + 42x^5 + 70x^6$$

coef. mod q

$$= 1 + 10x + 9x^2 + 8x^3 + 8x^4 + 8x^5 + 2x^6$$

modular reduction $/(x^n + 1)$

$$= 10 + 2x + 7x^2 + 8x^3$$

Polynomial vector R_q^k

- Matrix size k ;   e.g. k = 3
- Entries are polynomial ring R_q

$$\begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{k\times 1}$$

Polynomial vector $R_q^{k\times k}$

- Matrix size 3 × 3 ; e.g. k = 3
- Entries are polynomial ring R_q

$$\begin{bmatrix} R_q & R_q & R_q \\ R_q & R_q & R_q \\ R_q & R_q & R_q \end{bmatrix}_{k\times k}$$

$$R_q = a_0 + a_1 x + \cdots + a_{n-1} x^{n-1}$$

“ Small” Polynomial : S_η

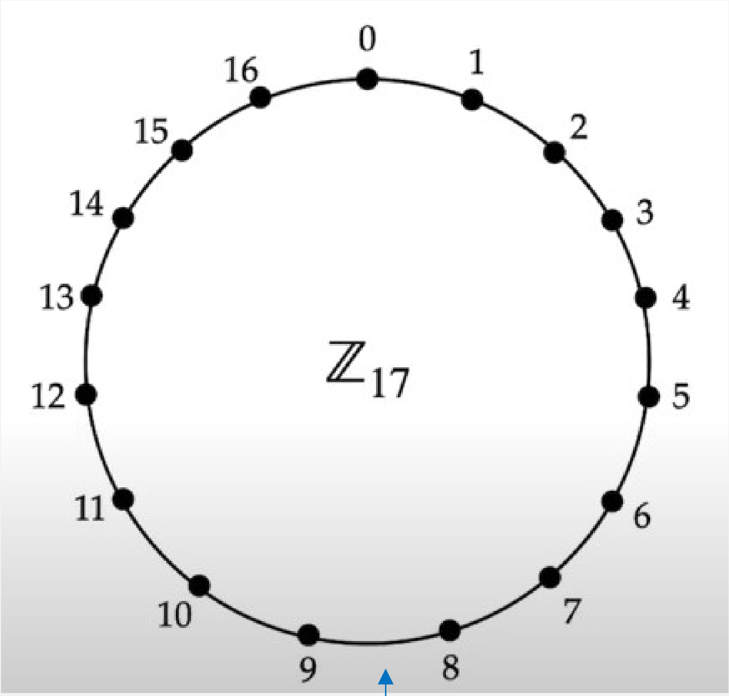
- η (eta) : coefficient $\in [-\eta, \eta]$ when written in symmetric mod
- e.g. $q = 17,\ n = 4, \eta = 2$
- Coefficient can be -2, -1, 0, 1, 2

$$s(x) = 1 + x - 2x^3$$

Noted : $s(x)$ is written is symmetric mods

$15x^3$

Symmetric mod → mods



$(q-1)/2$

Bit string

- Bits string $\{0,1\}^n$
- e.g. 01101101….0 ← n bits long

Kyber-768 domain Parameters

$q = 3329$	Polynomial coefficient can be : 0,1,2,…3328
$n = 256$	Each polynomial has degree 255 : $a_0 + a_1x + \cdots + a_{255}x^{255}$
$k = 3$	For matrix size
$\eta_1 = 2$	size of "small" polynomial
$\eta_2 = 2$	size of "small" polynomial
$d_u = 10$	for compression/decompression of ciphertext
$d_v = 4$	for compression/decompression of ciphertext

KYBER-PKE

- Key generation
- Encryption
- Decryption

Hash function: generate A : SHAKE 128

- Base on Learning with errors problem
- Public key encryption
- Security level : "secure against chosen plain text attack" → lower than Kyber-KEM

Kyber-PKE key generation: Alice does:

1. Select $\rho \in_R \{0,1\}^{256}$ and compute $A = \text{Expand}(\rho)$, where $A \in R_q^{k\times k}$.
2. Select $s \in_{CBD} S_{\eta_1}^k$ and $e \in_{CBD} S_{\eta_2}^k$.
3. Compute $t = As + e$.
4. Alice's **encryption (public) key** is (ρ, t); her **decryption (private) key** is s.

1

$\rho = \{0,1\}^{256}$ 256 bits string

A = Hash(ρ)

$$A = \begin{bmatrix} R_q & R_q & R_q \\ R_q & R_q & R_q \\ R_q & R_q & R_q \end{bmatrix}_{3\times3}$$

2

$$s = \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{3\times1} \qquad e = \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{3\times1}$$

Note : coefficient size has to be small according to $\eta_1 = 2$, $\eta_2 = 2$

3

$$t = As + e = \left(\begin{bmatrix} R_q & R_q & R_q \\ R_q & R_q & R_q \\ R_q & R_q & R_q \end{bmatrix}_{3\times3} * \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{3\times1} \right) + \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{3\times1} = \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{3\times1} + \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{3\times1}$$

As A e s t e

Kyber-PKE encryption: To encrypt a message $m \in \{0,1\}^n$ for Alice, Bob does:

1. Obtain an authentic copy of Alice's encryption key (ρ, t) and compute $A = \text{Expand}(\rho)$.
2. Select $r \in_{CBD} S_{\eta_1}^k$, $e_1 \in_{CBD} S_{\eta_2}^k$ and $e_2 \in_{CBD} S_{\eta_2}$.
3. Compute $u = A^T r + e_1$ and $v = t^T r + e_2 + \lceil \frac{q}{2} \rfloor m$.
4. Compute $c_1 = \text{Compress}_q(u, d_u)$ and $c_2 = \text{Compress}_q(v, d_v)$.
5. Output $c = (c_1, c_2)$. Ciphertext: send to Alice

1

$$m = \{0,1\}^{256}$$

$$t = \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_3, \rho = \{0,1\}^{256}$$

m, ρ can be written as polynomial

e.g. $\rho = x + x^3 + x^5 + \cdots + x^{255}$

2

$$r = \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_3 \quad e_1 = \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_3 \quad e_2 = R_q$$

R_q are small!

3

$$u = A^T r + e_1 = \begin{bmatrix} R_q & R_q & R_q \\ R_q & R_q & R_q \\ R_q & R_q & R_q \end{bmatrix}_{3\times3} \times \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{3\times1} + \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{3\times1}$$

A^T r e_1

$$v = t^T r + e_2 + \lceil q/2 \rfloor m = [R_q \quad R_q \quad R_q]_{1\times3} \times \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{3\times1} + R_q$$

t^T e t

Kyber-PKE decryption: To decrypt $c = (c_1, c_2)$, Alice does:

1. Compute
   $u' = \mathrm{Decompress}_q(c_1, d_u)$ and
   $v' = \mathrm{Decompress}_q(c_2, d_v)$.
2. Compute
   $m = \mathrm{Round}_q(v' - s^T u')$.

1

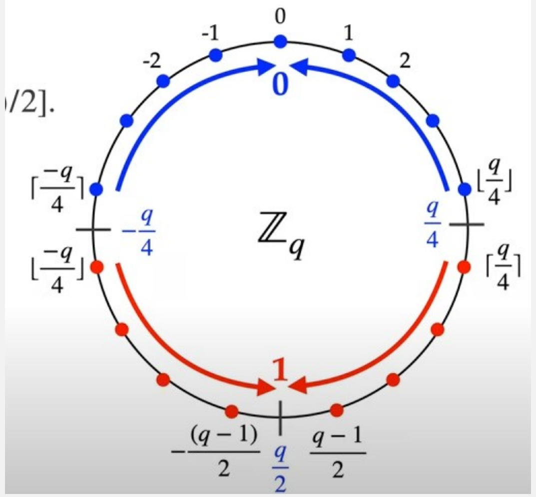
$$u' = \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{3\times 1}, \; v' = R_q$$

2

$$\begin{aligned} m &= Round_q(v' - s^T u') \\ &= Round_q\left(R_q - \begin{bmatrix} R_q & R_q & R_q \end{bmatrix}_{1\times 3} \times \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{3\times 1} \right) \\ &= Round_q(R_q) \end{aligned}$$

→ get plaintext Bob's original message

Rounding



Note: there is small chance that decryption fail but the probability is very low

KYBER-KEM

- Key generation
- Encapsulation
- Decapsulation

Hash function:
generate A : SHAKE 128
G is SHA3-512, H is SHA3-256, J is SHAKE256

- Kyber PKE alone is not secure enough. Applying "Fujisaki Okamoto" transform → Kyber KEM
- Key Encapsulation mechanism
- Using Kyber-PKE as base building block
- Use Hash() and seed to create Pseudo RNG alongside
  - pure random
  - Central binomial Distribution
- Security : "Secure against chosen ciphertext attack" (more secure than Kyber-PKE)

Kyber-KEM key generation: Alice does:

1. Use the Kyber-PKE key generation algorithm to select a Kyber-PKE encryption key (ρ, t) and decryption key s.
2. Select $z \in_R \{0,1\}^{256}$.
3. Alice's **encapsulation key** is $ek = (\rho, t)$; her **decapsulation key** is $dk = (s, ek, H(ek), z)$.

1

Follow Kyber-PKE Key-gen (page 15)

$$\rho = \{0,1\}^{256} \quad , t = \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{3\times 1}$$

$$s = \begin{bmatrix} R_q \\ R_q \\ R_q \end{bmatrix}_{3\times 1}$$

encryption key $ek = (\rho, t)$

2

$$z = \{0,1\}^{256}$$

<table>
<tr><th></th><th>Public key</th><th>Private key</th></tr>
<tr><td>Kyber-KEM</td>
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Encapsulation key = Encryption key

Decapsulation key != Decryptionkey

Kyber-KEM encapsulation: To establish a shared secret key with Alice, Bob does:

1. Obtain an authentic copy of Alice's encapsulation key *ek*.
2. Select $m \in_R \{0,1\}^{256}$.
3. Compute $h = H(ek)$ and $(K, R) = G(m, h)$, where $K, R \in \{0,1\}^{256}$.
4. Use the Kyber-PKE encryption algorithm to encrypt m with encryption key *ek*, and using R to generate the random quantities needed; call the resulting ciphertext c.
5. Output the secret key K and ciphertext c.

1

$ek = (\rho, t)$

2

$m = 11000101 \ldots 1 \leftarrow$ 256 bits

3

Using hash function H and G

$K = 11000101 \ldots 1 \leftarrow$ 256 bits string

$R = 11000101 \ldots 1 \leftarrow$ 256 bits string

K is secret key

R is seed for Kyber-PKE step 4

4

Use R as seed $\rightarrow r, e_1, e_2$ to encrypt m

Follow Kyber-PKE encryption (page 16)

Get ciphertext $c = (c_1, c_2)$

Send ciphertext c to Alice

Kyber-KEM decapsulation: To recover the secret key K from c using $dk = (s, ek, H(ek), z)$, Alice does:

1. Use the Kyber-PKE decryption algorithm to decrypt c using decryption key s; call the resulting plaintext m'.
2. Compute $(K', R') = G(m', H(ek))$.
3. Compute $\overline{K} = J(z, c)$.
4. Use the Kyber-PKE encryption algorithm to encrypt m' with encryption key ek, and using R' to generate the random quantities needed; call the resulting ciphertext c'.
5. If $c \neq c'$ then return($\overline{K}$). Decapsulation fail
6. Return(K'). Decapsulation successful

1

Follow Kyber-PKE decryption (page17)
$m' = \{0,1\}^{256}$

2

Get $K', R' = \{0,1\}^{256}$

K' is the candidate of secret key

R' is the seed use in step 4

3

$\overline{K}$ is return value when the decapsulation fail

4

Use R' as seed → r, e_1, e_2 to encrypt m' again!!

Follow Kyber-PKE encryption (page 16)

Get ciphertext c' = (c_1, c_2)

FUTURE WORK

- Literature review : implementation methods of Kyber KEM
  - Which part of algorithm have been implemented
  - Which part we could implement, and benefits?
- Next presentation : Kyber KEM implementation on FPGA using High Level Synthesis tools

REFERENCES

- https://eee.poriyaan.in/topic/fpga--field-programmable-gate-arrays--11689/
- https://www.researchgate.net/figure/Digital-circuit-with-two-inputs-OR-and-AND-gates_fig11_309907692
- https://cryptography101.ca/kyber-dilithium/