# Circuit Theorems in AC Analysis

# 19

## CHAPTER OUTLINE



## CHAPTER OBJECTIVES

- Apply the superposition theorem to ac circuit analysis
- Apply Thevenin's theorem to simplify reactive ac circuits for analysis
- Apply Norton's theorem to simplify reactive ac circuits
- Apply the maximum power transfer theorem

## KEY TERMS

- Equivalent circuit
- Complex conjugates

## APPLICATION ACTIVITY PREVIEW

In the application activity, you will evaluate a band-pass filter module to determine its internal component values. You will apply Thevenin's theorem to determine an optimum load impedance for maximum power transfer.

## VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/careersresources/

## INTRODUCTION

Four important theorems were covered in Chapter 8 with emphasis on their applications in the analysis of dc circuits. This chapter is a continuation of that coverage with emphasis on applications in the analysis of ac circuits with reactive components.

The theorems in this chapter make analysis easier for certain types of circuits. These methods do not replace Ohm's law and Kirchhoff's laws, but they are normally used in conjunction with those laws in certain situations.

The superposition theorem helps you to deal with circuits that have multiple sources. Thevenin's and Norton's theorems provide methods for reducing a circuit to a simple equivalent form for easier analysis. The maximum power transfer theorem is used in applications where it is important for a given circuit to provide maximum power to a load.

## 19–1 THE SUPERPOSITION THEOREM

The superposition theorem was introduced in Chapter 8 for use in dc circuit analysis. In this section, the superposition theorem is applied to circuits with ac sources and reactive components.

After completing this section, you should be able to

- **Apply the superposition theorem to ac circuit analysis**
  - State the superposition theorem
  - List the steps in applying the theorem

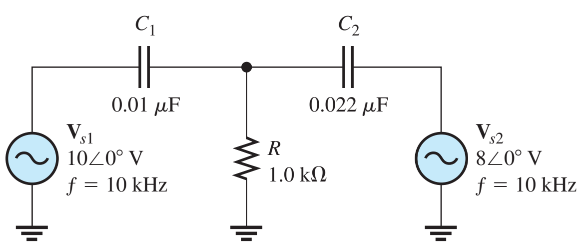
The superposition theorem was introduced Section 8–4 for dc circuits and is restated here for review with impedance replacing resistance. In ac circuits, the superposition theorem is applied by using complex numbers for the algebraic sum but otherwise is applied in the same way as for dc circuits. It is applied only in cases where all sources have exactly the same frequency.

The current or voltage in any given branch of a multiple-source circuit can be found by determining the current or voltage in that particular branch produced by each source acting independently, with all other sources replaced by their internal impedances. The total current or voltage in the branch is the algebraic sum of the responses in that branch.

The steps are reviewed in Example 19–1 with two ideal voltage sources.

**EXAMPLE 19–1**

Find the current in $R$ of Figure 19–1 using the superposition theorem. Assume the internal source impedances are zero.

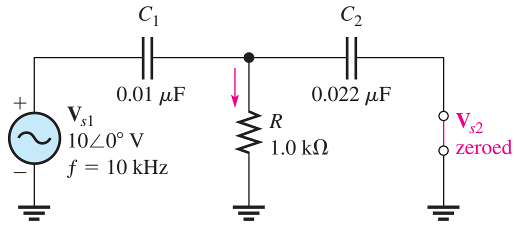



▲ FIGURE 19–1

*Solution* **Step 1:** Replace $V_{s2}$ with its internal impedance (zero in this case), and find the current in $R$ due to $V_{s1}$, as indicated in Figure 19–2.

$$X_{C1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi(10\text{ kHz})(0.01\ \mu\text{F})} = 1.59\text{ k}\Omega$$

$$X_{C2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi(10\text{ kHz})(0.022\ \mu\text{F})} = 723\ \Omega$$




▲ FIGURE 19–2

Looking from $V_{s1}$, the impedance is

$$\mathbf{Z} = \mathbf{X}_{C1} + \frac{\mathbf{R}\mathbf{X}_{C2}}{\mathbf{R} + \mathbf{X}_{C2}} = 1.59\angle -90^\circ \text{ k}\Omega + \frac{(1.0\angle 0^\circ \text{ k}\Omega)(723\angle -90^\circ \ \Omega)}{1.0 \text{ k}\Omega - j723 \ \Omega}$$
$$= 1.59\angle -90^\circ \text{ k}\Omega + 586\angle -54.1^\circ \ \Omega$$
$$= -j1.59 \text{ k}\Omega + 344 \ \Omega - j475 \ \Omega = 344 \ \Omega - j2.07 \text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z} = 2.09\angle -80.6^\circ \text{ k}\Omega$$

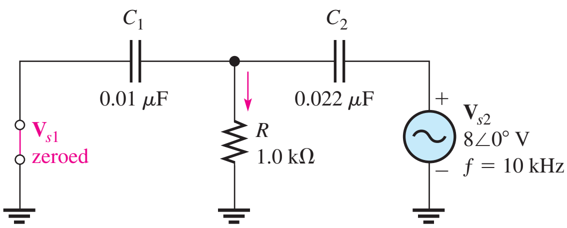
The total current from $V_{s1}$ is

$$\mathbf{I}_{s1} = \frac{\mathbf{V}_{s1}}{\mathbf{Z}} = \frac{10\angle 0^\circ \text{ V}}{2.09\angle -80.6^\circ \text{ k}\Omega} = 4.77\angle 80.6^\circ \text{ mA}$$

Use the current-divider formula. The current through $R$ due to $V_{s1}$ is

$$\mathbf{I}_{R1} = \left(\frac{X_{C2}\angle -90^\circ}{R - jX_{C2}}\right)\mathbf{I}_{s1} = \left(\frac{723\angle -90^\circ \ \Omega}{1.0 \text{ k}\Omega - j723 \ \Omega}\right)4.77\angle 80.6^\circ \text{ mA}$$
$$= 2.80\angle 26.4^\circ \text{ mA}$$

**Step 2:** Find the current in $R$ due to source $V_{s2}$ by replacing $V_{s1}$ with its internal impedance (zero), as shown in Figure 19–3.




▲ FIGURE 19–3

Looking from $V_{s2}$, the impedance is

$$\mathbf{Z} = \mathbf{X}_{C2} + \frac{\mathbf{R}\mathbf{X}_{C1}}{\mathbf{R} + \mathbf{X}_{C1}} = 723\angle -90^\circ \ \Omega + \frac{(1.0\angle 0^\circ \text{ k}\Omega)(1.59\angle -90^\circ \text{ k}\Omega)}{1.0 \text{ k}\Omega - j1.59 \text{ k}\Omega}$$
$$= 723\angle -90^\circ \ \Omega + 847\angle -32.1 \ \Omega$$
$$= -j723 \ \Omega + 717 \ \Omega - j450 \ \Omega = 717 \ \Omega - j1{,}174 \ \Omega$$

Converting to polar form yields

$$\mathbf{Z} = 1{,}376\angle -58.6^\circ\ \Omega$$

The total current from $V_{s2}$ is

$$\mathbf{I}_{s2} = \frac{\mathbf{V}_{s2}}{\mathbf{Z}} = \frac{8\angle 0^\circ\ \text{V}}{1{,}376\angle -58.6^\circ\ \Omega} = 5.82\angle 58.6^\circ\ \text{mA}$$

Use the current-divider formula. The current through $R$ due to $V_{s2}$ is

$$\mathbf{I}_{R2} = \left(\frac{X_{C1}\angle -90^\circ}{R - jX_{C1}}\right)\mathbf{I}_{s2}$$

$$= \left(\frac{1.59\angle -90^\circ\ \text{k}\Omega}{1.0\ \text{k}\Omega - j1.59\ \text{k}\Omega}\right)5.82\angle 58.6^\circ\ \text{mA} = 4.92\angle 26.4^\circ\ \text{mA}$$

**Step 3:** Convert the two individual resistor currents to rectangular form and add to get the total current through $R$.

$$\mathbf{I}_{R1} = 2.80\angle 26.4.6^\circ\ \text{mA} = 2.51\ \text{mA} + j1.25\ \text{mA}$$

$$\mathbf{I}_{R2} = 4.92\angle 26.4^\circ\ \text{mA} = 4.41\ \text{mA} + j2.19\ \text{mA}$$

$$\mathbf{I}_{R} = \mathbf{I}_{R1} + \mathbf{I}_{R2} = 6.92\ \text{mA} + j3.44\ \text{mA} = \mathbf{7.72\angle 26.4^\circ\ mA}$$

*Related Problem** Determine $\mathbf{I}_R$ if $\mathbf{V}_{s2} = 8\angle 180^\circ$ V in Figure 19–1.

MultiSim

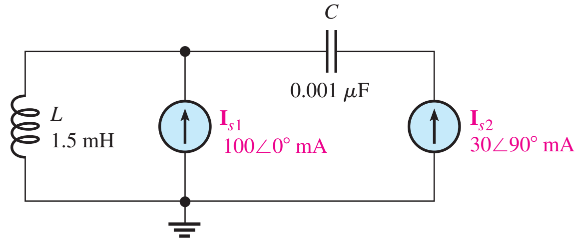
Use Multisim files E19-01A and E19-01B to verify the calculated results in this example and to confirm your calculation for the related problem.

*Answers are at the end of the chapter.

Example 19–2 illustrates the application of the superposition theorem for a circuit with two current sources, $I_{s1}$ and $I_{s2}$.

## EXAMPLE 19–2

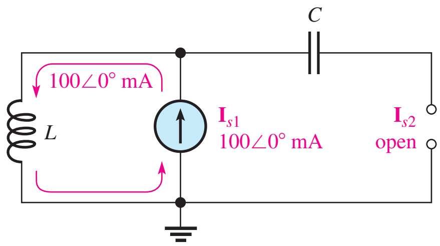
Find the inductor current in Figure 19–4. Assume the current sources are ideal.




▲ FIGURE 19–4

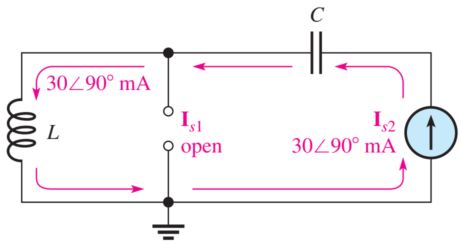
*Solution* **Step 1:** Find the current through the inductor due to current source $I_{s1}$ by replacing source $I_{s2}$ with an open, as shown in Figure 19–5. As you can see, the entire 100 mA from the current source $I_{s1}$ is through the inductor.

▶ FIGURE 19–5




**Step 2:** Find the current through the inductor due to current source $I_{s2}$ by replacing source $I_{s1}$ with an open, as indicated in Figure 19–6. Notice that all of the 30 mA from source $I_{s2}$ is through the inductor.

▶ FIGURE 19–6




**Step 3:** To get the total inductor current, superimpose the two individual currents and add as phasor quantities.

$$\mathbf{I}_L = \mathbf{I}_{L1} + \mathbf{I}_{L2}$$
$$= 100\angle 0^\circ \text{ mA} + 30\angle 90^\circ \text{ mA} = 100 \text{ mA} + j30 \text{ mA}$$
$$= \mathbf{104\angle 16.7^\circ \text{ mA}}$$

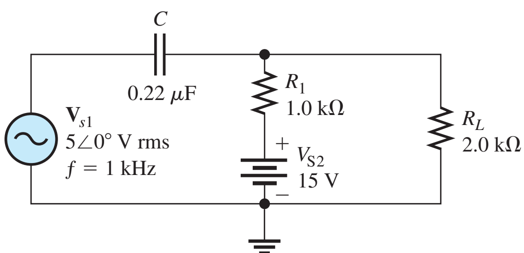
*Related Problem* Find the current through the capacitor in Figure 19–4.

Example 19–3 illustrates the analysis of a circuit with an ac voltage source and a dc voltage source. This situation is common in many amplifier applications.

## EXAMPLE 19–3

Find the total current in the load resistor, $R_L$, in Figure 19–7. Assume the sources are ideal.

▶ FIGURE 19–7



*Solution* **Step 1:** Find the current through $R_L$ due to the ac source $V_{s1}$ by zeroing (replacing with its internal impedance) the dc source $V_{S2}$, as shown in Figure 19–8. Looking from $V_{s1}$, the impedance is

$$\mathbf{Z} = \mathbf{X}_C + \frac{\mathbf{R}_1\mathbf{R}_L}{\mathbf{R}_1 + \mathbf{R}_L}$$

$$X_C = \frac{1}{2\pi(1.0\text{ kHz})(0.22\ \mu\text{F})} = 723\ \Omega$$

$$\mathbf{Z} = 723\angle -90^\circ\ \Omega + \frac{(1.0\angle 0^\circ\text{ k}\Omega)(2.0\angle 0^\circ\text{ k}\Omega)}{3.0\angle 0^\circ\text{ k}\Omega}$$

$$= -j723\ \Omega + 667\ \Omega = 984\angle -47.3^\circ\ \Omega$$

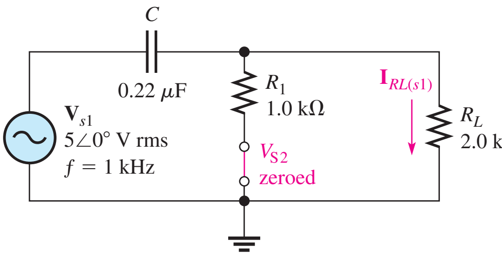
The total current from the ac source is

$$\mathbf{I}_{s1} = \frac{\mathbf{V}_{s1}}{\mathbf{Z}} = \frac{5\angle 0^\circ\text{ V}}{984\angle -47.3^\circ\ \Omega} = 5.08\angle 47.3^\circ\text{ mA}$$

Use the current-divider approach. The current in $R_L$ due to $V_{s1}$ is

$$\mathbf{I}_{RL(s1)} = \left(\frac{R_1}{R_1 + R_L}\right)\mathbf{I}_{s1} = \left(\frac{1.0\text{ k}\Omega}{3.0\text{ k}\Omega}\right)5.08\angle 47.3^\circ\text{ mA} = 1.69\angle 47.3^\circ\text{ mA}$$

▶ FIGURE 19–8



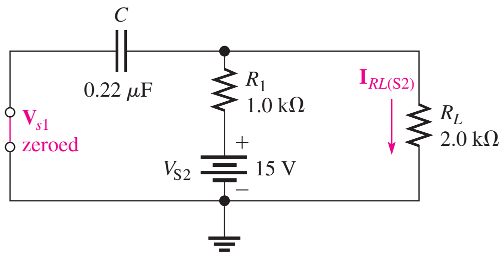

**Step 2:** Find the current in $R_L$ due to the dc source $V_{S2}$ by zeroing $V_{s1}$ (replacing with its internal impedance), as shown in Figure 19–9. Because $C$ appears as an open to dc, the impedance magnitude as seen by $V_{S2}$ is

$$Z = R_1 + R_L = 3.0\text{ k}\Omega$$

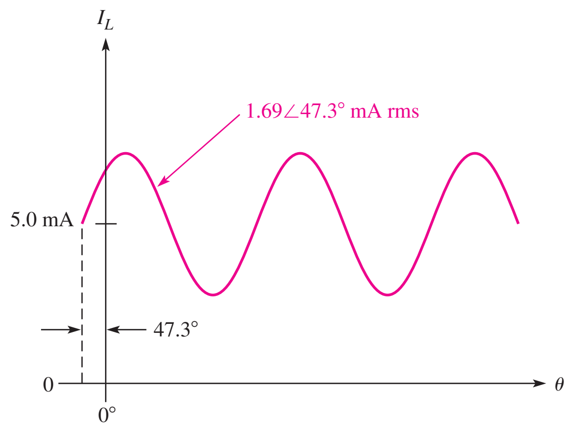
The current produced by $V_{S2}$ is

$$I_{RL(S2)} = \frac{V_{S2}}{Z} = \frac{15\text{ V}}{3.0\text{ k}\Omega} = 5.0\text{ mA dc}$$

▶ FIGURE 19–9



**Step 3:** By superposition, the total current in $R_L$ is $1.69\angle 47.3°$ mA riding on a dc level of 5.0 mA, as indicated in Figure 19–10.




▶ FIGURE 19–10

*Related Problem* Determine the current through $R_L$ if $V_{S2}$ is changed to 9 V.

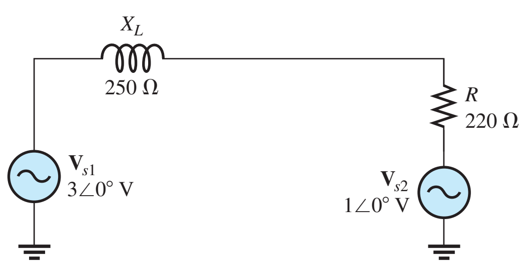
Use Multisim files E19-03A and E19-03B to verify the calculated results in this example and to confirm your calculation for the related problem.

**SECTION 19–1 CHECKUP**

Answers are at the end of the chapter.

1. Why is it necessary to know the phase relationship of two ac sources when applying the superposition theorem?
2. Why is the superposition theorem useful in the analysis of multiple-source circuits?
3. Using the superposition theorem, find the magnitude of the current through $R$ in Figure 19–11. Assume the sources are ideal.

▶ FIGURE 19–11

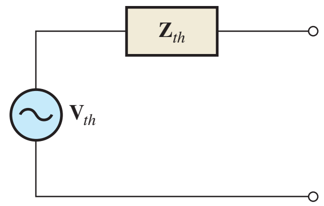



## 19–2 THEVENIN'S THEOREM

Thevenin's theorem was introduced in Section 8–5 for dc circuits. In this section, you will learn to apply Thevenin's theorem to linear ac circuits. Recall that with dc circuits, the Thevenin equivalent circuit consisted of a dc source in series with an equivalent resistance. In ac circuits, the source is an ac source in series with an equivalent impedance. You will need to use complex numbers with ac analysis.

After completing this section, you should be able to

- **Apply Thevenin's theorem to simplify reactive ac circuits for analysis**
  - Describe the form of a Thevenin equivalent circuit
  - Obtain the Thevenin equivalent ac voltage source
  - Obtain the Thevenin equivalent impedance
  - List the steps in applying Thevenin's theorem to an ac circuit



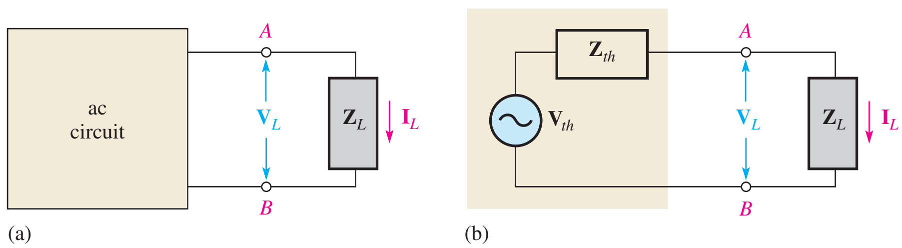

▲ FIGURE 19–12

Thevenin's equivalent circuit.

## Equivalency

The form of Thevenin's **equivalent circuit** for linear ac circuits is shown in Figure 19–12. Regardless of how complex the original circuit is, it can always be reduced to this equivalent form with respect to two output terminals. The equivalent voltage source is designated $\mathbf{V}_{th}$; the equivalent impedance is designated $\mathbf{Z}_{th}$ (lowercase italic subscript denotes ac quantity). Notice that the impedance is represented by a block in the circuit diagram. This is because the equivalent impedance can be of several forms: purely resistive, purely reactive, or a combination of a resistance and a reactance.

Figure 19–13(a) shows a block diagram that represents an ac circuit of any given complexity. This circuit has two output terminals, *A* and *B*. A load impedance, $\mathbf{Z}_L$, is connected to the terminals. The circuit produces a certain voltage, $\mathbf{V}_L$, and a certain current, $\mathbf{I}_L$, as illustrated.



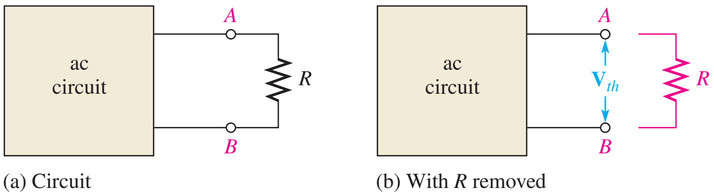

▲ FIGURE 19–13

An ac circuit of any complexity can be reduced to a Thevenin equivalent for analysis purposes.

By Thevenin's theorem, the circuit in the block can be reduced to an equivalent form, as indicated in the beige area of Figure 19–13(b). The term *equivalent* means that when the same value of load is connected to both the original circuit and Thevenin's equivalent circuit, the load voltages and currents are equal for both. Therefore, as far as the load is concerned, there is no difference between the original circuit and Thevenin's equivalent circuit. The load "sees" the same current and voltage regardless of whether it is connected to the original circuit or to the Thevenin equivalent. For ac circuits, the equivalent circuit is for one particular frequency. When the frequency is changed, the equivalent circuit must be recalculated.

## Thevenin's Equivalent Voltage ($\mathbf{V}_{th}$)

As you have seen, the equivalent voltage, $\mathbf{V}_{th}$, is one part of the complete Thevenin equivalent circuit.




▲ FIGURE 19–14

How $V_{th}$ is determined.

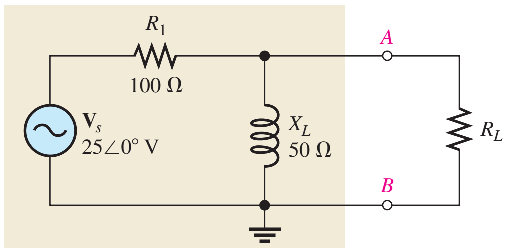
**Thevenin's equivalent voltage is defined as the open circuit voltage between two specified terminals in a circuit.**

To illustrate, let's assume that an ac circuit of some type has a resistor connected between two specified terminals, $A$ and $B$, as shown in Figure 19–14(a). We wish to find the Thevenin equivalent circuit for the circuit as "seen" by $R$. $\mathbf{V}_{th}$ is the voltage across terminals $A$ and $B$, with $R$ removed, as shown in part (b) of the figure. The circuit is viewed from the open terminals $A$ and $B$, and $R$ is considered external to the circuit for which the Thevenin equivalent is to be found.

The following three examples show how to find $\mathbf{V}_{th}$.

## EXAMPLE 19–4

Refer to Figure 19–15. Determine $\mathbf{V}_{th}$ for the circuit within the beige box as viewed from terminals $A$ and $B$.




▲ FIGURE 19–15

*Solution* Remove $R_L$ and determine the voltage from $A$ to $B$ ($\mathbf{V}_{th}$). In this case, the voltage from $A$ to $B$ is the same as the voltage across $X_L$. This is determined using the voltage-divider method.

$$\mathbf{V}_L = \left(\frac{X_L\angle 90^\circ}{R_1 + jX_L}\right)\mathbf{V}_s = \left(\frac{50\angle 90^\circ\ \Omega}{100\ \Omega + j50\ \Omega}\right)\mathbf{V}_s$$

$$= \left(\frac{50\angle 90^\circ\ \Omega}{112\angle 26.6^\circ\ \Omega}\right)25\angle 0^\circ\text{ V} = 11.2\angle 63.4^\circ\text{ V}$$

$$\mathbf{V}_{th} = \mathbf{V}_{AB} = \mathbf{V}_L = \mathbf{11.2\angle 63.4^\circ\ V}$$

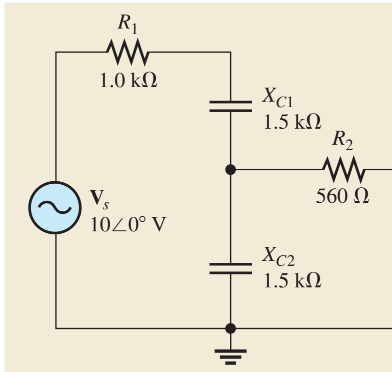
*Related Problem* Determine $\mathbf{V}_{th}$ if $R_1$ is changed to 47 Ω in Figure 19–15.

Use Multisim files E19-04A and E19-04B to verify the calculated results in this example and to confirm your calculation for the related problem.

**EXAMPLE 19–5**

Refer to Figure 19–16. Determine the Thevenin voltage for the circuit within the beige box as viewed from terminals *A* and *B*.

▶ **FIGURE 19–16**




*Solution* Thevenin's voltage for the circuit between terminals *A* and *B* is the voltage that appears across *A* and *B* with $R_L$ removed from the circuit.

There is no voltage drop across $R_2$ because the open between terminals *A* and *B* prevents current through it. Thus, $\mathbf{V}_{AB}$ is the same as $\mathbf{V}_{C2}$ and can be found by the voltage-divider formula.

$$\mathbf{V}_{AB} = \mathbf{V}_{C2} = \left(\frac{X_{C2}\angle -90^\circ}{R_1 - jX_{C1} - jX_{C2}}\right)\mathbf{V}_s = \left(\frac{1.5\angle -90^\circ\ \text{k}\Omega}{1.0\ \text{k}\Omega - j3.0\ \text{k}\Omega}\right)10\angle 0^\circ\ \text{V}$$

$$= \left(\frac{1.5\angle -90^\circ\ \text{k}\Omega}{3.16\angle -71.6^\circ\ \text{k}\Omega}\right)10\angle 0^\circ\ \text{V} = 4.74\angle -18.4^\circ\ \text{V}$$

$$\mathbf{V}_{th} = \mathbf{V}_{AB} = \mathbf{4.74\angle -18.4^\circ\ V}$$

*Related Problem* Determine $\mathbf{V}_{th}$ if $R_1$ is changed to 2.2 kΩ in Figure 19–16.

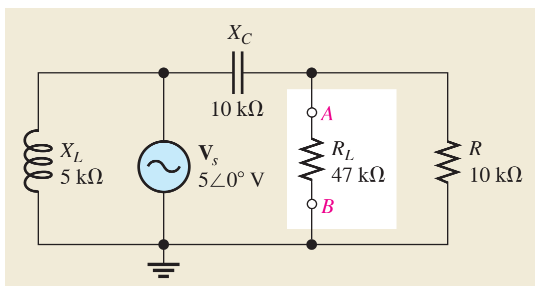
MultiSim

Use Multisim files E19-05A and E19-05B to verify the calculated results in this example and to confirm your calculation for the related problem.

**EXAMPLE 19–6**

Refer to Figure 19–17. Find $\mathbf{V}_{th}$ for the circuit within the beige box as viewed from terminals *A* and *B*.

▶ **FIGURE 19–17**



*Solution* First remove $R_L$ and determine the voltage across the resulting open terminals, which is $\mathbf{V}_{th}$. Find $\mathbf{V}_{th}$ by applying the voltage-divider formula to $X_C$ and $R$.

$$\mathbf{V}_{th} = \mathbf{V}_R = \left(\frac{R\angle 0^\circ}{R - jX_C}\right)\mathbf{V}_s = \left(\frac{10\angle 0^\circ \text{ k}\Omega}{10 \text{ k}\Omega - j10 \text{ k}\Omega}\right)5\angle 0^\circ \text{ V}$$

$$= \left(\frac{10\angle 0^\circ \text{ k}\Omega}{14.1\angle -45^\circ \text{ k}\Omega}\right)5\angle 0^\circ \text{ V} = \mathbf{3.54\angle 45^\circ \text{ V}}$$

Notice $X_L$ has no effect on the result, since the 5 V source appears across $X_C$ and $R$ in combination.

*Related Problem* Find $\mathbf{V}_{th}$ if $R$ is 22 kΩ and $R_L$ is 39 kΩ in Figure 19–17.

MultiSim

Use Multisim files E19-06A and E19-06B to verify the calculated results in this example and to confirm your calculation for the related problem.

## Thevenin's Equivalent Impedance ($\mathbf{Z}_{th}$)

The previous examples illustrated how to find $\mathbf{V}_{th}$. Now, let's determine the Thevenin equivalent impedance, $\mathbf{Z}_{th}$, the second part of a Thevenin equivalent circuit. As defined by Thevenin's theorem,

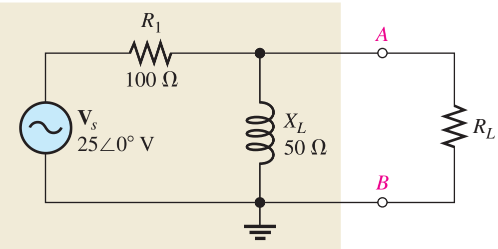
**Thevenin's equivalent impedance is the total impedance appearing between two specified terminals in a given circuit with all sources replaced by their internal impedances.**

To find $\mathbf{Z}_{th}$ between any two terminals in a circuit, replace all the voltage sources by a short (any internal impedance remains in series). Replace all the current sources by an open (any internal impedance remains in parallel). Then determine the total impedance between the two terminals. The following three examples illustrate how to find $\mathbf{Z}_{th}$.

**EXAMPLE 19–7**

Find $\mathbf{Z}_{th}$ for the part of the circuit in Figure 19–18 that is within the beige box as viewed from terminals $A$ and $B$. This is the same circuit used in Example 19–4. Assume $V_s$ is ideal.

► FIGURE 19–18



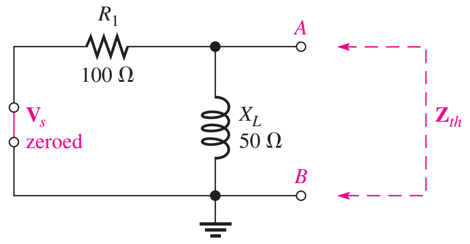

*Solution* First, replace $\mathbf{V}_s$ with its internal impedance (zero in this case), as shown in Figure 19–19.

Looking in between terminals $A$ and $B$, $R_1$ and $X_L$ are in parallel. Thus,

$$\mathbf{Z}_{th} = \frac{(R_1\angle 0^\circ)(X_L\angle 90^\circ)}{R_1 + jX_L} = \frac{(100\angle 0^\circ \ \Omega)(50\angle 90^\circ \ \Omega)}{100 \ \Omega + j50 \ \Omega}$$

$$= \mathbf{44.7\angle 63.4^\circ \ \Omega}$$

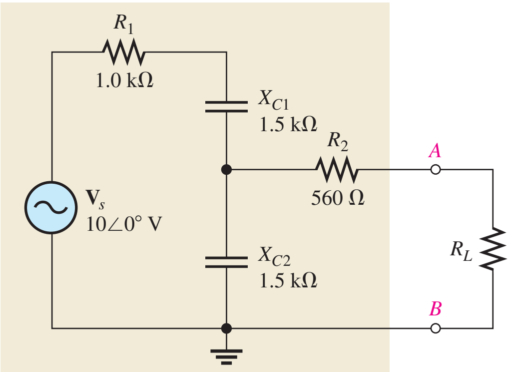
▶ FIGURE 19–19




*Related Problem* Change $R_1$ to 47 Ω and determine $\mathbf{Z}_{th}$.

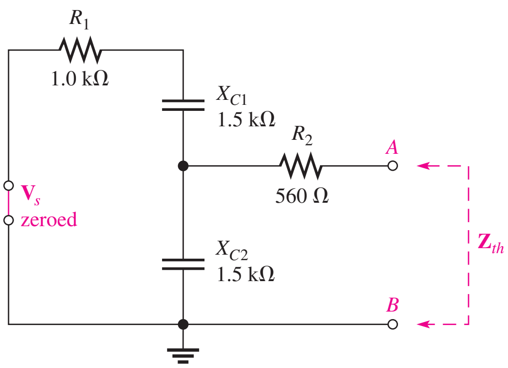
## EXAMPLE 19–8

Refer to Figure 19–20. Determine $\mathbf{Z}_{th}$ for the circuit within the beige box as viewed from terminals *A* and *B*. This is the same circuit used in Example 19–5.




▲ FIGURE 19–20

*Solution* First, replace the voltage source with its internal impedance (zero in this case), as shown in Figure 19–21.




▲ FIGURE 19–21

Looking from terminals $A$ and $B$, $C_2$ appears in parallel with the series combination of $R_1$ and $C_1$. This entire combination is in series with $R_2$. The calculation for $\mathbf{Z}_{th}$ is as follows:

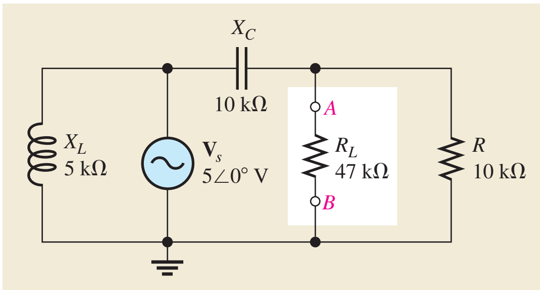
$$\mathbf{Z}_{th} = R_2\angle 0^\circ + \frac{(X_{C2}\angle -90^\circ)(R_1 - jX_{C1})}{R_1 - jX_{C1} - jX_{C2}}$$
$$= 560\angle 0^\circ\ \Omega + \frac{(1.5\angle -90^\circ\ \text{k}\Omega)(1.0\ \text{k}\Omega - j1.5\ \text{k}\Omega)}{1.0\ \text{k}\Omega - j3.0\ \text{k}\Omega}$$
$$= 560\angle 0^\circ\ \Omega + 855\angle -74.7^\circ\ \Omega$$
$$= 785\Omega - j824\Omega = \mathbf{1139\angle -46.4^\circ\ \Omega}$$

*Related Problem* Determine $\mathbf{Z}_{th}$ if $R_1$ is changed to 2.2 kΩ in Figure 19–20.

## EXAMPLE 19–9

Refer to Figure 19–22. Determine $\mathbf{Z}_{th}$ for the portion of the circuit within the beige box as viewed from terminals $A$ and $B$. This is the same circuit as in Example 19–6. Consider the source is ideal.

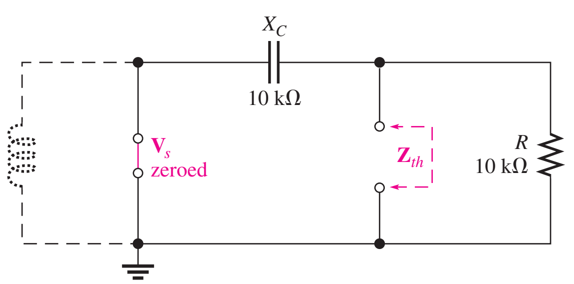
▶ FIGURE 19–22




*Solution* With the voltage source replaced by its internal impedance (zero in this case), $X_L$ is effectively out of the circuit. $R$ and $C$ appear in parallel when viewed from the open terminals, as indicated in Figure 19–23. $\mathbf{Z}_{th}$ is calculated as follows:

$$\mathbf{Z}_{th} = \frac{(R\angle 0^\circ)(X_C\angle -90^\circ)}{R - jX_C} = \frac{(10\angle 0^\circ\ \text{k}\Omega)(10\angle -90^\circ\ \text{k}\Omega)}{10\ \text{k}\Omega - j10\ \text{k}\Omega}$$
$$= \frac{(10\angle 0^\circ\ \text{k}\Omega)(10\angle -90^\circ\ \text{k}\Omega)}{14.1\angle -45^\circ\ \text{k}\Omega} = \mathbf{7.07\angle -45^\circ\ k\Omega}$$

▶ FIGURE 19–23



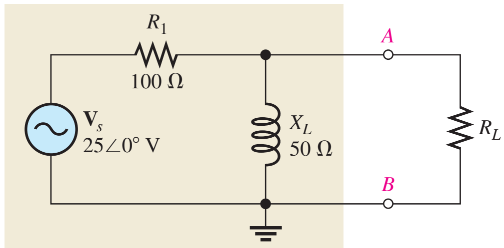

*Related Problem* Find $\mathbf{Z}_{th}$ if $R$ is 22 kΩ and $R_L$ is 39 kΩ in Figure 19–22.

## Thevenin's Equivalent Circuit

The previous six examples have shown how to find the two equivalent components of a Thevenin circuit, $\mathbf{V}_{th}$ and $\mathbf{Z}_{th}$. Keep in mind that you can find $\mathbf{V}_{th}$ and $\mathbf{Z}_{th}$ for any circuit. Once you have determined these equivalent values, you must connect them in series to form the Thevenin equivalent circuit. The following three examples use the previous examples to illustrate this final step.

### EXAMPLE 19–10

Refer to Figure 19–24. Draw the Thevenin equivalent for the circuit within the beige box as viewed from terminals *A* and *B*. This is the circuit used in Examples 19–4 and 19–7.

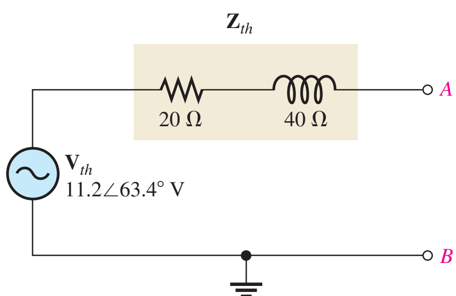



▲ FIGURE 19–24

*Solution* From Examples 19–4 and 19–7, respectively, $\mathbf{V}_{th} = 11.2\angle 63.4°$ V, and $\mathbf{Z}_{th} = 44.7\angle 63.4°\ \Omega$. In rectangular form, the impedance is

$$\mathbf{Z}_{th} = 20\ \Omega + j40\ \Omega$$

This form indicates that the impedance is a 20 Ω resistor in series with a 40 Ω inductive reactance. The Thevenin equivalent circuit is shown in Figure 19–25.

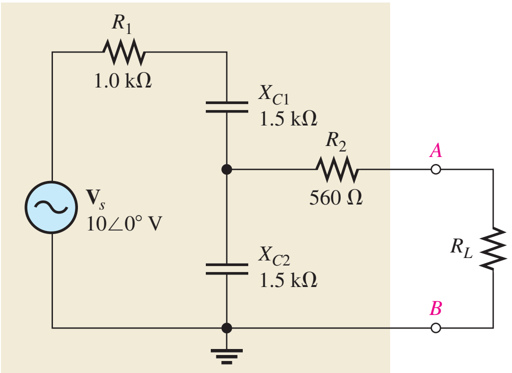



▲ FIGURE 19–25

*Related Problem* Draw the Thevenin equivalent circuit for Figure 19–24 with $R_1 = 47\ \Omega$.

## EXAMPLE 19–11

Refer to Figure 19–26. Draw the Thevenin equivalent for the circuit within the beige box as viewed from terminals $A$ and $B$. This is the circuit used in Examples 19–5 and 19–8.

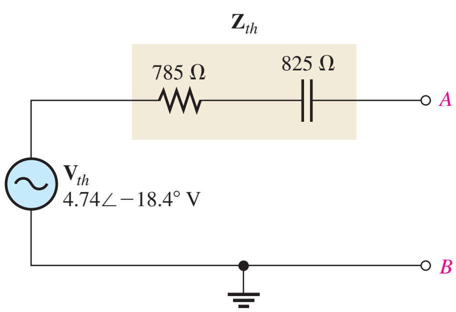



▲ FIGURE 19–26

*Solution* From Examples 19–5 and 19–8, respectively, $\mathbf{V}_{th} = 4.74\angle -18.4°$ V and $\mathbf{Z}_{th} = 11.4\angle -46.4°$ kΩ. In rectangular form, the impedance is

$$\mathbf{Z}_{th} = 785\ \Omega - j825\ \Omega$$

The Thevenin equivalent circuit is shown in Figure 19–27.

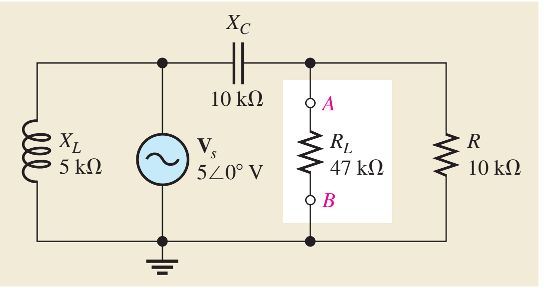



▲ FIGURE 19–27

*Related Problem* Draw the Thevenin equivalent for the circuit in Figure 19–26 with $R_1 = 2.2$ kΩ.

## EXAMPLE 19–12

Refer to Figure 19–28. Determine the Thevenin equivalent for the circuit within the beige box as viewed from terminals $A$ and $B$. This is the circuit used in Examples 19–6 and 19–9.

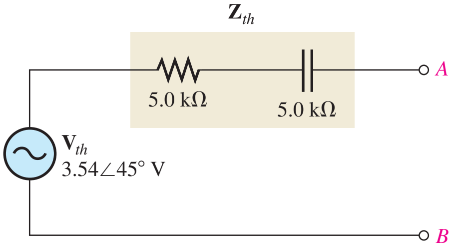



▲ FIGURE 19–28

*Solution* From Examples 19–6 and 19–9, respectively, $\mathbf{V}_{th} = 3.54\angle 45°$ V, and $\mathbf{Z}_{th} = 7.07\angle -45°$ kΩ. The impedance in rectangular form is

$$\mathbf{Z}_{th} = 5.0\ \text{k}\Omega - j5.0\ \text{k}\Omega$$

Thus, the Thevenin equivalent circuit is as shown in Figure 19–29.




▲ FIGURE 19–29

*Related Problem* Change $R$ to 22 kΩ and $R_L$ to 39 kΩ in Figure 19–28 and draw the Thevenin equivalent circuit.

## Summary of Thevenin's Theorem

Remember that the Thevenin equivalent circuit is always a voltage source in series with an impedance regardless of the original circuit that it replaces. The significance of Thevenin's theorem is that the equivalent circuit can replace the original circuit as far as any external load is concerned. Any load connected between the terminals of a Thevenin equivalent circuit experiences the same current and voltage as if it were connected to the terminals of the original circuit.

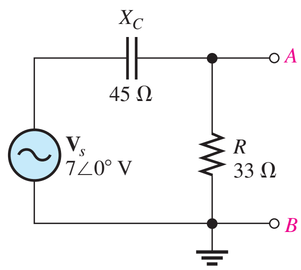
A summary of steps for applying Thevenin's theorem follows.

**Step 1:** Open the two terminals between which you want to find the Thevenin circuit. This is done by removing the component from which the circuit is to be viewed.
**Step 2:** Determine the voltage across the two open terminals.
**Step 3:** Determine the impedance viewed from the two open terminals with ideal voltage sources replaced with shorts and ideal current sources replaced with opens (zeroed).
**Step 4:** Connect $\mathbf{V}_{th}$ and $\mathbf{Z}_{th}$ in series to produce the complete Thevenin equivalent circuit.

**SECTION 19–2 CHECKUP**

1. What are the two basic components of a Thevenin equivalent ac circuit?
2. For a certain circuit, $Z_{th} = 25\ \Omega - j50\ \Omega$, and $V_{th} = 5\angle 0^\circ$ V. Draw the Thevenin equivalent circuit.
3. For the circuit in Figure 19–30, find the Thevenin equivalent looking from terminals $A$ and $B$. (Assume $V_S$ is ideal.)

▶ FIGURE 19–30




## 19–3 NORTON'S THEOREM

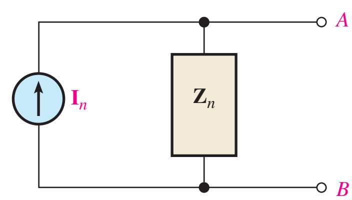
Like Thevenin's theorem, Norton's theorem provides a method of reducing a more complex circuit to a simpler, more manageable form for analysis. The basic difference is that **Norton's theorem** gives an equivalent current source (rather than a voltage source) in parallel (rather than in series) with an equivalent impedance.

After completing this section, you should be able to

- **Apply Norton's theorem to simplify reactive ac circuits**
  - Describe the form of a Norton equivalent circuit
  - Obtain the Norton equivalent ac current source
  - Obtain the Norton equivalent impedance

The form of Norton's equivalent circuit is shown in Figure 19–31. Regardless of how complex the original circuit is, it can be reduced to this equivalent form. The equivalent current source is designated $\mathbf{I}_n$, and the equivalent impedance is $\mathbf{Z}_n$ (lowercase italic subscript denotes ac quantity).

Norton's theorem shows you how to find $\mathbf{I}_n$ and $\mathbf{Z}_n$. Once they are known, simply connect them in parallel to get the complete Norton equivalent circuit.




▲ FIGURE 19–31

Norton equivalent circuit.

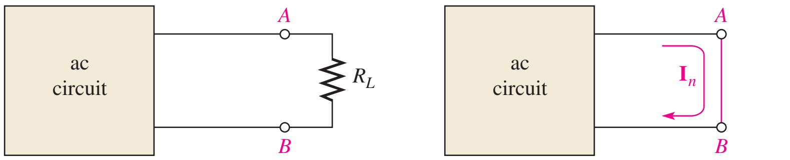
### Norton's Equivalent Current Source ($\mathbf{I}_n$)

$\mathbf{I}_n$ is one part of the Norton equivalent circuit; $\mathbf{Z}_n$ is the other part.

> **Norton's equivalent current is defined as the short-circuit current between two specified terminals in a given circuit.**

Any load connected between these two terminals effectively "sees" a current source $\mathbf{I}_n$ in parallel with $\mathbf{Z}_n$.

To illustrate, let's suppose that the circuit shown in Figure 19–32 has a load resistor connected to terminals $A$ and $B$, as indicated in part (a), and we wish to find the




(a) Circuit with load resistor

(b) Load is replaced by a short, resulting in a short circuit current, $\mathbf{I}_n$.

▲ FIGURE 19–32

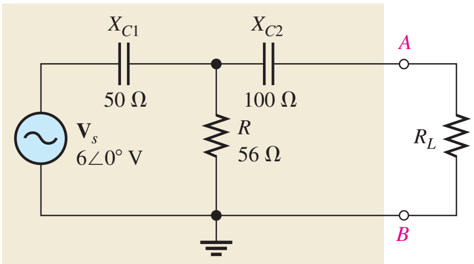
How $\mathbf{I}_n$ is determined.

Norton equivalent for the circuit as viewed from terminals *A* and *B*. To find $\mathbf{I}_n$, calculate the current between terminals *A* and *B* with those terminals shorted, as shown in part (b). Example 19–13 shows how to find $\mathbf{I}_n$.

## EXAMPLE 19–13

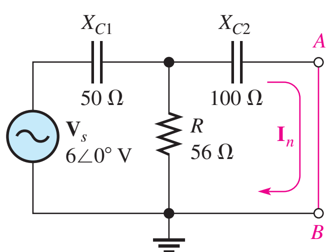
In Figure 19–33, determine $\mathbf{I}_n$ for the circuit as "seen" by the load resistor. The beige area identifies the portion of the circuit to be nortonized.

► FIGURE 19–33




*Solution* Short the terminals *A* and *B*, as shown in Figure 19–34.

► FIGURE 19–34




$\mathbf{I}_n$ is the current through the short and is calculated as follows. First, the total impedance viewed from the source is

$$\mathbf{Z} = \mathbf{X}_{C1} + \frac{\mathbf{R}\mathbf{X}_{C2}}{\mathbf{R} + \mathbf{X}_{C2}} = 50\angle{-90^\circ}\ \Omega + \frac{(56\angle 0^\circ\ \Omega)(100\angle{-90^\circ}\ \Omega)}{56\ \Omega - j100\ \Omega}$$

$$= 50\angle{-90^\circ}\ \Omega + 48.9\angle{-29.2^\circ}\ \Omega$$

$$= -j50\ \Omega + 42.6\ \Omega - j23.9\ \Omega = 42.6\ \Omega - j73.9\ \Omega$$

Converting to polar form yields

$$\mathbf{Z} = 85.3\angle{-60.0^\circ}\ \Omega$$

Next, the total current from the source is

$$\mathbf{I}_s = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{6\angle 0^\circ \text{ V}}{85.3\angle -60.0^\circ \ \Omega} = 70.3\angle 60.0^\circ \text{ mA}$$

Finally, apply the current-divider formula to get $\mathbf{I}_n$ (the current through the short between terminals $A$ and $B$).

$$\mathbf{I}_n = \left(\frac{\mathbf{R}}{\mathbf{R} + \mathbf{X}_{C2}}\right)\mathbf{I}_s = \left(\frac{56\angle 0^\circ \ \Omega}{56 \ \Omega - j100 \ \Omega}\right)70.3\angle 60.0^\circ \text{ mA} = \mathbf{34.4\angle 121^\circ \text{ mA}}$$

This is the value for the equivalent Norton current source.

*Related Problem* Determine $\mathbf{I}_n$ if $\mathbf{V}_s$ is changed to $2.5\angle 0^\circ$ V and $R$ is changed to 33 Ω in Figure 19–33.

## Norton's Equivalent Impedance ($\mathbf{Z}_n$)

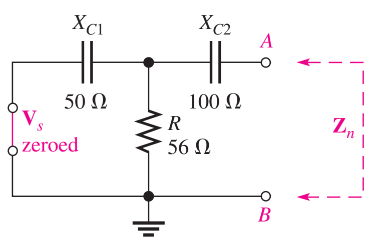
$\mathbf{Z}_n$ is defined the same as $\mathbf{Z}_{th}$: It is the total impedance appearing between two specified terminals of a given circuit viewed from the open terminals with all sources replaced by their internal impedances.

**EXAMPLE 19–14**

Find $\mathbf{Z}_n$ for the circuit in Figure 19–33 (Example 19–13) viewed from the open across terminals $A$ and $B$.

*Solution* First, replace $\mathbf{V}_s$ with its internal impedance (zero), as indicated in Figure 19–35.

▶ FIGURE 19–35




Looking in between terminals $A$ and $B$, $C_2$ is in series with the parallel combination of $R$ and $C_1$. Thus,

$$\mathbf{Z}_n = \mathbf{X}_{C2} + \frac{\mathbf{R}\mathbf{X}_{C1}}{\mathbf{R} + \mathbf{X}_{C1}} = 100\angle -90^\circ \ \Omega + \frac{(56\angle 0^\circ \ \Omega)(50\angle -90^\circ \ \Omega)}{56 \ \Omega - j50 \ \Omega}$$
$$= 100\angle -90^\circ \ \Omega + 37.3\angle -48.2^\circ \ \Omega$$
$$= -j100 \ \Omega + 24.8 \ \Omega - j27.8 \ \Omega = \mathbf{24.8 \ \Omega - j128 \ \Omega}$$

The Norton equivalent impedance is a 24.8 Ω resistance in series with a 128 Ω capacitive reactance.

*Related Problem* Find $\mathbf{Z}_n$ in Figure 19–33 if $R = 33$ Ω.

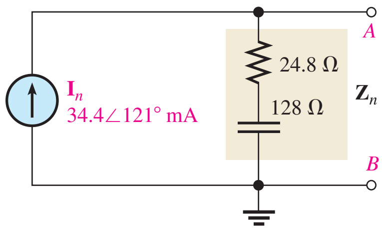
Examples 19–13 and 19–14 showed how to find the two equivalent components of a Norton equivalent circuit. Keep in mind that you can find these values for any given ac circuit. Once you know $\mathbf{I}_n$ and $\mathbf{Z}_n$, connect them in parallel to form the Norton equivalent circuit, as Example 19–15 illustrates.

**EXAMPLE 19–15**

Show the complete Norton equivalent circuit for the circuit in Figure 19–33 (Example 19–13).

*Solution* From Examples 19–13 and 19–14, respectively, $\mathbf{I}_n = 34.4\angle 121°$ mA and $\mathbf{Z}_n = 24.8\ \Omega - j128\ \Omega$. The Norton equivalent circuit is shown in Figure 19–36.

▶ **FIGURE 19–36**




*Related Problem* Show the Norton equivalent for the circuit in Figure 19–33 if $\mathbf{V}_s = 2.5\angle 0°$ V and $R = 33\ \Omega$.

## Summary of Norton's Theorem

Any load connected between the terminals of a Norton equivalent circuit will have the same current through it and the same voltage across it as it would when connected to the terminals of the original circuit. A summary of steps for theoretically applying Norton's theorem is as follows:

**Step 1:** Replace the load connected to the two terminals between which the Norton circuit is to be determined with a short.
**Step 2:** Determine the current through the short. This is $\mathbf{I}_n$.
**Step 3:** Open the terminals and determine the impedance between the two open terminals with all sources replaced with their internal impedances. This is $\mathbf{Z}_n$.
**Step 4:** Connect $\mathbf{I}_n$ and $\mathbf{Z}_n$ in parallel.

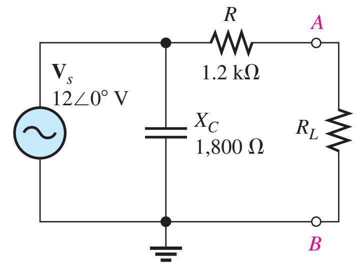
## Norton/Thevenin Equivalency

There is a simple relationship between a Norton equivalent circuit and a Thevenin equivalent circuit. You probably noticed that $Z_{th} = Z_n$. It is easy to show that $V_{th} = I_nR_n$ and $I_n = V_{th}/Z_{th}$ by simply applying Thevenin's theorem to a Norton circuit or by applying Norton's theorem to a Thevenin circuit. If one equivalent is desired, it is sometimes easier to initially derive the other circuit equivalent, and then using the equivalency obtain the equivalent you want. This equivalency is valid for both ac and dc circuits.

**SECTION 19–3 CHECKUP**

1. For a given circuit, $I_n = 5\angle 0°$ mA, and $Z_n = 150\ \Omega + j100\ \Omega$. Draw the Norton equivalent circuit.
2. Find the Norton circuit as seen by $R_L$ in Figure 19–37.

▶ FIGURE 19–37




## 19–4 MAXIMUM POWER TRANSFER THEOREM

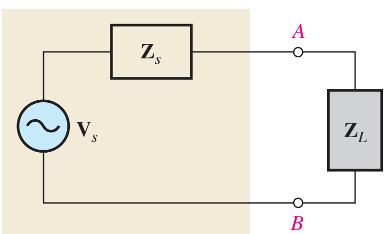
Maximum power is transferred to a load connected to a circuit when the load impedance is the complex conjugate of the circuit's source impedance.

After completing this section, you should be able to

- **Apply the maximum power transfer theorem**
    - Explain the maximum power transfer theorem
    - Determine the value of load impedance for which maximum power is transferred from a given circuit

The **complex conjugate** of $R - jX_C$ is $R + jX_L$ and vice versa, where the resistances are equal in magnitude and the reactances are equal in magnitude but opposite in sign. The source impedance is effectively Thevenin's equivalent impedance viewed from the output terminals. When $\mathbf{Z}_L$ is the complex conjugate of $\mathbf{Z}_{out}$, maximum power is transferred from an ac source to the load with a power factor of 1. An equivalent circuit with its source impedance and load is shown in Figure 19–38.

Example 19–16 shows that maximum power occurs when the impedances are conjugately matched.

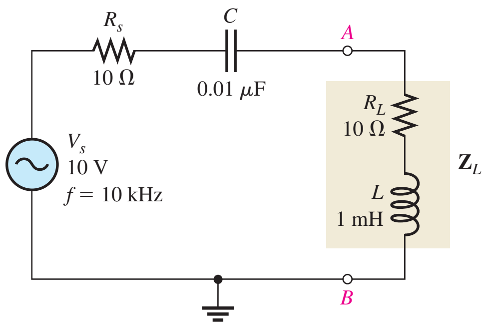



◀ FIGURE 19–38

Equivalent circuit with load.

**EXAMPLE 19–16**

The circuit to the left of terminals *A* and *B* in Figure 19–39 provides power to the load $\mathbf{Z}_L$. This can be viewed as simulating a power amplifier delivering power to a complex load. It is the Thevenin equivalent of a more complex circuit. Calculate and plot a graph of the power delivered to the load at 10 kHz intervals starting at 10 kHz going to 100 kHz.




▲ **FIGURE 19–39**

*Solution* For the first frequency (10 kHz), the solution is shown in detail. Then the TI-84 Plus CE calculator is used to solve the other frequencies.

For $f = 10$ kHz,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(10\text{ kHz})(0.01\ \mu\text{F})} = 1.59\text{ k}\Omega$$

$$X_L = 2\pi fL = 2\pi(10\text{ kHz})(1\text{ mH}) = 62.8\ \Omega$$

The magnitude of the total impedance is

$$Z_{tot} = \sqrt{(R_s + R_L)^2 + (X_L - X_C)^2} = \sqrt{(20\ \Omega)^2 + (1.53\text{ k}\Omega)^2} = 1.53\text{ k}\Omega$$

The current is

$$I = \frac{V_s}{Z_{tot}} = \frac{10\text{ V}}{1.53\text{ k}\Omega} = 6.54\text{ mA}$$

The load power is

$$P_L = I^2R_L = (6.54\text{ mA})^2(10\ \Omega) = \mathbf{428\ \mu W}$$

For the remaining frequencies, the equations for $X_C$, $X_L$, $Z_{tot}$, $I$, and $P_L$ are entered into the TI-84 Plus CE calculator as shown in Figure 19–40. To enter the equations, press the y= key. In this example, the equations represent $X_C$, $X_L$, $Z_{tot}$, $I$, and $P_L$ in that order. The independent variable is frequency (X in the equations). Since we are not particularly interested in the first four equations, they are deselected by selecting the = sign and pressing enter. The only plot that will be shown in the display is the plot of power as a function of frequency.

▶ **FIGURE 19–40**

Equations for plotting power to the load as a function of frequency.

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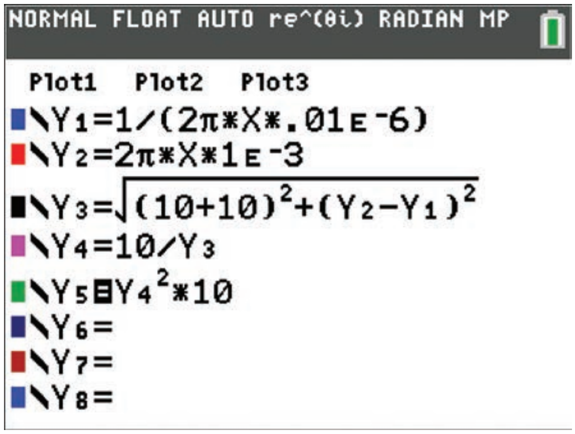
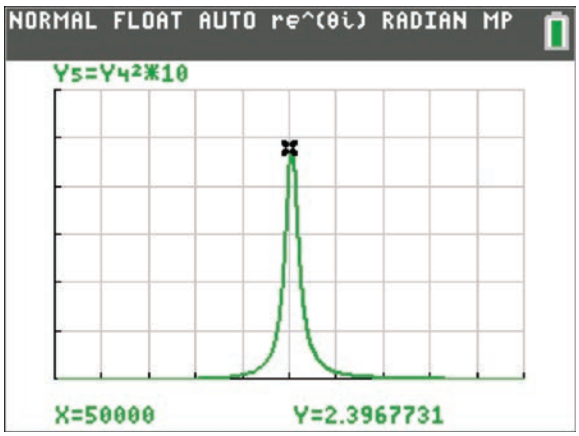



After entering the equations, set the scale factor for the plot by pressing the window key. Pressing the graph key will show the plot of output power as a function of frequency. Figure 19–41 shows the result.

▶ **FIGURE 19–41**

Plot of power as a function of frequency for the circuit in Figure 19–39. The power at 50 kHz is shown with a power to the load of 2.40 W.

Images used with permission by Texas Instruments, Inc.




You can see a table of the power at the other frequencies by pressing 2nd graph, which shows the table as shown in Figure 19–42.

The TI-84 Plus CE allows you to find the exact maximum point on the plot. Press 2nd trace and select maximum. The calculator will ask you to help by providing a left boundary and a right boundary that encompass the peak. The calculator will use these points to refine the maximum calculation. In this example the maximum power is at 50.3 kHz and is 2.5 W. Figure 19–43 shows the result. The load impedance is the complex conjugate of the output impedance at this frequency ($X_C = X_L$ at 50.3 kHz).

▶ **FIGURE 19–42**

Table of power in the load as a function of frequency.

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NORMAL FLOAT AUTO re^(θi) RADIAN MP
PRESS + FOR ΔTbl

| X | Y5 |
|---|---|
| 0 | ERROR |
| 10000 | 4.3E-4 |
| 20000 | 0.0022 |
| 30000 | 0.0085 |
| 40000 | 0.0457 |
| 50000 | 2.3968 |
| 60000 | 0.0776 |
| 70000 | 0.022 |
| 80000 | 0.0108 |
| 90000 | 0.0066 |
| 100000 | 0.0045 |

X=0

▶ **FIGURE 19–43**

Maximum power calculation.

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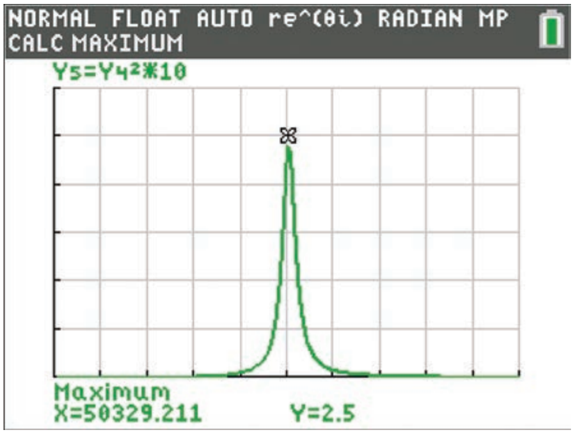



*Related Problem* If $R = 47\ \Omega$ and $C = 0.022\ \mu\text{F}$ in a series $RC$ circuit, what is the complex conjugate of the impedance at 100 kHz?

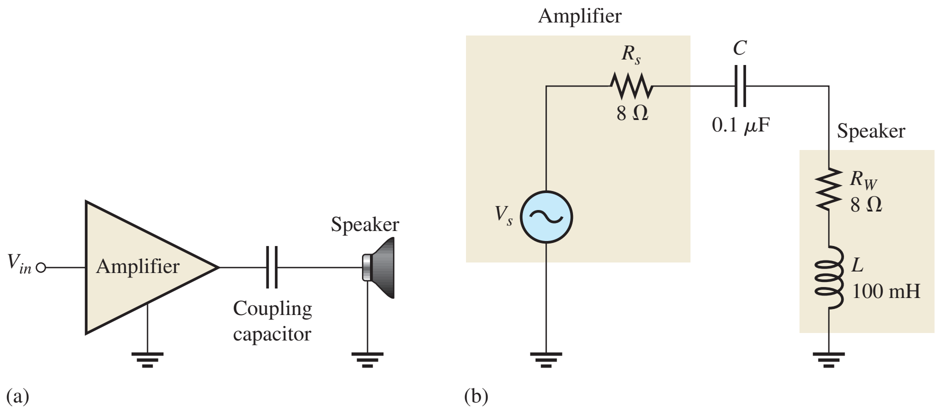
Use Multisim files E19-16A and E19-16B to verify the calculated results in this example.

Example 19–17 illustrates that the frequency at which maximum load power occurs is the value that makes the source and load impedances complex conjugates.

**EXAMPLE 19–17**

**(a)** Determine the frequency at which maximum power is transferred from the amplifier to the speaker in Figure 19–44(a). The amplifier and coupling capacitor are the source, and the speaker is the load, as shown in the equivalent circuit of Figure 19–44(b).

**(b)** How many watts of power are delivered to the speaker at this frequency if $V_s = 3.8$ V rms?




▲ **FIGURE 19–44**

*Solution* **(a)** When the power to the speaker is maximum, the source impedance ($R_s - jX_C$) and the load impedance ($R_W + jX_L$) are complex conjugates, so

$$X_C = X_L$$
$$\frac{1}{2\pi fC} = 2\pi fL$$

Solving for $f$,

$$f^2 = \frac{1}{4\pi^2 LC}$$
$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(100\text{ mH})(0.1\ \mu\text{F})}} \cong \mathbf{1.59\ kHz}$$

**(b)** Calculate the power to the speaker as follows:

$$Z_{tot} = R_s + R_W = 8\ \Omega + 8\ \Omega = 16\ \Omega$$
$$I = \frac{V_s}{Z_{tot}} = \frac{3.8\text{ V}}{16\ \Omega} = 238\text{ mA}$$
$$P_{max} = I^2 R_W = (238\text{ mA})^2(8\ \Omega) = \mathbf{451\ mW}$$

*Related Problem* Determine the frequency at which maximum power is transferred from the amplifier to the speaker in Figure 19–44 if the coupling capacitor is 1 μF.

**SECTION 19–4 CHECKUP**

1. If the source impedance of a certain driving circuit is 50 Ω − $j$10 Ω, what value of load impedance will result in the maximum power to the load?
2. For the circuit in Question 1, how much power is delivered to the load when the load impedance is the complex conjugate of the source impedance and when the load current is 2 A?

## Application Activity

In this application activity, you have a sealed band-pass filter module that has been removed from a system and two schematics. Both schematics indicate that the band-pass filter is implemented with a low-pass/high-pass combination. It is uncertain which schematic corresponds to the filter module, but one of them does. By certain measurements, you will determine which schematic represents the filter so that the filter circuit can be reproduced. Also, you will determine the proper load for maximum power transfer. The filter circuit contained in a sealed module and two schematics, one of which corresponds to the filter circuit, are shown in Figure 19–45.

### Filter Measurement and Analysis

1. Based on the oscilloscope measurement of the filter output shown in Figure 19–46, determine which schematic in Figure 19–45 represents the component values of the filter circuit in the module. A 10 V peak-to-peak voltage is applied to the input.
2. Based on the oscilloscope measurement in Figure 19–46, determine if the filter is operating at its approximate center frequency.
3. Using Thevenin's theorem, determine the load impedance that will provide for maximum power transfer at the center frequency when connected to the output of the filter. Assume the source impedance is zero.

### Review

4. Determine the peak-to-peak output voltage at the frequency shown in Figure 19–46 of the circuit in Figure 19–45 that was determined not to be in the module.
5. Find the center frequency of the circuit in Figure 19–45 that was determined not to be in the module.

**FIGURE 19–45**

Filter module and schematics.

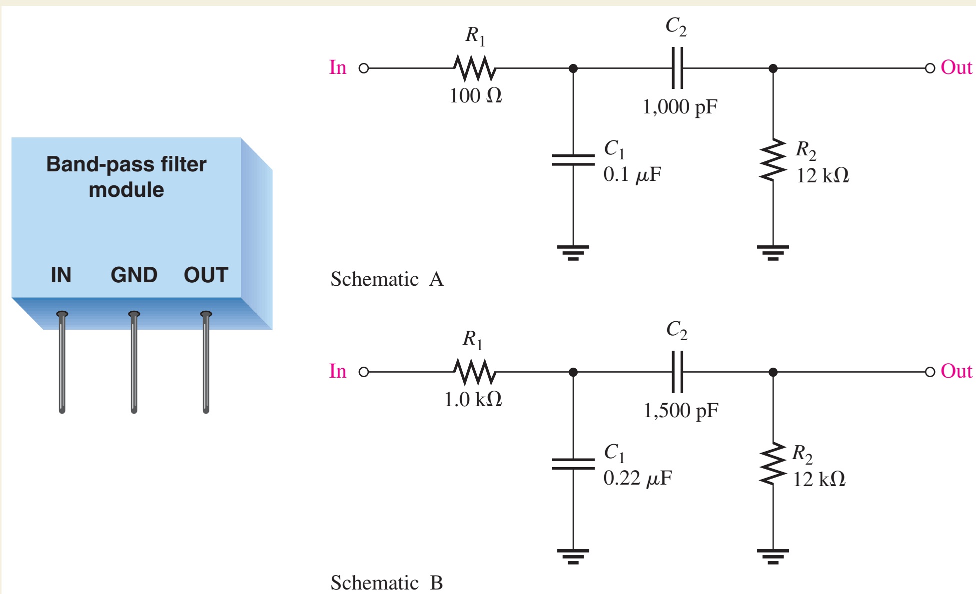
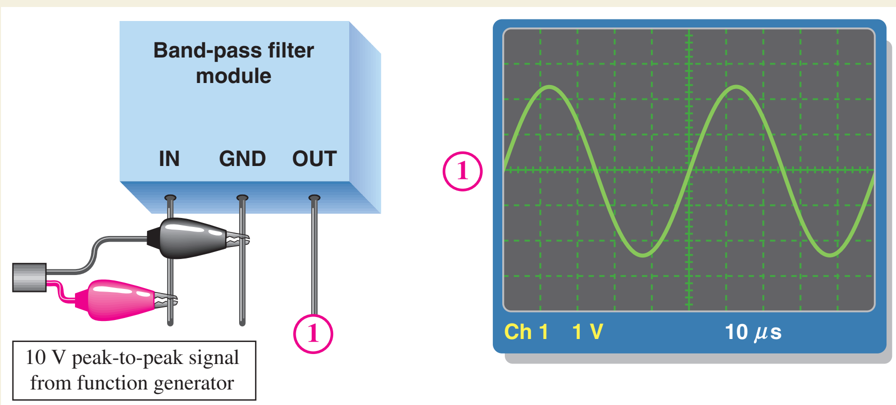



**FIGURE 19–46**

Filter module response.



## SUMMARY

- The superposition theorem is useful for the analysis of both ac and dc multiple-source circuits.
- Thevenin's theorem provides a method for the reduction of any linear ac circuit to an equivalent form consisting of an equivalent voltage source in series with an equivalent impedance.
- The term *equivalency,* as used in Thevenin's and Norton's theorems, means that when a given load impedance is connected to the equivalent circuit, it will have the same voltage across it and the same current through it as when it is connected to the original circuit.
- Norton's theorem provides a method for the reduction of any ac circuit to an equivalent form consisting of an equivalent current source in parallel with an equivalent impedance.
- Maximum power is transferred to a load when the load impedance is the complex conjugate of the impedance of the driving circuit.

## KEY TERMS

**These key terms are also in the end-of-book glossary.**

**Complex conjugates** Two complex numbers that have identical real parts and equal magnitude but oppositely signed imaginary parts.

**Equivalent circuit** A circuit that produces the same voltage and current to a given load as the original circuit that it replaces.

## TRUE/FALSE QUIZ

**Answers are at the end of the chapter.**

1. The superposition theorem can be applied to ac circuits with multiple sources.
2. If the current through a given component from one source is 100 mA and the current through the same component from a second source is 60 mA in the opposite direction, the total current through the component is 160 mA.
3. The purpose of Thevenin's theorem is to simplify a circuit to make analysis easier.
4. The Thevenin equivalent for an ac circuit consists of an equivalent impedance and an equivalent series voltage source.
5. If the output voltage across the open terminals of a certain ac circuit is 5 V, the Thevenin equivalent voltage must be less than 5 V.
6. To determine the Thevenin equivalent impedance of a circuit with an ac voltage source, the source must be replaced by an open.
7. Norton's theorem can be applied to a circuit with an ac current source.
8. The Norton equivalent impedance is found by replacing the ac current source with an open.
9. The complex conjugate of $10\ \Omega - j12\ \Omega$ is $12\ \Omega + j10\ \Omega$.
10. Maximum power is transferred from a circuit to the load when the load is the complex conjugate of the output impedance.

## SELF-TEST

**Answers are at the end of the chapter.**

1. In applying the superposition theorem,
   **(a)** all sources are considered simultaneously
   **(b)** all voltage sources are considered simultaneously
   **(c)** the sources are considered one at a time with all others replaced by a short
   **(d)** the sources are considered one at a time with all others replaced by their internal impedances

2. A Thevenin ac equivalent circuit always consists of an equivalent ac voltage source and an equivalent
   (a) capacitive reactance (b) inductive reactance
   (c) series impedance (d) parallel impedance
3. One circuit is equivalent to another when
   (a) the same load has the same voltage and current when connected to either circuit
   (b) different loads have the same voltage and current when connected to either circuit
   (c) the circuits have equal voltage sources and equal series impedances
   (d) the circuits produce the same output voltage under the same conditions.
4. The Thevenin equivalent voltage is
   (a) the open circuit voltage (b) the short circuit voltage
   (c) the voltage across an equivalent load (d) none of the above
5. The Thevenin equivalent impedance is the impedance looking from
   (a) the source with the output shorted
   (b) the source with the output open
   (c) any two specified open terminals with all sources replaced by their internal impedances
   (d) any two specified open terminals with all sources replaced by a short
6. A Norton ac equivalent circuit always consists of an equivalent
   (a) ac current source in series with an equivalent impedance
   (b) ac current source in parallel with an equivalent reactance
   (c) ac current source in parallel with an equivalent impedance
   (d) ac voltage source in parallel with an equivalent impedance
7. The Norton equivalent current is
   (a) the total current from the source (b) the short circuit current
   (c) the current to an equivalent load (d) none of the above
8. The complex conjugate of $50\ \Omega + j100\ \Omega$ is
   (a) $50\ \Omega - j50\ \Omega$ (b) $100\ \Omega + j50\ \Omega$ (c) $100\ \Omega - j50\ \Omega$ (d) $50\ \Omega - j100\ \Omega$
9. In order to get maximum power transfer from a capacitive source, the load must have
   (a) a capacitance equal to the source capacitance
   (b) an impedance equal in magnitude to the source impedance
   (c) larger resistance than the source
   (d) an impedance that is the complex conjugate of the source impedance
   (e) answers (a) and (d)

## CIRCUIT DYNAMICS QUIZ

**Answers are at the end of the chapter.**

**Refer to Figure 19–50.**

1. If the dc voltage source is shorted out, the voltage at point $A$ with respect to ground
   (a) increases (b) decreases (c) stays the same
2. If $C_2$ opens, the ac voltage across $R_5$
   (a) increases (b) decreases (c) stays the same
3. If $C_2$ opens, the dc voltage across $R_5$
   (a) increases (b) decreases (c) stays the same

**Refer to Figure 19–52.**

4. If $V_2$ is reduced to 0 V, the voltage across $R_L$
   (a) increases (b) decreases (c) stays the same

5. If the frequency of the voltage sources is increased, the current through $R_L$
   (a) increases (b) decreases (c) stays the same

**Refer to Figure 19–53.**

6. If the frequency of the source voltage is increased, the voltage across $R_1$
   (a) increases (b) decreases (c) stays the same
7. If $R_L$ opens, the voltage across it
   (a) increases (b) decreases (c) stays the same

**Refer to Figure 19–54.**

8. If the source frequency is increased, the voltage across $R_3$
   (a) increases (b) decreases (c) stays the same
9. If the capacitor value is reduced, the current from the source
   (a) increases (b) decreases (c) stays the same

**Refer to Figure 19–57.**

10. If $R_2$ opens, the current from the current source
    (a) increases (b) decreases (c) stays the same
11. If the frequency of both voltage sources increase in exactly the same way, $X_{C2}$
    (a) increases (b) decreases (c) stays the same
12. If the load is removed, the voltage across $R_3$
    (a) increases (b) decreases (c) stays the same
13. If the load is removed, the voltage across $R_2$
    (a) increases (b) decreases (c) stays the same

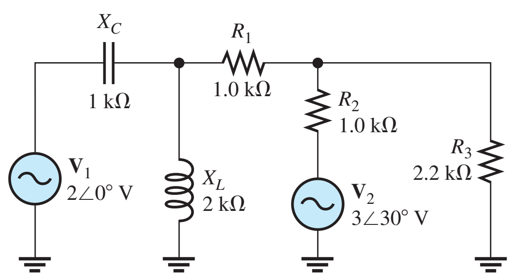
## PROBLEMS

**More difficult problems are indicated by an asterisk (*).**
**Answers to odd-numbered problems are at the end of the book.**

### SECTION 19–1 The Superposition Theorem

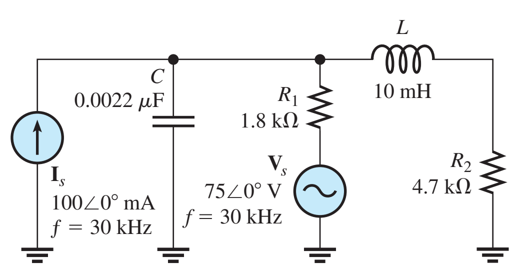
1. Using the superposition method, calculate the current through $R_3$ in Figure 19–47.
2. Use the superposition theorem to find the current in and the voltage across the $R_2$ branch of Figure 19–47.

▶ FIGURE 19–47

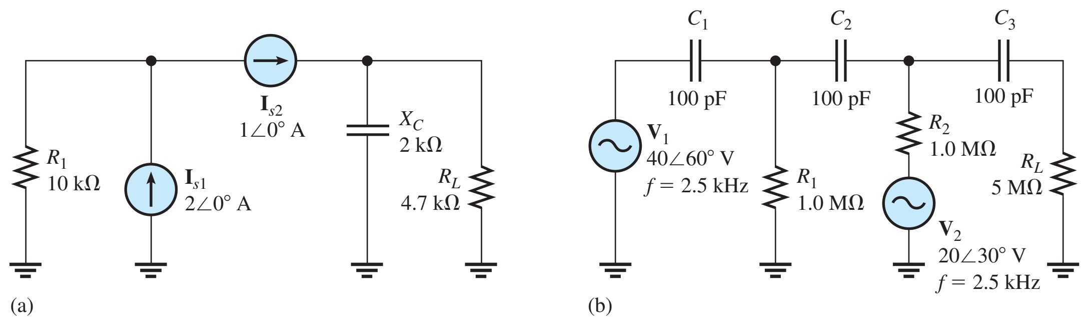



3. Using the superposition theorem, solve for the current through $R_1$ in Figure 19–48.

▶ FIGURE 19–48



**4.** Using the superposition theorem, find the current through $R_L$ in each circuit of Figure 19–49.

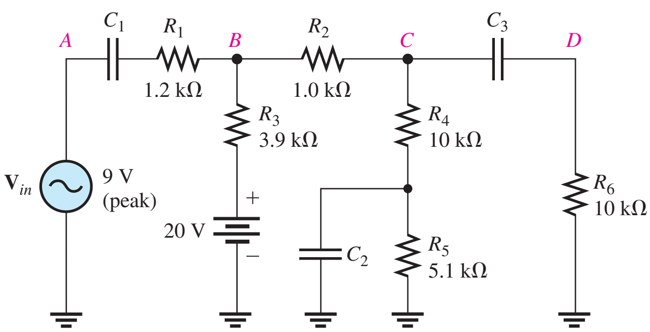



▲ FIGURE 19–49

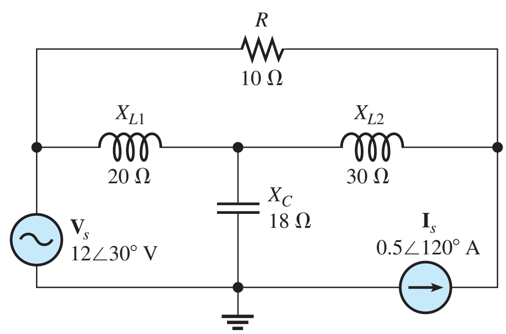
***5.** Determine the voltage at each point ($A$, $B$, $C$, $D$) in Figure 19–50. Assume $X_C = 0$ for all capacitors. Draw the voltage waveforms at each of the points.

***6.** Use the superposition theorem to find the capacitor current in Figure 19–51.

***7.** Use the superposition theorem to find the resistor current in Figure 19–51.




▲ FIGURE 19–50

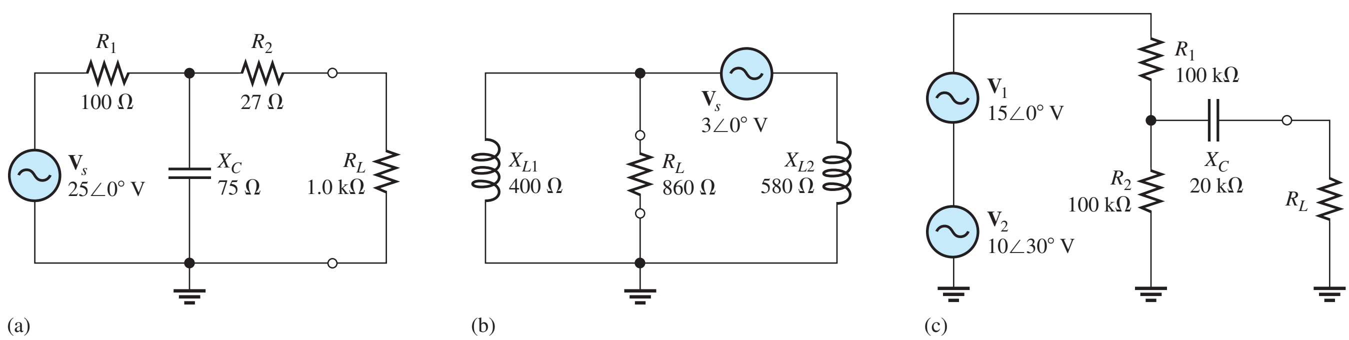



▲ FIGURE 19–51

## SECTION 19–2 Thevenin's Theorem

***8.** Using Thevenin's theorem, determine the ac equivalent circuit for the circuit in Figure 19–50 as viewed from $R_6$. Assume all $X_C = 0$.

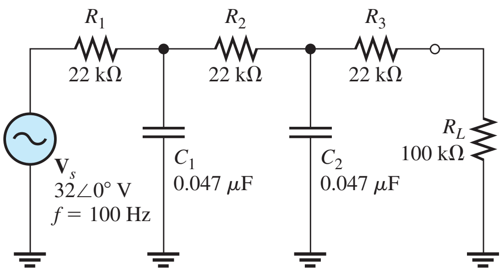
**9.** For each circuit in Figure 19–52, determine the Thevenin equivalent circuit for the portion of the circuit viewed by $R_L$.



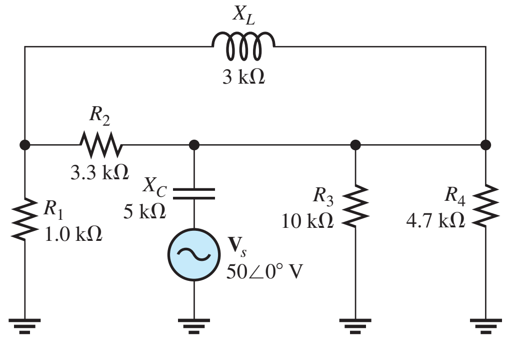

▲ FIGURE 19–52

**10.** Using Thevenin's theorem, determine the current through the load $R_L$ in Figure 19–53.

***11.** Using Thevenin's theorem, find the voltage across $R_4$ in Figure 19–54.

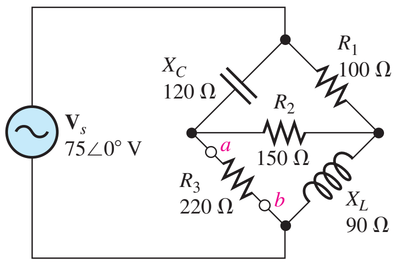



▲ FIGURE 19–53




▲ FIGURE 19–54

***12.** Simplify the circuit external to $R_3$ in Figure 19–55 to its Thevenin equivalent.




▲ FIGURE 19–55

## SECTION 19–3 Norton's Theorem

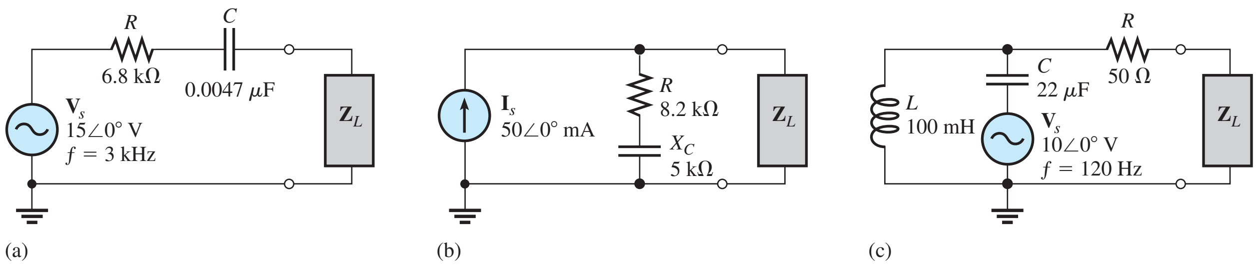
**13.** For each circuit in Figure 19–52, determine the Norton equivalent as seen by $R_L$.

**14.** Using Norton's theorem, find the current through the load resistor $R_L$ in Figure 19–53.

***15.** Using Norton's theorem, find the voltage across $R_4$ in Figure 19–54.

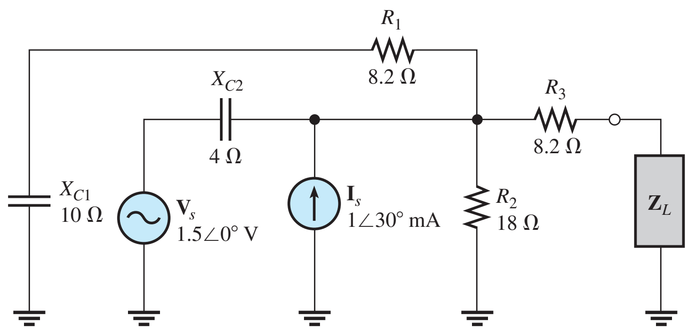
## SECTION 19–4 Maximum Power Transfer Theorem

**16.** For each circuit in Figure 19–56, maximum power is to be transferred to the load $R_L$. Determine the appropriate value for the load impedance in each case.




▲ FIGURE 19–56

*17. Determine $\mathbf{Z}_L$ for maximum power in Figure 19–57.

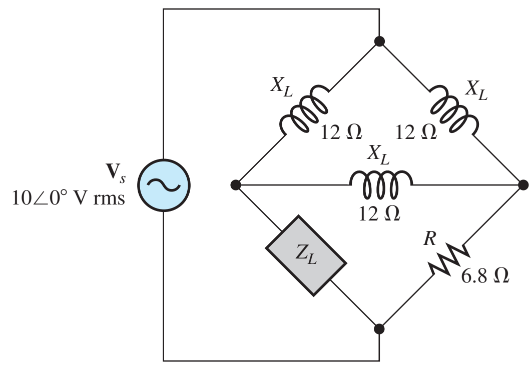



▲ FIGURE 19–57

18. Find the load impedance required for maximum power transfer to $Z_L$ in Figure 19–58. Determine the maximum true power.

*19. A load is to be connected in the place of $R_2$ in Figure 19–55 to achieve maximum power transfer. Determine the type of load, and express it in rectangular form.

► FIGURE 19–58




## Multisim Troubleshooting and Analysis

These problems require Multisim.

20. Open file P19-20 and determine if there is a fault. If so, find the fault.
21. Open file P19-21 and determine if there is a fault. If so, find the fault.
22. Open file P19-22 and determine if there is a fault. If so, find the fault.
23. Open file P19-23 and determine if there is a fault. If so, find the fault.
24. Open file P19-24 and determine the Thevenin equivalent circuit by measurement looking from Point $A$.
25. Open file P19-25 and determine the Norton equivalent circuit by measurement looking from Point $A$.

# ANSWERS

## SECTION CHECKUPS

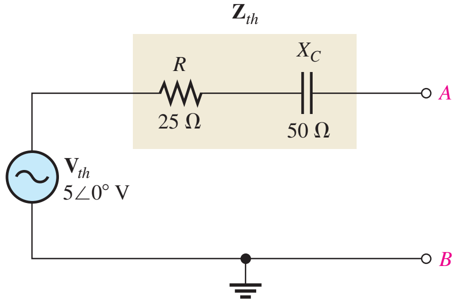
### SECTION 19–1 The Superposition Theorem

1. AC sources add as phasors; this includes the magnitude and the angle.
2. The circuit can be analyzed one source at a time using superposition.
3. $I_R = 6.01\angle -48.7^\circ$ mA

## SECTION 19–2 Thevenin's Theorem

1. The components of a Thevenin equivalent ac circuit are equivalent voltage source and equivalent series impedance.
2. See Figure 19–59.
3. $\mathbf{Z}_{th} = 21.5\ \Omega - j15.7\ \Omega$; $\mathbf{V}_{th} = 4.14\angle 53.7°\ \text{V}$

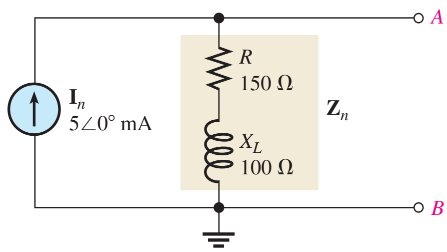
▶ FIGURE 19–59




## SECTION 19–3 Norton's Theorem

1. See Figure 19–60.
2. $\mathbf{Z}_n = R\angle 0° = 1.2\angle 0°\ \text{k}\Omega$; $\mathbf{I}_n = 10\angle 0°\ \text{mA}$

▶ FIGURE 19–60



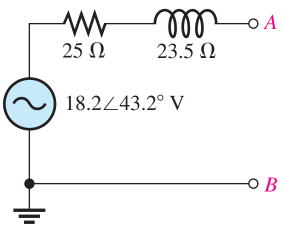

## SECTION 19–4 Maximum Power Transfer Theorem

1. $\mathbf{Z}_L = 50\ \Omega + j10\ \Omega$
2. $P_L = 200\ \text{W}$

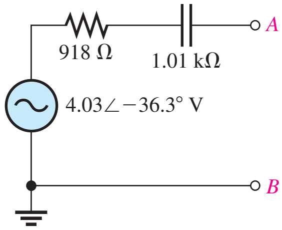
## RELATED PROBLEMS FOR EXAMPLES

**19–1** $2.16\angle -153°$ mA
**19–2** $30\angle 90°$ mA
**19–3** $1.69\angle 47.3°$ mA riding on a dc level of 3 mA
**19–4** $18.2\angle 43.2°$ V
**19–5** $4.03\angle -36.3°$ V
**19–6** $4.55\angle 24.4°$ V
**19–7** $34.2\angle 43.2°\ \Omega$
**19–8** $1.37\angle -47.8°\ \text{k}\Omega$
**19–9** $9.10\angle -65.6°\ \text{k}\Omega$

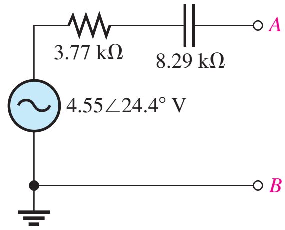
**19–10** See Figure 19–61.
**19–11** See Figure 19–62.
**19–12** See Figure 19–63.




▲ FIGURE 19–61



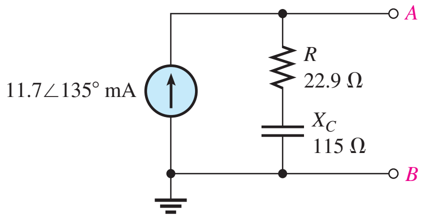

▲ FIGURE 19–62




▲ FIGURE 19–63

**19–13** $11.7\angle 135°$ mA
**19–14** $117\angle -78.7°\ \Omega$
**19–15** See Figure 19–64.

▶ FIGURE 19–64




**19–16** $47\ \Omega + j72.3\ \Omega$
**19–17** 503 Hz

## TRUE/FALSE QUIZ

**1.** T **2.** F **3.** T **4.** T **5.** F
**6.** T **7.** T **8.** F **9.** F **10.** T

## SELF-TEST

**1.** (d) **2.** (c) **3.** (a) **4.** (a) **5.** (c) **6.** (c) **7.** (b) **8.** (d) **9.** (d)

## CIRCUIT DYNAMICS QUIZ

**1.** (c) **2.** (a) **3.** (c) **4.** (b) **5.** (a) **6.** (a) **7.** (a)
**8.** (a) **9.** (b) **10.** (c) **11.** (b) **12.** (b) **13.** (a)