

## CHAPTER OUTLINE

- 18–1 Low-Pass Filters
- 18–2 High-Pass Filters
- 18–3 Band-Pass Filters
- 18–4 Band-Stop Filters
- Application Activity

## CHAPTER OBJECTIVES

- ▶ Analyze the operation of  $RC$  and  $RL$  low-pass filters
- ▶ Analyze the operation of  $RC$  and  $RL$  high-pass filters
- ▶ Analyze the operation of band-pass filters
- ▶ Analyze the operation of band-stop filters

## KEY TERMS

- |                                |                              |
|--------------------------------|------------------------------|
| ▶ Low-pass filter              | ▶ Bode plot                  |
| ▶ Passband                     | ▶ High-pass filter           |
| ▶ Critical frequency ( $f_c$ ) | ▶ Band-pass filter           |
| ▶ Roll-off                     | ▶ Center frequency ( $f_0$ ) |
| ▶ Attenuation                  | ▶ Band-stop filter           |
| ▶ Decade                       |                              |

## APPLICATION ACTIVITY PREVIEW

In the application activity, you will plot the frequency responses of filters based on oscilloscope measurements and identify the types of filters.

## VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at <http://www.pearsonhighered.com/careersresources/>

## INTRODUCTION

The concept of filters was introduced in Chapters 15, 16, and 17 to illustrate applications of  $RC$ ,  $RL$ , and  $RLC$  circuits. This chapter is essentially an extension of the earlier material and provides additional coverage of the important topic of filters.

Passive filters are discussed in this chapter. Passive filters use various combinations of resistors, capacitors, and inductors. In a later course, you will study active filters that use passive components combined with amplifiers. You have already seen how basic  $RC$ ,  $RL$ , and  $RLC$  circuits can be used as filters. Now, you will learn that passive filters can be placed in four general categories according to their response characteristics: low-pass, high-pass, band-pass, and band-stop. Within each category, there are several common types that will be examined.

## 18–1 LOW-PASS FILTERS

A **low-pass filter** allows signals with lower frequencies to pass from input to output while rejecting higher frequencies.

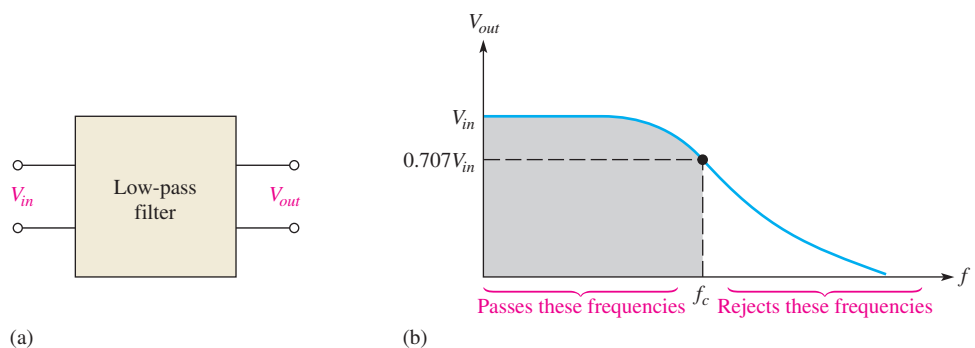
After completing this section, you should be able to

- ◆ **Analyze the operation of *RC* and *RL* low-pass filters**
  - ◆ Express the voltage and power ratios of a filter in decibels
  - ◆ Determine the critical frequency of a low-pass filter
  - ◆ Explain the difference between actual and ideal low-pass response curves
  - ◆ Define *roll-off*
  - ◆ Generate a Bode plot for a low-pass filter
  - ◆ Discuss phase shift in a low-pass filter

Figure 18–1 shows a block diagram and a general response curve for a low-pass filter. The range of frequencies passed by a filter within specified limits is called the **passband** of the filter. The point considered to be the upper end of the passband is at the critical frequency,  $f_c$ , as illustrated in Figure 18–1(b). The **critical frequency ( $f_c$ )** is the frequency at which the filter's output voltage is 70.7% of the maximum. The filter's critical frequency is also called the *cutoff frequency*, *break frequency*, or *–3 dB frequency* because the output voltage is down 3 dB from its passband amplitude at this frequency. The term *dB (decibel)* is a commonly used unit in filter measurements.

► **FIGURE 18–1**

Low-pass filter block diagram and general response curve.



### Decibels

The basis for the decibel unit stems from the logarithmic response of the human ear to the intensity of sound. The **decibel** is a logarithmic measurement of the ratio of one power to another or one voltage to another, which can be used to express the input-to-output relationship of a filter. The following equation expresses a power ratio in decibels:

Equation 18–1

$$\text{dB} = 10 \log \left( \frac{P_{out}}{P_{in}} \right)$$

where  $\log$  represents the base 10 logarithm.

From the properties of logarithms, the following decibel formula for a voltage ratio is derived. (This equation is based on the assumption that  $V_{in}$  and  $V_{out}$  are measured across the same value of resistance.)

$$\text{dB} = 20 \log\left(\frac{V_{out}}{V_{in}}\right) \quad \text{Equation 18-2}$$

### EXAMPLE 18-1

At a certain frequency, the output voltage of a filter is 5 V and the input is 10 V. Express the voltage ratio in decibels.

**Solution** 
$$20 \log\left(\frac{V_{out}}{V_{in}}\right) = 20 \log\left(\frac{5 \text{ V}}{10 \text{ V}}\right) = 20 \log(0.5) = -6.02 \text{ dB}$$

**Related Problem\*** Express the ratio  $V_{out}/V_{in} = 0.85$  in decibels.

\*Answers are at the end of the chapter.

### RC Low-Pass Filter

A basic RC low-pass filter is shown in Figure 18-2. Notice that the output voltage is taken across the capacitor.

When the input is dc (0 Hz), the output voltage equals the input voltage because  $X_C$  is infinitely large. As the input frequency is increased,  $X_C$  decreases and, as a result,  $V_{out}$  gradually decreases until a frequency is reached where  $X_C = R$ . This is the critical frequency,  $f_c$ , of the filter.

$$X_C = \frac{1}{2\pi f_c C} = R$$

Solving for  $f_c$ ,

$$f_c = \frac{1}{2\pi RC} \quad \text{Equation 18-3}$$

At any frequency, by application of the voltage-divider formula, the output voltage magnitude is

$$V_{out} = \left(\frac{X_C}{\sqrt{R^2 + X_C^2}}\right) V_{in}$$

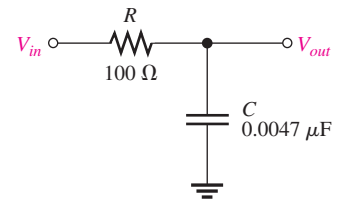
Since  $X_C = R$  at  $f_c$ , the output voltage at the critical frequency can be expressed as

$$V_{out} = \left(\frac{R}{\sqrt{R^2 + R^2}}\right) V_{in} = \left(\frac{R}{\sqrt{2R^2}}\right) V_{in} = \left(\frac{R}{R\sqrt{2}}\right) V_{in} = \left(\frac{1}{\sqrt{2}}\right) V_{in} = 0.707 V_{in}$$

These calculations show that the output is 70.7% of the input when  $X_C = R$ . The frequency at which this occurs is, by definition, the critical frequency.

The ratio of output voltage to input voltage at the critical frequency can be expressed in decibels as follows:

$$\begin{aligned} V_{out} &= 0.707 V_{in} \\ \frac{V_{out}}{V_{in}} &= 0.707 \\ 20 \log\left(\frac{V_{out}}{V_{in}}\right) &= 20 \log(0.707) = -3.01 \text{ dB} \end{aligned}$$



▲ FIGURE 18-2

**EXAMPLE 18–2**

Determine the critical frequency and  $V_{out}$  at  $f_c$  for an input of 10 V for the  $RC$  low-pass filter in Figure 18–2.

*Solution*

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(100\ \Omega)(0.0047\ \mu\text{F})} = 339\ \text{kHz}$$

The output voltage is 3 dB below  $V_{in}$  at this frequency ( $V_{out}$  has a maximum value of  $V_{in}$ ).

$$V_{out} = 0.707 V_{in} = 7.07\ \text{V}$$

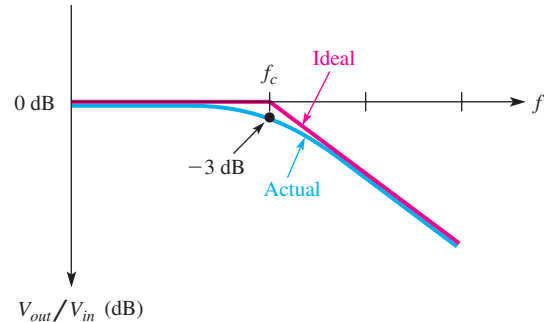
*Related Problem* A certain  $RC$  low-pass filter has  $R = 1.0\ \text{k}\Omega$  and  $C = 0.022\ \mu\text{F}$ . Determine its critical frequency.

### Roll-Off of the Response Curve

The blue line in Figure 18–3 shows an actual response curve for a low-pass filter. The maximum output is defined to be 0 dB as a reference. Zero decibels corresponds to  $V_{out} = V_{in}$  because  $20 \log(V_{out}/V_{in}) = 20 \log 1 = 0\ \text{dB}$ . The output drops from 0 dB to –3 dB at the critical frequency and then continues to decrease at a fixed rate. This pattern of decrease is called the **roll-off** of the frequency response. The red line shows an ideal output response that is considered to be “flat” out to the critical frequency. The output then decreases at the fixed rate.

► **FIGURE 18–3**

Actual and ideal response curves for a low-pass filter.



As you have seen, the output voltage of a low-pass filter decreases by 3 dB when the frequency is increased to the critical value  $f_c$ . As the frequency continues to increase above  $f_c$ , the output voltage continues to decrease. In fact, for each tenfold increase in frequency above  $f_c$ , there is a 20 dB reduction in the output, as shown in the following steps.

Let's take a frequency that is ten times the critical frequency ( $f = 10f_c$ ). Since  $R = X_C$  at  $f_c$ , then  $R = 10X_C$  at  $10f_c$  because of the inverse relationship of  $X_C$  and  $f$ .

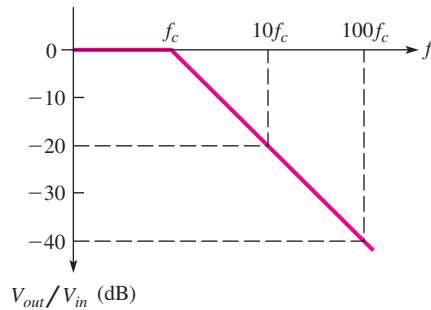
The **attenuation** is the reduction in voltage expressed as the ratio  $V_{out}/V_{in}$  and is developed as follows:

$$\begin{aligned} \frac{V_{out}}{V_{in}} &= \frac{X_C}{\sqrt{R^2 + X_C^2}} = \frac{X_C}{\sqrt{(10X_C)^2 + X_C^2}} \\ &= \frac{X_C}{\sqrt{100X_C^2 + X_C^2}} = \frac{X_C}{\sqrt{X_C^2(100 + 1)}} = \frac{X_C}{X_C\sqrt{101}} = \frac{1}{\sqrt{101}} \cong \frac{1}{10} = 0.1 \end{aligned}$$

The dB attenuation is

$$20 \log\left(\frac{V_{out}}{V_{in}}\right) = 20 \log(0.1) = -20\ \text{dB}$$

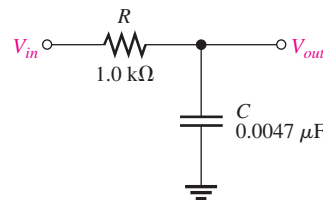
A tenfold change in frequency is called a **decade**. So, for an  $RC$  circuit, the output voltage is reduced by 20 dB for each decade increase in frequency. A similar result can be derived for a high-pass circuit. The roll-off is a constant  $-20$  dB/decade for a basic  $RC$  or  $RL$  filter. Figure 18–4 shows the ideal frequency response plot on a semilog scale, where each interval on the horizontal axis represents a tenfold increase in frequency. This response curve is called a **Bode plot**. There are two versions of the Bode plot, a magnitude plot (illustrated in Figure 18–4), and a phase plot. Notice that the ratio of  $V_{out}/V_{in}$  in dB is plotted on the  $y$ -axis and frequency is plotted on the  $x$ -axis.



**▲ FIGURE 18–4**  
Frequency roll-off for an  $RC$  low-pass filter (Bode plot).

### EXAMPLE 18–3

Make a Bode plot for the filter in Figure 18–5 for three decades of frequency. Use semilog graph paper.



**▲ FIGURE 18–5**

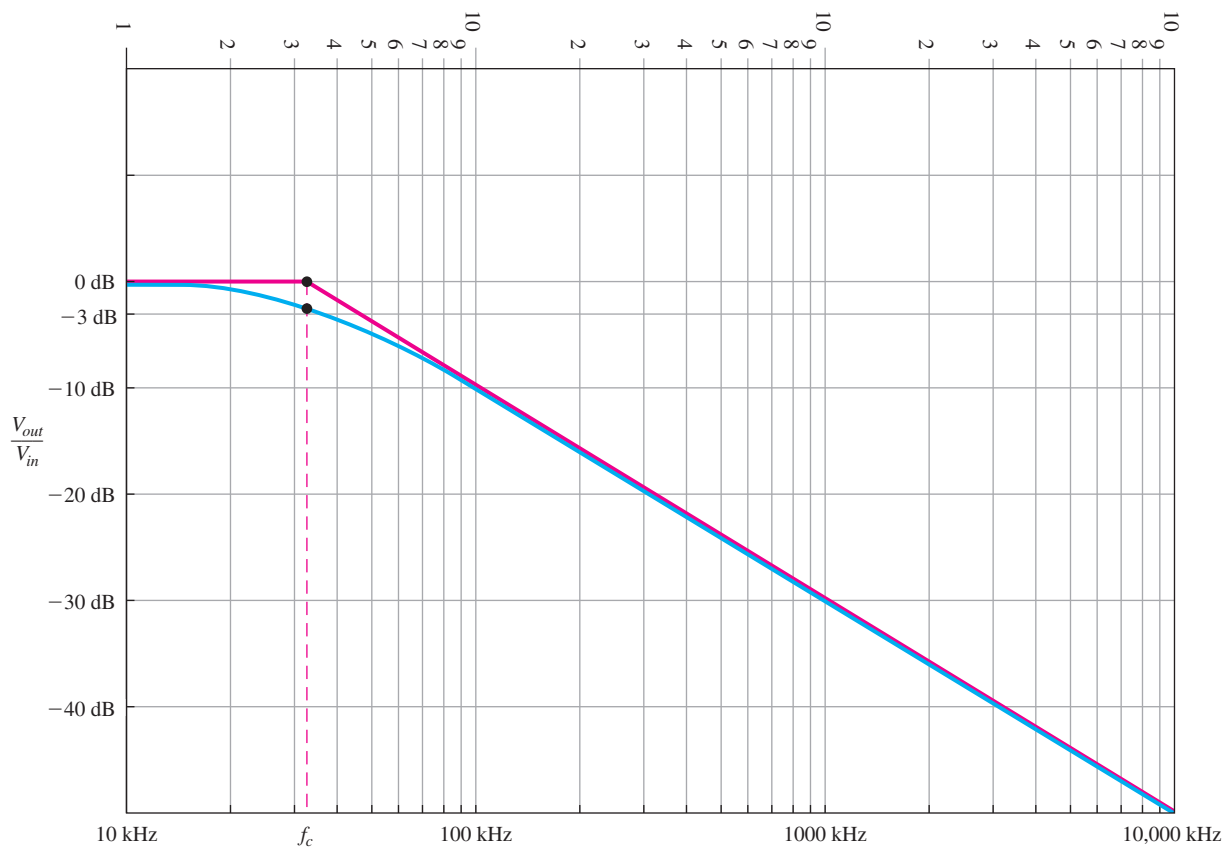
**Solution** The critical frequency for this low-pass filter is

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(1.0 \text{ k}\Omega)(0.0047 \text{ }\mu\text{F})} = 33.9 \text{ kHz}$$

The idealized Bode plot is shown with the red line on the semilog graph in Figure 18–6. The approximate actual response curve is shown with the blue line. Notice first that the horizontal scale is logarithmic and the vertical scale is linear. The frequency is on the logarithmic scale, and the filter output in decibels is on the linear scale.

The output is flat below  $f_c$  (33.9 kHz). As the frequency is increased above  $f_c$ , the output drops at a  $-20$  dB/decade rate. Thus, for the ideal curve, every time the frequency is increased by ten, the output is reduced by 20 dB. A slight variation from this occurs in actual practice. The output is actually at  $-3$  dB rather than 0 dB at the critical frequency.



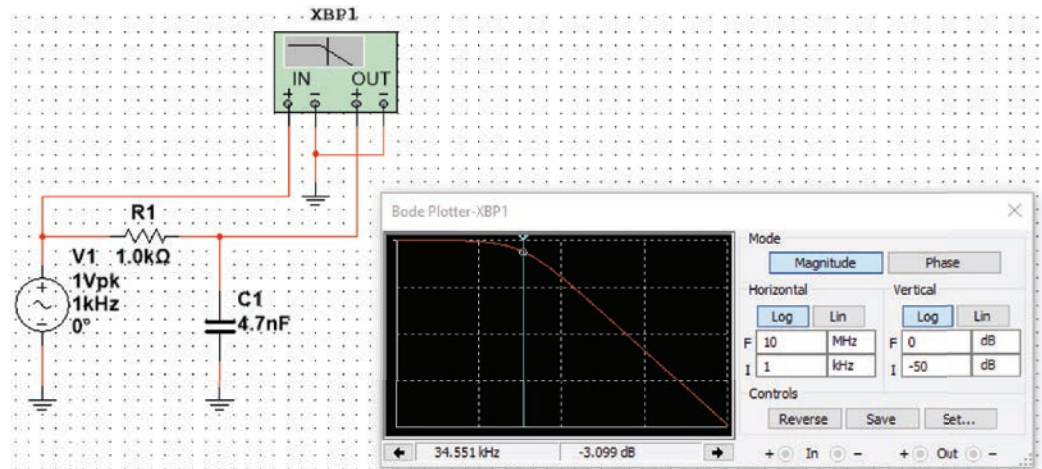


▲ **FIGURE 18-6**

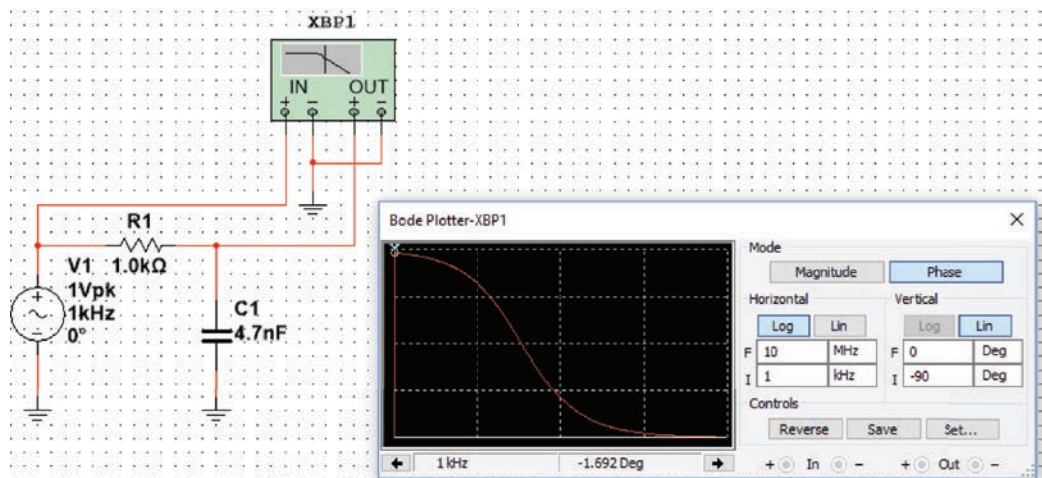
Bode plot for Figure 18-5. The red line represents the ideal response curve and the blue line represents the actual response.

If you have the Multisim program available, it is easy to obtain a Bode plot using a fictitious instrument called the Bode Plotter. Figure 18-7 shows the circuit with the Bode Plotter connected across the input and output and set for a range of 1 kHz to 10 MHz. It is not necessary to set the ac source to any particular frequency. Move the plotter's vertical cursor (on the left edge of the display) to read the frequency and the ratio of the output magnitude to input magnitude (in dB) or phase at any frequency on the plot. In Figure 18-7(a), the cursor is set approximately to the cutoff frequency. You can also plot the phase angle between the input and output by selecting the Phase button as shown in Figure 18-7(b).

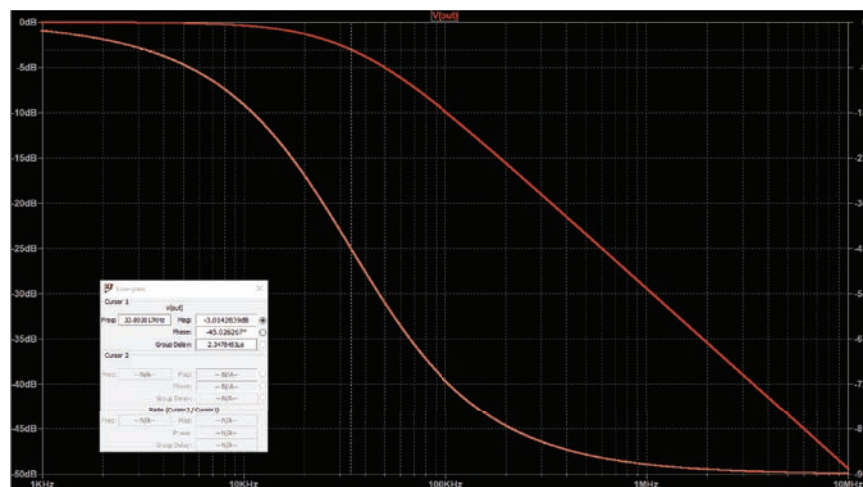
You can also obtain a Bode plot with LTSpice. Figure 18-7(c) shows an LTSpice simulation of the circuit with both the magnitude and phase shown in the same plot.



(a) Multisim Bode magnitude plot



(b) Multisim Bode phase plot

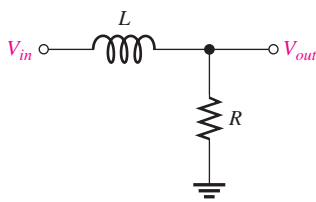


(c) LTSpice Bode magnitude and phase plot

### ▲ FIGURE 18-7

Computer Bode plots for the circuit in Figure 18-5.

**Related Problem** What happens to the critical frequency and roll-off rate if  $C$  is reduced to 0.001  $\mu\text{F}$  in Figure 18-5?



▲ FIGURE 18–8

RL low-pass filter.

## Equation 18–4

**RL Low-Pass Filter**

A basic  $RL$  low-pass filter is shown in Figure 18–8. Notice that the output voltage is taken across the resistor.

When the input is dc (0 Hz), the output voltage ideally equals the input voltage because  $X_L$  is a short (if  $R_W$  is neglected). As the input frequency is increased,  $X_L$  increases and, as a result,  $V_{out}$  gradually decreases until the critical frequency is reached. At this point,  $X_L = R$  and the frequency is

$$2\pi f_c L = R$$

$$f_c = \frac{R}{2\pi L}$$

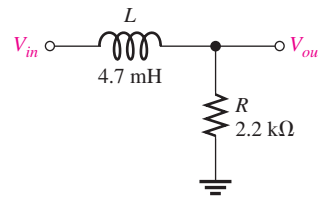
$$f_c = \frac{1}{2\pi(L/R)}$$

Just as in the  $RC$  low-pass filter,  $V_{out} = 0.707V_{in}$  and, thus, the output voltage is  $-3$  dB below the input voltage at the critical frequency.

**EXAMPLE 18–4**

Make a Bode plot for the filter in Figure 18–9 for three decades of frequency. Use semilog graph paper.

► FIGURE 18–9



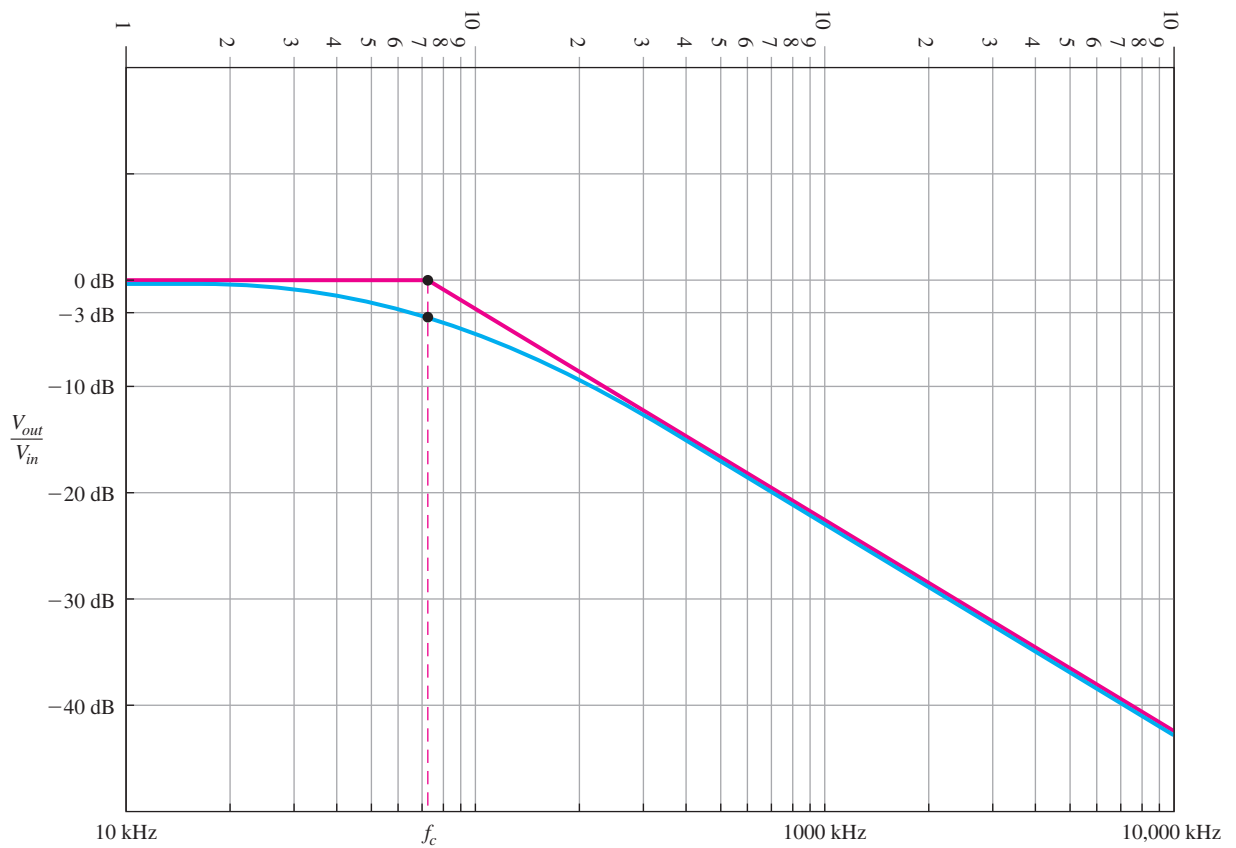
**Solution** The critical frequency for this low-pass filter is

$$f_c = \frac{1}{2\pi(L/R)} = \frac{1}{2\pi(4.7 \text{ mH}/2.2 \text{ k}\Omega)} = 74.5 \text{ kHz}$$

The idealized Bode plot is shown with the red line on the semilog graph in Figure 18–10. The approximate actual response curve is shown with the blue line. Notice first that the horizontal scale is logarithmic and the vertical scale is linear. The frequency is on the logarithmic scale, and the filter output in decibels is on the linear scale.

The output is flat below  $f_c$  (74.5 kHz). As the frequency is increased above  $f_c$ , the output drops at a  $-20$  dB/decade rate. Thus, for the ideal curve, every time the frequency is increased by ten, the output is reduced by 20 dB. A slight variation from this occurs in actual practice. The output is actually at  $-3$  dB rather than 0 dB at the critical frequency.





▲ FIGURE 18-10

Bode plot for Figure 18-9. The red line is the ideal response curve and the blue line is the actual response.

#### Related Problem

What happens to the critical frequency and roll-off rate if  $L$  is reduced to 1 mH in Figure 18-9?



Use Multisim files E18-04A and E18-04B to verify the calculated results in this example and to confirm your calculations for the related problem.

## Phase Shift in a Low-Pass Filter

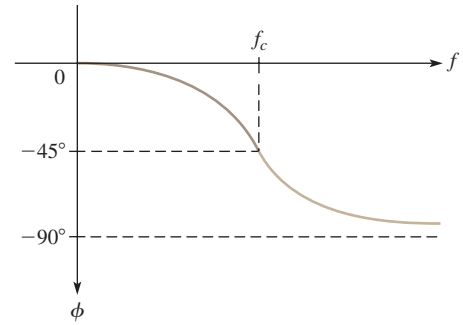
The  $RC$  low-pass filter acts as a lag circuit. Recall from Chapter 15 that the phase shift from input to output is expressed as

$$\phi = -\tan^{-1}\left(\frac{R}{X_C}\right)$$

At the critical frequency,  $X_C = R$  and, therefore,  $\phi = -45^\circ$ . As the input frequency is reduced,  $\phi$  decreases and approaches  $0^\circ$  when the frequency approaches zero. Figure 18-11, illustrates this phase characteristic.

► FIGURE 18–11

Phase characteristic of a low-pass filter.



The  $RL$  low-pass filter also acts as a lag circuit. Recall from Chapter 16 that the phase shift is expressed as

$$\phi = -\tan^{-1}\left(\frac{X_L}{R}\right)$$

As in the  $RC$  filter, the phase shift from input to output is  $-45^\circ$  at the critical frequency and decreases for frequencies below  $f_c$ .

### SECTION 18–1 CHECKUP

Answers are at the end of the chapter.

1. In a certain low-pass filter,  $f_c = 2.5$  kHz. What is its passband?
2. In a certain low-pass filter,  $R = 100\ \Omega$  and  $X_C = 2\ \Omega$  at a frequency,  $f_1$ . Determine  $V_{out}$  at  $f_1$  when  $V_{in} = 5\angle 0^\circ$  V rms.
3. Express the attenuation of the filter in question 2 in dB at the frequency  $f_1$ .
4.  $V_{out} = 400$  mV, and  $V_{in} = 1.2$  V. Express the ratio  $V_{out}/V_{in}$  in dB.

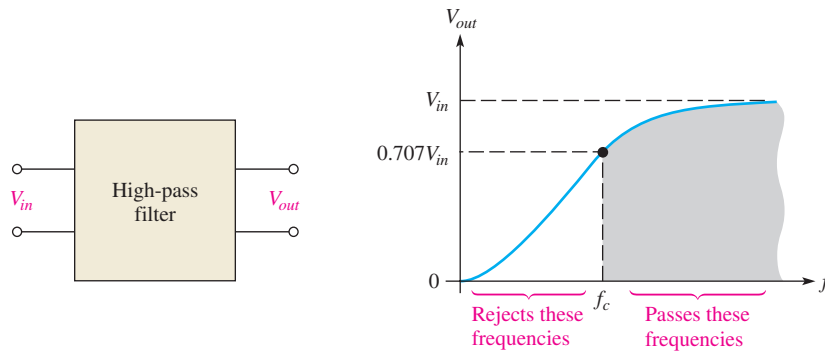
## 18–2 HIGH-PASS FILTERS

A **high-pass filter** allows signals with higher frequencies to pass from input to output while rejecting lower frequencies.

After completing this section, you should be able to

- ◆ **Analyze the operation of  $RC$  and  $RL$  high-pass filters**
  - ◆ Determine the critical frequency of a high-pass filter
  - ◆ Explain the difference between actual and ideal response curves
  - ◆ Generate a Bode plot for a high-pass filter
  - ◆ Discuss phase shift in a high-pass filter

Figure 18–12 shows a block diagram and a general response curve for a high-pass filter. The frequency considered to be the lower end of the passband is called the *critical frequency*. Just as in the low-pass filter, it is the frequency at which the output is 70.7% of the passband amplitude as indicated in the figure.

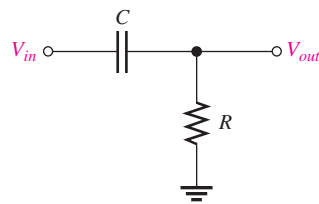


▲ FIGURE 18-12

High-pass filter block diagram and response curve.

### RC High-Pass Filter

A basic  $RC$  high-pass filter is shown in Figure 18-13. Notice that the output voltage is taken across the resistor.



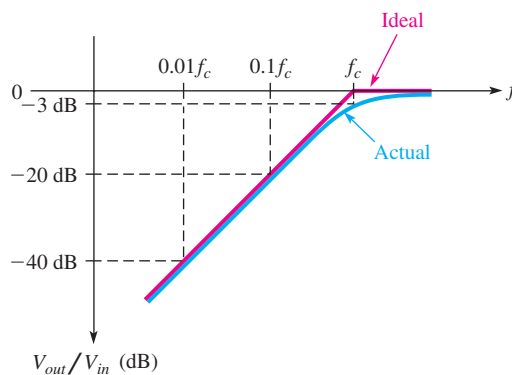
◀ FIGURE 18-13

 $RC$  high-pass filter.

When the input frequency is at its critical value,  $X_C = R$  and the output voltage is  $0.707V_{in}$ , just as in the case of the low-pass filter. As the input frequency increases above the critical frequency,  $X_C$  decreases and, as a result, the output voltage increases and approaches a value equal to  $V_{in}$ . The expression for the critical frequency of the high-pass filter is the same as for the low-pass filter.

$$f_c = \frac{1}{2\pi RC}$$

Below  $f_c$ , the output voltage decreases (rolls off) at a rate of  $-20$  dB/decade. Figure 18-14 shows an actual and an ideal response curve for a high-pass filter.



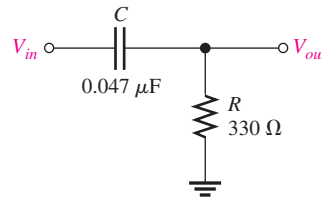
▲ FIGURE 18-14

Actual and ideal response curves for a high-pass filter.

## EXAMPLE 18–5

Draw a Bode plot for the filter in Figure 18–15 for three decades of frequency. Use semilog graph paper.

▶ FIGURE 18–15

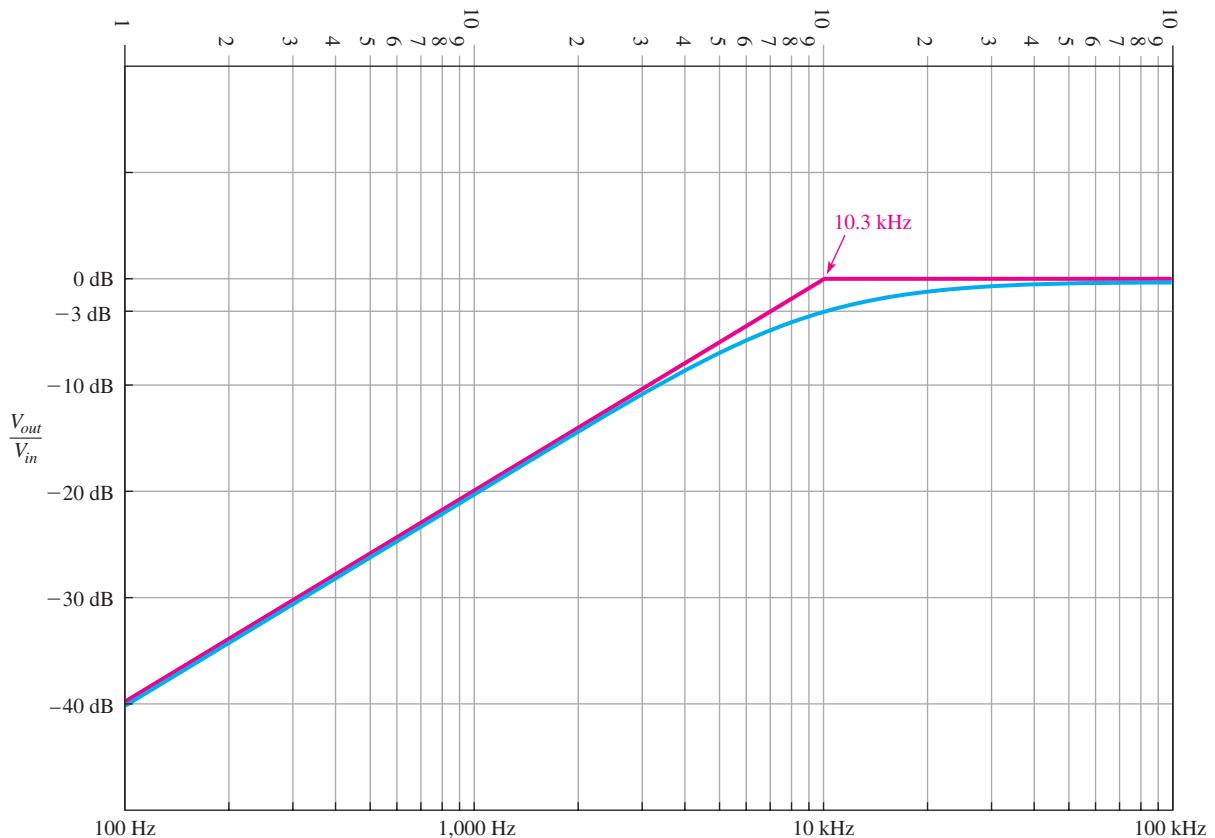


**Solution** The critical frequency for this high-pass filter is

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(330\ \Omega)(0.047\ \mu\text{F})} = 10.3\ \text{kHz} \approx 10\ \text{kHz}$$

The idealized Bode plot is shown with the red line on the semilog graph in Figure 18–16. The approximate actual response curve is shown with the blue line. Notice first that the horizontal scale is logarithmic and the vertical scale is linear. The frequency is on the logarithmic scale, and the filter output in decibels is on the linear scale.

The output is flat above  $f_c$  (approximately 10 kHz). As the frequency is reduced below  $f_c$ , the output drops at a  $-20\ \text{dB/decade}$  rate. Thus, for the ideal curve, every time the frequency is reduced by ten, the output is reduced by 20 dB. A slight variation from this occurs in actual practice. The output is actually at  $-3\ \text{dB}$  rather than 0 dB at the critical frequency.



▲ FIGURE 18–16

Bode plot for Figure 18–15. The red line is the ideal response curve and the blue line is the actual response curve.

**Related Problem**

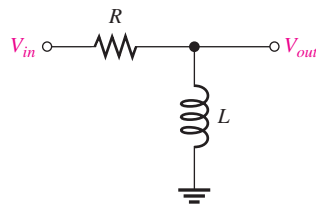
If the input frequency for the high-pass filter is decreased to 10 Hz, what is the output to input ratio in decibels?



Use Multisim file E18-05 to verify the calculated results in this example and to confirm your calculation for the related problem.

## RL High-Pass Filter

A basic  $RL$  high-pass filter is shown in Figure 18–17. Notice that the output is taken across the inductor.



◀ **FIGURE 18–17**

$RL$  high-pass filter.

When the input frequency is at its critical value,  $X_L = R$ , and the output voltage is  $0.707V_{in}$ . As the frequency increases above  $f_c$ ,  $X_L$  increases and, as a result, the output voltage increases until it equals  $V_{in}$ . The expression for the critical frequency of the high-pass filter is the same as for the low-pass filter.

$$f_c = \frac{1}{2\pi(L/R)}$$

## Phase Shift in a High-Pass Filter

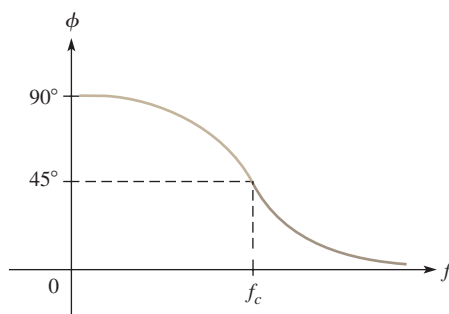
Both the  $RC$  and the  $RL$  high-pass filters act as lead circuits. Recall from Chapters 15 and 16 that the phase shift from input to output for the  $RC$  lead circuit is

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

and the phase shift for the  $RL$  lead circuit is

$$\phi = \tan^{-1}\left(\frac{R}{X_L}\right)$$

At the critical frequency,  $X_L = R$  and, therefore,  $\phi = 45^\circ$ . As the frequency is increased,  $\phi$  decreases toward  $0^\circ$ , as shown in Figure 18–18.



◀ **FIGURE 18–18**

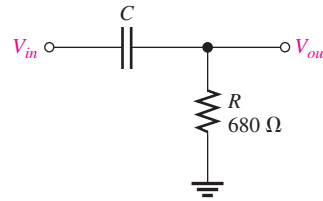
Phase characteristic of a high-pass filter.



## EXAMPLE 18–6

- (a) In Figure 18–19, find the value of  $C$  so that  $X_C$  is approximately ten times less than  $R$  at an input frequency of 10 kHz.
- (b) If a 5.0 V sine wave with a dc level of 10 V is applied, what are the output voltage magnitude and the phase shift?

► FIGURE 18–19



**Solution** (a) Determine the value of  $C$  as follows:

$$X_C = 0.1R = 0.1(680\ \Omega) = 68\ \Omega$$

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(10\ \text{kHz})(68\ \Omega)} = \mathbf{0.234\ \mu\text{F}}$$

Using the nearest standard value of  $C = 0.22\ \mu\text{F}$ ,

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(10\ \text{kHz})(0.22\ \mu\text{F})} = 72\ \Omega$$

- (b) Determine the magnitude of the sinusoidal output using the voltage-divider formula.

$$V_{out} = \left( \frac{R}{\sqrt{R^2 + X_C^2}} \right) V_{in} = \left( \frac{680\ \Omega}{\sqrt{(680\ \Omega)^2 + (72\ \Omega)^2}} \right) 5.0\ \text{V} = \mathbf{4.97\ \text{V}}$$

The phase shift is

$$\phi = \tan^{-1} \left( \frac{X_C}{R} \right) = \tan^{-1} \left( \frac{72\ \Omega}{680\ \Omega} \right) = \mathbf{6.1^\circ}$$

At  $f = 10\ \text{kHz}$ , which is a decade above the critical frequency, the sinusoidal output is almost equal to the input in magnitude, and the phase shift is very small. The 10 V dc level has been filtered out and does not appear at the output.

**Related Problem** Repeat parts (a) and (b) of the example if  $R$  is changed to 220  $\Omega$ .



Use Multisim files E18-06A and E18-06B to verify the calculated results in this example and to confirm your calculations for the related problem.

SECTION 18–2  
CHECKUP

1. The input voltage of a high-pass filter is 1.0 V. What is  $V_{out}$  at the critical frequency?
2. In a certain  $RL$  high-pass filter,  $V_{in} = 10\angle 0^\circ\ \text{V}$ ,  $R = 1.0\ \text{k}\Omega$ , and  $X_L = 15\ \text{k}\Omega$ . Determine  $V_{out}$ .
3. Assume a basic  $RC$  high-pass filter has an attenuation of  $-20\ \text{dB}$  at 200 kHz. What is the critical frequency for the filter?

### 18-3 BAND-PASS FILTERS

A **band-pass filter** allows a certain band of frequencies to pass and attenuates or rejects all frequencies below and above the passband.

After completing this section, you should be able to

- ♦ **Analyze the operation of band-pass filters**
  - ♦ Define *bandwidth*
  - ♦ Show how a band-pass filter is implemented with low-pass and high-pass filters
  - ♦ Explain the series-resonant band-pass filter
  - ♦ Explain the parallel-resonant band-pass filter
  - ♦ Calculate the bandwidth and output voltage of a band-pass filter

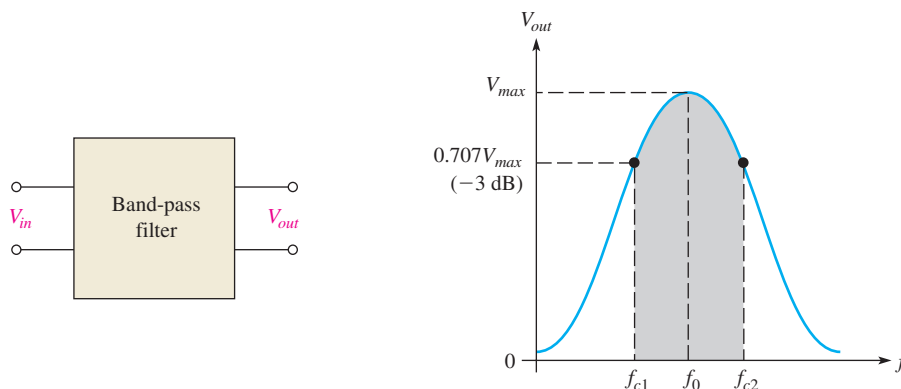
The bandwidth of a band-pass filter is the range of frequencies for which the current, and therefore the output voltage, is equal to or greater than 70.7% of its value at the resonant frequency.

As you know, bandwidth is often abbreviated  $BW$  and can be calculated as

$$BW = f_{c2} - f_{c1}$$

where  $f_{c1}$  is the lower cutoff frequency and  $f_{c2}$  is the upper cutoff frequency.

Figure 18-20 shows a typical band-pass response curve.

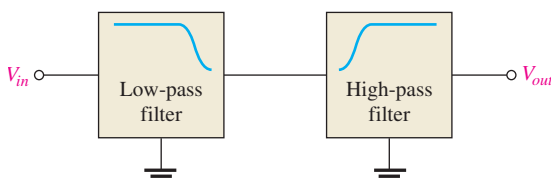


▲ **FIGURE 18-20**

Typical band-pass response curve.

#### Low-Pass/High-Pass Filter

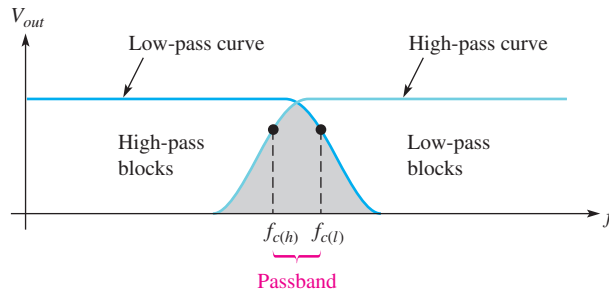
A combination of a low-pass and a high-pass filter can be used to form a band-pass filter, as illustrated in Figure 18-21. The loading effect of the second filter on the first must be taken into account.



◀ **FIGURE 18-21**

A low-pass and a high-pass filter are used to form a band-pass filter.

If the critical frequency of the low-pass filter,  $f_{c(l)}$ , is higher than the critical frequency of the high-pass filter,  $f_{c(h)}$ , the responses overlap. Thus, all frequencies except those between  $f_{c(h)}$  and  $f_{c(l)}$  are eliminated, as shown in Figure 18–22.



▲ FIGURE 18–22

Overlapping response curves of a low-pass/high-pass filter.

### EXAMPLE 18–7

A high-pass filter with  $f_c = 2.0$  kHz and a low-pass filter with  $f_c = 2.5$  kHz are used to construct a band-pass filter. Assuming no loading effect, what is the bandwidth of the passband?

*Solution*

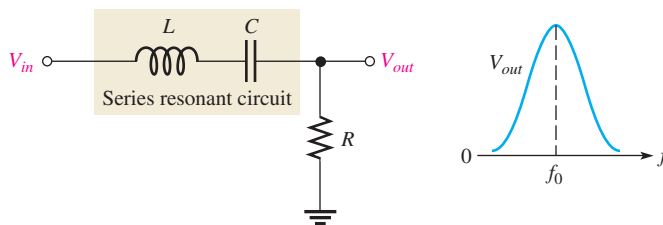
$$BW = f_{c(l)} - f_{c(h)} = 2.5 \text{ kHz} - 2.0 \text{ kHz} = \mathbf{500 \text{ Hz}}$$

*Related Problem*

If  $f_{c(l)} = 9.0$  kHz and the bandwidth is 1.5 kHz, what is  $f_{c(h)}$ ?

### Series Resonant Band-Pass Filter

A basic series resonant band-pass filter is shown in Figure 18–23. As you learned in Chapter 17, a series resonant circuit has minimum impedance and maximum current at the resonant frequency,  $f_r$ . Thus, most of the input voltage is dropped across the resistor at the resonant frequency. Therefore, the output across  $R$  has a band-pass characteristic with a maximum output at the resonant frequency. The resonant frequency is called the **center frequency**,  $f_0$ . The bandwidth is determined by the quality factor,  $Q$ , of the circuit and the resonant frequency, as was discussed in Chapter 17. Recall that  $Q = X_L/R$ , where  $R$  is the resistance in the circuit.



▲ FIGURE 18–23

Series resonant band-pass filter.

A higher value of  $Q$  results in a smaller bandwidth. A lower value of  $Q$  causes a larger bandwidth. A formula for the bandwidth of a resonant circuit in terms of  $Q$  is stated in the following equation:

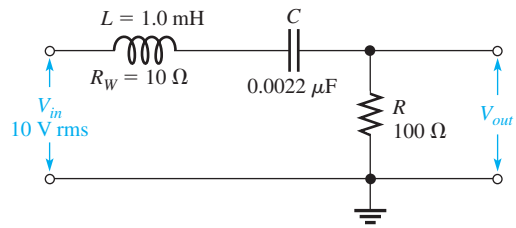
Equation 18–5

$$BW = \frac{f_0}{Q}$$

**EXAMPLE 18–8**

Determine the output voltage magnitude at the center frequency ( $f_0$ ) and the bandwidth for the filter in Figure 18–24.

► **FIGURE 18–24**



**Solution** At  $f_0$ ,  $X_C$  and  $X_L$  cancel and the impedance of the resonant circuit is equal to the winding resistance,  $R_W$ . The total circuit resistance is  $R_L + R_W$ . By the voltage-divider formula,

$$V_{out} = \left( \frac{R_L}{R_L + R_W} \right) V_{in} = \left( \frac{100 \Omega}{110 \Omega} \right) 10 \text{ V} = \mathbf{9.09 \text{ V}}$$

The center frequency is

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1.0 \text{ mH})(0.0022 \mu\text{F})}} = 107 \text{ kHz}$$

At  $f_0$ , the inductive reactance is

$$X_L = 2\pi fL = 2\pi(107 \text{ kHz})(1.0 \text{ mH}) = 674 \Omega$$

and the total resistance is

$$R_{tot} = R_L + R_W = 100 \Omega + 10 \Omega = 110 \Omega$$

Therefore, the circuit  $Q$  is

$$Q = \frac{X_L}{R_{tot}} = \frac{674 \Omega}{110 \Omega} = 6.13$$

The bandwidth is

$$BW = \frac{f_0}{Q} = \frac{107 \text{ kHz}}{6.13} = \mathbf{17.5 \text{ kHz}}$$

**Related Problem** If a 1.0 mH coil with a winding resistance of 18  $\Omega$  replaces the existing coil in Figure 18–24, how is the bandwidth affected?



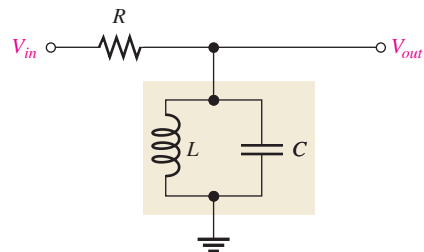
Use Multisim files E18-08A and E18-08B to verify the calculated results in this example and to confirm your answer for the related problem.

## Parallel Resonant Band-Pass Filter

A type of band-pass filter using a parallel resonant circuit is shown in Figure 18–25. Recall that a parallel resonant circuit has maximum impedance at resonance. The circuit in Figure 18–25 acts as a voltage divider. At resonance, the impedance of the tank circuit is much greater than the resistance. Thus, most of the input voltage is across the tank circuit, producing a maximum output voltage at the resonant (center) frequency.

► **FIGURE 18–25**

Parallel resonant band-pass filter.

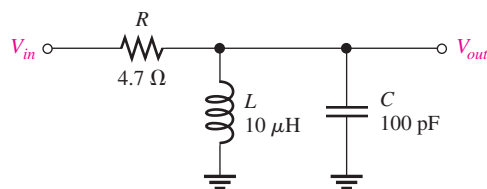


For frequencies above or below resonance, the tank circuit impedance drops off, and more of the input voltage is across  $R$ . As a result, the output voltage across the tank circuit drops off, creating a band-pass characteristic.

At very high frequencies, a factor to consider is that inductors do not act as pure inductors but have capacitance between windings. As a consequence, all inductors have a “self-resonant” frequency that acts like a parallel resonant circuit. When choosing an inductance for a high-frequency resonant filter, you should choose one that has a self-resonant frequency much higher than the cut-off frequency.

**EXAMPLE 18–9**

What is the center frequency of the filter in Figure 18–26? Assume  $R_W = 0\ \Omega$ .

► **FIGURE 18–26**

**Solution** The center frequency of the filter is its resonant frequency.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(10\ \mu\text{H})(100\ \text{pF})}} = 5.03\ \text{MHz}$$

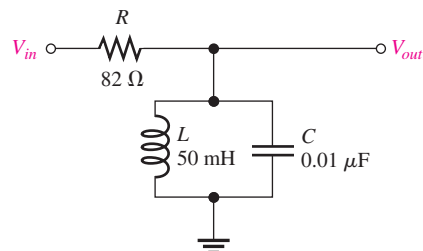
**Related Problem** Determine  $f_0$  in Figure 18–26 if  $C$  is changed to 1,000 pF.



Use Multisim files E18-09A and E18-09B to verify the calculated results in this example and to confirm your calculation for the related problem.

**EXAMPLE 18–10**

Determine the center frequency and bandwidth for the band-pass filter in Figure 18–27 if the inductor has a winding resistance of 15  $\Omega$ .

► **FIGURE 18–27**



**Solution** Recall from Chapter 17 (Eq. 17–13) that the resonant (center) frequency of a nonideal tank circuit is

$$f_0 = \frac{\sqrt{1 - (R_W^2 C/L)}}{2\pi\sqrt{LC}} = \frac{\sqrt{1 - (15\ \Omega)^2(0.01\ \mu\text{F})/50\ \text{mH}}}{2\pi\sqrt{(50\ \text{mH})(0.01\ \mu\text{F})}} = 7.12\ \text{kHz}$$

The  $Q$  of the coil at resonance is

$$Q = \frac{X_L}{R_W} = \frac{2\pi f_0 L}{R_W} = \frac{2\pi(7.12\ \text{kHz})(50\ \text{mH})}{15\ \Omega} = 149$$

The bandwidth of the filter is

$$BW = \frac{f_0}{Q} = \frac{7.12\ \text{kHz}}{149} = 47.7\ \text{Hz}$$

Note that since  $Q > 10$ , the simpler formula,  $f_0 = 1/(2\pi\sqrt{LC})$ , could have been used to calculate  $f_0$ .

**Related Problem** Knowing the value of  $Q$ , recalculate  $f_0$  using the simpler formula.

#### SECTION 18–3 CHECKUP

1. For a band-pass filter,  $f_{c(h)} = 29.8\ \text{kHz}$  and  $f_{c(l)} = 30.2\ \text{kHz}$ . What is the bandwidth?
2. A parallel resonant band-pass filter has the following values:  $R_W = 15\ \Omega$ ,  $L = 50\ \mu\text{H}$ , and  $C = 470\ \text{pF}$ . Determine the approximate center frequency.
3. What causes an inductor to have a self-resonant frequency?

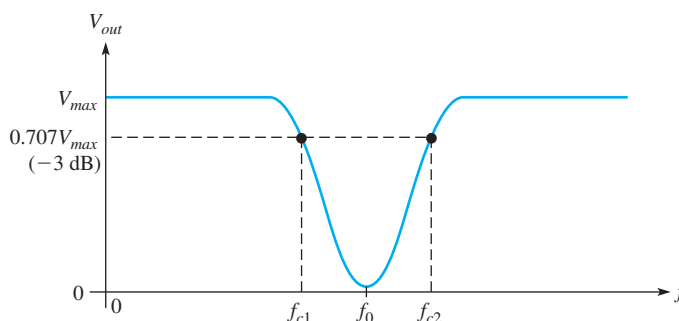
## 18–4 BAND-STOP FILTERS

A band-stop filter is essentially the opposite of a band-pass filter in terms of the responses. A **band-stop filter** allows all frequencies to pass except those lying within a certain stopband.

After completing this section, you should be able to

- ♦ **Analyze the operation of band-stop filters**
  - ♦ Show how a band-stop filter is implemented with low-pass and high-pass filters
  - ♦ Explain the series-resonant band-stop filter
  - ♦ Explain the parallel-resonant band-stop filter
  - ♦ Calculate the bandwidth and output voltage of a band-stop filter

Figure 18–28 shows a general band-stop response curve.

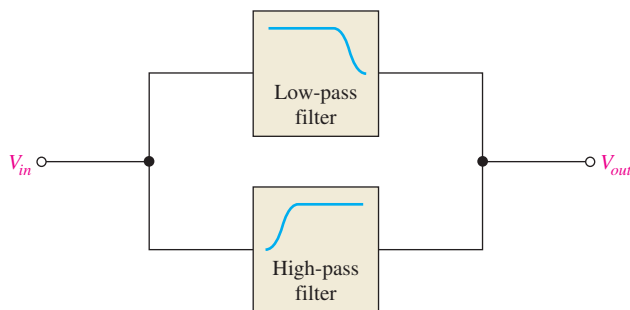


▲ FIGURE 18–28

General band-stop response curve.

### Low-Pass/High-Pass Filter

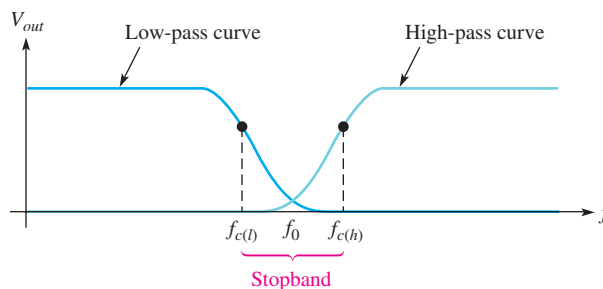
A band-stop filter can be formed from a low-pass and a high-pass filter, as shown in Figure 18–29.



▲ FIGURE 18–29

A low-pass and a high-pass filter are used to form a band-stop filter.

If the low-pass critical frequency,  $f_{c(l)}$ , is set lower than the high-pass critical frequency,  $f_{c(h)}$ , a band-stop characteristic is formed as illustrated in Figure 18–30.

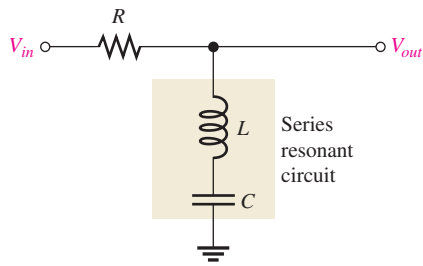


▲ FIGURE 18–30

Band-stop response curve.

### Series Resonant Band-Stop Filter

A series resonant circuit used in a band-stop configuration is shown in Figure 18–31. Basically, it works as follows: At the resonant frequency, the impedance is minimum, and therefore the output voltage is minimum. Most of the input voltage is dropped across  $R$ . At frequencies above and below resonance, the impedance increases, causing more voltage across the output.

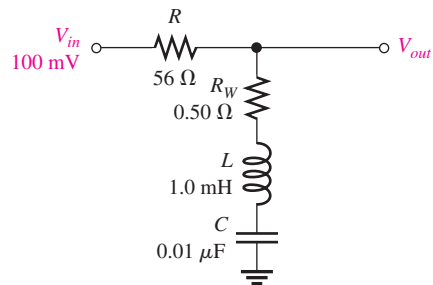


◀ **FIGURE 18-31**  
Series resonant band-stop filter.

### EXAMPLE 18-11

Find the output voltage magnitude at  $f_0$  and the bandwidth in Figure 18-32.

▶ **FIGURE 18-32**



**Solution** Since  $X_L = X_C$  at resonance, they cancel, leaving  $R_W$ . The output voltage is determined using the voltage-divider formula.

$$V_{out} = \left( \frac{R_W}{R + R_W} \right) V_{in} = \left( \frac{0.50 \, \Omega}{56.5 \, \Omega} \right) 100 \, \text{mV} = 885 \, \mu\text{V}$$

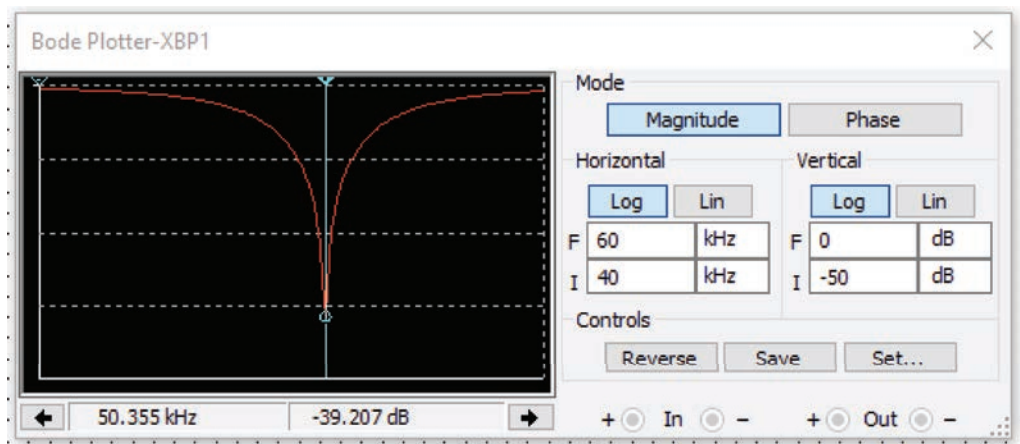
To determine the bandwidth, first calculate the center frequency and  $Q$  of the circuit.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1.0 \, \text{mH})(0.01 \, \mu\text{F})}} = 50.3 \, \text{kHz}$$

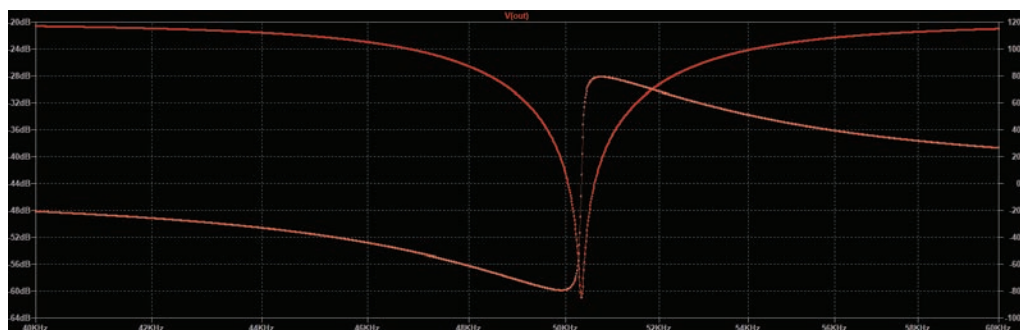
$$Q = \frac{X_L}{R_{tot}} = \frac{2\pi fL}{R + R_W} = \frac{2\pi(50.3 \, \text{kHz})(1.0 \, \text{mH})}{56.5 \, \Omega} = \frac{316 \, \Omega}{56.5 \, \Omega} = 5.56$$

$$BW = \frac{f_0}{Q} = \frac{50.3 \, \text{kHz}}{5.56} = 9.05 \, \text{kHz}$$

To see the magnitude or phase of the Bode plot, you can construct a Multisim circuit and use the Bode Plotter connected to the input and output. Multisim allows you to choose either magnitude or phase. To see details, change the resolution to 1,000 points in the “Set...” menu on the plotter. Choose an initial value (I) of 40 kHz and a final value (F) of 60 kHz. The result with these settings is shown in Figure 18-33(a). Optionally, you can construct the circuit using LTSpice and view the magnitude and phase Bode plot on the same diagram. The (b) figure shows the LTSpice output with both plots.



(a) Multisim Bode Plotter (magnitude plot)



(b) LTSpice (magnitude and phase)

▲ FIGURE 18–33

Bode plots for the circuit in Figure 18–32.

**Related Problem**

Assume you wanted to shift the peak to 50.0 kHz to provide maximum rejection at this frequency. Suggest a change to the circuit to accomplish this.

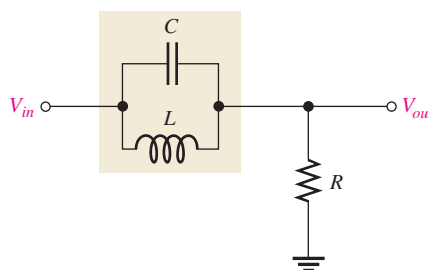
Use Multisim files E18-11A and E18-11B to verify the calculated results in this example and to confirm your calculation for the related problem.

**Parallel Resonant Band-Stop Filter**

A parallel resonant circuit used in a band-stop configuration is shown in Figure 18–34. At the resonant frequency, the tank impedance is maximum, and so most of the input voltage appears across it. Very little voltage is across  $R$  at resonance. As the tank impedance decreases above and below resonance, the output voltage increases.

► FIGURE 18–34

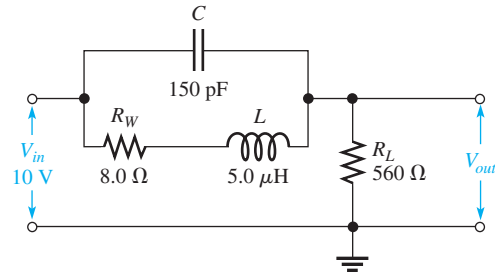
Parallel resonant band-stop filter.



## EXAMPLE 18-12

Find the center frequency of the filter in Figure 18-35. Draw the output response curve showing the minimum and maximum voltages.

▶ FIGURE 18-35



**Solution** The center frequency is

$$f_0 = \frac{\sqrt{1 - R_W^2 C/L}}{2\pi\sqrt{LC}} = \frac{\sqrt{1 - (8.0\ \Omega)^2(150\ \text{pF})/5.0\ \mu\text{H}}}{2\pi\sqrt{(5.0\ \mu\text{H})(150\ \text{pF})}} = 5.81\ \text{MHz}$$

At the center (resonant) frequency,

$$X_L = 2\pi f_0 L = 2\pi(5.79\ \text{MHz})(5.0\ \mu\text{H}) = 183\ \Omega$$

$$Q = \frac{X_L}{R_W} = \frac{183\ \Omega}{8.0\ \Omega} = 22.8$$

$$Z_r = R_W(Q^2 + 1) = 8.0\ \Omega(22.8^2 + 1) = 4.17\ \text{k}\Omega \quad (\text{purely resistive})$$

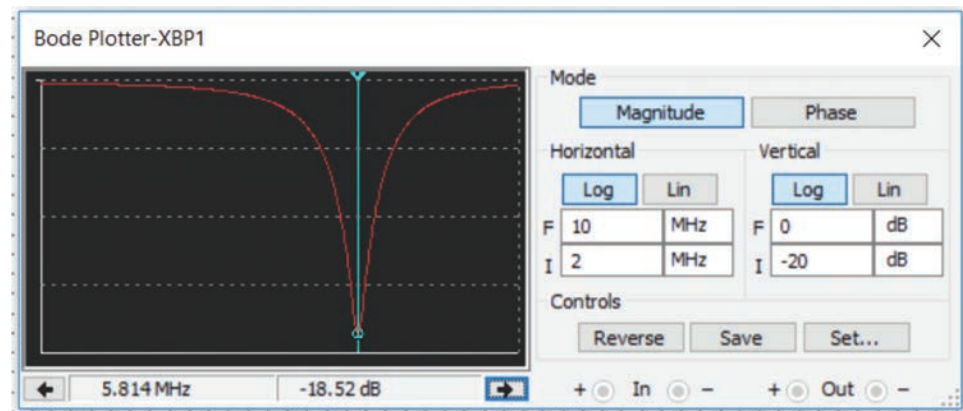
Next, use the voltage-divider formula to find the minimum output voltage magnitude.

$$V_{out(min)} = \left( \frac{R_L}{R_L + Z_r} \right) V_{in} = \left( \frac{560\ \Omega}{4.73\ \text{k}\Omega} \right) 10\ \text{V} = 1.18\ \text{V}$$

At zero frequency, the impedance of the tank circuit is  $R_W$  because  $X_C = \infty$  and  $X_L = 0\ \Omega$ . Therefore, the maximum output voltage below resonance is

$$V_{out(max)} = \left( \frac{R_L}{R_L + R_W} \right) V_{in} = \left( \frac{560\ \Omega}{568\ \Omega} \right) 10\ \text{V} = 9.86\ \text{V}$$

As the frequency increases much higher than  $f_0$ ,  $X_C$  approaches  $0\ \Omega$ , and  $V_{out}$  approaches  $V_{in}$  (10 V). Figure 18-36 shows the Bode plot from Multisim plotted between 2 MHz and 10 MHz.



▶ FIGURE 18-36

## Related Problem

What is the minimum output voltage if  $R_L = 1.0\ \text{k}\Omega$  in Figure 18-35?

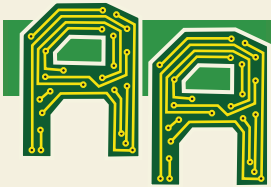
Use Multisim files E18-12A and E18-12B to verify the calculated results in this example and to confirm your calculation for the related problem.





## SECTION 18-4 CHECKUP

1. How does a band-stop filter differ from a band-pass filter?
2. Name three basic ways to construct a band-stop filter.
3. Explain how a series resonant band-stop filter could be converted to a band-pass filter.

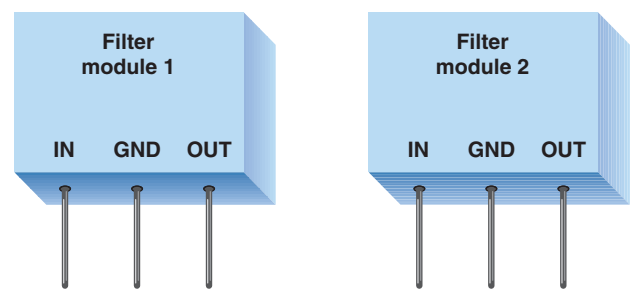


## Application Activity

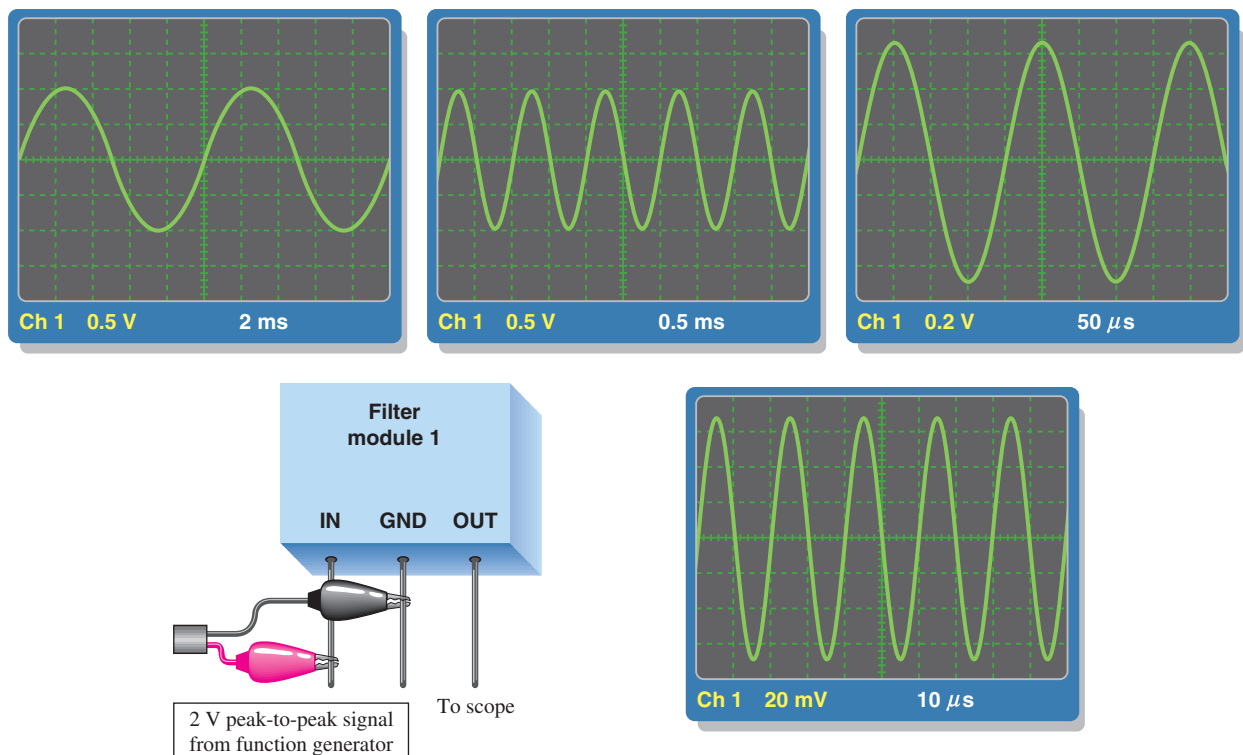
In this application activity, you will plot the frequency responses of two types of filters based on a series of oscilloscope measurements and identify the type of filter in each case. The filters are contained in sealed modules as shown in Figure 18-37. You are concerned only with determining the filter response characteristics and not the types of internal components.

### Filter Measurement and Analysis

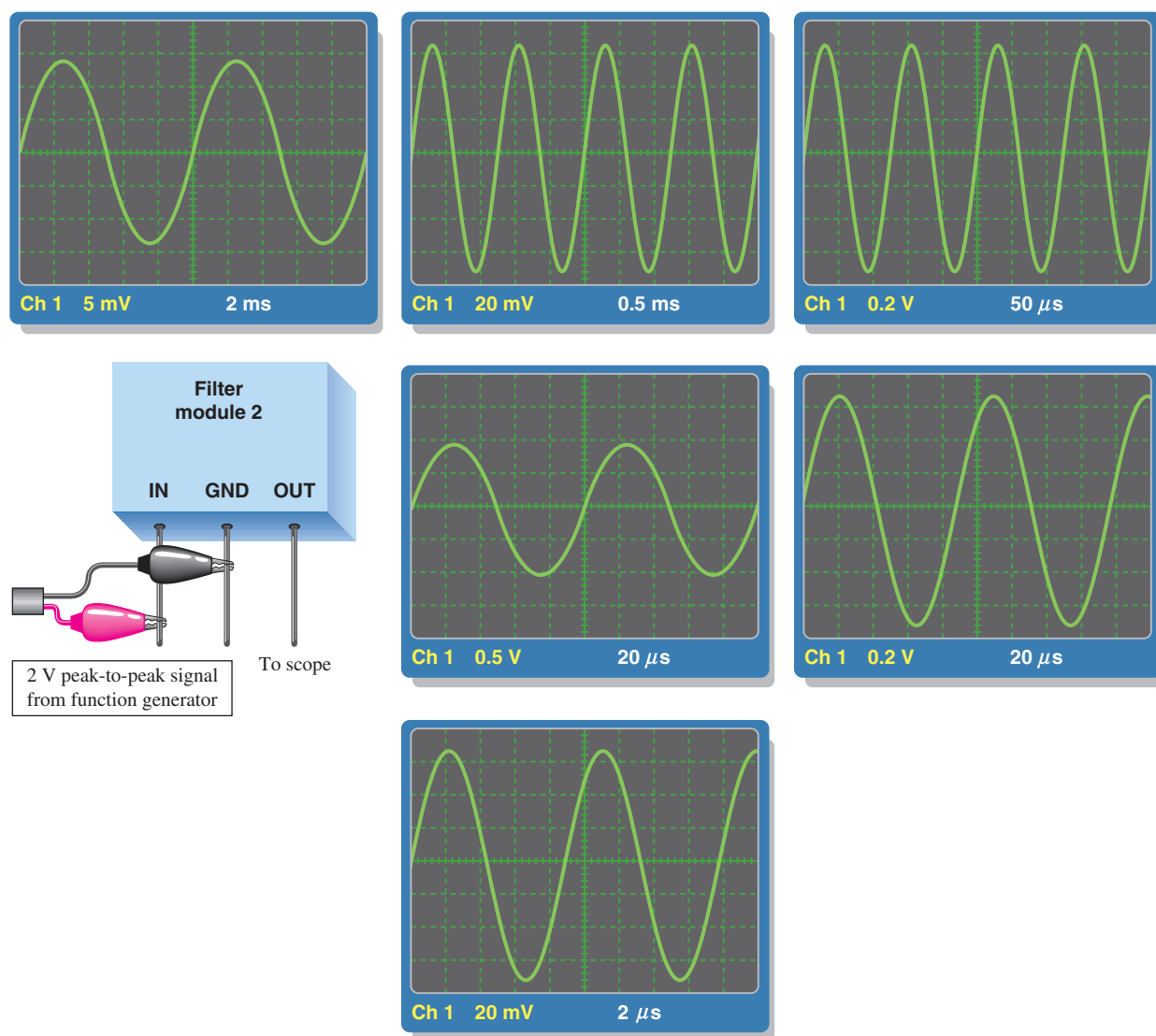
1. Refer to Figure 18-38. Based on the series of four oscilloscope measurements, create a Bode plot for the filter under test, specify applicable frequencies, and identify the type of filter.
2. Refer to Figure 18-39. Based on the series of six oscilloscope measurements, create a Bode plot for the filter under test, specify applicable frequencies, and identify the type of filter.



▲ **FIGURE 18-37**  
Filter modules.



▲ **FIGURE 18-38**



▲ FIGURE 18-39

**Review**

3. Explain how the waveforms in Figure 18-38 indicate the type of filter.
4. Explain how the waveforms in Figure 18-39 indicate the type of filter.

**SUMMARY**

- Four categories of passive filters according to their response characteristics are low-pass, high-pass, band-pass, and band-stop.
- In an  $RC$  low-pass filter, the output voltage is taken across the capacitor and the output lags the input.
- In an  $RL$  low-pass filter, the output voltage is taken across the resistor and the output lags the input.
- In an  $RC$  high-pass filter, the output is taken across the resistor and the output leads the input.
- In an  $RL$  high-pass filter, the output is taken across the inductor and the output leads the input.

- The roll-off rate of a basic  $RC$  or  $RL$  filter is  $-20$  dB per decade.
- A band-pass filter passes frequencies between the lower and upper critical frequencies and rejects all others.
- A band-stop filter rejects frequencies between its lower and upper critical frequencies and passes all others.
- The bandwidth of a resonant filter is determined by the quality factor ( $Q$ ) of the circuit and the resonant frequency.
- Critical frequencies are also called  $-3$  dB frequencies, cutoff frequencies, or break frequencies.
- The output voltage is 70.7% of its maximum at the critical frequencies.

## KEY TERMS

Key terms and other bold terms in the chapter are defined in the end-of-book glossary.

**Attenuation** A reduction of the output signal compared to the input signal, resulting in a ratio with a value of less than 1 for the output voltage to the input voltage of a circuit.

**Band-pass filter** A filter that passes a range of frequencies lying between two critical frequencies and rejects frequencies above and below that range.

**Band-stop filter** A filter that rejects a range of frequencies lying between two critical frequencies and passes frequencies above and below that range.

**Bode plot** A frequency response curve for a filter or circuit. The Bode magnitude plot is a graph of the ratio of  $V_{out}/V_{in}$  in dB plotted against frequency, which is plotted on a logarithmic scale. The Bode phase plot is the ratio of the output phase to the input phase expressed in angular units (typically degrees) plotted against frequency.

**Center frequency ( $f_0$ )** The resonant frequency of a band-pass or band-stop filter.

**Critical frequency ( $f_c$ )** The frequency at which a filter's output voltage is 70.7% of the maximum.

**Decade** A tenfold change in frequency or other parameter.

**High-pass filter** A type of filter that passes all frequencies above a critical frequency and rejects all frequencies below that critical frequency.

**Low-pass filter** A type of filter that passes all frequencies below a critical frequency and rejects all frequencies above that critical frequency.

**Passband** The range of frequencies passed by a filter.

**Roll-off** The rate of decrease of a filter's frequency response.

## FORMULAS

$$18-1 \quad \text{dB} = 10 \log \left( \frac{P_{out}}{P_{in}} \right) \quad \text{Power ratio in decibels}$$

$$18-2 \quad \text{dB} = 20 \log \left( \frac{V_{out}}{V_{in}} \right) \quad \text{Voltage ratio in decibels}$$

$$18-3 \quad f_c = \frac{1}{2\pi RC} \quad \text{Critical frequency}$$

$$18-4 \quad f_c = \frac{1}{2\pi(L/R)} \quad \text{Critical frequency}$$

$$18-5 \quad BW = \frac{f_0}{Q} \quad \text{Bandwidth}$$

## TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The bandwidth of a low-pass filter ideally extends from 0 Hz up to a specified critical frequency.
2. The decibel is a unit for expressing the ratio of one frequency to another.
3. In an  $RC$  low-pass filter, the output is taken across the capacitor.
4. Roll-off is the rate at which the output of a low-pass filter decreases above the critical frequency.
5. Attenuation is the ratio of input voltage to output voltage.

6. In an  $RL$  high-pass filter, the output is taken across the inductor.
7. A high-pass filter acts as a lag circuit.
8. If the output voltage of a band-pass filter is 1 V at the resonant frequency, it is 0.707 V at the critical frequencies.
9. The frequency at which resonance occurs in a band-pass filter is sometimes called the center frequency.
10. The output of a resonant band-stop filter is maximum at the resonant frequency.
11. The same series resonant circuit can be used for a band-pass or band-stop filter.
12. The  $BW$  of a band-stop filter is determined by the  $Q$ .

## SELF-TEST

Answers are at the end of the chapter.

1. The maximum output voltage of a certain low-pass filter is 10 V. The output voltage at the critical frequency is  
 (a) 10 V      (b) 0 V      (c) 7.07 V      (d) 1.414 V
2. A sinusoidal voltage with a peak-to-peak value of 15 V is applied to an  $RC$  low-pass filter. If the reactance at the input frequency is zero, the output voltage is  
 (a) 15 V peak-to-peak      (b) zero  
 (c) 10.6 V peak-to-peak      (d) 7.5 V peak-to-peak
3. The same signal in Question 2 is applied to an  $RC$  high-pass filter. If the reactance is zero at the input frequency, the output voltage is  
 (a) 15 V peak-to-peak      (b) zero  
 (c) 10.6 V peak-to-peak      (d) 7.5 V peak-to-peak
4. At the critical frequency, the output of a filter is down relative to its passband amplitude by  
 (a) 0 dB      (b)  $-3$  dB      (c)  $-20$  dB      (d)  $-6$  dB
5. If the output of a low-pass  $RC$  filter is 12 dB below its maximum at  $f = 1$  kHz, then at  $f = 10$  kHz, the output is below its maximum by  
 (a) 3 dB      (b) 10 dB      (c) 20 dB      (d) 32 dB
6. In a passive filter, the ratio  $V_{out}/V_{in}$  is called  
 (a) roll-off      (b) gain      (c) attenuation      (d) critical reduction
7. For each decade increase in frequency above the critical frequency, the output of a low-pass filter decreases by  
 (a) 20 dB      (b) 3 dB      (c) 10 dB      (d) 0 dB
8. At the critical frequency, the phase shift through a high-pass filter is  
 (a)  $90^\circ$       (b)  $0^\circ$       (c)  $45^\circ$       (d) dependent on the reactance
9. In a series resonant band-pass filter, a higher value of  $Q$  results in  
 (a) a higher resonant frequency      (b) a smaller bandwidth  
 (c) a higher impedance      (d) a larger bandwidth
10. At series resonance,  
 (a)  $X_C = X_L$       (b)  $X_C > X_L$       (c)  $X_C < X_L$
11. In a certain parallel resonant band-pass filter, the resonant frequency is 10 kHz. If the band-width is 2 kHz, the lower critical frequency is  
 (a) 5 kHz      (b) 12 kHz  
 (c) 9 kHz      (d) not determinable
12. In a band-pass filter, the output voltage at the resonant frequency is  
 (a) minimum      (b) maximum  
 (c) 70.7% of maximum      (d) 70.7% of minimum
13. In a band-stop filter, the output voltage at the critical frequencies is  
 (a) minimum      (b) maximum  
 (c) 70.7% of maximum      (d) 70.7% of minimum

14. At a sufficiently high value of  $Q$ , the resonant frequency for a parallel resonant filter is ideally
- (a) much greater than the resonant frequency of a series resonant filter
  - (b) much less than the resonant frequency of a series resonant filter
  - (c) equal to the resonant frequency of a series resonant filter

## CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 18–40(a).

1. If the frequency of the input voltage is increased,  $V_{out}$ 
  - (a) increases                      (b) decreases                      (c) stays the same
2. If  $C$  is increased, the output voltage
  - (a) increases                      (b) decreases                      (c) stays the same

Refer to Figure 18–40(d).

3. If the frequency of the input voltage is increased,  $V_{out}$ 
  - (a) increases                      (b) decreases                      (c) stays the same
4. If  $L$  is increased, the output voltage
  - (a) increases                      (b) decreases                      (c) stays the same

Refer to Figure 18–42.

5. If the switch is thrown from position 1 to position 2, the critical frequency
  - (a) increases                      (b) decreases                      (c) stays the same
6. If the switch is thrown from position 2 to position 3, the critical frequency
  - (a) increases                      (b) decreases                      (c) stays the same

Refer to Figure 18–43(a).

7. If the frequency of the input voltage is increased,  $V_{out}$ 
  - (a) increases                      (b) decreases                      (c) stays the same
8. If  $R$  is increased to  $180\ \Omega$ , the output voltage
  - (a) increases                      (b) decreases                      (c) stays the same

Refer to Figure 18–44.

9. If the switch is thrown from position 1 to position 2, the critical frequency
  - (a) increases                      (b) decreases                      (c) stays the same
10. If the switch is in position 3 and  $R_5$  opens,  $V_{out}$ 
  - (a) increases                      (b) decreases                      (c) stays the same

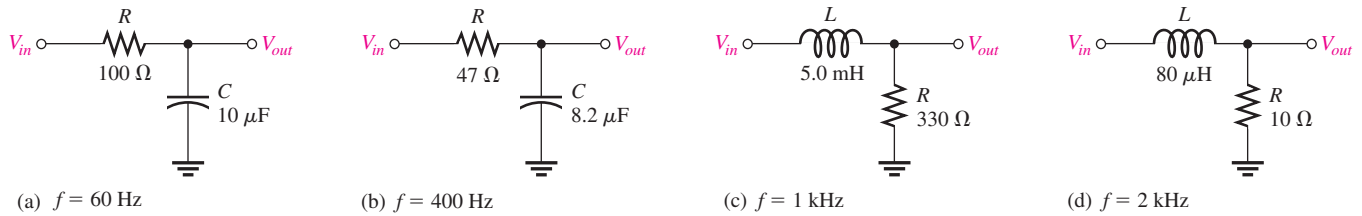
## PROBLEMS

More difficult problems are indicated by an asterisk (\*).  
Answers to odd-numbered problems are at the end of the book.

### SECTION 18–1 Low-Pass Filters

1. In a certain low-pass filter,  $X_C = 500\ \Omega$  and  $R = 2.2\ \text{k}\Omega$ . What is the output voltage ( $V_{out}$ ) when the input is  $10\ \text{V rms}$ ?
2. A certain low-pass filter has a critical frequency of  $3\ \text{kHz}$ . Determine which of the following frequencies are passed and which are rejected:
  - (a)  $100\ \text{Hz}$                       (b)  $1\ \text{kHz}$                       (c)  $2\ \text{kHz}$                       (d)  $5\ \text{kHz}$
3. Determine the output voltage ( $V_{out}$ ) of each filter in Figure 18–40 at the specified frequency when  $V_{in} = 10\ \text{V}$ .

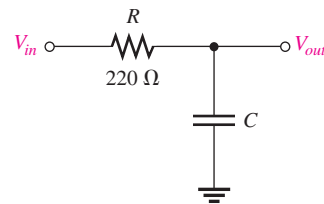




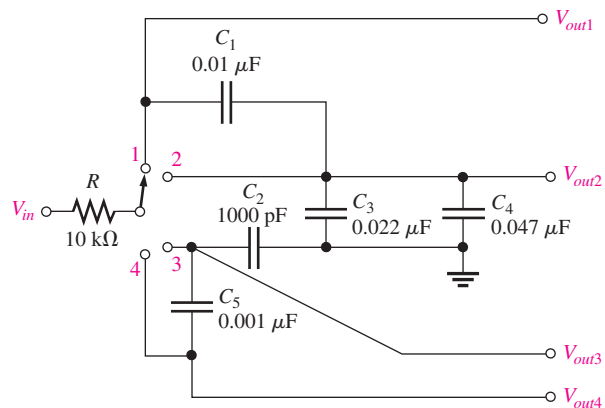
▲ FIGURE 18-40

4. What is  $f_c$  for each filter in Figure 18-40? Determine the output voltage at  $f_c$  in each case when  $V_{in} = 5 \text{ V}$ .
5. For the filter in Figure 18-41, calculate the value of  $C$  required for each of the following critical frequencies:
  - (a) 60 Hz
  - (b) 500 Hz
  - (c) 1 kHz
  - (d) 5 kHz

► FIGURE 18-41



- \*6. Determine the critical frequency for each switch position on the switched filter network of Figure 18-42.

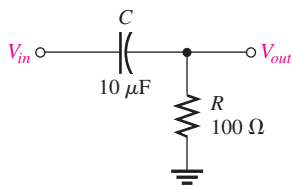
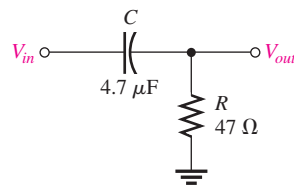
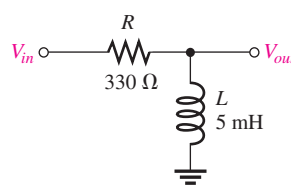
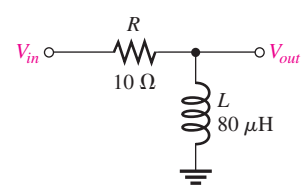


▲ FIGURE 18-42

7. Draw a Bode plot of the magnitude for each part of Problem 5.
8. For each following case, express the voltage ratio in dB:
  - (a)  $V_{in} = 1 \text{ V}$ ,  $V_{out} = 1 \text{ V}$
  - (b)  $V_{in} = 5 \text{ V}$ ,  $V_{out} = 3 \text{ V}$
  - (c)  $V_{in} = 10 \text{ V}$ ,  $V_{out} = 7.07 \text{ V}$
  - (d)  $V_{in} = 25 \text{ V}$ ,  $V_{out} = 5 \text{ V}$
9. The input voltage to a low-pass  $RC$  filter is 8 V rms. Find the output voltage at the following dB levels:
  - (a) -1 dB
  - (b) -3 dB
  - (c) -6 dB
  - (d) -20 dB
10. For a basic  $RC$  low-pass filter, find the output voltage in dB relative to a 0 dB input for the following frequencies ( $f_c = 1.0 \text{ kHz}$ ):
  - (a) 10 kHz
  - (b) 100 kHz
  - (c) 1 MHz
11. Repeat Problem 10 for  $f_c = 100 \text{ kHz}$ .

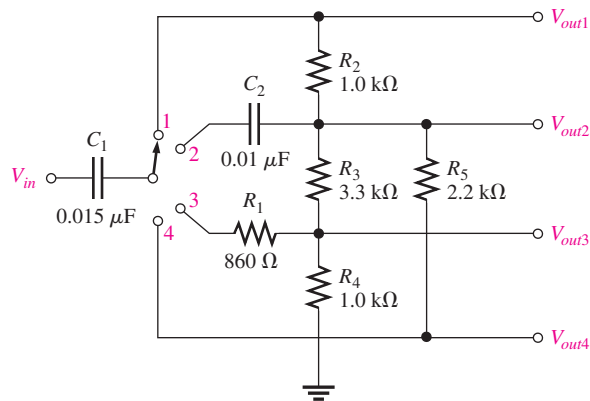
## SECTION 18-2 High-Pass Filters

12. Repeat Problem 11 for a high-pass filter.
13. In a high-pass filter,  $X_C = 500\ \Omega$  and  $R = 2.2\ \text{k}\Omega$ . What is the output voltage ( $V_{out}$ ) when  $V_{in} = 10\ \text{V rms}$ ?
14. A high-pass filter has a critical frequency of 50 Hz. Determine which of the following frequencies are passed and which are rejected:
  - (a) 1 Hz      (b) 20 Hz      (c) 50 Hz      (d) 60 Hz      (e) 30 kHz
15. Determine the output voltage of each filter in Figure 18-43 at the specified frequency when  $V_{in} = 10\ \text{V}$ .
16. What is  $f_c$  for each filter in Figure 18-43? Determine the output voltage at  $f_c$  in each case ( $V_{in} = 10\ \text{V}$ ).
17. Draw the Bode plot for each filter in Figure 18-43.

(a)  $f = 60\ \text{Hz}$ (b)  $f = 400\ \text{Hz}$ (c)  $f = 1\ \text{kHz}$ (d)  $f = 2\ \text{kHz}$ 

▲ FIGURE 18-43

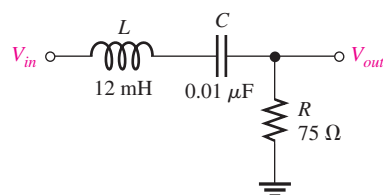
- \*18. Determine  $f_c$  for each switch position in Figure 18-44.



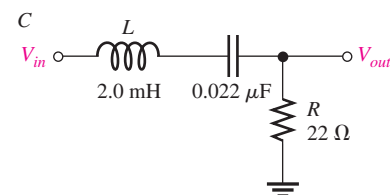
▲ FIGURE 18-44

## SECTION 18-3 Band-Pass Filters

19. Determine the center frequency for each filter in Figure 18-45.



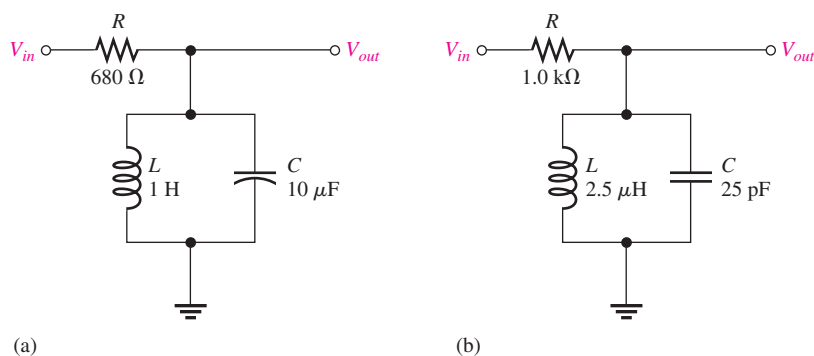
(a)



(b)

▲ FIGURE 18-45

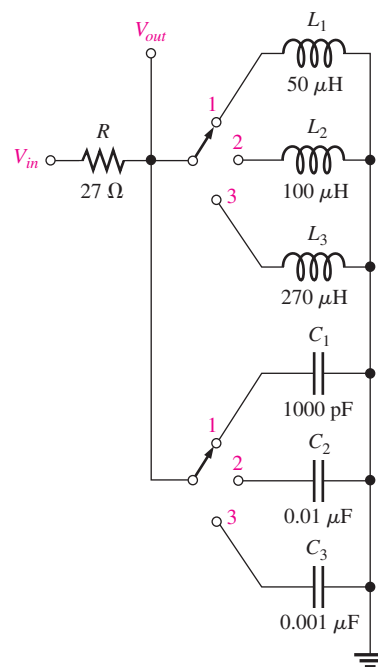
20. Assuming that the coils in Figure 18–45 have a winding resistance of  $10\ \Omega$ , find the bandwidth for each filter.
21. What are the upper and lower critical frequencies for each filter in Figure 18–45? Assume the response is symmetrical about  $f_0$ .
22. For each filter in Figure 18–46, find the center frequency of the passband. Neglect  $R_W$ .
23. If the coils in Figure 18–46 have a winding resistance of  $4\ \Omega$ , what is the output voltage at resonance when  $V_{in} = 120\text{ V}$ ?



▲ FIGURE 18–46

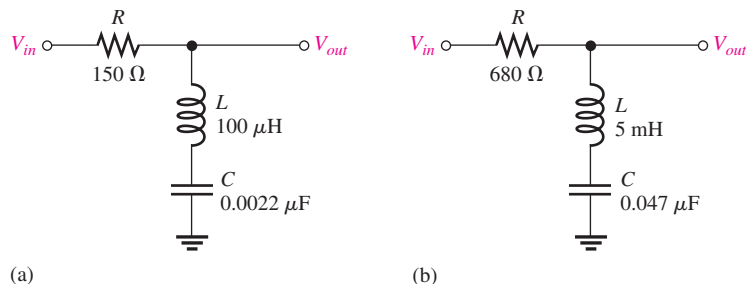
- \*24. Determine the separation of center frequencies for all switch positions in Figure 18–47. Do any of the responses overlap? Assume  $R_W = 0\ \Omega$  for each coil.
- \*25. Design a band-pass filter using a parallel resonant circuit to meet all of the following specifications:  $BW = 500\text{ Hz}$ ;  $Q = 40$ ; and  $I_{C(max)} = 20\text{ mA}$ ,  $V_{C(max)} = 2.5\text{ V}$ .

► FIGURE 18–47



## SECTION 18-4 Band-Stop Filters

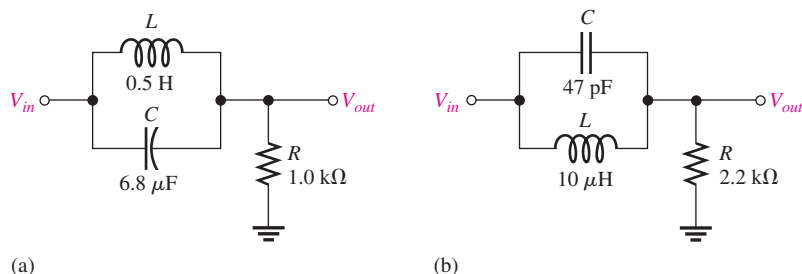
26. Determine the center frequency for each filter in Figure 18-48.



▲ FIGURE 18-48

27. For each filter in Figure 18-49, find the center frequency of the stopband.

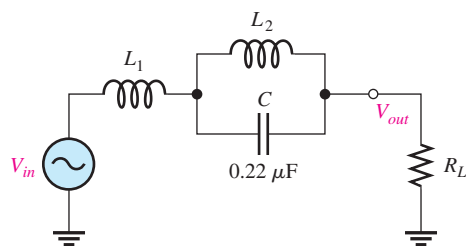
28. If the coils in Figure 18-49 have a winding resistance of  $8\ \Omega$ , what is the output voltage at resonance when  $V_{in} = 50\ \text{V}$ ?



▲ FIGURE 18-49

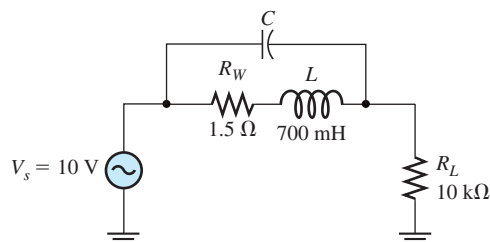
\*29. Determine the values of  $L_1$  and  $L_2$  in Figure 18-50 to pass a signal with a frequency of 1200 kHz and stop (reject) a signal with a frequency of 456 kHz.

► FIGURE 18-50



30. Assume you want to reject 60 Hz line noise by constructing the parallel resonant band stop filter shown in Figure 18-51. What size capacitor do you need to complete the filter?

► FIGURE 18-51





### Multisim Troubleshooting and Analysis

These problems require Multisim.

31. Open file P18-30 and determine if there is a fault. If so, find the fault.
32. Open file P18-31 and determine if there is a fault. If so, find the fault.
33. Open file P18-32 and determine if there is a fault. If so, find the fault.
34. Open file P18-33 and determine if there is a fault. If so, find the fault.
35. Open file P18-34 and determine if there is a fault. If so, find the fault.
36. Open file P18-35 and determine if there is a fault. If so, find the fault.
37. Open file P18-36 and determine the center frequency of the circuit.
38. Open file P18-37 and determine the bandwidth of the circuit.

## ANSWERS

### SECTION CHECKUPS

#### SECTION 18-1 Low-Pass Filters

1. The passband is 0 Hz to 2.5 kHz.
2.  $V_{out} = 100 \angle -88.9^\circ$  mV rms
3.  $-34.0$  dB
4.  $20 \log(V_{out}/V_{in}) = -9.54$  dB

#### SECTION 18-2 High-Pass Filters

1.  $V_{out} = 0.707$  V
2.  $V_{out} = 9.98 \angle 3.81^\circ$  V
3. 2.0 MHz

#### SECTION 18-3 Band-Pass Filters

1.  $BW = 30.2 \text{ kHz} - 29.8 \text{ kHz} = 400 \text{ Hz}$
2.  $f_0 \cong 1.04 \text{ MHz}$
3. The self-resonant frequency is due to capacitance between windings of the inductor, which forms a parallel resonant circuit.

#### SECTION 18-4 Band-Stop Filters

1. A band-stop filter rejects, rather than passes, a certain band of frequencies.
2. High-pass/low-pass combination, series resonant circuit, and parallel resonant circuit
3. Take the output across the resistor instead of the resonant circuit.

### RELATED PROBLEMS FOR EXAMPLES

- 18-1  $-1.41$  dB
- 18-2 7.23 kHz
- 18-3  $f_c$  increases to 159 kHz. Roll-off rate remains  $-20$  dB/decade.
- 18-4  $f_c$  increases to 350 kHz. Roll-off rate remains  $-20$  dB/decade.
- 18-5  $-60$  dB
- 18-6  $C = 0.723 \mu\text{F}$ ;  $V_{out} = 4.98$  V;  $\phi = 5.7^\circ$
- 18-7 10.5 kHz
- 18-8  $BW$  increases to 18.8 kHz.
- 18-9 1.59 MHz
- 18-10 7.12 kHz (no significant difference)
- 18-11 Place a small trimmer capacitor (200–300 pF) in parallel with  $C_1$ .
- 18-12 1.93 V

**TRUE/FALSE QUIZ**

- |      |      |      |       |       |       |
|------|------|------|-------|-------|-------|
| 1. T | 2. F | 3. T | 4. T  | 5. F  | 6. T  |
| 7. F | 8. T | 9. T | 10. F | 11. T | 12. T |

**SELF-TEST**

- |        |        |         |         |         |         |         |
|--------|--------|---------|---------|---------|---------|---------|
| 1. (c) | 2. (b) | 3. (a)  | 4. (b)  | 5. (d)  | 6. (c)  | 7. (a)  |
| 8. (c) | 9. (b) | 10. (a) | 11. (c) | 12. (b) | 13. (c) | 14. (c) |

**CIRCUIT DYNAMICS QUIZ**

- |        |        |        |        |         |
|--------|--------|--------|--------|---------|
| 1. (b) | 2. (b) | 3. (b) | 4. (b) | 5. (b)  |
| 6. (a) | 7. (a) | 8. (a) | 9. (a) | 10. (c) |