RLC Circuits and Resonance

17

CHAPTER OUTLINE

CHAPTER OBJECTIVES

PART 1: SERIES CIRCUITS

- Determine the impedance of a series *RLC* circuit
- Analyze series *RLC* circuits
- Analyze a circuit for series resonance

PART 2: PARALLEL CIRCUITS

- Determine the impedance of a parallel *RLC* circuit
- Analyze parallel *RLC* circuits
- Analyze a circuit for parallel resonance

PART 3: SERIES-PARALLEL CIRCUITS

- Analyze series-parallel *RLC* circuits

PART 4: SPECIAL TOPICS

- Determine the bandwidth of resonant circuits
- Discuss some applications of resonant circuits

KEY TERMS

- Series resonance
- Resonant frequency (f_r)
- Parallel resonance
- Tank circuit
- Half-power frequency
- Selectivity

APPLICATION ACTIVITY PREVIEW

In the application activity, the focus is on the resonant tuning circuit in the RF amplifier of an AM radio receiver. The tuning circuit is used to select any desired frequency within the AM band so that a desired station can be tuned in.

VISIT THE COMPANION WEBSITE

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http://www.pearsonhighered.com/careersresources/

INTRODUCTION

In this chapter, the analysis methods learned in Chapters 15 and 16 are extended to the coverage of circuits with combinations of resistive, inductive, and capacitive components. Series and parallel *RLC* circuits, plus series-parallel combinations, are studied.

Circuits with both inductance and capacitance can exhibit the property of resonance, which is important in many types of applications. Resonance is the basis for frequency selectivity in communication systems. For example, the ability of a radio or television receiver to select a certain frequency that is transmitted by a particular station and, at the same time, to eliminate frequencies from other stations is based on the principle of resonance. The conditions in *RLC* circuits that produce resonance and the characteristics of resonant circuits are covered in this chapter.

COVERAGE OPTIONS

If you chose Option 1 to cover all of Chapter 15 and all of Chapter 16, then all of this chapter should be covered next.

If you chose Option 2 to cover reactive circuits in Chapters 15 and 16 on the basis of the four major parts, then the appropriate part of this chapter should be covered next, followed by the next part in Chapter 15, if applicable.

Part 1 SERIES CIRCUITS

17–1 IMPEDANCE OF SERIES *RLC* CIRCUITS

A series *RLC* circuit contains resistance, inductance, and capacitance. Since inductive reactance and capacitive reactance have opposite effects on the circuit phase angle, the total reactance is less than either individual reactance.

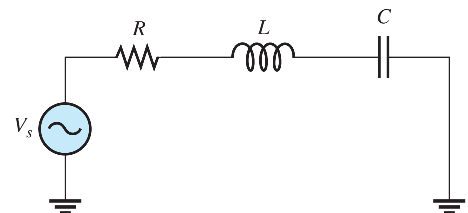
After completing this section, you should be able to

- **Determine the impedance of a series *RLC* circuit**
  - Calculate total reactance
  - Determine whether a circuit is predominately inductive or capacitive

A series *RLC* circuit is shown in Figure 17–1. It contains resistance, inductance, and capacitance.

▶ FIGURE 17–1

Series *RLC* circuit.



As you know, inductive reactance ($\mathbf{X}_L$) causes the total current to lag the applied voltage. Capacitive reactance ($\mathbf{X}_C$) has the opposite effect: It causes the current to lead the voltage. Thus $\mathbf{X}_L$ and $\mathbf{X}_C$ tend to offset each other. When they are equal, they cancel, and the total reactance is zero. In any case, the magnitude of the total reactance in the series circuit is

Equation 17–1

$$X_{tot} = |X_L - X_C|$$

The term $|X_L - X_C|$ means the absolute value of the difference of the two reactances. That is, the sign of the result is considered positive no matter which reactance is greater. For example, $3 - 7 = -4$, but the absolute value is

$$|3 - 7| = 4$$

In a series circuit, when $X_L > X_C$, the circuit is predominantly inductive, and when $X_C > X_L$, the circuit is predominantly capacitive.

The total impedance for the series *RLC* circuit is stated in rectangular form in Equation 17–2 and in polar form in Equation 17–3.

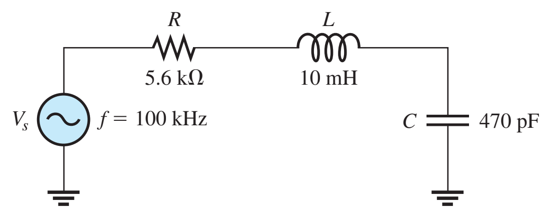
$$\mathbf{Z} = R + jX_L - jX_C \qquad \text{Equation 17–2}$$

$$\mathbf{Z} = \sqrt{R^2 + (X_L - X_C)^2}\angle \pm \tan^{-1}\left(\frac{X_{tot}}{R}\right) \qquad \text{Equation 17–3}$$

In Equation 17–3, $\sqrt{R^2 + (X_L - X_C)^2}$ is the magnitude and $\tan^{-1}(X_{tot}/R)$ is the phase angle between the total current and the applied voltage. If the circuit is predominately inductive, the phase angle is positive; and if predominately capacitive, the phase angle is negative.

EXAMPLE 17–1

For the series *RLC* circuit in Figure 17–2, determine the total impedance. Express it in both rectangular and polar forms.



▲ FIGURE 17–2

Solution First, find X_C and X_L.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100\text{ kHz})(470\text{ pF})} = 3.39\text{ k}\Omega$$

$$X_L = 2\pi fL = 2\pi(100\text{ kHz})(10\text{ mH}) = 6.28\text{ k}\Omega$$

In this case, X_L is greater than X_C, and thus the circuit is more inductive than capacitive. The magnitude of the total reactance is

$$X_{tot} = |X_L - X_C| = |6.28\text{ k}\Omega - 3.39\text{ k}\Omega| = 2.89\text{ k}\Omega \qquad \text{inductive}$$

The impedance in rectangular form is

$$\mathbf{Z} = R + (jX_L - jX_C) = 5.60\text{ k}\Omega + (j6.28\text{ k}\Omega - j3.39\text{ k}\Omega) = \mathbf{5.60\text{ k}\Omega + j2.90\text{ k}\Omega}$$

The impedance in polar form is

$$\mathbf{Z} = \sqrt{R^2 + X_{tot}^2}\angle\tan^{-1}\left(\frac{X_{tot}}{R}\right)$$

$$= \sqrt{(5.60\text{ k}\Omega)^2 + (2.90\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{2.90\text{ k}\Omega}{5.60\text{ k}\Omega}\right) = \mathbf{6.31\angle 27.4^\circ\text{ k}\Omega}$$

The positive angle shows that the circuit is inductive.

*Related Problem** Determine **Z** in polar form if *f* is increased to 200 kHz.

*Answers are at the end of the chapter.

As you have seen, in a series circuit when the inductive reactance is greater than the capacitive reactance, the circuit appears inductive; so the current lags the applied voltage. When the capacitive reactance is greater, the circuit appears capacitive, and the current leads the applied voltage.

SECTION 17–1 CHECKUP
Answers are at the end of the chapter.

1. In a given series *RLC* circuit, X_C is 150 Ω and X_L is 80 Ω. What is the total reactance in ohms? Is the circuit inductive or capacitive?
2. Determine the impedance in polar form for the circuit in Question 1 when $R = 47\ \Omega$. What is the magnitude of the impedance? What is the phase angle? Is the current leading or lagging the applied voltage?

17–2 Analysis of Series *RLC* Circuits

Recall that capacitive reactance varies inversely with frequency and that inductive reactance varies directly with frequency. In this section, the combined effects of the reactances as a function of frequency are examined.

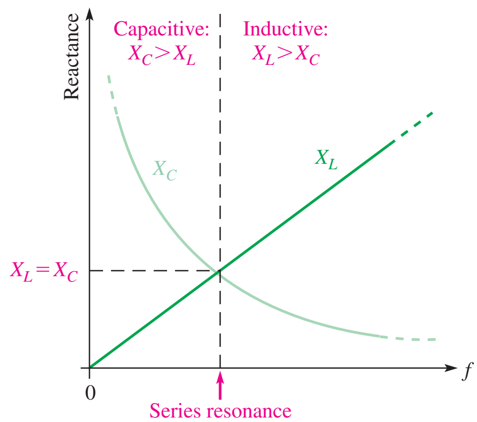
After completing this section, you should be able to

- **Analyze series *RLC* circuits**
  - Determine current in a series *RLC* circuit
  - Determine the voltages in a series *RLC* circuit
  - Determine the phase angle

Figure 17–3 shows that for a typical series *RLC* circuit, the total reactance behaves as follows: Starting at a very low frequency, X_C is high, and X_L is low, and the circuit is predominantly capacitive. As the frequency is increased, X_C decreases and X_L increases until a value is reached where $X_C = X_L$ and the two reactances cancel, making the circuit purely resistive. This condition is **series resonance** and will be studied in Section 17–3. As the frequency is increased further, X_L becomes greater than X_C, and the circuit is predominantly inductive. Example 17–2 illustrates how the impedance and phase angle change as the source frequency is varied.

FIGURE 17–3

How X_C and X_L vary with frequency.



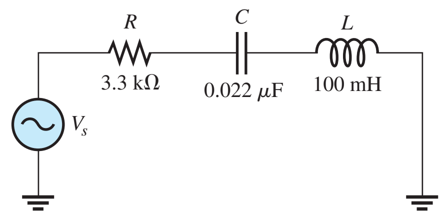
The graph of X_L is a straight line, and the graph of X_C is curved, as shown in Figure 17–3. The general equation for a straight line is $y = mx + b$, where m is the slope of the line and b is the y-axis intercept point. The formula $X_L = 2\pi fL$ fits this general straight-line formula, where $y = X_L$ (a variable), $m = 2\pi L$ (a constant), $x = f$ (a variable), and $b = 0$ as follows: $X_L = 2\pi Lf + 0$.

The X_C curve is called a *hyperbola,* and the general equation of a hyperbola is $xy = k$. The equation for capacitive reactance, $X_C = 1/2\pi fC$, can be rearranged as $X_C f = 1/2\pi C$ where $x = X_C$ (a variable), $y = f$ (a variable), and $k = 1/2\pi C$ (a constant).

EXAMPLE 17–2

Calculate the impedance in polar form at 1.0 kHz, 2.0 kHz, 4.0 kHz, and 8.0 kHz for the circuit in Figure 17–4. Note the change in magnitude and phase angle with frequency.

► **FIGURE 17–4**



Solution The simplest way to solve a problem like this is with a graphing calculator. The first frequency (1.0 kHz) is solved without the graphing calculator to clarify the method. At $f = 1$ kHz,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1\text{ kHz})(0.022\ \mu\text{F})} = 7.23\text{ k}\Omega$$

$$X_L = 2\pi fL = 2\pi(1\text{ kHz})(100\text{ mH}) = 628\ \Omega$$

The circuit is clearly capacitive, and the impedance is

$$\mathbf{Z} = \sqrt{R^2 + (X_L - X_C)^2}\angle -\tan^{-1}\left(\frac{X_{tot}}{R}\right)$$

$$= \sqrt{(3.3\text{ k}\Omega)^2 + (628\ \Omega - 7.23\text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{6.60\text{ k}\Omega}{3.3\text{ k}\Omega}\right) = \mathbf{7.38\angle -63.5^\circ\ k\Omega}$$

The negative sign for the angle is used to indicate that the circuit is capacitive.

The TI-84 Plus CE is used to illustrate the solution. Start by entering the equations for capacitive reactance, inductive reactance, total impedance, and phase angle. Use the y= key to enter the equations as shown in Figure 17–5. Y_1 is the equation for capacitive reactance, Y_2 is inductive reactance, Y_3 is the total impedance and Y_4 is the phase angle. The independent variable X stands for frequency. Set up the table start and increments using the window menu. Start at X (frequency) = 0 and use increments (Xscl) of 1,000.

► **FIGURE 17–5**

Equations for capacitive reactance, inductive reactance, total impedance, and phase angle.

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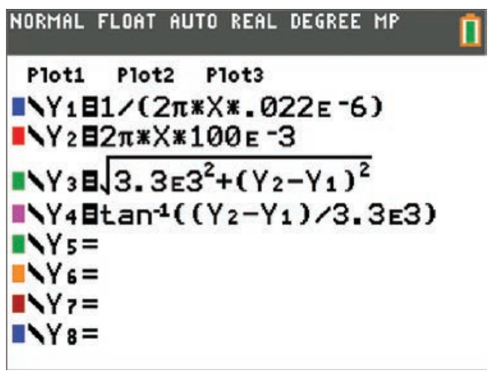
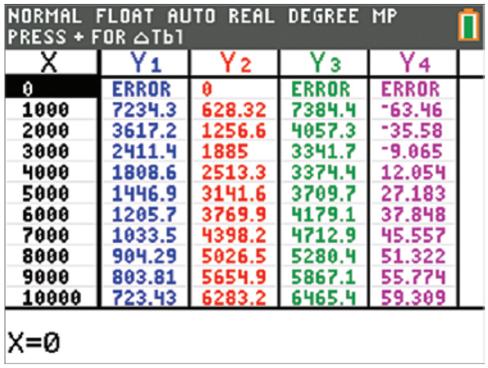


Press 2nd graph to see the table values for each variable as shown in Figure 17–6. (Note that error at a frequency of zero is a divide by zero error.)

▶ **FIGURE 17–6**


NORMAL FLOAT AUTO REAL DEGREE MP
PRESS + FOR △Tbl

X	Y1	Y2	Y3	Y4
0	ERROR	0	ERROR	ERROR
1000	7234.3	628.32	7384.4	-63.46
2000	3617.2	1256.6	4057.3	-35.58
3000	2411.4	1885	3341.7	-9.065
4000	1808.6	2513.3	3374.4	12.054
5000	1446.9	3141.6	3709.7	27.183
6000	1205.7	3769.9	4179.1	37.848
7000	1033.5	4398.2	4712.9	45.557
8000	904.29	5026.5	5280.4	51.322
9000	803.81	5654.9	5867.1	55.774
10000	723.43	6283.2	6465.4	59.309

X=0

Table values for capacitive reactance, inductive reactance, total impedance, and phase angle.

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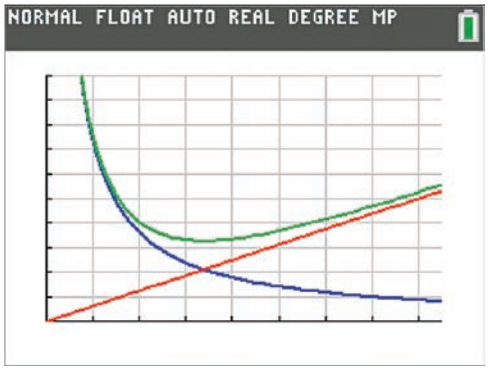
Notice that the row for 1,000 Hz confirms the result of the previous "hand" calculation. You can graph the results of Y_1, Y_2, and Y_3 by selecting these equations and pressing [graph]. Figure 17–7 shows the result. Grid lines are shown every 1 kHz in X and every 1 kΩ in Y.

▶ **FIGURE 17–7**



Reactance and impedance curves for capacitive reactance (blue), inductive reactance (red), and total impedance (green).

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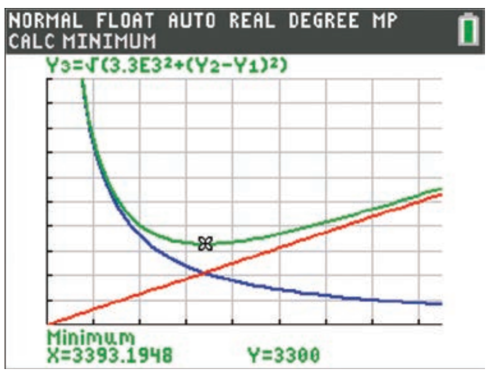
The calculator solution allows you to examine specific value on the response easily and can be used to find the minimum value on the total impedance curve. To find the minimum, press [2nd] [trace] and select option 3, which is minimum. You will need to "help" the calculator by selecting an arbitrary point on the green curve to the left of the minimum (LeftBound) and a point to the right of the minimum (RightBound). Press [enter] and the calculator displays the minimum as shown in Figure 17–8. We find the minimum in this case is at 3,300 Ω, where $X_C = X_L$. (More on this is in Section 17–3, when you study series resonance.)

▶ **FIGURE 17–8**



Finding the minimum for the impedance curve.

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Notice how the circuit changed from capacitive to inductive as the frequency increased. The phase condition changed from the current leading to the current lagging as indicated by the sign of the angle. Note that the impedance magnitude decreased to a minimum equal to the resistance and then began increasing again. Also, notice that the negative phase angle decreased as the frequency increased and the angle became positive when the circuit became inductive and then increased with increasing frequency.

Related Problem Determine **Z** in polar form for $f = 7$ kHz if the inductor is changed to a 50 mH inductor. Determine if the circuit is inductive or capacitive.

In a series *RLC* circuit, the capacitor voltage and the inductor voltage are always 180° out of phase with each other. For this reason, V_C and V_L subtract from each other, and thus the voltage across L and C combined is always less than the larger individual voltage across either element, as illustrated in Figure 17–9 and in the waveform diagram of Figure 17–10.

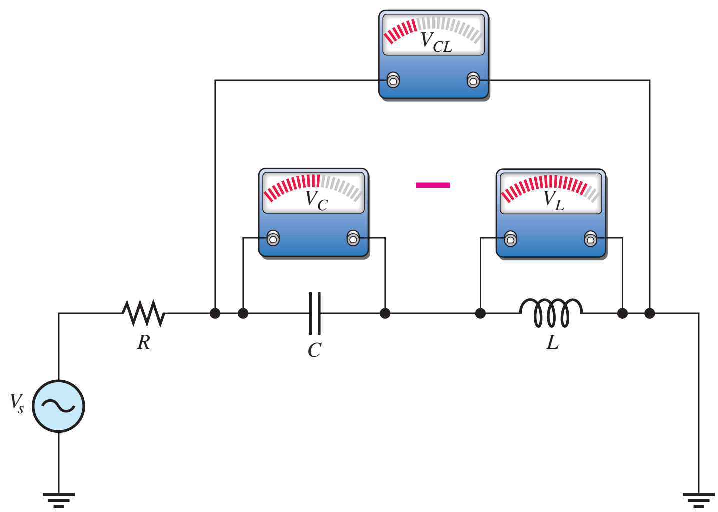


FIGURE 17–9

The voltage across the series combination of C and L is always less than the larger individual voltage across either C or L.

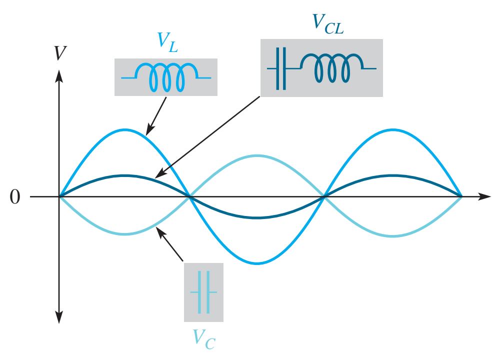


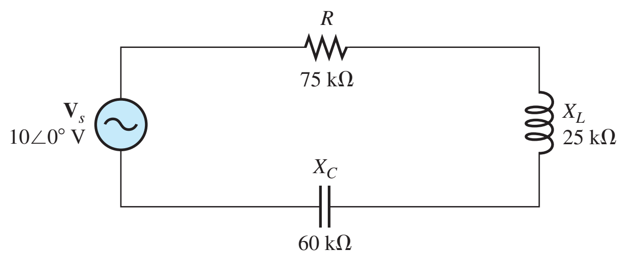
FIGURE 17–10

V_{CL} is the algebraic sum of V_L and V_C. Because of the phase relationship, V_L and V_C effectively subtract.

In the next example, Ohm's law is used to find the current and voltages in a series *RLC* circuit.

EXAMPLE 17–3

Find the current and the voltages across each component in Figure 17–11. Express each quantity in polar form, and draw a complete voltage phasor diagram.



▲ FIGURE 17–11

Solution First, find the total impedance.

$$\mathbf{Z} = R + jX_L - jX_C = 75\text{ k}\Omega + j25\text{ k}\Omega - j60\text{ k}\Omega = 75\text{ k}\Omega - j35\text{ k}\Omega$$

Convert to polar form for convenience in applying Ohm's law.

$$\mathbf{Z} = \sqrt{R^2 + X_{tot}^2}\angle -\tan^{-1}\left(\frac{X_{tot}}{R}\right)$$

$$= \sqrt{(75\text{ k}\Omega)^2 + (35\text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{35\text{ k}\Omega}{75\text{ k}\Omega}\right) = 82.8\angle -25^\circ\text{ k}\Omega$$

where $X_{tot} = |X_L - X_C|$.

Apply Ohm's law to find the current.

$$\mathbf{I} = \frac{\mathbf{V}_s}{\mathbf{Z}} = \frac{10\angle 0^\circ\text{ V}}{82.8\angle -25^\circ\text{ k}\Omega} = \mathbf{121\angle 25.0^\circ\ \mu A}$$

Now, apply Ohm's law to find the voltages across ***R***, ***L***, and ***C***.

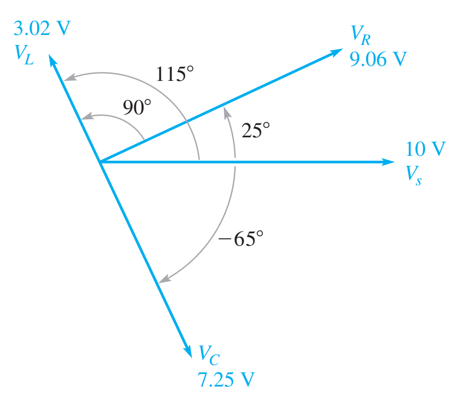
$$\mathbf{V}_R = \mathbf{IR} = (121\angle 25.0^\circ\ \mu\text{A})(75\angle 0^\circ\text{ k}\Omega) = \mathbf{9.06\angle 25.0^\circ\ V}$$

$$\mathbf{V}_L = \mathbf{IX}_L = (121\angle 25.0^\circ\ \mu\text{A})(25\angle 90^\circ\text{ k}\Omega) = \mathbf{3.02\angle 115^\circ\ V}$$

$$\mathbf{V}_C = \mathbf{IX}_C = (121\angle 25.0^\circ\ \mu\text{A})(60\angle -90^\circ\text{ k}\Omega) = \mathbf{7.25\angle -65.0^\circ\ V}$$

The phasor diagram is shown in Figure 17–12. The magnitudes represent rms values. Notice that V_L is leading V_R by 90°, and V_C is lagging V_R by 90°. Also, there is a 180° phase difference between V_L and V_C. If the current phasor were shown,

► FIGURE 17–12



it would be at the same angle as V_R. The current is leading V_s, the source voltage, by 25°, indicating a capacitive circuit ($X_C > X_L$). The phasor diagram is rotated 25° from its usual position because the reference is the source voltage, V_s, which is shown oriented along the x-axis.

Related Problem What will happen to the current as the frequency of the source voltage in Figure 17–11 is increased?

SECTION 17–2 CHECKUP

1. The following voltages occur in a certain series *RLC* circuit. Determine the source voltage: $V_R = 24\angle 30° \text{ V}$, $V_L = 15\angle 120° \text{ V}$, and $V_C = 45\angle -60° \text{ V}$.
2. In a series *RLC* circuit, with $R = 1.0 \text{ k}\Omega$, $X_C = 1.8 \text{ k}\Omega$, and $X_L = 1.2 \text{ k}\Omega$, does the current lead or lag the applied voltage?
3. Determine the total reactance in Question 2.

17–3 Series Resonance

In a series *RLC* circuit, series resonance occurs when $X_C = X_L$. The frequency at which resonance occurs is called the **resonant frequency** and is designated f_r.

After completing this section, you should be able to

- **Analyze a circuit for series resonance**
  - Define *series resonance*
  - Determine the impedance at resonance
  - Explain why the reactances cancel at resonance
  - Determine the series resonant frequency
  - Calculate the current, voltages, and phase angle at resonance

Figure 17–13 illustrates the series resonant condition.

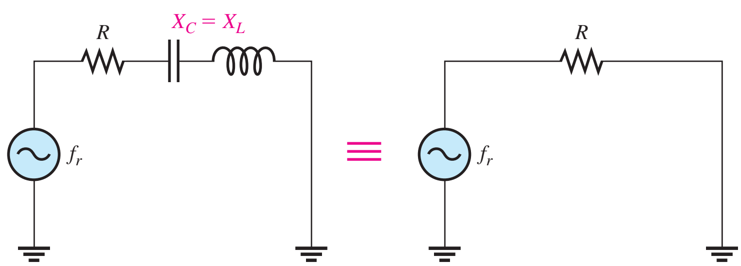


FIGURE 17–13

Series resonance. X_C and X_L cancel each other resulting in a purely resistive load.

Resonance is a condition in a series *RLC* circuit in which the capacitive and inductive reactances are equal in magnitude; thus, they cancel each other and result in a purely resistive impedance. In a series *RLC* circuit, the total impedance was given in Equation 17–2 as

$$\mathbf{Z} = R + jX_L - jX_C$$

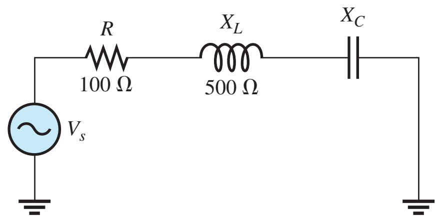
At resonance, $X_L = X_C$ and the j terms cancel; thus, the impedance is purely resistive. These resonant conditions are stated in the following equations:

$$X_L = X_C$$
$$Z_r = R$$

EXAMPLE 17–4

For the series RLC circuit in Figure 17–14, determine X_C and $\mathbf{Z}$ at resonance.

▶ FIGURE 17–14



Solution $X_L = X_C$ at the resonant frequency. Thus, $X_C = X_L = \mathbf{500\ \Omega}$. The impedance at resonance is

$$\mathbf{Z}_r = R + jX_L - jX_C = 100\ \Omega + j500\ \Omega - j500\ \Omega = \mathbf{100\angle 0^\circ\ \Omega}$$

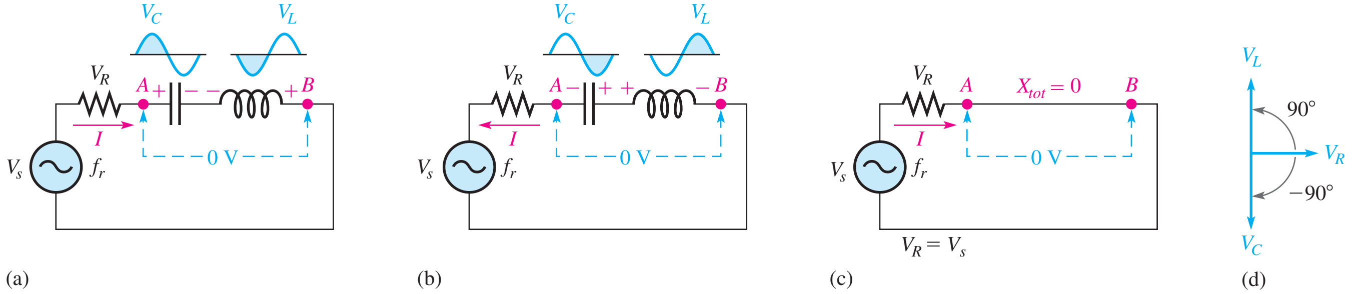
This shows that the impedance at resonance is equal to the resistance because the reactances are equal in magnitude and therefore cancel.

Related Problem Just below the resonant frequency, is the circuit more inductive or more capacitive?

X_L and X_C Cancel at Resonance

At the series resonant frequency (f_r), the voltages across C and L are equal in magnitude because the reactances are equal. The same current is through both since they are in series ($IX_C = IX_L$). Also, V_L and V_C are always 180° out of phase with each other.

During any given cycle, the polarities of the voltages across C and L are opposite, as shown in parts (a) and (b) of Figure 17–15. The equal and opposite voltages across C and L cancel, leaving zero volts from point A to point B as shown. Since there is no voltage drop from A to B but there is still current, the total reactance must be zero, as indicated in part (c). Also, the voltage phasor diagram in part (d) shows that V_C and V_L are equal in magnitude and 180° out of phase with each other.



▲ FIGURE 17–15

At the resonant frequency, f_r, the voltages across C and L are equal in magnitude. Since they are 180° out of phase with each other, they cancel, leaving 0 V across the LC combination (point A to point B). The section of the circuit from A to B effectively looks like a short at resonance.

Series Resonant Frequency

For a given series *RLC* circuit, resonance occurs at only one specific frequency. A formula for this resonant frequency is developed as follows:

$$X_L = X_C$$

Substitute the reactance formulas.

$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

Then, multiplying both sides by $f_r/2\pi L$,

$$f_r^2 = \frac{1}{4\pi^2 LC}$$

Take the square root of both sides. The formula for series resonant frequency is

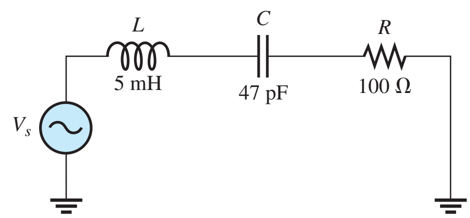
$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Equation 17–4

EXAMPLE 17–5

Find the series resonant frequency for the circuit in Figure 17–16.

► **FIGURE 17–16**



Solution The resonant frequency is

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(5\text{ mH})(47\text{ pF})}} = \mathbf{328\ kHz}$$

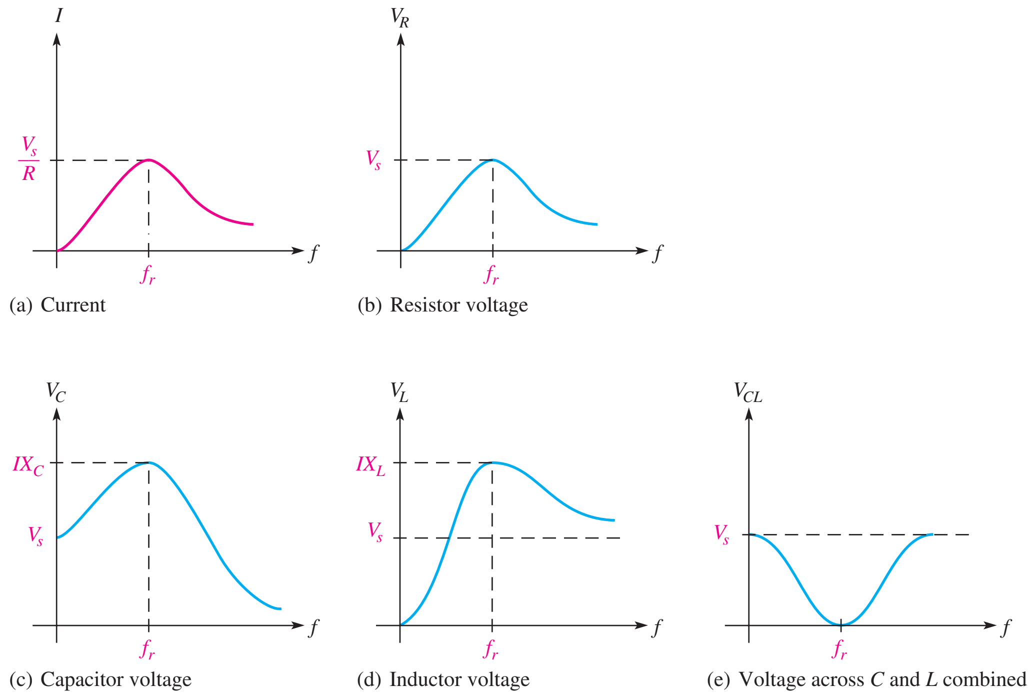
Related Problem If $C = 0.01\ \mu\text{F}$ in Figure 17–16, what is the resonant frequency?

MultiSim

Use Multisim files E17-05A and E17-05B to verify the calculated results in this example and to confirm your calculation for the related problem.

Current and Voltages in a Series *RLC* Circuit

At the series resonant frequency, the current is maximum ($I_{max} = V_s/R$). Above and below resonance, the current decreases because the impedance increases. This is because at resonance X_T is zero and the total impedance is R. Above and below resonance, the total series reactance X_T is not zero. A response curve showing the plot of current versus frequency is shown in Figure 17–17(a). The resistor voltage, V_R, follows the current and is maximum (equal to V_s) at resonance and zero at $f = 0$ and at $f = \infty$, as shown in Figure 17–17(b). The general shapes of the V_C and V_L curves are indicated in Figure 17–17(c) and (d). Notice that $V_C = V_s$ when $f = 0$, because the capacitor appears open. Also notice that V_L approaches V_s as f approaches infinity, because the inductor appears open. The voltage across the C and L combination decreases as the frequency increases below resonance, reaching a minimum of zero at the resonant frequency; then it increases above resonance, as shown in Figure 17–17(e).



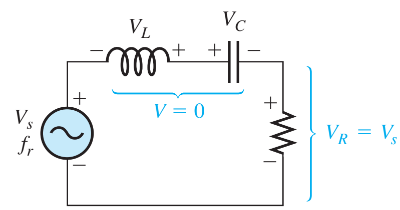
▲ **FIGURE 17–17**

Generalized current and voltage magnitudes as a function of frequency in a series *RLC* circuit. V_C and V_L can be much larger than the source voltage. The shapes of the graphs depend on specific circuit values.

The voltages are maximum at resonance but drop off above and below f_r. The voltages across L and C at resonance are exactly equal in magnitude but 180° out of phase; so they cancel. Thus, the total voltage across both L and C is zero, and $V_R = V_s$ at resonance, as indicated in Figure 17–18. Individually, V_L and V_C can be much greater than the source voltage, as you will see later. Keep in mind that V_L and V_C are always opposite in polarity regardless of the frequency, but only at resonance are their magnitudes equal.

► **FIGURE 17–18**

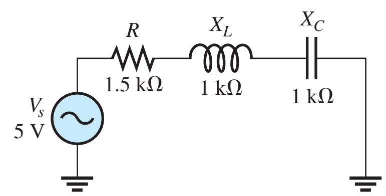
Series *RLC* circuit at resonance.



EXAMPLE 17–6

Find I, V_R, V_L, and V_C at resonance for the circuit in Figure 17–19. The resonant values of X_L and X_C are shown.

► **FIGURE 17–19**



Solution At resonance, I is maximum and equal to V_s/R.

$$I = \frac{V_s}{R} = \frac{5\text{ V}}{1.5\text{ k}\Omega} = \mathbf{3.33\text{ mA}}$$

Apply Ohm's law to obtain the voltage magnitudes.

$$V_R = IR = (3.33\text{ mA})(1.5\text{ k}\Omega) = \mathbf{5.08\text{ V}}$$
$$V_L = IX_L = (3.33\text{ mA})(1\text{ k}\Omega) = \mathbf{3.33\text{ V}}$$
$$V_C = IX_C = (3.33\text{ mA})(1\text{ k}\Omega) = \mathbf{3.33\text{ V}}$$

Notice that all of the source voltage is dropped across the resistor. Also, of course, V_L and V_C are equal in magnitude but opposite in phase. This causes these voltages to cancel, making the total reactive voltage zero.

Related Problem What is the phase angle if the frequency is doubled?

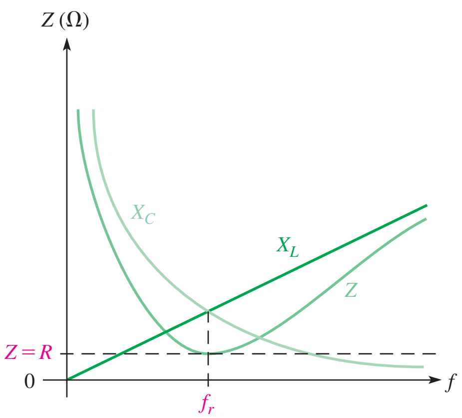
MultiSim

Use Multisim files E17-06A and E17-06B to verify the calculated results in this example and to confirm your calculation for the related problem.

Series *RLC* Impedance

At frequencies below f_r, $X_C > X_L$; thus, the circuit is capacitive. At the resonant frequency, $X_C = X_L$, so the circuit is purely resistive. At frequencies above f_r, $X_L > X_C$; thus, the circuit is inductive. Notice that this is the result observed in Example 17–2.

The impedance magnitude is minimum at resonance ($Z = R$) and increases in value above and below the resonant point. The graph in Figure 17–20 illustrates how impedance changes with frequency. At zero frequency, both X_C and Z are infinitely large and X_L is zero because the capacitor looks like an open at 0 Hz and the inductor looks like a short. As the frequency increases, X_C decreases and X_L increases. Since X_C is larger than X_L at frequencies below f_r, Z decreases along with X_C. At f_r, $X_C = X_L$ and $Z = R$. At frequencies above f_r, X_L becomes increasingly larger than X_C, causing Z to increase.



◀ **FIGURE 17–20**

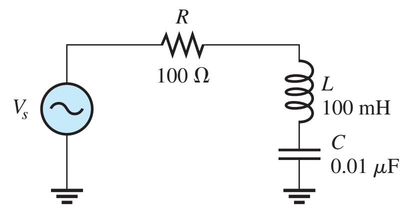
Series *RLC* impedance as a function of frequency.

EXAMPLE 17–7

For the circuit in Figure 17–21, determine the impedance magnitude at the following frequencies:

(a) f_r **(b)** 1,000 Hz below f_r **(c)** 1,000 Hz above f_r

▶ FIGURE 17–21



Solution **(a)** At f_r, the impedance is equal to R.

$$Z = R = \mathbf{100\ \Omega}$$

To determine the impedance above and below f_r, first calculate the resonant frequency.

$$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(100\text{ mH})(0.01\ \mu\text{F})}} = 5.03\text{ kHz}$$

(b) At 1,000 Hz below f_r, the frequency and reactances are as follows:

$$f = f_r - 1\text{ kHz} = 5.03\text{ kHz} - 1\text{ kHz} = 4.03\text{ kHz}$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(4.03\text{ kHz})(0.01\ \mu\text{F})} = 3.95\text{ k}\Omega$$

$$X_L = 2\pi fL = 2\pi(4.03\text{ kHz})(100\text{ mH}) = 2.53\text{ k}\Omega$$

Therefore, the impedance at $f_r - 1$ kHz is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(100\ \Omega)^2 + (2.53\text{ k}\Omega - 3.95\text{ k}\Omega)^2} = \mathbf{1.42\ k\Omega}$$

(c) At 1,000 Hz above f_r,

$$f = 5.03\text{ kHz} + 1\text{ kHz} = 6.03\text{ kHz}$$

$$X_C = \frac{1}{2\pi(6.03\text{ kHz})(0.01\ \mu\text{F})} = 2.64\text{ k}\Omega$$

$$X_L = 2\pi(6.03\text{ kHz})(100\text{ mH}) = 3.79\text{ k}\Omega$$

Therefore, the impedance at $f_r + 1$ kHz is

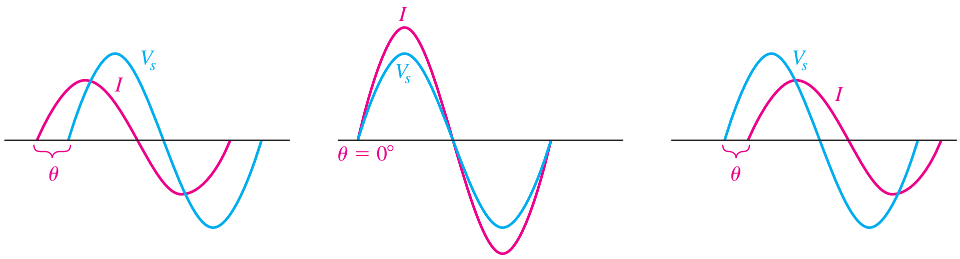
$$Z = \sqrt{(100\ \Omega)^2 + (3.79\text{ k}\Omega - 2.64\text{ k}\Omega)^2} = \mathbf{1.16\ k\Omega}$$

In part (b) Z is capacitive, and in part (c) Z is inductive.

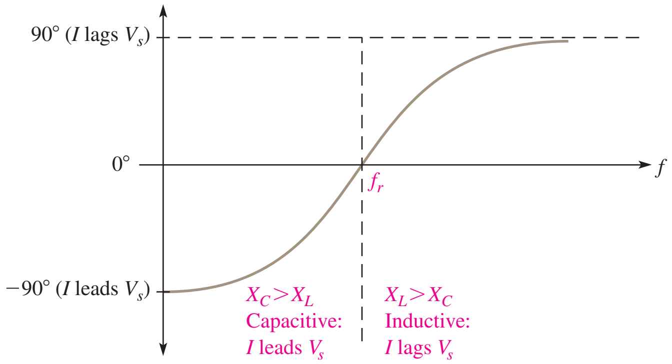
Related Problem What happens to the impedance magnitude if f is decreased below 4.03 kHz? Above 6.03 kHz?

The Phase Angle of a Series *RLC* Circuit

At frequencies below resonance, $X_C > X_L$, and the current leads the source voltage, as indicated in Figure 17–22(a). The phase angle decreases as the frequency approaches the resonant value and is 0° at resonance, as indicated in part (b). At frequencies above resonance, $X_L > X_C$, and the current lags the source voltage, as indicated in part (c). As the frequency goes higher, the phase angle approaches 90°. A plot of phase angle versus frequency is shown in part (d) of the figure.



(a) Below f_r, I leads V_s. (b) At f_r, I is in phase with V_s. (c) Above f_r, I lags V_s.



(d) Phase angle versus frequency.

▲ **FIGURE 17–22**

The phase angle as a function of frequency in a series *RLC* circuit.

SECTION 17–3 CHECKUP

1. What is the condition for series resonance?
2. Why is the current maximum at the resonant frequency?
3. Calculate the resonant frequency for $C = 1{,}000$ pF and $L = 1{,}000\ \mu$H.
4. In Question 3, is the circuit inductive or capacitive at 50 kHz?

OPTION 2 NOTE

This completes the coverage of series reactive circuits. Coverage of parallel reactive circuits begins in Chapter 15, Part 2, on page 673.

Part 2 Parallel Circuits

17–4 Impedance of Parallel *RLC* Circuits

In this section, you will study the impedance and phase angle of a parallel *RLC* circuit. Also, conductance, susceptance, and admittance of a parallel *RLC* circuit are covered.

After completing this section, you should be able to

- **Determine the impedance of a parallel *RLC* circuit**
    - Calculate the conductance, susceptance, and admittance
    - Determine whether a circuit is predominately inductive or capacitive

Figure 17–23 shows a parallel *RLC* circuit. The total impedance can be calculated using the reciprocal of the sum-of-reciprocals method, just as was done for circuits with resistors in parallel.

$$\frac{1}{\mathbf{Z}} = \frac{1}{R\angle 0^\circ} + \frac{1}{X_L\angle 90^\circ} + \frac{1}{X_C\angle -90^\circ}$$

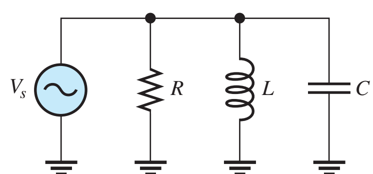
or

Equation 17–5

$$\mathbf{Z} = \frac{1}{\dfrac{1}{R\angle 0^\circ} + \dfrac{1}{X_L\angle 90^\circ} + \dfrac{1}{X_C\angle -90^\circ}}$$

► FIGURE 17–23

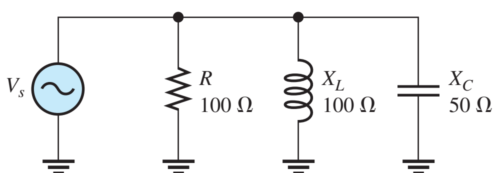
Parallel *RLC* circuit.



EXAMPLE 17–8

Find **Z** in polar form for the parallel *RLC* circuit in Figure 17–24.

► FIGURE 17–24



Solution Use the sum-of-reciprocals formula.

$$\frac{1}{\mathbf{Z}} = \frac{1}{R\angle 0^\circ} + \frac{1}{X_L\angle 90^\circ} + \frac{1}{X_C\angle -90^\circ} = \frac{1}{100\angle 0^\circ\ \Omega} + \frac{1}{100\angle 90^\circ\ \Omega} + \frac{1}{50\angle -90^\circ\ \Omega}$$

Apply the rule for division of polar numbers.

$$\frac{1}{\mathbf{Z}} = 10\angle 0^\circ \text{ mS} + 10\angle -90^\circ \text{ mS} + 20\angle 90^\circ \text{ mS}$$

Recall that the sign of the denominator angle changes when dividing.

Next, convert each term to its rectangular equivalent and combine.

$$\frac{1}{\mathbf{Z}} = 10 \text{ mS} - j10 \text{ mS} + j20 \text{ mS} = 10 \text{ mS} + j10 \text{ mS}$$

Take the reciprocal to obtain **Z** and then convert to polar form.

$$\mathbf{Z} = \frac{1}{10 \text{ mS} + j10 \text{ mS}} = \frac{1}{\sqrt{(10 \text{ mS})^2 + (10 \text{ mS})^2}\angle \tan^{-1}\left(\frac{10 \text{ mS}}{10 \text{ mS}}\right)}$$

$$= \frac{1}{14.14\angle 45^\circ \text{ mS}} = \mathbf{70.7\angle -45^\circ\ \Omega}$$

The negative angle shows that the circuit is capacitive. This may surprise you because $X_L > X_C$. However, in a parallel circuit, the smaller quantity has the greater effect on the total current because its current is the greatest. Similar to the case of resistances in parallel, the smaller reactance draws more current and has the greater effect on the total Z.

In this circuit, the total current leads the total voltage by a phase angle of 45°.

Related Problem If the frequency in Figure 17–24 increases, does the impedance increase or decrease?

Conductance, Susceptance, and Admittance

The concepts of conductance (G), capacitive susceptance (B_C), inductive susceptance (B_L), and admittance (Y) were discussed in Chapters 15 and 16. The phasor formulas are restated here.

$$\mathbf{G} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ$$ **Equation 17–6**

$$\mathbf{B}_C = \frac{1}{X_C\angle -90^\circ} = B_C\angle 90^\circ = jB_C$$ **Equation 17–7**

$$\mathbf{B}_L = \frac{1}{X_L\angle 90^\circ} = B_L\angle -90^\circ = -jB_L$$ **Equation 17–8**

Equation 17–9

$$\mathbf{Y} = \frac{1}{Z\angle \pm\theta} = Y\angle \pm\theta = G + jB_C - jB_L$$

Equation 17–9 in polar form is

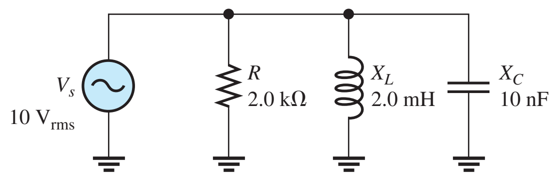
$$\mathbf{Y} = \sqrt{G^2 + (B_C - B_L)^2}\,\angle\tan^{-1}\left(\frac{B_C - B_L}{G}\right)$$

As you know, the unit of each of these quantities is the siemens (S).

EXAMPLE 17–9

For the *RLC* circuit in Figure 17–25 determine the conductance, capacitive susceptance, inductive susceptance, and total admittance at 20 kHz. Draw the admittance phasor diagram at 20 kHz and show a diagram of the frequency response of the circuit to 80 kHz.

▶ FIGURE 17–25



Solution The response of the circut at 20 kHz is solved by showing the explicit steps. Then the TI-84 Plus CE is then used to show the frequency response by a graphing calculator.

At 20 kHz:

$$X_C = \frac{1}{2\pi fC}$$

$$B_C = \frac{1}{X_C} = 2\pi fC = 2\pi(20\text{ kHz})(10\text{ nF}) = 1.26\text{ mS}$$

$$X_L = 2\pi fL$$

$$B_L = \frac{1}{X_L} = \frac{1}{2\pi fL} = \frac{1}{2\pi(20\text{ kHz})(2.0\text{ mH})} = 3.98\text{ mS}$$

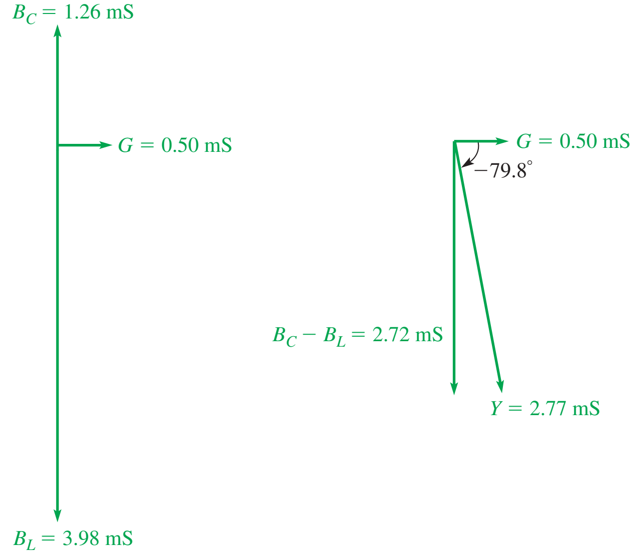
$$G = \frac{1}{R} = \frac{1}{2.0\text{ k}\Omega} = 0.50\text{ mS}$$

$$\mathbf{Y} = \sqrt{G^2 + (B_C - B_L)^2}\,\angle\tan^{-1}\left(\frac{B_C - B_L}{G}\right)$$

$$= \sqrt{0.50\text{ mS}^2 + (1.26\text{ mS} - 3.88\text{ mS})^2}\,\angle\tan^{-1}\left(\frac{1.26\text{ mS} - 3.98\text{ mS}}{0.50\text{ mS}}\right)$$

$$= \mathbf{2.77\text{ mS}\angle -79.6^\circ}$$

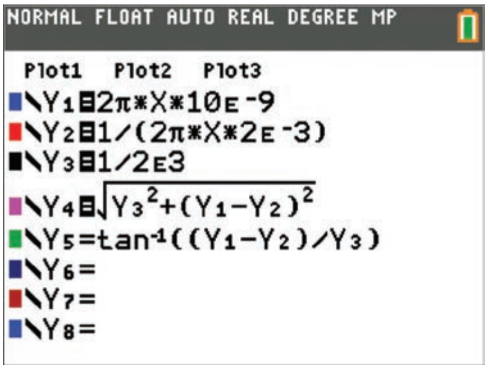
From the calculated values at 20 kHz, the admittance phasor diagram can be drawn as Figure 17–26.



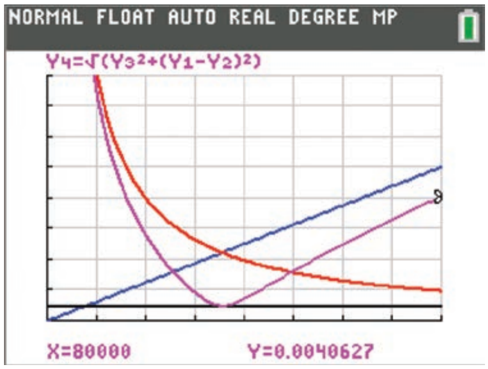
▲ **FIGURE 17–26**

Admittance phasors at 20 kHz.

For solving the frequency response on a graphing calculator, enter the equations as shown in Figure 17–27(a). Y_1 represents B_C, Y_2 represents B_L, Y_3 represents G, and Y_4 represents Y. Notice that Y_4 uses previous results (Y_1, Y_2, and Y_3). To insert these quantities into a new equation, press alpha trace and select the variable you want. The independent variable for all equations is the frequency, shown as X. Y_5 is the phase angle, but is deselected from the graph in part (b). (Deselect the equation by moving the cursor to the equal sign and press enter.) To view a graph of the equations, set up the plot parameters in window and press graph. Figure 17–27(b) shows the graph with the calculation of Y at 80 kHz shown at the bottom. The grid lines are present every 10 kHz in X and every 1 mS in Y.



(a) Equations for B_C, B_L, G, Y, and θ



(b) Graphs of B_C (blue), B_L (red), G (black), and Y (magenta). The readout (at the bottom) is showing Y is calculated at 80 kHz as 4.0627 mS.

▲ **FIGURE 17–27**

Images used with permission by Texas Instruments, Inc.

Related Problem Is the circuit in Figure 17–25 predominately inductive or predominately capacitive at 40 kHz?

SECTION 17–4 CHECKUP

1. In a certain parallel *RLC* circuit, the capacitive reactance is 600 Ω, and the inductive reactance is 1,000 Ω. Is the circuit predominantly capacitive or inductive?
2. Determine the admittance of a parallel circuit in which $R = 1.0\ \text{k}\Omega$, $X_C = 500\ \Omega$, and $X_L = 1.2\ \text{k}\Omega$.
3. In Question 2, what is the impedance?

17–5 ANALYSIS OF PARALLEL *RLC* CIRCUITS

As you have learned, the smaller reactance in a parallel circuit dominates because it results in the larger branch current.

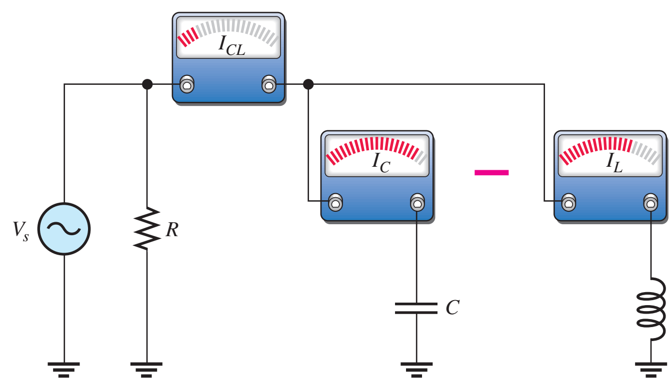
After completing this section, you should be able to

- **Analyze parallel *RLC* circuits**
  - Explain how the currents are related in terms of phase
  - Calculate impedance, currents, and voltages

Recall that capacitive reactance varies inversely with frequency and that inductive reactance varies directly with frequency. In a parallel *RLC* circuit at low frequencies, the inductive reactance is less than the capacitive reactance; therefore, the circuit is inductive. As the frequency is increased, X_L increases and X_C decreases until a value is reached where $X_L = X_C$. This is the point of **parallel resonance**. As the frequency is increased further, X_C becomes smaller than X_L, and the circuit becomes capacitive.

Current Relationships

In a parallel *RLC* circuit, the current in the capacitive branch and the current in the inductive branch are *always* 180° out of phase with each other (neglecting any coil resistance). Because I_C and I_L add algebraically, the total current is actually the difference in their magnitudes. Thus, the total current into the parallel branches of L and C is always less than the largest individual branch current, as illustrated in Figure 17–28 and in the waveform diagram of Figure 17–29. Of course, the current in the resistive branch is always 90° out of phase with both reactive currents, as shown in the current phasor diagram of Figure 17–30. Notice that I_C is plotted on the positive y-axis and I_L is plotted on the negative y-axis.



▲ FIGURE 17–28

The total current into the parallel combination of C and L is the difference of the two branch currents.

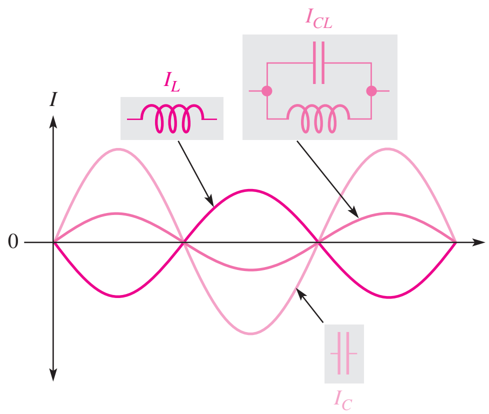


FIGURE 17–29

I_C and I_L effectively subtract.

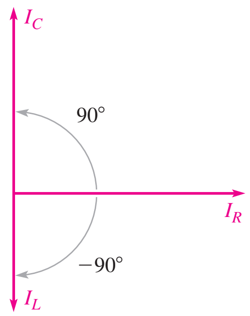


FIGURE 17–30

Typical current phasor diagram for a parallel *RLC* circuit.

The total current can be expressed as

$$\mathbf{I}_{tot} = \sqrt{I_R^2 + (I_C - I_L)^2}\angle\tan^{-1}\left(\frac{I_{CL}}{I_R}\right)$$

Equation 17–10

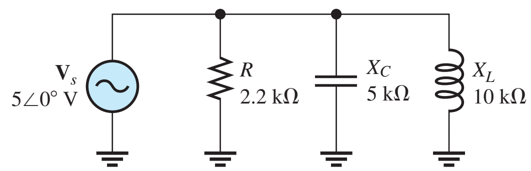
where I_{CL} is $I_C - I_L$, the total current into the L and C branches.

In Example 17–9, the admittance phasor diagram was drawn at a frequency of 20 kHz. To draw the current phasors for this example, you simply multiply each phasor by the applied voltage (10 V). In this case, the total current is $V_S \times Y = (10\text{ V})(2.77\text{ mS}) = 27.7\text{ mA}$.

EXAMPLE 17–10

For the circuit in Figure 17–31 find each branch current and the total current. Draw a diagram of their relationship.

FIGURE 17–31



Solution Use Ohm's law to find each branch current in phasor form.

$$\mathbf{I}_R = \frac{\mathbf{V}_s}{\mathbf{R}} = \frac{5\angle 0^\circ\text{ V}}{2.2\angle 0^\circ\text{ k}\Omega} = \mathbf{2.27\angle 0^\circ\text{ mA}}$$

$$\mathbf{I}_C = \frac{\mathbf{V}_s}{\mathbf{X}_C} = \frac{5\angle 0^\circ\text{ V}}{5\angle -90^\circ\text{ k}\Omega} = \mathbf{1\angle 90^\circ\text{ mA}}$$

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{X}_L} = \frac{5\angle 0^\circ \text{ V}}{10\angle 90^\circ \text{ k}\Omega} = \mathbf{0.5\angle -90^\circ \text{ mA}}$$

The total current is the phasor sum of the branch currents. By Kirchhoff's law,

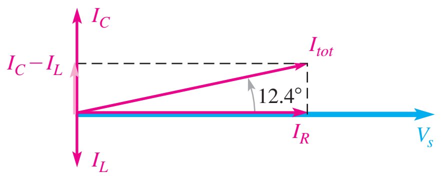
$$\mathbf{I}_{tot} = \mathbf{I}_R + \mathbf{I}_C + \mathbf{I}_L$$
$$= 2.27\angle 0^\circ \text{ mA} + 1\angle 90^\circ \text{ mA} + 0.5\angle -90^\circ \text{ mA}$$
$$= 2.27 \text{ mA} + j1 \text{ mA} - j0.5 \text{ mA} = 2.27 \text{ mA} + j0.5 \text{ mA}$$

Converting to polar form yields

$$\mathbf{I}_{tot} = \sqrt{I_R^2 + (I_C - I_L)^2}\angle \tan^{-1}\left(\frac{I_{CL}}{I_R}\right)$$
$$= \sqrt{(2.27 \text{ mA})^2 + (0.5 \text{ mA})^2}\angle \tan^{-1}\left(\frac{0.5 \text{ mA}}{2.27 \text{ mA}}\right) = \mathbf{2.33\angle 12.4^\circ \text{ mA}}$$

The total current is 2.32 mA leading V_s by 12.4°. Figure 17–32 is the current phasor diagram for the circuit.

▶ FIGURE 17–32



Related Problem Will total current increase or decrease if the frequency in Figure 17–31 is increased?

SECTION 17–5 CHECKUP

1. In a three-branch parallel circuit, $R = 150\ \Omega$, $X_C = 100\ \Omega$, and $X_L = 50\ \Omega$. Determine the current in each branch when $V_s = 12$ V.
2. The impedance of a parallel *RLC* circuit is $2.8\angle -38.9^\circ$ kΩ. Is the circuit capacitive or inductive?

17–6 PARALLEL RESONANCE

In this section, we will first look at the resonant condition in an ideal parallel *LC* circuit (no winding resistance). Then, we will examine the more realistic case where the resistance of the coil is taken into account.

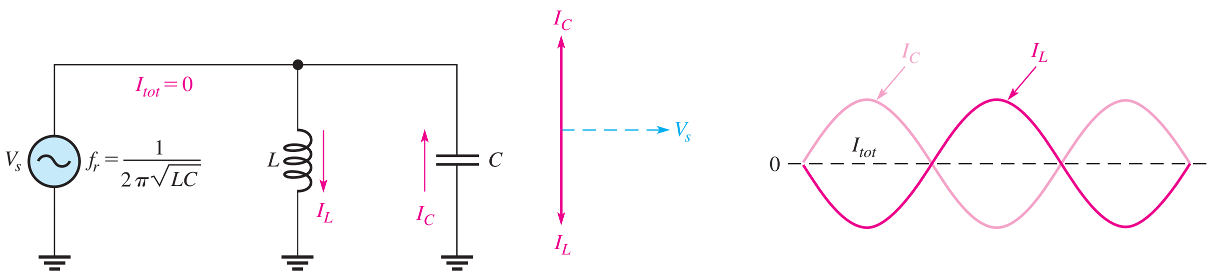
After completing this section, you should be able to

- **Analyze a circuit for parallel resonance**
  - Describe parallel resonance in an ideal circuit
  - Describe parallel resonance in a nonideal circuit

- Explain how impedance varies with frequency
- Determine current and phase angle at resonance
- Determine parallel resonant frequency

Condition for Ideal Parallel Resonance

Ideally, parallel resonance occurs when $X_C = X_L$. The frequency at which resonance occurs is called the *resonant frequency,* just as in the series case. When $X_C = X_L$, the two branch currents, I_C and I_L, are equal in magnitude, and, of course, they are always 180° out of phase with each other. Thus, the two currents cancel and the total current is zero, as shown in Figure 17–33.



(a) Parallel circuit at resonance ($X_C = X_L$, $Z = \infty$) (b) Current phasors (c) Current waveforms

▲ **FIGURE 17–33**

An ideal parallel *LC* circuit at resonance.

Since the total current is zero, the impedance of the ideal parallel LC circuit is infinitely large (∞). These ideal resonant conditions are stated as follows:

$$X_L = X_C$$
$$Z_r = \infty$$

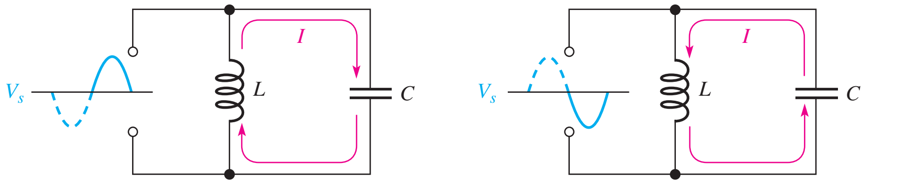
Parallel Resonant Frequency

For an ideal (no resistance) parallel resonant circuit, the frequency at which resonance occurs is determined by the same formula as in series resonant circuits; that is,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

Tank Circuit

The parallel resonant LC circuit is often called a **tank circuit**. The term *tank circuit* refers to the fact that the parallel resonant circuit stores energy in the magnetic field of the coil and in the electric field of the capacitor. The stored energy is transferred back and forth between the capacitor and the coil on alternate half-cycles as the current goes first one way and then the other when the inductor deenergizes and the capacitor charges, and vice versa. This concept is illustrated in Figure 17–34.



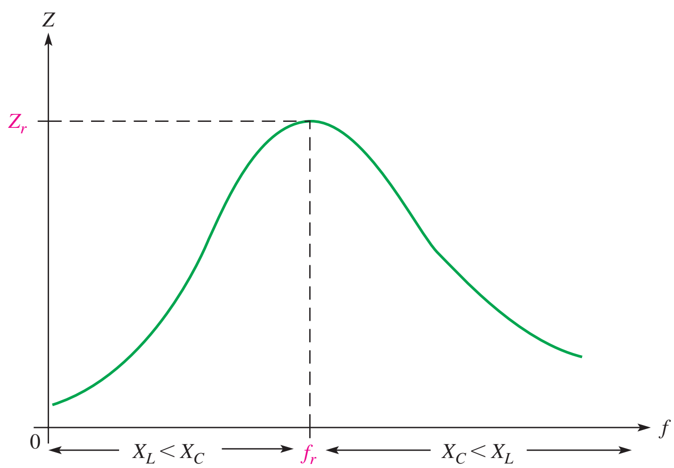
(a) The coil deenergizes as the capacitor charges. (b) The capacitor discharges as the coil energizes.

▲ FIGURE 17–34

Energy storage in an ideal parallel resonant tank circuit.

Variation of the Impedance with Frequency

Ideally, the impedance of a parallel resonant circuit is infinite. In practice, the impedance is maximum at the resonant frequency and decreases at lower and higher frequencies, as indicated by the curve in Figure 17–35.

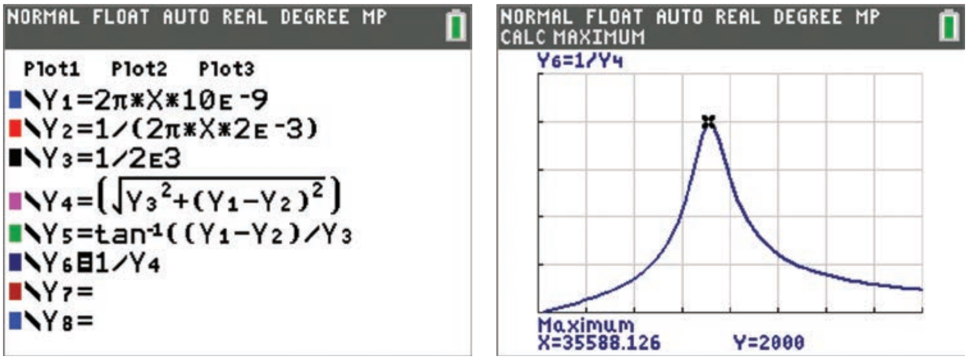


▲ FIGURE 17–35

Generalized impedance curve for a parallel resonant circuit. The circuit is inductive below f_r, resistive at f_r, and capacitive above f_r.

At very low frequencies, X_L is very small and X_C is very high, so the total impedance is essentially equal to that of the inductive branch. As the frequency goes up, the impedance also increases, and the inductive reactance dominates (because it is less than X_C) until the resonant frequency is reached. At this point, of course, $X_L \cong X_C$ (for $Q > 10$) and the impedance is at its maximum. As the frequency goes above resonance, the capacitive reactance dominates (because it is less than X_L) and the impedance decreases.

Recall that Example 17–9 illustrated the admittance curve as a function of frequency. The impedance curve is obtained easily by calculating the reciprocal of the admittance curve. Figure 17–36(a) shows the equations from Example 17–9 with the addition of the equation for the impedance, which is Y_6 (which has been selected). You can see a plot of impedance for the circuit by changing the plot parameters and pressing graph. The grid lines are set at 10 kHz per division in X and 500 Ω per division in Y. The calculator can also find the resonant frequency by looking for the maximum point using the calculate menu (press 2nd trace and select maximum).



(a) Equations from Example 17–9 with the addition of Y_6 for impedance

(b) Impedance plot for Example 17–9

▲ **FIGURE 17–36**

Equations and impedance plot for Example 17–9.

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Current and Phase Angle at Resonance

In the ideal tank circuit, the total current from the source at resonance is zero because the impedance is infinite. In the nonideal case when the winding resistance is considered, there is some total current at the resonant frequency, and it is determined by the impedance at resonance.

$$I_{tot} = \frac{V_s}{Z_r}$$

Equation 17–11

The phase angle of the parallel resonant circuit is 0° because the impedance is purely resistive at the resonant frequency.

Effect of Winding Resistance on the Parallel Resonant Frequency

When the winding resistance is considered, the resonant condition can be expressed as

$$2\pi f_r L\left(\frac{Q^2 + 1}{Q^2}\right) = \frac{1}{2\pi f_r C}$$

where Q is the **quality factor** of the coil, X_L/R_W. Solving for f_r in terms of Q yields

$$f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{Q^2}{Q^2 + 1}}$$

Equation 17–12

When $Q \geq 10$, the term with the Q factors is approximately 1.

$$\sqrt{\frac{Q^2}{Q^2 + 1}} = \sqrt{\frac{100}{101}} = 0.995 \cong 1$$

Therefore, the parallel resonant frequency is approximately the same as the series resonant frequency as long as Q is equal to or greater than 10.

$$f_r \cong \frac{1}{2\pi\sqrt{LC}} \qquad \text{for } Q \geq 10$$

A precise expression for f_r in terms of the circuit component values is

$$f_r = \frac{\sqrt{1 - (R_W^2 C/L)}}{2\pi\sqrt{LC}}$$

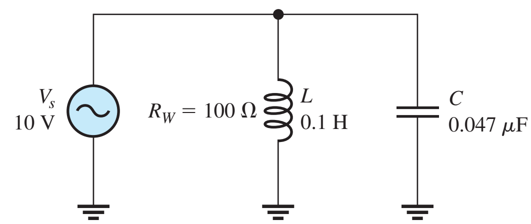
Equation 17–13

This precise formula is seldom necessary and the simpler equation $f_r = 1/(2\pi\sqrt{LC})$ is sufficient for most practical situations. A derivation of Equation 17–13 is given in Appendix B.

EXAMPLE 17–11

Find the precise frequency and the value of Q at resonance for the circuit in Figure 17–37.

▶ FIGURE 17–37



Solution Use Equation 17–13 to find the resonant frequency.

$$f_r = \frac{\sqrt{1 - (R_W^2 C/L)}}{2\pi\sqrt{LC}} = \frac{\sqrt{1 - [(100\ \Omega)^2(0.047\ \mu\text{F})/0.1\ \text{H}]}}{2\pi\sqrt{(0.047\ \mu\text{F})(0.1\ \text{H})}} = \mathbf{2.31\ kHz}$$

To calculate the quality factor, Q, first find X_L.

$$X_L = 2\pi f_r L = 2\pi(2.32\ \text{kHz})(0.1\ \text{H}) = \mathbf{1.45\ k\Omega}$$

$$Q = \frac{X_L}{R_W} = \frac{1.45\ \text{k}\Omega}{100\ \Omega} = \mathbf{14.5}$$

Note that since $Q > 10$, the approximate formula, $f_r \cong 1/(2\pi\sqrt{LC})$, can be used.

Related Problem For a smaller R_W, will f_r be less than or greater than 2.31 kHz?

Use Multisim files E17-11A and E17-11B to verify the calculated results in this example and to confirm your answer for the related problem.

SECTION 17–6 CHECKUP

1. Is the impedance minimum or maximum at parallel resonance?
2. Is the current minimum or maximum at parallel resonance?
3. For ideal parallel resonance, assume $X_L = 1500\ \Omega$. What is X_C?
4. A parallel tank circuit has the following values: $R_W = 4\ \Omega$, $L = 50$ mH, and $C = 10$ pF. Calculate f_r.
5. If $Q = 25$, $L = 50$ mH, and $C = 1{,}000$ pF, what is f_r?
6. In Question 5, if $Q = 2.5$, what is f_r?

Option 2 Note

This completes the coverage of parallel reactive circuits. Coverage of series-parallel reactive circuits begins in Chapter 15, Part 3, on page 682.

SERIES-PARALLEL CIRCUITS

Part 3

17–7 ANALYSIS OF SERIES-PARALLEL *RLC* CIRCUITS

In this section, series and parallel combinations of R, L, and C components are analyzed in specific examples. Also, conversion of a series-parallel circuit to an equivalent parallel circuit is covered and resonance in a nonideal parallel circuit is considered.

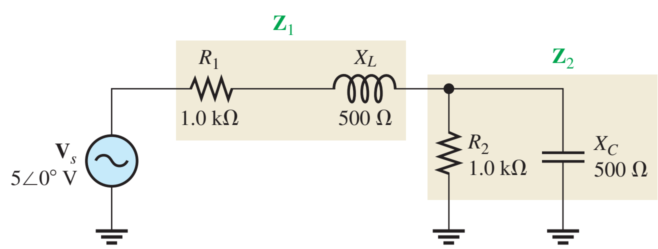
After completing this section, you should be able to

- **Analyze series-parallel *RLC* circuits**
  - Determine currents and voltages
  - Convert a series-parallel circuit to an equivalent parallel form
  - Analyze nonideal (with coil resistance) parallel circuits for parallel resonance
  - Examine the effect of a resistive load on a tank circuit

The following two examples illustrate an approach to the analysis of circuits with both series and parallel combinations of resistance, inductance, and capacitance.

EXAMPLE 17–12

In Figure 17–38, find the voltage across the capacitor in polar form. Is this circuit predominantly inductive or capacitive?



▲ FIGURE 17–38

Solution Use the voltage-divider formula in this analysis. The impedance of the series combination of R_1 and X_L is called $\mathbf{Z}_1$. In rectangular form,

$$\mathbf{Z}_1 = R_1 + jX_L = 1{,}000\ \Omega + j500\ \Omega$$

Converting to polar form yields

$$\mathbf{Z}_1 = \sqrt{R_1^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R_1}\right)$$

$$= \sqrt{(1{,}000\ \Omega)^2 + (500\ \Omega)^2}\angle\tan^{-1}\left(\frac{500\ \Omega}{1{,}000\ \Omega}\right) = 1{,}118\angle 26.6^\circ\ \Omega$$

The impedance of the parallel combination of R_2 and X_C is called $\mathbf{Z}_2$. In polar form,

$$\mathbf{Z}_2 = \left(\frac{R_2X_C}{\sqrt{R_2^2 + X_C^2}}\right)\angle -\tan^{-1}\left(\frac{R_2}{X_C}\right)$$

$$= \left[\frac{(1{,}000\ \Omega)(500\ \Omega)}{\sqrt{(1{,}000\ \Omega)^2 + (500\ \Omega)^2}}\right]\angle -\tan^{-1}\left(\frac{1{,}000\ \Omega}{500\ \Omega}\right) = 447\angle -63.4^\circ\ \Omega$$

Converting to rectangular form yields

$$\mathbf{Z}_2 = Z_2\cos\theta + jZ_2\sin\theta$$

$$= (447\ \Omega)\cos(-63.4^\circ) + j447\sin(-63.4^\circ) = 200\ \Omega - j400\ \Omega$$

The total impedance $\mathbf{Z}_{tot}$ in rectangular form is

$$\mathbf{Z}_{tot} = \mathbf{Z}_1 + \mathbf{Z}_2 = (1{,}000\ \Omega + j500\ \Omega) + (200\ \Omega - j400\ \Omega) = 1{,}200\ \Omega + j100\ \Omega$$

Converting to polar form yields

$$\mathbf{Z}_{tot} = \sqrt{(1{,}200\ \Omega)^2 + (100\ \Omega)^2}\angle\tan^{-1}\left(\frac{100\ \Omega}{1{,}200\ \Omega}\right) = 1204\angle 4.76^\circ\ \Omega$$

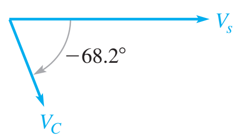
Now apply the voltage-divider formula to get $\mathbf{V}_C$.

$$\mathbf{V}_C = \left(\frac{\mathbf{Z}_2}{\mathbf{Z}_{tot}}\right)\mathbf{V}_s = \left(\frac{447\angle -63.4^\circ\ \Omega}{1204\angle 4.76^\circ\ \Omega}\right)5\angle 0^\circ\ \text{V} = \mathbf{1.86\angle -68.2^\circ\ V}$$

Therefore, V_C is 1.86 V and lags V_s by 68.2°.

The $+j$ term in $\mathbf{Z}_{tot}$, or the positive angle in its polar form, indicates that the circuit is more inductive than capacitive. However, it is just slightly more inductive because the angle is small. This result may surprise you, because $X_C = X_L = 500\ \Omega$. However, the capacitor is in parallel with a resistor, so the capacitor actually has less effect on the total impedance than does the inductor. Figure 17–39 shows the phasor relationship of V_C and V_s. Although $X_C = X_L$, this circuit is not at resonance because the j term of the total impedance is not zero due to the parallel combination of R_2 and X_C. You can see this by noting that the phase angle associated with $\mathbf{Z}_{tot}$ is 4.76° and not zero.

▶ FIGURE 17–39



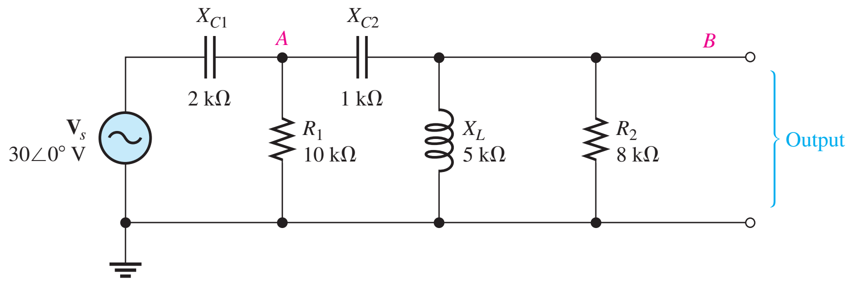
Related Problem Determine the voltage across the capacitor in polar form if R_1 is increased to 2.2 kΩ.

Open Multisim file E17-12 and verify the capacitor voltage.

EXAMPLE 17–13

For the reactive circuit in Figure 17–40, find the voltage at point B with respect to ground.

▶ FIGURE 17–40



Solution The voltage ($\mathbf{V}_B$) at point B is the voltage across the open output terminals. Use the voltage-divider approach. To do so, you must know the voltage ($\mathbf{V}_A$) at point A first; so you need to find the impedance from point A to ground as a starting point.

The parallel combination of X_L and R_2 is in series with X_{C2}. This combination is in parallel with R_1. Call this impedance from point A to ground, $\mathbf{Z}_A$. To find $\mathbf{Z}_A$, take the following steps. The impedance of the parallel combination of R_2 and X_L is called $\mathbf{Z}_1$.

$$\mathbf{Z}_1 = \left(\frac{R_2X_L}{\sqrt{R_2^2 + X_L^2}}\right)\angle\tan^{-1}\left(\frac{R_2}{X_L}\right)$$

$$= \left(\frac{(8\text{ k}\Omega)(5\text{ k}\Omega)}{\sqrt{(8\text{ k}\Omega)^2 + (5\text{ k}\Omega)^2}}\right)\angle\tan^{-1}\left(\frac{8\text{ k}\Omega}{5\text{ k}\Omega}\right) = 4.24\angle 58.0^\circ\text{ k}\Omega$$

Next, combine $\mathbf{Z}_1$ in series with $\mathbf{X}_{C2}$ to get an impedance $\mathbf{Z}_2$.

$$\mathbf{Z}_2 = \mathbf{X}_{C2} + \mathbf{Z}_1$$
$$= 1\angle{-90^\circ}\text{ k}\Omega + 4.24\angle 58^\circ\text{ k}\Omega = -j1\text{ k}\Omega + 2.25\text{ k}\Omega + j3.6\text{ k}\Omega$$
$$= 2.25\text{ k}\Omega + j2.6\text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z}_2 = \sqrt{(2.25\text{ k}\Omega)^2 + (2.6\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{2.6\text{ k}\Omega}{2.25\text{ k}\Omega}\right) = 3.43\angle 49.1^\circ\text{ k}\Omega$$

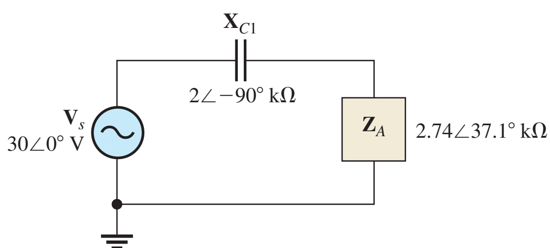
Finally, combine $\mathbf{Z}_2$ and $\mathbf{R}_1$ in parallel to get $\mathbf{Z}_A$.

$$\mathbf{Z}_A = \frac{\mathbf{R}_1\mathbf{Z}_2}{\mathbf{R}_1 + \mathbf{Z}_2} = \frac{(10\angle 0^\circ\text{ k}\Omega)(3.43\angle 49.1^\circ\text{ k}\Omega)}{10\text{ k}\Omega + 2.25\text{ k}\Omega + j2.6\text{ k}\Omega}$$

$$= \frac{34.3\angle 49.1^\circ\text{ k}\Omega}{12.25\text{ k}\Omega + j2.6\text{ k}\Omega} = \frac{34.3\angle 49.1^\circ\text{ k}\Omega}{12.5\angle 12.0^\circ\text{ k}\Omega} = 2.74\angle 37.1^\circ\text{ k}\Omega$$

The simplified circuit is shown in Figure 17–41.

▶ FIGURE 17–41



Next, use the voltage-divider principle to find the voltage ($\mathbf{V}_A$) at point A in Figure 17–40. The total impedance is

$$\mathbf{Z}_{tot} = \mathbf{X}_{C1} + \mathbf{Z}_A$$
$$= 2\angle -90° \text{ k}\Omega + 2.74\angle 37.1° \text{ k}\Omega = -j2 \text{ k}\Omega + 2.19 \text{ k}\Omega + j1.66 \text{ k}\Omega$$
$$= 2.19 \text{ k}\Omega - j0.344 \text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z}_{tot} = \sqrt{(2.19 \text{ k}\Omega)^2 + (0.344 \text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{0.344 \text{ k}\Omega}{2.19 \text{ k}\Omega}\right) = 2.21\angle -8.9° \text{ k}\Omega$$

The voltage at point A is

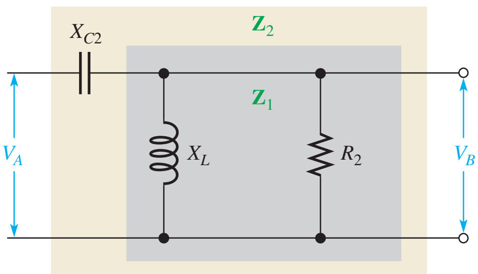
$$\mathbf{V}_A = \left(\frac{\mathbf{Z}_A}{\mathbf{Z}_{tot}}\right)\mathbf{V}_s = \left(\frac{2.74\angle 37.1° \text{ k}\Omega}{2.22\angle -8.9° \text{ k}\Omega}\right)30\angle 0° \text{ V} = 37.2\angle 46.0° \text{ V}$$

Next, find the voltage ($\mathbf{V}_B$) at point B by dividing $\mathbf{V}_A$ down, as indicated in Figure 17–42. $\mathbf{V}_B$ is the open terminal output voltage.

$$\mathbf{V}_B = \left(\frac{\mathbf{Z}_1}{\mathbf{Z}_2}\right)\mathbf{V}_A = \left(\frac{4.24\angle 58.0° \text{ k}\Omega}{3.44\angle 49.1° \text{ k}\Omega}\right)37.2\angle 46.0° \text{ V} = \mathbf{45.9\angle 54.9° \text{ V}}$$

Surprisingly, V_A is greater than V_s, and V_B is greater than V_A! This result is possible because of the out-of-phase relationship of the reactive voltages. Remember that X_C and X_L tend to cancel each other.

▶ FIGURE 17–42



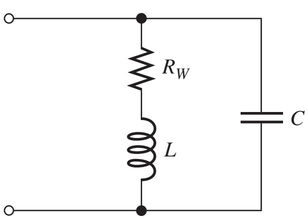
Related Problem What is the voltage in polar form across C_1 in Figure 17–40?

Conversion of Series-Parallel to Parallel

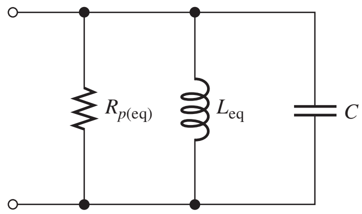
The particular series-parallel configuration shown in Figure 17–43 is important because it represents a circuit having parallel L and C branches, with the winding resistance of the coil taken into account as a series resistance in the L branch.

▶ FIGURE 17–43

A series-parallel RLC circuit ($Q = X_L/R_W$).



It is helpful to view the series-parallel circuit in Figure 17–43 in an equivalent parallel form, as indicated in Figure 17–44.



◀ FIGURE 17–44

Parallel equivalent form of the circuit in Figure 17–43.

The equivalent inductance, L_{eq}, and the equivalent parallel resistance, $R_{p(eq)}$, are given by the following formulas:

$$L_{eq} = L\left(\frac{Q^2 + 1}{Q^2}\right)$$ **Equation 17–14**

$$R_{p(eq)} = R_W(Q^2 + 1)$$ **Equation 17–15**

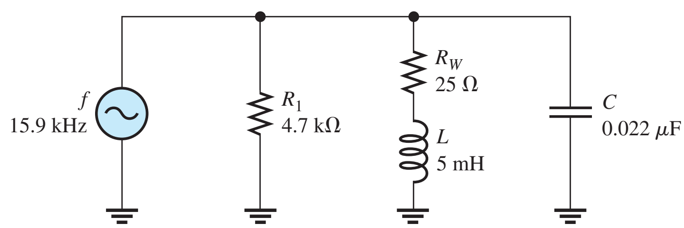
where Q is the quality factor of the coil, X_L/R_W. Derivations of these formulas are quite involved and thus are not given here. Notice in the equations that for a $Q \geq 10$, the value of L_{eq} is approximately the same as the original value of L. For example, if $L = 10$ mH and $Q = 10$, then

$$L_{eq} = 10\text{ mH}\left(\frac{10^2 + 1}{10^2}\right) = 10\text{ mH}(1.01) = 10.1\text{ mH}$$

The equivalency of the two circuits means that at a given frequency, when the same value of voltage is applied to both circuits, the same total current is in both circuits and the phase angles are the same. Basically, an equivalent circuit simply makes circuit analysis more convenient.

EXAMPLE 17–14

Convert the series-parallel circuit in Figure 17–45 to an equivalent parallel form at the given frequency.



▲ FIGURE 17–45

Solution Determine the inductive reactance.

$$X_L = 2\pi fL = 2\pi(15.9\text{ kHz})(5\text{ mH}) = 500\ \Omega$$

The Q of the coil is

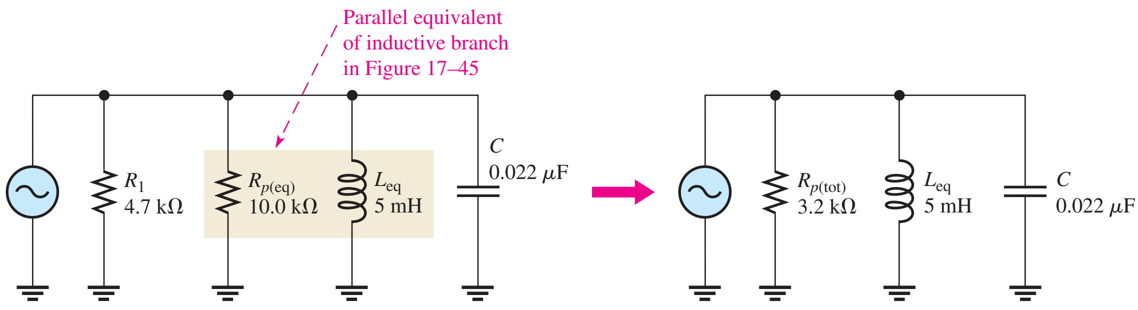
$$Q = \frac{X_L}{R_W} = \frac{500\ \Omega}{25\ \Omega} = 20$$

Since $Q > 10$, then $L_{eq} \cong L = 5$ mH.

The equivalent parallel resistance is

$$R_{p(\text{eq})} = R_W(Q^2 + 1) = (25\ \Omega)(20^2 + 1) = 10\ \text{k}\Omega$$

This equivalent resistance appears in parallel with R_1 as shown in Figure 17–46(a). When combined, they give a total parallel resistance ($R_{p(tot)}$) of 3.2 kΩ, as indicated in Figure 17–46(b).



(a) Parallel equivalent of the circuit in Figure 17–45

(b) $R_{p(tot)} = R_1 \parallel R_{p(\text{eq})} = 3.2\ \text{k}\Omega$

▲ **FIGURE 17–46**

Related Problem Find the equivalent parallel circuit if $R_W = 10\ \Omega$ in Figure 17–45.

Parallel Resonant Conditions in a Nonideal Circuit

The resonance of an ideal parallel *LC* circuit was examined in Section 17–6. Now, let's consider resonance in a tank circuit with the resistance of the coil taken into account. Figure 17–47 shows a nonideal tank circuit and its parallel *RLC* equivalent.

Recall that the quality factor, Q, of the circuit at resonance is simply the Q of the coil if no other resistance is in the circuit.

$$Q = \frac{X_L}{R_W}$$

The expressions for the equivalent inductance and the equivalent parallel resistance were given in Equations 17–14 and 17–15 as

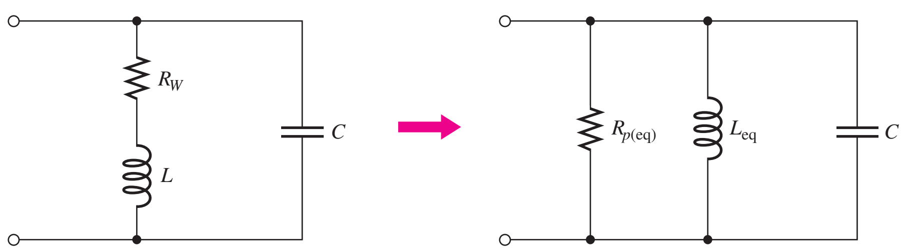
$$L_{\text{eq}} = L\left(\frac{Q^2 + 1}{Q^2}\right)$$

$$R_{p(\text{eq})} = R_W(Q^2 + 1)$$

For $Q \geq 10$, $L_{\text{eq}} \cong L$.

► **FIGURE 17–47**

A practical treatment of parallel resonant circuits must include the coil resistance.



(a) Nonideal tank circuit

(b) Parallel *RLC* equivalent

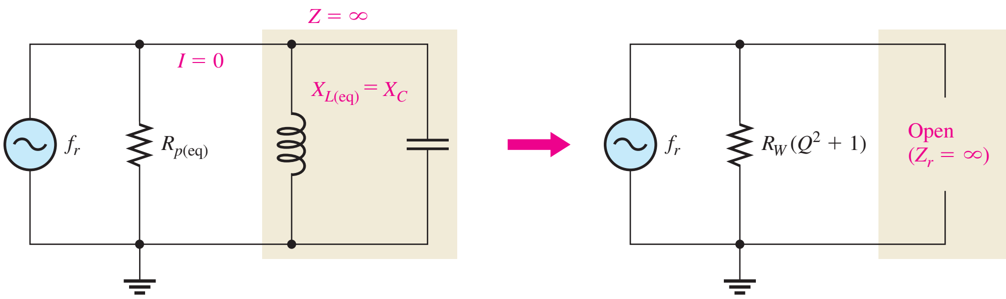
At parallel resonance,

$$X_{L(\mathrm{eq})} = X_C$$

In the parallel equivalent circuit, $R_{p(\mathrm{eq})}$ is in parallel with an ideal coil and a capacitor, so the L and C branches act as an ideal tank circuit which has an infinite impedance at resonance as shown in Figure 17–48. Therefore, the total impedance of the nonideal tank circuit at resonance can be expressed as simply the equivalent parallel resistance.

$$Z_r = R_W(Q^2 + 1)$$ **Equation 17–16**

A derivation of Equation 17–16 is given in Appendix B.



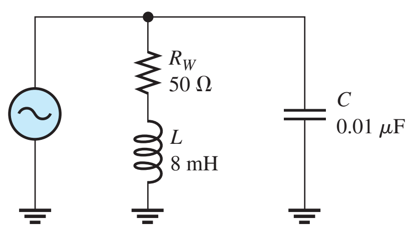
▲ **FIGURE 17–48**

At resonance, the parallel *LC* portion appears open and the source sees only $R_{p(\mathrm{eq})}$.

EXAMPLE 17–15

Determine the impedance of the circuit in Figure 17–49 at the resonant frequency ($f_r \cong$ 17,794 Hz).

► **FIGURE 17–49**



Solution Before you can calculate the impedance using Equation 17–16, you must find the quality factor. To get Q, first find the inductive reactance.

$$X_L = 2\pi f_r L = 2\pi(17{,}794\text{ Hz})(8\text{ mH}) = 894\ \Omega$$

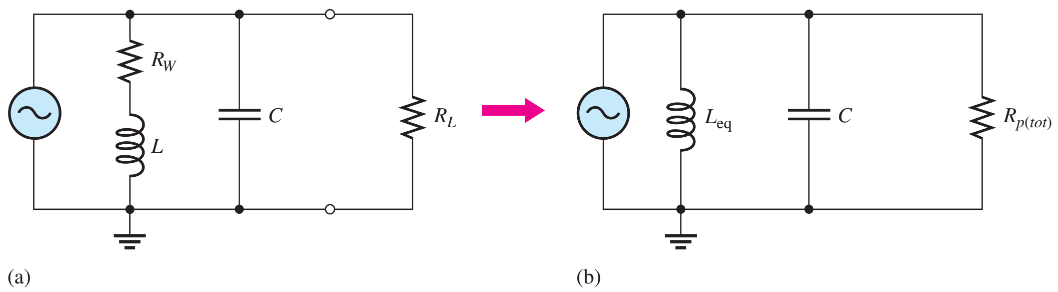
$$Q = \frac{X_L}{R_W} = \frac{894\ \Omega}{50\ \Omega} = 17.9$$

$$Z_r = R_W(Q^2 + 1) = 50\ \Omega(17.9^2 + 1) = \mathbf{16.1\ k\Omega}$$

Related Problem Determine Z_r for $R_W = 10\ \Omega$.

An External Load Resistance Affects a Tank Circuit

In most practical situations, an external load resistance appears in parallel with a non-ideal tank circuit, as shown in Figure 17–50(a). Obviously, the external resistor (R_L) will dissipate more of the energy delivered by the source and thus will lower the overall



▲ **FIGURE 17–50**

Tank circuit with a parallel load resistor and its equivalent circuit.

Q of the circuit. The external resistor effectively appears in parallel with the equivalent parallel resistance of the coil, $R_{p(\text{eq})}$, and both are combined to determine a total parallel resistance, $R_{p(tot)}$, as indicated in Figure 17–50(b).

$$R_{p(tot)} = R_L \parallel R_{p(\text{eq})}$$

The overall Q, designated Q_O, for a parallel RLC circuit is expressed differently from the Q of a series circuit.

Equation 17–17

$$Q_O = \frac{R_{p(tot)}}{X_{L(\text{eq})}}$$

As you can see, the effect of loading the tank circuit is to reduce its overall Q (which is equal to the coil Q when unloaded).

SECTION 17–7 CHECKUP

1. A certain resonant circuit has a 100 μH inductor with a 2 Ω winding resistance in parallel with a 0.22 μF capacitor. If $Q = 8$, determine the parallel equivalent of this circuit.
2. Find the equivalent parallel inductance and resistance for a 20 mH coil with a winding resistance of 10 Ω at a frequency of 1 kHz.

Option 2 Note

This completes the coverage of series-parallel circuits. Coverage of special topics begins in Chapter 15, Part 4, on page 689.

Part 4

Special Topics

17–8 Bandwidth of Resonant Circuits

The current in a series *RLC* is maximum at the resonant frequency because the reactances cancel. The current in a parallel *RLC* is minimum at the resonant frequency because the inductive and capacitive currents cancel. This circuit behavior relates to a characteristic called bandwidth.

After completing this section, you should be able to

- **Determine the bandwidth of resonant circuits**
  - Discuss the bandwidth of series and parallel resonant circuits
  - State the formula for bandwidth
  - Define *half-power frequency*
  - Define *selectivity*
  - Explain how the Q affects the bandwidth

Series Resonant Circuits

The current in a series *RLC* circuit is maximum at the resonant frequency (also known as *center frequency*) and drops off on either side of this frequency. Bandwidth, sometimes abbreviated *BW*, is an important characteristic of a resonant circuit. The bandwidth is the range of frequencies for which the current is equal to or greater than 70.7% of its resonant value.

Figure 17–51 illustrates bandwidth on the response curve of a series *RLC* circuit. Notice that the frequency f_1 below f_r is the point at which the current is $0.707I_{max}$ and is commonly called the *lower critical frequency*. The frequency f_2 above f_r, where the current is again $0.707I_{max}$, is the *upper critical frequency*. Other names for f_1 and f_2 are *−3 dB frequencies, cutoff frequencies,* and *half-power frequencies.* The significance of the latter term is discussed later in the chapter.

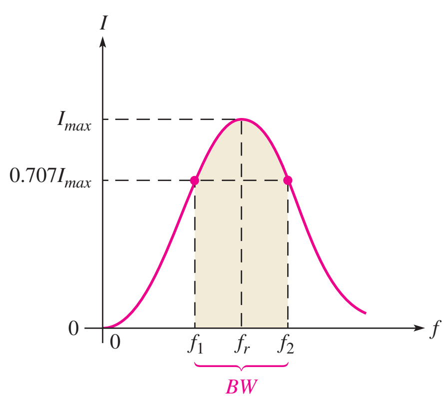


FIGURE 17–51

Bandwidth on series resonant response curve for *I*.

EXAMPLE 17–16

A certain series resonant circuit has a maximum current of 100 mA at the resonant frequency. What is the value of the current at the critical frequencies?

Solution Current at the critical frequencies is 70.7% of maximum.

$$I_{f1} = I_{f2} = 0.707 I_{max} = 0.707(100 \text{ mA}) = \mathbf{70.7\ mA}$$

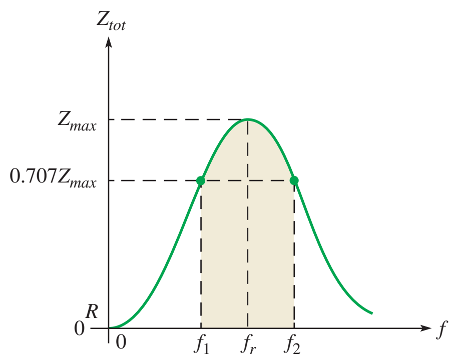
Related Problem A certain series resonant circuit has a current of 25 mA at the critical frequencies. What is the current at resonance?

Parallel Resonant Circuits

For a parallel resonant circuit, the impedance is maximum at the resonant frequency; so the total current is minimum. The bandwidth can be defined in relation to the impedance curve in the same manner that the current curve was used in the series circuit. Of course, f_r is the frequency at which Z is maximum; f_1 is the lower critical frequency at which $Z = 0.707Z_{max}$; and f_2 is the upper critical frequency at which again $Z = 0.707Z_{max}$. The bandwidth is the range of frequencies between f_1 and f_2, as shown in Figure 17–52.

▶ **FIGURE 17–52**

Bandwidth of the parallel resonant response curve for Z_{tot}.



Recall that in a parallel resonant circuit, the admittance is a minimum at resonance. Because admittance is the reciprocal of impedance, you can also use the admittance curve to determine the bandwidth. In this case, the critical frequencies are at $Y = 1.41Y_{min}$.

Formula for Bandwidth

The bandwidth for either series or parallel resonant circuits is the range of frequencies between the critical frequencies for which the response curve (I or Z) is 0.707 of the maximum value. Thus, the bandwidth is actually the difference between f_2 and f_1.

Equation 17–18

$$BW = f_2 - f_1$$

Ideally, f_r is the center frequency and can be calculated as follows:

Equation 17–19

$$f_r = \frac{f_1 + f_2}{2}$$

EXAMPLE 17–17

The circuit in Example 17–9 has a lower critical frequency of 31.8 kHz and an upper critical frequency of 39.8 kHz.From this information, determine the bandwidth and the ideal center (resonant) frequency.

Solution

$$BW = f_2 - f_1 = 39.8\text{ kHz} - 31.8\text{ kHz} = \mathbf{8.0\ kHz}$$

$$f_r = \frac{f_1 + f_2}{2} = \frac{39.8\text{ kHz} + 31.8\text{ kHz}}{2} = \mathbf{35.8\ kHz}$$

Related Problem If the bandwidth of a resonant circuit is 2.5 kHz and its center frequency is 8 kHz, what are the lower and upper critical frequencies?

Half-Power Frequencies

As previously mentioned, the upper and lower critical frequencies are sometimes called the **half-power frequencies**. This term is derived from the fact that the power from the source at these frequencies is one-half the power delivered at the resonant frequency. The following shows that this is true for a series circuit. The same end result also applies to a parallel circuit. At resonance,

$$P_{max} = I_{max}^2 R$$

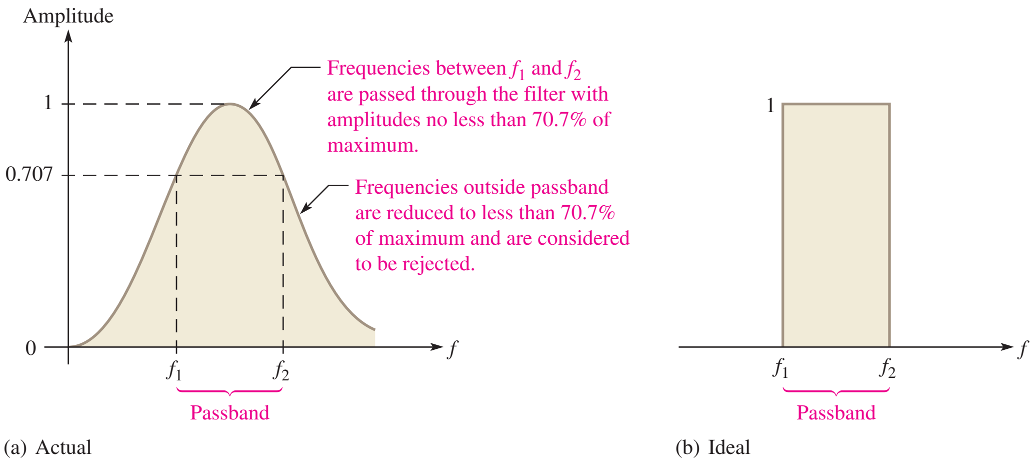
The power at f_1 or f_2 is

$$P_{f1} = I_{f1}^2 R = (0.707 I_{max})^2 R = (0.707)^2 I_{max}^2 R = 0.5 I_{max}^2 R = 0.5 P_{max}$$

Selectivity

The response curves in Figures 17–51 and 17–52 are also called *selectivity curves.* **Selectivity** defines how well a resonant circuit responds to a certain frequency and discriminates against all others. *The narrower the bandwidth, the greater the selectivity.*

Ideally, we assume that a resonant circuit accepts frequencies within its bandwidth and completely eliminates frequencies outside the bandwidth. Such is not actually the case, however, because signals with frequencies outside the bandwidth are not completely eliminated. Their magnitudes, however, are greatly reduced. The further the frequencies are from the critical frequencies, the greater the reduction, as illustrated in Figure 17–53(a). An ideal selectivity curve is shown in Figure 17–53(b).



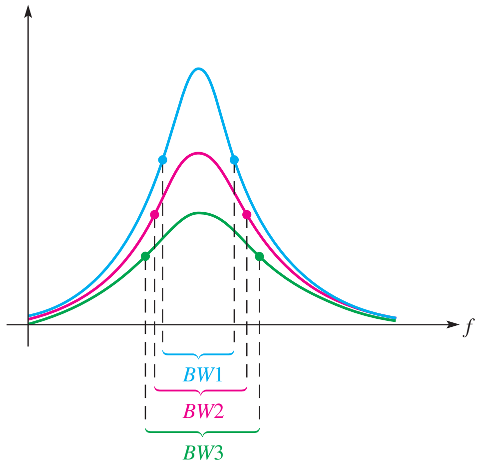
▲ **FIGURE 17–53**

Generalized selectivity curve.

Another factor that influences selectivity is the steepness of the slopes of the response curve. The faster the curve drops off at the critical frequencies, the more selective the circuit is because it tends to respond only to the frequencies within

▶ FIGURE 17–54

Comparative selectivity curves. The blue curve represents the greatest selectivity.



the bandwidth. Figure 17–54 shows a general comparison of three response curves with varying degrees of selectivity.

Q Affects Bandwidth

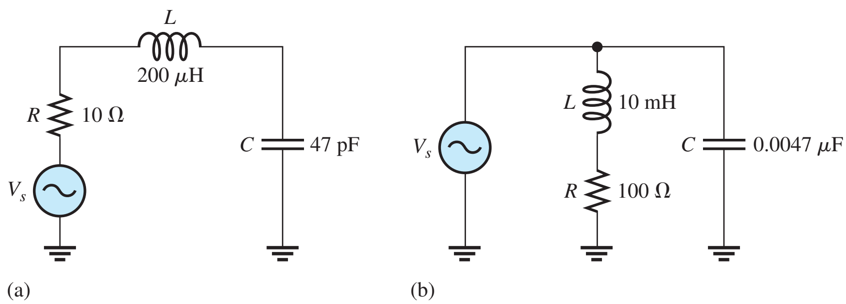
A higher value of circuit Q results in a narrower bandwidth. A lower value of Q causes a wider bandwidth. A formula for the bandwidth of a resonant circuit in terms of Q is stated in the following equation:

Equation 17–20

$$BW = \frac{f_r}{Q}$$

EXAMPLE 17–18

What is the bandwidth of each circuit in Figure 17–55?



▲ FIGURE 17–55

Solution For the circuit in Figure 17–55(a), determine the bandwidth as follows:

$$fr = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(200\,\mu\text{H})(47\,\text{pF})}} = 1.64\,\text{MHz}$$

$$X_L = 2\pi f_r L = 2\pi(1.64\,\text{MHz})(200\,\mu\text{H}) = 2.06\,\text{k}\Omega$$

$$Q = \frac{X_L}{R} = \frac{2.06\,\text{k}\Omega}{10\,\Omega} = 206$$

$$BW = \frac{f_r}{Q} = \frac{1.64\,\text{MHz}}{206} = \mathbf{7.96\ kHz}$$

For the circuit in Figure 17–55(b),

$$f_r = \frac{\sqrt{1 - (R_W^2 C/L)}}{2\pi\sqrt{LC}} \cong \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(10\,\text{mH})(0.0047\,\mu\text{F})}} = 23.2\,\text{kHz}$$

$$X_L = 2\pi f_r L = 2\pi(23.2\,\text{kHz})(10\,\text{mH}) = 1.46\,\text{k}\Omega$$

$$Q = \frac{X_L}{R} = \frac{1.46\,\text{k}\Omega}{100\,\Omega} = 14.6$$

$$BW = \frac{f_r}{Q} = \frac{23.2\,\text{kHz}}{14.6} = \mathbf{1.59\ kHz}$$

Related Problem Change *C* in Figure 17–55(a) to 1,000 pF and determine the bandwidth.

MultiSim

Use Multisim files E17-18A, E17-18B, and E17-18C to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 17–8 CHECKUP

1. What is the bandwidth when $f_2 = 2.2\,\text{MHz}$ and $f_1 = 1.8\,\text{MHz}$?
2. For a resonant circuit with the critical frequencies in Question 1, what is the center frequency?
3. The power-dissipated at resonance is 1.8 W. What is the power at the upper critical frequency?
4. Does a larger *Q* mean a narrower or a wider bandwidth?

17–9 APPLICATIONS

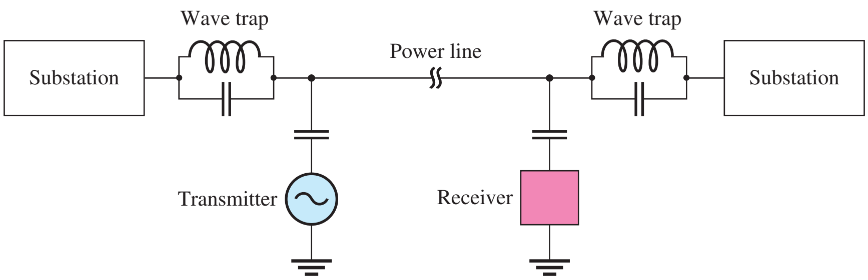
Resonant circuits are used in a wide variety of applications, particularly in communication systems. In this section, we will look briefly at a few common communication systems applications to illustrate the importance of resonant circuits in electronic communication.

After completing this section, you should be able to

- **Discuss some applications of resonant circuits**
  - Describe a tuned amplifier application
  - Describe antenna coupling
  - Describe tuned amplifiers
  - Describe audio crossover networks
  - Describe a radio receiver

Wave Trap

A **wave trap** is resonant circuit designed to pass certain frequencies and block others. They are commonly used in communication systems to block interference. Wave traps are also used by electric utilities to allow power lines to double as communication lines between substations. The power line frequency (50 or 60 Hz) will pass but the high frequency communication signal is blocked. The wave trap is part of a system called power line carrier communications (PLCC). In the case of electric utilities, wave traps are basically parallel resonant circuits installed on the power lines to block the communication signals from being passed to the power distribution portion of the substation. Figure 17–56 illustrates basic wave traps for a PLCC applications.

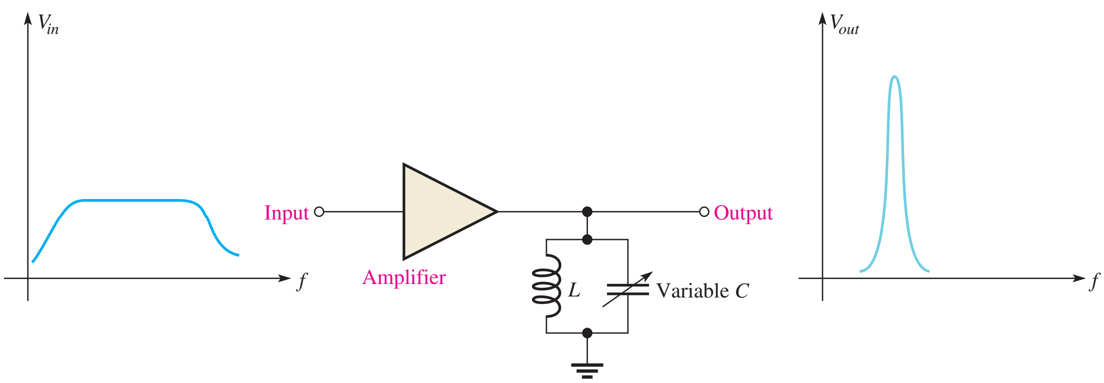


▲ FIGURE 17–56

Basic wave traps for PLCC.

Tuned Amplifiers

A *tuned amplifier* is a circuit that amplifies signals within a specified band. Typically, a parallel resonant circuit is used in conjunction with an amplifier to achieve the selectivity. In terms of the general operation, input signals with frequencies that range over a wide band are accepted on the amplifier's input and are amplified. The resonant circuit allows only a relatively narrow band of those frequencies to be passed on. The variable capacitor allows tuning over the range of input frequencies so that a desired frequency can be selected, as indicated in Figure 17–57.

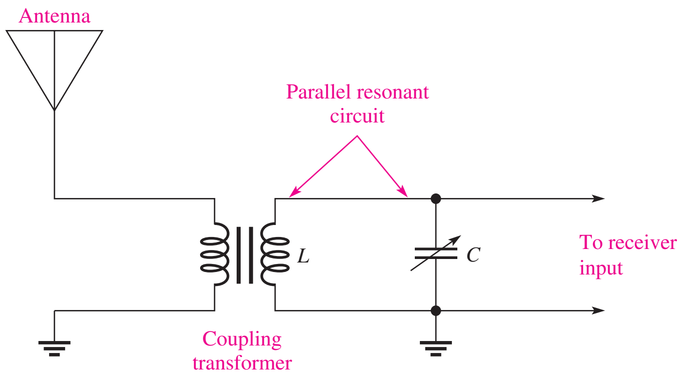


▲ FIGURE 17–57

A basic tuned band-pass amplifier.

Antenna Input to a Receiver

Radio signals are sent out from a transmitter via electromagnetic waves that propagate through the atmosphere. When the electromagnetic waves cut across the receiving antenna, small voltages are induced. Out of all the wide range of electromagnetic frequencies, only one frequency or a limited band of frequencies must be extracted.



▲ FIGURE 17–58

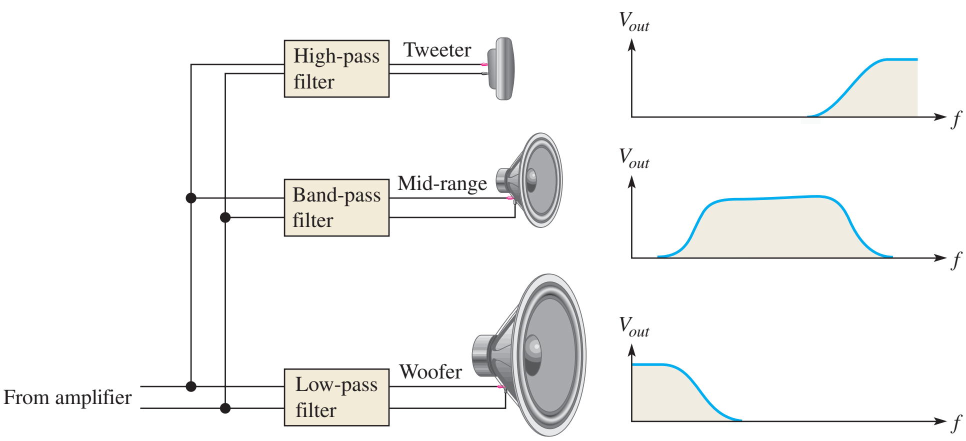
Resonant coupling from an antenna.

Figure 17–58 shows a typical arrangement of an antenna coupled to the receiver input by a transformer. A variable capacitor is connected across the transformer secondary to form a parallel resonant circuit.

Audio Crossover Networks

Most stereo systems have speakers designed for specific parts of the audio frequency spectrum. Audio crossover networks are filter networks that separate the audio signal into different frequency bands for the speakers while maintaining an overall flat response. Crossover networks can be passive (meaning only inductors, capacitors, and resistors are used) or active (meaning transistor and operational amplifiers are used).

A three-way passive network includes a band-pass filter that is essentially a low-Q resonant filter designed to have a broad flat response. The design of passive crossover networks is complicated because of various factors that affect the overall response (such as variations of the speaker's impedance as a function of frequency), but the basic concept is simple. The network separates the audio spectrum into three parts and passes each part to the appropriate speaker, as indicated in Figure 17–59. The network has three filters: a high-pass filter (usually only a single mylar or polystyrene capacitor) that passes high frequencies to the tweeter (or high-frequency speaker), a band-pass filter that passes middle frequencies to the mid-range speaker, and a low-pass filter that passes low frequencies to the woofer (or low-frequency speaker).

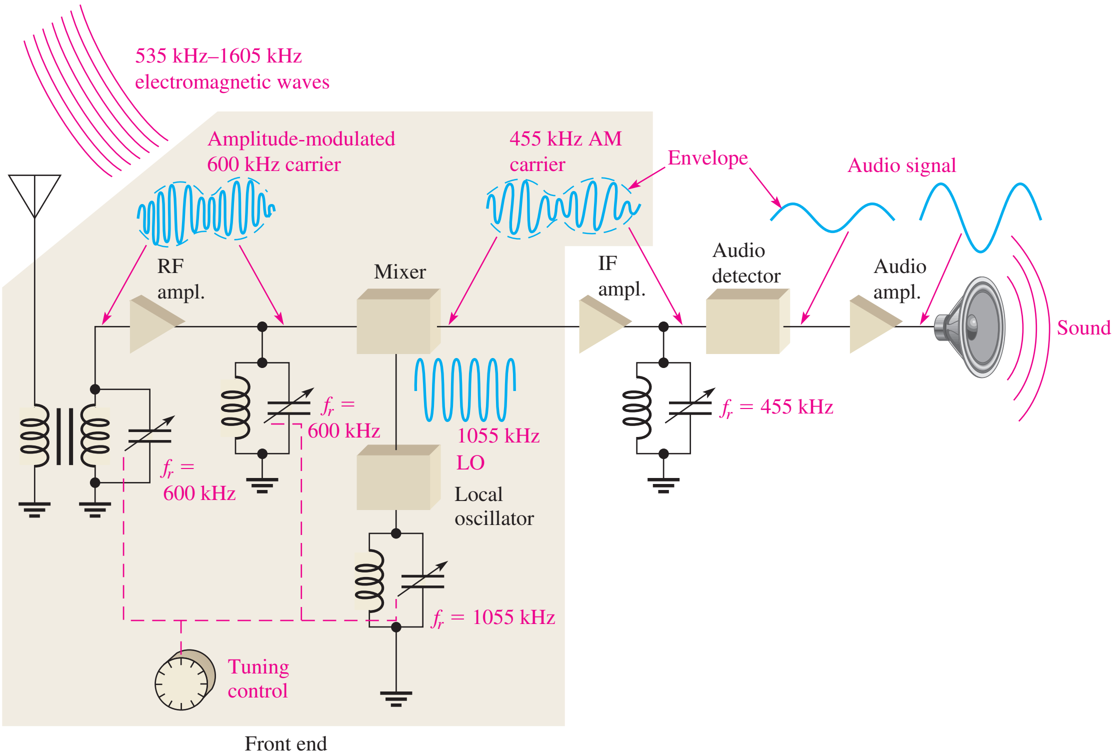


▲ FIGURE 17–59

A crossover network uses filters to separate the audio frequency bands.

Superheterodyne Receiver

Another example of resonant circuit (filter) applications is in the common AM (amplitude modulation) receiver. The AM broadcast band ranges from 535 kHz to 1605 kHz. Each AM station is assigned a 10 kHz bandwidth within that range. The tuned circuits are



▲ **FIGURE 17–60**

A simplified diagram of a superheterodyne AM radio broadcast receiver showing an example of the application of tuned resonant circuits.

designed to pass only the signals from the desired radio station, rejecting all others. To reject stations outside the one that is tuned, the tuned circuits must be selective, passing on only the signals in the 10 kHz band and rejecting all others. Too much selectivity is not desirable either, however. If the bandwidth is too narrow, some of the higher frequency modulated signals will be rejected, resulting in a loss of fidelity. Ideally, the resonant circuit must reject signals that are not in the desired passband. A simplified block diagram of a superheterodyne AM receiver is shown in Figure 17–60.

There are basically three parallel resonant circuits in the front end of the receiver. Each of these resonant circuits is gang-tuned by capacitors; that is, the capacitors are mechanically or electronically linked together so that they change together as the tuning knob is turned. The front end is tuned to receive a desired station, for example, one that transmits at 600 kHz. The input resonant circuit from the antenna and the RF (radio frequency) amplifier resonant circuit select only a frequency of 600 kHz out of all the frequencies crossing the antenna.

The actual audio (sound) signal is carried by the 600 kHz carrier frequency by modulating the amplitude of the carrier so that it follows the audio signal as indicated. The variation in the amplitude of the carrier corresponding to the audio signal is called the *envelope.* The 600 kHz is then applied to a circuit called the *mixer.*

The *local oscillator* (LO) is tuned to a frequency that is 455 kHz above the selected frequency (1055 kHz, in this case). By a process called *heterodyning* or *beating,* the AM signal and the local oscillator signal are mixed together, and the 600 kHz AM signal is converted by the mixer to a 455 kHz AM signal (1055 kHz − 600 kHz = 455 kHz).

The 455 kHz is the intermediate frequency (IF) for standard AM receivers. No matter which station within the broadcast band is selected, its frequency is always converted to the 455 kHz IF. The amplitude-modulated IF is amplified by the IF amplifier which is tuned to 455 kHz. The output of the IF amplifier is applied to an *audio detector* which removes the IF, leaving only the envelope which is the audio signal. The audio signal is then amplified and applied to the speaker.

SECTION 17–9 CHECKUP

1. What is the function of a wave trap on an electric utility power line?
2. Generally, why is a tuned filter necessary when a signal is coupled from an antenna to the input of a receiver?
3. What is a crossover network?
4. What is meant by *ganged tuning*?

Application Activity

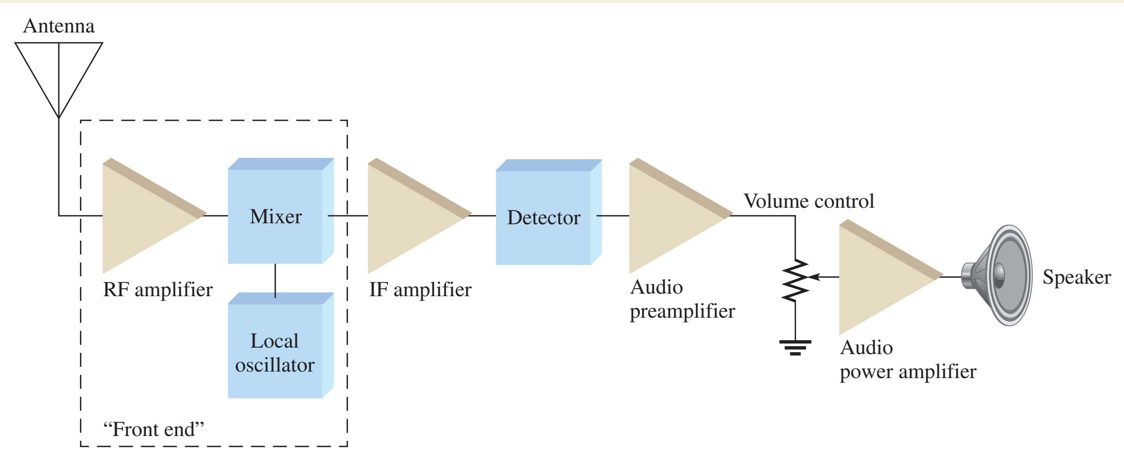
In the Chapter 11 application activity, you worked with a receiver system to learn basic ac measurements. In this chapter, the receiver is again used to illustrate one application of resonant circuits. We will focus on a part of the "front end" of the receiver system that contains resonant circuits. Generally, the front end includes the RF amplifier, the local oscillator, and the mixer. In this application activity, the RF amplifier is the focus. A knowledge of amplifier circuits is not necessary at this time.

A basic block diagram of an AM radio receiver is shown in Figure 17–61. In this particular system, the "front end" includes the circuitry used for tuning in a desired broadcasting station by frequency selection and then converting that selected frequency to a standard intermediate frequency (IF). AM radio stations transmit in the frequency range from 535 kHz to 1605 kHz. The purpose of the RF amplifier is to take the signals picked up by the antenna, reject all but the signal from the desired station, and amplify it to a higher level.

A schematic of the RF amplifier is shown in Figure 17–62. The parallel resonant tuning circuit consists of L, C_1, and C_2. This particular RF amplifier does not have a resonant circuit on the output. C_1 is a varactor, which is a semiconductor device that you will learn more about in a later course. All that you need to know at this point is that the varactor is basically a variable capacitor whose capacitance is varied by changing the dc voltage across it. In this circuit, the dc voltage comes from the wiper of the potentiometer used for tuning the receiver.

The voltage from the potentiometer can be varied from +1 V to +9 V. The particular varactor used in this circuit can be varied from 200 pF at 1 V to 5 pF at 9 V. The capacitor C_2 is a trimmer capacitor that is used for initially adjusting the resonant circuit. Once it is preset, it is left at that value. C_1 and C_2 are in parallel and their capacitances add to produce the total capacitance for the resonant circuit. C_3 has a minimal effect on the resonant circuit and can be ignored. The purpose of C_3 is to allow the dc voltage to be applied to the varactor while providing an ac ground.

In this application activity, you will focus on the RF amplifier circuit board in Figure 17–63. Although all of the amplifier components are on the board, the part that you are to focus on is the resonant circuit indicated by the highlighted area.

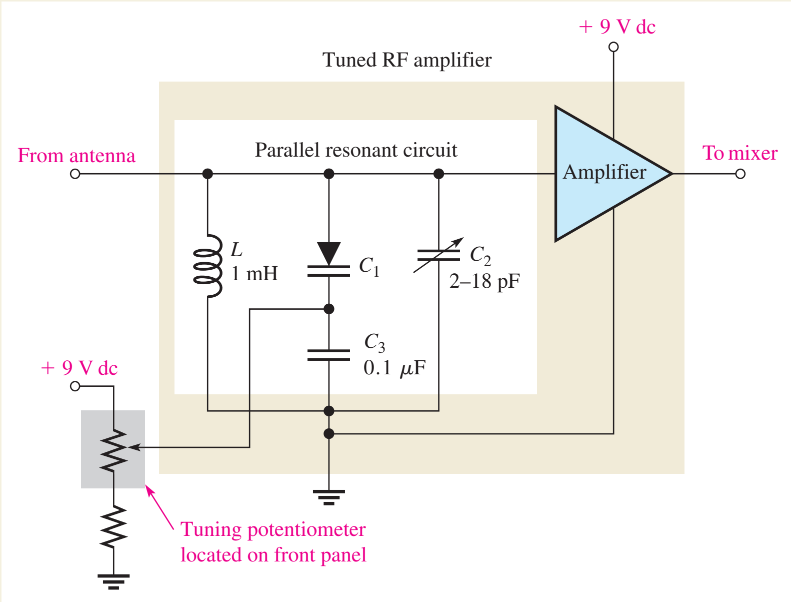


▲ **FIGURE 17–61**

Simplified block diagram of a basic radio receiver.

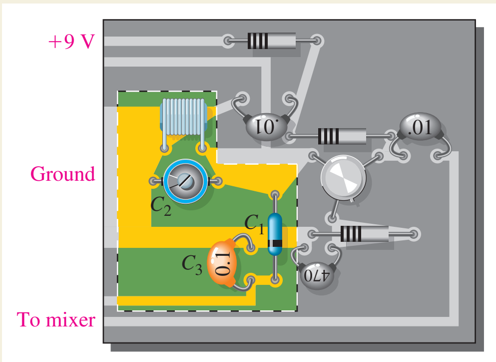
▶ FIGURE 17–62

Partial schematic of the RF amplifier showing the resonant tuning circuit.



▶ FIGURE 17–63

RF amplifier circuit board.



Capacitance in the Resonant Circuit

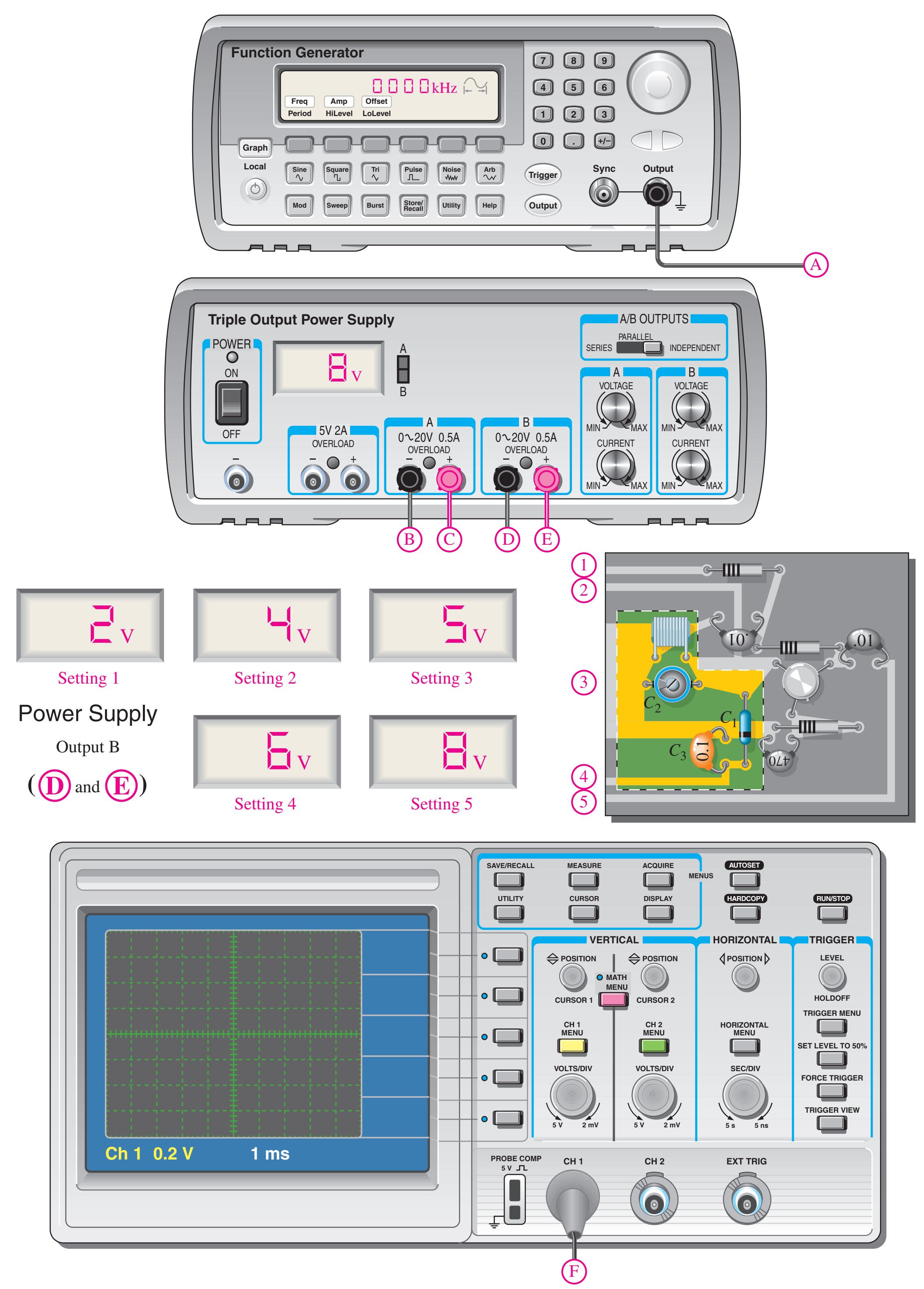
1. Calculate a capacitance setting for C_2 that will ensure a complete coverage of the AM frequency band as the varactor is varied over its capacitance range. C_3 can be ignored. The full range of resonant frequencies for the tuning circuit should more than cover the AM band, so that at the maximum varactor capacitance, the resonant frequency will be less than 535 kHz and at the minimum varactor capacitance, the resonant frequency will be greater than 1605 kHz.
2. Using the value of C_2 that you have calculated, determine the values of the varactor capacitance that will produce a resonant frequency of 535 kHz and 1605 kHz, respectively.

Testing the Resonant Circuit

3. Suggest a procedure for testing the resonant circuit using the instruments in the test bench setup of Figure 17–64. Develop a test setup by creating a point-to-point hookup of the board and the instruments.
4. Using the graph in Figure 17–65 that shows the variation in varactor capacitance versus varactor voltage, determine the resonant frequency for each indicated setting from the *B* outputs of the dc power supply (rightmost output terminals). The *A* output of the power supply is used to provide 9 V to the amplifier. The *B* output of the power supply is used to simulate the potentiometer voltage.

Review

5. What is the AM frequency range?
6. State the purpose of the RF amplifier.
7. How is a particular frequency in the AM band selected?

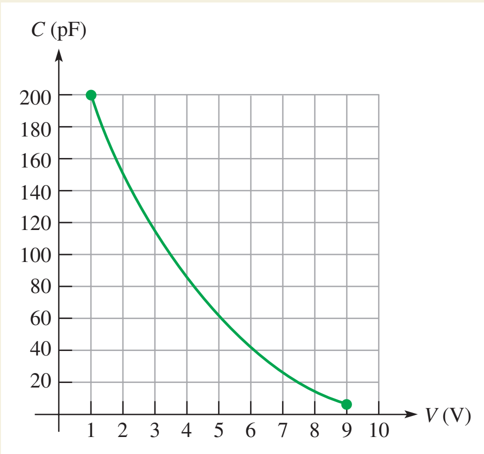


▲ FIGURE 17–64

Test bench setup.

► FIGURE 17–65

Varactor capacitance versus voltage.



SUMMARY

- X_L and X_C have opposing effects in an *RLC* circuit.
- In a series *RLC* circuit, the larger reactance determines the net reactance of the circuit.
- At series resonance, the inductive and capacitive reactances are ideally equal.
- The impedance of a series *RLC* circuit is purely resistive at resonance.
- In a series *RLC* circuit, the current is maximum at resonance.
- The reactive voltages V_L and V_C cancel at resonance in a series *RLC* circuit because they are equal in magnitude and 180° out of phase.
- In a parallel *RLC* circuit, the smaller reactance determines the net reactance of the circuit.
- In a parallel resonant circuit, the impedance is maximum at the resonant frequency.
- A parallel resonant circuit is commonly called a *tank circuit.*
- The impedance of a parallel *RLC* circuit is purely resistive at resonance.
- The bandwidth of a series resonant circuit is the range of frequencies for which the current is $0.707I_{max}$ or greater.
- The bandwidth of a parallel resonant circuit is the range of frequencies for which the impedance is $0.707Z_{max}$ or greater.
- The critical frequencies are the frequencies above and below resonance where the circuit response is 70.7% of the maximum response.
- A higher Q produces a narrower bandwidth.

KEY TERMS

Key terms and other bold terms in the chapter are defined in the end-of-book glossary.

Half-power frequency The frequency at which the output power of a resonant circuit is 50% of the maximum (the output voltage is 70.7% of maximum); another name for *critical* or *cutoff frequency.*

Parallel resonance A condition in a parallel *RLC* circuit in which the reactances ideally are equal and the impedance is maximum.

Resonant frequency The frequency at which resonance occurs; also known as the *center frequency.*

Selectivity A measure of how effectively a resonant circuit passes certain desired frequencies and rejects all others. Generally, the narrower the bandwidth, the greater the selectivity.

Series resonance A condition in a series *RLC* circuit in which the reactances ideally cancel and the impedance is minimum.

Tank circuit A parallel resonant circuit.

FORMULAS

Series *RLC* Circuits

17–1 $X_{tot} = |X_L - X_C|$

17–2 $\mathbf{Z} = R + jX_L - jX_C$

17–3 $\mathbf{Z} = \sqrt{R^2 + (X_L - X_C)^2}\angle \pm\tan^{-1}\left(\frac{X_{tot}}{R}\right)$

Series Resonance

17–4 $f_r = \frac{1}{2\pi\sqrt{LC}}$

Parallel *RLC* Circuits

17–5 $\mathbf{Z} = \frac{1}{\frac{1}{R\angle 0^\circ} + \frac{1}{X_L\angle 90^\circ} + \frac{1}{X_C\angle -90^\circ}}$

17–6 $\mathbf{G} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ$

17–7 $\mathbf{B}_C = \frac{1}{X_C\angle -90^\circ} = B_C\angle 90^\circ = jB_C$

17–8 $\mathbf{B}_L = \frac{1}{X_L\angle 90^\circ} = B_L\angle -90^\circ = -jB_L$

17–9 $\mathbf{Y} = \frac{1}{Z\angle \pm\theta} = Y\angle \mp\theta = G + jB_C - jB_L$

17–10 $\mathbf{I}_{tot} = \sqrt{I_R^2 + (I_C - I_L)^2}\angle\tan^{-1}\left(\frac{I_{CL}}{I_R}\right)$

Parallel Resonance

17–11 $I_{tot} = \frac{V_s}{Z_r}$

17–12 $f_r = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{Q^2}{Q^2 + 1}}$

17–13 $f_r = \frac{\sqrt{1 - (R_W^2 C/L)}}{2\pi\sqrt{LC}}$

17–14 $L_{eq} = L\left(\frac{Q^2 + 1}{Q^2}\right)$

17–15 $R_{p(eq)} = R_W(Q^2 + 1)$

17–16 $Z_r = R_W(Q^2 + 1)$

17–17 $Q_O = \frac{R_{p(tot)}}{X_{L(eq)}}$

17–18 $BW = f_2 - f_1$

17–19 $f_r = \frac{f_1 + f_2}{2}$

17–20 $BW = \frac{f_r}{Q}$

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The total reactance of a series *RLC* circuit is the difference between the capacitive reactance and the inductive reactance.
2. The total impedance of a series *RLC* circuit is the algebraic sum of R, X_C, and X_L.
3. Series resonance occurs when $X_L = X_C$.
4. Below the resonant frequency, a series *RLC* circuit is predominately capacitive.
5. In a parallel *RLC* circuit, the circuit is said to be inductive when $X_L > X_C$.
6. Multiplying the admittance phasors in a parallel *RLC* circuit by the applied source voltage results in the current phasors.
7. At the series resonant frequency, the *RLC* circuit is resistive and the current is minimum.
8. In an ideal parallel *RLC* circuit, the total current is zero at resonance.
9. At parallel resonance, the total impedance of an *RLC* circuit is maximum.
10. Below the parallel resonant frequency, the circuit is predominately capacitive.
11. The bandwidth of a parallel resonant circuit is the difference between the upper and lower critical frequencies.
12. Selectivity of a resonant circuit is better when the bandwidth is narrow.
13. A wave trap on a power line passes communication signals to the substation.
14. An audio cross-over network uses different filters to separate audio frequencies for the speakers.

SELF-TEST

Answers are at the end of the chapter.

1. The total reactance of a series *RLC* circuit at resonance is
   (a) zero **(b)** equal to the resistance
   (c) infinity **(d)** capacitive
2. The phase angle between the source voltage and current of a series *RLC* circuit at resonance is
   (a) $-90°$ **(b)** $+90°$
   (c) $0°$ **(d)** dependent on the reactance
3. The impedance at the resonant frequency of a series *RLC* circuit with $L = 15$ mH, $C = 0.015\ \mu$F, and $R_W = 80\ \Omega$ is
   (a) 15 kΩ **(b)** 80 Ω **(c)** 30 Ω **(d)** 0 Ω
4. In a series *RLC* circuit that is operating below the resonant frequency, the current
   (a) is in phase with the applied voltage **(b)** lags the applied voltage
   (c) leads the applied voltage
5. If the value of *C* in a series *RLC* circuit is increased, the resonant frequency
   (a) is not affected **(b)** increases
   (c) remains the same **(d)** decreases
6. In a certain series resonant circuit, $V_C = 150$ V, $V_L = 150$ V, and $V_R = 50$ V. The value of the source voltage is
   (a) 150 V **(b)** 300 V **(c)** 50 V **(d)** 350 V
7. A certain series resonant circuit has a bandwidth of 1 kHz. If the existing coil is replaced with one having a lower value of Q, the bandwidth will
   (a) increase **(b)** decrease
   (c) remain the same **(d)** be more selective

8. At frequencies below resonance in a parallel RLC circuit, the current
   (a) leads the source voltage **(b)** lags the source voltage
   (c) is in phase with the source voltage
9. The total current into the L and C branches of a parallel circuit at resonance is ideally
   (a) maximum **(b)** low **(c)** high **(d)** zero
10. To tune a parallel resonant circuit to a lower frequency, the capacitance should be
    (a) increased **(b)** decreased
    (c) left alone **(d)** replaced with inductance
11. The resonant frequency of a parallel circuit is approximately the same as a series circuit when
    (a) the Q is very low **(b)** the Q is very high
    (c) there is no resistance **(d)** either answer (b) or (c)
12. If the resistance in parallel with a parallel resonant circuit is reduced, the bandwidth
    (a) disappears **(b)** decreases
    (c) becomes sharper **(d)** increases

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 17–67.

1. If R_1 opens, the total current
   (a) increases **(b)** decreases **(c)** stays the same
2. If C_1 opens, the voltage across C_2
   (a) increases **(b)** decreases **(c)** stays the same
3. If L_2 opens, the voltage across it
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 17–70.

4. If L opens, the voltage across R
   (a) increases **(b)** decreases **(c)** stays the same
5. If f is adjusted to its resonant value, the current through R
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 17–71.

6. If L is increased to 100 mH, the resonant frequency
   (a) increases **(b)** decreases **(c)** stays the same
7. If C is increased to 100 pF, the resonant frequency
   (a) increases **(b)** decreases **(c)** stays the same
8. If L becomes open, the voltage across C
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 17–73.

9. If R_2 becomes open, the voltage across L
   (a) increases **(b)** decreases **(c)** stays the same
10. If C becomes shorted, the voltage across R_1
    (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 17–86.

11. If L_1 opens, the voltage from point a to point b

(a) increases **(b)** decreases **(c)** stays the same

12. If the frequency of the source is increased, the voltage from a to b

(a) increases **(b)** decreases **(c)** stays the same

13. If the frequency of the source voltage is increased, the current through R_1

(a) increases **(b)** decreases **(c)** stays the same

14. If the frequency of the source voltage is decreased, the voltage across C

(a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

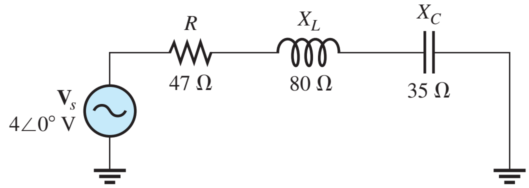
More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the book.

PART 1: SERIES CIRCUITS

SECTION 17–1 Impedance of Series *RLC* Circuits

1. A certain series *RLC* circuit has the following values: $R = 10\ \Omega$, $C = 0.047\ \mu\text{F}$, and $L = 5\text{ mH}$. Determine the impedance in polar form. What is the net reactance? The source frequency is 5 kHz.
2. Find the impedance in Figure 17–66, and express it in polar form.

▶ FIGURE 17–66



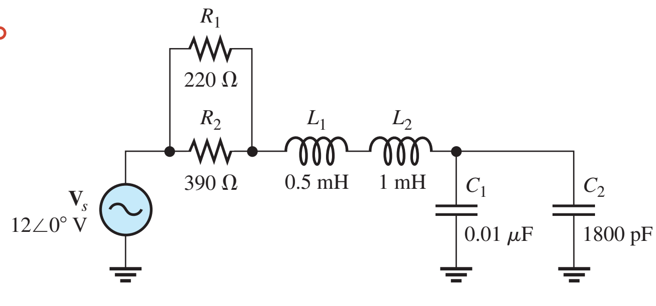
3. If the frequency of the source voltage in Figure 17–66 is doubled from the value that produces the indicated reactances, how does the magnitude of the impedance change?
4. For the circuit of Figure 17–66, determine the net reactance that will make the impedance magnitude equal to 100 Ω.

SECTION 17–2 Analysis of Series *RLC* Circuits

5. For the circuit in Figure 17–66, find $\mathbf{I}_{tot}$, $\mathbf{V}_R$, $\mathbf{V}_L$, and $\mathbf{V}_C$ in polar form.
6. Draw the voltage phasor diagram for the circuit in Figure 17–66.
7. Analyze the circuit in Figure 17–67 for the following ($f = 25$ kHz):

   (a) $\mathbf{I}_{tot}$ **(b)** P_{true} **(c)** P_r **(d)** P_a

▶ FIGURE 17–67



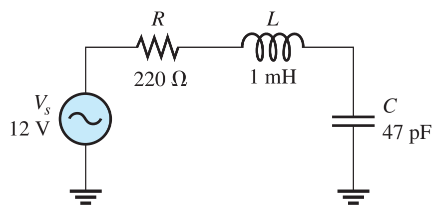
SECTION 17–3 Series Resonance

8. For the circuit in Figure 17–66, is the resonant frequency higher or lower than the setting indicated by the reactance values?

9. For the circuit in Figure 17–68, what is the voltage across R at resonance?

10. Find X_L, X_C, Z, and I at the resonant frequency in Figure 17–68.

► FIGURE 17–68



11. A certain series resonant circuit has a maximum current of 50 mA and a V_L of 100 V. The applied voltage is 10 V. What is Z? What are X_L and X_C?

12. For the *RLC* circuit in Figure 17–69, determine the resonant frequency.

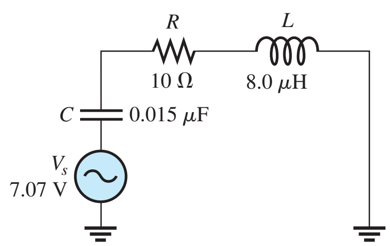
13. What is the value of the current at the half-power points in Figure 17–69?

14. Determine the phase angle between the applied voltage and the current at the critical frequencies in Figure 17–69. What is the phase angle at resonance?

***15.** Design a circuit in which the following series resonant frequencies are switch-selectable:

(a) 500 kHz **(b)** 1000 kHz **(c)** 1500 kHz **(d)** 2000 kHz

► FIGURE 17–69



PART 2: PARALLEL CIRCUITS

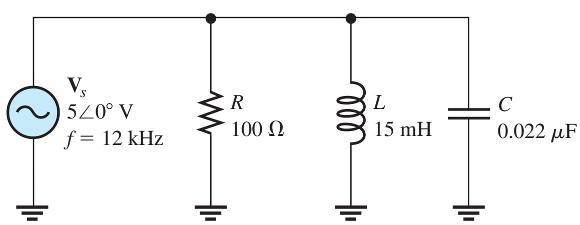
SECTION 17–4 Impedance of Parallel *RLC* Circuits

16. Express the impedance of the circuit in Figure 17–70 in polar form.

17. Is the circuit in Figure 17–70 capacitive or inductive? Explain.

18. At what frequency does the circuit in Figure 17–70 change its reactive characteristic (from inductive to capacitive or vice versa)?

► FIGURE 17–70



SECTION 17–5 Analysis of Parallel *RLC* Circuits

19. For the circuit in Figure 17–70, find all the currents and voltages in polar form.

20. Find the total impedance of the circuit in Figure 17–70 at 50 kHz.

21. C hange the frequency to 100 kHz in Figure 17–70 and repeat Problem 19.

SECTION 17–6 Parallel Resonance

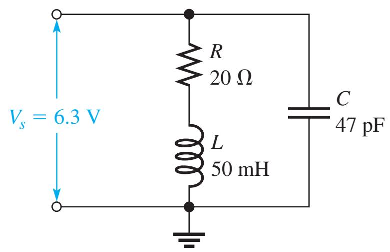
22. What is the impedance of an ideal parallel resonant circuit (no resistance in either branch)?

23. Find Z at resonance and f_r for the tank circuit in Figure 17–71.

24. How much current is drawn from the source in Figure 17–71 at resonance? What are the inductive current and the capacitive current at the resonant frequency?

25. Find P_{true}, P_r, and P_a in the circuit of Figure 17–71 at resonance.

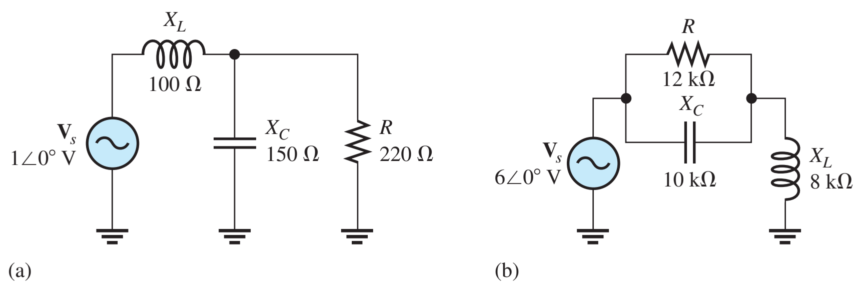
▶ FIGURE 17–71



PART 3: SERIES-PARALLEL CIRCUITS

SECTION 17–7 Analysis of Series-Parallel *RLC* Circuits

26. Find the total impedance for each circuit in Figure 17–72.



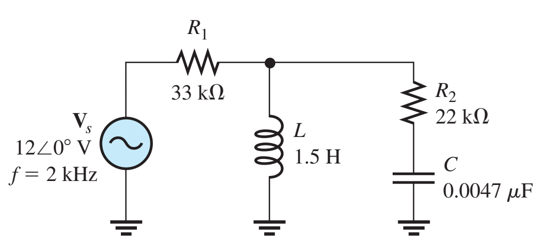
▲ FIGURE 17–72

27. For each circuit in Figure 17–72, determine the phase angle between the source voltage and the total current.

28. Determine the voltage across each element in Figure 17–73, and express each in polar form.

29. Convert the circuit in Figure 17–73 to an equivalent series form.

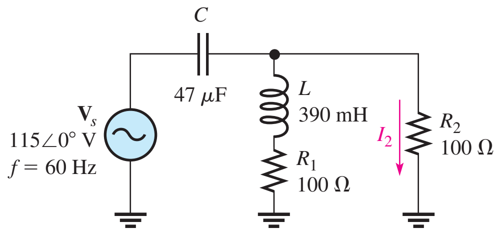
▶ FIGURE 17–73



30. What is the current through R_2 in Figure 17–74?

31. In Figure 17–74, what is the phase angle between I_2 and the source voltage?

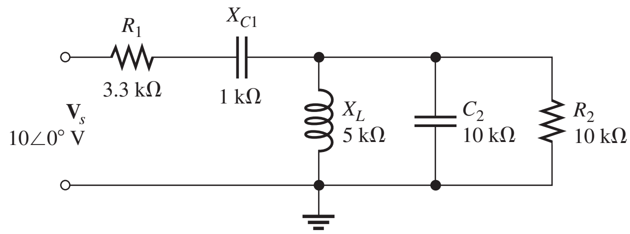
▶ FIGURE 17–74



*32. Determine the total resistance and the total reactance in Figure 17–75.

*33. Find the current through each component in Figure 17–75. Find the voltage across each component.

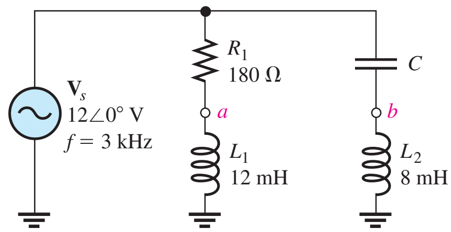
▶ FIGURE 17–75



34. Determine if there is a value of C that will make $V_{ab} = 0$ V in Figure 17–76. If not, explain.

*35. If the value of C is 0.22 μF, what is the current through a 100 Ω resistor connected from a to b in Figure 17–76?

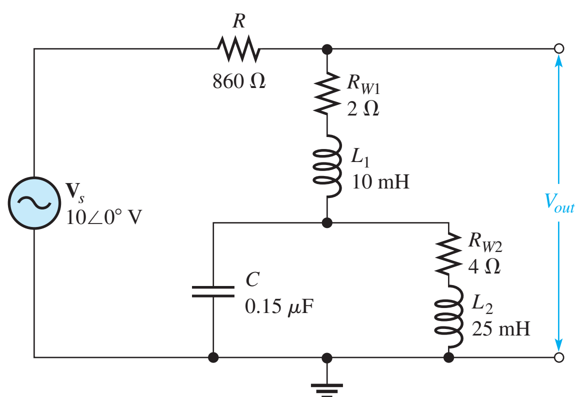
▶ FIGURE 17–76



*36. How many resonant frequencies are there in the circuit of Figure 17–77? Why?

*37. Determine the resonant frequencies and the output voltage at each frequency in Figure 17–77.

▶ FIGURE 17–77



*38. Design a parallel-resonant network using a single coil and switch-selectable capacitors to produce the following resonant frequencies: 8 MHz, 9 MHz, 10 MHz, and 11 MHz. Assume a 10 μH coil with a winding resistance of 5 Ω.

PART 4: SPECIAL TOPICS

SECTION 17–8 Bandwidth of Resonant Circuits

39. At resonance, $X_L = 2\text{ k}\Omega$ and $R_W = 25\ \Omega$ in a parallel *RLC* circuit. The resonant frequency is 5 kHz. Determine the bandwidth.

40. If the lower critical frequency is 2,400 Hz and the upper critical frequency is 2,800 Hz, what is the bandwidth? What is the resonant frequency?

41. In a certain *RLC* circuit, the power at resonance is 2.75 W. What is the power at the lower critical frequency?

***42.** What values of L and C should be used in a tank circuit to obtain a resonant frequency of 8 kHz? The bandwidth must be 800 Hz. The winding resistance of the coil is 10 Ω.

43. A parallel resonant circuit has a Q of 50 and a BW of 400 Hz. If Q is doubled, what is the bandwidth for the same f_r?

Multisim Troubleshooting and Analysis

MultiSim

These problems require Multisim.

44. Open file P17-44 and determine if there is a fault. If so, find the fault.

45. Open file P17-45 and determine if there is a fault. If so, find the fault.

46. Open file P17-46 and determine if there is a fault. If so, find the fault.

47. Open file P17-47 and determine if there is a fault. If so, find the fault.

48. Open file P17-48 and determine if there is a fault. If so, find the fault.

49. Open file P17-49 and determine if there is a fault. If so, find the fault.

50. Open file P17-50 and determine the resonant frequency of the circuit.

51. Open file P17-51 and determine the resonant frequency of the circuit.

ANSWERS

SECTION CHECKUPS

SECTION 17–1 Impedance of Series *RLC* Circuits

1. $X_{tot} = 70\ \Omega$; capacitive

2. $\mathbf{Z} = 84.3\angle -56.1°\ \Omega$; $Z = 84.3\ \Omega$; $\theta = -56.1°$; current is leading V_s.

SECTION 17–2 Analysis of Series *RLC* Circuits

1. $\mathbf{V}_s = 38.4\angle -21.3°\text{ V}$

2. Current leads the voltage.

3. $X_{tot} = 600\ \Omega$

SECTION 17–3 Series Resonance

1. For series resonance, $X_L = X_C$.

2. The current is maximum because the impedance is minimum.

3. $f_r = 159\text{ kHz}$

4. The circuit is capacitive.

SECTION 17–4 Impedance of Parallel *RLC* Circuits

1. The circuit is capacitive.

2. $\mathbf{Y} = 1.54\angle 49.4°\text{ mS}$

3. $\mathbf{Z} = 651\angle -49.4°\ \Omega$

SECTION 17–5 Analysis of Parallel *RLC* Circuits

1. $I_R = 80$ mA, $I_C = 120$ mA, $I_L = 240$ mA
2. The circuit is capacitive.

SECTION 17–6 Parallel Resonance

1. Impedance is maximum at parallel resonance.
2. The current is minimum.
3. $X_C = 1{,}500\ \Omega$
4. $f_r = 225$ kHz
5. $f_r = 22.5$ kHz
6. $f_r = 20.9$ kHz

SECTION 17–7 Analysis of Series-Parallel *RLC* Circuits

1. $R_{p(\text{eq})} = 130\ \Omega$, $L_{\text{eq}} = 102\ \mu\text{H}$, $C = 0.22\ \mu\text{F}$
2. $L_{(\text{eq})} = 20.1$ mH, $R_{p(\text{eq})} = 1.59\ \text{k}\Omega$

SECTION 17–8 Bandwidth of Resonant Circuits

1. $BW = f_2 - f_1 = 400$ kHz
2. $f_r = 2$ MHz
3. $P_{f2} = 0.9$ W
4. Larger Q means narrower BW.

SECTION 17–9 Applications

1. The wave trap passes the electrical frequency while blocking the high-frequency communication signal.
2. A tuned filter is used to select a narrow band of frequencies of interest.
3. A crossover network is a filter network that separates the audio signal into different frequency bands for the speakers while maintaining an overall flat response.
4. Ganged tuning is done with several capacitors (or inductors) whose values can be varied simultaneously with a common control.

RELATED PROBLEMS FOR EXAMPLES

17–1 $\mathbf{Z} = 12.7\angle 82.3^\circ\ \text{k}\Omega$

17–2 $\mathbf{Z} = 3.50\angle 19.5^\circ\ \text{k}\Omega$

17–3 Current will increase with frequency to a certain point and then it will decrease.

17–4 The circuit is more capacitive.

17–5 $f_r = 22.5$ kHz

17–6 45°

17–7 Z increases; Z increases.

17–8 Z decreases.

17–9 Inductive

17–10 I_{tot} increases.

17–11 Greater

17–12 $\mathbf{V}_C = 0.931\angle -65.8^\circ$ V

17–13 $\mathbf{V}_{C1} = 27.1\angle -81.1^\circ$ V

17–14 $R_{p(\text{eq})} = 25.1\ \text{k}\Omega$, $L_{\text{eq}} = 5$ mH; $C = 0.022\ \mu\text{F}$

17–15 $Z_r = 80.0\ \text{k}\Omega$

17–16 $I = 35.4$ mA

17–17 $f_1 = 6.75$ kHz; $f_2 = 9.25$ kHz

17–18 $BW = 7.96$ kHz

TRUE/FALSE QUIZ

1. T **2.** F **3.** T **4.** T **5.** F **6.** T **7.** F
8. T **9.** T **10.** F **11.** T **12.** T **13.** F **14.** T

SELF-TEST

1. (a) **2.** (c) **3.** (b) **4.** (c) **5.** (d) **6.** (c) **7.** (a) **8.** (b)
9. (d) **10.** (a) **11.** (b) **12.** (d)

CIRCUIT DYNAMICS QUIZ

1. (b) **2.** (a) **3.** (a) **4.** (c) **5.** (c) **6.** (b) **7.** (b) **8.** (c)
9. (a) **10.** (a) **11.** (d) **12.** (a) **13.** (b) **14.** (a)