RL Circuits

16

Chapter Outline

Part 1: Series Circuits

16–1 Sinusoidal Response of Series *RL* Circuits
16–2 Impedance of Series *RL* Circuits
16–3 Analysis of Series *RL* Circuits

Part 2: Parallel Circuits

16–4 Impedance and Admittance of Parallel *RL* Circuits
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Application Activity

Chapter Objectives

Part 1: Series Circuits

- Describe the relationship between current and voltage in a series *RL* circuit
- Determine the impedance of a series *RL* circuit
- Analyze a series *RL* circuit

Part 2: Parallel Circuits

- Determine impedance and admittance in a parallel *RL* circuit
- Analyze a parallel *RL* circuit

Part 3: Series-Parallel Circuits

- Analyze series-parallel *RL* circuits

Part 4: Special Topics

- Determine power in *RL* circuits
- Describe two examples of *RL* circuit applications
- Troubleshoot *RL* circuits

Key Terms

- *RL* lead circuit
- *RL* lag circuit
- Inductive susceptance (B_L)

Application Activity Preview

In the application activity, you will use your knowledge of *RL* circuits to determine, based on parameter measurements, the type of filter circuits that are encapsulated in sealed modules and their component values.

Visit the Companion Website

Study aids for this chapter are available at http://www.pearsonhighered.com/careersresources/

Introduction

In this chapter you will study series and parallel *RL* circuits. The analyses of *RL* and *RC* circuits are similar. The major difference is that the phase responses are opposite; inductive reactance increases with frequency, while capacitive reactance decreases with frequency.

An *RL* circuit contains both resistance and inductance. In this chapter, basic series and parallel *RL* circuits and their responses to sinusoidal ac voltages are presented. Series-parallel combinations are also analyzed. True, reactive, and apparent power in *RL* circuits are discussed and some basic *RL* circuit applications are introduced. Applications of *RL* circuits include filters and switching regulators. Troubleshooting is also covered in this chapter.

Coverage Options

If you chose Option 1 to cover all of Chapter 15 on *RC* circuits, then all of this chapter should be covered next.

If you chose Option 2 to cover reactive circuits beginning in Chapter 15 on the basis of the four major parts, then the appropriate part of this chapter should be covered next, followed by the corresponding part in Chapter 17.

SERIES CIRCUITS

Part 1

16–1 SINUSOIDAL RESPONSE OF SERIES *RL* CIRCUITS

As with the *RC* circuit, all currents and voltages in a series *RL* circuit are sinusoidal when the input voltage is sinusoidal. The inductance causes a phase shift between the voltage and the current that depends on the relative values of the resistance and the inductive reactance.

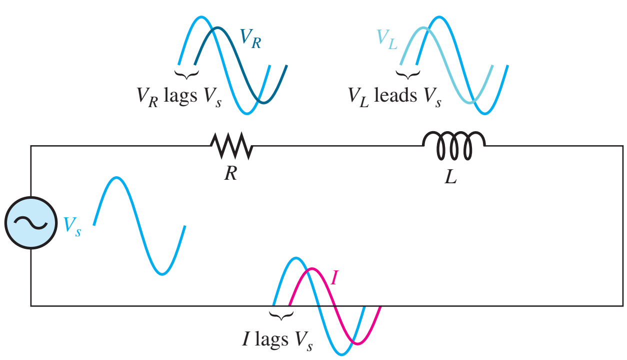
After completing this section, you should be able to

- **Describe the relationship between current and voltage in a series *RL* circuit**
  - Discuss voltage and current waveforms
  - Discuss phase shift

In an *RL* circuit, the resistor voltage and the current lag the source voltage. The inductor voltage leads the source voltage. Ideally, the phase angle between the current and the inductor voltage is always 90°. These generalized phase relationships are indicated in Figure 16–1. Notice how they differ from those of the *RC* circuit that was discussed in Chapter 15.

The amplitudes and the phase relationships of the voltages and current depend on the values of the resistance and the **inductive reactance**. When a circuit is purely inductive, the phase angle between the applied voltage and the total current is 90°, with the current lagging the voltage. When there is a combination of both resistance and inductive reactance in a circuit, the phase angle is somewhere between 0° and 90°, depending on the relative values of the resistance and the inductive reactance.

Recall that practical inductors have winding resistance, capacitance between windings, and other factors that prevent an inductor from behaving as an ideal component.



▲ **FIGURE 16–1**

Illustration of sinusoidal response with general phase relationships of V_R, V_L, and I relative to the source voltage. V_R and I are in phase, while V_R and V_L are 90° out of phase with each other.

In practical circuits, these effects can be significant; however, for the purpose of isolating the inductive effects, we will treat inductors in this chapter as ideal (except in the Application Activity).

SECTION 16–1 CHECKUP
Answers are at the end of the chapter.

1. A 1 kHz sinusoidal voltage is applied to an *RL* circuit. What is the frequency of the resulting current?
2. When the resistance in an *RL* circuit is greater than the inductive reactance, is the phase angle between the applied voltage and the total current closer to 0° or to 90°?

16–2 IMPEDANCE OF SERIES *RL* CIRCUITS

The impedance of a series *RL* circuit consists of resistance and inductive reactance and is the total opposition to sinusoidal current. Its unit is the ohm. The impedance also results in a phase difference between the total current and the source voltage. Therefore, the impedance consists of a magnitude component and a phase angle component and is represented as a phasor quantity.

After completing this section, you should be able to

- **Determine the impedance of a series *RL* circuit**
  - Express inductive reactance in complex form
  - Express total impedance in complex form
  - Calculate impedance magnitude and the phase angle

The impedance of a series *RL* circuit is determined by the resistance and the inductive reactance. Inductive reactance is expressed as a phasor quantity in rectangular form as

$$\mathbf{X}_L = jX_L$$

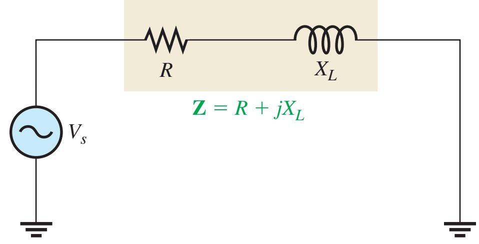
In the series *RL* circuit of Figure 16–2, the total impedance is the phasor sum of R and jX_L and is expressed as

Equation 16–1

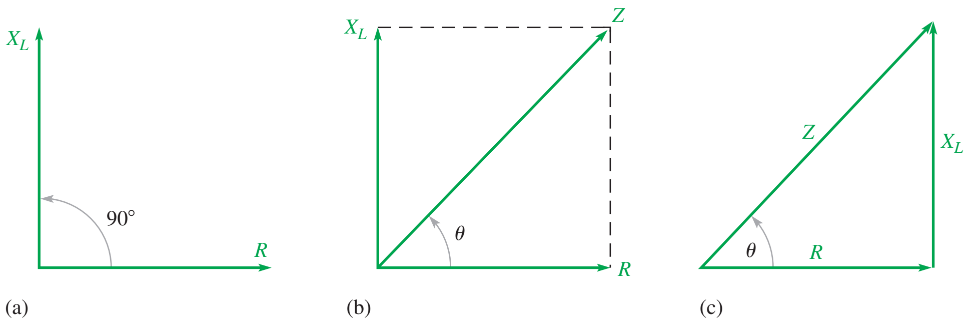
$$\mathbf{Z} = R + jX_L$$

▶ FIGURE 16–2

Impedance in a series *RL* circuit.



In ac analysis, both R and X_L are as shown in the phasor diagram of Figure 16–3(a), with X_L appearing at a +90° angle with respect to R. This relationship comes from the fact that the inductor voltage leads the current, and thus the resistor voltage, by 90°. Since $\mathbf{Z}$ is the phasor sum of R and jX_L, its phasor representation is as shown in Figure 16–3(b). A repositioning of the phasors, as shown in part (c), forms a right triangle called the *impedance triangle.* The length of each phasor represents the magnitude of the quantity, and θ is the phase angle between the applied voltage and the current in the *RL* circuit.



▲ FIGURE 16–3

Development of the impedance triangle for a series *RL* circuit.

The impedance magnitude of the series *RL* circuit can be expressed in terms of the resistance and reactance as

$$Z = \sqrt{R^2 + X_L^2}$$

The magnitude of the impedance is expressed in ohms.

The phase angle, θ, is expressed as

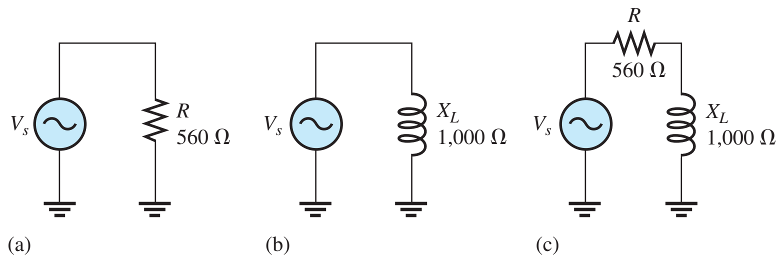
$$\theta = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Combining the magnitude and the angle, the impedance can be expressed in polar form as

$$\mathbf{Z} = \sqrt{R^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)$$ **Equation 16–2**

EXAMPLE 16–1

For each circuit in Figure 16–4, write the phasor expression for the impedance in both rectangular and polar forms.



▲ FIGURE 16–4

Solution For the circuit in Figure 16–4(a), the impedance is

$$\mathbf{Z} = R + j0 = R = \mathbf{560\ \Omega} \quad \text{in rectangular form } (X_L = 0)$$
$$\mathbf{Z} = R\angle 0° = \mathbf{560\angle 0°\ \Omega} \quad \text{in polar form}$$

The impedance is simply equal to the resistance, and the phase angle is zero because pure resistance does not introduce a phase shift.

For the circuit in Figure 16–4(b), the impedance is

$$\mathbf{Z} = 0 + jX_L = \mathbf{j1{,}000\ \Omega} \quad \text{in rectangular form } (R = 0)$$
$$\mathbf{Z} = X_L\angle 90^\circ = \mathbf{1{,}000\angle 90^\circ\ \Omega} \quad \text{in polar form}$$

The impedance equals the inductive reactance in this case, and the phase angle is $+90^\circ$ because the inductance causes the current to lag the voltage by 90°.

For the circuit in Figure 16–4(c), the impedance in rectangular form is

$$\mathbf{Z} = R + jX_L = \mathbf{560\ \Omega + j1{,}000\ \Omega}$$

The impedance in polar form is

$$\mathbf{Z} = \sqrt{R^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)$$
$$= \sqrt{(560\ \Omega)^2 + (1{,}000\ \Omega)^2}\angle\tan^{-1}\left(\frac{1{,}000\ \Omega}{560\ \Omega}\right) = \mathbf{1{,}150\angle 60.8^\circ\ \Omega}$$

In this case, the impedance is the phasor sum of the resistance and the inductive reactance. The phase angle is fixed by the relative values of X_L and R.

*Related Problem** In a series *RL* circuit, $R = 1.8\ \text{k}\Omega$ and $X_L = 950\ \Omega$. Express the impedance in both rectangular and polar forms.

*Answers are at the end of the chapter.

SECTION 16–2 CHECKUP

1. The impedance of a certain series *RL* circuit is $150\ \Omega + j220\ \Omega$. What is the value of the resistance? The inductive reactance?
2. What is the phase angle for circuit in Question 1?
3. A series *RL* circuit has a total resistance of $33\ \text{k}\Omega$ and an inductive reactance of $50\ \text{k}\Omega$. Write the phasor expression for the impedance in rectangular form. Convert the impedance to polar form.

16–3 ANALYSIS OF SERIES *RL* CIRCUITS

In this section, Ohm's law and Kirchhoff's voltage law are used in the analysis of series *RL* circuits to determine voltage, current, and impedance. Also, *RL* lead and lag circuits are examined.

After completing this section, you should be able to

- **Analyze a series *RL* circuit**
  - Apply Ohm's law and Kirchhoff's voltage law to series *RL* circuits
  - Express the voltages and current as phasor quantities
  - Show how impedance and phase angle vary with frequency
  - Discuss and analyze the *RL* lead circuit
  - Discuss and analyze the *RL* lag circuit

Ohm's Law

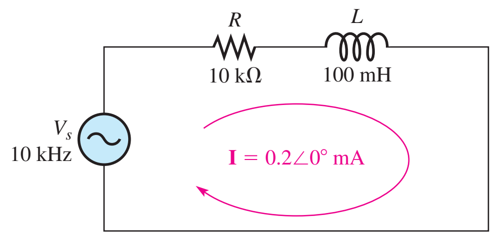
The application of Ohm's law to series *RL* circuits involves the use of the phasor quantities of **Z**, **V**, and **I**. The three equivalent forms of Ohm's law were stated in Chapter 15 for *RC* circuits. They apply also to *RL* circuits and are restated here:

$$\mathbf{V} = \mathbf{IZ} \qquad \mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} \qquad \mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

Recall that since Ohm's law calculations involve multiplication and division operations, you should express the voltage, current, and impedance in polar form.

EXAMPLE 16–2

The current in Figure 16–5 is expressed in polar form as $\mathbf{I} = 0.2\angle 0^\circ$ mA. Determine the source voltage expressed in polar form, and draw a phasor diagram showing the relationship between the source voltage and the current.



▲ FIGURE 16–5

Solution The magnitude of the inductive reactance is

$$X_L = 2\pi fL = 2\pi(10\text{ kHz})(100\text{ mH}) = 6.28\text{ k}\Omega$$

The impedance in rectangular form is

$$\mathbf{Z} = R + jX_L = 10\text{ k}\Omega + j6.28\text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z} = \sqrt{R^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)$$

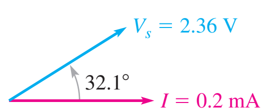
$$= \sqrt{(10\text{ k}\Omega)^2 + (6.28\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{6.28\text{ k}\Omega}{10\text{ k}\Omega}\right) = \mathbf{11.8\angle 32.1^\circ\text{ k}\Omega}$$

Use Ohm's law to determine the source voltage.

$$\mathbf{V}_s = \mathbf{IZ} = (0.2\angle 0^\circ\text{ mA})(11.8\angle 32.1^\circ\text{ k}\Omega) = \mathbf{2.36\angle 32.1^\circ\text{ V}}$$

The magnitude of the source voltage is 2.36 V at an angle of 32.1° with respect to the current; that is, the voltage leads the current by 32.1°, as shown in the phasor diagram of Figure 16–6.

► FIGURE 16–6



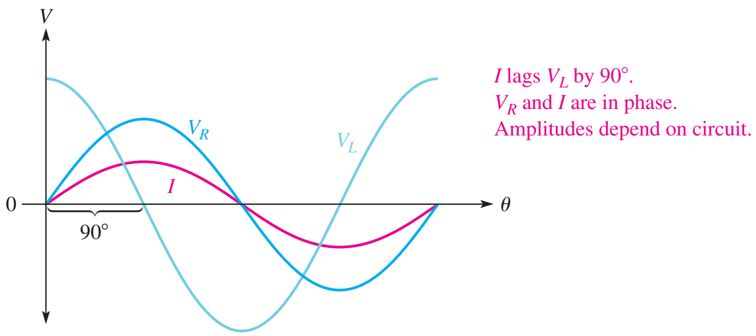
Related Problem If the source voltage in Figure 16–5 were $5\angle 0^\circ$ V, what would be the current expressed in polar form?

MultiSim

Use Multisim files E16-02A and E16-02B to verify the calculated results in this example and to confirm your calculation for the related problem.

Phase Relationships of Current and Voltages

In a series *RL* circuit, the current is the same through both the resistor and the inductor. Thus, the resistor voltage is in phase with the current, and the inductor voltage leads the current by 90°. Therefore, there is a phase difference of 90° between the resistor voltage, V_R, and the inductor voltage, V_L, as shown in the waveform diagram of Figure 16–7.



▶ **FIGURE 16–7**

Phase relation of voltages and current in a series *RL* circuit.

From Kirchhoff's voltage law, the sum of the voltage drops must equal the applied voltage. However, since V_R and V_L are not in phase with each other, they must be added as phasor quantities with V_L leading V_R by 90°, as shown in Figure 16–8(a). As shown in part (b), $\mathbf{V}_s$ is the phasor sum of V_R and V_L.

Equation 16–3

$$\mathbf{V}_s = V_R + jV_L$$

This equation can be expressed in polar form as

Equation 16–4

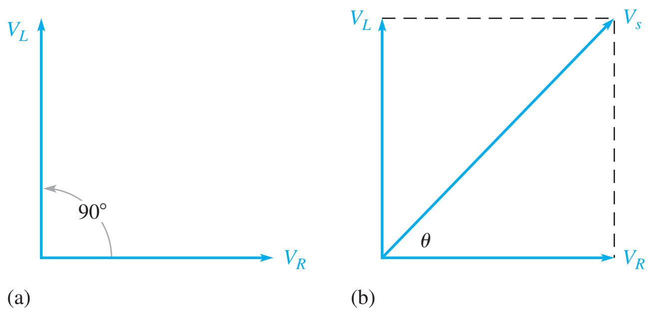
$$\mathbf{V}_s = \sqrt{V_R^2 + V_L^2}\angle\tan^{-1}\left(\frac{V_L}{V_R}\right)$$

where the magnitude of the source voltage is

$$V_s = \sqrt{V_R^2 + V_L^2}$$

and the phase angle between the resistor voltage and the source voltage is

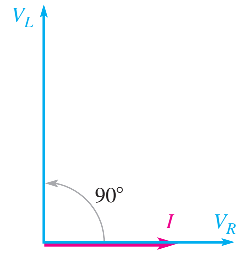
$$\theta = \tan^{-1}\left(\frac{V_L}{V_R}\right)$$



▶ **FIGURE 16–8**

Voltage phasor diagram for a series *RL* circuit.

Since the resistor voltage and the current are in phase, θ is also the phase angle between the source voltage and the current. Figure 16–9 shows a voltage and current phasor diagram that represents the waveform diagram of Figure 16–7.



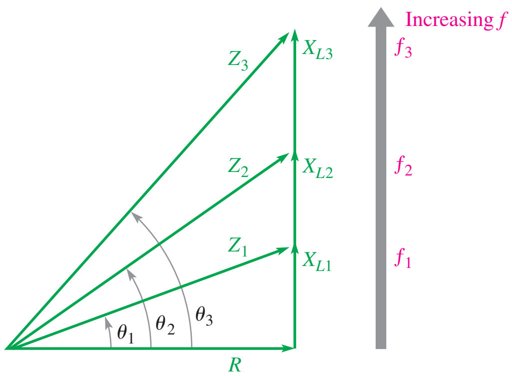
▲ FIGURE 16–9

Voltage and current phasor diagram for the waveforms in Figure 16–7.

Variation of Impedance and Phase Angle with Frequency

The impedance triangle is useful in visualizing how the frequency of the applied voltage affects the *RL* circuit response. As you know, inductive reactance varies directly with frequency. When X_L increases, the magnitude of the total impedance also increases; and when X_L decreases, the magnitude of the total impedance decreases. Thus, Z is directly dependent on frequency. The phase angle θ also varies directly with frequency because $\theta = \tan^{-1}(X_L/R)$. As X_L increases with frequency, so does θ, and vice versa.

The impedance triangle is used in Figure 16–10 to illustrate the variations in X_L, Z, and θ as the frequency changes. Of course, R remains constant. The main point is that because X_L varies directly with the frequency, so also do the magnitude of the total impedance and the phase angle. Example 16–3 illustrates this.



◄ FIGURE 16–10

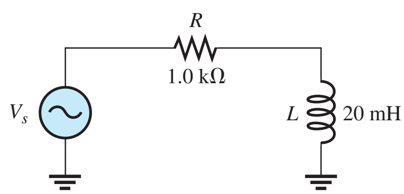
As the frequency increases, X_L increases, Z increases, and θ increases. Each value of frequency can be visualized as forming a different impedance triangle.

EXAMPLE 16–3

For the series *RL* circuit in Figure 16–11, determine the magnitude of the total impedance and the phase angle for each of the following frequencies:

(a) 10 kHz **(b)** 20 kHz **(c)** 30 kHz

► FIGURE 16–11



Solution **(a)** For $f = 10$ kHz,

$$X_L = 2\pi fL = 2\pi(10\text{ kHz})(20\text{ mH}) = 1.26\text{ k}\Omega$$

$$\mathbf{Z} = \sqrt{R^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)$$

$$= \sqrt{(1.0\text{ k}\Omega)^2 + (1.26\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{1.26\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = 1.61\angle 51.5^\circ\text{ k}\Omega$$

Thus, $Z = \mathbf{1.61\text{ k}\Omega}$ and $\theta = \mathbf{51.5^\circ}$.

(b) For $f = 20\text{ kHz}$,

$$X_L = 2\pi(20\text{ kHz})(20\text{ mH}) = 2.51\text{ k}\Omega$$

$$\mathbf{Z} = \sqrt{(1.0\text{ k}\Omega)^2 + (2.51\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{2.51\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = 2.70\angle 68.3^\circ\text{ k}\Omega$$

Thus, $Z = \mathbf{2.70\ k\Omega}$ and $\theta = \mathbf{68.3^\circ}$.

(c) For $f = 30\text{ kHz}$,

$$X_L = 2\pi(30\text{ kHz})(20\text{ mH}) = 3.77\text{ k}\Omega$$

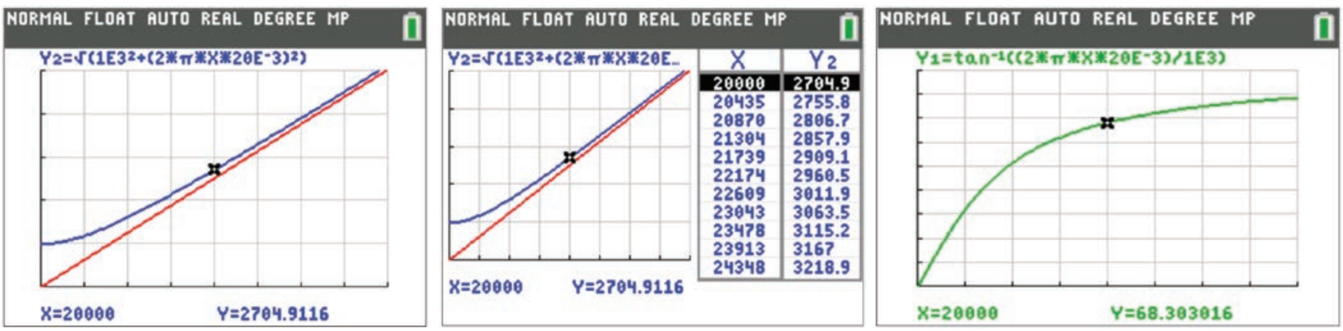
$$\mathbf{Z} = \sqrt{(1.0\text{ k}\Omega)^2 + (3.77\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{3.77\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = \mathbf{3.90\angle 75.1^\circ\ k\Omega}$$

Thus, $Z = \mathbf{3.90\ k\Omega}$ and $\theta = \mathbf{75.1^\circ}$.

Notice that as the frequency increases, X_L, Z, and θ also increase.

Related Problem Determine $\mathbf{Z}$ and θ in Figure 16–11 if f is 100 kHz.

A graphing calculator can be used to show how X_L, Z, and θ are dependent on frequency. The values from Example 16–3 are used to illustrate this idea. For X_L and Z, enter each equation separately and plot them both as shown in Figure 16–12(a). The red line represents the magnitude of X_L, and the blue line represents the magnitude of Z. Using the trace function, you can read values for frequency and reactance or impedance. You can view a table of values the calculator uses to draw the graph by selecting GRAPH-TABLE in the mode menu. (Set up table parameters using 2nd window.) Figure 16–12(b) shows the result. Notice that at low frequencies, R dominates the magnitude of the impedance; and at high frequencies, X_L dominates the impedance. (Grid lines in [a] and [b] are at increments of $f = 5{,}000\text{ Hz}$ and $Z = 1{,}000\ \Omega$.) You can also use the graphing calculator to view how phase angle is affected by frequency in this example as shown in Figure 16–12(c). Grid lines in (c) are at increments of $f = 5{,}000\text{ Hz}$ and $\theta = 10^\circ$.



(a) Impedance (red) and reactance (blue) curves (b) Impedance (red) and reactance (blue) curves with Table data (c) Phase angle as a function of frequency

▲ **FIGURE 16–12**

Graphs for Example 16–3.

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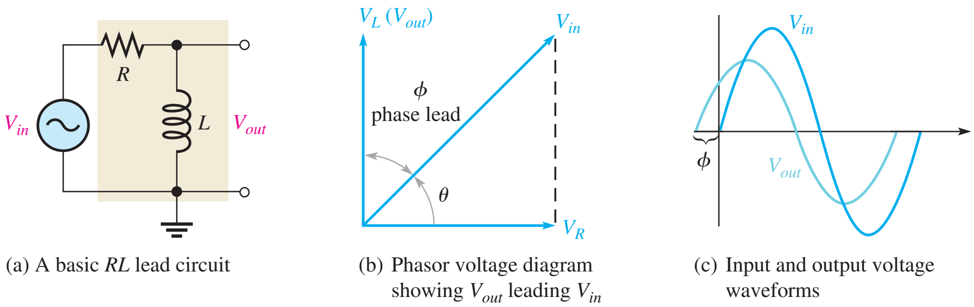
The *RL* Lead Circuit

An ***RL* lead circuit** is a phase shift circuit in which the output voltage leads the input voltage by a specified amount. Figure 16–13(a) shows a series *RL* circuit with the output voltage taken across the inductor. Note that in the *RC* lead circuit, the output



▲ **FIGURE 16–13**

The *RL* lead circuit ($V_{out} = V_L$).

was taken across the resistor. The source voltage is the input, V_{in}. As you know, θ is the angle between the current and the input voltage; it is also the angle between the resistor voltage and the input voltage because V_R and I are in phase.

Since V_L leads V_R by 90°, the phase angle between the inductor voltage and the input voltage is the difference between 90° and θ, as shown in Figure 16–13(b). The inductor voltage is the output; it leads the input, thus creating a basic lead circuit.

The input and output voltage waveforms of the lead circuit are shown in Figure 16–13(c). The amount of phase difference, designated ϕ, between the input and the output is dependent on the relative values of the inductive reactance and the resistance, as is the magnitude of the output voltage.

Phase Difference Between Input and Output The angle between V_{out} and V_{in} is designated ϕ (phi) and is developed as follows. The polar expressions for the input voltage and the current are $V_{in}\angle 0°$ and $I\angle -\theta$, respectively. The output voltage in polar form is

$$\mathbf{V}_{out} = (I\angle -\theta)(X_L\angle 90°) = IX_L\angle(90° - \theta)$$

This expression shows that the output voltage is at an angle of $90° - \theta$ with respect to the input voltage. Since $\theta = \tan^{-1}(X_L/R)$, the angle ϕ between the input and output is

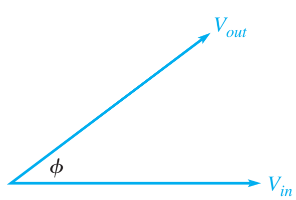
$$\phi = 90° - \tan^{-1}\left(\frac{X_L}{R}\right)$$

Equivalently, this angle can be expressed as

$$\phi = \tan^{-1}\left(\frac{R}{X_L}\right)$$

Equation 16–5

The angle ϕ between the output and input is always positive, indicating that the output voltage leads the input voltage, as indicated in Figure 16–14.

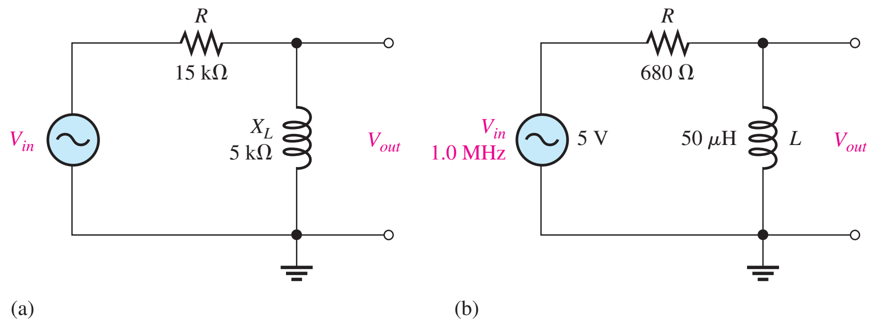


◀ **FIGURE 16–14**

EXAMPLE 16–4

Determine the amount of phase lead from input to output in each lead circuit in Figure 16–15.

▶ FIGURE 16–15



Solution For the lead circuit in Figure 16–15(a),

$$\phi = \tan^{-1}\left(\frac{R}{X_L}\right) = \tan^{-1}\left(\frac{15\text{ k}\Omega}{5\text{ k}\Omega}\right) = \mathbf{71.6^\circ}$$

The output leads the input by 71.6°.

For the lead circuit in Figure 16–15(b), first determine the inductive reactance.

$$X_L = 2\pi fL = 2\pi(1.0\text{ MHz})(50\ \mu\text{H}) = 314\ \Omega$$

$$\phi = \tan^{-1}\left(\frac{R}{X_L}\right) = \tan^{-1}\left(\frac{680\ \Omega}{314\ \Omega}\right) = \mathbf{65.2^\circ}$$

The output leads the input by 65.2°.

Related Problem In a certain lead circuit, $R = 2.2\text{ k}\Omega$ and $X_L = 1\text{ k}\Omega$. What is the phase lead?

MultiSim

Use Multisim files E16-04A, E16-04B, and E16-04C to verify the calculated results in this example and to confirm your calculation for the related problem.

Magnitude of the Output Voltage To evaluate the output voltage in terms of its magnitude, visualize the *RL* lead circuit as a voltage divider. A portion of the total input voltage is dropped across the resistor and a portion across the inductor. Because the output voltage is the voltage across the inductor, it can be calculated using either Ohm's law ($V_{out} = IX_L$) or the voltage-divider formula.

Equation 16–6

$$V_{out} = \left(\frac{X_L}{\sqrt{R^2 + X_L^2}}\right)V_{in}$$

The phasor expression for the output voltage of an *RL* lead circuit is

$$\mathbf{V}_{out} = V_{out}\angle\phi$$

EXAMPLE 16–5

For the lead circuit in Figure 16–15(b) (Example 16–4), determine the output voltage in phasor form when the input voltage has an rms value of 5 V. Draw the input and output voltage waveforms showing their peak values. The inductive reactance X_L (314 Ω) and ϕ (65.2°) were found in Example 16–4.

Solution The output voltage in phasor form is

$$\mathbf{V}_{out} = V_{out}\angle\phi = \left(\frac{X_L}{\sqrt{R^2 + X_L^2}}\right)V_{in}\angle\phi$$

$$= \left(\frac{314\ \Omega}{\sqrt{(680\ \Omega)^2 + (314\ \Omega)^2}}\right)5\angle 65.2^\circ\text{ V} = \mathbf{2.10\angle 65.2^\circ\ V}$$

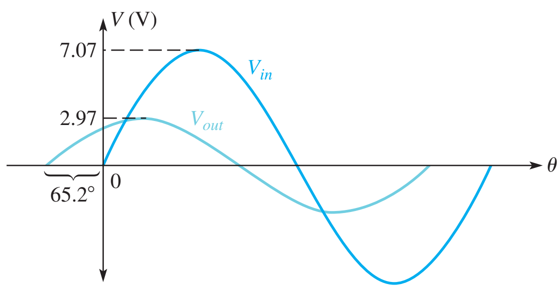
The peak values of voltage are

$$V_{in(p)} = 1.414V_{in(rms)} = 1.414(5\text{ V}) = 7.07\text{ V}$$

$$V_{out(p)} = 1.414V_{out(rms)} = 1.414(2.10\text{ V}) = 2.97\text{ V}$$

The waveforms with their peak values are shown in Figure 16–16. Notice that the output voltage leads the input voltage by 65.2°.

▶ FIGURE 16–16



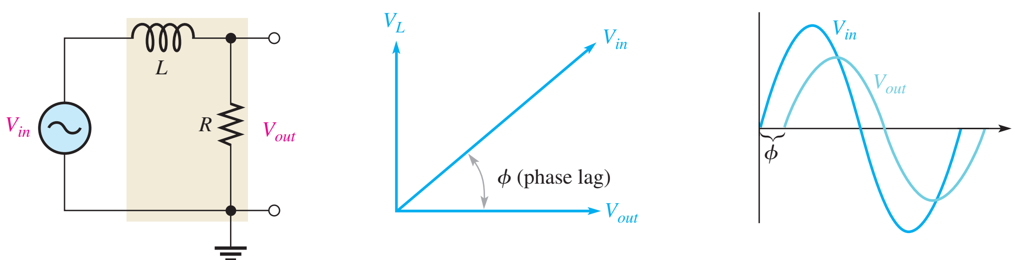
Related Problem In a lead circuit, does the output voltage increase or decrease when the frequency increases?

MultiSim

Use Multisim files E16-05A and E16-05B to verify the calculated results in this example and to confirm your calculation for the related problem.

The *RL* Lag Circuit

An ***RL* lag circuit** is a phase shift circuit in which the output voltage lags the input voltage by a specified amount. When the output of a series *RL* circuit is taken across the resistor rather than the inductor, as shown in Figure 16–17(a), it becomes a lag circuit.



(a) A basic *RL* lag circuit

(b) Phasor voltage diagram showing phase lag between V_{in} and V_{out}

(c) Input and output waveforms

▲ FIGURE 16–17

The *RL* lag circuit ($V_{out} = V_R$).

Phase Difference Between Input and Output In a series RL circuit, the current lags the input voltage. Since the output voltage is taken across the resistor, the output lags the input, as indicated by the phasor diagram in Figure 16–17(b). The waveforms are shown in Figure 16–17(c).

As in the lead circuit, the amount of phase difference between the input and output and the magnitude of the output voltage in the lag circuit are dependent on the relative values of the resistance and the inductive reactance. When the input voltage is assigned a reference angle of 0°, the angle of the output voltage (ϕ) with respect to the input voltage equals θ because the resistor voltage (output) and the current are in phase with each other. The expression for the angle between the input voltage and the output voltage is

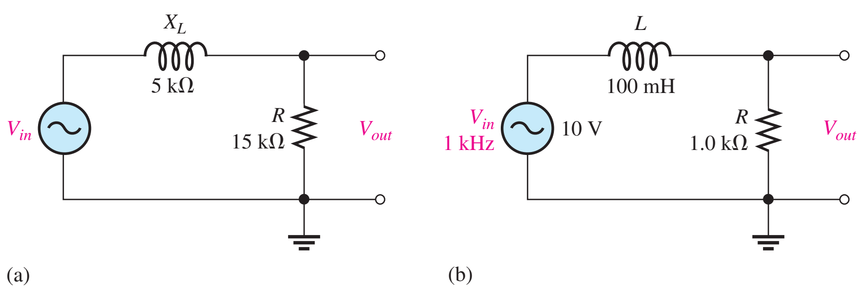
Equation 16–7

$$\phi = -\tan^{-1}\left(\frac{X_L}{R}\right)$$

This angle is negative because the output lags the input.

EXAMPLE 16–6

Calculate the output phase angle for each circuit in Figure 16–18.



▲ FIGURE 16–18

Solution For the lag circuit in Figure 16–18(a).

$$\phi = -\tan^{-1}\left(\frac{X_L}{R}\right) = -\tan^{-1}\left(\frac{5\text{ k}\Omega}{15\text{ k}\Omega}\right) = \mathbf{-18.4°}$$

The output lags the input by 18.4°.

For the lag circuit in Figure 16–18(b), first determine the inductive reactance.

$$X_L = 2\pi fL = 2\pi(1\text{ kHz})(100\text{ mH}) = 628\ \Omega$$

$$\phi = -\tan^{-1}\left(\frac{X_L}{R}\right) = -\tan^{-1}\left(\frac{628\ \Omega}{1.0\text{ k}\Omega}\right) = \mathbf{-32.1°}$$

The output lags the input by 32.1°.

Related Problem In a certain lag circuit, $R = 5.6\text{ k}\Omega$ and $X_L = 3.5\text{ k}\Omega$. Determine the phase angle.

Use Multisim files E16-06A, E16-06B, and E16-06C to verify the calculated results in this example and to confirm your calculation for the related problem.

Magnitude of the Output Voltage Since the output voltage of an RL lag circuit is taken across the resistor, the magnitude can be calculated using either Ohm's law ($V_{out} = IR$) or the voltage-divider formula.

$$V_{out} = \left(\frac{R}{\sqrt{R^2 + X_L^2}}\right)V_{in}$$

Equation 16–8

The expression for the output voltage in phasor form is

$$\mathbf{V}_{out} = V_{out}\angle\phi$$

EXAMPLE 16–7

The input voltage in Figure 16–18(b) (Example 16–6) has an rms value of 10 V. Determine the phasor expression for the output voltage. Draw the waveform relationships for the input and output voltages. The phase angle (−32.1°) and X_L (628 Ω) were found in Example 16–6.

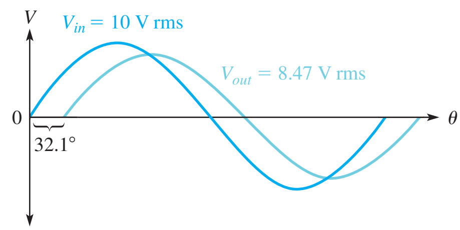
Solution The phasor expression for the output voltage is

$$\mathbf{V}_{out} = V_{out}\angle\phi = \left(\frac{R}{\sqrt{R^2 + X_L^2}}\right)V_{in}\angle\phi$$

$$= \left(\frac{1.0\ \text{k}\Omega}{1181\ \Omega}\right)10\angle-32.1^\circ\ \text{V} = \mathbf{8.47\angle-32.1^\circ\ V\ rms}$$

The waveforms are shown in Figure 16–19.

▶ FIGURE 16–19



Related Problem In a lag circuit, $R = 4.7\ \text{k}\Omega$ and $X_L = 6\ \text{k}\Omega$. If the rms input voltage is 20 V, what is the output voltage?

Use Multisim files E16-07A and E16-07B to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 16–3 CHECKUP

1. In a certain series RL circuit, $V_R = 2$ V and $V_L = 3$ V. What is the magnitude of the source voltage?
2. In Question 1, what is the phase angle between the source voltage and the current?
3. When the frequency of the applied voltage in a series RL circuit is increased, what happens to the inductive reactance? What happens to the magnitude of the total impedance? What happens to the phase angle?
4. A certain RL lead circuit consists of a 3.3 kΩ resistor and a 15 mH inductor. Determine the phase shift between input and output at a frequency of 5 kHz.
5. An RL lag circuit has the same component values as the lead circuit in Question 4. What is the magnitude of the output voltage at 5 kHz when the input is 10 V rms?

OPTION 2 NOTE

Coverage of series reactive circuits continues in Chapter 17, Part 1, on page 778.

Part 2

Parallel Circuits

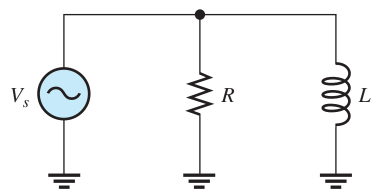
16–4 Impedance and Admittance of Parallel *RL* Circuits

In this section, you will learn how to determine the impedance and phase angle of a parallel *RL* circuit. The impedance consists of a magnitude component and a phase angle component. Also, inductive susceptance and admittance of a parallel *RL* circuit are introduced.

After completing this section, you should be able to

- **Determine impedance and admittance in a parallel *RL* circuit**
    - Express total impedance in complex form
    - Define and calculate *inductive susceptance* and *admittance*

Figure 16–20 shows a basic parallel *RL* circuit connected to an ac voltage source.



▲ FIGURE 16–20

Parallel *RL* circuit.

The expression for the total impedance of a two-component parallel *RL* circuit is developed as follows, using the product-over-sum rule.

$$\mathbf{Z} = \frac{(R\angle 0°)(X_L\angle 90°)}{R + jX_L} = \frac{RX_L\angle(0° + 90°)}{\sqrt{R^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)}$$

$$= \left(\frac{RX_L}{\sqrt{R^2 + X_L^2}}\right)\angle\left(90° - \tan^{-1}\left(\frac{X_L}{R}\right)\right)$$

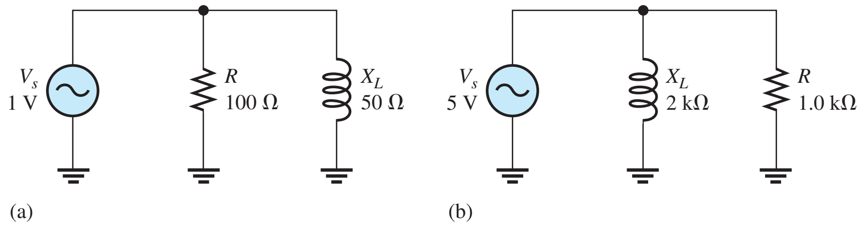
Equivalently, this equation can be expressed as

Equation 16–9

$$\mathbf{Z} = \left(\frac{RX_L}{\sqrt{R^2 + X_L^2}}\right)\angle\tan^{-1}\left(\frac{R}{X_L}\right)$$

EXAMPLE 16–8

For each circuit in Figure 16–21, determine the magnitude and phase angle of the total impedance.



▲ FIGURE 16–21

Solution For the circuit in Figure 16–21(a), the total impedance is

$$\mathbf{Z} = \left(\frac{RX_L}{\sqrt{R^2 + X_L^2}}\right)\angle\tan^{-1}\left(\frac{R}{X_L}\right)$$

$$= \left(\frac{(100\ \Omega)(50\ \Omega)}{\sqrt{(100\ \Omega)^2 + (50\ \Omega)^2}}\right)\angle\tan^{-1}\left(\frac{100\ \Omega}{50\ \Omega}\right) = \mathbf{44.7\angle 63.4^\circ\ \Omega}$$

Thus, $Z = 44.7\ \Omega$ and $\theta = 63.4°$.

For the circuit in Figure 16–21(b), the total impedance is

$$\mathbf{Z} = \left(\frac{(1.0\text{ k}\Omega)(2\text{ k}\Omega)}{\sqrt{(1.0\text{ k}\Omega)^2 + (2\text{ k}\Omega)^2}}\right)\angle\tan^{-1}\left(\frac{1.0\text{ k}\Omega}{2\text{ k}\Omega}\right) = \mathbf{894\angle 26.6^\circ\ \Omega}$$

Thus, $Z = 894\ \Omega$ and $\theta = 26.6°$.

Notice that the positive angle indicates that the voltage leads the current, in contrast to the *RC* case where the voltage lags the current.

Related Problem In a parallel circuit, $R = 10\text{ k}\Omega$ and $X_L = 14\text{ k}\Omega$. Determine the total impedance in polar form.

Conductance, Susceptance, and Admittance

As you know from the previous chapter, conductance (G) is the reciprocal of resistance, susceptance (B) is the reciprocal of reactance, and admittance (Y) is the reciprocal of impedance.

For parallel *RL* circuits, the phasor expression for **inductive susceptance (B_L)** is

$$\mathbf{B}_L = \frac{1}{X_L\angle 90^\circ} = B_L\angle -90^\circ = -jB_L$$

and the phasor expression for **admittance** is

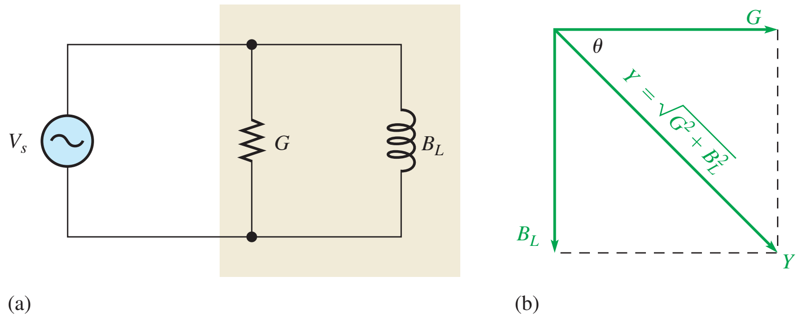
$$\mathbf{Y} = \frac{1}{Z\angle \pm\theta} = Y\angle\mp\theta$$

For the basic parallel *RL* circuit shown in Figure 16–22(a), the total admittance is the phasor sum of the conductance and the inductive susceptance, as shown in part (b).

Equation 16–10

$$\mathbf{Y} = G - jB_L$$

As with the *RC* circuit, the unit for conductance (G), inductive susceptance (B_L), and admittance (Y) is the siemens (S).



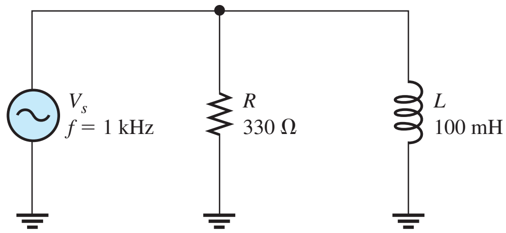
▲ FIGURE 16–22

Admittance in a parallel *RL* circuit.

EXAMPLE 16–9

Determine the total admittance and then convert it to total impedance in Figure 16–23. Draw the admittance phasor diagram.

▶ FIGURE 16–23



Solution First, determine the conductance magnitude. $R = 330\ \Omega$; thus,

$$G = \frac{1}{R} = \frac{1}{330\ \Omega} = 3.03\text{ mS}$$

Then, determine the inductive reactance.

$$X_L = 2\pi fL = 2\pi(1{,}000\text{ Hz})(100\text{ mH}) = 628\ \Omega$$

The inductive susceptance magnitude is

$$B_L = \frac{1}{X_L} = \frac{1}{628\ \Omega} = 1.59\text{ mS}$$

The total admittance is

$$\mathbf{Y}_{tot} = G - jB_L = 3.03\text{ mS} - j1.59\text{ mS}$$

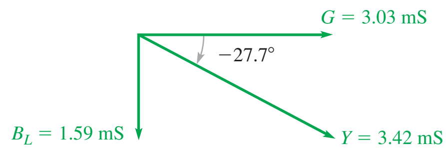
which can be expressed in polar form as

$$\mathbf{Y}_{tot} = \sqrt{G^2 + B_L^2}\angle -\tan^{-1}\left(\frac{B_L}{G}\right)$$

$$= \sqrt{(3.03\text{ mS})^2 + (1.59\text{ mS})^2}\angle -\tan^{-1}\left(\frac{1.59\text{ mS}}{3.03\text{ mS}}\right) = \mathbf{3.42\angle -27.7^\circ\ mS}$$

The admittance phasor diagram is shown in Figure 16–24.

▶ **FIGURE 16–24**



Convert total admittance to total impedance as follows:

$$\mathbf{Z}_{tot} = \frac{1}{\mathbf{Y}_{tot}} = \frac{1}{3.42\angle -27.7^\circ \text{ mS}} = \mathbf{292\angle 27.7^\circ\ \Omega}$$

The positive phase angle indicates that the voltage leads the current.

Related Problem What is the total admittance of the circuit in Figure 16–23 if f is increased to 2 kHz?

SECTION 16–4 CHECKUP

1. If $Z = 500\ \Omega$, what is the value of the magnitude of the admittance Y?
2. In a certain parallel RL circuit, $R = 470\ \Omega$ and $X_L = 750\ \Omega$. Determine Y.
3. In the circuit of Question 2, does the total current lead or lag the applied voltage? By what phase angle?

16–5 ANALYSIS OF PARALLEL *RL* CIRCUITS

Ohm's law and Kirchhoff's current law are used in the analysis of RL circuits. Current and voltage relationships in a parallel RL circuit are examined.

After completing this section, you should be able to

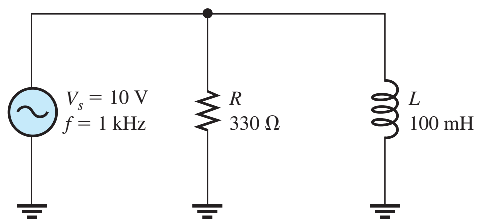
- **Analyze a parallel RL circuit**
  - Apply Ohm's law and Kirchhoff's current law to parallel RL circuits
  - Express the voltages and currents as phasor quantities

The following two examples apply Ohm's law to the analysis of a parallel RL circuit.

EXAMPLE 16–10

Determine I_R, I_L, I_{tot} and the phase angle in the circuit of Figure 16–25. Notice that this is the same circuit given in Example 16–9 but with a source voltage defined as 10.0 V. Draw a phasor diagram of the currents.

▶ **FIGURE 16–25**



Solution In Example 16–9, the conductance, susceptance, and admittance were found as $G = 3.03\,\text{ms}$, $B_L = 1.59\,\text{mS}$, and $Y_{tot} = 3.42\,\text{mS}$. Applying Ohm's law using conductance, susceptance, and admittance, we find that:

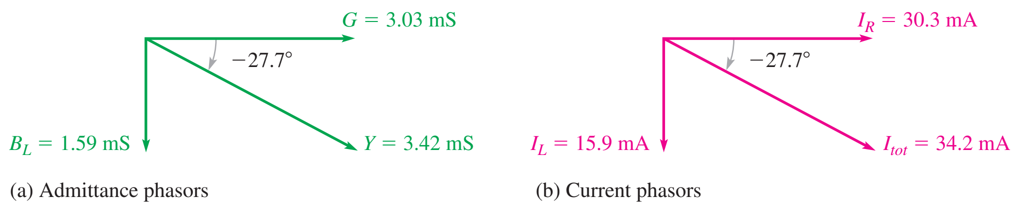
$$I_R = V_sG = (10\text{ V})(3.03\text{ mS}) = 30.3\text{ mA}$$

$$I_L = V_sB_L = (10\text{ V})(1.59\text{ mS}) = 15.9\text{ mA}$$

$$I_{tot} = V_sY_{tot} = (10\text{ V})(3.42\text{ mS}) = 34.2\text{ mS}$$

The phase angle between G and Y_{tot} was found in Example 16–9 as -27.7°.

The admittance phasor diagram was drawn as Figure 16–24. The current phasor diagram is proportional to the admittance phasor diagram with the voltage acting as a scaling factor. For comparison, both the admittance phasor diagram and the current phasor diagram are shown in Figure 16–26.



▲ **FIGURE 16–26**

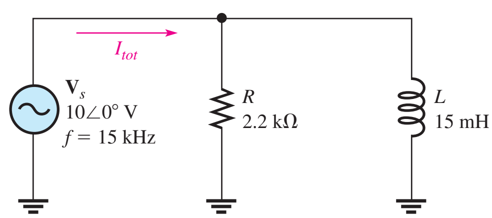
The current phasors are found by multiplying the admittance phasors by the voltage.

Related Problem If you multiply the *current* phasors by the voltage, what type of phasor diagram will result?

EXAMPLE 16–11

Determine the total current and the phase angle in the circuit of Figure 16–27. Draw a phasor diagram showing the relationship of $\mathbf{V}_s$ and $\mathbf{I}_{tot}$.

▶ **FIGURE 16–27**



Solution The inductive reactance is

$$X_L = 2\pi fL = 2\pi(15\text{ kHz})(15\text{ mH}) = 1.41\text{ k}\Omega$$

The inductive susceptance magnitude is

$$B_L = \frac{1}{X_L} = \frac{1}{1.41\text{ k}\Omega} = 707\ \mu\text{S}$$

The conductance magnitude is

$$G = \frac{1}{R} = \frac{1}{2.2\text{ k}\Omega} = 455\ \mu\text{S}$$

The total admittance is

$$\mathbf{Y}_{tot} = G - jB_L = 455\ \mu\text{S} - j707\ \mu\text{S}$$

Converting to polar form yields

$$\mathbf{Y}_{tot} = \sqrt{G^2 + B_L^2}\angle -\tan^{-1}\left(\frac{B_L}{G}\right)$$

$$= \sqrt{(455\ \mu\text{S})^2 + (707\ \mu\text{S})^2}\angle -\tan^{-1}\left(\frac{707\ \mu\text{S}}{455\ \mu\text{S}}\right) = 841\angle -57.3^\circ\ \mu\text{S}$$

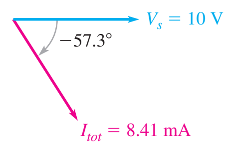
The phase angle is -57.3°.

Use Ohm's law to determine the total current.

$$\mathbf{I}_{tot} = \mathbf{V}_s\mathbf{Y}_{tot} = (10\angle 0^\circ\text{ V})(841\angle -57.3^\circ\ \mu\text{S}) = \mathbf{8.41\angle -57.3^\circ\ mA}$$

The magnitude of the total current is 8.41 mA, and it lags the applied voltage by **57.3°**, as indicated by the negative angle associated with it. The phasor diagram in Figure 16–28 shows these relationships.

▶ FIGURE 16–28



Related Problem Determine the current in polar form if *f* is reduced to 8.0 kHz in Figure 16–27.

MultiSim

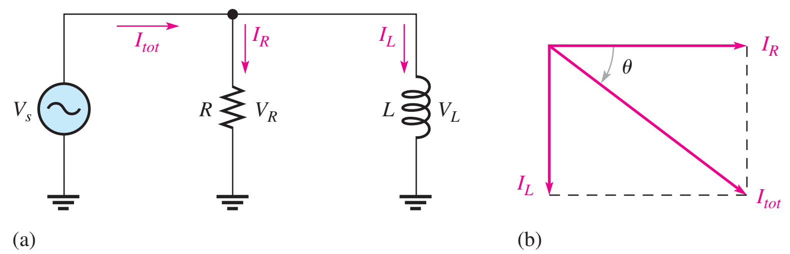
Use Multisim files E16-11A and E16-11B to verify the calculated results in this example and to confirm your calculation for the related problem.

Phase Relationships of Currents and Voltages

Figure 16–29(a) shows all the currents in a basic parallel *RL* circuit. The total current, I_{tot}, divides at the junction into two branch currents, I_R and I_L. The applied voltage, V_s, appears across both the resistive and the inductive branches, so V_s, V_R, and V_L are all in phase and of the same magnitude.

The current through the resistor is in phase with the voltage. The current through the inductor lags the voltage and the resistor current by 90°. By Kirchhoff's current law, the total current is the phasor sum of the two branch currents, as shown by the phasor diagram in Figure 16–29(b). The total current is expressed as

$$\mathbf{I}_{tot} = I_R - jI_L \qquad \textbf{Equation 16–11}$$



▲ FIGURE 16–29

Currents in a parallel *RL* circuit. The current directions shown in part (a) are instantaneous and, of course, reverse when the source voltage reverses during each cycle.

This equation can be expressed in polar form as

Equation 16–12

$$\mathbf{I}_{tot} = \sqrt{I_R^2 + I_L^2}\angle -\tan^{-1}\left(\frac{I_L}{I_R}\right)$$

where the magnitude of the total current is

$$I_{tot} = \sqrt{I_R^2 + I_L^2}$$

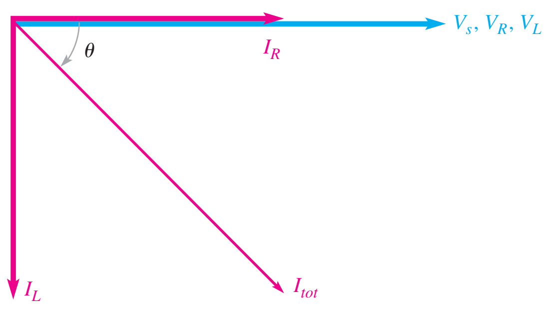
and the phase angle between the resistor current and the total current is

$$\theta = -\tan^{-1}\left(\frac{I_L}{I_R}\right)$$

Since the resistor current and the applied voltage are in phase, θ also represents the phase angle between the total current and the applied voltage. Figure 16–30 shows a complete current and voltage phasor diagram.

▶ FIGURE 16–30

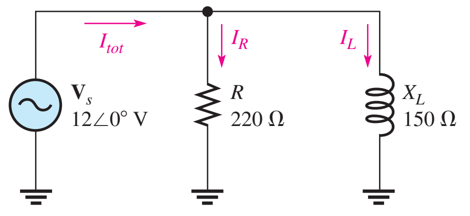
Current and voltage phasor diagram for a parallel *RL* circuit (amplitudes are arbitrary).



EXAMPLE 16–12

Determine the value of each current in Figure 16–31, and describe the phase relationship of each with the applied voltage. Draw the current phasor diagram.

▶ FIGURE 16–31



Solution The resistor current, the inductor current, and the total current are expressed as follows:

$$\mathbf{I}_R = \frac{\mathbf{V}_s}{\mathbf{R}} = \frac{12\angle 0^\circ \text{ V}}{220\angle 0^\circ\ \Omega} = \mathbf{54.5\angle 0^\circ\ mA}$$

$$\mathbf{I}_L = \frac{\mathbf{V}_s}{\mathbf{X}_L} = \frac{12\angle 0^\circ \text{ V}}{150\angle 90^\circ\ \Omega} = \mathbf{80\angle -90^\circ\ mA}$$

$$\mathbf{I}_{tot} = I_R - jI_L = 54.5 \text{ mA} - j80 \text{ mA}$$

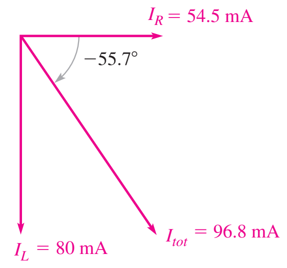
Converting $\mathbf{I}_{tot}$ to polar form yields

$$\mathbf{I}_{tot} = \sqrt{I_R^2 + I_L^2}\angle -\tan^{-1}\left(\frac{I_L}{I_R}\right)$$

$$= \sqrt{(54.5 \text{ mA})^2 + (80 \text{ mA})^2}\angle -\tan^{-1}\left(\frac{80 \text{ mA}}{54.5 \text{ mA}}\right) = \mathbf{96.8\angle -55.7^\circ\ mA}$$

As the results show, the resistor current is 54.5 mA and is in phase with the applied voltage. The inductor current is 80 mA and lags the applied voltage by 90°. The total current is 96.8 mA and lags the voltage by 55.7°. The phasor diagram in Figure 16–32 shows these relationships.

▶ FIGURE 16–32



Related Problem Find the magnitude of $\mathbf{I}_{tot}$ and the circuit phase angle if $X_L = 300\ \Omega$ in Figure 16–31.

Conversion from Parallel to Series Form

In Section 15–6, you saw how to convert a parallel *RC* circuit to an equivalent series *RC* circuit for a given frequency. The same basic procedure can be accomplished for a parallel *RL* circuit at a specific frequency. To obtain the equivalent series circuit for a given parallel *RL* circuit, first find the impedance. Writing the impedance in rectangular notation results in the equivalent series resistor and the equivalent series inductive reactance as given by the following equations:

$$R_{\text{eq}} = Z\cos\theta \qquad \textbf{Equation 16–13}$$

$$X_{L(\text{eq})} = Z\sin\theta \qquad \textbf{Equation 16–14}$$

EXAMPLE 16–13

For the parallel RL circuit in Example 16–12, determine the equivalent series resistance and equivalent series inductive reactance.

Solution The impedance in Example 16–12 can be found by applying Ohm's law:

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{12\angle 0^\circ \text{ V}}{96.8\angle -55.7^\circ \text{ mA}} = 124\angle 55.7^\circ \text{ k}\Omega$$

The equivalent series resistance and inductive reactance are

$$R_{eq} = Z\cos\theta = 124\cos(55.7^\circ) = \mathbf{69.9\ \Omega}$$

$$X_{L(eq)} = Z\sin\theta = 124\sin(55.7^\circ) = \mathbf{102\ \Omega}$$

Related Problem What happens to the equivalent resistance if the frequency is increased?

SECTION 16–5 CHECKUP

1. The admittance of an RL circuit is 4 mS, and the applied voltage is 8 V. What is the total current?
2. In a certain parallel RL circuit, the resistor current is 12 mA, and the inductor current is 20 mA. Determine the magnitude and phase angle of the total current. This phase angle is measured with respect to what?
3. What is the phase angle between the inductor current and the applied voltage in a parallel RL circuit?
4. Given the phasor current diagram and the source voltage for a parallel RL circuit, how do you find the admittance phasor diagram?

OPTION 2 NOTE

Coverage of series reactive circuits continues in Chapter 17, Part 2, on page 792.

Part 3

SERIES-PARALLEL CIRCUITS

16–6 ANALYSIS OF SERIES-PARALLEL *RL* CIRCUITS

The concepts studied with respect to series and parallel circuits are used to analyze circuits with combinations of both series and parallel R and L components.

After completing this section, you should be able to

- **Analyze series-parallel *RL* circuits**
    - Determine total impedance
    - Calculate currents and voltages

Analysis of series-parallel RL circuits uses complex mathematics that was introduced in Section 15–1. Graphing calculators such as the TI-84 Plus CE, can handle the standard add, subtract, multiply and divide operations of complex numbers in either rectangular or polar form, so they do not require making tedious conversions between forms. In this section, we show the procedure for detailed solution of series-parallel RL circuits for cases where you are not using a calculator that can do complex math.

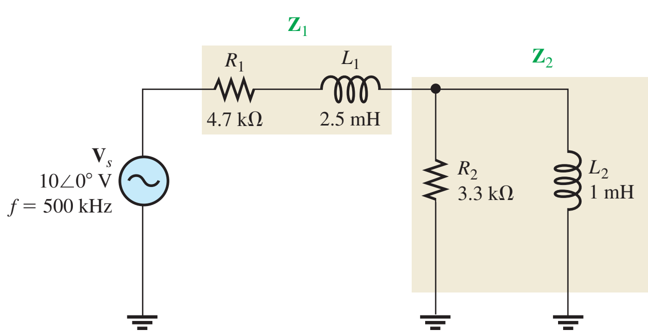
Recall that the impedance of series components is most easily expressed in rectangular form and that the impedance of parallel components is best found by using polar form. The steps for analyzing a circuit with a series and a parallel component are illustrated in Example 16–14. First express the impedance of the series part of the circuit in rectangular form and the impedance of the parallel part in polar form. Next, convert the impedance of the parallel part to rectangular form and add it to the impedance of the series part. Once you determine the rectangular form of the total impedance, you can convert it to polar form in order to see the magnitude and phase angle and to calculate the current.

EXAMPLE 16–14

In the circuit of Figure 16–33, determine the following values:

(a) $\mathbf{Z}_{tot}$ **(b)** $\mathbf{I}_{tot}$ **(c)** θ

▶ FIGURE 16–33



Solution **(a)** First, calculate the magnitudes of inductive reactance.

$$X_{L1} = 2\pi f L_1 = 2\pi(500\text{ kHz})(2.5\text{ mH}) = 7.85\text{ k}\Omega$$
$$X_{L2} = 2\pi f L_2 = 2\pi(500\text{ kHz})(1\text{mH}) = 3.14\text{ k}\Omega$$

One approach is to find the impedance of the series portion and the impedance of the parallel portion and combine them to get the total impedance. The impedance of the series combination of R_1 and L_1 is

$$\mathbf{Z}_1 = R_1 + jX_{L1} = 4.7\text{ k}\Omega + j7.85\text{ k}\Omega$$

To determine the impedance of the parallel portion, first determine the admittance of the parallel combination of R_2 and L_2.

$$G_2 = \frac{1}{R_2} = \frac{1}{3.3\text{ k}\Omega} = 303\ \mu\text{S}$$
$$B_{L2} = \frac{1}{X_{L2}} = \frac{1}{3.14\text{ k}\Omega} = 318\ \mu\text{S}$$
$$\mathbf{Y}_2 = G_2 - jB_L = 303\ \mu\text{S} - j318\ \mu\text{S}$$

Converting to polar form yields

$$\mathbf{Y}_2 = \sqrt{G_2^2 + B_{L2}^2}\angle -\tan^{-1}\left(\frac{B_{L2}}{G_2}\right)$$
$$= \sqrt{(303\ \mu\text{S})^2 + (318\ \mu\text{S})^2}\angle -\tan^{-1}\left(\frac{318\ \mu\text{S}}{303\ \mu\text{S}}\right) = 439\angle -46.4^\circ\ \mu\text{S}$$

Then, the impedance of the parallel portion is

$$\mathbf{Z}_2 = \frac{1}{\mathbf{Y}_2} = \frac{1}{439\angle -46.4^\circ\ \mu\text{S}} = 2.28\angle 46.4^\circ\text{ k}\Omega$$

Converting to rectangular form yields

$$\mathbf{Z}_2 = Z_2\cos\theta + jZ_2\sin\theta$$
$$= (2.28\text{ k}\Omega)\cos(46.4^\circ) + j(2.28\text{ k}\Omega)\sin(46.4^\circ) = 1.57\text{ k}\Omega + j1.65\text{ k}\Omega$$

The series portion and the parallel portion are in series with each other. Combine $\mathbf{Z}_1$ and $\mathbf{Z}_2$ to get the total impedance.

$$\mathbf{Z}_{tot} = \mathbf{Z}_1 + \mathbf{Z}_2$$
$$= (4.7\text{ k}\Omega + j7.85\text{ k}\Omega) + (1.57\text{ k}\Omega + j1.65\text{ k}\Omega) = 6.27\text{ k}\Omega + j9.50\text{ k}\Omega$$

Expressing $\mathbf{Z}_{tot}$ in polar form yields

$$\mathbf{Z}_{tot} = \sqrt{Z_1^2 + Z_2^2}\angle\tan^{-1}\left(\frac{Z_2}{Z_1}\right)$$
$$= \sqrt{(6.27\text{ k}\Omega)^2 + (9.50\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{9.50\text{ k}\Omega}{6.27\text{ k}\Omega}\right) = \mathbf{11.4\angle 56.6^\circ\ k\Omega}$$

(b) Use Ohm's law to find the total current.

$$\mathbf{I}_{tot} = \frac{\mathbf{V}_s}{\mathbf{Z}_{tot}} = \frac{10\angle 0^\circ\text{ V}}{11.4\angle 56.6^\circ\text{ k}\Omega} = \mathbf{878\angle -56.6^\circ\ \mu A}$$

(c) The total current lags the applied voltage by **56.6°**.

Related Problem **(a)** Determine the voltage across the series part of the circuit in Figure 16–33.

(b) Determine the voltage across the parallel part of the circuit in Figure 16–33.

MultiSim

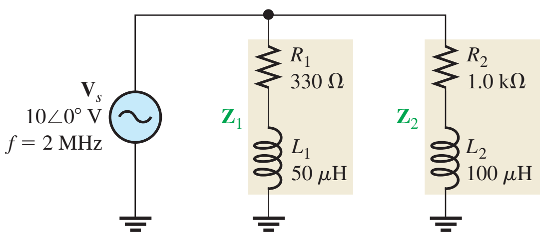
Use Multisim files E16-14A and E16-14B to verify the calculated results in the (b) part of this example and to confirm your calculation for the related problem, part (b).

Example 16–15 shows two sets of series components in parallel. The approach is to first express each branch impedance in rectangular form and then convert each of these impedances to polar form. Next, calculate each branch current using polar notation. Once you know the branch currents, you can find the total current by adding the two branch currents in rectangular form. In this particular case, the total impedance is not required.

EXAMPLE 16–15

Determine the voltage across each component in Figure 16–34. Draw a voltage phasor diagram and a current phasor diagram.

▶ FIGURE 16–34



Solution First, calculate X_{L1} and X_{L2}.

$$X_{L1} = 2\pi f L_1 = 2\pi(2\text{ MHz})(50\ \mu\text{H}) = 628\ \Omega$$

$$X_{L2} = 2\pi f L_2 = 2\pi(2\text{ MHz})(100\ \mu\text{H}) = 1.26\text{ k}\Omega$$

Next, determine the impedance of each branch.

$$\mathbf{Z}_1 = R_1 + jX_{L1} = 330\ \Omega + j628\ \Omega$$

$$\mathbf{Z}_2 = R_2 + jX_{L2} = 1.0\text{ k}\Omega + j1.26\text{ k}\Omega$$

Convert these impedances to polar form.

$$\mathbf{Z}_1 = \sqrt{R_1^2 + X_{L1}^2}\angle\tan^{-1}\left(\frac{X_{L1}}{R_1}\right)$$

$$= \sqrt{(330\ \Omega)^2 + (628\ \Omega)^2}\angle\tan^{-1}\left(\frac{628\ \Omega}{330\ \Omega}\right) = 710\angle 62.3^\circ\ \Omega$$

$$\mathbf{Z}_2 = \sqrt{R_2^2 + X_{L2}^2}\angle\tan^{-1}\left(\frac{X_{L2}}{R_2}\right)$$

$$= \sqrt{(1.0\text{ k}\Omega)^2 + (1.26\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{1.26\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = 1.61\angle 51.5^\circ\text{ k}\Omega$$

Calculate each branch current.

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{\mathbf{Z}_1} = \frac{10\angle 0^\circ\text{ V}}{710\angle 62.3^\circ\ \Omega} = 14.1\angle -62.3^\circ\text{ mA}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_s}{\mathbf{Z}_2} = \frac{10\angle 0^\circ\text{ V}}{1.61\angle 51.5^\circ\text{ k}\Omega} = 6.23\angle -51.5^\circ\text{ mA}$$

Now, use Ohm's law to get the voltage across each element.

$$\mathbf{V}_{R1} = \mathbf{I}_1\mathbf{R}_1 = (14.1\angle -62.3^\circ\ \text{mA})(330\angle 0^\circ\ \Omega) = \mathbf{4.65\angle -62.3^\circ\ V}$$

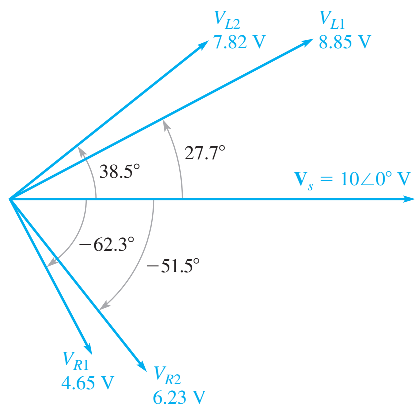
$$\mathbf{V}_{L1} = \mathbf{I}_1\mathbf{X}_{L1} = (14.1\angle -62.3^\circ\ \text{mA})(628\angle 90^\circ\ \Omega) = \mathbf{8.85\angle 27.7^\circ\ V}$$

$$\mathbf{V}_{R2} = \mathbf{I}_1\mathbf{R}_2 = (6.23\angle -51.5^\circ\ \text{mA})(1\angle 0^\circ\ \text{k}\Omega) = \mathbf{6.23\angle -51.5^\circ\ V}$$

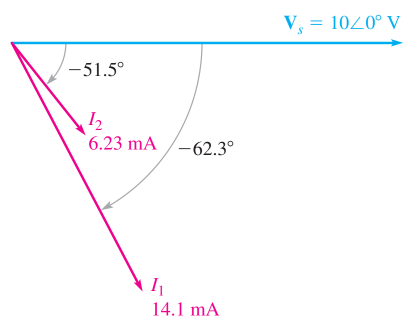
$$\mathbf{V}_{L2} = \mathbf{I}_2\mathbf{X}_{L2} = (6.23\angle -51.5^\circ\ \text{mA})(1.26\angle 90^\circ\ \text{k}\Omega) = \mathbf{7.82\angle 38.5^\circ\ V}$$

The voltage phasor diagram is shown in Figure 16–35, and the current phasor diagram is shown in Figure 16–36.

▶ FIGURE 16–35



▶ FIGURE 16–36



Related Problem What is the total current in polar form in Figure 16–34?

MultiSim

Use Multisim files E16-15A through E16-15E to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 16–6 CHECKUP

1. What is the total impedance in polar form of the circuit in Figure 16–34?
2. Determine the total current in rectangular form for the circuit in Figure 16–34.

OPTION 2 NOTE

Coverage of series-parallel reactive circuits continues in Chapter 17, Part 3, on page 803.

Special Topics

Part 4

16–7 Power in *RL* Circuits

In a purely resistive ac circuit, all of the energy delivered by the source is dissipated in the form of heat by the resistance. In a purely inductive ac circuit, all of the energy delivered by the source is stored by the inductor in its magnetic field during a portion of the voltage cycle and then returned to the source during another portion of the cycle so that there is no net energy conversion to heat. When there is both resistance and inductance, some of the energy is stored and returned by the inductance and some is dissipated by the resistance on each alteration. The amount of energy converted to heat is determined by the relative values of the resistance and the inductive reactance.

After completing this section, you should be able to

- **Determine power in *RL* circuits**
  - Explain true and reactive power
  - Draw the power triangle
  - Explain power factor correction

When the resistance in a series *RL* circuit is greater than the inductive reactance, more of the total energy delivered by the source is converted to heat by the resistance than is stored by the inductor. Likewise, when the reactance is greater than the resistance, more of the total energy is stored and returned than is converted to heat.

As you know, the power dissipation in a resistance is called the *true power*. The power in an inductor is reactive power and is expressed as

$$P_r = I^2 X_L$$

Equation 16–15

The Power Triangle for *RL* Circuits

The generalized power triangle for a series *RL* circuit is shown in Figure 16–37. The **apparent power, P_a**, is the resultant of the average power, P_{true}, and the reactive power, P_r.

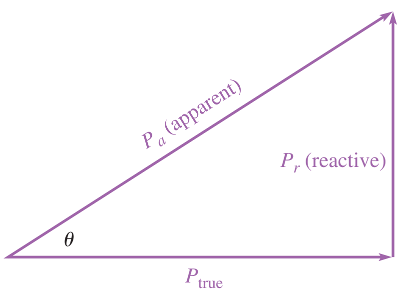


FIGURE 16–37

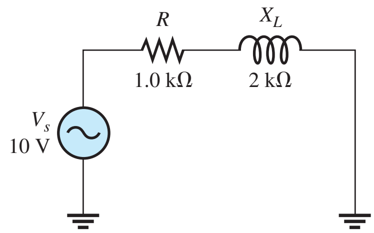
Power triangle for an *RL* circuit.

Recall that the **power factor** equals the cosine of θ ($PF = \cos\theta$). As the phase angle between the applied voltage and the total current increases, the power factor decreases, indicating an increasingly reactive circuit. A smaller power factor indices less true power and more reactive power.

EXAMPLE 16–16

Determine the power factor, the true power, the reactive power, and the apparent power in the circuit in Figure 16–38.

▶ FIGURE 16–38



Solution The total impedance of the circuit in rectangular form is

$$\mathbf{Z} = R + jX_L = 1.0\text{ k}\Omega + j2\text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z} = \sqrt{R_2^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)$$

$$= \sqrt{(1.0\text{ k}\Omega)^2 + (2\text{ k}\Omega)^2}\angle\tan^{-1}\left(\frac{2\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = 2.24\angle 63.4^\circ\text{ k}\Omega$$

The current magnitude is

$$I = \frac{V_s}{Z} = \frac{10\text{ V}}{2.24\text{ k}\Omega} = 4.47\text{ mA}$$

The phase angle, indicated in the expression for **Z**, is

$$\theta = 63.4^\circ$$

The power factor is, therefore,

$$PF = \cos\theta = \cos(63.4^\circ) = \mathbf{0.447}$$

The true power is

$$P_{\text{true}} = V_s I\cos\theta = (10\text{ V})(4.46\text{ mA})(0.448) = \mathbf{20.0\text{ mW}}$$

The reactive power is

$$P_r = I^2X_L = (4.47\text{ mA})^2(2\text{ k}\Omega) = \mathbf{40.0\text{ mVAR}}$$

The apparent power is

$$P_a = I^2Z = (4.47\text{ mA})^2(2.24\text{ k}\Omega) = \mathbf{44.7\text{ mVA}}$$

Related Problem If the frequency in Figure 16–38 is increased, what happens to P_{true}, P_r, and P_a?

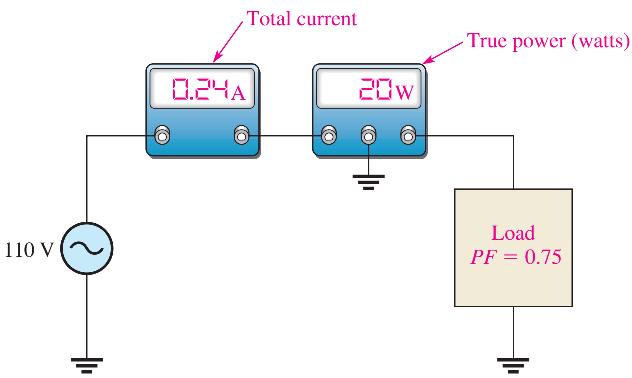
Significance of the Power Factor

As you learned in Chapter 15, the power factor (PF) is important in determining how much useful power (true power) is transferred to a load. The highest power factor is 1, which indicates that all of the current to a load is in phase with the voltage (resistive). When the power factor is 0, all of the current to a load is 90° out of phase with the voltage (reactive).

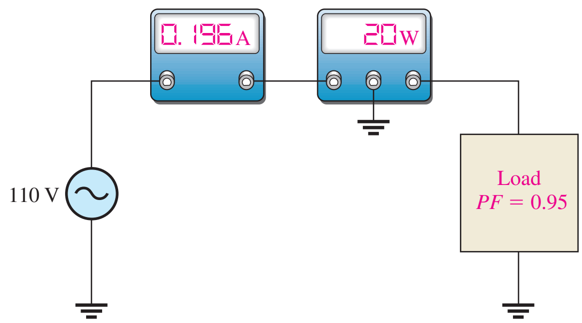
In industrial systems with inductive loads (such as motors), a certain amount of reactive power is needed that is stored in the magnetic field. However, in general a power factor as close to 1 as possible is desirable because then most of the power transferred from the source to the load is the useful or true power. True power goes only one way—from source to load—and performs work on the load in terms of energy dissipation. Reactive power simply goes back and forth between the source and the load with no net work being done. Energy must be used in order for work to be done.

Many practical loads have inductance as a result of their particular function, and it is essential for their proper operation. Examples are transformers, electric motors, light ballasts, and speakers, to name a few. Therefore, inductive (and capacitive) loads are important considerations.

To see the effect of the power factor on system requirements, refer to Figure 16–39. This figure shows a representation of a typical inductive load consisting effectively of inductance and resistance in parallel. Part (a) shows a load with a relatively low power factor (0.75), and part (b) shows a load with a relatively high power factor (0.95). Both loads dissipate equal amounts of power as indicated by the wattmeters. Thus, an equal amount of work is done on both loads.



(a) A lower power factor means more total current for a given power dissipation (watts). A larger source VA rating is required to deliver the true power (watts).



(b) A higher power factor means less total current for a given power dissipation. A smaller source can deliver the same true power (watts).

▲ **FIGURE 16–39**

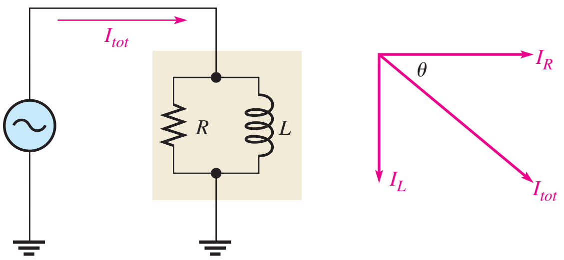
Illustration of the effect of the power factor on system requirements such as source rating (VA) and conductor size.

Although both loads are equivalent in terms of the amount of work done (true power), the low power factor load in Figure 16–39(a) draws more current from the source than does the high power factor load in Figure 16–39(b), as indicated by the ammeters. Therefore, the source in part (a) must have a higher VA rating than the one in part (b). Also, the lines connecting the source to the load in part (a) must be a larger wire gauge than those in part (b), a condition that becomes significant when very long transmission lines are required, such as in power distribution. In this case, the reactive power can be converted to I^2R loss in wiring, another undesirable aspect to a low *PF*.

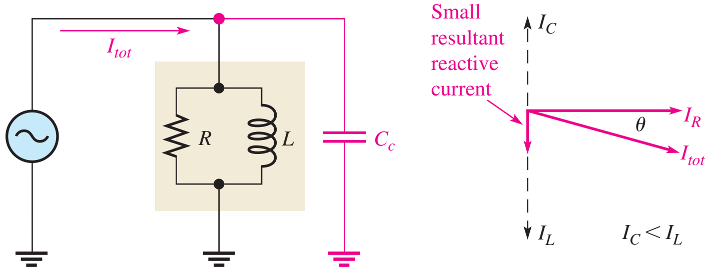
Figure 16–39 has demonstrated that a higher power factor is an advantage in delivering power more efficiently to a load.

Power Factor Correction

The power factor of an inductive load can be increased by the addition of a capacitor in parallel, as shown in Figure 16–40. The capacitor compensates for the phase lag of the total current by creating a capacitive component of current that is 180° out of phase with the inductive component. This has a canceling effect and reduces the phase angle and the total current and increases the power factor, as illustrated in the figure.



(a) Total current is the resultant of I_R and I_L.



(b) I_C subtracts from I_L, leaving only a small reactive current, thus decreasing I_{tot} and the phase angle.

▲ **FIGURE 16–40**

Example of how the power factor can be increased by the addition of a compensating capacitor (C_c).

SECTION 16–7 CHECKUP

1. To which component in an *RL* circuit is the power dissipation due?
2. Calculate the power factor when $\theta = 50°$.
3. Assume a motor that is connected to a 120 V supply has a current of 12 A and delivers a true power of 1,200 W to a load. What is the *PF*?
4. A certain *RL* circuit consists of a 470 Ω resistor and an inductive reactance of 620 Ω at the operating frequency. Determine P_{true}, P_r, and P_a when $I = 100$ mA.

16–8 Basic Applications

Two applications of *RL* circuits are covered in this section. The first application is a basic frequency selective (filter) circuit. The second application is the switching regulator, a widely used circuit in power supplies because of its high efficiency. The switching regulator uses other components, but the *RL* circuit is emphasized.

After completing this section, you should be able to

- **Describe two examples of *RL* circuit applications**
  - Discuss how the *RL* circuit operates as a filter
  - Discuss the advantage of an inductor in a switching regulator

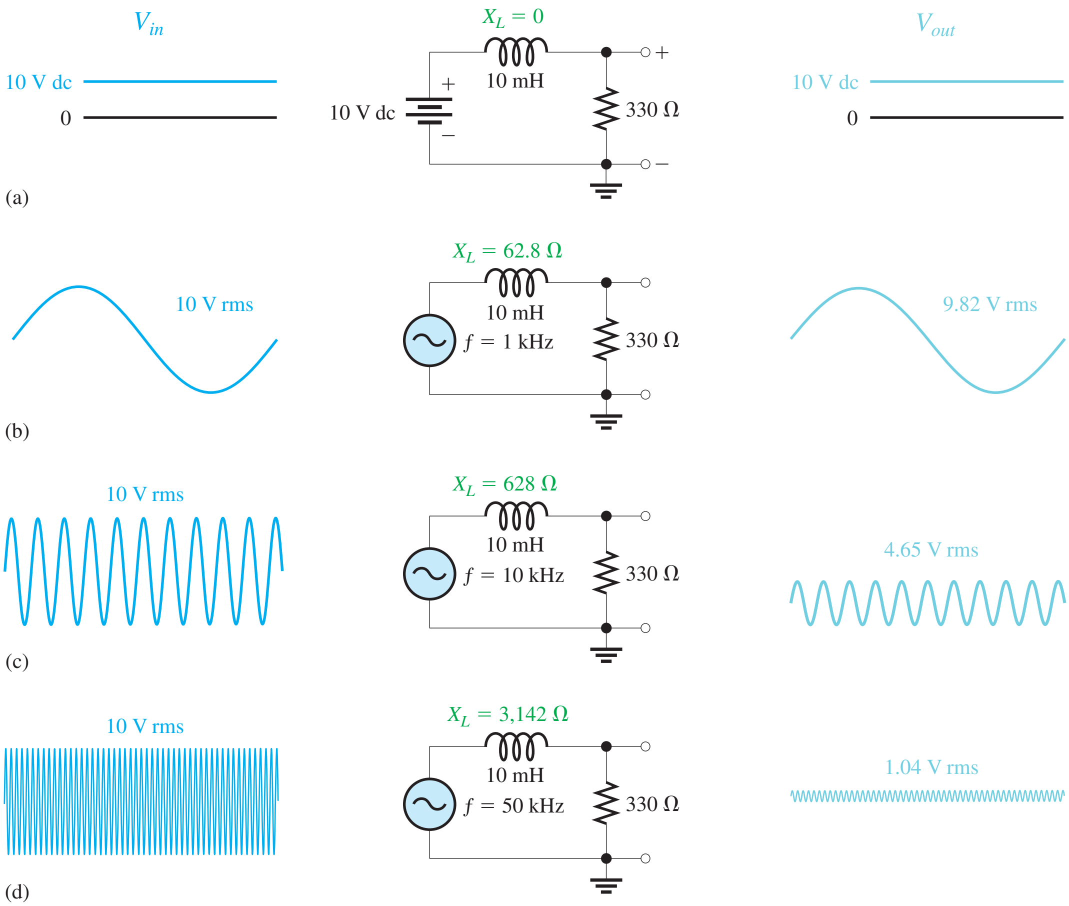
The *RL* Circuit as a Filter

As with *RC* circuits, series *RL* circuits also exhibit a frequency-selective characteristic and therefore act as basic filters.

Low-Pass Filter You have seen what happens to the output magnitude and phase angle in a lag circuit. In terms of the filtering action, the variation of the magnitude of the output voltage as a function of frequency is important.

Figure 16–41 shows the filtering action of a series *RL* circuit using specific values for purposes of illustration. In part (a) of the figure, the input is zero frequency (dc). Since the inductor ideally acts as a short to constant direct current, the output voltage equals the full value of the input voltage (neglecting the winding resistance). Therefore, the circuit passes all of the input voltage to the output (10 V in, 10 V out).

In Figure 16–41(b), the frequency of the input voltage has been increased to 1 kHz, causing the inductive reactance to increase to 62.8 Ω. For an input voltage of 10 V rms, the output voltage is approximately 9.9 V rms, which can be calculated using the voltage-divider approach or Ohm's law.



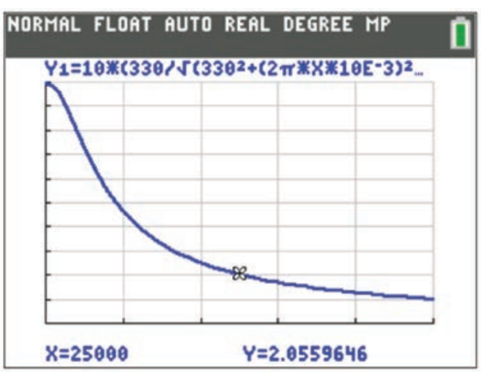
▲ **FIGURE 16–41**

Low-pass filtering action of an *RL* circuit (phase shift from input to output is not indicated).

In Figure 16–41(c), the input frequency has been increased to 10 kHz, causing the inductive reactance to increase further to 628 Ω. For a constant input voltage of 10 V rms, the output voltage is now 4.65 V rms.

As the input frequency is increased further, the output voltage continues to decrease as shown in Figure 16–41(d) for $f = 50$ kHz. To see the frequency response, you can input the values given into the basic voltage divider equation (using complex math). This is shown in Figure 16–42 for illustration. Grid lines are drawn in 10 kHz increments on the *x*-axis and in 1.0 V increments on the *y*-axis.

A description of the circuit action is as follows: As the frequency of the input increases, the inductive reactance increases. Because the resistance is constant and the inductive



◄ **FIGURE 16–42**

Response of the circuit in Figure 16–41 on a TI-84 Plus CE graphing calculator.

Images used with permission by Texas Instruments, Inc.

reactance increases, the voltage across the inductor increases and that across the resistor (output voltage) decreases. The input frequency can be increased until it reaches a value at which the reactance is so large compared to the resistance that the output voltage can be neglected because it becomes very small compared to the input voltage.

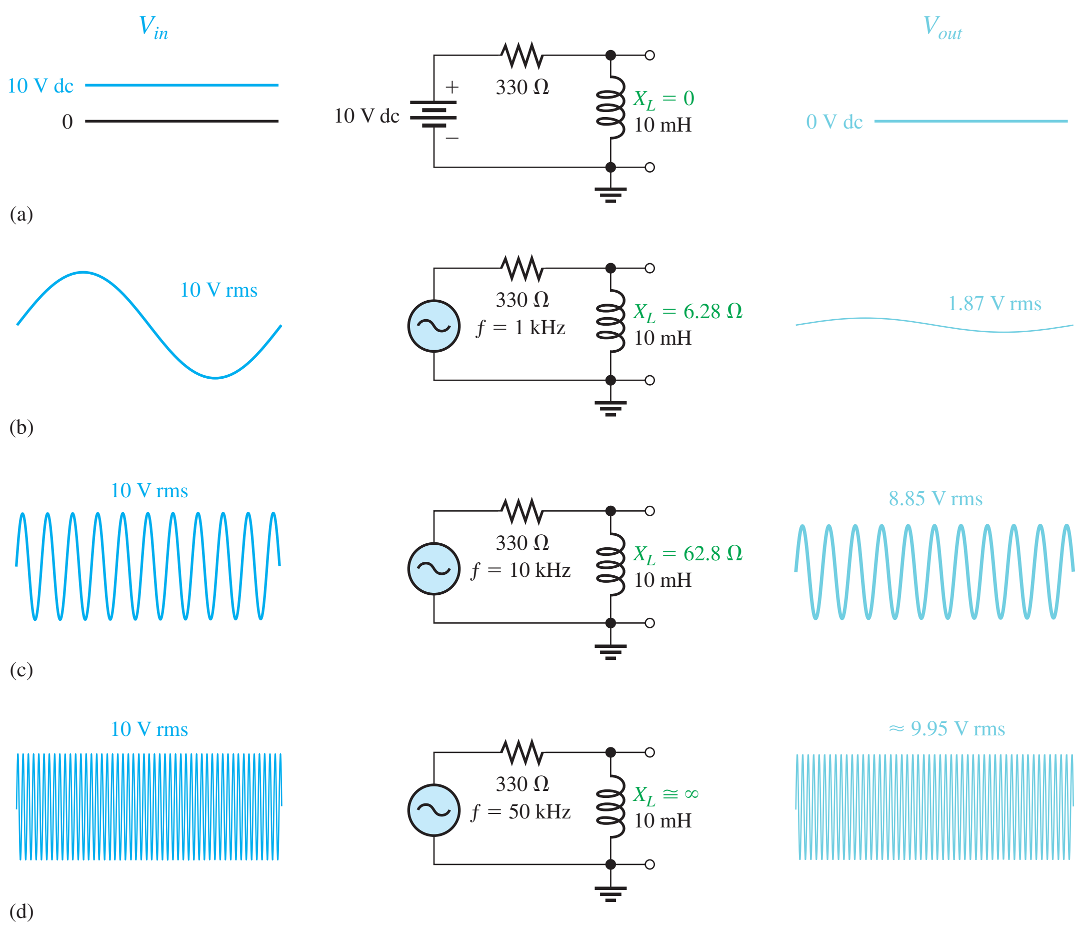
As shown in Figure 16–41, the circuit passes dc (zero frequency) completely. As the frequency of the input increases, less of the input voltage is passed through to the output. That is, the output voltage decreases as the frequency increases. This *RL* circuit is therefore a basic type of low-pass filter.

High-Pass Filter Figure 16–43 illustrates high-pass filtering action, where the output is taken across the inductor. When the input voltage is dc (zero frequency) in part (a), the output is zero volts because the inductor ideally appears as a short across the output.

In Figure 16–43(b), the frequency of the input signal has been increased to 1 kHz with an rms value of 10 V. The output voltage is 1.87 V rms. Thus, only a small percentage of the input voltage appears at the output at this frequency.

In Figure 16–43(c), the input frequency is increased further to 10 kHz, causing more voltage to be developed as a result of the increase in the inductive reactance. The output voltage at this frequency is 8.85 V rms. As you can see, the output voltage increases as the frequency increases. A value of frequency is reached at which the reactance is very large compared to the resistance and most of the input voltage appears across the inductor. This is the case at 50 kHz, where the output is 9.95 V as shown in Figure 16–43(d).

This circuit tends to prevent lower frequency signals from appearing on the output but permits higher frequency signals to pass through from input to output; thus, it is a basic type of high-pass filter.



▲ **FIGURE 16–43**

High-pass filtering action of an *RL* circuit (phase shift from input to output is not indicated).

A simple modification of the equation used for the low pass filter in Figure 16–42 enables you to plot the response for the circuit in Figure 16–43 on a graphing calculator. Grid lines are drawn as before in 10 kHz increments on the x-axis and in 1.0 V increments on the y-axis. Notice that output voltage increases and then levels off as it approaches the value of the input voltage as the frequency increases.

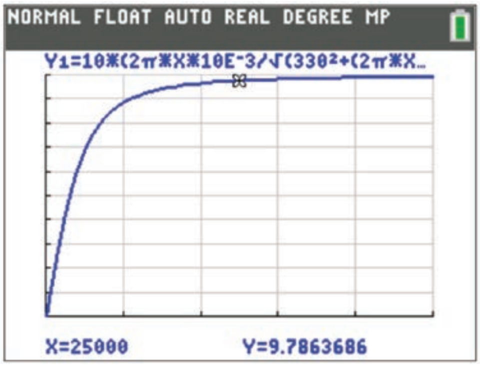


FIGURE 16–44

Response of the circuit in Figure 16–43 on a TI-84 Plus CE graphing calculator.

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The Switching Regulator

In high-frequency switching power supplies, small inductors are used as an essential part of the filter section. A switching power supply is much more efficient at converting ac to dc than any other type of supply. For this reason it is widely used in computers and other electronic systems. A switching regulator precisely controls the dc voltage. One type of switching regulator is shown in Figure 16–45. It uses an electronic switch to change unregulated dc to high-frequency pulses. The output is the average value of the pulses. The pulse width is controlled by the pulse width modulator, which rapidly turns on and off a transistor switch and then is filtered by the filter section to produce regulated dc. (Ripple in the figure is exaggerated to show the cycle.) The pulse width modulator can increase pulse width if the output drops, or decrease it if the output rises, thus maintaining a constant average output for varying conditions.

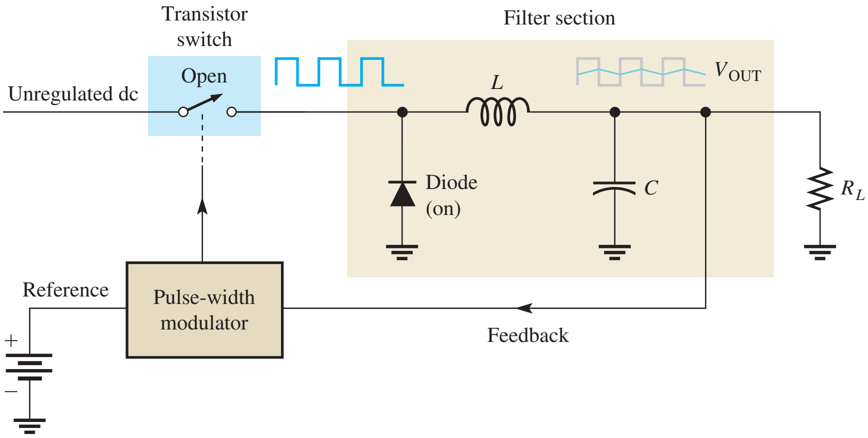
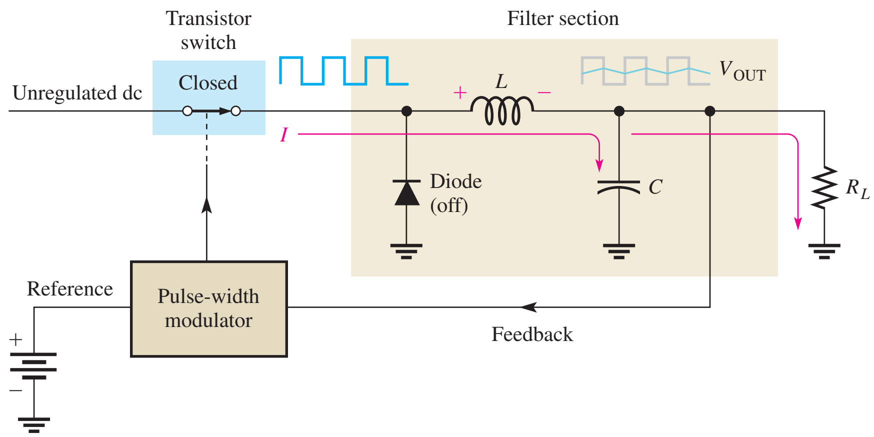


FIGURE 16–45

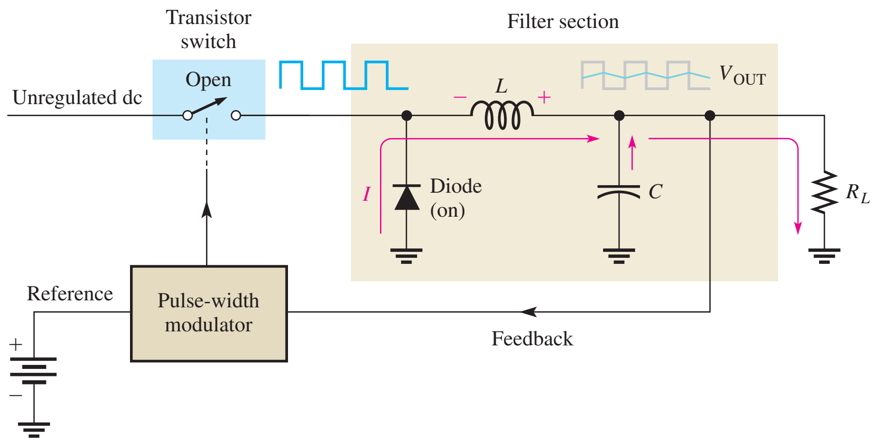
Diagram showing an inductor in a switching regulator.

Figure 16–46 illustrates the basic filtering action. The filter consists of a diode, an inductor, and a capacitor. The diode is a one-way device for current that you will study in a devices course. In this application, the diode acts as an on-off switch that allows current in only one direction.

An important component of the filter section is the inductor, which in this type of regulator will always have current in it. The average voltage and the load resistor determine the amount of current. Recall that Lenz's law states that an induced voltage is created across a coil that opposes a *change* in current. When the transistor switch is closed, the pulse is high and current is passed through the inductor and the load, as shown in Figure 16–46(a). The diode is off at this time. Notice that the inductor has an induced voltage across it that opposes a change in current. When the pulse goes low, as in Figure 16–46(b), the transistor is off and the inductor develops a voltage in the opposite direction than before. The diode acts as a closed switch, which provides a path for current. This action tends to keep the load current constant. The capacitor adds to this smoothing action by charging and discharging a small amount during the process.



(a) Pulse high



(b) Pulse low

▲ **FIGURE 16–46**

Switching regulator action.

SECTION 16–8 CHECKUP

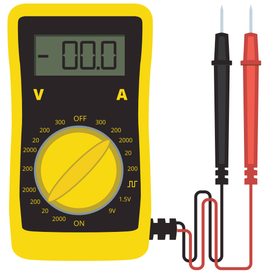
1. When an *RL* circuit is used as a low-pass filter, across which component is the output taken?
2. What is the major advantage of a switching regulator?
3. What happens to the pulse width of a switching regulator if the output voltage drops?

16–9 TROUBLESHOOTING

Typical component failures have an effect on the frequency response of basic *RL* circuits.

After completing this section, you should be able to

- **Troubleshoot *RL* circuits**
  - Find an open inductor
  - Find an open resistor
  - Find an open in a parallel circuit



Effect of an Open Inductor The most common failure mode for inductors occurs when the winding opens as a result of excessive current or a mechanical contact failure. It is easy to see how an open coil affects the operation of a basic series *RL* circuit, as shown in Figure 16–47. Obviously, there is no current path; therefore, the resistor voltage is zero, and the total applied voltage appears across the inductor. An open connection or a shorted resistor could also account for the source voltage appearing across the inductor. If you suspect an open coil, remove one or both leads from the circuit and check continuity with an ohmmeter.

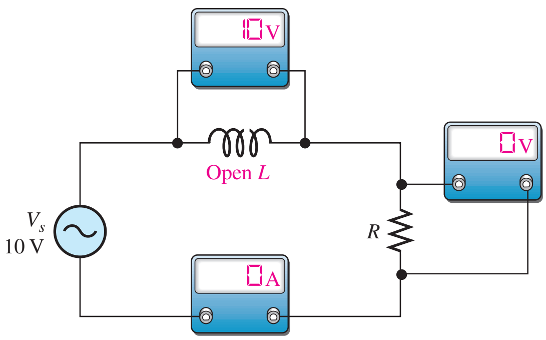


FIGURE 16–47

Effect of an open coil.

Effect of an Open Resistor When the resistor is open, there is no current and the inductor voltage is zero. The total input voltage is across the open resistor, as shown in Figure 16–48.

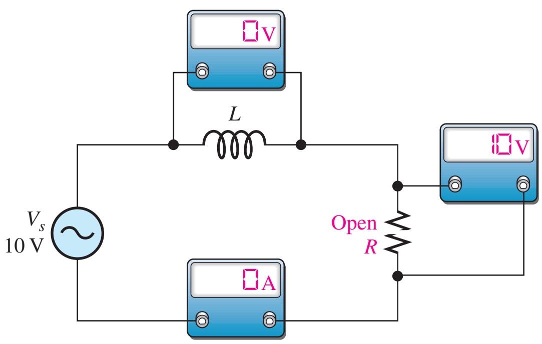
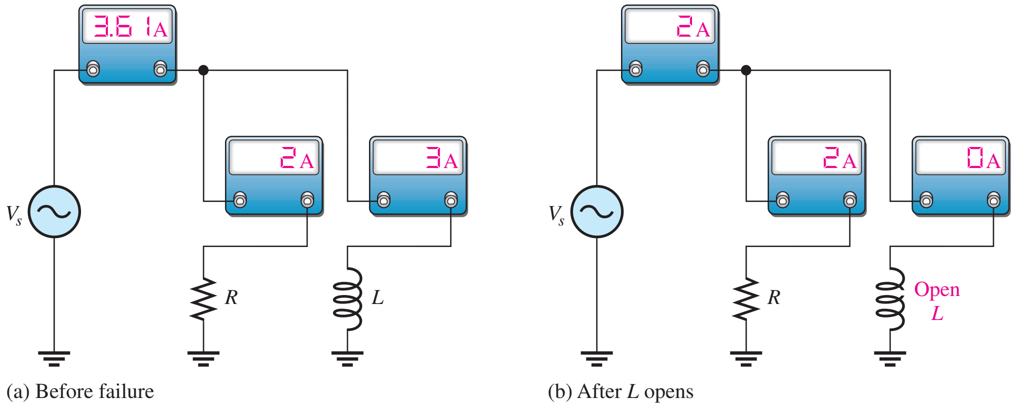


FIGURE 16–48

Effect of an open resistor.

Open Components in Parallel Circuits In a parallel *RL* circuit, an open resistor or inductor will cause the total current to decrease because the total impedance will increase. Obviously, the branch with the open component will have zero current. Figure 16–49 illustrates these conditions.



▲ **FIGURE 16–49**

Effect of an open component in a parallel circuit with V_s constant.

Effect of an Inductor with Shorted Windings Although a rare occurrence, it is possible for some of the windings of coils to short together as a result of damaged insulation. This failure mode is much less likely than the open coil and is difficult to detect. Shorted windings may result in a reduction in inductance because the inductance of a coil is proportional to the square of the number of turns. A short between windings effectively reduces the number of turns, which may or may not have an adverse effect on the circuit depending on the number of turns that are shorted.

Other Troubleshooting Considerations

The failure of a circuit to work properly is not always the result of a faulty component. A loose wire, a bad contact, or a poor solder joint can cause an open circuit. A short can be caused by a wire clipping or solder splash. Things that can go wrong include not plugging in a power supply or a function generator or setting up a measuring instrument incorrectly such as setting a DMM to dc when ac is wanted. Wrong values in a circuit (such as an incorrect resistor value), the function generator set at the wrong frequency, or the wrong output connected to the circuit can also cause improper operation.

An *LCR* meter is a quick way to check an inductor for an incorrect value. Figure 16–50 shows an example of an auto-ranging source-measurement type of meter. The meter generates an ac signal that is across the component under test and measures the voltage and current. The impedance is calculated indirectly from the voltage and current measurements and can be shown on the display or converted to an inductance value for the display. To measure an inductor, the user must isolate the component from the circuit and select a test frequency; the test frequency should be as close as possible to the normal operating frequency. The user also selects an equivalent series or parallel circuit for the inductor's resistance. From this, the *LCR* meter can determine and show the *Q* for the inductor at the test frequency as well as other information.

Always check to make sure that the instruments are properly connected to the circuits and to a power outlet if required. Also, look for obvious things such as a broken or loose contact, a connector that is not completely plugged in, or a piece of wire or a solder bridge that could be shorting something out.

A consideration in high-speed circuits is the circuit traces can look like transmission lines, where inductance and resistance are distributed continuously along the line rather than acting as discrete components. The circuit in the following example represents a simulation of a very long transmission line using a series of discrete elements to represent the line. The repeated elements model the distributed reactance of the line

▲ **FIGURE 16–50**

A handheld *LCR* meter.

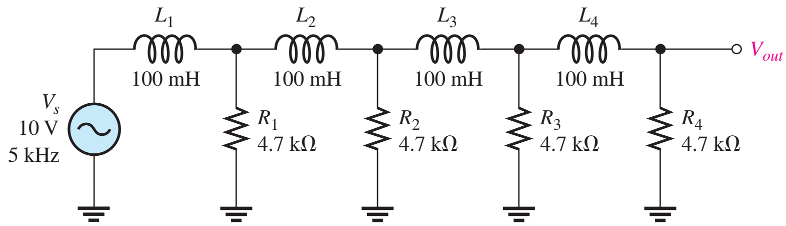
(Courtesy of B+K Precision).

and allow observation of the propagation of a signal along the line. The simulation was constructed on a protoboard. The fact that it is constructed on a protoboard is a consideration in troubleshooting it. The troubleshooting approach to the is to use APM (analysis, planning, and measurement) method and half-splitting.

EXAMPLE 16–17

The circuit represented by the schematic in Figure 16–51 has no output voltage. The circuit is physically constructed on a protoboard. Use your trouble-shooting skills to find the problem.

▶ FIGURE 16–51



Solution Apply the APM method to this troubleshooting problem.

Analysis: First think of the possible causes for the circuit to have no output voltage.

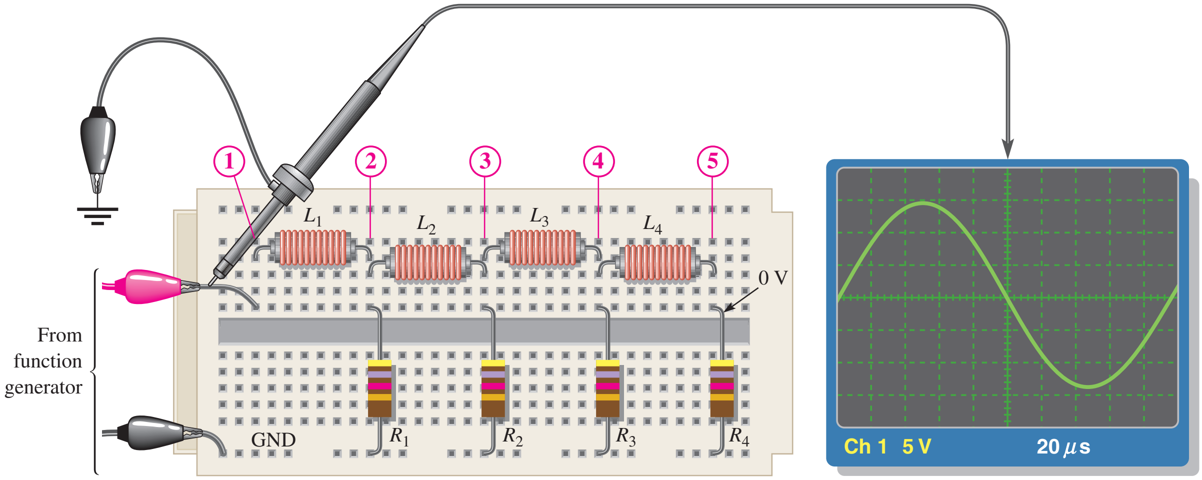
1. There is no source voltage or the frequency is so high that the inductors appear to be open because their reactances are extremely high compared to the resistance values.
2. There is a short between one of the resistors and ground. Either a resistor could be shorted, or there could be some physical short. A shorted resistor is not a common fault.
3. There is an open between the source and the output. This would prevent current and thus cause the output voltage to be zero. An inductor could be open, or the conductive path could be open due to a broken or loose connecting wire or a bad protoboard contact.
4. There is an incorrect component value. A resistor could be so small that the voltage across it is negligible. An inductor could be so large that its reactance at the input frequency is extremely high.
5. If the circuit is newly constructed and has never worked, it could be mis-wired. A careful visual check may show the problem.

Planning: You decide to make some visual checks for problems such as the function generator power cord not plugged in or the frequency set at an incorrect value. Also, broken leads, shorted leads, as well as an incorrect resistor color code or inductor value often can be found visually. If nothing is discovered after a visual check, then you will make voltage measurements to track down the cause of the problem. You decide to use a digital oscilloscope and a DMM to make the measurements using the half-splitting technique to more quickly isolate the fault.

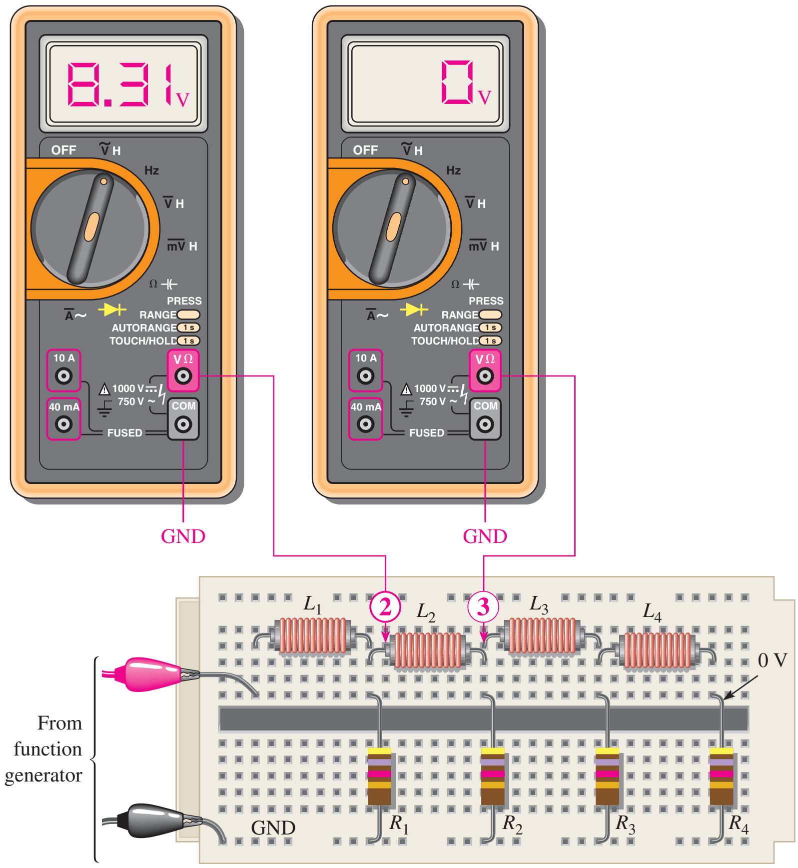
Measurement: Assume that you find the function generator is plugged in and the frequency setting appears to be correct. Also, you find no visible opens or shorts during your visual check, and the component values are correct.

The first step in the measurement process is to check the voltage from the source with the scope. Assume a 10 V rms sine wave with a frequency of 5 kHz is observed at the circuit input as shown in Figure 16–52(a). The correct ac voltage is present, so *the first possible cause has been eliminated.*

Next, check for a short by disconnecting the source and placing the DMM (set on the ohmmeter function) across each resistor. If any resistor is shorted



(a) Scope shows the correct voltage at the input.



(b) A zero voltage at point 3 indicates the fault is between point 3 and the source. A reading of 8.31 V at point 2 shows that L_2 is open.

▲ **FIGURE 16–52**

(unlikely), the meter will read zero or a very small resistance. Assuming the meter readings are okay, *the second possible cause has been eliminated.*

Since the voltage has been "lost" somewhere between the input and the output, you must now look for the voltage. You reconnect the source and, using the half-splitting approach, measure the voltage at point 3 (the middle of the circuit) with respect to ground. The DMM test lead is placed on the right lead of inductor L_2, as indicated in Figure 16–52(b). Assume the voltage at this point is zero. This tells you that the part of the circuit to the right of point ③ is probably okay and the fault is in the circuit between point ③ and the source.

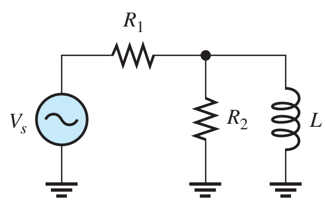
Now, you begin tracing the circuit back toward the source looking for the voltage (you could also start from the source and work forward). Placing the meter test lead on point ②, at the left lead of inductor L_2, results in a reading of 8.31 V as shown in Figure 16–52(b). This, of course, indicates that L_2 is open. Fortunately, in this case, a component, and not a contact on the board, is faulty. It is usually easier to replace a component than to repair a bad contact.

Related Problem Suppose you had measured 0 V at the left lead of L_2 and 10 V at the right lead of L_1. What would this have indicated?

SECTION 16–9 CHECKUP

1. **Describe the effect of an inductor with shorted windings on the response of a series *RL* circuit.**
2. **In the circuit of Figure 16–53, indicate whether I_{tot}, V_{R1}, and V_{R2} increase or decrease as a result of *L* opening.**

▶ FIGURE 16–53



Application Activity

Two sealed modules have been removed from a communications system that is being modified. Each module has three terminals and is labeled as an *RL* filter, but no specifications are given. You are asked to test the modules to determine the type of filters and the component values.

The sealed modules have three terminals labeled IN, GND, and OUT as shown in Figure 16–54. You will apply your knowledge of series *RL* circuits and some basic measurements to determine the internal circuit configuration and the component values.

Resistance Measurements of Module 1

1. Determine the arrangement of the two components and the values of the resistor and winding resistance for module 1 indicated by the meter readings in Figure 16–54.

AC Measurements of Module 1

2. Determine the inductance value for module 1 indicated by the test setup in Figure 16–55.

Resistance Measurements of Module 2

3. Determine the arrangement of the two components and the values of the resistor and the winding resistance for module 2 indicated by the meter readings in Figure 16–56.

AC Measurements of Module 2

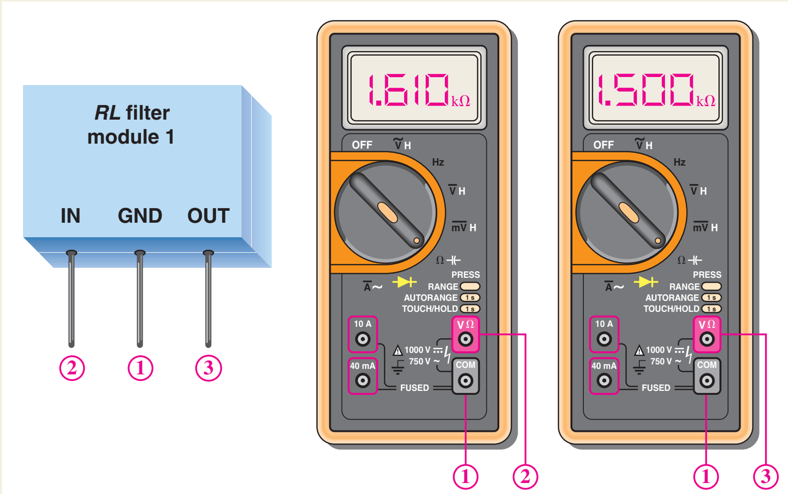
4. Determine the inductance value for module 2 indicated by the test setup in Figure 16–57.

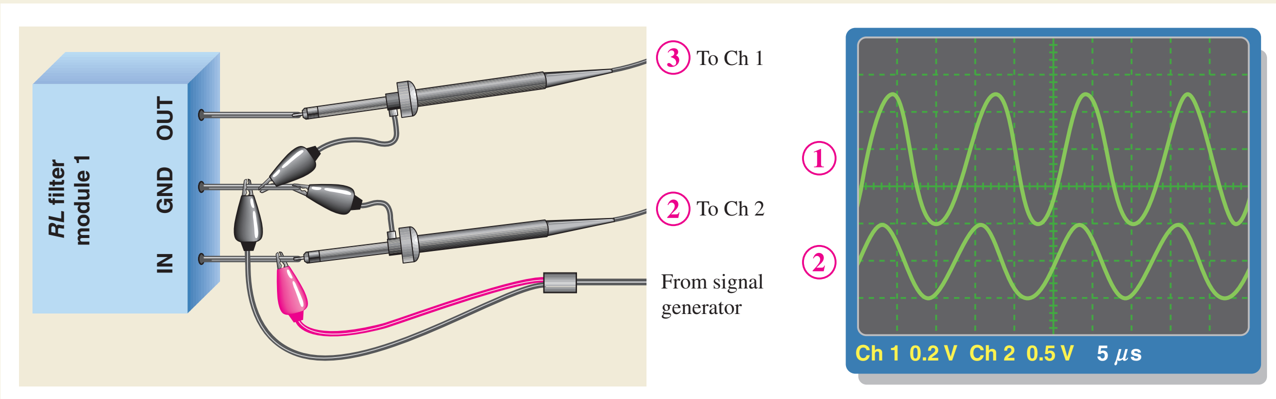
Review

5. If the inductor in module 1 were open, what would you measure on the output with the test setup of Figure 16–55?
6. If the inductor in module 2 were open, what would you measure on the output with the test setup of Figure 16–57?

▶ FIGURE 16–54

Resistance measurements of module 1.



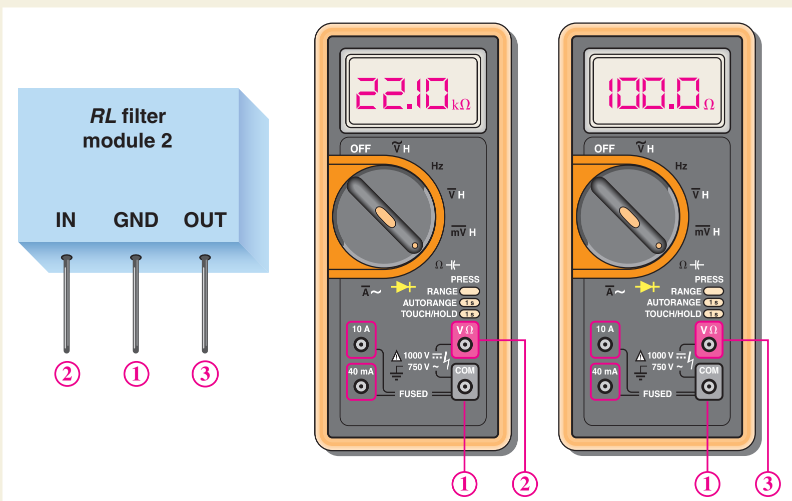


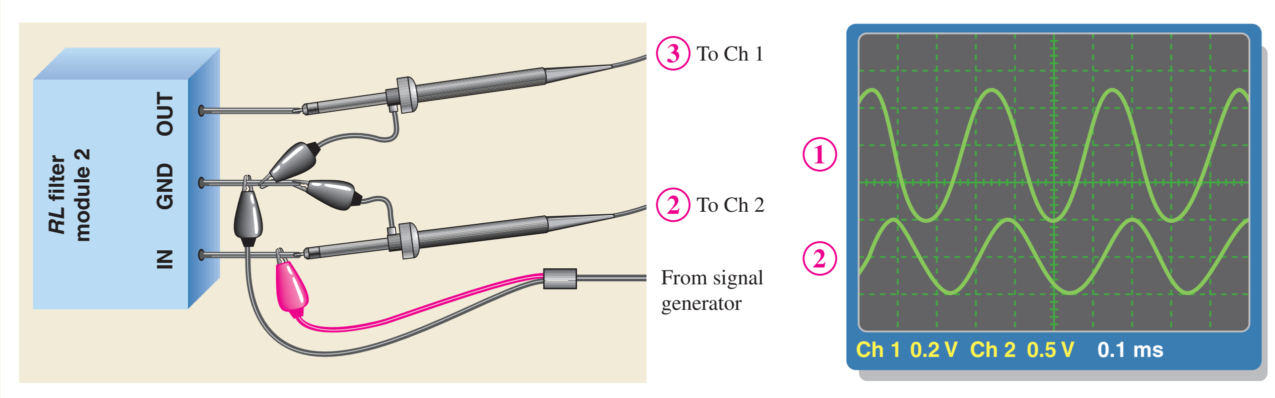
▲ FIGURE 16–55

AC measurements for module 1.

▶ FIGURE 16–56

Resistance measurements of module 2.





▲ FIGURE 16–57

AC measurements for module 2.

Option 2 Note

Coverage of special topics continues in Chapter 17, Part 4, on page 811.

SUMMARY

- When a sinusoidal voltage is applied to an *RL* circuit, the current and all the voltage drops are also sine waves.
- Total current in a series or parallel *RL* circuit always lags the source voltage.
- The resistor voltage is always in phase with the current.
- In an ideal inductor, the voltage always leads the current by 90°.
- In an *RL* circuit, the impedance is determined by the combination of the resistance and the inductive reactance.
- Impedance is expressed in units of ohms.
- The impedance of an *RL* circuit varies directly with frequency.
- The phase angle (θ) of a series *RL* circuit varies directly with frequency.
- You can determine the impedance of a circuit by measuring the applied voltage and the total current and then applying Ohm's law.
- In an *RL* circuit, part of the power is resistive and part reactive.
- The power factor indicates how much of the apparent power is true power.
- A power factor of 1 indicates a purely resistive circuit, and a power factor of 0 indicates a purely reactive circuit.
- A filter passes certain frequencies and rejects others.

KEY TERMS

Key terms and other bold terms in the chapter are defined in the end-of-book glossary.

Inductive susceptance (B_L) The ability of an inductor to permit current; the reciprocal of inductive reactance. The unit is the siemens (S).

***RL* lag circuit** A phase shift circuit in which the output voltage, taken across the resistor, lags the input voltage by a specified angle.

***RL* lead circuit** A phase shift circuit in which the output voltage, taken across the inductor, leads the input voltage by a specified angle.

FORMULAS

Series *RL* Circuits

16–1 $\mathbf{Z} = R + jX_L$

16–2 $\mathbf{Z} = \sqrt{R^2 + X_L^2}\angle\tan^{-1}\left(\frac{X_L}{R}\right)$

16–3 $\mathbf{V}_s = V_R + jV_L$

16–4 $\mathbf{V}_s = \sqrt{V_R^2 + V_L^2}\angle\tan^{-1}\left(\frac{V_L}{V_R}\right)$

Lead Circuit

16–5 $\phi = \tan^{-1}\left(\frac{R}{X_L}\right)$

16–6 $\mathbf{V}_{out} = \left(\frac{X_L}{\sqrt{R^2 + X_L^2}}\right)V_{in}$

Lag Circuit

16–7 $\phi = -\tan^{-1}\left(\frac{X_L}{R}\right)$

16–8 $\mathbf{V}_{out} = \left(\frac{R}{\sqrt{R^2 + X_L^2}}\right)V_{in}$

Parallel *RL* Circuits

16–9 $\mathbf{Z} = \left(\frac{RX_L}{\sqrt{R^2 + X_L^2}}\right)\angle\tan^{-1}\left(\frac{R}{X_L}\right)$

16–10 $\mathbf{Y} = G - jB_L$

16–11 $\mathbf{I}_{tot} = I_R - jI_L$

16–12 $\mathbf{I}_{tot} = \sqrt{I_R^2 + I_L^2}\angle-\tan^{-1}\left(\frac{I_L}{I_R}\right)$

16–13 $R_{\text{eq}} = Z\cos\theta$

16–14 $X_{L(\text{eq})} = Z\sin\theta$

Power in *RL* Circuits

16–15 $P_r = I^2X_L$

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. If a sine wave voltage is applied to a series *RL* circuit, the current is also a sine wave.
2. The total impedance of a series *RL* circuit is the algebraic sum of the magnitudes of the resistance and the inductive reactance.
3. Ohm's law cannot be applied to reactive circuits.
4. The impedance of an *RL* circuit can be expressed as a phasor quantity using complex numbers.
5. The total impedance of a series *RL* circuit can be expressed as $R + jX_L$.

6. Voltage leads the current in terms of phase in a series *RL* circuit.
7. The impedance in any *RL* circuit increases with frequency.
8. Multiplying the impedance phasors in a series *RL* circuit by the current will give the power phasors.
9. In a parallel *RL* circuit, the total impedance is the phasor sum of the conductance and the susceptance.
10. In a series *RL* lag circuit, the output is taken across the resistor.
11. Susceptance is the reciprocal of impedance.
12. Power factor is determined by the magnitudes of the voltage and current.

SELF-TEST

Answers are at the end of the chapter.

1. In a series *RL* circuit, the resistor voltage
   (a) leads the applied voltage **(b)** lags the applied voltage
   (c) is in phase with the applied voltage **(d)** is in phase with the current
   (e) answers (a) and (d) **(f)** answers (b) and (d)
2. When the frequency of the voltage applied to a series *RL* circuit is increased, the impedance
   (a) decreases **(b)** increases **(c)** does not change
3. When the voltage applied to a series *RL* circuit is decreased, the phase angle
   (a) decreases **(b)** increases **(c)** does not change
4. If the frequency is doubled and the resistance is halved, the impedance of a series *RL* circuit
   (a) doubles **(b)** halves
   (c) remains constant **(d)** cannot be determined without values
5. To reduce the current in a series *RL* circuit, the frequency should be
   (a) increased **(b)** decreased **(c)** constant
6. In a series *RL* circuit, 10 V rms is measured across the resistor, and 10 V rms is measured across the inductor. The peak value of the source voltage is
   (a) 14.14 V **(b)** 28.28 V **(c)** 10 V **(d)** 20 V
7. The voltages in Problem 6 are measured at a certain frequency. To make the resistor voltage greater than the inductor voltage, the frequency is
   (a) increased **(b)** decreased **(c)** doubled **(d)** not a factor
8. When the resistor voltage in a series *RL* circuit becomes greater than the inductor voltage, the phase angle
   (a) increases **(b)** decreases **(c)** is not affected
9. When the frequency of the source voltage is increased, the impedance of a parallel *RL* circuit
   (a) increases **(b)** decreases **(c)** remains constant
10. In a parallel *RL* circuit, there are 2 mA rms in the resistive branch and 2 mA rms in the inductive branch. The total rms current is
    (a) 4 mA **(b)** 5.66 mA **(c)** 2 mA **(d)** 2.83 mA
11. You are observing two voltage waveforms on an oscilloscope. The time base (time/division) of the scope is adjusted so that one-half cycle of the waveforms covers the ten horizontal divisions. The positive-going zero crossing of one waveform is at the leftmost division, and the positive-going zero crossing of the other is three divisions to the right. The phase angle between these two waveforms is
    (a) 18° **(b)** 36° **(c)** 54° **(d)** 180°
12. Which of the following power factors results in less energy being converted to heat in an *RL* circuit?
    (a) 1 **(b)** 0.9 **(c)** 0.5 **(d)** 0.1

13. If a load is purely inductive and the reactive power is 10 VAR, the apparent power is
    (a) 0 VA **(b)** 10 VA **(c)** 14.14 VA **(d)** 3.16 VA
14. For a certain load, the true power is 10 W and the reactive power is 10 VAR. The apparent power is
    (a) 5 VA **(b)** 20 VA **(c)** 14.14 VA **(d)** 100 VA

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 16–60.

1. If L opens, the voltage across it
   (a) increases **(b)** decreases **(c)** stays the same
2. If R opens, the voltage across L
   (a) increases **(b)** decreases **(c)** stays the same
3. If the frequency is increased, the voltage across R
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 16–67.

4. If L opens, the voltage across R
   (a) increases **(b)** decreases **(c)** stays the same
5. If f is increased, the current through R
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 16–73.

6. If R_1 becomes open, the current through L_1
   (a) increases **(b)** decreases **(c)** stays the same
7. If L_2 opens, the voltage across R_2
   (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 16–74.

8. If L_2 opens, the voltage from point B to ground
   (a) increases **(b)** decreases **(c)** stays the same
9. If L_1 opens, the voltage from point B to ground
   (a) increases **(b)** decreases **(c)** stays the same
10. If the frequency of the source voltage is increased, the current through R_1
    (a) increases **(b)** decreases **(c)** stays the same
11. If the frequency of the source voltage is decreased, the voltage from point A to ground
    (a) increases **(b)** decreases **(c)** stays the same

Refer to Figure 16–77.

12. If L_2 opens, the voltage across L_1
    (a) increases **(b)** decreases **(c)** stays the same
13. If R_1 opens, the output voltage
    (a) increases **(b)** decreases **(c)** stays the same
14. If R_3 becomes open, the output voltage
    (a) increases **(b)** decreases **(c)** stays the same
15. If a partial short develops in L_1, the source current
    (a) increases **(b)** decreases **(c)** stays the same
16. If the source frequency increases, the output voltage
    (a) increases **(b)** decreases **(c)** stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the book.

PART 1: SERIES CIRCUITS

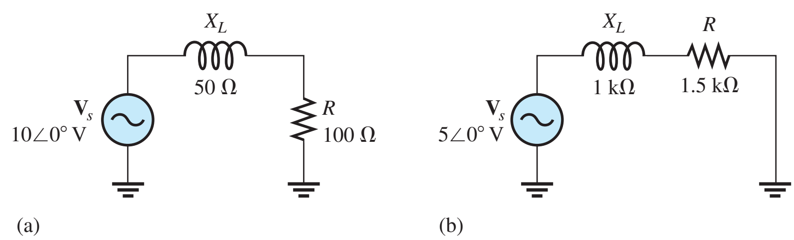
SECTION 16–1 Sinusoidal Response of Series *RL* Circuits

1. A 15 kHz sinusoidal voltage is applied to a series *RL* circuit. What is the frequency of I, V_R, and V_L?

2. What are the wave shapes of I, V_R, and V_L in Problem 1?

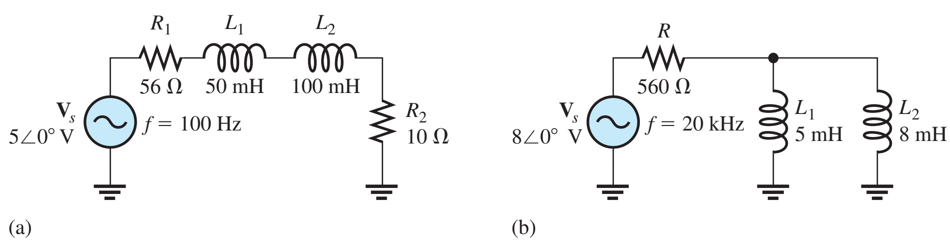
SECTION 16–2 Impedance of Series *RL* Circuits

3. Express the total impedance of each circuit in Figure 16–58 in both polar and rectangular forms.



▲ FIGURE 16–58

4. Determine the impedance magnitude and phase angle in each circuit in Figure 16–59. Draw the impedance diagrams.

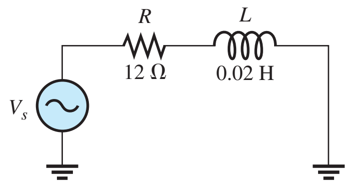


▲ FIGURE 16–59

5. In Figure 16–60, determine the impedance at each of the following frequencies:

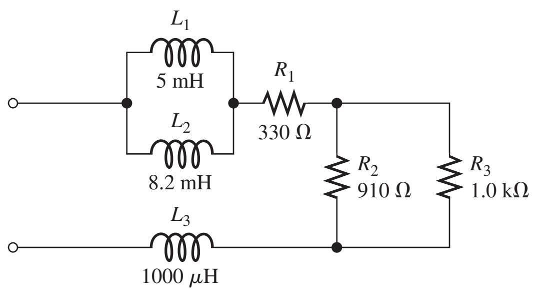
(a) 100 Hz **(b)** 500 Hz **(c)** 1 kHz **(d)** 2 kHz

▶ FIGURE 16–60



6. Determine the values of R and X_L in a series RL circuit for the following values of total impedance:
   (a) $\mathbf{Z} = 20\ \Omega + j45\ \Omega$ **(b)** $\mathbf{Z} = 500\angle 35°\ \Omega$
   (c) $\mathbf{Z} = 2.5\angle 72.5°\ \text{k}\Omega$ **(d)** $\mathbf{Z} = 998\angle 45°\ \Omega$
7. Reduce the circuit in Figure 16–61 to a single resistance and inductance in series.

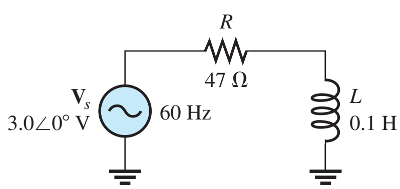
▶ FIGURE 16–61



SECTION 16–3 Analysis of Series *RL* Circuits

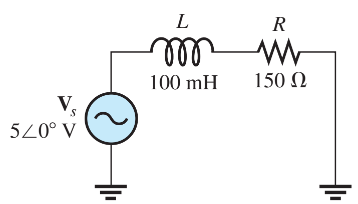
8. A 5 V, 10 kHz sinusoidal voltage is applied to the circuit in Figure 16–61. Calculate the voltage across the total resistance found in Problem 7.
9. For the same applied voltage in Problem 8, determine the voltage across L_3 for the circuit in Figure 16–61.
10. Express the current in polar form for each circuit of Figure 16–58.
11. Calculate the total current in each circuit of Figure 16–59 and express in polar form.
12. Determine θ for the circuit in Figure 16–62.

▶ FIGURE 16–62



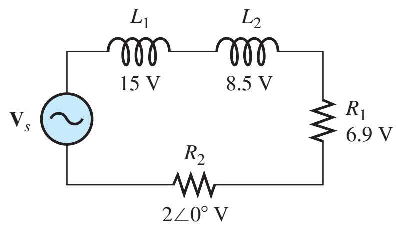
13. If the inductance in Figure 16–62 is doubled, does θ increase or decrease, and by how many degrees?
14. Draw the waveforms for $\mathbf{V}_s$, $\mathbf{V}_R$, and $\mathbf{V}_L$ in Figure 16–62. Show the proper phase relationships.
15. For the circuit in Figure 16–63, find $\mathbf{V}_R$ and $\mathbf{V}_L$ for each of the following frequencies:
   (a) 60 Hz **(b)** 200 Hz **(c)** 500 Hz **(d)** 1 kHz

▶ FIGURE 16–63



16. Determine the magnitude and phase angle of the source voltage in Figure 16–64.

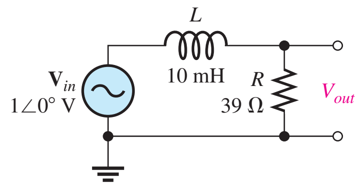
▶ FIGURE 16–64



17. For the lag circuit in Figure 16–65, determine the phase lag of the output voltage with respect to the input for the following frequencies:

(a) 1 Hz **(b)** 100 Hz **(c)** 1 kHz **(d)** 10 kHz

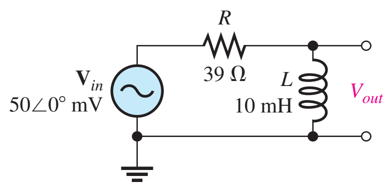
▶ FIGURE 16–65



18. Repeat Problem 17 for the lead circuit to find the phase lead in Figure 16–66.

19. Express $\mathbf{V}_{out}$ in Figure 16–66 in polar form for each frequency in Problem 17.

▶ FIGURE 16–66



PART 2: PARALLEL CIRCUITS

SECTION 16–4 Impedance and Admittance of Parallel *RL* Circuits

20. What is the impedance in polar form for the circuit in Figure 16–67?

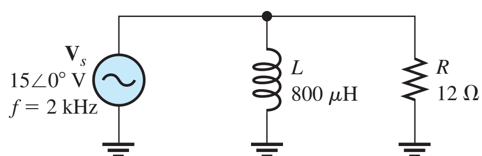
21. What is the impedance in rectangular form for the circuit in Figure 16–67?

22. Repeat Problem 20 for the following frequencies:

(a) 1.5 kHz **(b)** 3 kHz **(c)** 5 kHz **(d)** 10 kHz

23. At what frequency does X_L equal R in Figure 16–67?

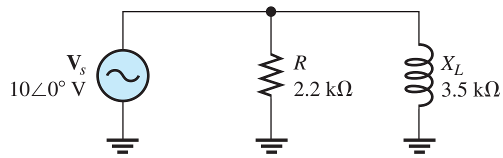
▶ FIGURE 16–67



SECTION 16–5 Analysis of Parallel *RL* Circuits

24. Find the total current and each branch current in Figure 16–68.

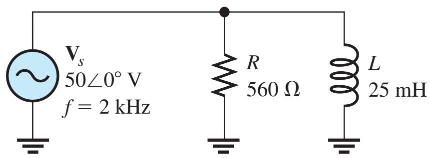
▶ FIGURE 16–68



25. Determine the following quantities in Figure 16–69:

(a) $\mathbf{Z}$ **(b)** $\mathbf{I}_R$ **(c)** $\mathbf{I}_L$ **(d)** $\mathbf{I}_{tot}$ **(e)** θ

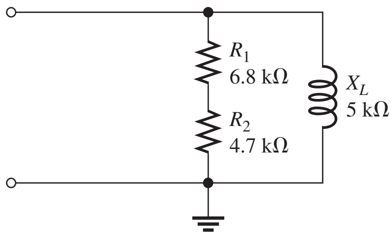
▶ FIGURE 16–69



26. Repeat Problem 25 for $R = 56\ \Omega$ and $L = 330\ \mu$H.

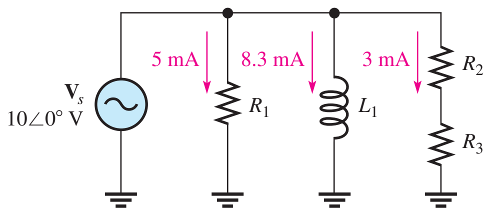
27. Convert the circuit in Figure 16–70 to an equivalent series form.

▶ FIGURE 16–70



28. Find the magnitude and phase angle of the total current in Figure 16–71.

▶ FIGURE 16–71

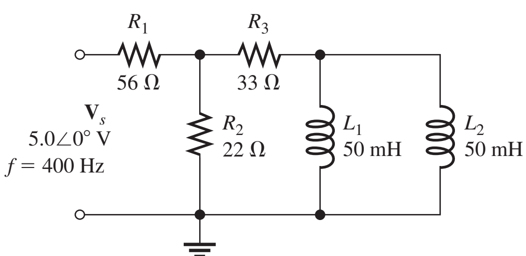


PART 3: SERIES-PARALLEL CIRCUITS

SECTION 16–6 Analysis of Series-Parallel *RL* Circuits

29. Determine the voltages in polar form across each element in Figure 16–72. Draw the voltage phasor diagram.

▶ FIGURE 16–72



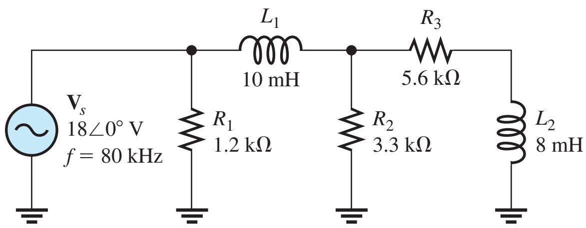
30. Is the circuit in Figure 16–72 predominantly resistive or predominantly inductive?

31. Find the current in each branch and the total current in Figure 16–72. Express the currents in polar form. Draw the current phasor diagram.

32. For the circuit in Figure 16–73, determine the following:

(a) $\mathbf{I}_{tot}$ **(b)** θ **(c)** $\mathbf{V}_{R1}$ **(d)** $\mathbf{V}_{R2}$ **(e)** $\mathbf{V}_{R3}$ **(f)** $\mathbf{V}_{L1}$ **(g)** $\mathbf{V}_{L2}$

▶ FIGURE 16–73

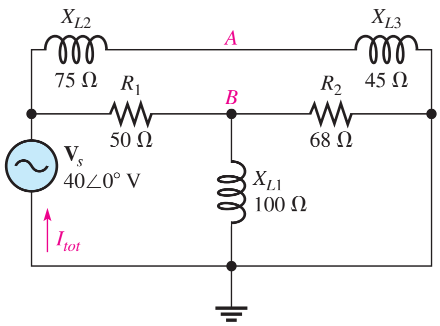


***33.** For the circuit in Figure 16–74, determine the following:

(a) $\mathbf{I}_{tot}$ **(b)** $\mathbf{V}_{L1}$ **(c)** $\mathbf{V}_{AB}$

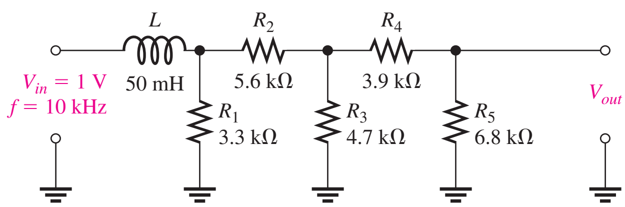
***34.** Draw the phasor diagram of all voltages and currents in Figure 16–74.

▶ FIGURE 16–74

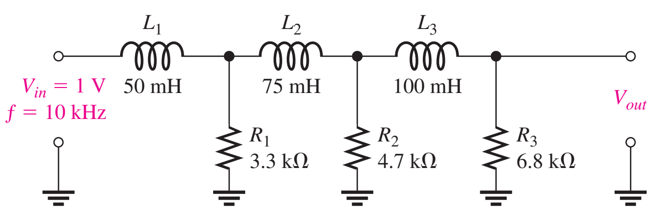


35. Determine the phase shift and attenuation (ratio of V_{out} to V_{in}) from the input to the output for the circuit in Figure 16–75.

▶ FIGURE 16–75



***36.** Determine the phase shift and attenuation from the input to the output for the ladder network in Figure 16–76.



▲ FIGURE 16–76

*37. Design an ideal inductive switching circuit that will provide a momentary voltage of 2.5 kV from a 12 V dc source when a switch is thrown instantaneously from one position to another. The drain on the source must not exceed 1 A.

PART 4: SPECIAL TOPICS

SECTION 16–7 Power in *RL* Circuits

38. In a certain *RL* circuit, the true power is 100 mW, and the reactive power is 340 mVAR. What is the apparent power?

39. Determine the true power and the reactive power in Figure 16–62.

40. What is the power factor in Figure 16–68?

41. Determine P_{true}, P_r, P_a, and *PF* for the circuit in Figure 16–73. Sketch the power triangle.

*42. Find the true power for the circuit in Figure 16–74.

SECTION 16–8 Basic Applications

43. Draw the response curve for the circuit in Figure 16–65. Show the output voltage versus frequency in 1 kHz increments from 0 Hz to 5 kHz.

44. Using the same procedure as in Problem 41, draw the response curve for Figure 16–66.

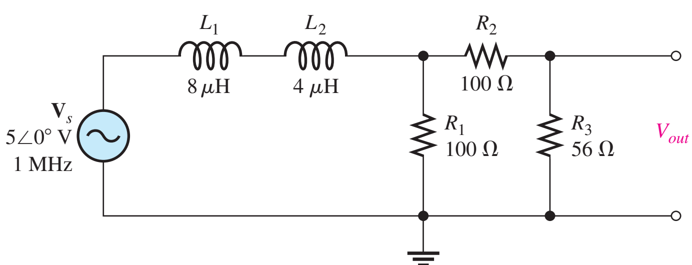
45. Draw the voltage phasor diagram for each circuit in Figures 16–65 and 16–66 for a frequency of 8 kHz.

SECTION 16–9 Troubleshooting

46. Determine the voltage across each component in Figure 16–77 if L_1 is open.

47. Determine the output voltage in Figure 16–77 for each of the following failure modes:

(a) L_1 open **(b)** L_2 open **(c)** R_1 open **(d)** a short across R_2



▲ FIGURE 16–77

Multisim Troubleshooting and Analysis

These problems require Multisim.

48. Open file P16-48 and determine if there is a fault. If so, find the fault.

49. Open file P16-49 and determine if there is a fault. If so, find the fault.

50. Open file P16-50 and determine if there is a fault. If so, find the fault.

51. Open file P16-51 and determine if there is a fault. If so, find the fault.

52. Open file P16-52 and determine if there is a fault. If so, find the fault.

53. Open file P16-53 and determine if there is a fault. If so, find the fault.

54. Open file P16-54 and determine the frequency response for the filter.

55. Open file P16-55 and determine the frequency response for the filter.

ANSWERS

SECTION CHECKUPS

SECTION 16–1 Sinusoidal Response of Series *RL* Circuits

1. The current frequency is 1 kHz.
2. The phase angle is closer to 0°.

SECTION 16–2 Impedance of Series *RL* Circuits

1. $R = 150\ \Omega$; $X_L = 220\ \Omega$
2. 55.7°
3. $\mathbf{Z} = R + jX_L = 33\ \text{k}\Omega + j50\ \text{k}\Omega$; $\mathbf{Z} = \sqrt{R^2 + X_L^2}\angle\tan^{-1}(X_L/R) = 59.9\angle 56.6°\ \text{k}\Omega$

SECTION 16–3 Analysis of Series *RL* Circuits

1. $V_s = \sqrt{V_R^2 + V_L^2} = 3.61\ \text{V}$
2. $\theta = \tan^{-1}(V_L/V_R) = 56.3°$
3. When f increases, X_L increases, Z increases, and θ increases.
4. $\phi = 81.9°$
5. $V_{out} = 9.90\ \text{V}$

SECTION 16–4 Impedance and Admittance of Parallel *RL* Circuits

1. $Y = \dfrac{1}{Z} = 2\ \text{mS}$
2. $Y = \sqrt{G^2 + B_L^2} = 2.51\ \text{mS}$
3. $\mathbf{I}$ lags V_s; $\theta = 32.1°$

SECTION 16–5 Analysis of Parallel *RL* Circuits

1. $I_{tot} = 32\ \text{mA}$
2. $\mathbf{I}_{tot} = 23.3\angle -59.0°\ \text{mA}$; θ is with respect to the input voltage.
3. $\theta = -90°$
4. Divide each of the current phasors by the voltage.

SECTION 16–6 Analysis of Series-Parallel *RL* Circuits

1. $\mathbf{Z} = 494\angle 59.0°\ \Omega$
2. $I_{tot} = 10.4\ \text{mA} - j17.4\ \text{mA}$

SECTION 16–7 Power in *RL* Circuits

1. Power dissipation is due to resistance.
2. $PF = 0.643$
3. $PF = 0.833$
4. $P_{\text{true}} = 4.7\ \text{W}$; $P_r = 6.2\ \text{VAR}$; $P_a = 7.78\ \text{VA}$

SECTION 16–8 Basic Applications

1. The output is across the resistor.
2. It is more efficient than other types.
3. It is adjusted by the pulse width modulator to be longer.

SECTION 16–9 Troubleshooting

1. Shorted windings reduce L and thereby reduce X_L at any given frequency.
2. I_{tot} decreases, V_{R1} decreases, V_{R2} increases.

RELATED PROBLEMS FOR EXAMPLES

16–1 $\mathbf{Z} = 1.8\text{ k}\Omega + j950\ \Omega$; $\mathbf{Z} = 2.04\angle 27.8°\text{ k}\Omega$

16–2 $\mathbf{I} = 423\angle -32.1°\ \mu\text{A}$

16–3 $Z = 12.6\text{ k}\Omega$; $\theta = 85.5°$

16–4 $\phi = 65.6°$

16–5 V_{out} increases.

16–6 $\phi = -32.0°$

16–7 $V_{out} = 12.3$ V rms

16–8 $\mathbf{Z} = 8.14\angle 35.5°\text{ k}\Omega$

16–9 $\mathbf{Y} = 3.03\text{ mS} - j0.796\text{ mS}$

16–10 Power phasors

16–11 $\mathbf{I} = 14.0\angle -71.1°\text{ mA}$

16–12 $I_{tot} = 67.6\text{ mA}$; $\theta = 36.3°$

16–13 The equivalent resistance increases.

16–14 **(a)** $\mathbf{V}_1 = 8.04\angle 2.52°\text{ V}$ **(b)** $\mathbf{V}_2 = 2.00\angle -10.2°\text{ V}$

16–15 $\mathbf{I}_{tot} = 20.2\angle -59.0°\text{ mA}$

16–16 P_{true}, P_r, and P_a decrease.

16–17 Open connection between L_1 and L_2

TRUE/FALSE QUIZ

1. T **2.** F **3.** F **4.** T **5.** T **6.** T **7.** T **8.** F **9.** F **10.** T **11.** T **12.** F

SELF-TEST

1. (f) **2.** (b) **3.** (c) **4.** (d) **5.** (a) **6.** (d) **7.** (b)
8. (b) **9.** (a) **10.** (d) **11.** (c) **12.** (d) **13.** (b) **14.** (c)

CIRCUIT DYNAMICS QUIZ

1. (a) **2.** (b) **3.** (b) **4.** (c) **5.** (c) **6.** (c) **7.** (a) **8.** (c)
9. (a) **10.** (b) **11.** (c) **12.** (b) **13.** (a) **14.** (a) **15.** (a) **16.** (b)