TIME RESPONSE OF REACTIVE CIRCUITS

20

CHAPTER OUTLINE

CHAPTER OBJECTIVES

- Explain the operation of an *RC* integrator
- Analyze an *RC* integrator with a single input pulse
- Analyze an *RC* integrator with repetitive input pulses
- Analyze an *RC* differentiator with a single input pulse
- Analyze an *RC* differentiator with repetitive input pulses
- Analyze the operation of an *RL* integrator
- Analyze the operation of an *RL* differentiator
- Explain the relationship of time response to frequency response
- Troubleshoot *RC* integrators and *RC* differentiators

KEY TERMS

- Integrator
- Transient time
- Steady-state
- Differentiator
- DC component

APPLICATION ACTIVITY PREVIEW

In the application activity, you will have to complete the wiring for a time-delay circuit. You will also determine component values to meet certain specifications and then determine instrument settings to properly test the circuit.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/careersresources/

INTRODUCTION

In Chapters 15 and 16, the frequency response of *RC* and *RL* circuits was covered. In this chapter, the time response of *RC* and *RL* circuits with pulse inputs is examined. Before starting this chapter, you should review the material in Sections 12–5 and 13–4. Understanding exponential changes of voltages and currents in capacitors and inductors is crucial to the study of time response. Throughout this chapter, exponential formulas that were given in Chapters 12 and 13 are used.

With pulse inputs, the time responses of circuits are important. In the areas of pulse and digital circuits, technicians are often concerned with how a circuit responds over an interval of time to rapid changes in voltages or current. The relationship of the circuit time constant to the input pulse characteristics, such as pulse width and period, determines the wave shapes of voltages in the circuit.

Integrator and *differentiator*, terms used throughout this chapter, refer to mathematical functions that are approximated by these circuits under certain conditions. Mathematical integration is a summation process, and mathematical differentiation is a process for establishing an instantaneous rate of change of a quantity.

20–1 The *RC* Integrator

In terms of time response, a series *RC* circuit in which the output voltage is taken across the capacitor is a type of circuit known as an **integrator**. Recall that in terms of frequency response, this circuit is a basic low-pass filter. The term *integrator* is derived from the mathematical process of integration, which this type of circuit approximates under certain conditions.

After completing this section, you should be able to

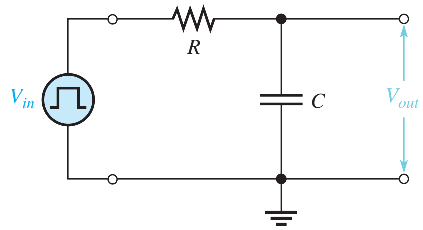
- **Explain the operation of an *RC* integrator**
  - Describe how a capacitor charges and discharges
  - Explain how a capacitor reacts to an instantaneous change in voltage or current
  - Describe the basic output voltage waveform

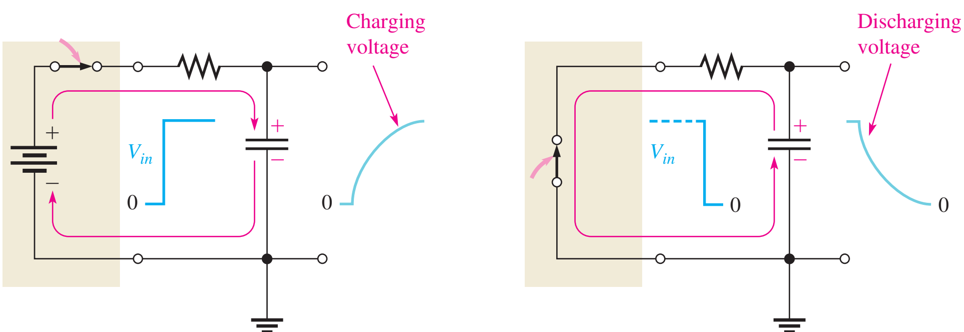
Charging and Discharging of a Capacitor

When an ideal pulse generator is connected to the input of an *RC* integrator, as shown in Figure 20–1, the capacitor will charge and discharge in response to the pulses. When the input goes from its low level to its high level, the capacitor charges toward the high level of the pulse through the resistor. This charging action is analogous to connecting a battery through a closed switch to the *RC* circuit, as illustrated in Figure 20–2(a). When the pulse goes from its high level back to its low level, the capacitor discharges back through the source. Compared to the resistance of the resistor, the resistance of the source is assumed to be negligible. This discharging action is analogous to replacing the source with a closed switch, as illustrated in Figure 20–2(b).

▶ FIGURE 20–1

An *RC* integrator with a pulse generator connected.





(a) When the input pulse goes HIGH, the source effectively acts as a battery in series with a closed switch, thereby charging the capacitor.

(b) When the input pulse goes back LOW, the source effectively acts as a closed switch, providing a discharge path for the capacitor.

▲ FIGURE 20–2

The equivalent action when a pulse source charges and discharges the capacitor.

As you learned in Chapter 12, a capacitor will charge and discharge following an exponential curve. Its rate of charging and discharging, of course, depends on the *RC* **time constant**, a fixed time interval determined by *R* and *C* ($\tau = RC$).

For an ideal pulse, both edges are considered to be instantaneous. Two basic rules of capacitor behavior help in understanding the response of *RC* circuits to pulse inputs.

1. The capacitor appears as a short to an instantaneous change in current and as an open to dc.
2. The voltage across the capacitor cannot change instantaneously—it can change only exponentially.

Capacitor Voltage

In an *RC* integrator, the output is the capacitor voltage. The capacitor charges during the time that the pulse is high. If the pulse is at its high level long enough, the capacitor will fully charge to the voltage amplitude of the pulse, as illustrated in Figure 20–3. The capacitor discharges during the time that the pulse is low. If the low time between pulses is long enough, the capacitor will fully discharge to zero, as shown in the figure. Then when the next pulse occurs, it will charge again.

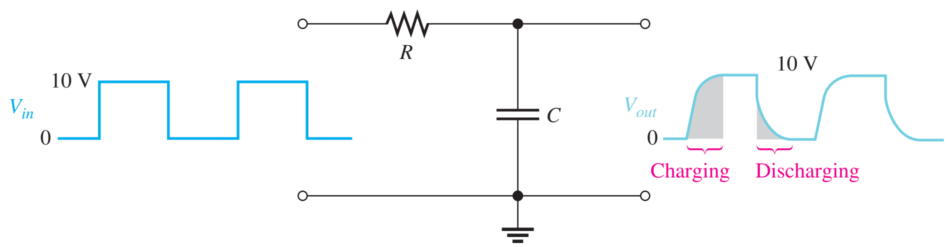


FIGURE 20–3
Illustration of a capacitor fully charging and discharging in response to a pulse input.

SECTION 20–1 CHECKUP
Answers are at the end of the chapter.

1. Define the term *integrator* in relation to an *RC* circuit.
2. What causes the capacitor in an *RC* circuit to charge and discharge?

20–2 RESPONSE OF AN *RC* INTEGRATOR TO A SINGLE PULSE

From the previous section, you have a general idea of how an *RC* integrator responds to an input pulse. In this section, the response to a single pulse is examined in detail.

After completing this section, you should be able to

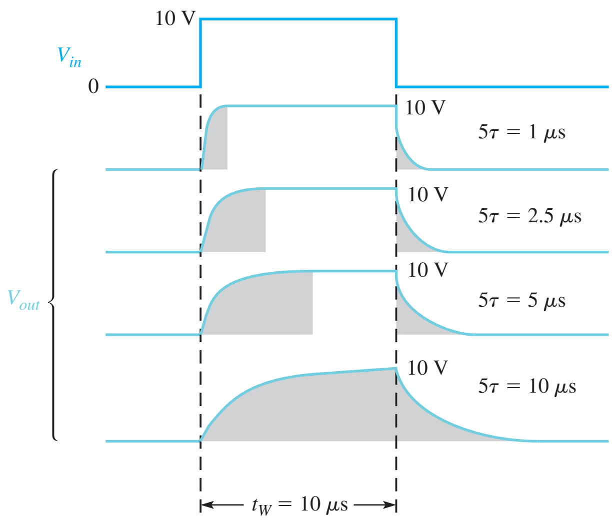
- **Analyze an *RC* integrator with a single input pulse**
  - Discuss the importance of the circuit time constant
  - Define *transient time*
  - Determine the response when the pulse width is equal to or greater than five time constants
  - Determine the response when the pulse width is less than five time constants

Two conditions of pulse response must be considered:

1. When the input pulse width (t_W) is equal to or greater than five time constants ($t_W \geq 5\tau$)
2. When the input pulse width is less than five time constants ($t_W < 5\tau$)

Recall that five time constants is accepted as the time for a capacitor to fully charge or fully discharge; this time is often called the **transient time** or *transient response time*. A capacitor will fully charge if the pulse width is equal to or greater than five time constants (5τ). This condition is expressed as $t_W \geq 5\tau$. At the end of the pulse, the capacitor fully discharges back through the source.

Figure 20–4 illustrates the output waveforms for various *RC* transient times and a fixed input pulse width. Notice that as the transient time becomes shorter, compared to the pulse width, the shape of the output pulse approaches that of the input. In each case, the output reaches the full amplitude of the input.



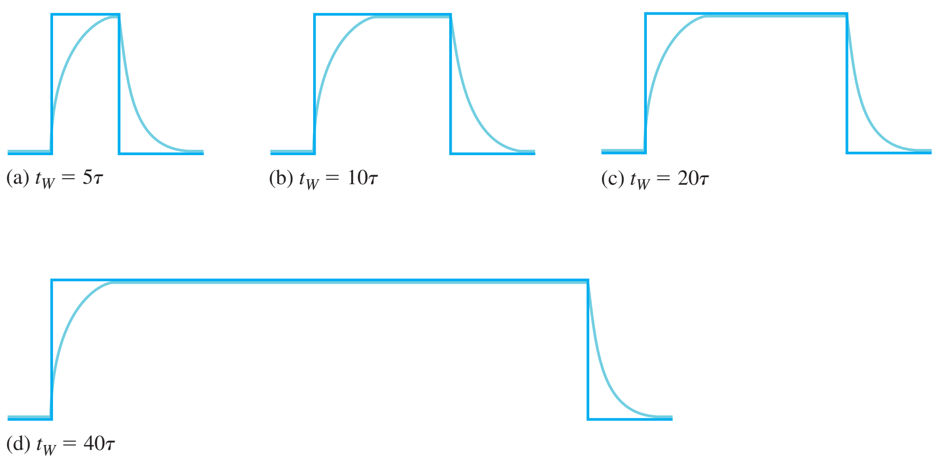
▲ **FIGURE 20–4**

Variation of an *RC* integrator's output pulse shape with time constant. The shaded areas indicate when the capacitor is charging or discharging.

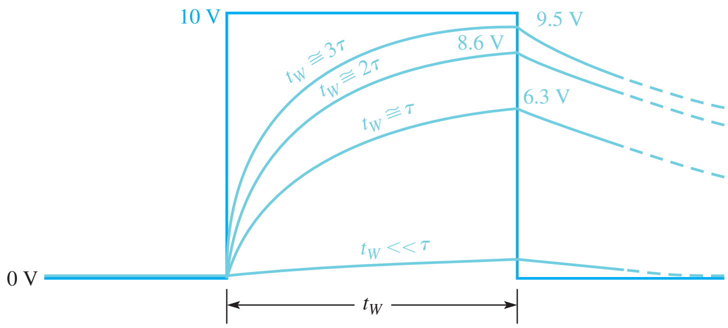
Figure 20–5 shows how a fixed time constant and a variable input pulse width affect the integrator output. Notice that as the pulse width is increased, the shape of the output pulse approaches that of the input. Again, this means that the transient time is short compared to the pulse width.

▶ **FIGURE 20–5**

Variation of an *RC* integrator's output pulse shape with input pulse width (the time constant is fixed). Dark blue is input and light blue is output.



Now let's examine the case in which the width of the input pulse is less than five time constants of the *RC* integrator. This condition is expressed as $t_W < 5\tau$. As you know, the capacitor charges for the duration of the pulse. However, because the pulse width is less than the time it takes the capacitor to fully charge (5τ), the output voltage will *not* reach the full input voltage before the end of the pulse. The capacitor only partially charges, as illustrated in Figure 20–6 for several values of *RC* time constants. Notice that for longer time constants, the output reaches a lower voltage because the capacitor cannot charge as much. Of course, in the examples with a single pulse input, the capacitor fully discharges after the pulse ends.

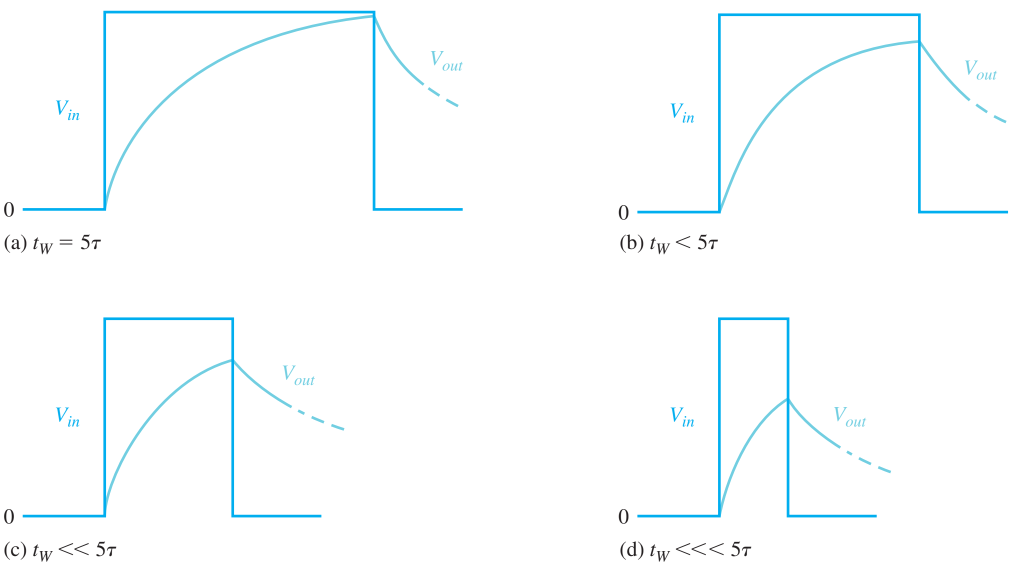


▲ **FIGURE 20–6**

Capacitor voltage for various time constants that are longer than the input pulse width. Dark blue is input and light blue is output.

When the time constant is much greater than the input pulse width, the capacitor charges very little, and, as a result, the output voltage becomes almost negligible, as indicated in Figure 20–6.

Figure 20–7 illustrates the effect of reducing the input pulse width for a fixed time constant value. As the input pulse width is reduced, the output voltage decreases because the capacitor has less time to charge. However, it takes the capacitor approximately the same length of time (5τ) to discharge back to zero for each condition after the pulse is removed.



▲ **FIGURE 20–7**

The capacitor charges less and less as the input pulse width is reduced. The time constant is fixed.

EXAMPLE 20–1

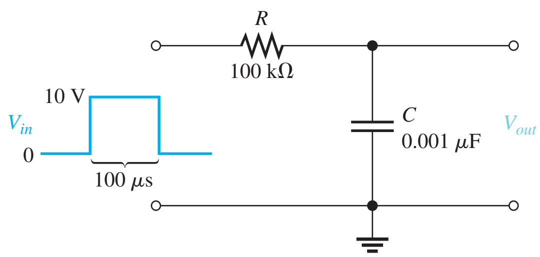
A single 10 V pulse with a width of 100 μs is applied to the *RC* integrator in Figure 20–8.

(a) To what voltage will the capacitor charge?

(b) How long will it take the capacitor to discharge if the internal resistance of the pulse source is 50 Ω?

(c) Draw the output voltage waveform.

▶ **FIGURE 20–8**



Solution

(a) The circuit time constant is

$$\tau = RC = (100\,\text{k}\Omega)(0.001\,\mu\text{F}) = 100\,\mu\text{s}$$

Notice that the pulse width is exactly equal to the time constant. Thus, the capacitor will charge approximately 63% of the full input amplitude in one time constant, so the output will reach a maximum voltage of

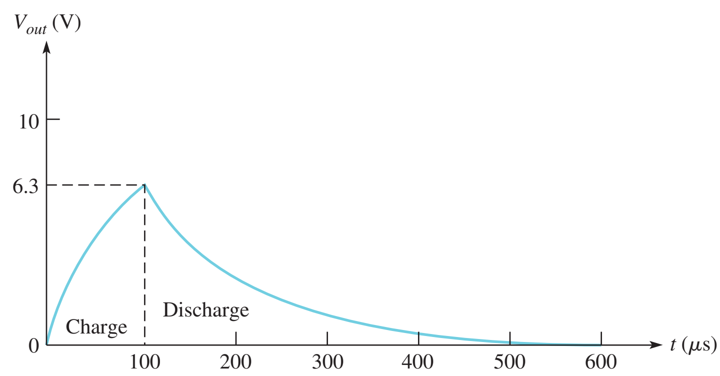
$$V_{out} = (0.63)10\text{ V} = \mathbf{6.3\text{ V}}$$

(b) The capacitor discharges back through the source when the pulse ends. You can neglect the 50 Ω source resistance in series with 100 kΩ. The total approximate discharge time, therefore, is

$$5\tau = 5(100\,\mu\text{s}) = \mathbf{500\,\mu s}$$

(c) The output charging and discharging curve is shown in Figure 20–9.

▶ **FIGURE 20–9**



*Related Problem**

If the input pulse width in Figure 20–8 is increased to 200 μs, to what maximum voltage will the capacitor charge?

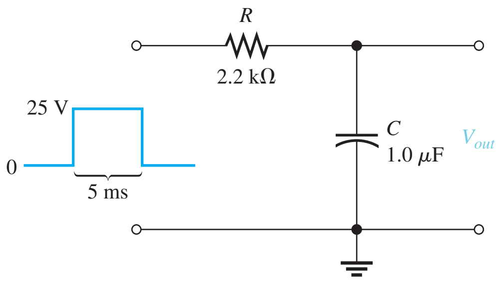
Use Multisim files E20-01A and E20-01B to verify the calculated results in this example and the related problem.

*Answers are at the end of the chapter.

EXAMPLE 20–2

Determine how much the capacitor in Figure 20–10 will charge when the single pulse is applied to the input.

▶ FIGURE 20–10



Solution Calculate the time constant.

$$\tau = RC = (2.2\,\text{k}\Omega)(1.0\,\mu\text{F}) = 2.2\,\text{ms}$$

Because the pulse width is 5 ms, the capacitor charges for 2.27 time constants (5 ms/2.2 ms = 2.27). Use the exponential formula from Chapters 12 (Eq. 12–19) to find the voltage to which the capacitor will charge. With $V_F = 25$ V and $t = 5$ ms, the calculation is as follows:

$$\begin{aligned} v &= V_F(1 - e^{-t/RC}) \\ &= (25\text{ V})(1 - e^{-5\text{ms}/2.2\text{ms}}) \\ &= (25\text{ V})(1 - 0.103) = (25\text{ V})(0.897) = \mathbf{22.4\text{ V}} \end{aligned}$$

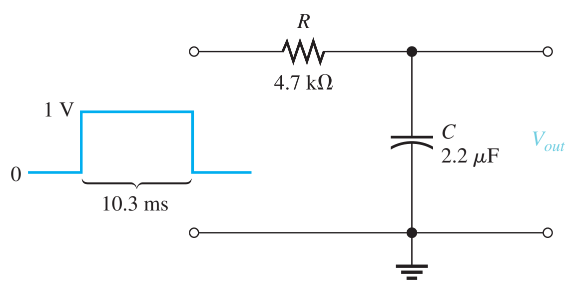
These calculations show that the capacitor charges to 22.4 V during the 5 ms duration of the input pulse. It will discharge back to zero in five time constants when the pulse goes back to zero.

Related Problem Determine how much *C* will charge if the pulse width is increased to 10 ms.

SECTION 20–2 CHECKUP

1. When an input pulse is applied to an *RC* integrator, what condition must exist in order for the output voltage to reach full amplitude?
2. For the circuit in Figure 20–11, which has a single input pulse, find the maximum output voltage and determine how long the capacitor will discharge.
3. For Figure 20–11, draw the approximate shape of the output voltage with respect to the input pulse.
4. If an integrator time constant equals the input pulse width, will the capacitor fully charge?
5. Describe the condition under which the output voltage has the approximate shape of a rectangular input pulse.

▶ FIGURE 20–11



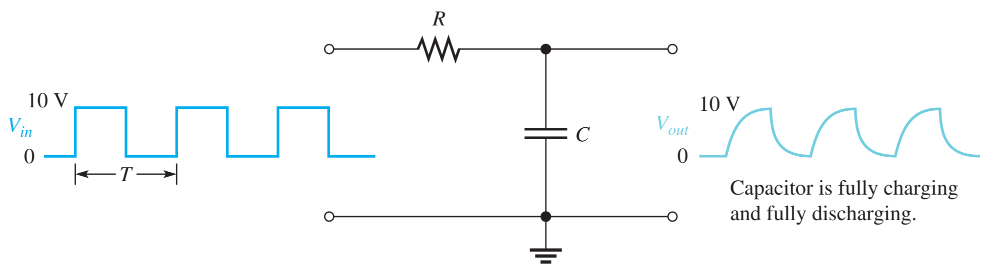
20–3 Response of *RC* Integrators to Repetitive Pulses

In electronic systems, you will encounter waveforms with repetitive pulses much more often than single pulses. However, an understanding of the integrator's response to single pulses is necessary in order to understand how these circuits respond to repeated pulses.

After completing this section, you should be able to

- **Analyze an *RC* integrator with repetitive input pulses**
  - Determine the response when the capacitor does not fully charge or discharge
  - Define *steady state*
  - Describe the effect of an increase in time constant on circuit response

If a **periodic** pulse waveform is applied to an *RC* integrator, as shown in Figure 20–12, the output waveshape depends on the relationship of the circuit time constant and the frequency (period) of the input pulses. The capacitor, of course, charges and discharges in response to a pulse input. The amount of charge and discharge of the capacitor depends both on the circuit time constant and on the input frequency, as mentioned.



▲ FIGURE 20–12

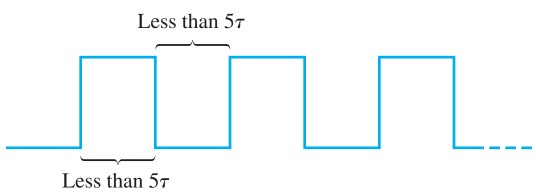
RC integrator with a repetitive pulse waveform input ($T = 10\tau$).

If the pulse width and the time between pulses are each equal to or greater than five time constants, the capacitor will fully charge and fully discharge during each period (T) of the input waveform. This case is shown in Figure 20–12.

When the pulse width and the time between pulses are shorter than five time constants, as illustrated in Figure 20–13 for a square wave, the capacitor will *not* completely charge or discharge. We will now examine the effects of this situation on the output voltage of the *RC* integrator.

► FIGURE 20–13

Input waveform that does not allow full charge or discharge of the capacitor in an *RC* integrator.



For illustration, let's use an *RC* integrator with a charging and discharging time constant equal to the pulse width of a 10 V square wave input, as shown in Figure 20–14. This choice will simplify the analysis and will demonstrate the basic action of the integrator under these conditions. At this point, we do not care what the exact time constant value is because we know that an *RC* circuit charges approximately 63% during one time constant interval.

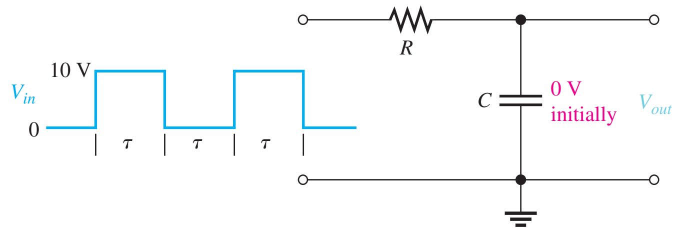


FIGURE 20–14

RC integrator with a square wave input having a period equal to two time constants ($T = 2\tau$).

Let's assume that the capacitor in Figure 20–14 begins initially uncharged and examine the output voltage on a pulse-by-pulse basis. Figure 20–15 shows the charging and discharging shapes of five pulses.

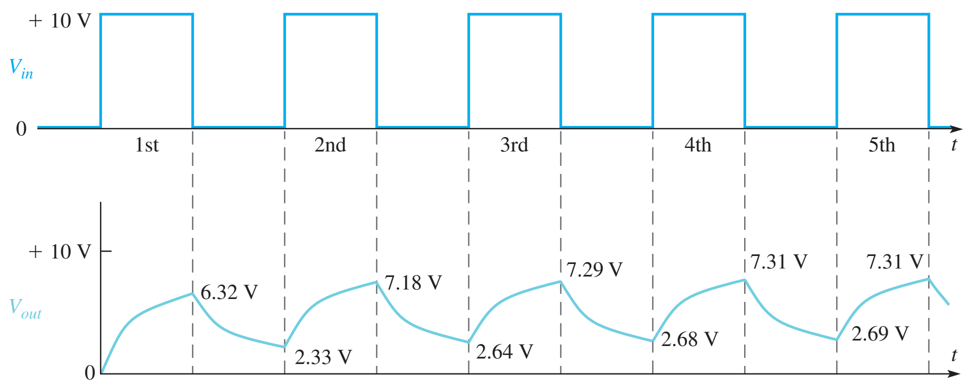


FIGURE 20–15

Input and output for the initially uncharged integrator in Figure 20–14.

First pulse During the first pulse, the capacitor charges. The output voltage reaches 6.32 V (63.2% of 10 V), as shown in Figure 20–15.

Between first and second pulses The capacitor discharges, and the voltage decreases to 36.8% of the voltage at the beginning of this interval: 0.368(6.32 V) = 2.33 V.

Second pulse The capacitor voltage begins at 2.33 V and increases 63.2% of the way to 10 V. This calculation is as follows: The total charging range is 10 V − 2.33 V = 7.67 V. The capacitor voltage will increase an additional 63.2% of 7.67 V, which is 4.85 V. Thus, at the end of the second pulse, the output voltage is 2.33 V + 4.85 V = 7.18 V, as shown in Figure 20–15. Notice that the average is building up.

Between second and third pulses The capacitor discharges during this time, and therefore the voltage decreases to 36.8% of the initial voltage by the end of the second pulse: 0.368(7.18 V) = 2.64 V.

Third pulse At the start of the third pulse, the capacitor voltage begins at 2.64 V. The capacitor charges 63.2% of the way from 2.64 V to 10 V: 0.632(10 V − 2.64 V) = 4.65 V. Therefore, the voltage at the end of the third pulse is 2.64 V + 4.65 V = 7.29 V.

Between third and fourth pulses The voltage during this interval decreases due to capacitor discharge. It will decrease to 36.8% of its value by the end of the third pulse. The final voltage in this interval is 0.368(7.29 V) = 2.68 V.

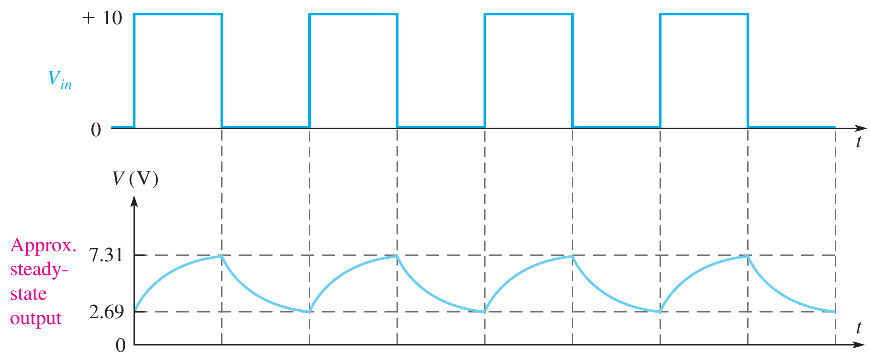
Fourth pulse At the start of the fourth pulse, the capacitor voltage is 2.68 V. The voltage increases by 0.632(10 V − 2.68 V) = 4.63 V. Therefore, at the end of the fourth pulse, the capacitor voltage is 2.68 V + 4.63 V = 7.31 V. Notice that the values are leveling off as the pulses continue.

Between fourth and fifth pulses Between these pulses, the capacitor voltage drops to 0.368(7.31 V) = 2.69 V.

Fifth pulse During the fifth pulse, the capacitor charges 0.632(10 V − 2.69 V) = 4.62 V. Since it started at 2.69 V, the voltage at the end of the pulse is 2.69 V + 4.62 V = 7.31 V.

Steady-State Time Response

In the preceding discussion, the output voltage gradually built up and then began leveling off. It takes approximately 5τ for the output voltage to build up to a constant average value. This interval is the transient time of the circuit. Once the output voltage reaches the average value of the input voltage, a **steady-state** condition is reached that continues as long as the periodic input continues. This condition is illustrated in Figure 20–16 based on the values obtained in the preceding discussion.



▲ **FIGURE 20–16**

Output reaches steady state after 5τ.

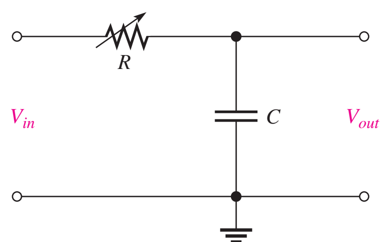
The transient time for our example circuit is the time from the beginning of the first pulse to the end of the third pulse. The reason for this interval is that the capacitor voltage at the end of the third pulse is 7.29 V, which is about 99% of the final voltage.

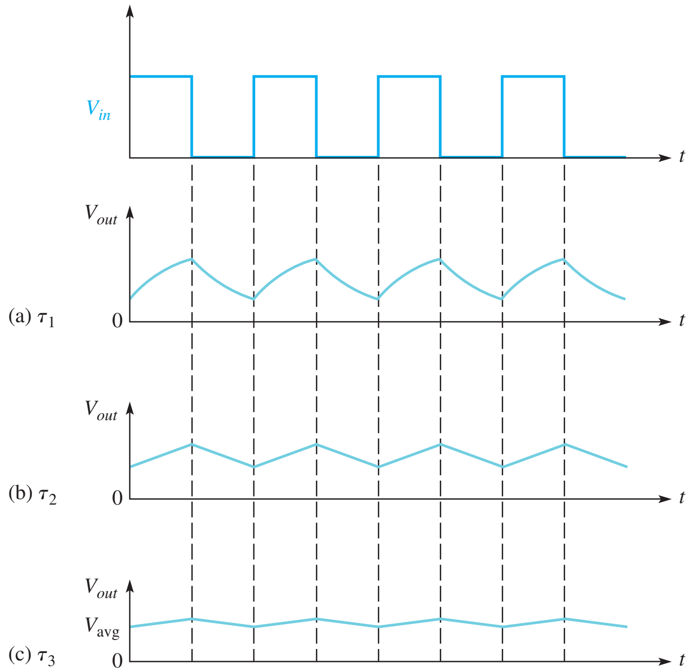
The Effect of an Increase in Time Constant

What happens to the output voltage if the RC time constant of the integrator is increased with a variable resistor, as indicated in Figure 20–17? As the time constant is increased, the capacitor charges less during a pulse and discharges less between pulses. The result is a smaller fluctuation in the output voltage for increasing values of time constant, as shown in Figure 20–18.

▶ **FIGURE 20–17**

Integrator with a variable RC time constant.





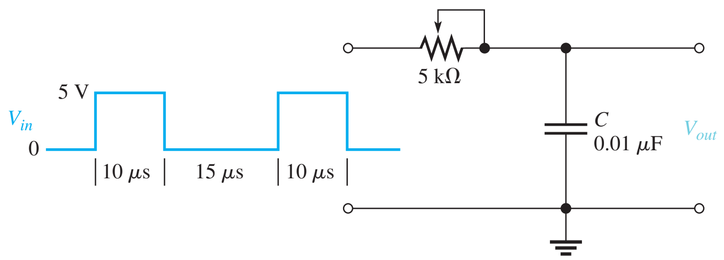
▲ FIGURE 20–18

Effect of longer time constants on the output of an *RC* integrator ($\tau_3 > \tau_2 > \tau_1$).

As the time constant becomes extremely long compared to the pulse width, the output voltage approaches a constant dc voltage, as shown in Figure 20–18(c). This value is the average value of the input. For a square wave, it is one-half the amplitude.

EXAMPLE 20–3

Determine the output voltage waveform for the first two pulses applied to the *RC* integrator in Figure 20–19. Assume that the capacitor is initially uncharged and the rheostat is set to 5 kΩ.



▲ FIGURE 20–19

Solution First, calculate the circuit time constant.

$$\tau = RC = (5\ \text{k}\Omega)(0.01\ \mu\text{F}) = 50\ \mu\text{s}$$

Obviously, the time constant is much longer than the input pulse width or the interval between pulses (notice that the input is not a square wave). In this case, the exponential formulas must be applied, and the analysis is relatively difficult. Follow the solution carefully.

1. *Calculation for first pulse:* Use the equation for an increasing exponential because C is charging. Note that V_F is 5 V, and t equals the pulse width of 10 μs. Therefore,

$$v_C = V_F(1 - e^{-t/RC}) = (5\text{ V})(1 - e^{-10\,\mu\text{s}/50\,\mu\text{s}}) = \mathbf{906\ mV}$$

   This result is plotted in Figure 20–20(a).

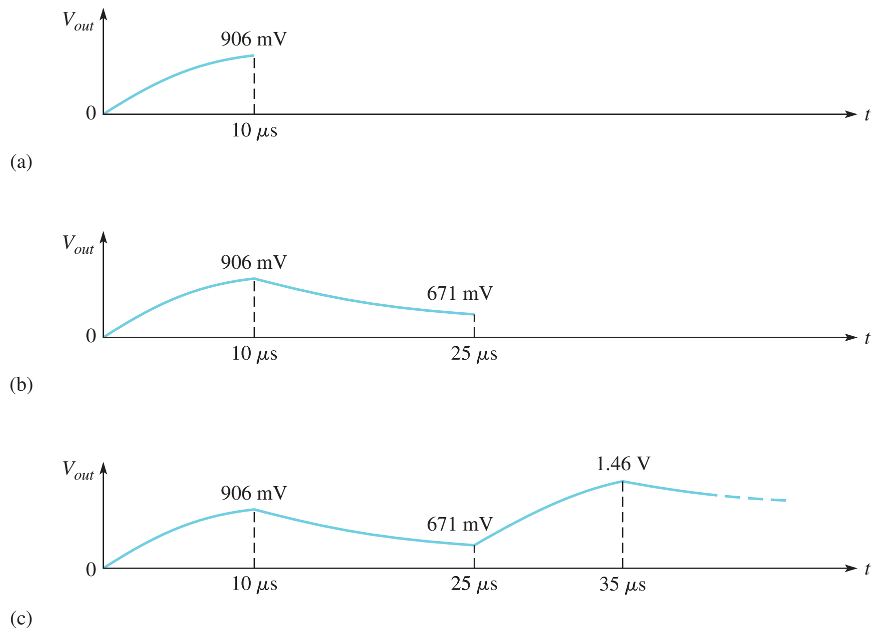
2. *Calculation for interval between first and second pulse:* Use the equation for a decreasing exponential because C is discharging. Note that V_i is 906 mV because C begins to discharge from this value at the end of the first pulse. The discharge time is 15 μs. Therefore,

$$v_C = V_i e^{-t/RC} = (906\text{ mV})e^{-15\,\mu\text{s}/50\,\mu\text{s}} = \mathbf{671\ mV}$$

   This result is shown in Figure 20–20(b).

3. *Calculation for second pulse:* At the beginning of the second pulse, the output voltage is 671 mV. During the second pulse, the capacitor will again charge. In this case, it does not begin at zero volts. It already has 671 mV from the previous charge and discharge. To handle this situation, you must use the general exponential formula.

$$v = V_F + (V_i - V_F)e^{-t/\tau}$$



▲ FIGURE 20–20

Using this equation, you can calculate the voltage across the capacitor at the end of the second pulse as follows:

$$\begin{aligned} v_C &= V_F + (V_i - V_F)e^{-t/RC} \\ &= 5\text{ V} + (671\text{ mV} - 5\text{ V})e^{-10\,\mu\text{s}/50\,\mu\text{s}} \\ &= 5\text{ V} - 3.44 = \mathbf{1.46\ V} \end{aligned}$$

This result is shown in Figure 20–20(c).

Notice that the output waveform builds up on successive input pulses. After approximately 5τ, it will reach its steady state and will fluctuate between a constant maximum and a constant minimum, with an average equal to the average value of the input. You can see this pattern by carrying the analysis in this example further.

Related Problem Determine V_{out} at the beginning of the third pulse.

Use Multisim file E20-03 to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 20–3 CHECKUP

1. What conditions allow an *RC* integrator capacitor to fully charge and discharge when a periodic pulse waveform is applied to the input?
2. For an *RC* integrator circuit, what will the output waveform look like if the circuit time constant is extremely short compared to the pulse width of a square wave input?
3. What is the time called that is required for the output voltage to build up to a constant average value when 5τ is greater than the pulse width of an input square wave?
4. Define *steady-state response*.
5. Describe the output of an *RC* integrator when the input is a square wave with a period much less than 1τ.

20–4 RESPONSE OF AN *RC* DIFFERENTIATOR TO A SINGLE PULSE

In terms of time response, a series *RC* circuit in which the output voltage is taken across the resistor is known as a **differentiator**. Recall that in terms of frequency response, it is a high-pass filter. The term *differentiator* is derived from the mathematical process of differentiation, which this type of circuit approximates under certain conditions.

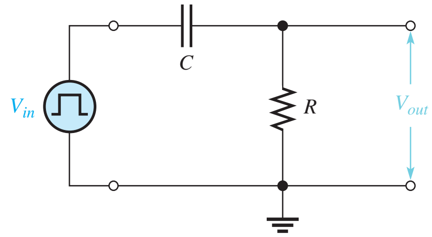
After completing this section, you should be able to

- **Analyze an *RC* differentiator with a single input pulse**
  - Describe the response at the rising edge of the input pulse
  - Determine the response during and at the end of a pulse for various pulse width-time constant relationships

Figure 20–21 shows an *RC* differentiator with a pulse input. The same action occurs in a differentiator as in an integrator, except the output voltage is taken across the resistor rather than across the capacitor. The capacitor charges exponentially at a rate depending on the *RC* time constant. The shape of the differentiator's resistor voltage is determined by the charging and discharging action of the capacitor.

▶ FIGURE 20–21

An *RC* differentiator with a pulse generator connected.

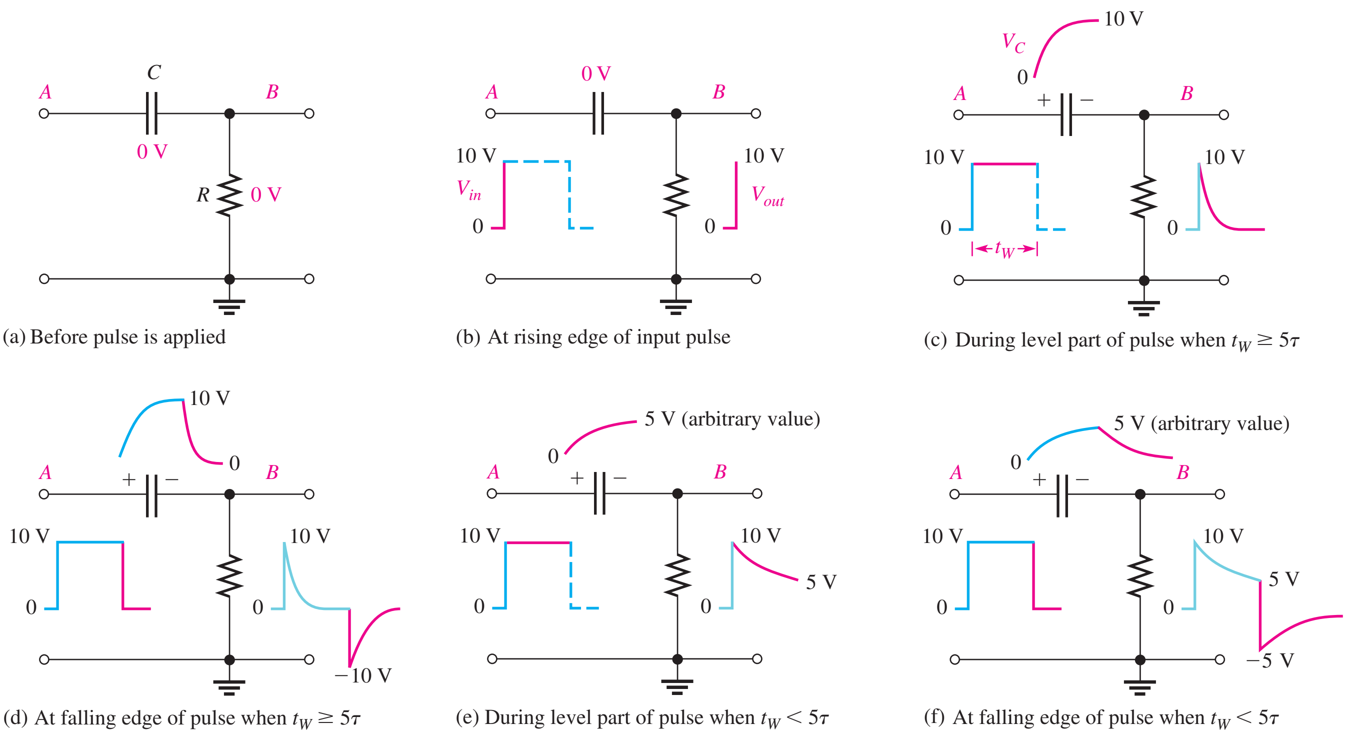


Pulse Response

To understand how the output voltage is shaped by a differentiator, you must consider the following:

1. The response to the rising pulse edge
2. The response between the rising and falling edges
3. The response to the falling pulse edge

Let's assume that the capacitor is initially uncharged prior to the rising pulse edge. Prior to the pulse, the input is zero volts. Thus, there are zero volts across the capacitor and also zero volts across the resistor, as indicated in Figure 20–22(a).



▲ FIGURE 20–22

Examples of the response of an *RC* differentiator to a single input pulse under two conditions: $t_W \geq 5\tau$ and $t_W < 5\tau$.

Response to the Rising Edge of the Input Pulse Let's also assume that a 10 V pulse is applied to the input. When the rising edge occurs, point *A* goes to +10 V. Recall that the voltage across a capacitor cannot change instantaneously, and thus the capacitor appears instantaneously as a short. Therefore, if point *A* goes instantly to +10 V, then point *B* *must* also go instantly to +10 V, keeping the capacitor voltage zero for the instant of the rising edge. The capacitor voltage is the voltage from point A to point B.

The voltage at point B with respect to ground is the voltage across the resistor (and the output voltage). Thus, the output voltage suddenly goes to +10 V in response to the rising pulse edge, as indicated in Figure 20–22(b).

Response During Pulse When $t_W \geq 5\tau$ While the pulse is at its high level between the rising edge and the falling edge, the capacitor is charging. When the pulse width is equal to or greater than five time constants ($t_W \geq 5\tau$), the capacitor has time to fully charge.

As the voltage across the capacitor builds up exponentially, the voltage across the resistor decreases exponentially until it reaches zero volts at the time the capacitor reaches full charge (+10 V in this case). This decrease in the resistor voltage occurs because the sum of the capacitor voltage and the resistor voltage at any instant must be equal to the applied voltage, in compliance with Kirchhoff's voltage law ($v_C + v_R = v_{in}$). This part of the response is illustrated in Figure 20–22(c).

Response to Falling Edge When $t_W \geq 5\tau$ Let's examine the case in which the capacitor is fully charged at the end of the pulse ($t_W \geq 5\tau$). Refer to Figure 20–22(d). On the falling edge, the input pulse suddenly goes from +10 V back to zero. An instant before the falling edge, the capacitor is charged to 10 V, so point A is +10 V and point B is 0 V. The voltage across a capacitor cannot change instantaneously, so when point A makes a transition from +10 V to zero on the falling edge, point B *must* also make a 10 V transition from zero to −10 V so that the sum of voltages around the closed loop equals zero as required by Kirchhoff's Voltage Law. This keeps the voltage across the capacitor at 10 V for the instant of the falling edge.

The capacitor now begins to discharge exponentially. As a result, the resistor voltage goes from −10 V to zero in an exponential curve, as indicated in red in Figure 20–22(d).

Response During Pulse When $t_W < 5\tau$ When the pulse width is less than five time constants ($t_W < 5\tau$), the capacitor does not have time to fully charge. Its partial charge depends on the relation of the time constant and the pulse width.

Because the capacitor does not reach the full +10 V, the resistor voltage will not reach zero volts by the end of the pulse. For example, if the capacitor charges to +5 V during the pulse interval, the resistor voltage will decrease to +5 V, as illustrated in Figure 20–22(e).

Response to Falling Edge When $t_W < 5\tau$ Now, let's examine the case in which the capacitor is only partially charged at the end of the pulse ($t_W < 5\tau$). For example, if the capacitor charges to +5 V, the resistor voltage at the instant before the falling edge is also +5 V because the capacitor voltage plus the resistor voltage must add up to +10 V, as illustrated in Figure 20–22(e).

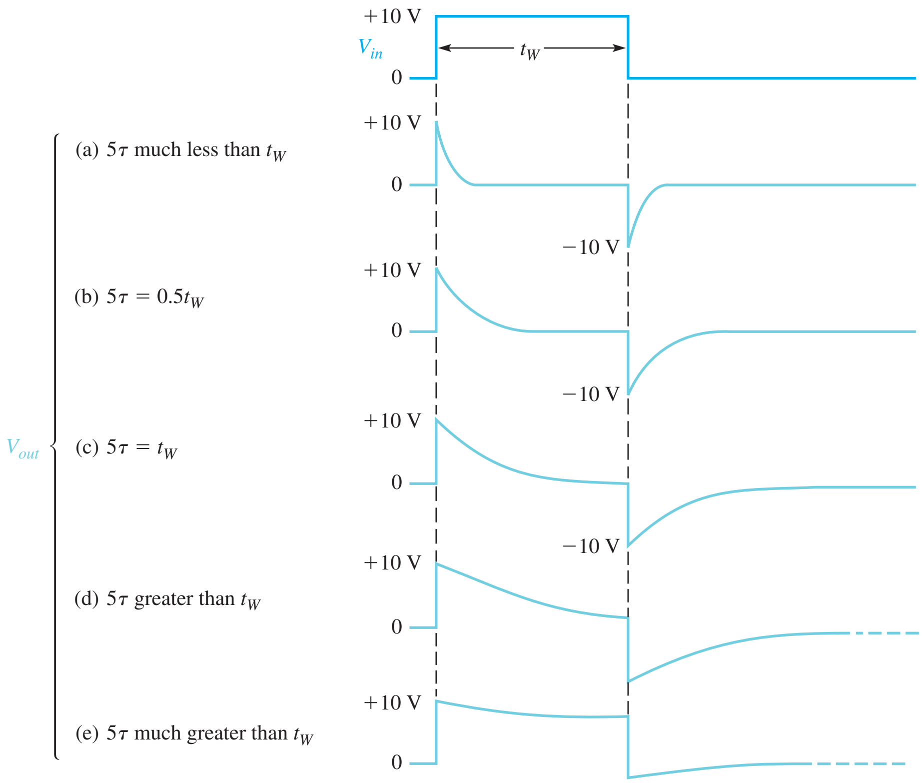
When the falling edge occurs, point A goes from +10 V to zero. As a result, point B goes from +5 V to −5 V, as illustrated in Figure 20–22(f). This decrease occurs, of course, because the capacitor voltage cannot change at the instant of the falling edge and Kirchhoff's Voltage Law requires that the sum of voltages around the closed loop is equal to zero. Immediately after the falling edge, the capacitor begins to discharge to zero. As a result, the resistor voltage goes from −5 V to zero, as shown.

Summary of *RC* Differentiator Response to a Single Pulse

A good way to summarize this section is to look at the general output waveforms of a differentiator as the time constant is varied from one extreme, when 5τ is much less than the pulse width, to the other extreme, when 5τ is much greater than the pulse width. These situations are illustrated in Figure 20–23. In part (a) of the figure, the output consists of narrow positive and negative "spikes." In part (e), the output approaches the shape of the input. Various conditions between these extremes are illustrated in parts (b), (c), and (d).

TECH TIP

You may have observed a pulse that looks similar to Figure 20–23(e) when you ac couple a pulse to an oscilloscope. In this case the capacitor in the oscilloscope coupling circuit can act as an unwanted differentiating circuit, causing the pulse to droop. To avoid this, you can dc couple the scope and check the probe compensation.

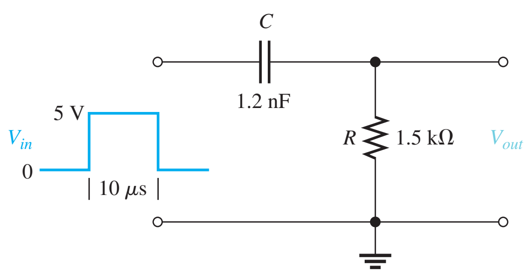


▲ **FIGURE 20–23**

Effects of a change in time constant on the shape of the output voltage of an *RC* differentiator.

EXAMPLE 20–4

Draw the output voltage for the *RC* differentiator in Figure 20–24.



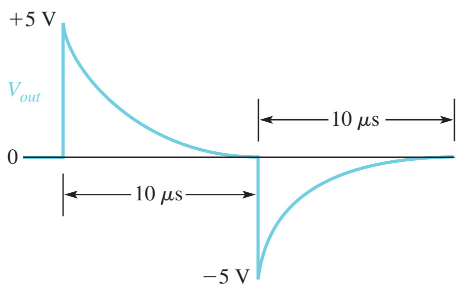
▲ **FIGURE 20–24**

Solution First, calculate the time constant.

$$\tau = RC = 1.8\ \mu s$$

In this case, $t_W > 5\tau$, so the capacitor reaches full charge before the end of the pulse.

On the rising edge, the resistor voltage jumps to +5 V and then decreases exponentially to zero by the end of the pulse. On the falling edge, the resistor voltage jumps to −5 V and then goes back to zero exponentially. The resistor voltage is, of course, the output, and its shape is shown in Figure 20–25.



▲ FIGURE 20–25

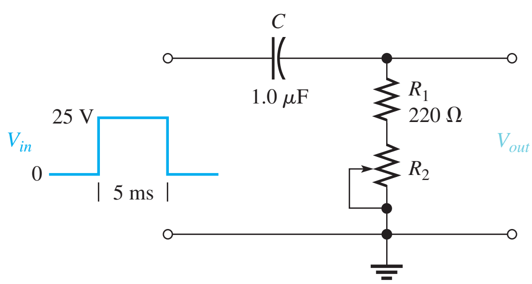
Related Problem Draw the output voltage if C is changed to 120 pF in Figure 20–24.

MultiSim

Use Multisim files E20-04A and E20-04B to verify the calculated results in this example and to confirm your answer for the related problem. To simulate a single pulse, specify a waveform with the given pulse width but a small duty cycle (long period).

EXAMPLE 20–5

Determine the output voltage waveform for the *RC* differentiator in Figure 20–26 with the rheostat set so that the total resistance of R_1 and R_2 is 2.0 kΩ.



▲ FIGURE 20–26

Solution First, calculate the time constant.

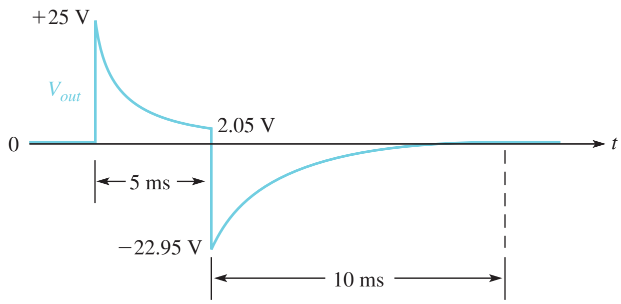
$$\tau = R_{tot}C = (2.0\ \text{k}\Omega)(1.0\ \mu\text{F}) = 2.0\ \text{ms}$$

On the rising edge, the resistor voltage immediately jumps to +25 V. Because the pulse width is 5 ms, the capacitor charges for 2.5 time constants and therefore does not reach full charge. Thus, you must use the formula for a decreasing exponential in order to calculate to what voltage the output decreases by the end of the pulse.

$$v_{out} = V_i e^{-t/RC} = 25\ \text{V}e^{-5\,\text{ms}/2\,\text{ms}} = (25\ \text{V})(0.0821) = 2.05\ \text{V}$$

where $V_i = 25$ V and $t = 5$ ms. This calculation gives the resistor voltage (v_{out}) at the end of the 5 ms pulse width interval.

On the falling edge, the resistor voltage immediately jumps from +2.05 V down to −22.95 V (a 25 V transition). The resulting waveform of the output voltage is shown in Figure 20–27.



▲ **FIGURE 20–27**

Related Problem Determine the voltage at the end of the pulse in Figure 20–26 if the rheostat is set so that the total resistance is 1.5 kΩ.

MultiSim

Use Multisim files E20-05A and E20-05B to verify the calculated results in this example and to confirm your calculation for the related problem. To simulate a single pulse, specify a waveform with the given pulse width but a small duty cycle.

SECTION 20–4 CHECKUP

1. Draw the output of a differentiator for a 10 V input pulse when $5\tau = 0.5t_W$.
2. Under what condition does the output pulse shape most closely resemble the input pulse for a differentiator?
3. What does the differentiator output look like when 5τ is much less than the pulse width of the input?
4. If the resistor voltage in a differentiating circuit is down to +5 *V* at the end of a 15 V input pulse, to what negative value will the resistor voltage go in response to the falling edge of the input?

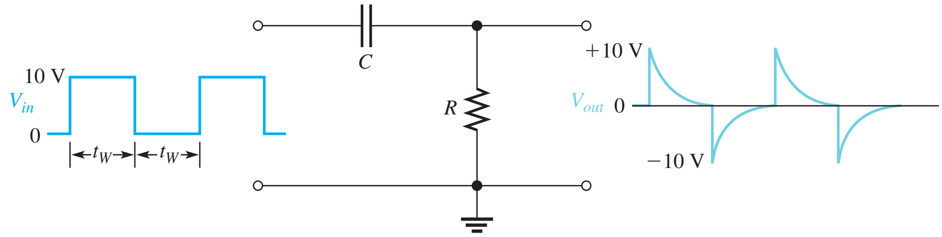
20–5 RESPONSE OF *RC* DIFFERENTIATORS TO REPETITIVE PULSES

The *RC* differentiator response to a single pulse, covered in the last section, is extended in this section to repetitive pulses.

After completing this section, you should be able to

- **Analyze an *RC* differentiator with repetitive input pulses**
    - Determine the response when the pulse width is less than five time constants

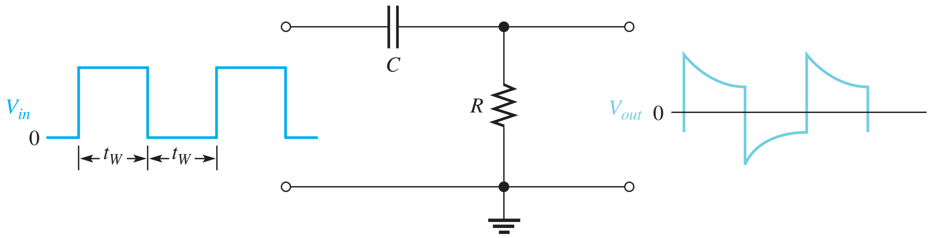
If a periodic pulse waveform is applied to an *RC* differentiating circuit, two conditions again are possible: $t_W \geq 5\tau$ or $t_W < 5\tau$. Figure 20–28 shows the output when $t_W = 5\tau$. As the time constant is reduced, both the positive and the negative portions of the output become narrower. Notice that the average value of the output is zero. An average value of zero means that the waveform has equal positive and negative portions. The average value of a waveform is its **dc component**. Because a capacitor blocks dc, the dc component of the input is prevented from passing through to the output, resulting in an average value of zero.



▲ FIGURE 20–28

Example of differentiator response when $t_W = 5\tau$.

Figure 20–29 shows the steady-state output when $t_W < 5\tau$. As the time constant is increased, the positively and negatively sloping portions become flatter. For a very long time constant, the output approaches the shape of the input, but with an average value of zero.



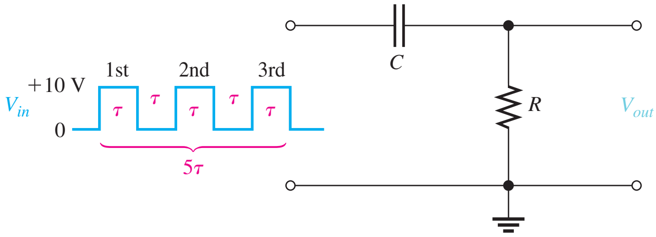
▲ FIGURE 20–29

Example of differentiator response when $t_W < 5\tau$.

Analysis of a Repetitive Waveform

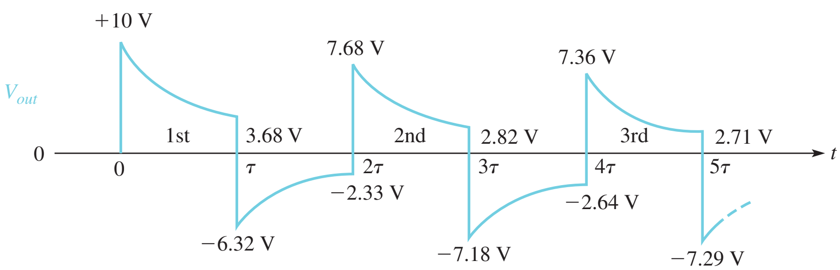
Like the integrator, the differentiator output takes time (5τ) to reach steady state. To illustrate the response, let's take an example in which the time constant equals the input pulse width. At this point, we do not care what the circuit time constant is because we know that the resistor voltage will decrease to approximately 37% of its maximum value during one pulse (1τ).

Let's assume that the capacitor in Figure 20–30 begins initially uncharged and then examine the output voltage on a pulse-by-pulse basis. The results of the analysis that follows are shown in Figure 20–31.



◀ FIGURE 20–30

RC differentiator with $\tau = t_W$.



◀ FIGURE 20–31

Differentiator output waveform during transient time for the circuit in Figure 20–30.

First pulse On the rising edge, the output instantaneously jumps to +10 V. Then the capacitor partially charges to 63.2% of 10 V, which is 6.32 V. Thus, the output voltage must decrease to 3.68 V, as shown in Figure 20–31. On the falling edge, the output instantaneously makes a negative-going 10 V transition to −6.32 V(−10 V + 3.68 V = −6.32 V).

Between first and second pulses The capacitor discharges to 36.8% of 6.32 V, which is 2.33 V. Thus, the resistor voltage, which starts at −6.32 V, must increase to −2.33 V. Why? Because at the instant prior to the next pulse, the input voltage is zero. Therefore, the sum of v_C and v_R must be zero (2.33 V − 2.33 V = 0). Remember that $v_C + v_R = v_{in}$ at all times, in accordance with Kirchhoff's voltage law.

Second pulse On the rising edge, the output makes an instantaneous, positive-going, 10 V transition from −2.33 V to 7.68 V. Then by the end of the pulse the capacitor charges 0.632 × (10 V − 2.33 V) = 4.85 V. Thus, the capacitor voltage increases from 2.33 V to 2.33 V + 4.85 V = 7.18 V. The output voltage drops to 0.368 × 7.68 V = 2.82 V.

On the falling edge, the output instantaneously makes a negative-going transition from 2.82 V to −7.18 V, as shown in Figure 20–31.

Between second and third pulses The capacitor discharges to 36.8% of 7.18 V, which is 2.64 V. Thus, the output voltage starts at −7.18 V and increases to −2.64 V because the capacitor voltage and the resistor voltage must add up to zero at the instant prior to the third pulse (the input is zero).

Third pulse On the rising edge, the output makes an instantaneous 10 V transition from −2.64 V to +7.36 V. Then the capacitor charges 0.632 × (10 V − 2.64 V) = 4.65 V to 2.64 V + 4.65 V = 7.29 V. As a result, the output voltage drops to 0.368 × 7.36 V = 2.71 V. On the falling edge, the output instantly goes from +2.71 V down to −7.29 V.

After the third pulse, five time constants have elapsed, and the output voltage is close to its steady state. Thus, it will continue to vary from a positive maximum of about +7.3 V to a negative maximum of about −7.3 V, with an average value of zero.

SECTION 20–5 CHECKUP

1. What conditions allow an *RC* differentiator to fully charge and discharge when a periodic pulse waveform is applied to the input?
2. What will the output waveform look like if the circuit time constant is extremely short compared to the pulse width of a square wave input?
3. What is the average value of the differentiator output voltage during steady state?

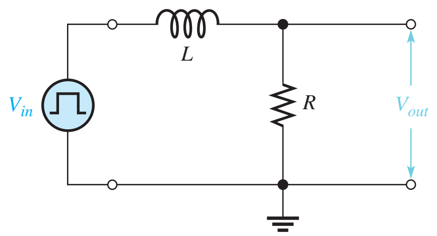
20–6 Response of *RL* Integrators to Pulse Inputs

A series *RL* circuit in which the output voltage is taken across the resistor is known as an integrator in terms of the time response. *RL* integrators are not as common as *RC* integrators because of cost and inductors are not as close to an ideal response as are capacitors. Although only the response to a single pulse is discussed, it can be extended to repetitive pulses, as described for the *RC* integrator.

After completing this section, you should be able to

- **Analyze the operation of an *RL* integrator**
  - Determine the response to a single input pulse

Figure 20–32 shows an *RL* integrator. The output waveform is taken across the resistor and, under equivalent conditions, is the same shape as that for the *RC* integrator. Recall that in the *RC* case, the output was across the capacitor.



◀ **FIGURE 20–32**

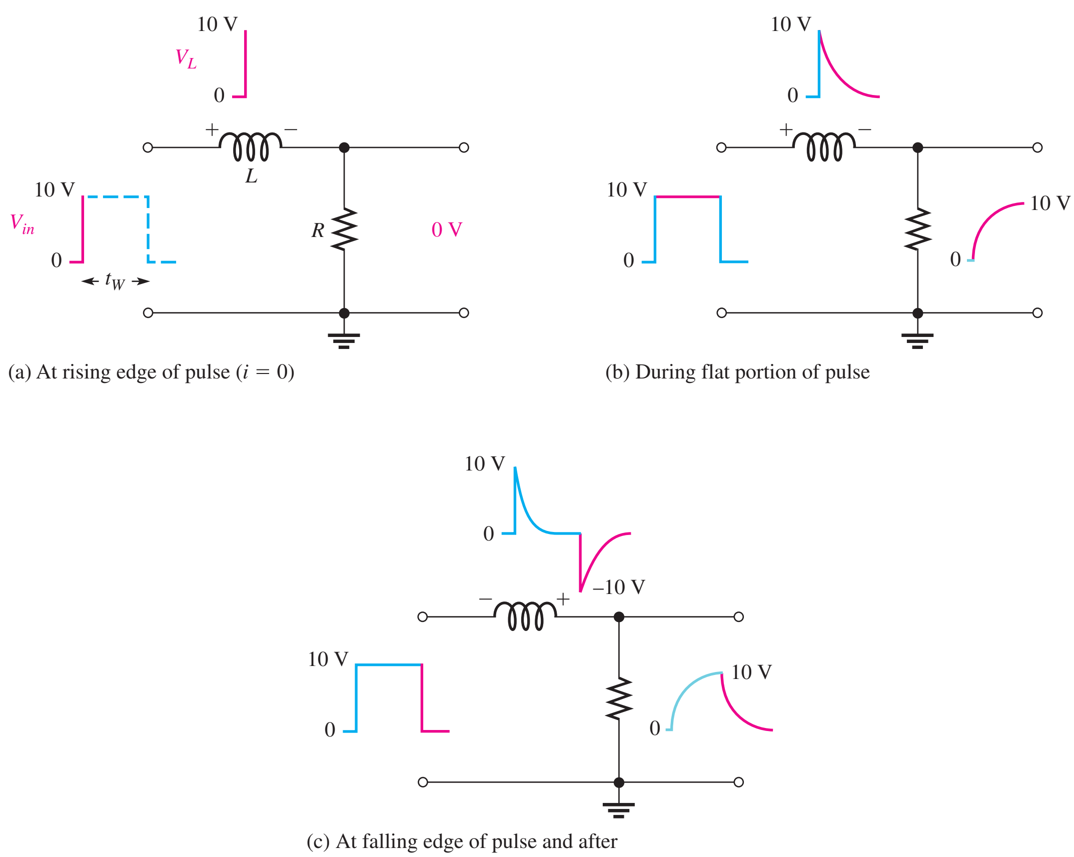
An *RL* integrator with a pulse generator connected.

As you know, each edge of an ideal pulse is considered to be instantaneous. Two basic rules for inductor behavior will aid in analyzing *RL* circuit responses to pulse inputs:

1. The inductor appears as an open to an instantaneous change in current and as a short (ideally) to dc.
2. The current in an inductor cannot change instantaneously—it can change only exponentially.

Response of the *RL* Integrator to a Single Pulse

When a pulse generator is connected to the input of the integrator and the voltage pulse goes from its low level to its high level, the inductor prevents a sudden change in current. As a result, the inductor acts as an open, and all of the input voltage is across it at the instant of the rising pulse edge. This situation is indicated in Figure 20–33(a).



▲ **FIGURE 20–33**

Illustration of the pulse response of an *RL* integrator ($t_W > 5\tau$).

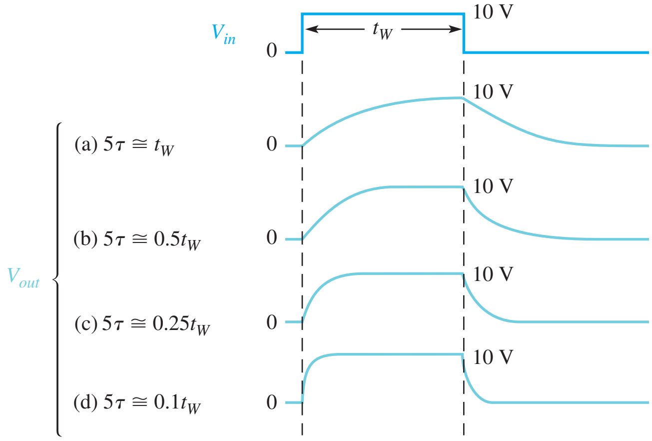
After the rising edge, the current builds up, and the output voltage follows the current as it increases exponentially, as shown in Figure 20–33(b). The current can reach a maximum of V_p/R if the transient time is shorter than the pulse width ($V_p = 10$ V in this example).

When the pulse goes from its high level to its low level, an induced voltage with reversed polarity is created across the coil in an effort to keep the current equal to V_p/R. The output voltage begins to decrease exponentially, as shown in Figure 20–33(c).

The exact shape of the output depends on the L/R time constant as summarized in Figure 20–34 for various relationships between the time constant and the pulse width. You should note that the response of this RL circuit in terms of the shape of the output is identical to that of the RC integrator. The relationship of the L/R time constant to the input pulse width has the same effect as the RC time constant that was shown in Figure 20–4. For example, when $t_W < 5\tau$, the output voltage will not reach its maximum possible value.

▶ **FIGURE 20–34**

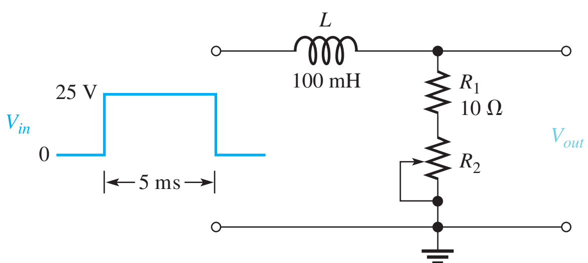
Illustration of the variation in *RL* integrator output pulse shape with time constant.



EXAMPLE 20–6

Determine the maximum output voltage for the RL integrator in Figure 20–35 when a single pulse is applied as shown. The rheostat is set so that the total resistance is 50 Ω. Assume the inductor is ideal.

▶ **FIGURE 20–35**



Solution Calculate the time constant.

$$\tau = \frac{L}{R} = \frac{100\text{ mH}}{50\ \Omega} = 2\text{ ms}$$

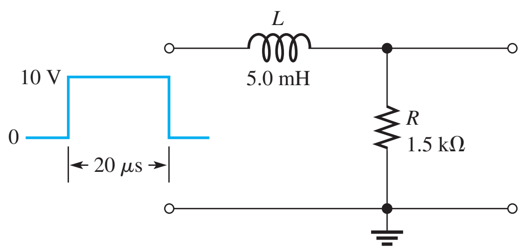
Because the pulse width is 5 ms, the inductor charges for 2.5τ. Use the exponential formula derived from Equation 13–8 with $V_i = 0$ and $\tau = L/R$ to calculate the voltage.

$$\begin{aligned} v_{out(max)} &= V_F(1 - e^{-t/\tau}) = 25(1 - e^{-5\text{ ms}/2\text{ ms}}) \\ &= 25(1 - e^{-2.5}) = 25(1 - 0.082) = 25(0.918) = \mathbf{23.0\ V} \end{aligned}$$

Related Problem To what maximum resistance can the rheostat, R_2, be set for the output voltage to reach 25 V by the end of the pulse in Figure 20–35?

EXAMPLE 20–7

A pulse is applied to the *RL* integrator in Figure 20–36. Determine the complete waveshapes and the values for I, V_R, and V_L.



▲ FIGURE 20–36

Solution The circuit time constant is

$$\tau = \frac{L}{R} = \frac{5.0\text{ mH}}{1.5\text{ k}\Omega} = 3.33\ \mu\text{s}$$

Since $5\tau = 16.7\ \mu\text{s}$ is less than t_W, the current will reach its maximum value and remain there until the end of the pulse.

At the rising edge of the pulse,

$$i = 0\text{ A}$$
$$v_R = 0\text{ V}$$
$$v_L = 10\text{ V}$$

The inductor initially appears as an open, so all of the input voltage appears across L.

During the pulse,

i increases exponentially to $\dfrac{V_p}{R} = \dfrac{10\text{ V}}{1.5\text{ k}\Omega} = 6.67\text{ mA in }16.7\ \mu\text{s}$

v_R increases exponentially to 10 V in 16.7 μs

v_L decreases exponentially to zero in 16.7 μs

At the falling edge of the pulse,

$$i = 6.67\text{ mA}$$
$$v_R = 10\text{ V}$$
$$v_L = -10\text{ V}$$

After the pulse,

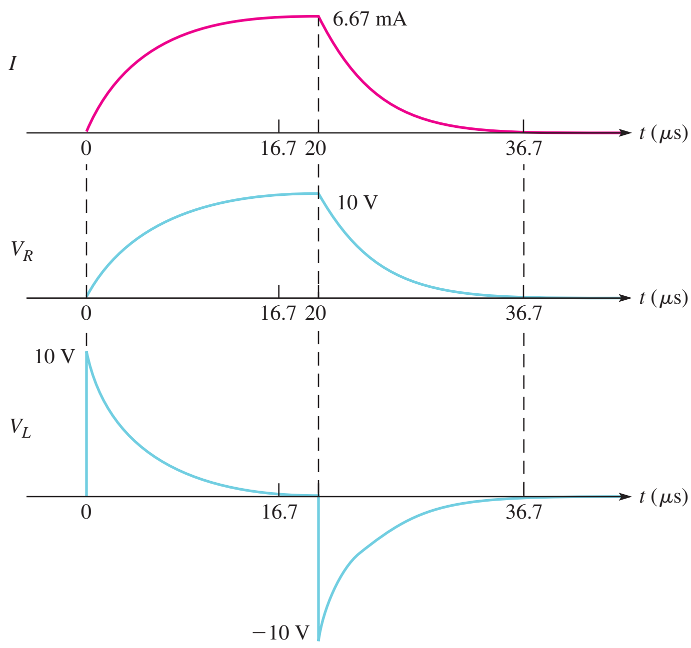
i decreases exponentially to zero in 16.7 μs

v_R decreases exponentially to zero in 16.7 μs

v_L increases exponentially to zero in 16.7 μs

The waveforms are shown in Figure 20–37.

▶ FIGURE 20–37

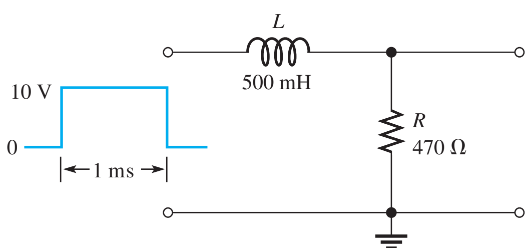


Related Problem What will be the maximum output voltage if the amplitude of the input pulse is increased to 20 V in Figure 20–36?

Use Multisim files E20-07A and E20-07B to verify the calculated results in this example and to confirm your calculation for the related problem. To simulate a single pulse, specify a waveform with the given pulse width but a small duty cycle.

EXAMPLE 20–8

A 10 V pulse with a width of 1 ms is applied to the *RL* integrator in Figure 20–38. Determine the voltage level that the output will reach during the pulse. If the source has an internal resistance of 30 Ω, how long will it take the output to decay to zero? Draw the output voltage waveform. Assume the inductor is ideal.



▲ FIGURE 20–38

Solution The inductor charges through the 30 Ω source resistance plus the 470 Ω external resistor. The time constant is

$$\tau = \frac{L}{R_{tot}} = \frac{500\text{ mH}}{470\ \Omega + 30\ \Omega} = \frac{500\text{ mH}}{500\ \Omega} = 1.0\text{ ms}$$

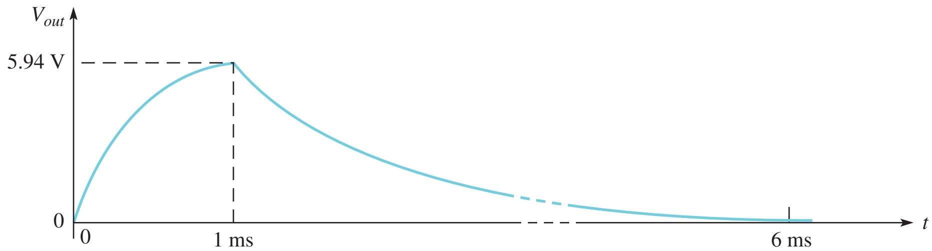
Notice that in this case the pulse width is exactly equal to τ. Thus, the voltage across the circuit resistance will reach approximately 63% of the full input amplitude in 1τ. Therefore, the voltage across the circuit's resistance reaches 6.32 V at the end of the pulse. This voltage is divided between the source resistance and R. Applying the voltage divider rule, the output will reach $(470\ \Omega/500\ \Omega)6.32\text{ V} = \mathbf{5.94\ V}$

After the pulse is gone, the inductor discharges back through the 30 Ω source resistance and the 470 Ω resistor. The output voltage takes $\mathbf{5\tau}$ to completely decay to zero.

$$5\tau = 5(1\text{ ms}) = 5\text{ ms}$$

The output voltage is shown in Figure 20–39.

▶ **FIGURE 20–39**



An alternate solution to the problem is to note that the maximum current is 20 mA (10 V/500 Ω = 20 mA) and solve for the current at 1 ms. At 1 ms, the output current is

$$i_{out(max)} = I_f(1 - e^{-t/\tau}) = 20\text{ mA}(1 - e^{-1\text{ ms}/1\text{ ms}}) = 12.6\text{ mA}$$

Applying Ohm's law, the output current at 1 ms is

$$v = iR = (12.6\text{ mA})(470\ \Omega) = \mathbf{5.94\ V}$$

Related Problem If the source has an internal resistance of 50 Ω, what is the maximum output voltage?

MultiSim

Use Multisim files E20-08A and E20-08B to verify the calculated results in this example and to confirm your calculation for the related problem. To simulate a single pulse, specify a waveform with the given pulse width but a small duty cycle.

SECTION 20–6 CHECKUP

1. In an *RL* integrator, across which component is the output voltage taken?
2. When a pulse is applied to an *RL* integrator, what condition must exist in order for the output voltage to reach the amplitude of the input?
3. Under what condition will the output voltage have the approximate shape of the input pulse?

20–7 Response of *RL* Differentiators to Pulse Inputs

A series *RL* circuit in which the output voltage is taken across the inductor is known as a differentiator in terms of time response. Although only the response to a single pulse is discussed, it can be extended to repetitive pulses, as was described for the *RC* differentiator.

After completing this section, you should be able to

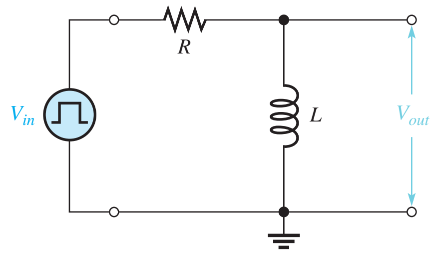
- **Analyze the operation of an *RL* differentiator**
  - Determine the response to a single input pulse

Response of the *RL* Differentiator to a Single Pulse

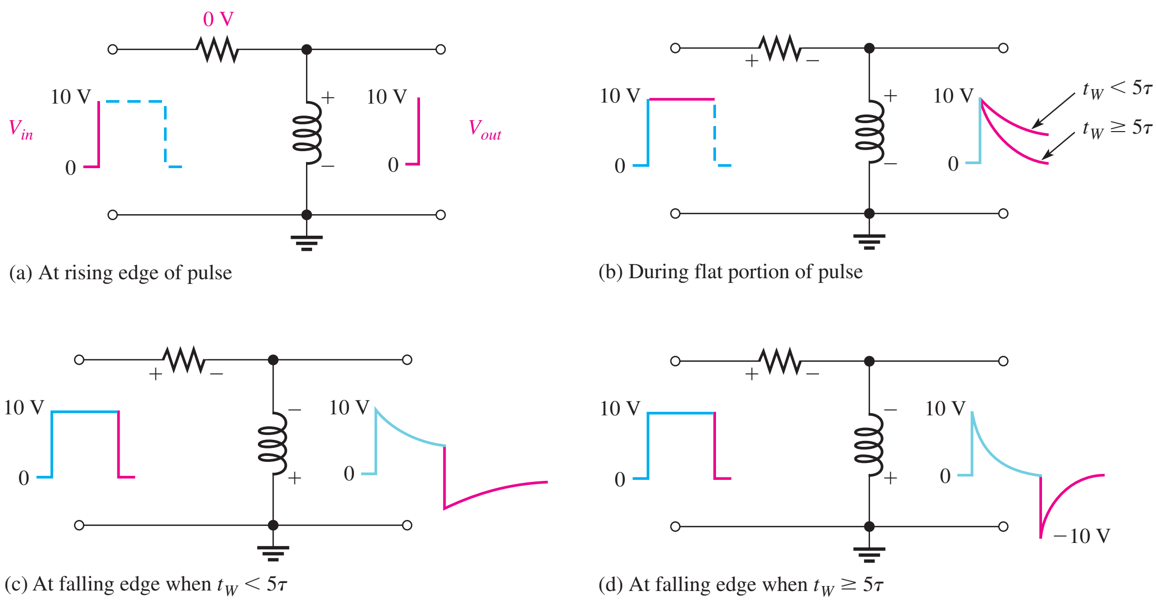
Figure 20–40 shows an *RL* differentiator with a pulse generator connected to the input.

▶ FIGURE 20–40

An *RL* differentiator with a pulse generator connected.



Initially, before the pulse, there is no current in the circuit. When the input pulse goes from its low level to its high level, the inductor prevents a sudden change in current. It does so, as you know, with an induced voltage equal and opposite to the input. As a result, *L* looks like an open, and all of the input voltage appears across it at the instant of the rising edge, as shown in Figure 20–41(a) with a 10 V pulse.



▲ FIGURE 20–41

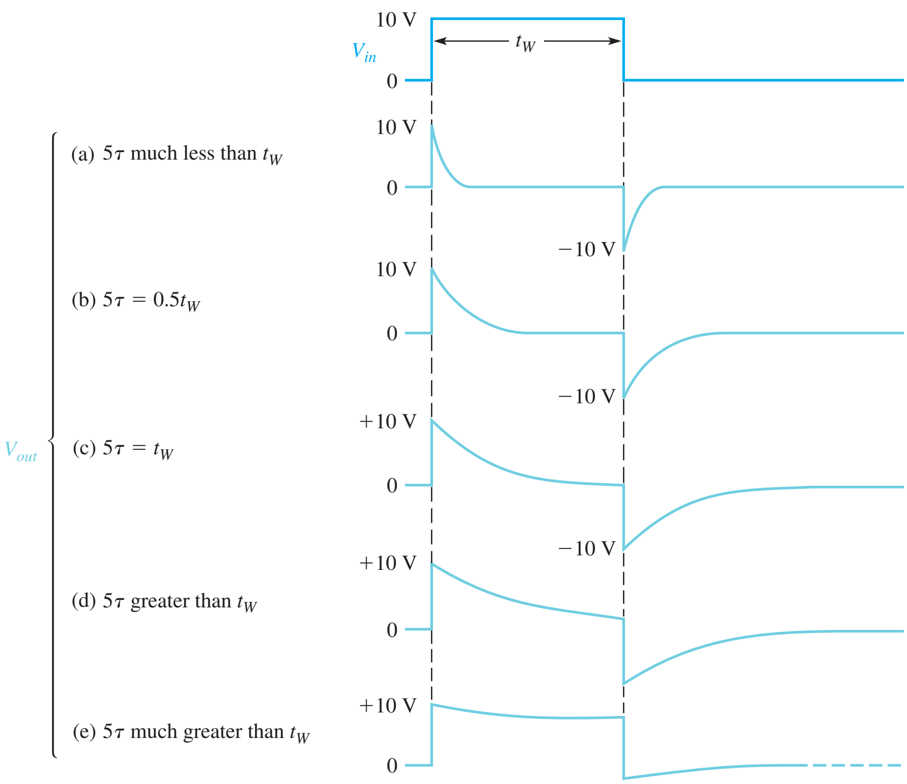
Illustration of the response of an *RL* differentiator for both time constant conditions.

During the pulse, the current exponentially builds up. As a result, the inductor voltage decreases, as shown in Figure 20–41(b). The rate of decrease, as you know, depends on the *L*/*R* time constant. When the falling edge of the input occurs, the inductor reacts to keep the current as is, by creating an induced voltage in a direction as indicated in Figure 20–41(c). This reaction is seen as a sudden negative-going transition of the inductor voltage, as indicated in Figure 20–41(c) and (d).

Two conditions are possible, as indicated in Figure 20–41(c) and (d). In part (c), 5τ is greater than the input pulse width, and the output voltage does not have time to decay to zero. In part (d), 5τ is less than or equal to the pulse width, and so the output decays to zero before the end of the pulse. In this case, a –10 V transition occurs at the trailing edge.

Keep in mind that as far as the input and output waveforms are concerned, the *RL* integrator and differentiator perform the same as their *RC* counterparts.

A summary of the *RL* differentiator response for relationships of various time constants and pulse widths is shown in Figure 20–42.



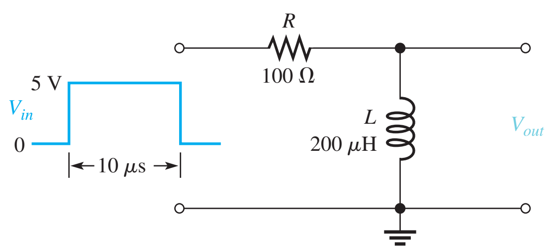
▲ **FIGURE 20–42**

Illustration of the variation in output pulse shape with the *RL* time constant.

EXAMPLE 20–9

Draw the output voltage for the *RL* differentiator in Figure 20–43.

▶ **FIGURE 20–43**



Solution First, calculate the time constant.

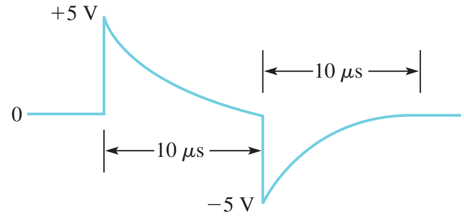
$$\tau = \frac{L}{R} = \frac{200\ \mu\text{H}}{100\ \Omega} = 2.0\ \mu\text{s}$$

In this case, $t_W = 5\tau$, so the output will decay to zero at the end of the pulse.

On the rising edge, the inductor voltage jumps to +5 V and then decays exponentially to zero. It reaches approximately zero at the instant of the falling edge.

On the falling edge of the input, the inductor voltage jumps to −5 V and then goes back to zero. The output waveform is shown in Figure 20–44.

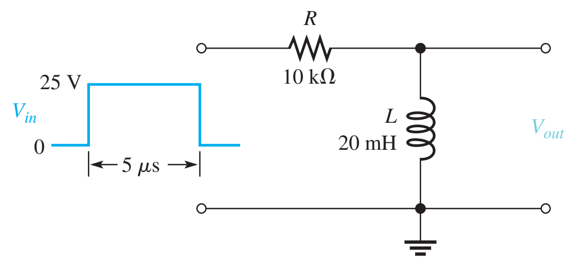
▶ FIGURE 20–44



Related Problem Draw the output voltage if the pulse width is reduced to 5 μs in Figure 20–43.

EXAMPLE 20–10

Determine the output voltage waveform for the *RL* differentiator in Figure 20–45.



▲ FIGURE 20–45

Solution First, calculate the time constant.

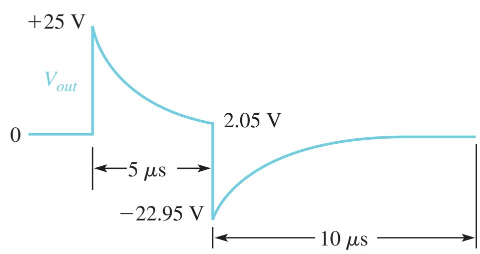
$$\tau = \frac{L}{R} = \frac{20\text{ mH}}{10\text{ k}\Omega} = 2.0\ \mu\text{s}$$

On the rising edge, the inductor voltage immediately jumps to +25 V. Because the pulse width is 5 μs, the inductor charges for only 2.5τ, so you must use the formula for a decreasing exponential derived from Equation 13–8 with $V_F = 0$ and $\tau = L/R$.

$$v_L = V_i e^{-t/\tau} = 25e^{-5\,\mu\text{s}/2\,\mu\text{s}} = 25e^{-2.5} = 2.05\text{ V}$$

This result is the inductor voltage at the end of the 5 μs input pulse.

On the falling edge, the output immediately jumps from +2.05 V down to −22.95 V (a 25 V negative-going transition). The complete output waveform is shown in Figure 20–46.



▲ FIGURE 20–46

Related Problem What must be the value of R for the output voltage to reach zero by the end of the pulse in Figure 20–45?

MultiSim

Use Multisim files E20-10A and E20-10B to verify the calculated results in this example and to confirm your calculation for the related problem. To simulate a single pulse, specify a waveform with the given pulse width but a small duty cycle.

SECTION 20–7 CHECKUP

1. In an *RL* differentiator, across which component is the output taken?
2. Under what condition does the output pulse shape most closely resemble the input pulse?
3. If the inductor voltage in an *RL* differentiator is down to +2 V at the end of a +10 V input pulse, to what negative voltage will the output go in response to the falling edge of the input?

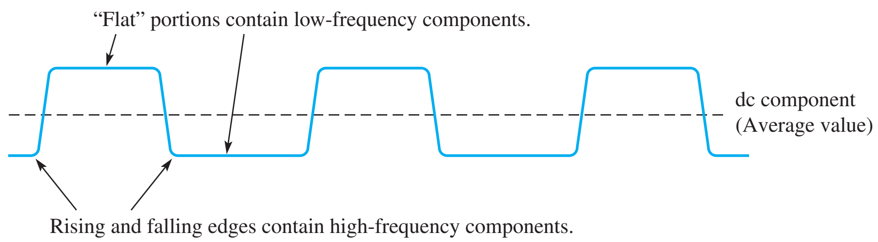
20–8 RELATIONSHIP OF TIME RESPONSE TO FREQUENCY RESPONSE

A definite relationship exists between time (pulse) response and frequency response. The fast rising and falling edges of a pulse waveform contain the higher frequency components. The flatter portions of the pulse waveform, which are the tops and the baseline of the pulses, represent slow changes or lower frequency components. The average value of the pulse waveform is its dc component.

After completing this section, you should be able to

- **Explain the relationship of time response to frequency response**
  - Describe a pulse waveform in terms of its frequency components
  - Explain how *RC* and *RL* integrators act as filters
  - Explain how *RC* and *RL* differentiators act as filters
  - State the formulas that relate rise and fall times to frequency

The relationships of pulse characteristics and frequency content of pulse waveforms are indicated in Figure 20–47.

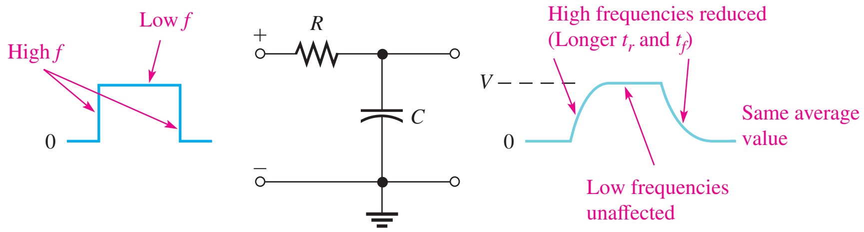


▲ **FIGURE 20–47**

Frequency content of a pulse waveform.

The Integrator

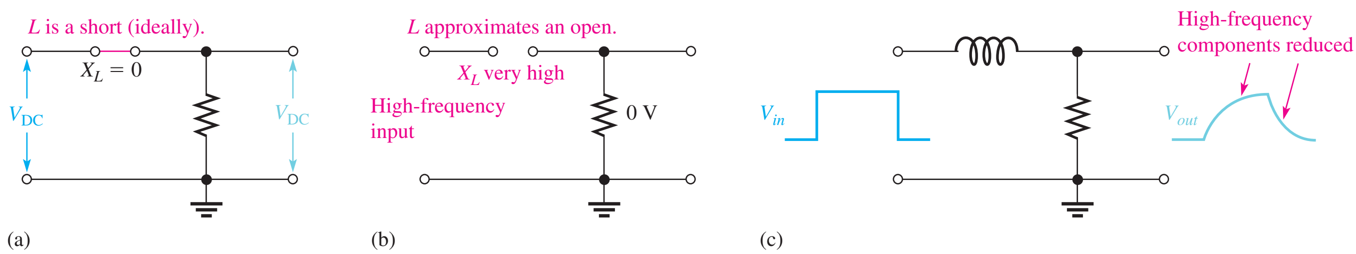
***RC* Integrator** In terms of frequency response, the *RC* integrator acts as a low-pass filter. As you learned, the *RC* integrator tends to exponentially "round off" the edges of the applied pulses. This rounding off occurs to varying degrees, depending on the relationship of the time constant to the pulse width and period. The rounding off of the edges indicates that the integrator tends to reduce the higher frequency components of the pulse waveform, as illustrated in Figure 20–48.



▲ FIGURE 20–48

Time and frequency response relationship in an *RC* integrator (one pulse in a repetitive waveform shown).

***RL* Integrator** Like the *RC* integrator, the *RL* integrator also acts as a basic low-pass filter because *L* is in series between the input and output. The inductive reactance, X_L, is small for low frequencies and offers little opposition. It increases with frequency, so at higher frequencies most of the total voltage is dropped across *L* and very little across *R*, the output. If the input is dc, *L* is like a short ($X_L = 0$). At high frequencies, *L* becomes like an open, as illustrated in Figure 20–49.



▲ FIGURE 20–49

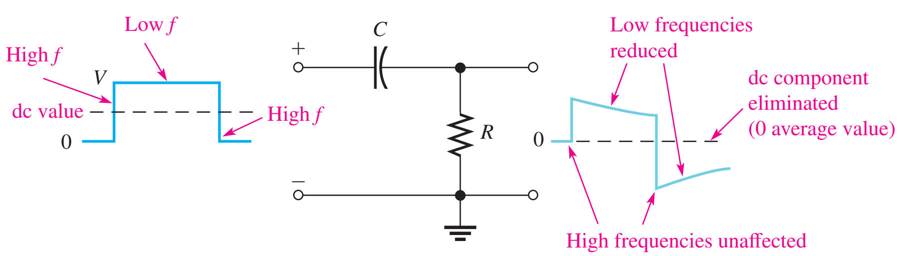
Low-pass filtering action.

The Differentiator

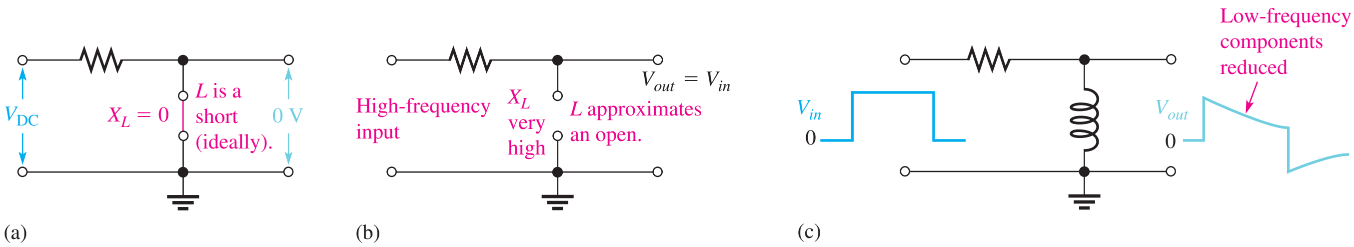
***RC* Differentiator** In terms of frequency response, the *RC* differentiator acts as a high-pass filter. As you know, the differentiator tends to introduce tilt to the flat portion of a pulse. That is, it tends to reduce the lower frequency components of a pulse waveform. Also, it completely eliminates the dc component of the input and produces a zero average-value output. This action is illustrated in Figure 20–50.

▶ FIGURE 20–50

Time and frequency response relationship in an *RC* differentiator (one pulse in a repetitive waveform shown).



***RL* Differentiator** Again like the *RC* differentiator, the *RL* differentiator also acts as a basic high-pass filter. Because L is connected across the output, less voltage is developed across it at lower frequencies than at higher ones. There are zero volts across the output for dc (ignoring winding resistance). For high frequencies, most of the input voltage is dropped across the output coil ($X_L = 0$ for dc; $X_L \cong$ open for high frequencies). Figure 20–51 shows high-pass filter action.



▲ **FIGURE 20–51**

High-pass filtering action.

Formulas Relating Time Response to Frequency Response

The fast transitions of a pulse (rise time, t_r, and fall time, t_f) are related to the highest frequency component, f_h, in that pulse by the following approximate formula:

$$t_r = \frac{0.35}{f_h} \qquad \textbf{Equation 20–1}$$

This formula also applies to fall time, and the fastest transition determines the highest frequency in the pulse waveform.

Equation 20–1 can be rearranged to give the highest frequency as follows:

$$f_h = \frac{0.35}{t_r} \qquad \textbf{Equation 20–2}$$

also,

$$f_h = \frac{0.35}{t_f} \qquad \textbf{Equation 20–3}$$

Equations 20–1 through 20–4 are based on a specific type of low-pass filter response. In some cases, such as high-end digital scopes, the constant is 0.45 rather than 0.35. In any case, the formulas should be treated as approximate.

TECH TIP

If you want to measure a fast rising pulse with an oscilloscope, you need to be aware of the bandwidth of the instrument or you may be measuring the rise time of the oscilloscope and probe rather than the signal of interest. For example, an oscilloscope/probe combination with a bandwidth of 100 MHz has a rise time of approximately 3.5 ns. Ideally, the scope should have a rise time that is 4X faster than the rise time of the signal to be measured or accuracy is sacrificed.

EXAMPLE 20–11

What is the highest frequency contained in a pulse that has rise and fall times equal to 10 nanoseconds (10 ns)?

Solution

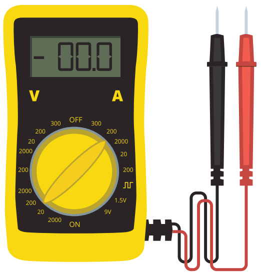
$$f_h = \frac{0.35}{t_r} = \frac{0.35}{10 \times 10^{-9}\text{ s}} = 0.035 \times 10^9\text{ Hz}$$
$$= 35 \times 10^6\text{ Hz} = \mathbf{35\ MHz}$$

Related Problem What is the highest frequency in a pulse with $t_r = 20$ ns and $t_f = 15$ ns?

SECTION 20–8 CHECKUP

1. What type of filter is an integrator?
2. What type of filter is a differentiator?
3. What is the highest frequency component in a pulse waveform having t_r and t_f equal to 1 μs?

20–9 Troubleshooting



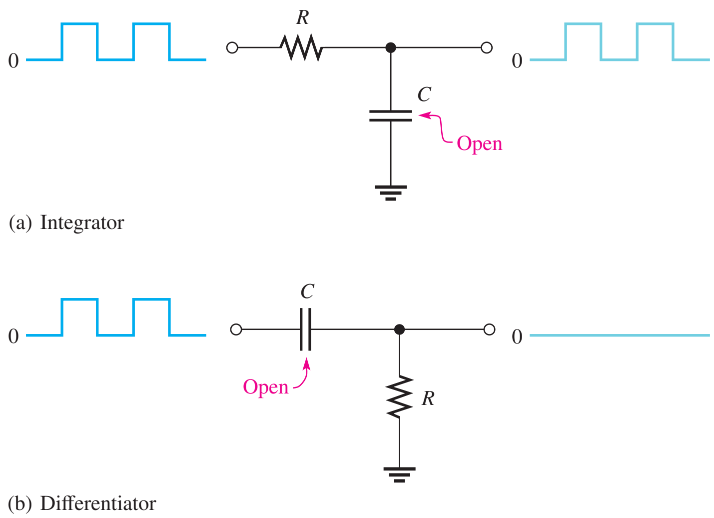
In this section, *RC* circuits with pulse inputs are used to demonstrate the effects of common component failures in selected cases. The concepts can then be easily related to *RL* circuits.

After completing this section, you should be able to

- **Troubleshoot *RC* integrators and *RC* differentiators**
  - Recognize the effect of an open capacitor
  - Recognize the effect of a leaky capacitor
  - Recognize the effect of a shorted capacitor
  - Recognize the effect of an open resistor

Open Capacitor

If the capacitor in an *RC* integrator opens, the output has the same waveshape as the input, as shown in Figure 20–52(a). If the capacitor in a differentiator opens, the output is zero because it is held at ground through the resistor, as illustrated in part (b).

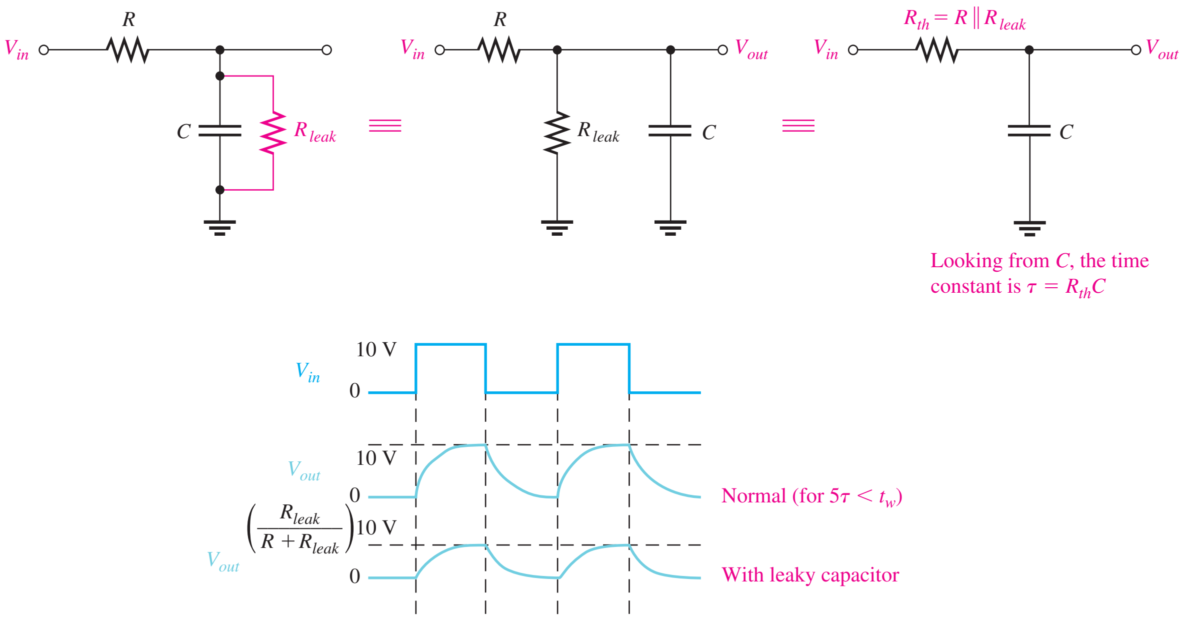


▲ FIGURE 20–52

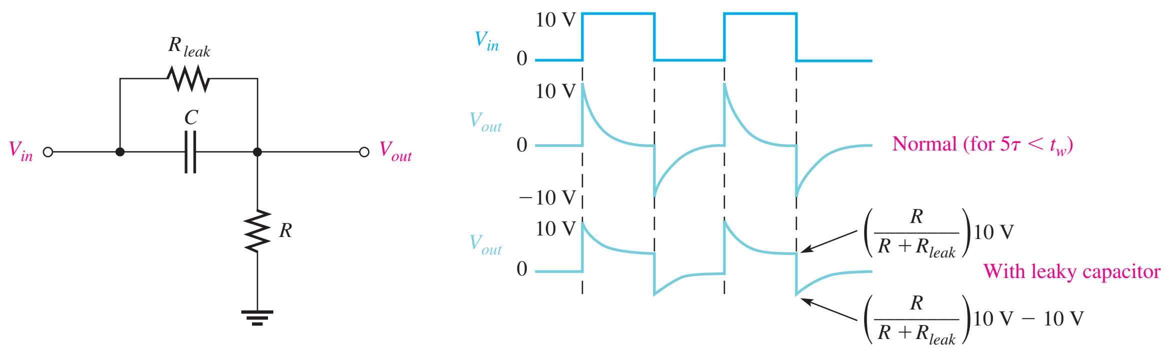
Examples of the effect of an open capacitor.

Leaky Capacitor

If the capacitor in an RC integrator becomes leaky, three things happen: (a) the time constant will be effectively reduced by the leakage resistance (when thevenized, looking from C it appears in parallel with R); (b) the waveshape of the output voltage (across C) is altered from normal by a shorter charging time; and (c) the amplitude of the output is reduced because R and R_{leak} effectively act as a voltage divider. These effects are illustrated in Figure 20–53(a).



(a)



(b)

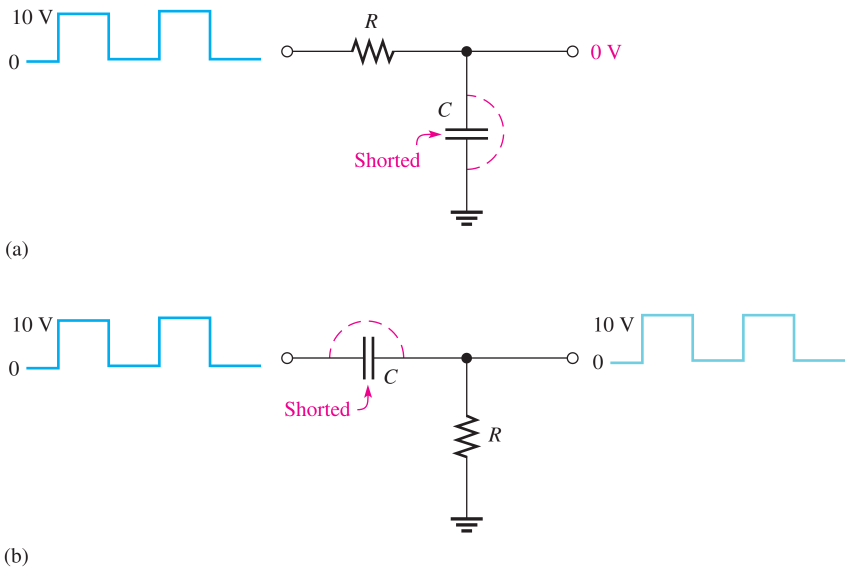
▲ **FIGURE 20–53**

Examples of the effect of a leaky capacitor.

If the capacitor in a differentiator becomes leaky, the time constant is reduced, just as in the integrator (they are both simply series RC circuits). When the capacitor reaches full charge, the output voltage (across R) is set by the effective voltage-divider action of R and R_{leak}, as shown in Figure 20–53(b).

Shorted Capacitor

If the capacitor in an *RC* integrator shorts, the output is at ground, as shown in Figure 20–54(a). If the capacitor in a *RC* differentiator shorts, the output is the same as the input, as shown in part (b).

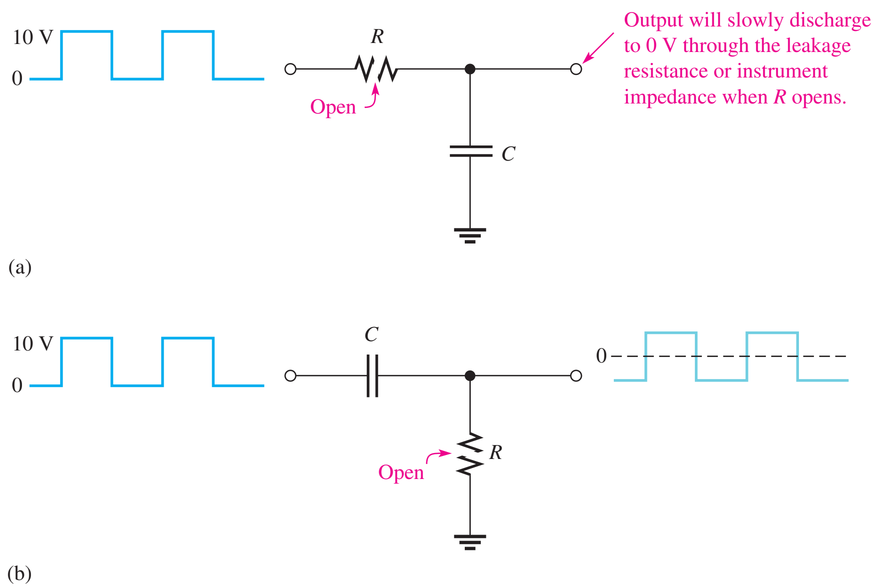


▲ FIGURE 20–54

Examples of the effect of a shorted capacitor.

Open Resistor

If the resistor in an *RC* integrator opens, the capacitor has no discharge path, and, ideally, it will hold its charge. In an actual situation, the charge will gradually leak off or the capacitor will discharge slowly through a measuring instrument connected to the output. This is illustrated in Figure 20–55(a).



▲ FIGURE 20–55

Examples of the effects of an open resistor.

If the resistor in a differentiator opens, the output looks like the input except for the dc level because the capacitor now must charge and discharge through the extremely high resistance of the oscilloscope, as shown in Figure 20–55(b).

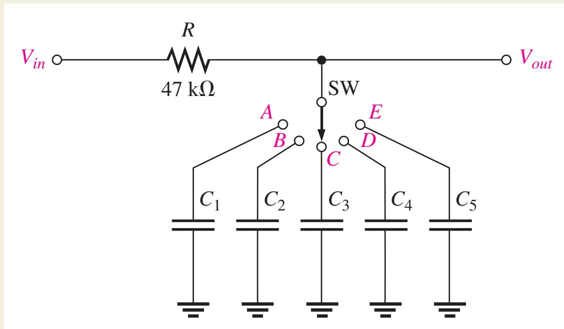
SECTION 20–9 CHECKUP

1. An *RC* integrator has a zero output with a square wave input. What are the possible causes of this problem?
2. If the capacitor in a differentiator is shorted, what is the output for a square wave input?

Application Activity

In this application activity, you are asked to build and test a time-delay circuit that will provide five switch-selectable delay times. An *RC* integrator is selected for this application. The input is a 5 V pulse of long duration, and the output goes to a threshold trigger circuit that is used to turn the power on to a portion of a system at any of the five selected time intervals after the occurrence of the original pulse.

A schematic of the selectable time-delay integrating circuit is shown in Figure 20–56. The *RC* integrator is driven by a pulse input; and the output is an exponentially increasing voltage that is used to trigger a threshold circuit at the 3.5 V level, which then turns power on to part of a system. The basic concept is shown in Figure 20–57. In this application, the delay time of the integrator is specified to be the time from the rising edge of the input pulse to the point where the output voltage reaches 3.5 V. The specified delay times are as listed in Table 20–1.

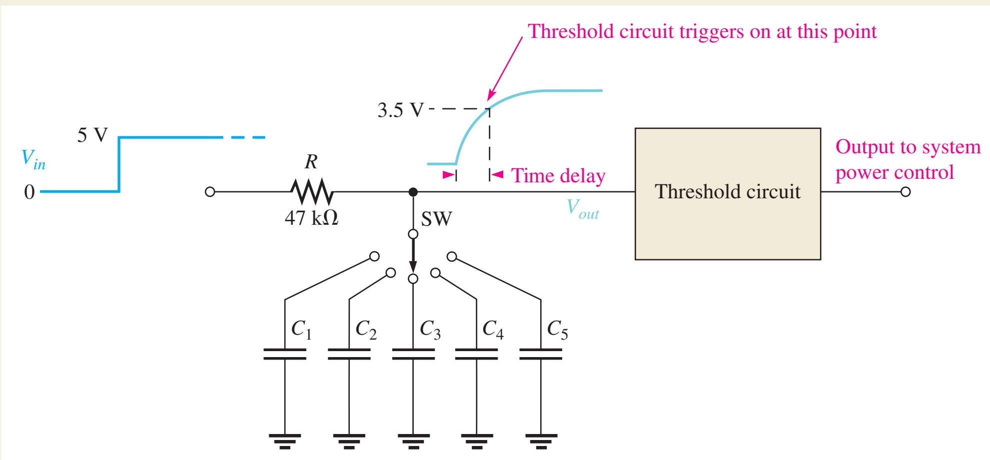


▲ **FIGURE 20–56**

Integrator delay circuit.

Capacitor Values

1. Determine a value for each capacitor that will provide the specified delay times within 10%. Select from the following list of standard values (all are in μF): 0.1,



▲ **FIGURE 20–57**

Illustration of the time-delay application.

▼ TABLE 20–1

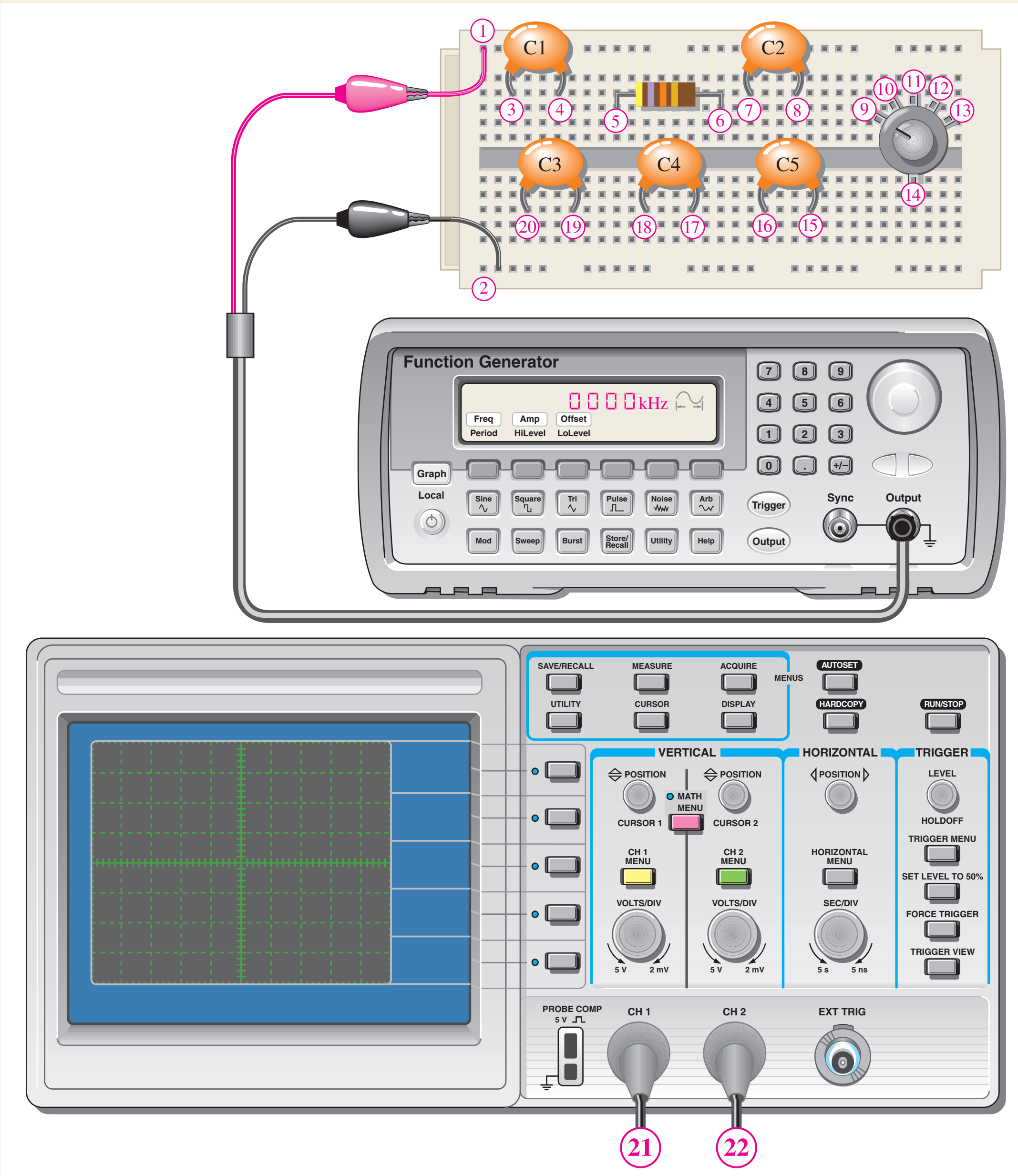
SWITCH POSITION	DELAY TIME
A	10 ms
B	25 ms
C	40 ms
D	65 ms
E	85 ms

0.12, 0.15, 0.18, 0.22, 0.27, 0.33, 0.39, 0.47, 0.56, 0.68, 0.82, 1.0, 1.2, 1.5, 1.8, 2.2, 2.7, 3.3, 3.9, 4.7, 5.6, 6.8, 8.2.

Circuit Connections

Refer to Figure 20–58. The components for the *RC* integrator in Figure 20–56 are assembled, but not interconnected, on the circuit board.

2. Using the circled numbers, develop a point-to-point wiring list to properly connect the circuit on the board.



▲ FIGURE 20–58

Selectable time delay integrating circuit.

3. Indicate, using the appropriate circled numbers, how you would connect the instruments to test the circuit.

Test Procedure

4. Specify the function, amplitude, and minimum frequency settings for the function generator in order to test all output delay times in Figure 20–58.
5. Specify the minimum oscilloscope settings for measuring each of the specified delay times in Figure 20–58.

Review

6. To add an additional time delay to the circuit of Figure 20–57, what changes must be made?
7. An additional time delay of 100 ms is required for the time-delay circuit. Determine the capacitor value that should be added.

SUMMARY

- In an *RC* integrating circuit, the output voltage is taken across the capacitor.
- In an *RC* differentiating circuit, the output voltage is taken across the resistor.
- In an *RL* integrating circuit, the output voltage is taken across the resistor.
- In an *RL* differentiating circuit, the output voltage is taken across the inductor.
- In an integrator, when the pulse width (t_W) of the input is much less than the transient time, the output voltage approaches a steady-state average value of the input.
- In an integrator, when the pulse width of the input is much greater than the transient time, the output voltage approaches the shape of the input.
- In a differentiator, when the pulse width of the input is much less than the transient time, the output voltage approaches the shape of the input but with an average value of zero.
- In a differentiator, when the pulse width of the input is much greater than the transient time, the output voltage consists of narrow, positive-going, and negative-going spikes occurring on the leading and trailing edges of the input pulses.
- The rising and falling edges of a pulse waveform contain the higher frequency components.
- The flat portion of the pulse contains the lower frequency components.

KEY TERMS

Key terms and other bold terms in the chapter are defined in the end-of-book glossary.

DC component The average value of a pulse waveform.

Differentiator A circuit producing an output that approaches the mathematical derivative of the input.

Integrator A circuit producing an output that approaches the mathematical integral of the input.

Steady state The equilibrium condition of a circuit that occurs after an initial transient time.

Transient time An interval equal to approximately five time constants.

FORMULAS

20–1 $$t_r = \frac{0.35}{f_h}$$ Rise time

20–2 $$f_h = \frac{0.35}{t_r}$$ Highest frequency in relation to rise time

20–3 $$f_h = \frac{0.35}{t_f}$$ Highest frequency in relation to fall time

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The output of an *RC* integrator is taken across the capacitor.
2. In an *RC* integrator, the capacitor voltage follows an exponential curve in response to a step input.
3. The time constant of a circuit is the delay between the input voltage and the output voltage.
4. For the output of a differentiator to reach the amplitude of the input pulse, the time constant must be very large compared to the pulse width.
5. The transient time of an *RC* circuit is the time for the capacitor to fully charge or discharge when there is a pulse input.
6. It takes five transient times to make up one time constant.
7. The output of an *RC* differentiator is taken across the resistor.
8. For the output of an integrator to approximate an input pulse, the time constant must be very small compared to the pulse width.
9. The output of an *RL* integrator is taken across the inductor.
10. In terms of frequency response, an integrator acts as a low-pass filter and a differentiator acts as a high-pass filter.

SELF-TEST

Answers are at the end of the chapter.

1. The output of an *RC* integrator is taken across the
   (a) resistor **(b)** capacitor **(c)** source **(d)** coil
2. When a 10 V input pulse with a width equal to one time constant is applied to an *RC* integrator, the capacitor charges to
   (a) 10 V **(b)** 5 V **(c)** 6.3 V **(d)** 3.7 V
3. When a 10 V pulse with a width equal to one time constant is applied to an *RC* differentiator, the capacitor charges to
   (a) 6.3 V **(b)** 10 V **(c)** 0 V **(d)** 3.7 V
4. In an *RC* integrator, the output pulse closely resembles the input pulse when
   (a) τ is much larger than the pulse width **(b)** τ is equal to the pulse width
   (c) τ is less than the pulse width **(d)** τ is much less than the pulse width
5. In an *RC* differentiator, the output pulse closely resembles the input pulse when
   (a) τ is much larger than the pulse width **(b)** τ is equal to the pulse width
   (c) τ is less than the pulse width **(d)** τ is much less than the pulse width
6. The positive and negative portions of a differentiator's output voltage are equal when
   (a) $5\tau < t_W$ **(b)** $5\tau > t_W$ **(c)** $5\tau = t_W$
   (d) $5\tau > 0$ **(e)** both (a) and (c) **(f)** both (b) and (d)
7. The output of an *RL* integrator is taken across the
   (a) resistor **(b)** coil **(c)** source **(d)** capacitor
8. The maximum possible current in an *RL* integrator is
   (a) $I = V_p/X_L$ **(b)** $I = V_p/Z$ **(c)** $I = V_p/R$
9. The current in an *RL* differentiator reaches its maximum possible value when
   (a) $5\tau = t_W$ **(b)** $5\tau < t_W$ **(c)** $5\tau > t_W$ **(d)** $\tau = 0.5t_W$
10. If you have an *RC* and an *RL* differentiator with equal time constants sitting side-by-side and you apply the same input pulse to both,
    (a) the *RC* has the widest output pulse
    (b) the *RL* has the most narrow spikes on the output
    (c) the output of one is an increasing exponential and the output of the other is a decreasing exponential
    (d) you can't tell the difference by observing the output waveforms

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 20–60.

1. If R_2 opens, the amplitude of the output voltage
   (a) increases (b) decreases (c) stays the same
2. If C doubled in value, the time constant
   (a) increases (b) decreases (c) stays the same
3. If R_1 is reduced in value, the output voltage amplitude
   (a) increases (b) decreases (c) stays the same

Refer to Figure 20–63.

4. If R_3 opens, the amplitude of the output voltage
   (a) increases (b) decreases (c) stays the same
5. If a 1,000 pF capacitor replaces C, the output voltage
   (a) increases (b) decreases (c) stays the same
6. If R_1 is 3.3 kΩ instead of 2.2 kΩ, the time constant
   (a) increases (b) decreases (c) stays the same

Refer to Figure 20–66.

7. If L is increased, the rise time of the output
   (a) increases (b) decreases (c) stays the same
8. If the width of the input pulse is increased to 10 μs, the amplitude of the output pulse
   (a) increases (b) decreases (c) stays the same

Refer to Figure 20–68.

9. If R_1 opens, the maximum amplitude of the output
   (a) increases (b) decreases (c) stays the same
10. If R_2 is shorted, the maximum amplitude of the output
    (a) increases (b) decreases (c) stays the same

PROBLEMS

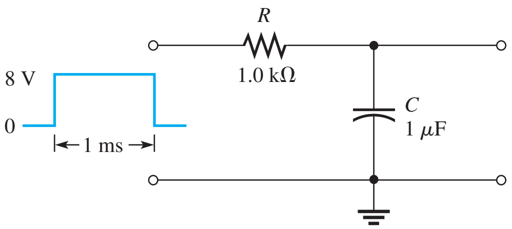
Answers to odd-numbered problems are at the end of the book.

SECTION 20–1 The *RC* Integrator

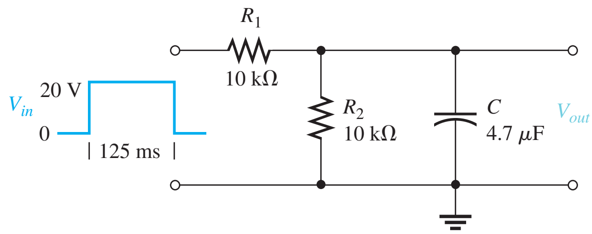
1. An integrating circuit has $R = 2.2\ \text{k}\Omega$ in series with $C = 0.047\ \mu\text{F}$. What is the time constant?
2. Determine how long it takes the capacitor in an integrating circuit to reach full charge for each of the following series *RC* combinations:
   (a) $R = 56\ \Omega$, $C = 47\ \mu\text{F}$ (b) $R = 3{,}300\ \Omega$, $C = 0.015\ \mu\text{F}$
   (c) $R = 22\ \text{k}\Omega$, $C = 100\ \text{pF}$ (d) $R = 5.6\ \text{M}\Omega$, $C = 10\ \text{pF}$

SECTION 20–2 Response of an *RC* Integrator to a Single Pulse

3. A 20 V pulse is applied to an *RC* integrator. The pulse width equals one time constant. To what voltage does the capacitor charge during the pulse? Assume that it is initially uncharged.
4. Repeat Problem 3 for the following values of t_W:
   (a) 2τ (b) 3τ (c) 4τ (d) 5τ
5. Draw the approximate shape of an integrator output voltage where 5τ is much less than the pulse width of a 10 V square-wave input. Repeat for the case in which 5τ is much larger than the pulse width.
6. Determine the output voltage for an *RC* integrator with a single input pulse, as shown in Figure 20–59. For repetitive pulses, how long will it take this circuit to reach steady state?



▲ FIGURE 20–59

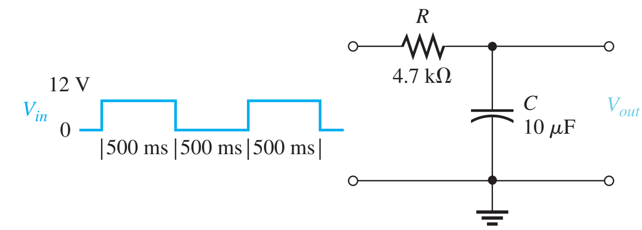


▲ FIGURE 20–60

7. **(a)** What is τ in Figure 20–60?
   (b) Draw the output voltage.
8. Sketch the output voltage in Figure 20–60 if the pulse width is increased to 1.25 s.
9. Repeat Problem 8 if the pulse width is reduced to 23.5 ms.

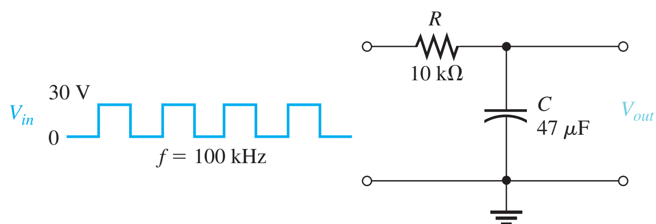
SECTION 20–3 Response of *RC* Integrators to Repetitive Pulses

10. Determine the transient time for Figure 20–61.
11. Draw the integrator output voltage in Figure 20–61, showing maximum voltages.
12. Sketch the output voltage if the pulse width of V_{in} in Figure 20–60 is changed to 47 ms and the frequency is the same.



▲ FIGURE 20–61

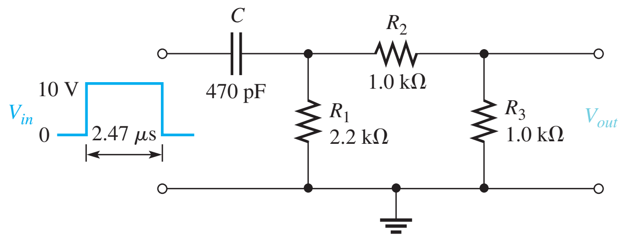
13. A 1 V, 10 kHz pulse waveform with a duty cycle of 25% is applied to an integrator with $\tau = 25\ \mu s$. Graph the output voltage for three initial pulses. *C* is initially uncharged.
14. What is the steady-state output voltage of the *RC* integrator with a square-wave input shown in Figure 20–62?



▲ FIGURE 20–62

SECTION 20–4 Response of an *RC* Differentiator to a Single Pulse

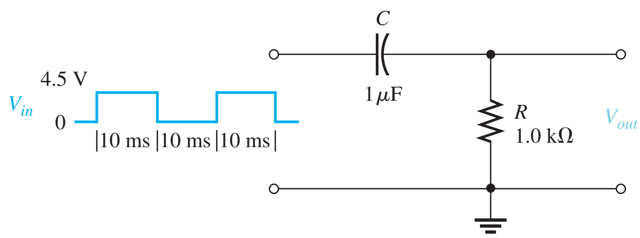
15. Repeat Problem 5 for an *RC* differentiator.
16. Redraw the circuit in Figure 20–59 to make it a differentiator, and repeat Problem 6.
17. **(a)** What is τ in Figure 20–63?
    (b) Draw the output voltage.



▲ FIGURE 20–63

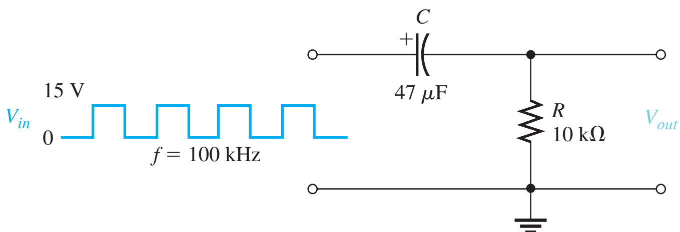
SECTION 20–5 Response of *RC* Differentiators to Repetitive Pulses

18. Draw the differentiator output in Figure 20–64, showing maximum voltages.



▲ FIGURE 20–64

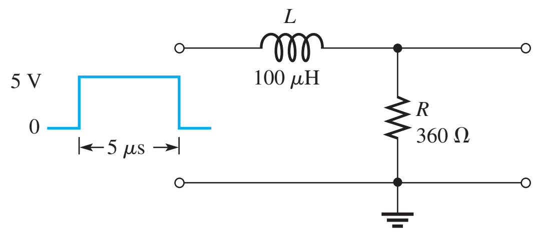
19. What is the steady-state output voltage of the differentiator with the square-wave input shown in Figure 20–65?



▲ FIGURE 20–65

SECTION 20–6 Response of *RL* Integrators to Pulse Inputs

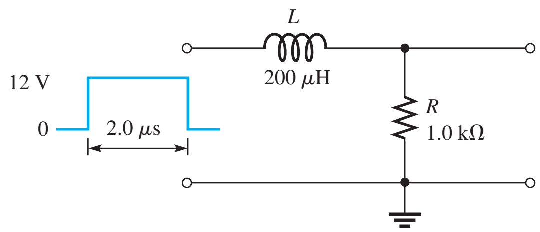
20. Determine the output voltage for the circuit in Figure 20–66. A single input pulse is applied as shown.



▲ FIGURE 20–66

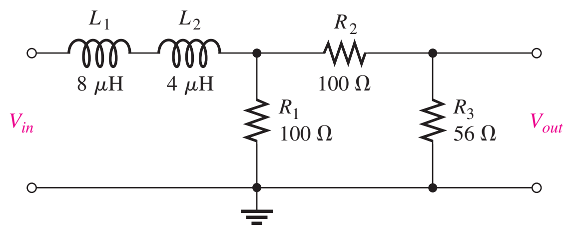
21. Draw the integrator output voltage in Figure 20–67, showing maximum voltages.

▶ FIGURE 20–67



22. Determine the time constant in Figure 20–68. Is this circuit an integrator or a differentiator?

▶ FIGURE 20–68

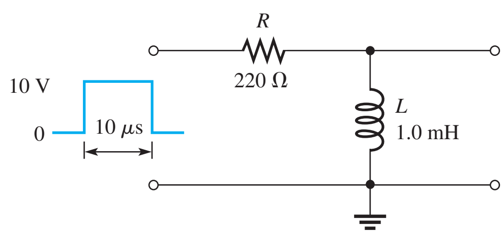


SECTION 20–7 Response of *RL* Differentiators to Pulse Inputs

23. **(a)** What is τ in Figure 20–69?

(b) Draw the output voltage.

▶ FIGURE 20–69



24. Draw the output waveform if a periodic pulse waveform with $t_W = 25\ \mu s$ and $T = 60\ \mu s$ is applied to the circuit in Figure 20–69.

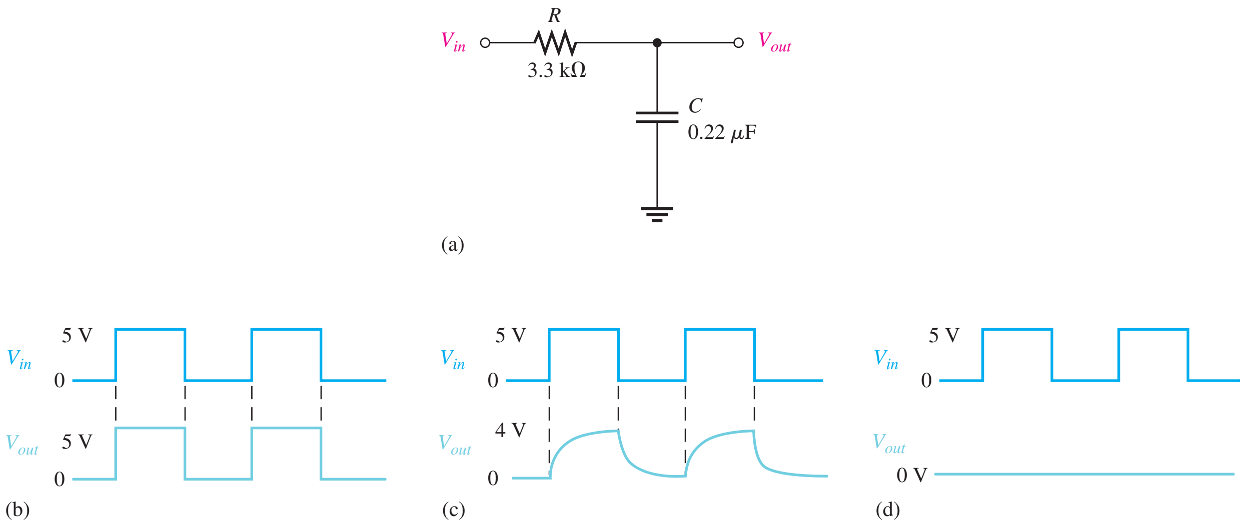
SECTION 20–8 Relationship of Time Response to Frequency Response

25. What is the highest frequency component in the output of an integrator with $\tau = 10\ \mu s$? Assume that $5\tau < t_W$.

26. A certain pulse waveform has a rise time of 55 ns and a fall time of 42 ns. What is the highest frequency component in the waveform?

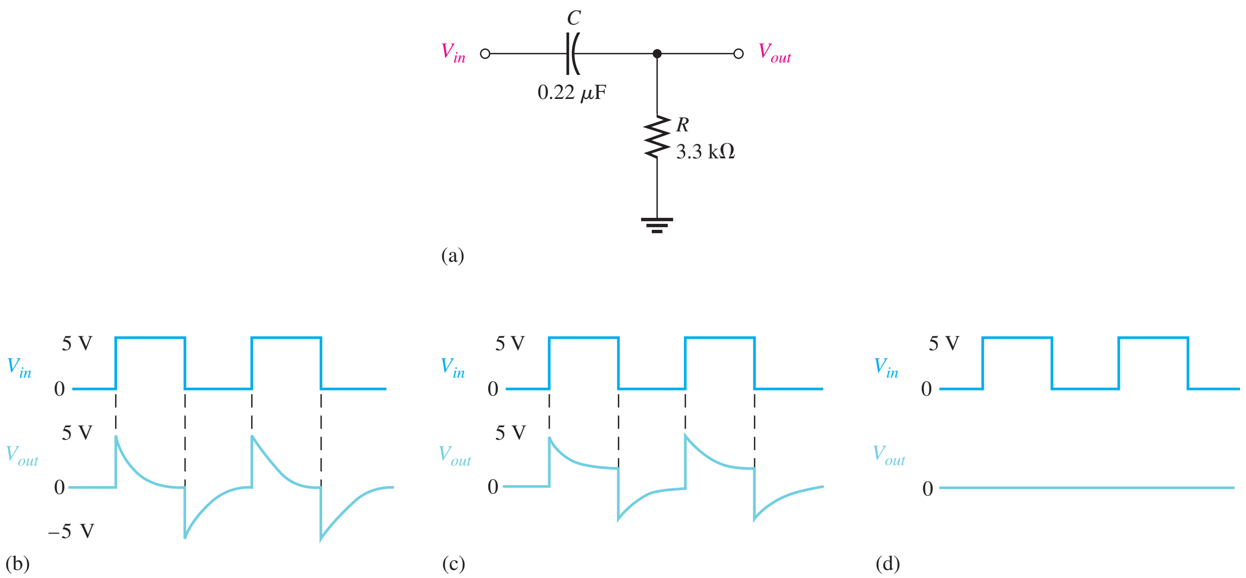
SECTION 20–9 Troubleshooting

27. Determine the most likely fault(s) in the circuit of Figure 20–70(a) for each set of waveforms in parts (b) through (d). V_{in} is a square wave with a period of 8 ms.



▲ **FIGURE 20–70**

28. Determine the most likely fault(s), if any, in the circuit of Figure 20–71(a) for each set of waveforms in parts (b) through (d). V_{in} is a square wave with a period of 8 ms.



▲ **FIGURE 20–71**

Multisim Troubleshooting and Analysis

These problems require Multisim.

29. Open file P20-29 and determine if there is a fault. If so, find the fault.

30. Open file P20-30 and determine if there is a fault. If so, find the fault.

31. Open file P20-31 and determine if there is a fault. If so, find the fault.

32. Open file P20-32 and determine if there is a fault. If so, find the fault.

ANSWERS

SECTION CHECKUPS

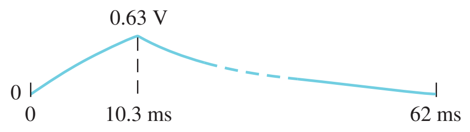
SECTION 20–1 The *RC* Integrator

1. An integrator is a series *RC* circuit in which the output is across the capacitor.
2. A voltage applied to the input causes the capacitor to charge. A short across the input causes the capacitor to discharge.

SECTION 20–2 Response of an *RC* Integrator to a Single Pulse

1. For the output of an integrator to reach amplitude, $5\tau \leq t_W$.
2. $V_{out(max)} = 630\text{ mV}$; $t_{\text{disch}} = 51.7\text{ ms}$
3. See Figure 20–72.

▶ FIGURE 20–72



4. No, *C* will not fully charge.
5. The output has approximately the shape of the input when $5\tau << t_W$ (5τ much less than t_W).

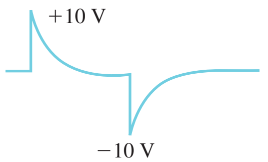
SECTION 20–3 Response of *RC* Integrators to Repetitive Pulses

1. *C* will fully charge and discharge when $5\tau \leq t_W$ and $5\tau \leq$ time between pulses.
2. When $\tau << t_W$, the output is approximately like the input.
3. Transient time
4. Steady-state response is an equilibrium condition of a circuit that occurs after an initial transient time.
5. An approximate dc voltage that is the average value of the input

SECTION 20–4 Response of an *RC* Differentiator to a Single Pulse

1. See Figure 20–73.

▶ FIGURE 20–73



2. The output resembles the input when $5\tau >> t_W$.
3. The output appears to be positive and negative spikes corresponding to the rising and falling edges of the input.
4. V_R will go to -10 V.

SECTION 20–5 Response of *RC* Differentiators to Repetitive Pulses

1. *C* will fully charge and discharge when $5\tau \leq t_W$ and $5\tau \leq$ time between pulses.
2. The output appears to be positive and negative spikes.
3. The average value is 0 V.

SECTION 20–6 Response of *RL* Integrators to Pulse Inputs

1. The output is taken across the resistor.
2. The output reaches the input amplitude when $5\tau \leq t_W$.
3. The output has the approximate shape of the input when $5\tau << t_W$.

SECTION 20–7 Response of *RL* Differentiators to Pulse Inputs

1. The output is taken across the inductor.

2. The output has the approximate shape of the input when $5\tau >> t_W$.

3. V_L will go to -8 V.

SECTION 20–8 Relationship of Time Response to Frequency Response

1. An integrator is a low-pass filter.

2. A differentiator is a high-pass filter.

3. $f_{max} = 350$ kHz

SECTION 20–9 Troubleshooting

1. A 0 V output may be caused by an open resistor or shorted capacitor.

2. If C is shorted, the output is the same as the input.

RELATED PROBLEMS FOR EXAMPLES

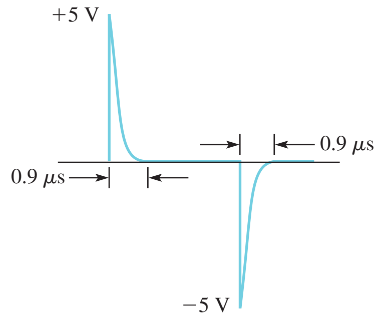
20–1 8.65 V

20–2 24.7 V

20–3 1.08 V

20–4 See Figure 20–74.

▶ FIGURE 20–74



20–5 892 mV

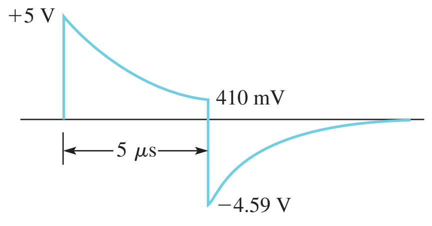
20–6 90 Ω

20–7 20 V

20–8 5.71 V

20–9 See Figure 20–75.

▶ FIGURE 20–75



20–10 20 kΩ

20–11 23.3 MHz

TRUE/FALSE QUIZ

1. T	**2.** T	**3.** F	**4.** F	**5.** T
6. F	**7.** T	**8.** T	**9.** F	**10.** T

SELF-TEST

1. (b)	**2.** (c)	**3.** (a)	**4.** (d)	**5.** (a)
6. (e)	**7.** (a)	**8.** (c)	**9.** (b)	**10.** (d)

CIRCUIT DYNAMICS QUIZ

1. (a)	**2.** (a)	**3.** (a)	**4.** (a)	**5.** (b)
6. (a)	**7.** (a)	**8.** (c)	**9.** (c)	**10.** (a)