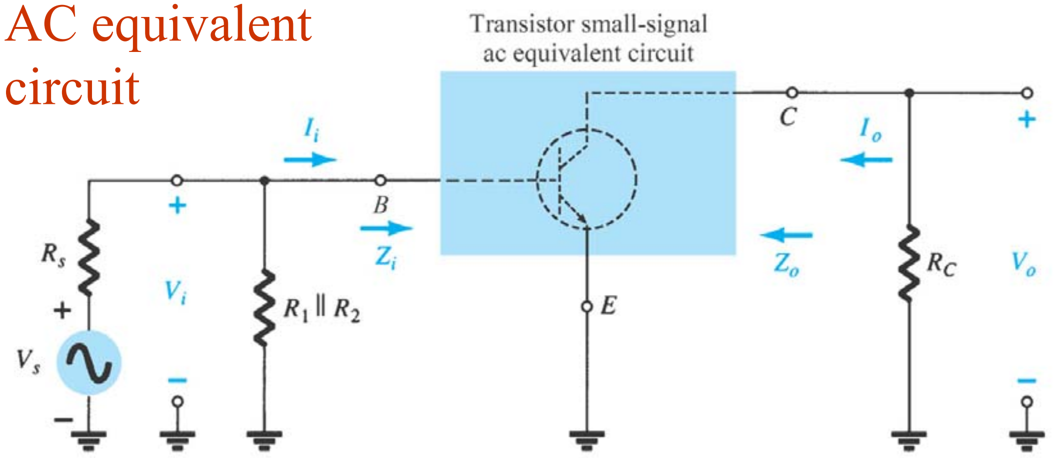
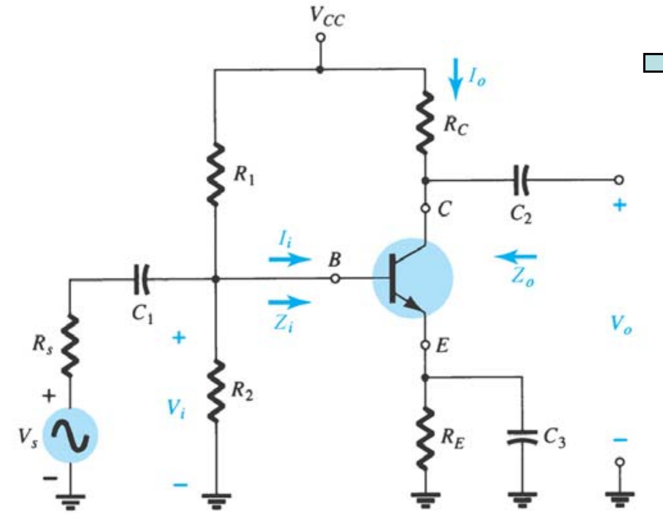
Chapter 5:
BJT AC Analysis

5.3 BJT Transistor Modeling

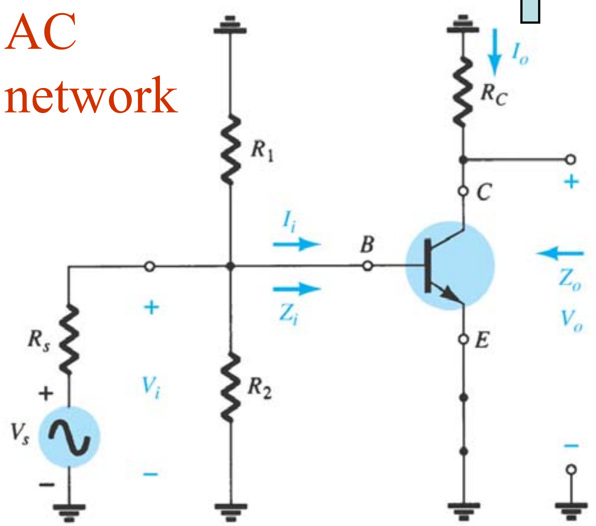
- **A model is an equivalent circuit that represents the AC characteristics of the transistor.**
- **A model uses circuit elements that approximate the behavior of the transistor.**
- **There are three models commonly used in small signal AC analysis of a transistor:**
  - **r_e model**
  - **Hybrid equivalent model**
  - **Hybrid ∏ model**

AC equivalent circuit





AC network



Sketch an AC network:

1. Remove DC supplies (replaced by short)
2. The coupling capacitor and bypass capacitor can be replaced by a short

5.4 The r_e Transistor Model

BJTs are basically current-controlled devices, therefore the r_e model uses a diode and a current source to duplicate the behavior of the transistor.

One disadvantage to this model is its sensitivity to the DC level. This model is designed for specific circuit conditions.

Common-Base Configuration

$$I_c = \alpha I_e \qquad r_e = \frac{26\,\text{mV}}{I_E}$$

Input impedance: Low

$$Z_i = r_e$$

Output impedance: High

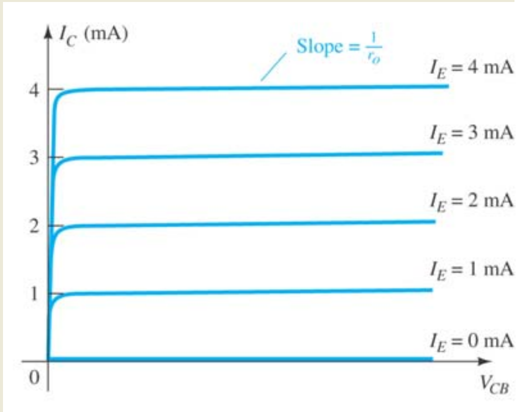
$$Z_o \cong \infty\Omega$$

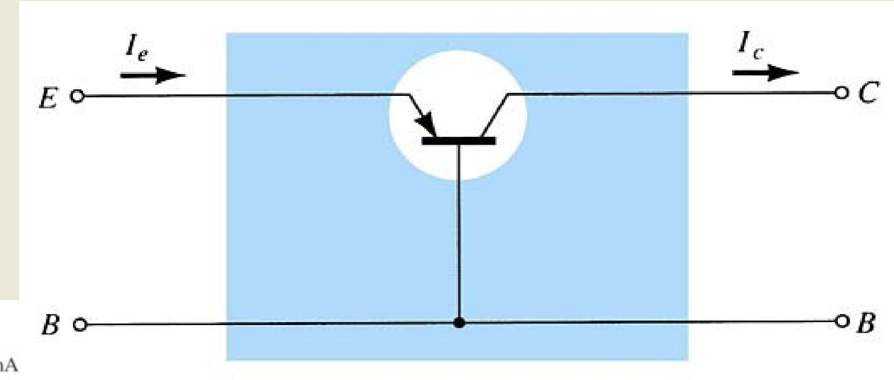
Voltage gain: voltage amplification

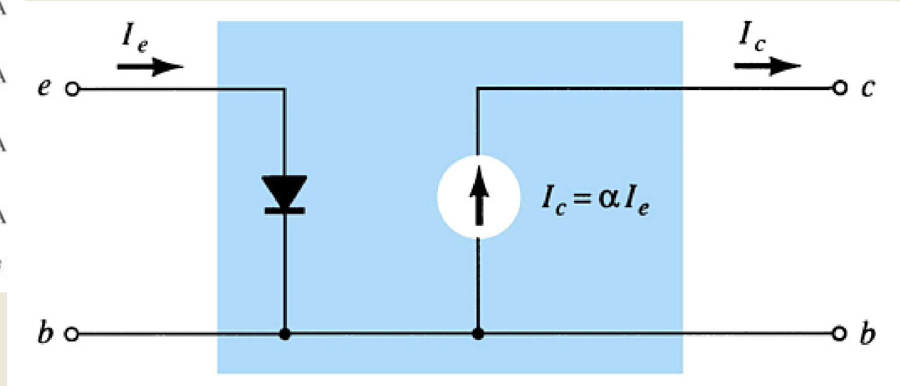
$$A_V = \frac{\alpha R_L}{r_e} \cong \frac{R_L}{r_e}$$

Current gain: No current amplification

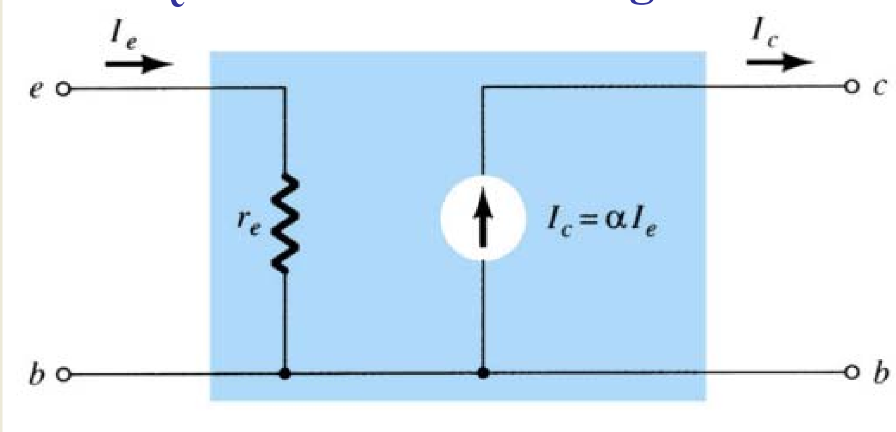
$$A_i = -\alpha \cong -1$$







r_e model for CB configuration



Common-Emitter Configuration

The diode r_e model can be replaced by the resistor r_e.

$$I_e = (\beta + 1) I_b \cong \beta I_b$$

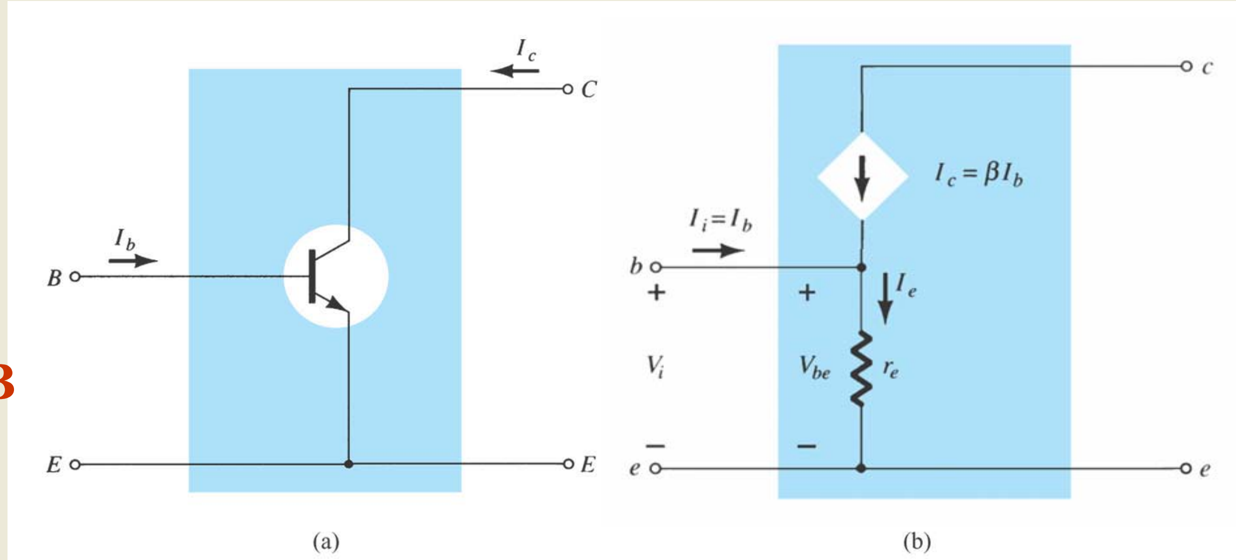
$$r_e = \frac{26\,\text{mV}}{I_e}$$

Input impedance: higher than CB

$$Z_i = \beta r_e$$

Output impedance: lower than CB

$$Z_o = r_o \cong \infty \Omega$$

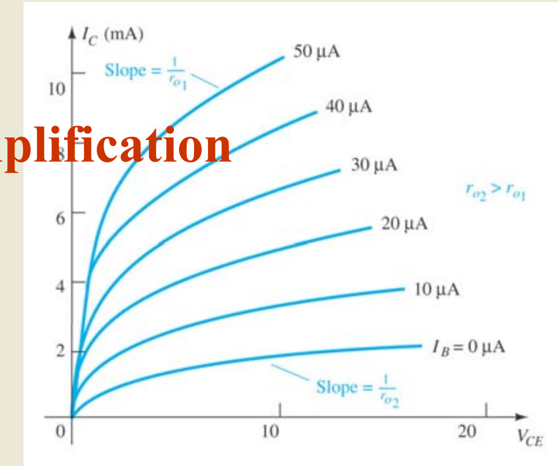


Voltage gain: Voltage amplification, V_o and V_i are 180° out of phase

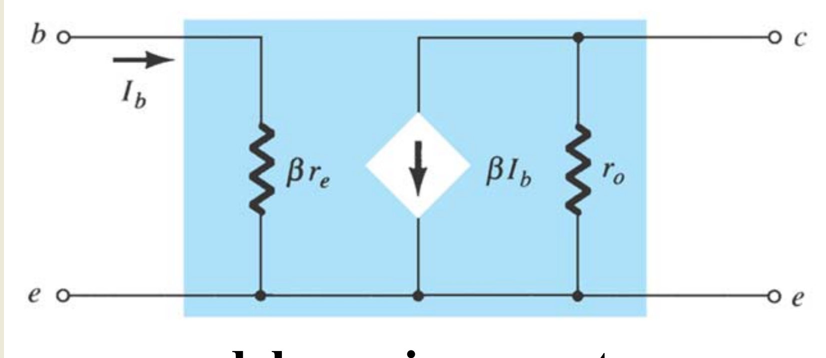
$$A_V = -\frac{R_L}{r_e}$$

Current gain: Current amplification

$$A_i = \beta \big|_{r_o = \infty}$$



r_e model for CE configuration

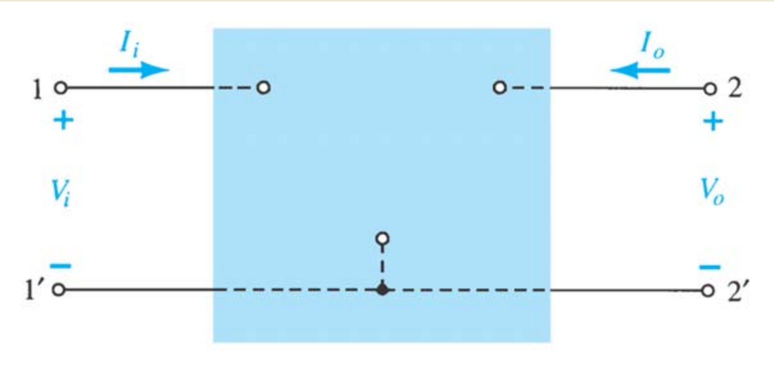


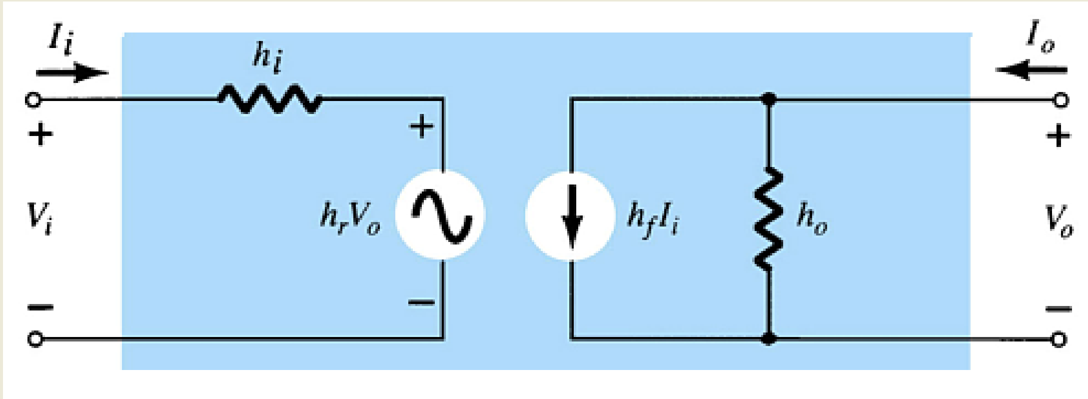
r_e model requires you to determine β, r_e, and r_o.

Use the common-emitter model for the common-collector configuration.

5.5 The Hybrid Equivalent Model

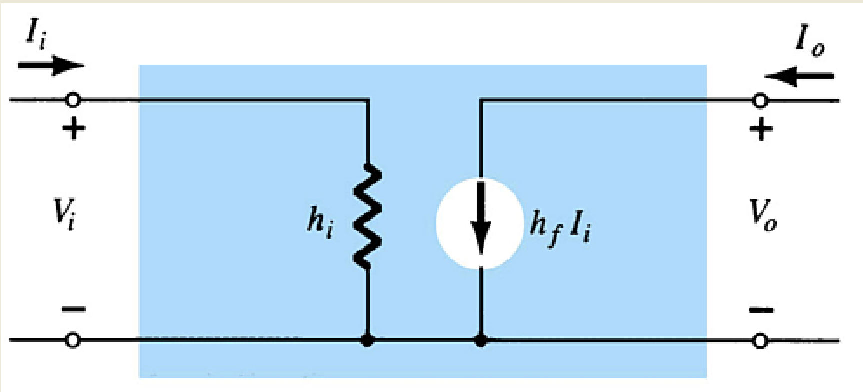
The following hybrid parameters are developed and used for modeling the transistor. These parameters can be found in a specification sheet for a transistor.





- h_i = input resistance
- h_r = reverse transfer voltage ratio (V_i/V_o) ≅ 0
- h_f = forward transfer current ratio (I_o/I_i)
- h_o = output conductance ≅ ∞

Simplified General H-Parameter Model: Approximate hybrid equivalent model



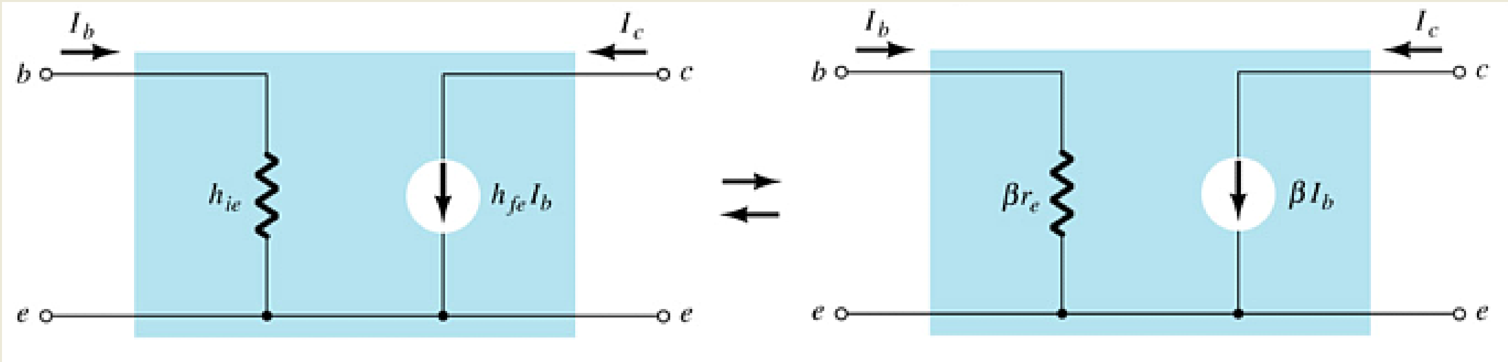
- h_i = input resistance
- h_r = reverse transfer voltage ratio (V_i/V_o) ≅ 0
- h_f = forward transfer current ratio (I_o/I_i)
- h_o = output conductance ≅ ∞

r_e Model vs. h-Parameter Model

Common-Emitter

$$h_{ie} = \beta r_e$$

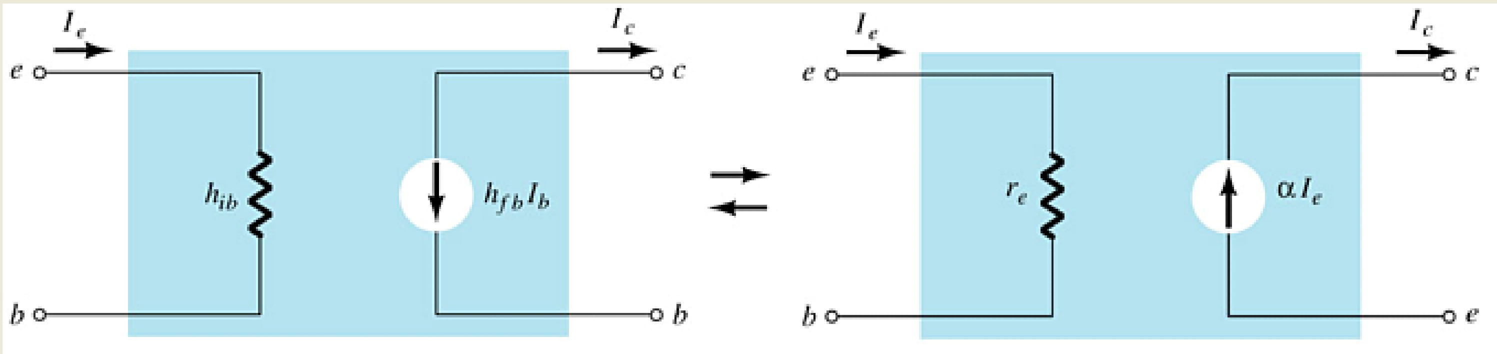
$$h_{fe} = \beta_{ac}$$



Common-Base

$$h_{ib} = r_e$$

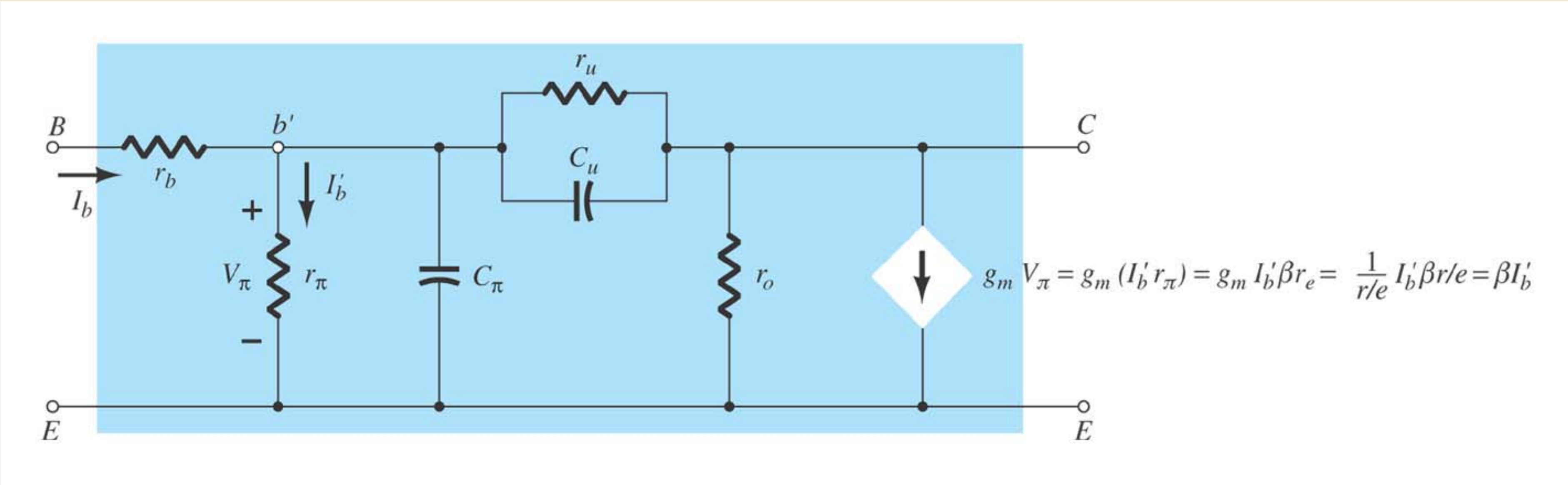
$$h_{fb} = -\alpha \cong -1$$



5.6 The Hybrid π Model

The hybrid π model is most useful for analysis of high-frequency transistor applications.

At lower frequencies the hybrid π model closely approximate the r_e parameters, and can be replaced by them.



AC Analysis with Equivalent models

Section 5.8 CE with fix-bias

Section 5.9 CE with voltage-divider bias

Section 5.10 CE with emitter bias

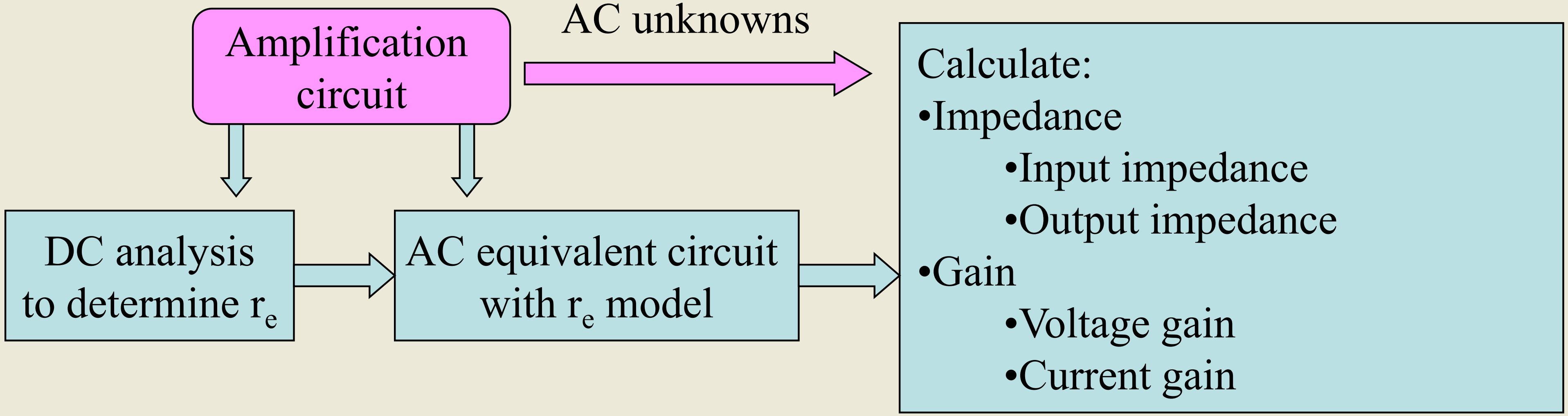
Section 5.14 CE with dc collector feedback bias

Section 5.13 CE with collector feedback

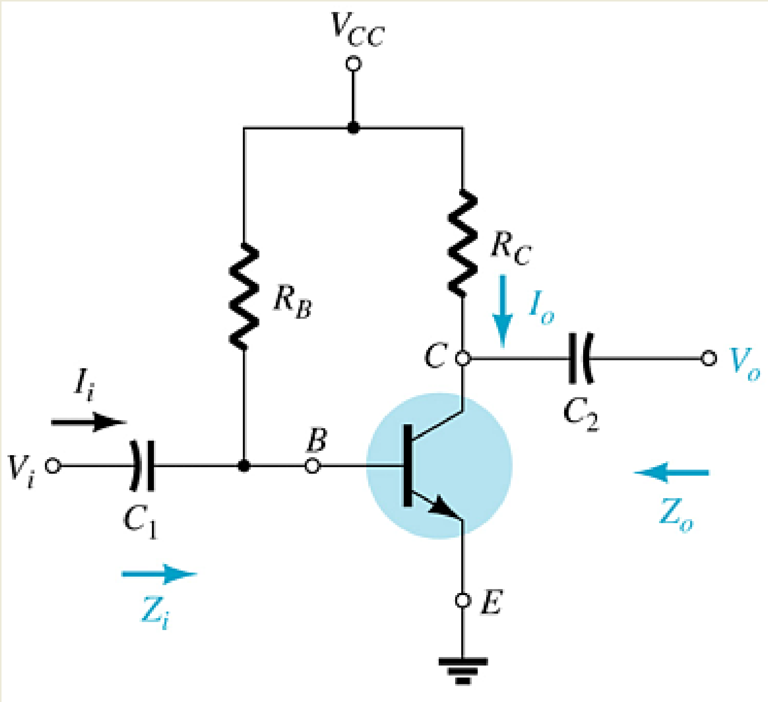
CE

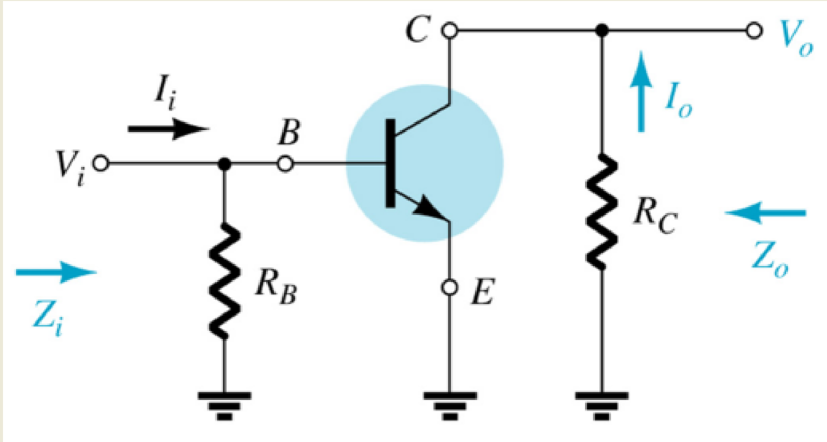
Section 5.11 CC: Emitter follower

Section 5.12 CB

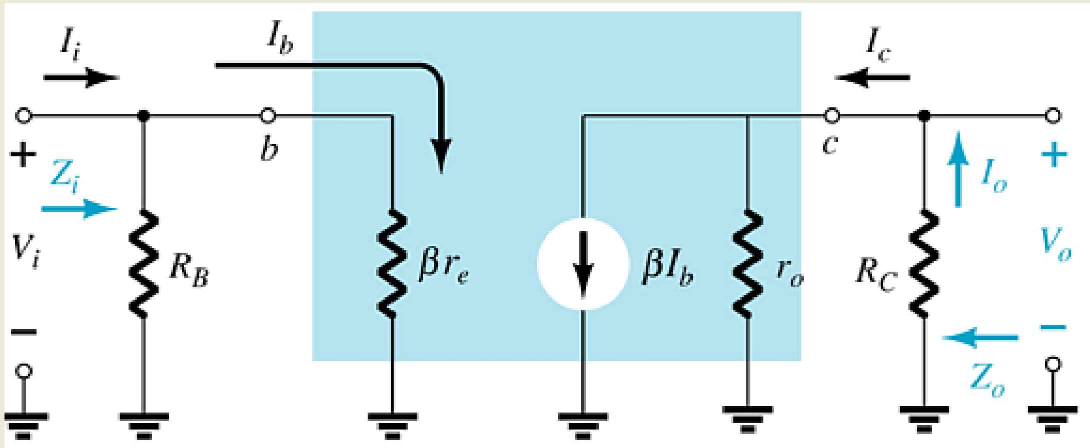


5.8 Common-Emitter Fixed-Bias Configuration





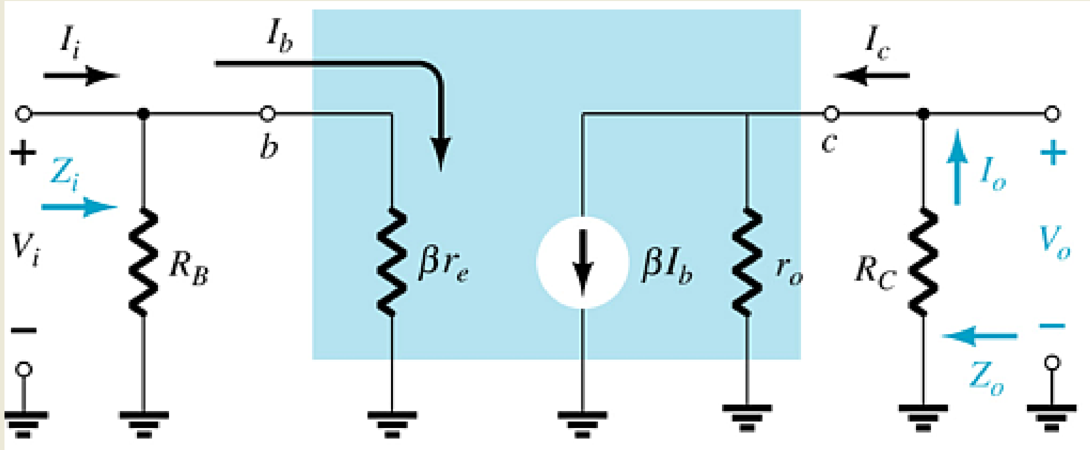
AC network



AC equivalent with r_e model

- The input is applied to the base
- The output is from the collector
- High input impedance
- Low output impedance
- High voltage and current gain
- Phase shift between input and output is 180°

Common-Emitter Fixed-Bias Calculations



Input impedance:

$$Z_i = R_B \parallel \beta r_e$$

$$Z_i \cong \beta r_e \Big|_{R_E \geq 10\beta r_e}$$

Output impedance:

$$Z_o = R_C \parallel r_O$$

$$Z_o \cong R_C \Big|_{r_o \geq 10 R_C}$$

Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{(R_C \parallel r_o)}{r_e}$$

$$A_v = -\frac{R_C}{r_e} \Big|_{r_o \geq 10 R_C}$$

Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_B r_o}{(r_o + R_C)(R_B + \beta r_e)}$$

$$A_i \cong \beta \Big|_{r_o \geq 10 R_C,\, R_B \geq 10\beta r_e}$$

Current gain from voltage gain:

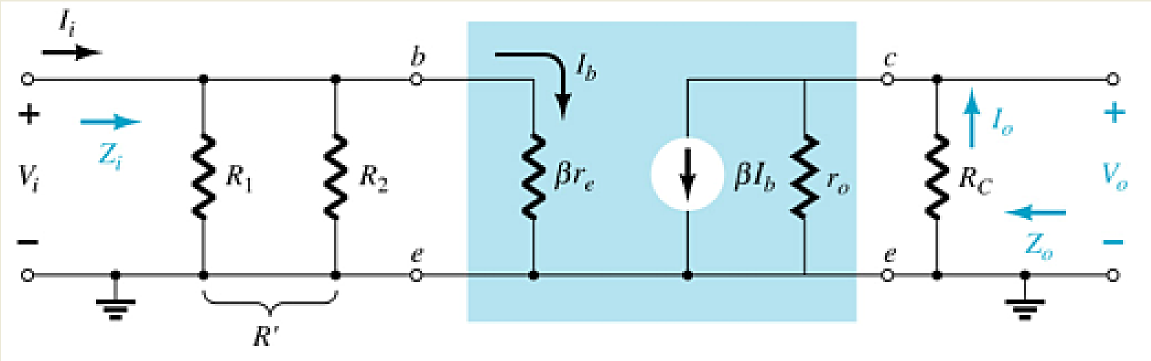
$$A_i = -A_v \frac{Z_i}{R_C}$$

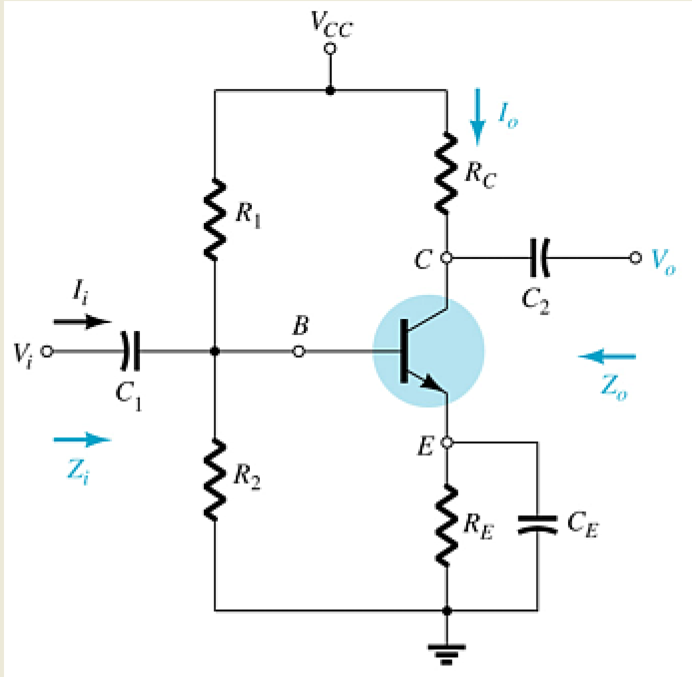
CE amplifiers:

- **High input impedance**
- **Low output impedance**
- **High voltage and current gain**
- **Phase shift between input and output is 180°**

5.9 Common-Emitter Voltage-Divider Bias

r_e model requires you to determine β, r_e, and r_o.





Input impedance:

$$R' = R_1 \parallel R_2$$

$$Z_i = R' \parallel \beta r_e$$

Output impedance:

$$Z_o = R_C \parallel r_o$$

$$Z_o \cong R_C \Big|_{r_o \geq 10R_C}$$

Voltage gain:

$$A_v = \frac{V_o}{V_i} = \frac{-R_C \parallel r_o}{r_e}$$

$$A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{r_e} \Big|_{r_o \geq 10R_C}$$

Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{\beta R' r_o}{(r_o + R_C)(R' + \beta r_e)}$$

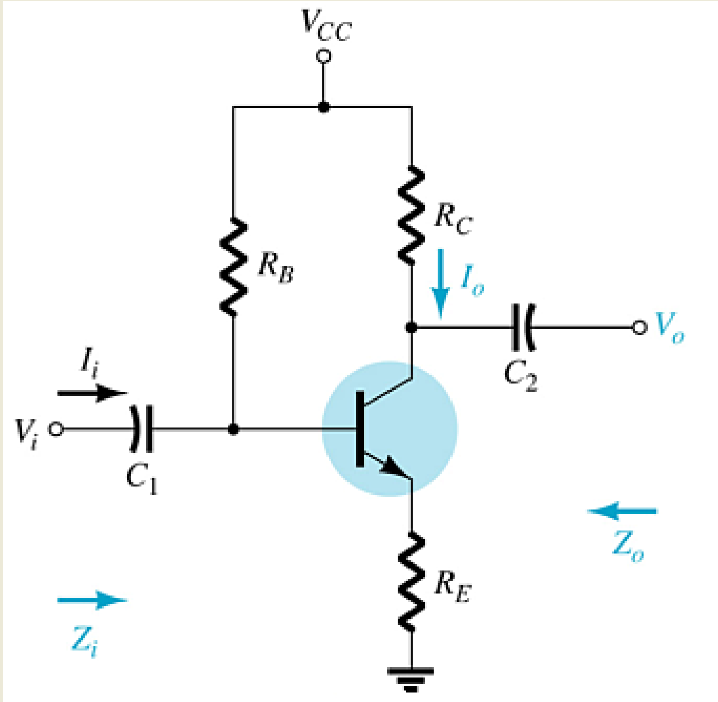
$$A_i = \frac{I_o}{I_i} \cong \frac{\beta R'}{R' + \beta r_e} \Big|_{r_o \geq 10R_C}$$

$$A_i = \frac{I_o}{I_i} \cong \beta \Big|_{r_o \geq 10R_C,\, R' \geq 10\beta r_e}$$

Current gain from voltage gain:

$$A_i = -A_v \frac{Z_i}{R_C}$$

5.10 Common-Emitter Emitter-Bias Configuration (Unbypassed R_E)



Input impedance:

$$Z_i = R_B \parallel Z_b$$

$$Z_b = \beta r_e + (\beta + 1) R_E$$

Output impedance:

$$Z_o = R_C$$

Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{\beta R_C}{Z_b}$$

$$A_v = \frac{V_o}{V_i} = -\frac{R_C}{r_e + R_E}\Bigg|_{Z_b = \beta(r_e + R_E)}$$

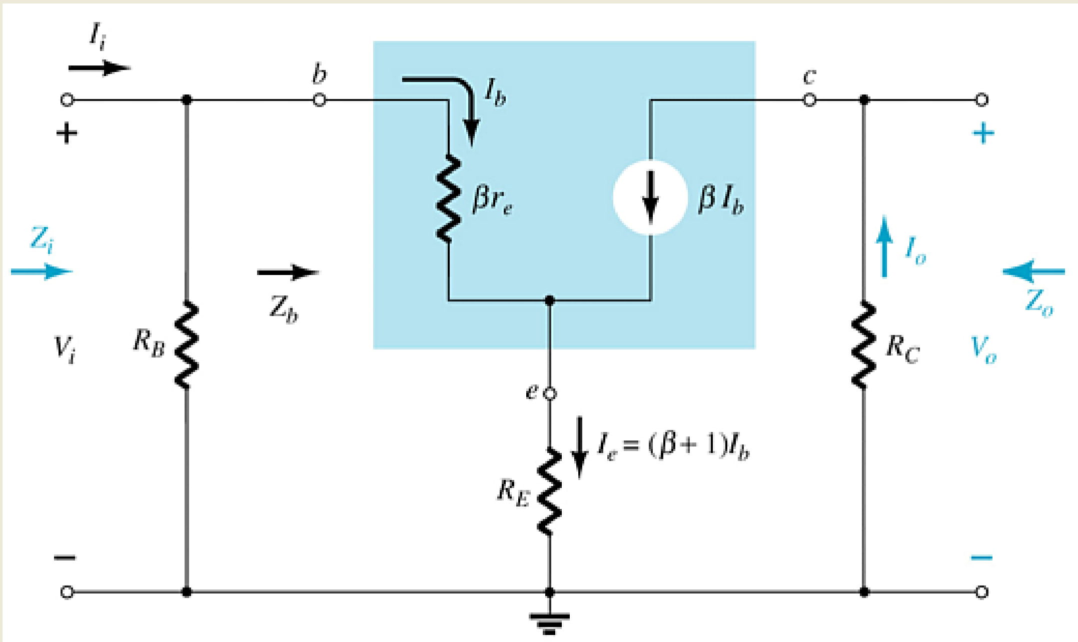
$$A_v = \frac{V_o}{V_i} \cong -\frac{R_C}{R_E}\Bigg|_{Z_b \cong \beta R_E}$$

Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_B}{R_B + Z_b}$$

Current gain from voltage gain:

$$A_i = -A_v \frac{Z_i}{R_C}$$



5.13 Common-Emitter Collector Feedback Configuration

Input impedance:

$$Z_i = \frac{r_e}{\frac{1}{\beta} + \frac{R_C}{R_F}}$$

Output impedance:

$$Z_o \cong R_C \parallel R_F$$

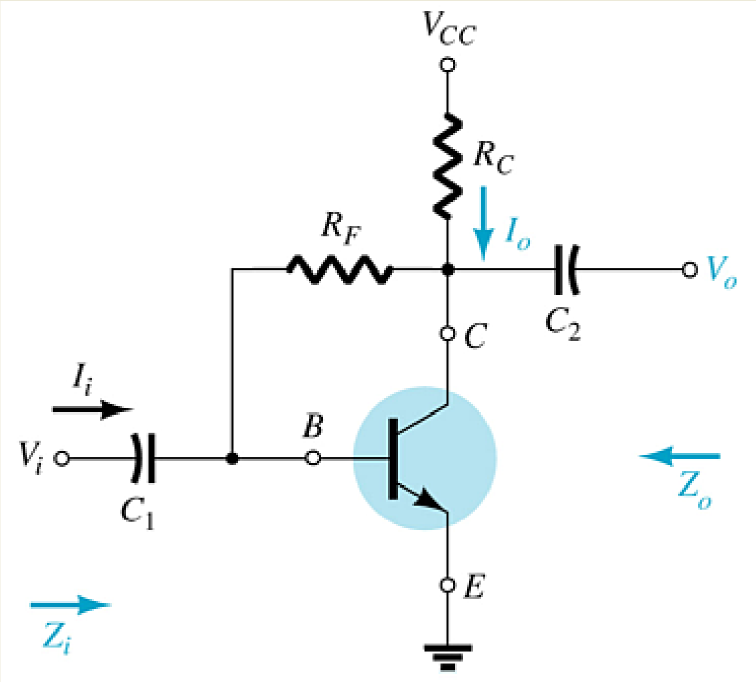
Voltage gain:

$$A_v = \frac{V_o}{V_i} = -\frac{R_C}{r_e}$$

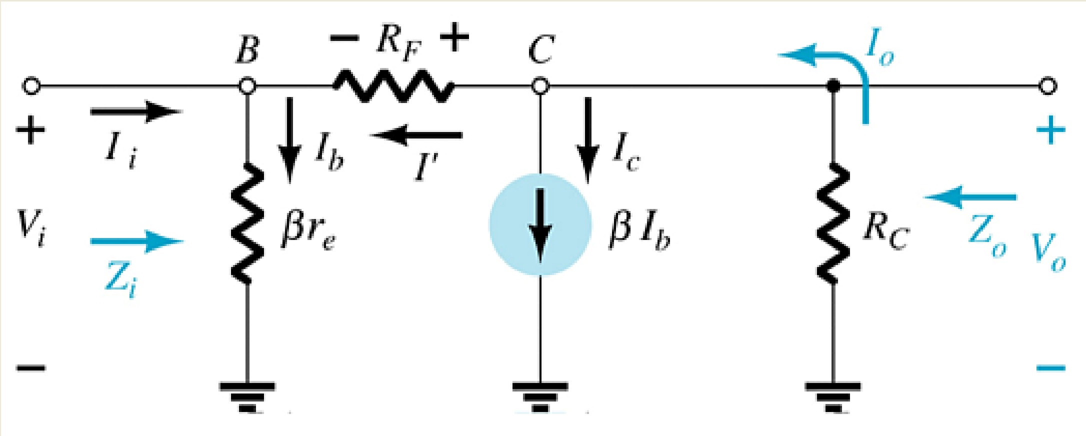
Current gain:

$$A_i = \frac{I_o}{I_i} = \frac{\beta R_F}{R_F + \beta R_C}$$

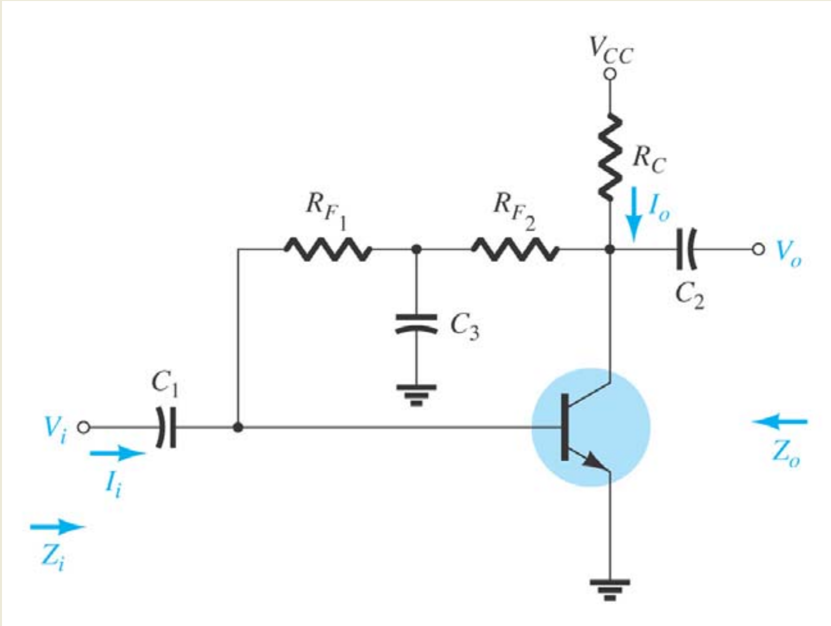
$$A_i = \frac{I_o}{I_i} \cong \frac{R_F}{R_C}$$

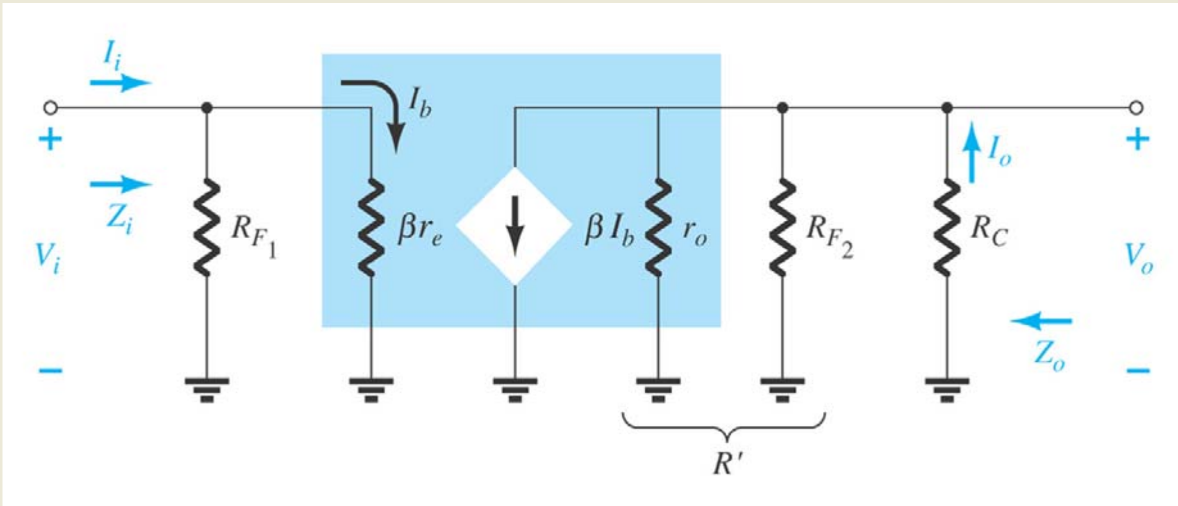


- **This is a variation of the common-emitter fixed-bias configuration**
- **Input is applied to the base**
- **Output is taken from the collector**
- **There is a 180° phase shift between input and output**

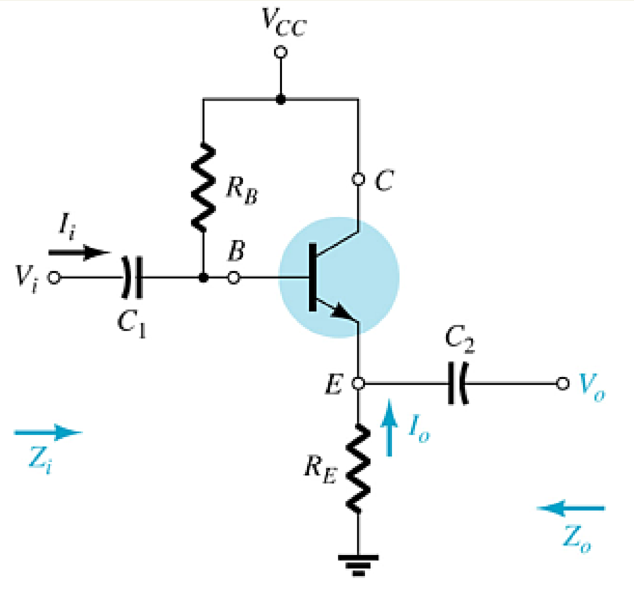


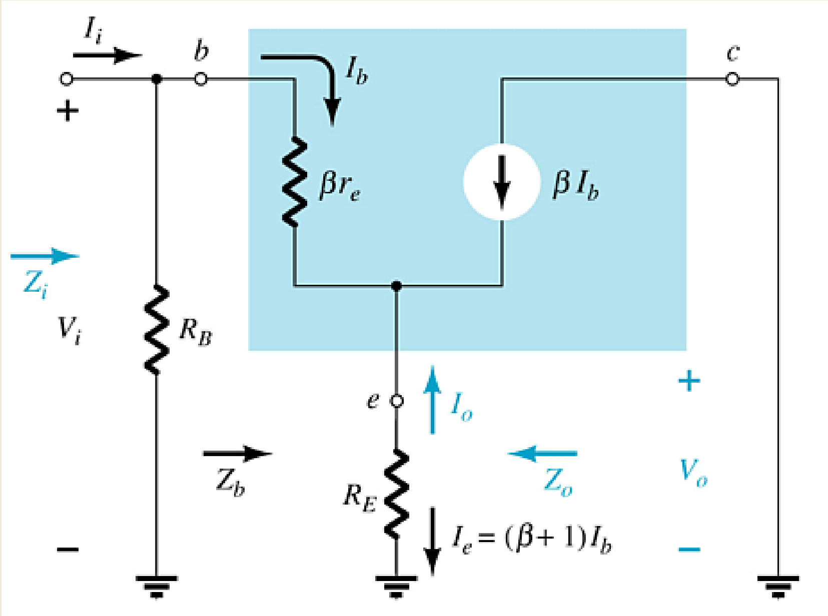
5.14 Collector DC Feedback Configuration



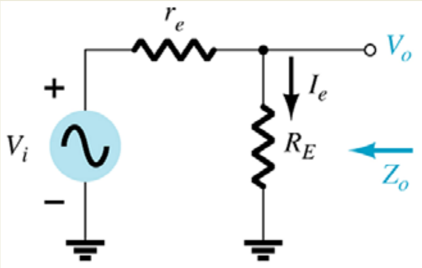


5.11 Emitter-Follower Configuration (CC)





- Emitter-follower is also known as the common-collector configuration.
- The input is applied to the base and the output is taken from the emitter.
- There is no phase shift between input and output.



Input impedance:

$$Z_i = R_B \parallel Z_b$$

$$Z_b = \beta r_e + (\beta + 1)R_E$$

$$Z_b \cong \beta(r_e + R_E)$$

$$Z_b \cong \beta R_E$$

Output impedance:

$$Z_o = R_E \parallel r_e$$

$$Z_o \cong r_e \Big|_{R_E >> r_e}$$

Voltage gain:

$$A_v = \frac{V_o}{V_i} = \frac{R_E}{R_E + r_e}$$

$$A_v = \frac{V_o}{V_i} \cong 1 \Big|_{R_E >> r_e,\, R_E + r_e \cong R_E}$$

Current gain:

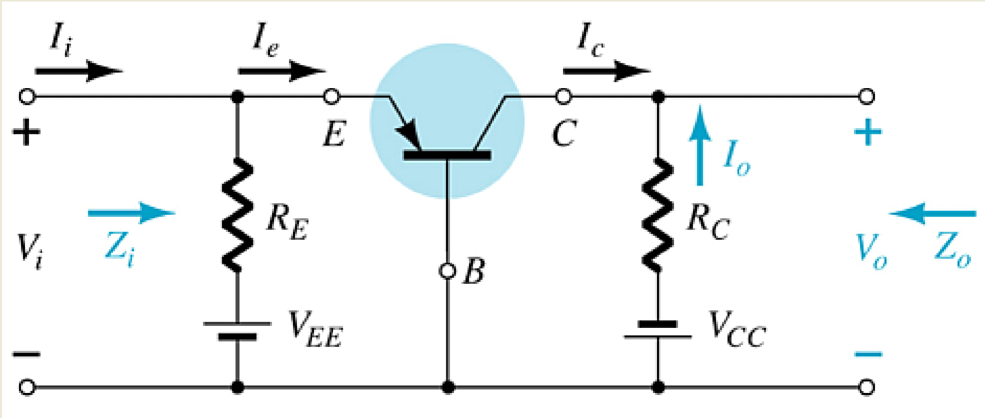
$$A_i \cong -\frac{\beta R_B}{R_B + Z_b}$$

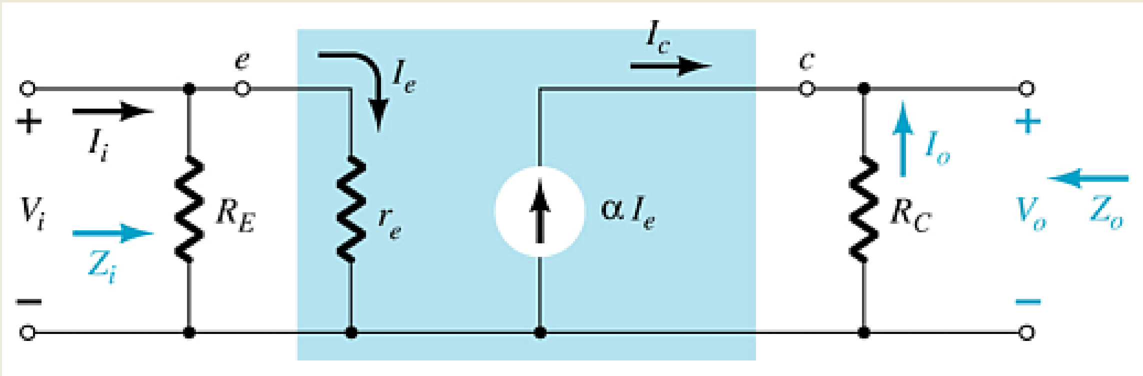
Current gain from voltage gain:

$$A_i = -A_v \frac{Z_i}{R_E}$$

5.12 Common-Base Configuration

- The input is applied to the emitter.
- The output is taken from the collector.
- Low input impedance.
- High output impedance.
- Current gain less than unity.
- Very high voltage gain.
- No phase shift between input and output.





Input impedance:

$$Z_i = R_E \parallel r_e$$

Output impedance:

$$Z_o = R_C$$

Voltage gain:

$$A_v = \frac{V_o}{V_i} = \frac{\alpha R_C}{r_e} \cong \frac{R_C}{r_e}$$

Current gain:

$$A_i = \frac{I_o}{I_i} = -\alpha \cong -1$$

5.17 Two-Port Systems Approach

This approach:

- **Reduces a circuit to a two-port system**
- **Provides a "Thévenin look" at the output terminals**
- **Makes it easier to determine the effects of a changing load**

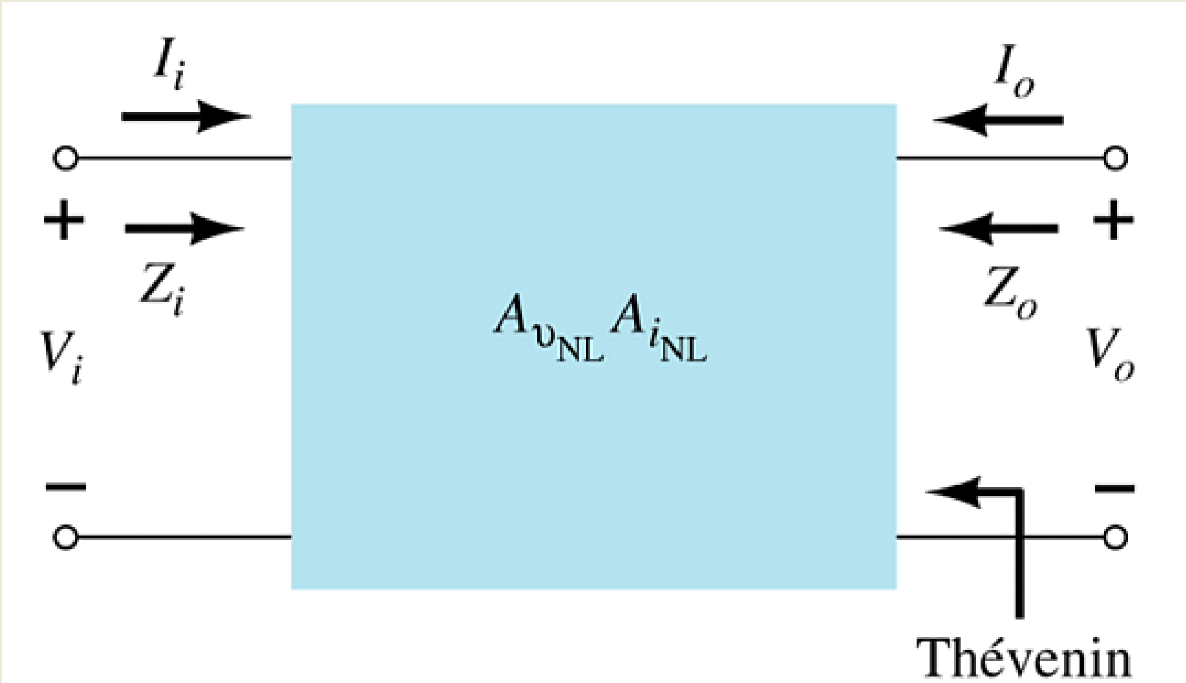
With V_i set to 0 V:

$$Z_{Th} = Z_o = R_o$$

The voltage across the open terminals is:

$$E_{Th} = A_{vNL} V_i$$

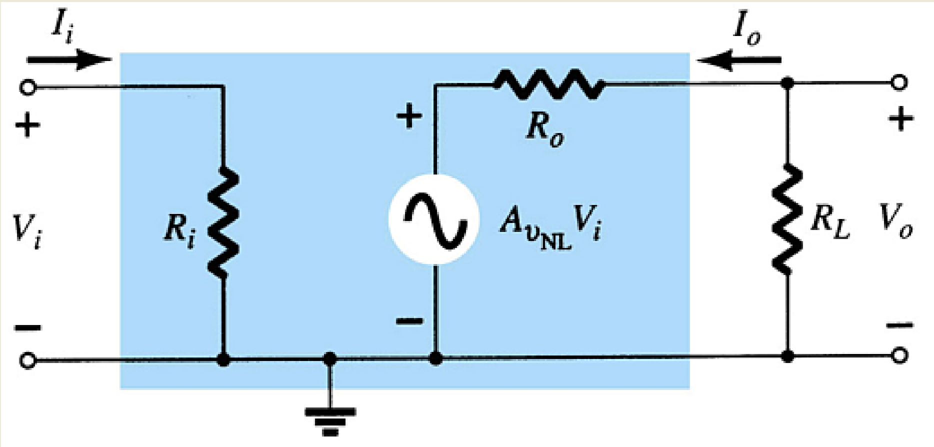
where A_{vNL} is the no-load voltage gain.



5.16 Effect of Load Impedance on Gain

This model can be applied to any current- or voltage-controlled amplifier.

Adding a load reduces the gain of the amplifier:

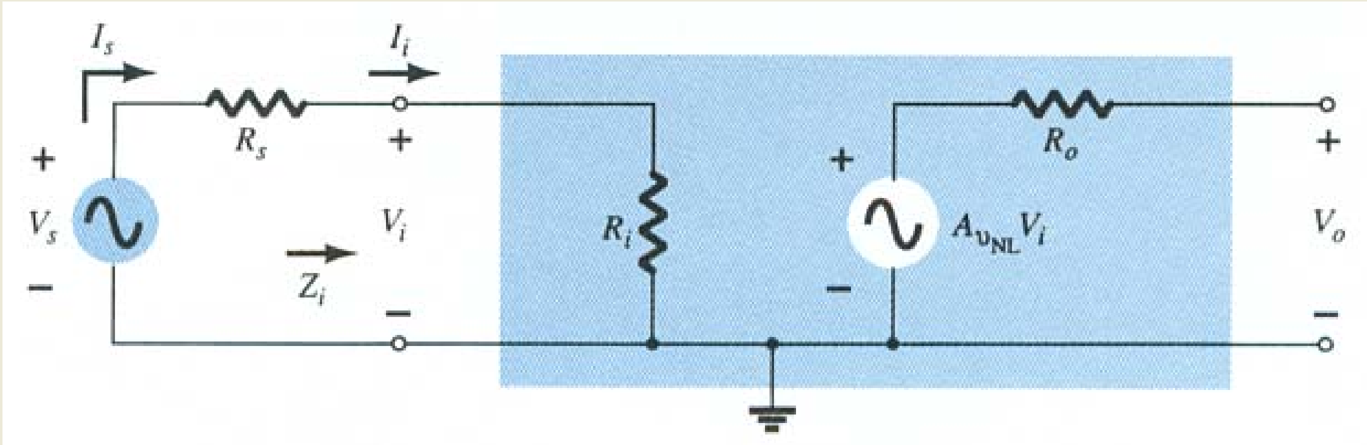


$$V_o = \frac{R_L}{R_L + R_o} A_{v_{NL}} V_i$$

$$A_v = \frac{V_o}{V_i} = \frac{R_L}{R_L + R_o} A_{vNL}$$

$$A_i = -A_v \frac{Z_i}{R_L}$$

5.16 Effect of Source Impedance on Gain



The fraction of applied signal that reaches the input of the amplifier is:

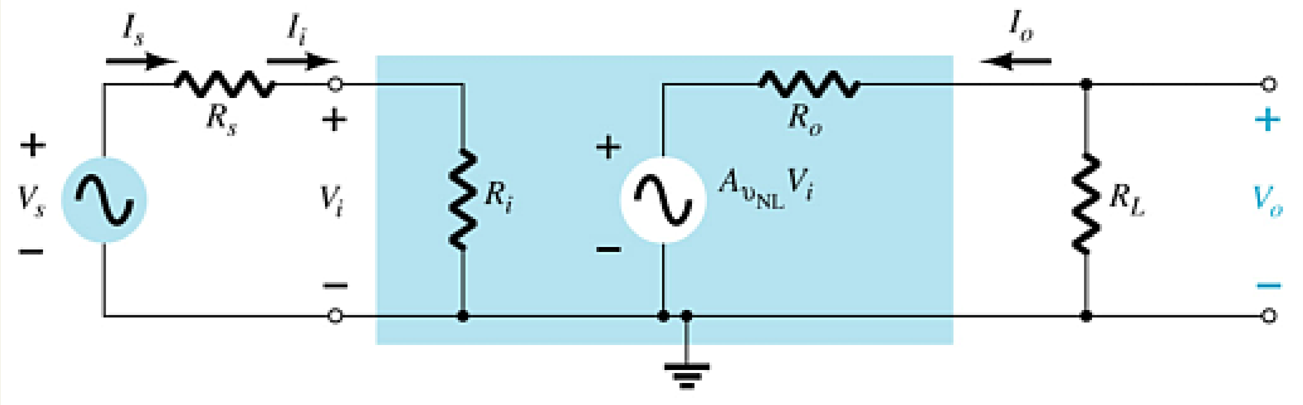
$$V_i = \frac{R_i V_s}{R_i + R_s}$$

$$V_o = A_{v_{NL}} V_i = A_{v_{NL}} \frac{R_i V_s}{R_i + R_s}$$

The internal resistance of the signal source reduces the overall gain:

$$A_{vs} = \frac{V_o}{V_s} = \frac{R_i}{R_i + R_s} A_{vNL}$$

5.16 Combined Effects of R_S and R_L on Voltage Gain



Effects of R_L:

$$A_v = \frac{V_o}{V_i} = \frac{R_L A_{vNL}}{R_L + R_o}$$

$$A_i = -A_v \frac{R_i}{R_L}$$

$$V_o = \frac{R_L}{R_L + R_o} A_{vNL} V_i = \frac{R_L}{R_L + R_o} A_{vNL} \frac{R_i V_s}{R_i + R_s}$$

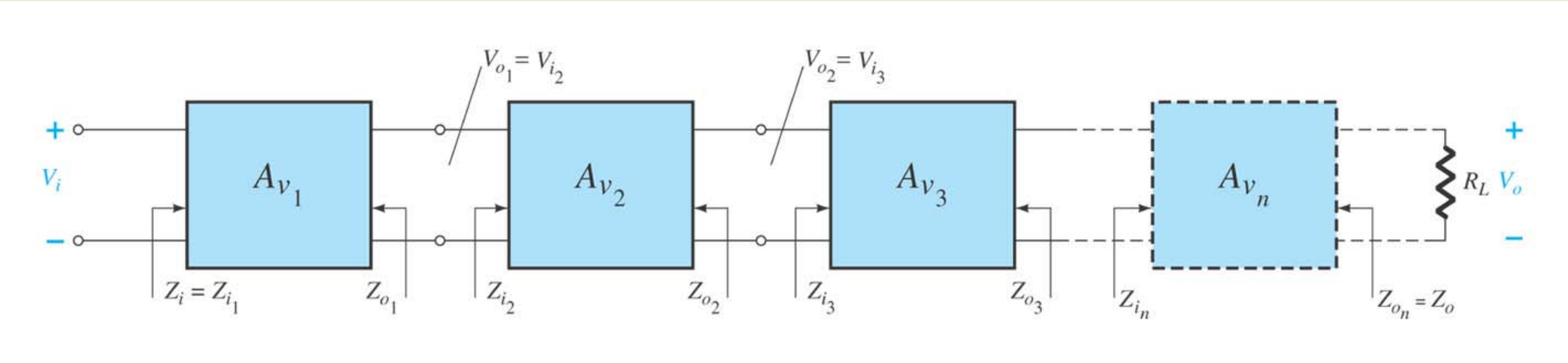
Effects of R_L and R_S:

$$A_{vs} = \frac{V_o}{V_s} = \frac{R_i}{R_i + R_s} \frac{R_L}{R_L + R_o} A_{vNL}$$

$$A_{is} = -A_{vs} \frac{R_s + R_i}{R_L}$$

5.19 Cascaded Systems

- The output of one amplifier is the input to the next amplifier
- The overall voltage gain is determined by the product of gains of the individual stages
- The DC bias circuits are isolated from each other by the coupling capacitors
- The DC calculations are independent of the cascading
- The AC calculations for gain and impedance are interdependent



$$\mathbf{Z_i = Z_{i_1}}$$

$$\mathbf{A_{v_T} = A_{v_1} \cdot A_{v_2} \cdot A_{v_3} \cdots A_{v_n}}$$

$$\mathbf{A_{v_1}, A_{v_2}, A_{v_3}, \cdots, A_{v_n}}$$

are loaded gains

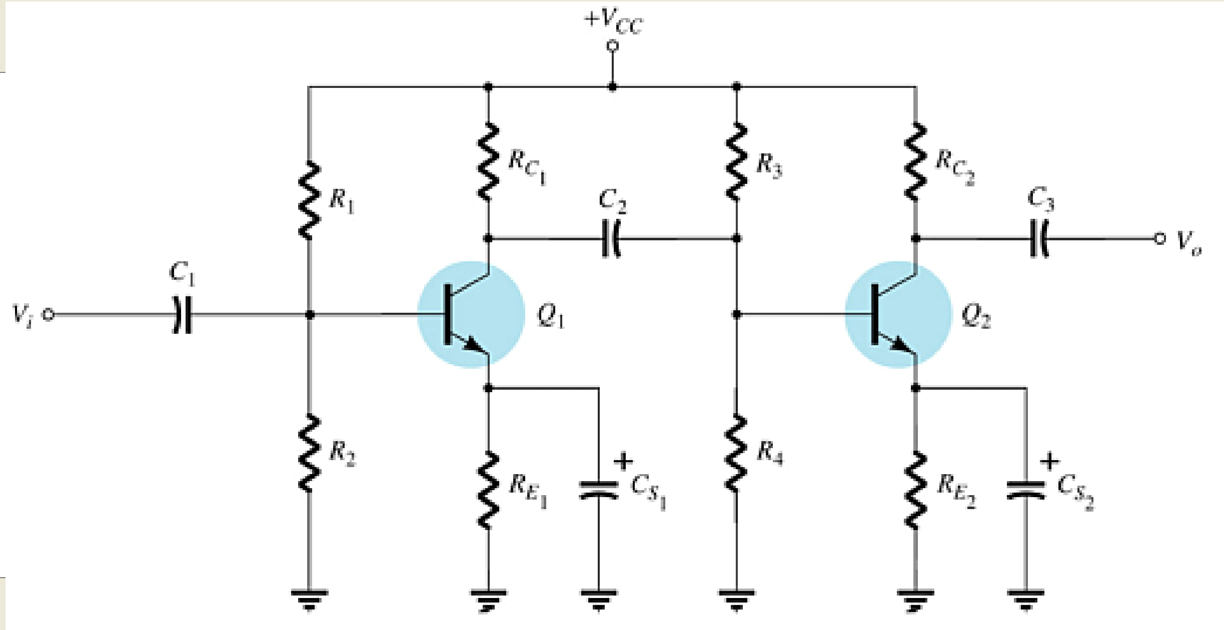
$$\mathbf{Z_o = Z_{o_n}}$$

$$\mathbf{A_{i_T} = -A_{v_T} \frac{Z_{i_1}}{R_L}}$$

R-C Coupled BJT Amplifiers

Input impedance, first stage:

$$Z_i = Z_{i1} = R_1 \parallel R_2 \parallel \beta r_{e1}$$



Output impedance, second stage:

$$Z_o = R_{C2}$$

Voltage gain:

$$A_{V1} = -\frac{R_{C1} \parallel Z_{i2}}{r_{e1}} = \frac{R_{C1} \parallel R_3 \parallel R_4 \parallel \beta r_{e2}}{r_{e1}}$$

$$Z_{i2} = R_3 \parallel R_4 \parallel \beta r_{e2}$$

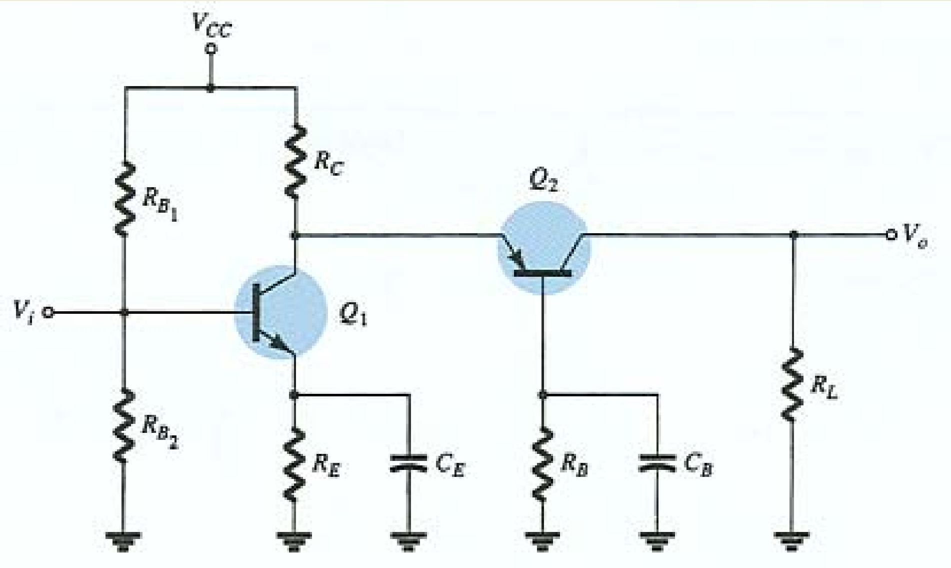
$$A_{V2} = -\frac{R_{C2}}{r_{e2}}$$

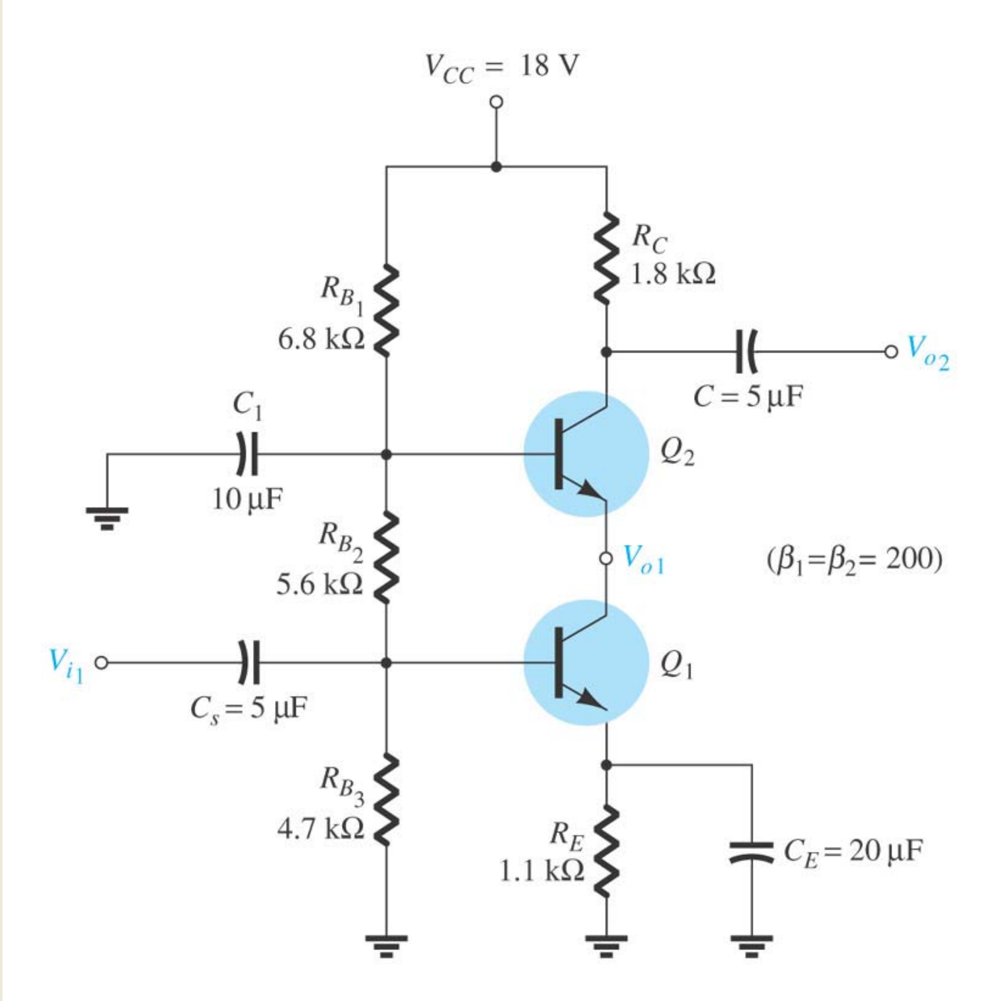
$$A_V = A_{V1} A_{V2}$$

Cascode Connection: CE–CB

This example is a CE–CB combination. This arrangement provides high input impedance but a low voltage gain.

The low voltage gain of the input stage reduces the Miller input capacitance, making this combination suitable for high-frequency applications.



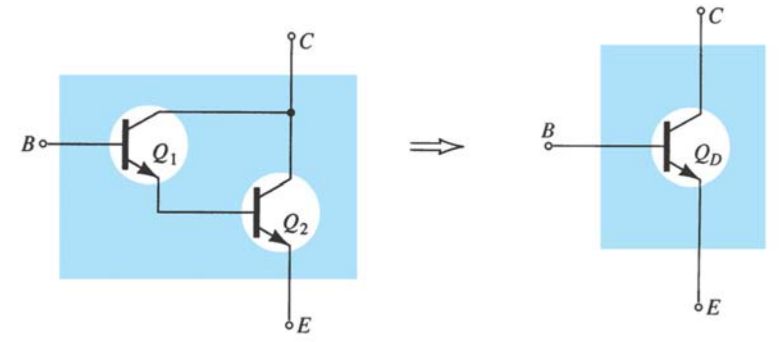


5.20 Darlington Connection

The Darlington circuit provides a very high current gain—the product of the individual current gains:

$$\beta_D = \beta_1\beta_2$$

The practical significance is that the circuit provides a very high input impedance.

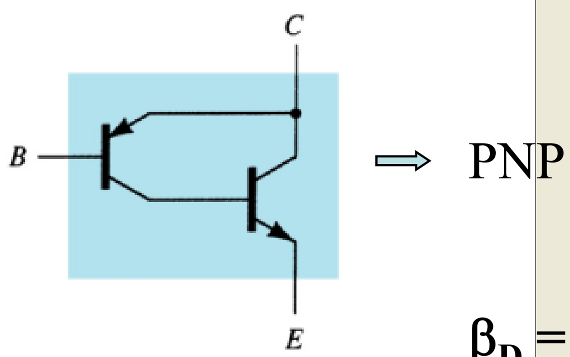


5.21 Feedback Pair

This is a two-transistor circuit that operates like a Darlington pair, *but it is not a Darlington pair*.

It has similar characteristics:

- **High current gain**
- **Low Voltage gain (near unity)**
- **Low output impedance**
- **High input impedance**

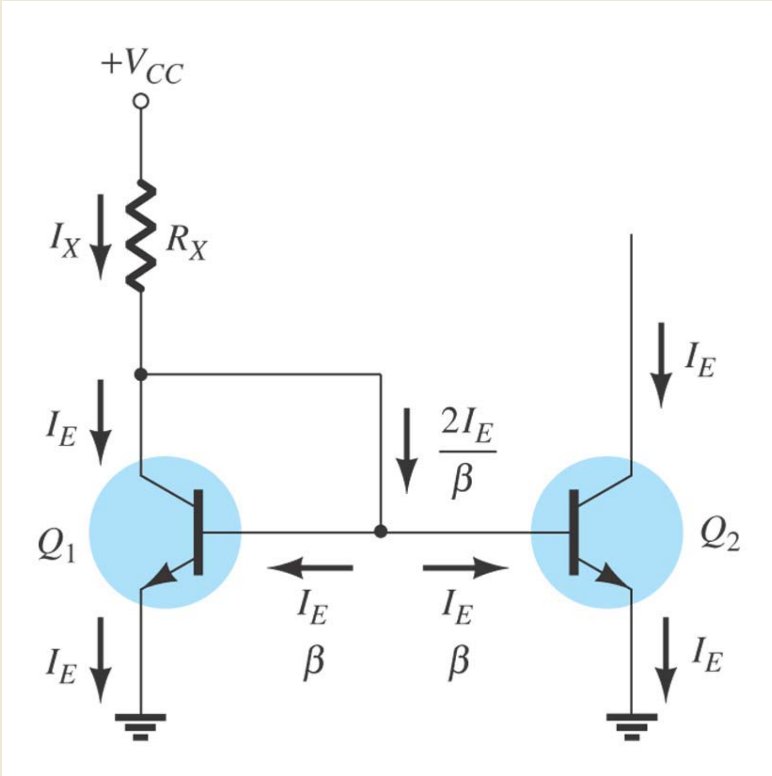


$$\beta_D = \beta_1\beta_2$$

The difference is that a Darlington uses a pair of like transistors, whereas the feedback-pair configuration uses complementary transistors.

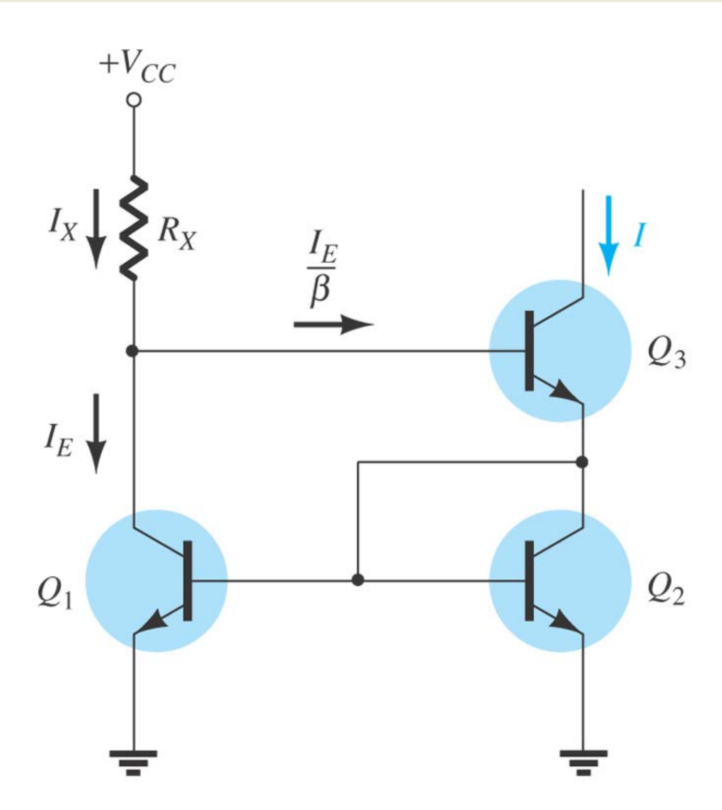
5.22 Current Mirror Circuits

Current mirror circuits provide constant current in integrated circuits.



$$I_X = I_E + 2I_B = I_E + 2\frac{I_E}{\beta} = \frac{\beta+2}{\beta} I_E$$

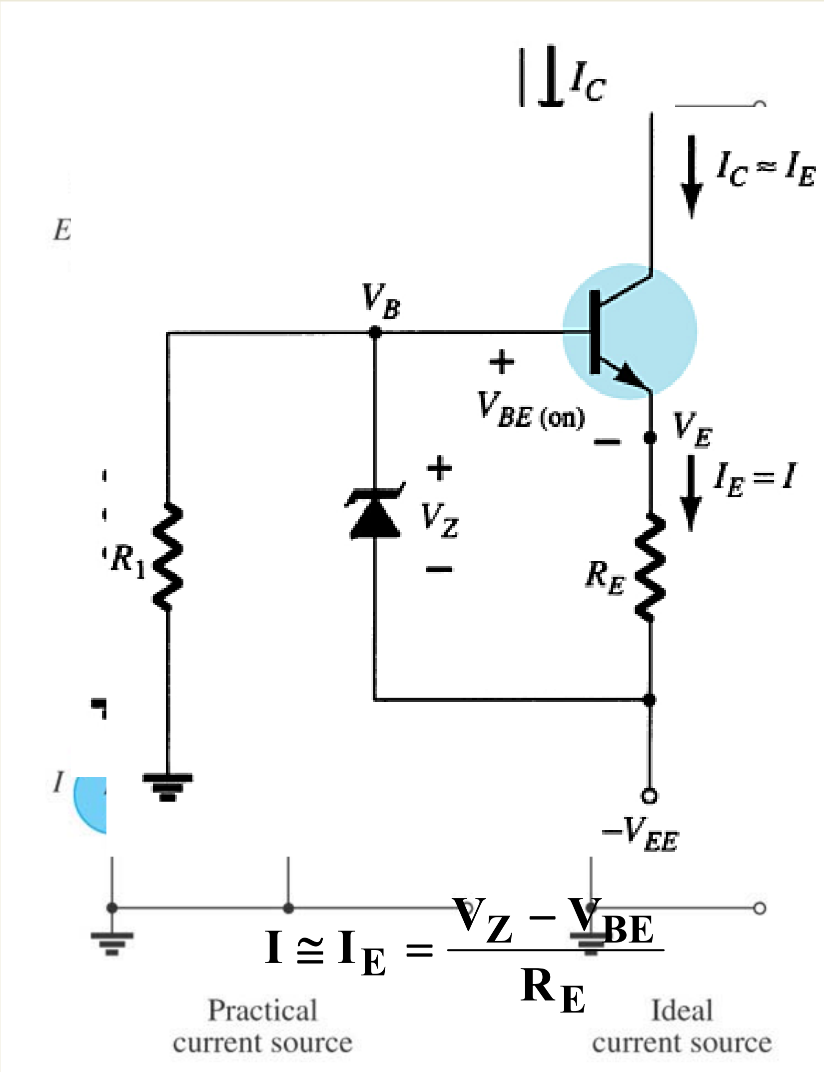
$$I_E = \frac{\beta}{\beta+2} I_X \approx I_X = \frac{V_{CC} - V_{BE}}{R_X}$$



Current mirror circuit with higher output impedance.

5.23 Current Source Circuits

Constant-current sources can be built using FETs, BJTs, and combinations of these devices.



$$I \cong I_E = \frac{V_Z - V_{BE}}{R_E}$$

(b)

Summary of Chapter 5

- AC analysis
  - Load line analysis
  - Mathematical analysis by small signal model
- AC analysis method by small signal model
  - DC analysis to determine r_e
  - AC equivalent circuit by re model
  - Calculation impedance and gain
- CE amplifier
- CB amplifier
- CC amplifier
- Cascaded amplifier system
  - Effect of R_s and R_L
  - CE-CB