INDUCTORS

13

CHAPTER OUTLINE

CHAPTER OBJECTIVES

- Describe the basic construction and characteristics of an inductor
- Discuss various types of inductors
- Analyze series and parallel inductors
- Analyze inductive dc switching circuits
- Analyze inductive ac circuits
- Discuss some inductor applications

KEY TERMS

- Inductor
- Winding
- Induced voltage
- Inductance
- Henry (H)
- *RL* time constant
- Inductive reactance
- Quality factor (**Q**)

APPLICATION ACTIVITY PREVIEW

In this application activity, the inductance of coils is determined by measuring the time constant of a test circuit using oscilloscope waveforms.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/careersresources/

INTRODUCTION

You have learned about the resistor and the capacitor. In this chapter, you will learn about a third type of basic passive component, the *inductor,* and study its characteristics.

The basic construction and electrical properties of inductors are discussed, and the effects of connecting them in series and in parallel are analyzed. How an inductor works in both dc and ac circuits is an important part of this coverage and forms the basis for the study of reactive circuits in terms of both frequency response and time response. You will also learn how to check for a faulty inductor.

The inductor, which is basically a coil of wire, is based on the principle of electromagnetic induction, which you studied in Chapter 10. Inductance is the property of a coil of wire that opposes a change in current. The basis for inductance is the electromagnetic field that surrounds any conductor when there is current through it. The electrical component designed to have the property of inductance is called an *inductor, coil,* or in certain high-frequency applications a *choke*. All of these terms refer to the same type of device.

13–1 The Basic Inductor

An **inductor** is a passive electrical component formed by a coil of wire and which exhibits the property of inductance. It is frequently wound on a ferromagnetic core to increase the inductance.

After completing this section, you should be able to

- **Describe the basic construction and characteristics of an inductor**
  - Define *inductance* and state its unit
  - Discuss induced voltage
  - Explain how an inductor stores energy
  - Specify how the physical characteristics affect inductance
  - Discuss winding resistance and winding capacitance
  - State Faraday's law
  - State Lenz's law

When a length of wire is formed into a coil, as shown in Figure 13–1, it becomes an inductor. The terms *coil* and *inductor* are used interchangeably. Current through the coil produces an electromagnetic field, as illustrated. The magnetic lines of force around each loop (turn) in the **winding** of the coil effectively add to the lines of force around the adjoining loops, forming a strong electromagnetic field within and around the coil. The net direction of the total electromagnetic field creates a north and a south pole.

HISTORY NOTE

Joseph Henry 1797–1878

Henry began his career as a professor at a small school in Albany, NY, and later became the first director of the Smithsonian Institution. He was the first American since Franklin to undertake original scientific experiments. He was the first to superimpose coils of wire wrapped on an iron core and first observed the effects of electromagnetic induction in 1830, a year before Faraday, but he did not publish his findings. Henry did obtain credit for the discovery of self-induction, however. The unit of inductance is named in his honor. (Photo credit: Courtesy of the Smithsonian Institution. Photo number 59,054.)

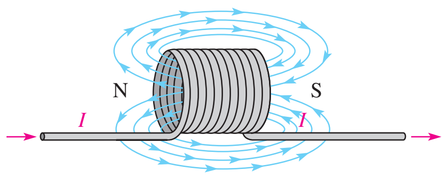


FIGURE 13–1

A coil of wire forms an inductor. When there is current through it, a three-dimensional electromagnetic field is created, surrounding the coil in all directions.

To understand the formation of the total electromagnetic field in a coil, consider the interaction of the electromagnetic fields around two adjacent loops. The magnetic lines of force around adjacent loops are each deflected into a single outer path when the loops are brought close together. This effect occurs because the magnetic lines of force are in opposing directions between adjacent loops and therefore cancel out when the loops are close together, as illustrated in Figure 13–2. The total electromagnetic field for the two loops is depicted in part (b). This effect is additive for many closely

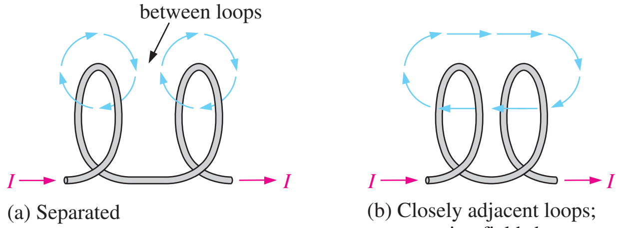
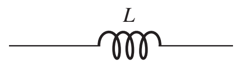


FIGURE 13–2

Interaction of magnetic lines of force in two adjacent loops of a coil.

adjacent loops in a coil; that is, each additional loop adds to the strength of the electromagnetic field. For simplicity, only single lines of force are shown, although there are many. Figure 13–3 shows a schematic symbol for an inductor.



▲ FIGURE 13–3

Symbol for inductor.

Inductance

When there is current through an inductor, an electromagnetic field is established. When the current changes, the electromagnetic field also changes. An increase in current expands the electromagnetic field, and a decrease in current reduces it. Therefore, a changing current produces a changing electromagnetic field around the inductor. In turn, the changing electromagnetic field causes an **induced voltage** across the coil in a direction to oppose the change in current. This property is called *self-inductance* but is usually referred to as simply *inductance*, symbolized by L.

Inductance is a measure of a coil's ability to establish an induced voltage as a result of a change in its current, and that induced voltage is in a direction to oppose the change in current.

The inductance (L) of a coil and the time rate of change of the current (di/dt) determine the induced voltage (v_{ind}). A change in current causes a change in the electromagnetic field, which, in turn, induces a voltage across the coil, as you know. The induced voltage is directly proportional to L and di/dt, as stated by the following formula:

$$v_{\text{ind}} = L\left(\frac{di}{dt}\right)$$

Equation 13–1

This formula indicates that the greater the inductance, the greater the induced voltage. Also, it shows that the faster the coil current changes (greater di/dt), the greater the induced voltage. Notice the similarity of Equation 13–1 to Equation 12–24: $i = C(dv/dt)$.

The Unit of Inductance The **henry (H)** is the basic unit of inductance. By definition, the inductance of a coil is one henry when current through the coil, changing at the rate of one ampere per second, induces one volt across the coil. The henry is a large unit, so in practical applications, millihenries (mH) and microhenries (μH) are the more common units.

EXAMPLE 13–1

Determine the induced voltage across a 1 henry (1 H) inductor when the current is changing at a rate of 2 A/s.

Solution

$$v_{\text{ind}} = L\left(\frac{di}{dt}\right) = (1\text{ H})(2\text{ A/s}) = \mathbf{2\ V}$$

*Related Problem** Determine the inductance when a current changing at a rate of 40 mA/ms causes 1.0 V to be induced.

*Answers are at the end of the chapter.

Energy Storage An inductor stores energy in the electromagnetic field created by the current. The energy stored is expressed as follows:

$$W = \frac{1}{2}LI^2$$

Equation 13–2

As you can see, the energy stored is proportional to the inductance and the square of the current. When current (I) is in amperes and inductance (L) is in henries, energy (W) is in joules.

Physical Characteristics of an Inductor

The following parameters are important in establishing the inductance of a coil: permeability of the core material, number of turns of wire, core length, and cross-sectional area of the core.

Core Material An inductor is basically a coil of wire that surrounds a magnetic or nonmagnetic material called the **core**. Examples of magnetic materials are iron, nickel, steel, cobalt, or alloys. These materials have permeabilities that are hundreds or thousands of times greater than that of a vacuum and are classified as *ferromagnetic*. Ferrites (introduced in Section 10–2) are one of the most common types of ferromagnetic core materials used with inductors. They concentrate the magnetic field lines, which permits a stronger magnetic field and larger inductance. Examples of nonmagnetic materials are air, cardboard, plastic, and glass. The permeabilities of these materials are the same as for a vacuum so they are classed with air-core inductors. These cores have no effect on the inductance but are used as a structure to wind the coils on and provide mechanical support. Some inductors do not have a core and are true air-core inductors.

As you learned in Chapter 10, the permeability (μ) of the core material determines how easily a magnetic field can be established, and is measured in Wb/At·m, which is the same as H/m. The inductance is directly proportional to the permeability of the core material.

Physical Parameters As indicated in Figure 13–4, the number of turns of wire, the length, and the cross-sectional area of the core are factors in setting the value of inductance. The inductance is inversely proportional to the length of the core and directly proportional to the cross-sectional area. Also, the inductance is directly related to the number of turns squared. This relationship is as follows:

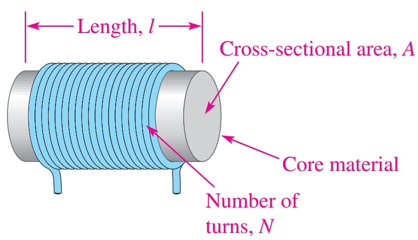
Equation 13–3

$$L = \frac{N^2 \mu A}{l}$$

where L is the inductance in henries (H), N is the number of turns of wire, μ is the permeability in henries per meter (H/m), A is the cross-sectional area in meters squared, and l is the core length in meters (m).

► FIGURE 13–4

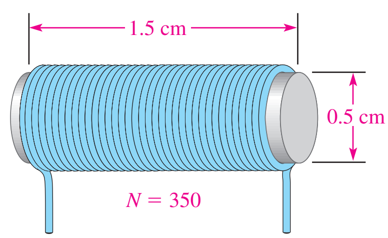
Physical parameters of an inductor.



EXAMPLE 13–2

Determine the inductance of the coil in Figure 13–5. The permeability of the core is 0.25×10^{-3} H/m.

► FIGURE 13–5



Solution First determine the length and area in meters.

$$l = 1.5 \text{ cm} = 0.015 \text{ m}$$
$$A = \pi r^2 = \pi(0.25 \times 10^{-2} \text{ m})^2 = 1.96 \times 10^{-5} \text{ m}^2$$

The inductance of the coil is

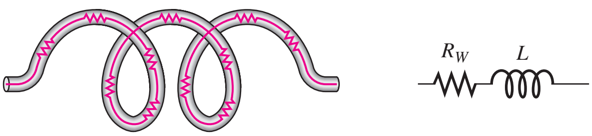
$$L = \frac{N^2 \mu A}{l} = \frac{(350)^2(0.25 \times 10^{-3} \text{ H/m})(1.96 \times 10^{-5} \text{ m}^2)}{0.015 \text{ m}} = \mathbf{40 \text{ mH}}$$

Related Problem Determine the inductance of a coil with 90 turns around a core that is 1.0 cm long and has a diameter of 0.8 cm. The permeability is 0.25×10^{-3} H/m.

Winding Resistance

When a coil is made of a certain material, for example, insulated copper wire, that wire has a certain resistance per unit of length. When many turns of wire are used to construct a coil, the total resistance may be significant. This inherent resistance is called the *dc resistance* or the *winding resistance* (R_W).

Although this resistance is distributed along the length of the wire, it effectively appears in series with the inductance of the coil, as shown in Figure 13–6. In many applications, the winding resistance may be small enough to be ignored and the coil can be considered an ideal inductor. In other cases, the resistance must be considered.



(a) The wire has resistance distributed along its length.

(b) Equivalent circuit

▲ **FIGURE 13–6**

Winding resistance of a coil.

Winding Capacitance

When two conductors are placed side by side, there is always some capacitance between them. Thus, when many turns of wire are placed close together in a coil, a certain amount of stray capacitance, called *winding capacitance* (C_W), is a natural side effect. In many applications, this winding capacitance is very small and has no significant effect. In other cases, particularly at high frequencies, it may become quite important.

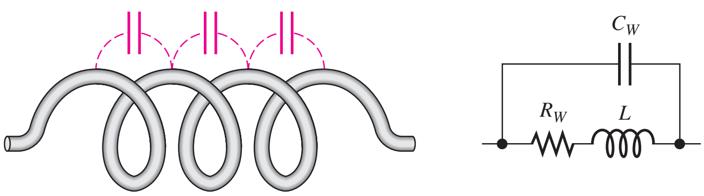
The equivalent circuit for an inductor with both its winding resistance (R_W) and its winding capacitance (C_W) is shown in Figure 13–7. The capacitance effectively acts in parallel. The total of the stray capacitances between each loop of the winding is indicated in a schematic as a capacitance appearing in parallel with the coil and its winding resistance, as shown in Figure 13–7(b).

SAFETY NOTE

Be careful when working with inductors because high induced voltages can be developed due to a rapidly changing magnetic field. This occurs when the current is interrupted or its value abruptly changed.

◀ **FIGURE 13–7**

Winding capacitance of a coil.



(a) Stray capacitance between each loop appears as a total parallel capacitance (C_W).

(b) Equivalent circuit

Measuring Inductors

The value of an inductor can be measured by several means, including its response to a square wave, a bridge measurement, or by a special meter called an LCR meter. An LCR meter is used to measure the inductance (L), capacitance (C), or resistance (R). To measure inductance, some meters can also display the impedance at a selected test frequency. From this measurement, the inductance is determined and displayed. An LCR meter is discussed further in Section 16–9.

Review of Faraday's Law

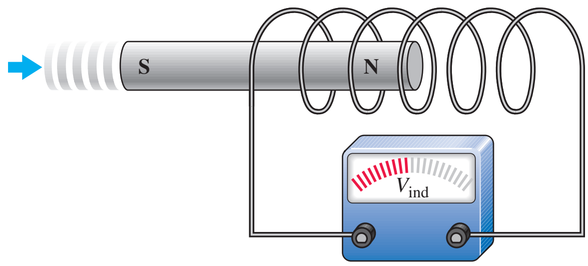
Recall from Section 10–5 that Faraday's law has two parts. The first part states that:

> **The amount of voltage induced in a coil is directly proportional to the rate of change of the magnetic field with respect to the coil.**

This principle is illustrated in Figure 13–8, where a bar magnet is moved through a coil of wire. An induced voltage is indicated by the voltmeter connected across the coil. The faster the magnet is moved, the greater the induced voltage.

▶ **FIGURE 13–8**

Induced voltage created by a changing magnetic field.



When a wire is formed into a certain number of loops or turns and is exposed to a changing magnetic field, a voltage is induced across the coil. The induced voltage is proportional to the number of turns of wire in the coil, N, and to the rate at which the magnetic field changes. The rate of change of the magnetic field is designated $d\phi/dt$, where ϕ is the magnetic flux. The ratio $d\phi/dt$, is expressed in webers/second (Wb/s). Faraday's law states that the induced voltage across a coil is equal to the number of turns (loops) times the rate of flux change. It was given in Equation 10–8 and is restated here.

$$v_{ind} = N\left(\frac{d\phi}{dt}\right)$$

EXAMPLE 13–3

Apply Faraday's law to find the induced voltage across a coil with 500 turns located in a magnetic field that is changing at a rate of 5 Wb/s.

Solution

$$v_{ind} = N\left(\frac{d\phi}{dt}\right) = (500\ \text{t})(5\ \text{Wb/s}) = \mathbf{2.5\ kV}$$

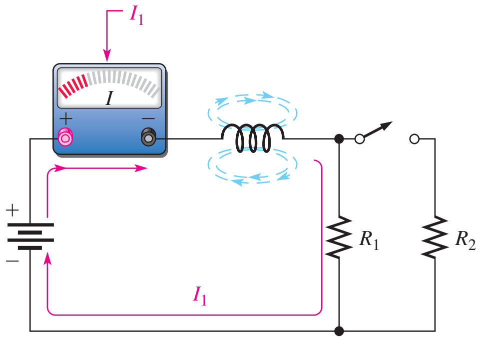
Related Problem A 1,000 turn coil has an induced voltage of 500 V across it. What is the rate of change of the magnetic field?

Lenz's Law

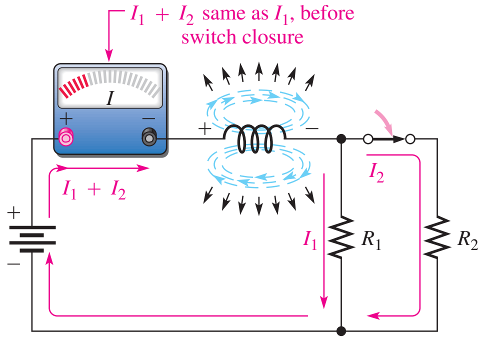
Lenz's law was introduced in Chapter 10 and is restated here.

> **When the current through a coil changes, an induced voltage is created as a result of the changing electromagnetic field and the polarity of the induced voltage is such that it always opposes the change in current.**

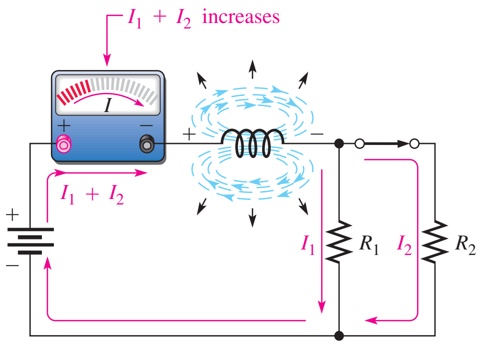
Figure 13–9 illustrates Lenz's law. In part (a), the current is constant and is limited by R_1. There is no induced voltage because the electromagnetic field is unchanging. In part (b), the switch suddenly is closed, placing R_2 in parallel with R_1 and thus reducing the resistance. Naturally, the current tries to increase and the electromagnetic field begins to expand, but the induced voltage opposes this attempted increase in current for an instant.



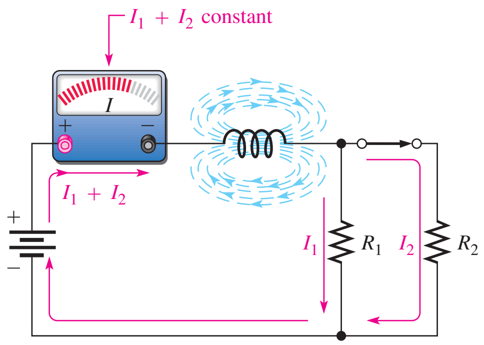
(a) Switch open: Constant current and constant magnetic field; no induced voltage.



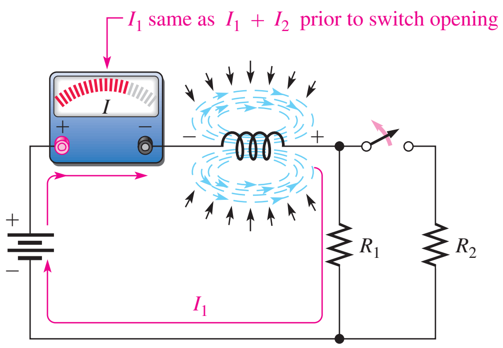
(b) At instant of switch closure: Expanding magnetic field induces voltage, which opposes an increase in total current. The total current remains the same at this instant.



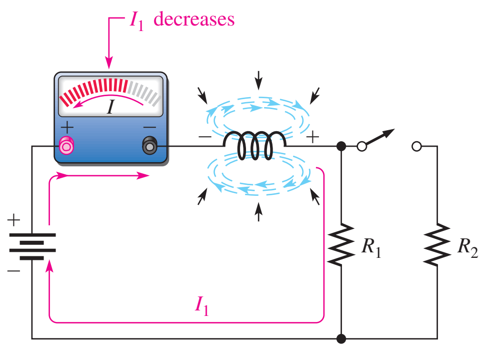
(c) Right after switch closure: The rate of expansion of the magnetic field decreases, allowing the current to increase exponentially as induced voltage decreases.



(d) Switch remains closed: Current and magnetic field reach constant value.



(e) At instant of switch opening: Magnetic field begins to collapse, creating an induced voltage, which opposes a decrease in current.



(f) After switch opening: Rate of collapse of magnetic field decreases, allowing current to decrease exponentially back to original value.

▲ **FIGURE 13–9**

Demonstration of Lenz's law in an inductive circuit: When the current tries to change suddenly, the electromagnetic field changes and induces a voltage in a direction that opposes that change in current.

In Figure 13–9(c), the induced voltage gradually decreases, allowing the current to increase. In part (d), the current has reached a constant value as determined by the parallel resistors, and the induced voltage is zero. In part (e), the switch has been suddenly opened, and, for an instant, the induced voltage prevents any decrease in current, and arcing between the switch contacts results. In part (f), the induced voltage gradually decreases, allowing the current to decrease back to a value determined by R_1. Notice that the induced voltage has a polarity that opposes any current change. The polarity of the induced voltage is opposite that of the battery voltage for an increase in current and aids the battery voltage for a decrease in current.

SECTION 13–1 CHECKUP

Answers are at the end of the chapter.

1. List the parameters that contribute to the inductance of a coil.
2. The current through a 15 mH inductor is changing at the rate of 500 mA/s. What is the induced voltage?
3. Describe what happens to L when
   (a) N is increased
   (b) The core length is increased
   (c) The cross-sectional area of the core is decreased
   (d) A ferromagnetic core is replaced by an air core
4. Explain why inductors have some winding resistance.
5. Explain why inductors have some winding capacitance.

13–2 Types of Inductors

Inductors normally are classified according to the type of core material.

After completing this section, you should be able to

- **Discuss various types of inductors**
  - Describe the basic types of fixed inductors
  - Distinguish between fixed and variable inductors

Inductors are made in a variety of shapes and sizes. Basically, they fall into two general categories: fixed and variable. The standard schematic symbols are shown in Figure 13–10.

Both fixed and variable inductors can be classified according to the type of core material. Three common types are the air core, the iron core, and the ferrite core. Each has a unique symbol, as shown in Figure 13–11.

Adjustable (variable) inductors usually have a screw-type adjustment that moves a sliding ferrite core in and out, thus changing the inductance. A wide variety of inductors exist, and some are shown in Figure 13–12. Small fixed inductors are frequently encapsulated in an insulating material that protects the fine wire in the coil. Encapsulated inductors have an appearance similar to a resistor.

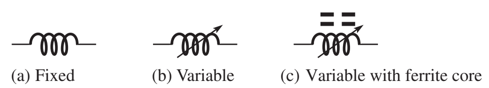
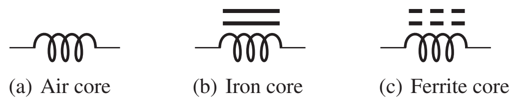


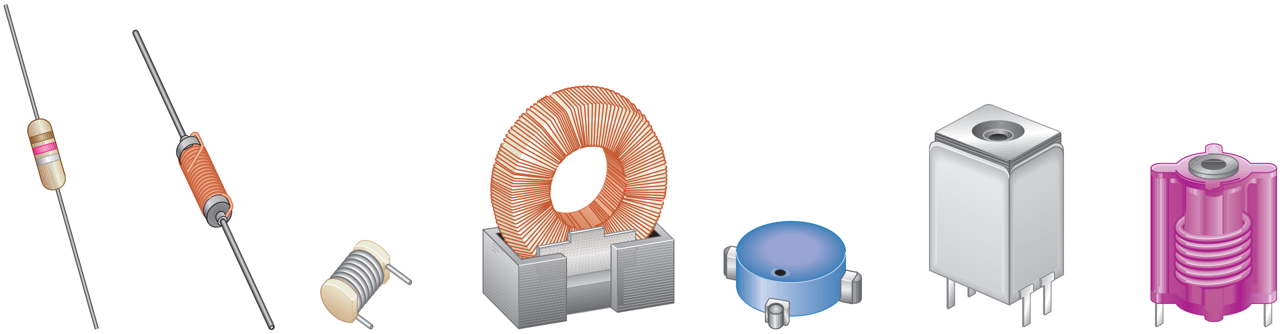
FIGURE 13–10

Symbols for fixed and variable inductors.



▲ FIGURE 13–11

Inductor symbols.



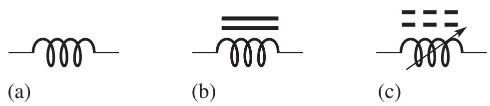
▲ FIGURE 13–12

Typical inductors.

SECTION 13–2 CHECKUP

1. Name two general categories of inductors.
2. Identify the inductor symbols in Figure 13–13.

► FIGURE 13–13



13–3 SERIES AND PARALLEL INDUCTORS

When inductors are connected in series, the total inductance increases. When inductors are connected in parallel, the total inductance decreases.

After completing this section, you should be able to

- **Analyze series and parallel inductors**
  - Determine total series inductance
  - Determine total parallel inductance

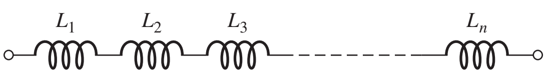
Total Series Inductance

When inductors are connected in series, as in Figure 13–14, the total inductance, L_T, is the sum of the individual inductances. The formula for L_T is expressed in the following equation for the general case of n inductors in series:

$$L_T = L_1 + L_2 + L_3 + \cdots + L_n$$

Equation 13–4

Notice that the calculation of total inductance in series is analogous to the calculations of total resistance in series (Chapter 5) and total capacitance in parallel (Chapter 12).

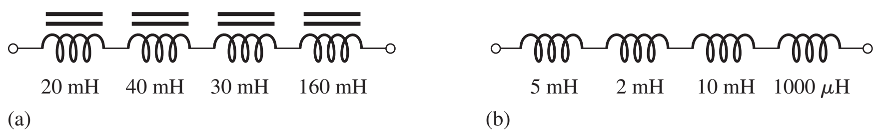


◄ FIGURE 13–14

Inductors in series.

EXAMPLE 13–4

Determine the total inductance for each of the series connections in Figure 13–15.



▲ **FIGURE 13–15**

Solution In Figure 13–15(a),

$$L_T = 20\text{ mH} + 40\text{ mH} + 30\text{ mH} + 160\text{ mH} = \mathbf{250\text{ mH}}$$

In Figure 13–15(b),

$$L_T = 5\text{ mH} + 2\text{ mH} + 10\text{ mH} + 1\text{ mH} = \mathbf{18\text{ mH}}$$

Note: $1{,}000\ \mu\text{H} = 1\text{ mH}$

Related Problem What is the total inductance of three 50 μH inductors in series?

Total Parallel Inductance

When inductors are connected in parallel, as in Figure 13–16, the total inductance is less than the smallest inductance. The general formula states that the reciprocal of the total inductance is equal to the sum of the reciprocals of the individual inductances.

Equation 13–5

$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n}$$

You can calculate total inductance, L_T, by taking the reciprocal of both sides of Equation 13–5.

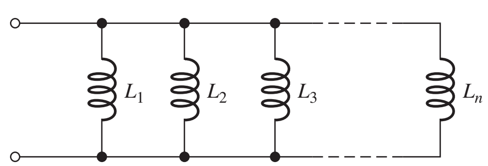
Equation 13–6

$$L_T = \frac{1}{\left(\frac{1}{L_1}\right) + \left(\frac{1}{L_2}\right) + \left(\frac{1}{L_3}\right) + \cdots + \left(\frac{1}{L_n}\right)}$$

The calculation for total inductance in parallel is analogous to the calculations for total parallel resistance (Chapter 6) and total series capacitance (Chapter 12). For series-parallel combinations of inductors, determine the total inductance in the same way as total resistance in series-parallel resistive circuits (Chapter 7).

► **FIGURE 13–16**

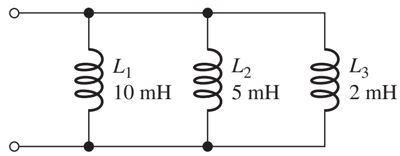
Inductors in parallel.



EXAMPLE 13–5

Determine L_T in Figure 13–17.

► **FIGURE 13–17**



Solution Use Equation 13–6 to determine the total inductance.

$$L_T = \frac{1}{\left(\frac{1}{L_1}\right) + \left(\frac{1}{L_2}\right) + \left(\frac{1}{L_3}\right)} = \frac{1}{\frac{1}{10\text{ mH}} + \frac{1}{5\text{ mH}} + \frac{1}{2\text{ mH}}} = \mathbf{1.25\ mH}$$

Related Problem Determine L_T for a parallel connection of 50 μH, 80 μH, 100 μH, and 150 μH.

SECTION 13–3 CHECKUP

1. State the rule for combining inductors in series.
2. What is L_T for a series connection of 100 μH, 500 μH, and 2 mH?
3. Five 100 mH coils are connected in series. What is the total inductance?
4. Compare the total inductance in parallel with the smallest-value individual inductor.
5. The calculation of total parallel inductance is analogous to that for total parallel resistance. (T or F)
6. Determine L_T for each parallel combination:
   (a) 40 μH and 60 μH
   (b) 100 mH, 50 mH, and 10 mH

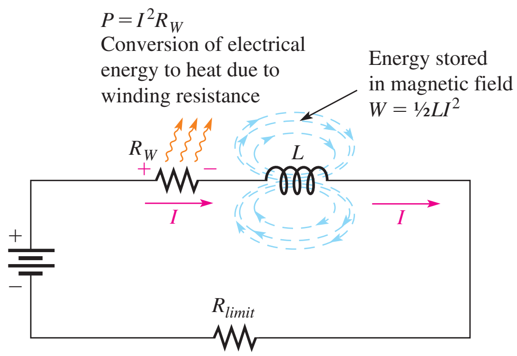
13–4 INDUCTORS IN DC CIRCUITS

Energy is stored in the electromagnetic field of an inductor when it is connected to a dc voltage source. The buildup of current through the inductor occurs in a predictable manner, which is dependent on the time constant of the circuit. The time constant is determined by the inductance and the resistance in a circuit.

After completing this section, you should be able to

- **Analyze inductive dc switching circuits**
  - Describe the increase and decrease of current in an inductor when switching occurs.
  - Define *RL time constant*
  - Describe induced voltage
  - Write the exponential equations for current in an inductor

When there is constant direct current in an inductor, there is no induced voltage. There is, however, a voltage drop due to the winding resistance of the coil. The inductance itself appears as a short to dc. Energy is stored in the electromagnetic field according to the formula previously stated in Equation 13–2, $W = \frac{1}{2}LI^2$. For the inductor, the only energy conversion to heat occurs in the winding resistance ($P = I^2R_W$). This condition is illustrated in Figure 13–18. Normally inductors are wound with fine wire. If the current is excessive, the inductor will open, so the dissipated power must not exceed ratings. Current can be limited with an external resistor as shown below:



▲ FIGURE 13–18

Energy storage and conversion to heat in an inductor in a dc circuit. The current limiting resistor, R_{limit}, will also dissipate energy in the form of heat.

The *RL* Time Constant

Because the inductor's basic action is to develop a voltage that opposes a change in its current, it follows that current cannot change instantaneously in an inductor. A certain time is required for the current to make a change from one value to another. The rate at which the current changes is determined by the *RL* time constant.

The *RL* time constant is a fixed time interval that equals the ratio of the inductance to the resistance.

The formula is

Equation 13–7
$$\tau = \frac{L}{R}$$

where τ is in seconds when inductance (L) is in henries and resistance (R) is in ohms.

It is easy to show that the unit of inductance (henry), divided by the unit of resistance (ohm) is the second. From Equation 13–1, the henry can be written as a (volt-second)/ampere. The ohm (from Ohm's law) is a volt/ampere. Substituting the units into Equation 13–7,

$$\frac{L}{R} = \frac{(\text{henry})}{(\text{ohm})} = \frac{\left(\frac{\cancel{\text{volt}} - \text{second}}{\cancel{\text{ampere}}}\right)}{\left(\frac{\cancel{\text{volt}}}{\cancel{\text{ampere}}}\right)} = \text{second}$$

EXAMPLE 13–6 A series RL circuit has a resistance of 1.0 kΩ and an inductance of 2.5 mH. What is the time constant?

Solution

$$\tau = \frac{L}{R} = \frac{2.5\text{ mH}}{1.0\text{ k}\Omega} = \frac{2.5 \times 10^{-3}\text{ H}}{1.0 \times 10^{3}\ \Omega} = 2.5 \times 10^{-6}\text{ s} = \mathbf{2.5\ \mu s}$$

Related Problem Find the time constant for $R = 2.2$ kΩ and $L = 500$ μH.

Current in an Inductor

Increasing Current In a series RL circuit, the current will increase to approximately 63% of its full value in one time-constant interval after voltage is applied. This buildup of current is analogous to the buildup of capacitor voltage during the charging in an RC circuit; they both follow an exponential curve and reach the approximate percentages of the final current as indicated in Table 13–1 and as illustrated in Figure 13–19.

The change in current over five time-constant intervals is illustrated in Figure 13–20. As a practical matter, when the current reaches its final value at 5τ, it ceases to change. At this time, the inductor acts as a short (except for winding resistance) to the constant current. The final value of the current is

$$I_F = \frac{V_S}{R} = \frac{10\text{ V}}{1.0\text{ k}\Omega} = 10\text{ mA}$$

NUMBER OF TIME CONSTANTS	APPROXIMATE % OF FINAL CURRENT
1	63.2
2	86.5
3	95.0
4	98.2
5	99.3 (considered 100%)

TABLE 13–1

Percentage of the final current after each time-constant interval during current buildup.

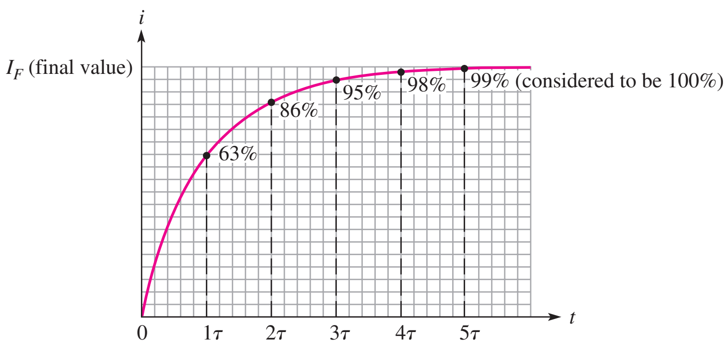


FIGURE 13–19

Increasing current in an inductor.

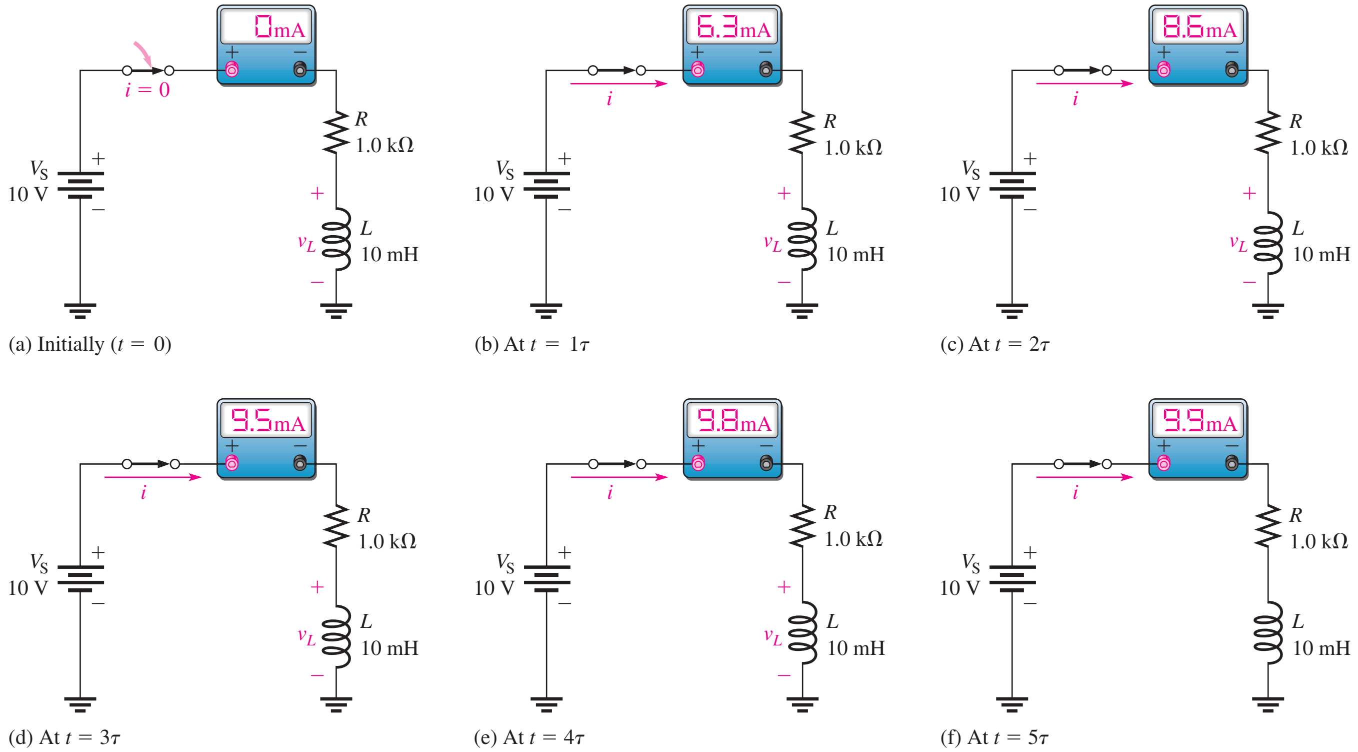


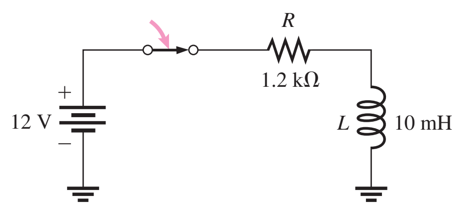
FIGURE 13–20

Illustration of the exponential buildup of current in an inductor. The current increases approximately 63% during each time-constant interval after the switch is closed. A voltage (v_L) is induced in the coil that tends to oppose the increase in current.

EXAMPLE 13–7

Calculate the RL time constant for Figure 13–21. Then determine the current and the time at each time-constant interval, measured from the instant the switch is closed.

FIGURE 13–21



Solution The RL time constant is

$$\tau = \frac{L}{R} = \frac{10\text{ mH}}{1.2\text{ k}\Omega} = \mathbf{8.33\ \mu s}$$

The current at each time-constant interval is a certain percentage of the final current. The final current is

$$I_F = \frac{V_S}{R} = \frac{12\text{ V}}{1.2\text{ k}\Omega} = 10\text{ mA}$$

Using the time-constant percentage values from Table 13–1,

At 1τ: $i = 0.632(10\text{ mA}) = \mathbf{6.32\text{ mA}}$; $t = \mathbf{8.33\ \mu s}$
At 2τ: $i = 0.865(10\text{ mA}) = \mathbf{8.65\text{ mA}}$; $t = \mathbf{16.7\ \mu s}$
At 3τ: $i = 0.950(10\text{ mA}) = \mathbf{9.50\text{ mA}}$; $t = \mathbf{25.0\ \mu s}$
At 4τ: $i = 0.982(10\text{ mA}) = \mathbf{9.82\text{ mA}}$; $t = \mathbf{33.3\ \mu s}$
At 5τ: $i = 0.993(10\text{ mA}) = 9.93\text{ mA} \cong \mathbf{10\text{ mA}}$; $t = \mathbf{41.7\ \mu s}$

Related Problem Repeat the calculations if R is 680 Ω and L is 100 μH.

MultiSim

Use Multisim file E13-07 to verify the calculated results in this example and to confirm your calculation for the related problem. Use a square wave to replace the dc voltage source and the switch.

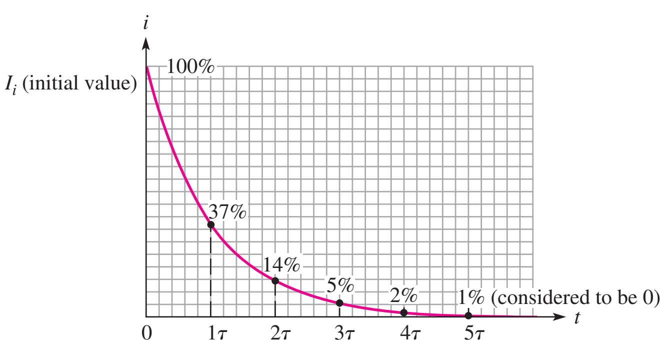
Decreasing Current Current in an inductor decreases exponentially according to the approximate percentage values in Table 13–2 and in Figure 13–22.

The change in current over five time-constant intervals is illustrated in Figure 13–23. As a practical matter, when the current reaches its final value of approximately 0 A, it ceases to change. Before the switch is opened, the current through L is at a constant value of 10 mA, which is determined by R_1 because L acts ideally as a short. When the switch is opened, the induced inductor voltage initially provides 10 mA through R_2. The current then decreases by 63% during each time constant interval.

NUMBER OF TIME CONSTANTS	APPROXIMATE % OF INITIAL CURRENT
1	36.8
2	13.5
3	5.0
4	1.8
5	0.07 (considered 0)

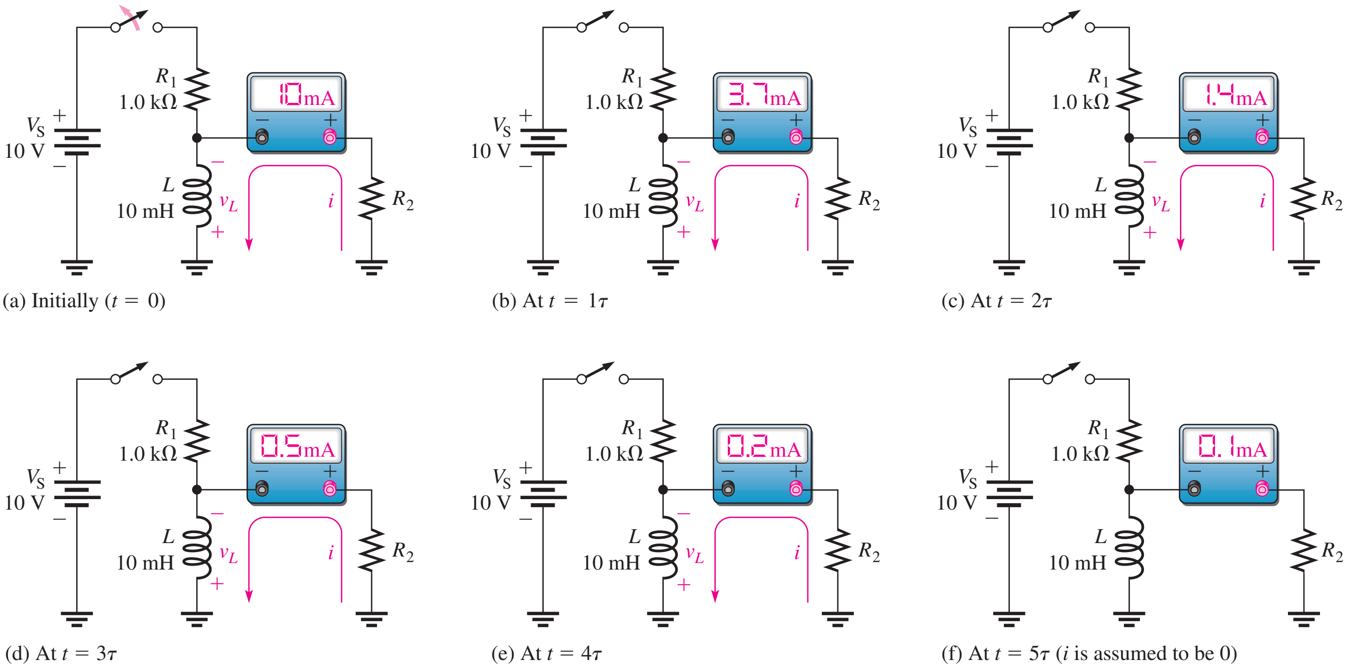
◀ **TABLE 13–2**

Percentage of initial current after each time-constant interval while current is decreasing.



▲ **FIGURE 13–22**

Decreasing current in an inductor.



▲ FIGURE 13–23

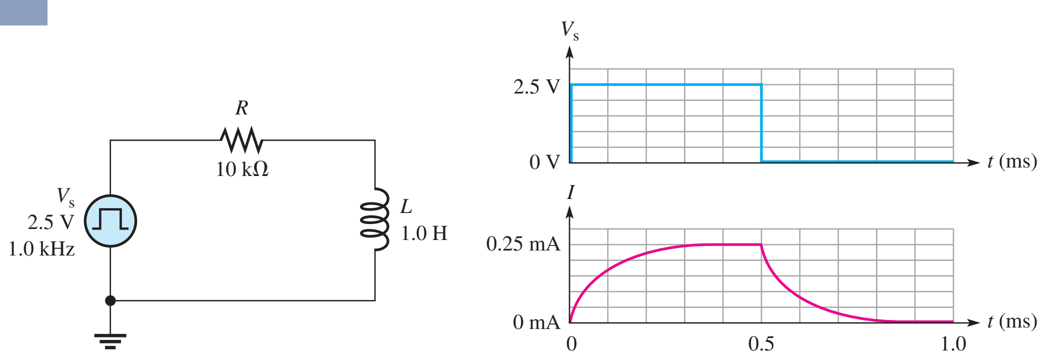
Illustration of the exponential decrease of current in an inductor. The current decreases approximately 63% during each time-constant interval after the switch is opened. A voltage (V_L) is induced in the coil that tends to oppose the decrease in current.

TECH NOTE

To measure the current waveform in a series *RL* circuit, you can measure the voltage across the resistor and apply Ohm's law. If the resistor is ungrounded, as in Figure 13–24, you can use a different measurement to view the resistor voltage by connecting one oscilloscope probe across each end of the resistor and selecting ADD and Invert on the oscilloscope. Both channels on the oscilloscope should be set to the same VOLTS/DIV setting. Alternatively, you can reverse the components (see the Multisim problem in Example 13–8).

Response to a Square Wave

A good way to demonstrate both increasing and decreasing current in an *RL* circuit is to use a square wave voltage as the input. The square wave is a useful signal for observing the dc response of a circuit because it automatically provides on and off action similar to a switch. (Time response will be covered further in Chapter 20.) When the square wave goes from its low level to its high level, the current in the circuit responds by exponentially rising to its final value. When the square wave returns to the zero level, the current in the circuit responds by exponentially decreasing to its zero value. Figure 13–24 shows input voltage and current waveforms.



▲ FIGURE 13–24

Current response of an ideal inductor to a square wave input.

EXAMPLE 13–8 For the circuit in Figure 13–24, what is the current at 0.1 ms and 0.6 ms?

Solution The RL time constant for the circuit is

$$\tau = \frac{L}{R} = \frac{1.0\text{ H}}{10\text{ k}\Omega} = 0.1\text{ ms}$$

If the square-wave generator period is long enough for the current to reach its maximum value in 5τ, the current will increase exponentially and during each time constant interval will have a value equal to the percentage of the final current given in Table 13–1. The final current is

$$I_F = \frac{V_S}{R} = \frac{2.5\text{ V}}{10\text{ k}\Omega} = 0.25\text{ mA}$$

The current at 0.1 ms is

$$i = 0.632(0.25\text{ mA}) = \mathbf{0.158\text{ mA}}$$

At 0.6 ms, the square-wave input has been at the 0 V level for 0.1 ms, or 1τ, and the current decreases from the maximum value toward its final value of 0 mA by 63.2%. Therefore,

$$i = 0.25\text{ mA} - 0.632(0.25\text{ mA}) = \mathbf{0.092\text{ mA}}$$

Related Problem What is the current at 0.2 ms and 0.8 ms?

MultiSim

Open the Multisim file E13-08. Notice that the inductor and resistor are reversed in order to ground one side of the resistor and make the measurement of the resistor voltage simpler. The shape of the current in the circuit is the same as the shape of the resistor voltage. By applying Ohm's law to the resistor voltage, you can find the current in the circuit at any instant in time. Confirm that the current at a time 0.1 ms is close to the calculated value.

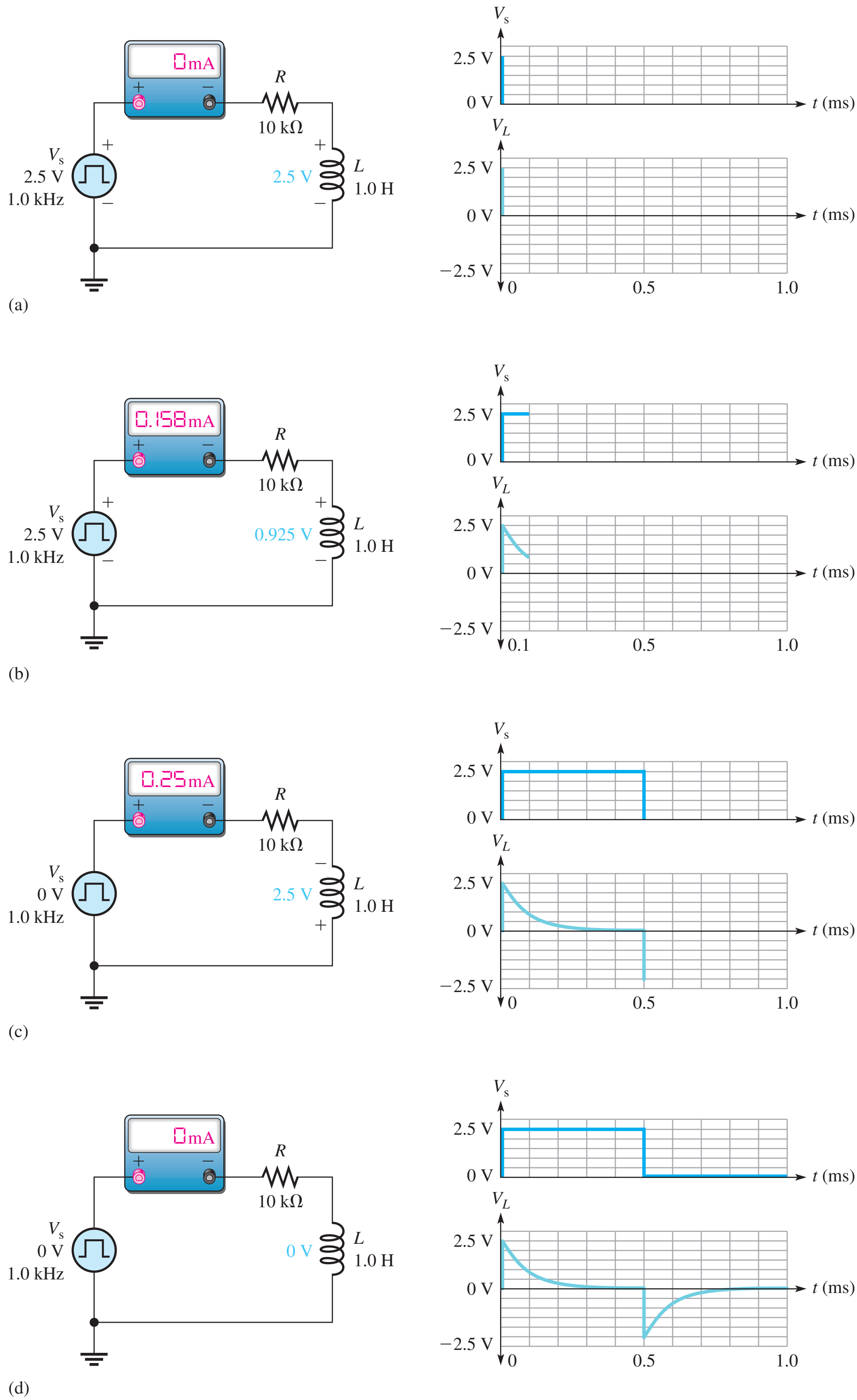
Voltages in a Series *RL* Circuit

As you know, when current changes in an inductor, a voltage is induced. Let's examine what happens to the induced voltage across the inductor in the series circuit in Figure 13–25 during one complete cycle of a square wave input. Keep in mind that the generator produces a level that is like switching a dc source on and then puts an "automatic" low resistance (ideally zero) path across the source when it returns to its zero level.

An ammeter placed in the circuit shows the current in the circuit at any instant in time. The V_L waveform is the voltage across the inductor. In Figure 13–25(a), the square wave has just transitioned from zero to its maximum value of 2.5 V. In accordance with Lenz's law, a voltage is induced across the inductor that opposes this *change* as the magnetic field surrounding the inductor builds up. There is no current in the circuit due to the equal but opposing voltages.

As the magnetic field builds up, the induced voltage across the inductor decreases, and current is in the circuit. After 1τ, the induced voltage across the inductor has decreased by 63%, which causes the current to increase by 63% to 0.158 mA. This is shown in Figure 13–25(b) at the end of one time constant (0.1 ms).

The voltage on the inductor continues to exponentially decrease to zero, at which point the current is limited only by the circuit resistance. Then the square wave goes back to zero (at $t = 0.5$ ms) as shown in Figure 13–25(c). Again a voltage is induced



▲ **FIGURE 13–25**

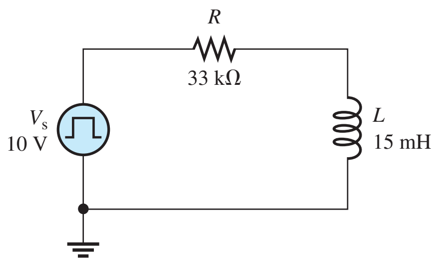
Voltage response of an ideal inductor to a square-wave input.

across the inductor opposing this change. This time, the polarity of the inductor voltage is reversed due to the collapsing magnetic field. Although the source voltage is 0, the collapsing magnetic field maintains current in the same direction until the current decreases to zero, as shown in Figure 13–25(d).

EXAMPLE 13–9

(a) The circuit in Figure 13–26 has a square-wave input. What is the highest frequency that can be used and still observe the complete waveform across the inductor?

(b) Assume the generator is set to the frequency determined in (a). Describe the voltage waveform across the resistor?



▲ FIGURE 13–26

Solution **(a)** $\tau = \dfrac{L}{R} = \dfrac{15\text{ mH}}{33\text{ k}\Omega} = 0.455\ \mu\text{s}$

The period needs to be 10 times longer than τ to observe the entire wave.

$$T = 10\tau = 4.55\ \mu\text{s}$$

$$f = \frac{1}{T} = \frac{1}{4.55\ \mu\text{s}} = \mathbf{220\ kHz}$$

(b) The voltage across the resistor has the same shape as the current waveform. The general shape was shown in Figure 13–24 and has a maximum value of 10 V (the same V_S assuming no winding resistance).

Related Problem What is the maximum voltage across the resistor for $f = 220$ kHz?

Use Multisim file E13-09 to verify the calculated results in this example and to confirm your calculation for the related problem.

The Exponential Formulas

The formulas for the exponential current and voltage in an *RL* circuit are similar to those used in Chapter 12 for the *RC* circuit, and the universal exponential curves in Figure 12–35 apply to inductors as well as capacitors. The general formulas for *RL* circuits are stated as follows:

$$v = V_F + (V_i - V_F)e^{-Rt/L} \qquad \text{Equation 13–8}$$

$$i = I_F + (I_i - I_F)e^{-Rt/L} \qquad \text{Equation 13–9}$$

where V_F and I_F are the final values of voltage and current, V_i and I_i are the initial values of voltage and current. The lowercase italic letters v and i are the instantaneous values of the inductor voltage and current at time t.

Increasing Current The formula for the special case in which an increasing exponential current curve begins at zero is derived by setting $I_i = 0$ in Equation 13–9.

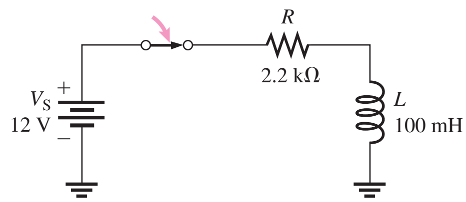
Equation 13–10

$$i = I_F(1 - e^{-Rt/L})$$

Using Equation 13–10, you can calculate the value of the increasing inductor current at any instant of time. You can calculate voltage by substituting v for i and V_F for I_F in Equation 13–10. Notice that the exponent Rt/L can also be written as $t/(L/R) = t/\tau$.

EXAMPLE 13–10

In Figure 13–27, determine the inductor current 30 μs after the switch is closed.



▲ FIGURE 13–27

Solution The RL time constant is

$$\tau = \frac{L}{R} = \frac{100\text{ mH}}{2.2\text{ k}\Omega} = 45.5\ \mu\text{s}$$

The final current is

$$I_F = \frac{V_S}{R} = \frac{12\text{ V}}{2.2\text{ k}\Omega} = 5.45\text{ mA}$$

The initial current is zero. Notice that 30 μs is less than one time constant, so the current will reach less than 63% of its final value in that time.

$$i_L = I_F(1 - e^{-Rt/L}) = 5.45\text{ mA}(1 - e^{-0.66}) = 5.45\text{ mA}(1 - 0.517) = \mathbf{2.64\ mA}$$

Related Problem In Figure 13–27, determine the inductor current 55 μs after the switch is closed.

Decreasing Current The formula for the special case in which a decreasing exponential current has a final value of zero is derived by setting $I_F = 0$ in Equation 13–9.

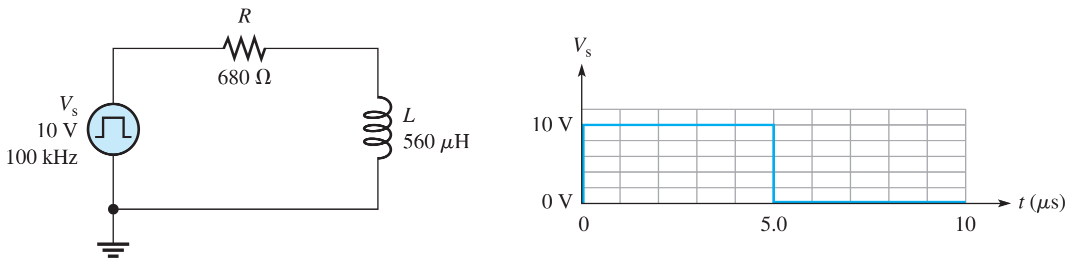
Equation 13–11

$$i = I_i e^{-Rt/L}$$

This formula can be used to calculate the decreasing inductor current at any instant, as the next example shows.

EXAMPLE 13–11

In Figure 13–28, what is the current at each microsecond interval for one complete cycle of the input square wave, V_S after calculating the current at each time, sketch the current waveform.



▲ **FIGURE 13–28**

Solution

$$\tau = \frac{L}{R} = \frac{560\ \mu\text{H}}{680\ \Omega} = 0.824\ \mu\text{s}$$

When the pulse goes from 0 V to 10 V at $t = 0$, the final current is

$$I_F = \frac{V_S}{R} = \frac{10\ \text{V}}{680\ \Omega} = 14.7\ \text{mA}$$

For the increasing current,

$$i = I_F(1 - e^{-Rt/L}) = I_F(1 - e^{-t/\tau})$$

$$\text{At } 1\ \mu\text{s: } i = 14.7\ \text{mA}(1 - e^{-1\mu\text{s}/0.824\mu\text{s}}) = \mathbf{10.3\ mA}$$
$$\text{At } 2\ \mu\text{s: } i = 14.7\ \text{mA}(1 - e^{-2\mu\text{s}/0.824\mu\text{s}}) = \mathbf{13.4\ mA}$$
$$\text{At } 3\ \mu\text{s: } i = 14.7\ \text{mA}(1 - e^{-3\mu\text{s}/0.824\mu\text{s}}) = \mathbf{14.3\ mA}$$
$$\text{At } 4\ \mu\text{s: } i = 14.7\ \text{mA}(1 - e^{-4\mu\text{s}/0.824\mu\text{s}}) = \mathbf{14.6\ mA}$$
$$\text{At } 5\ \mu\text{s: } i = 14.7\ \text{mA}(1 - e^{-5\mu\text{s}/0.824\mu\text{s}}) = \mathbf{14.7\ mA}$$

When the pulse goes from 10 V to 0 V at $t = 5\ \mu$s, the current decreases exponentially. For the decreasing current,

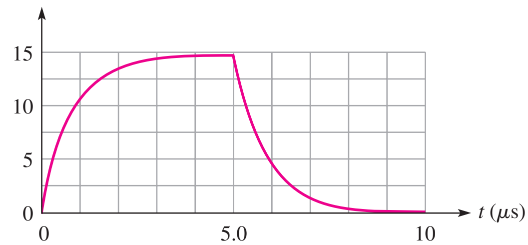
$$i = I_i(e^{-Rt/L}) = I_i(e^{-t/\tau})$$

The initial current is the value at 5 μs, which is 14.7 mA.

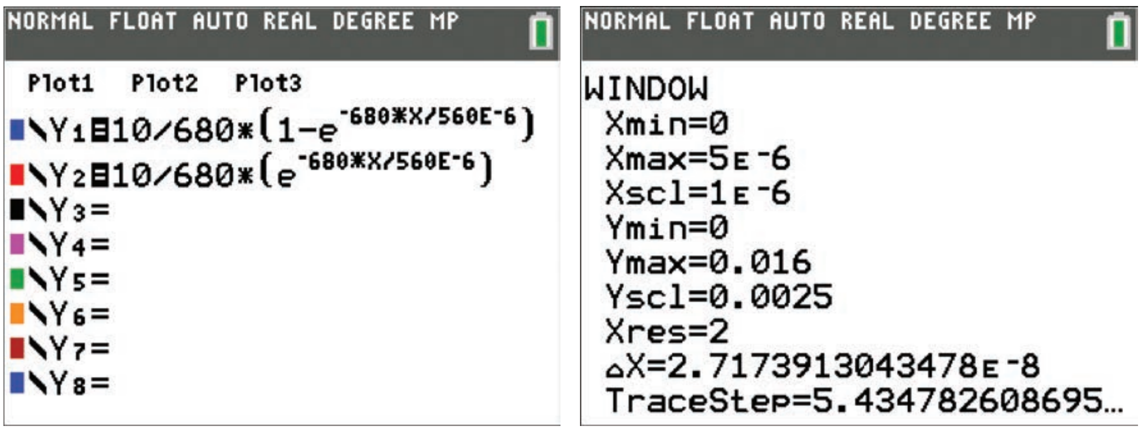
$$\text{At } 6\ \mu\text{s: } i = 14.7\ \text{mA}(1 - e^{-1\mu\text{s}/0.824\mu\text{s}}) = \mathbf{4.37\ mA}$$
$$\text{At } 7\ \mu\text{s: } i = 14.7\ \text{mA}(1 - e^{-2\mu\text{s}/0.824\mu\text{s}}) = \mathbf{1.30\ mA}$$
$$\text{At } 8\ \mu\text{s: } i = 14.7\ \text{mA}(1 - e^{-3\mu\text{s}/0.824\mu\text{s}}) = \mathbf{0.38\ mA}$$
$$\text{At } 9\ \mu\text{s: } i = 14.7\ \text{mA}(1 - e^{-4\mu\text{s}/0.824\mu\text{s}}) = \mathbf{0.11\ mA}$$
$$\text{At } 10\ \mu\text{s: } i = 14.7\ \text{mA}(1 - e^{-5\mu\text{s}/0.824\mu\text{s}}) = \mathbf{0.03\ mA}$$

Figure 13–29 is a graph of these results.

► **FIGURE 13–29**



You can simplify calculating a series of values and plotting them by using a graphing calculator. Enter separate equations for Y_1 and Y_2. Y_1 represents the rising current during the pulse on time; Y_2 represents the current when the pulse returns to zero. The equations for this example are entered as shown in Figure 13–30(a). Figure 13–30(b) shows parameters that will produce a plot similar to Figure 13–29. Set up the plot using the window key. Set up table parameters using 2nd window.



(a) Equations

(b) Plot parameters

▲ **FIGURE 13–30**

Input for graphing calculator solution of Example 13–11.

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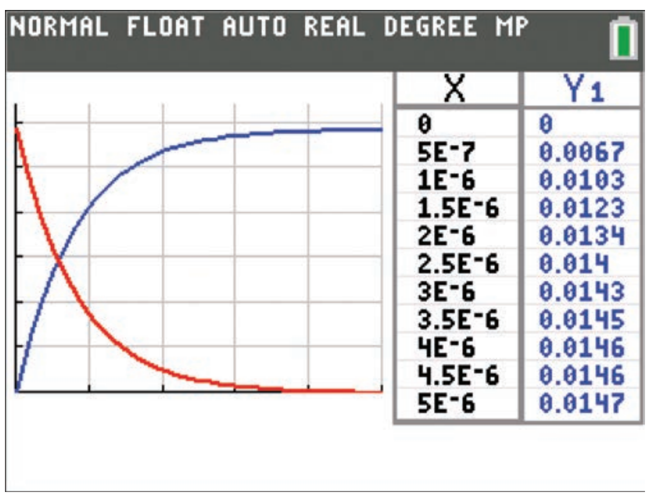
You can view a table of values and the graph by selecting GRAPH-TABLE in the mode menu. Figure 13–3 shows the result. The blue plot represents the rising current; the red plot represents the falling current. The current (Y_1) for the rising curve as shown on the table is given every 0.5 μs up to 5 μs (time is the X variable).



▲ **FIGURE 13–31**

Rising (blue) and falling (red) current and table of rising current values.

Images used with permission by Texas Instruments, Inc.

Related Problem What is the current at 0.5 μs?

Use Multisim file E13-11 to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 13–4 CHECKUP

1. A 15 mH inductor with a winding resistance of 60 Ω has a constant direct current of 10 mA through it. What is the voltage drop across the inductor?
2. A 20 V dc source is connected to a series *RL* circuit with a switch. At the instant of switch closure, what are the values of i and v_L?
3. In the same circuit as in Question 2, after a time interval equal to 5τ from switch closure, what is v_L?
4. In a series *RL* circuit where $R = 1.0\ \text{k}\Omega$ and $L = 500\ \mu\text{H}$, what is the time constant? Determine the current 0.25 μs after a switch connects 10 V across the circuit.

13–5 INDUCTORS IN AC CIRCUITS

An inductor passes ac with an amount of opposition called inductive reactance that depends on the frequency of the ac. The concept of the derivative was introduced in Chapter 12, and the expression for induced voltage in an inductor was stated earlier in Equation 13–1. These are used again in this section.

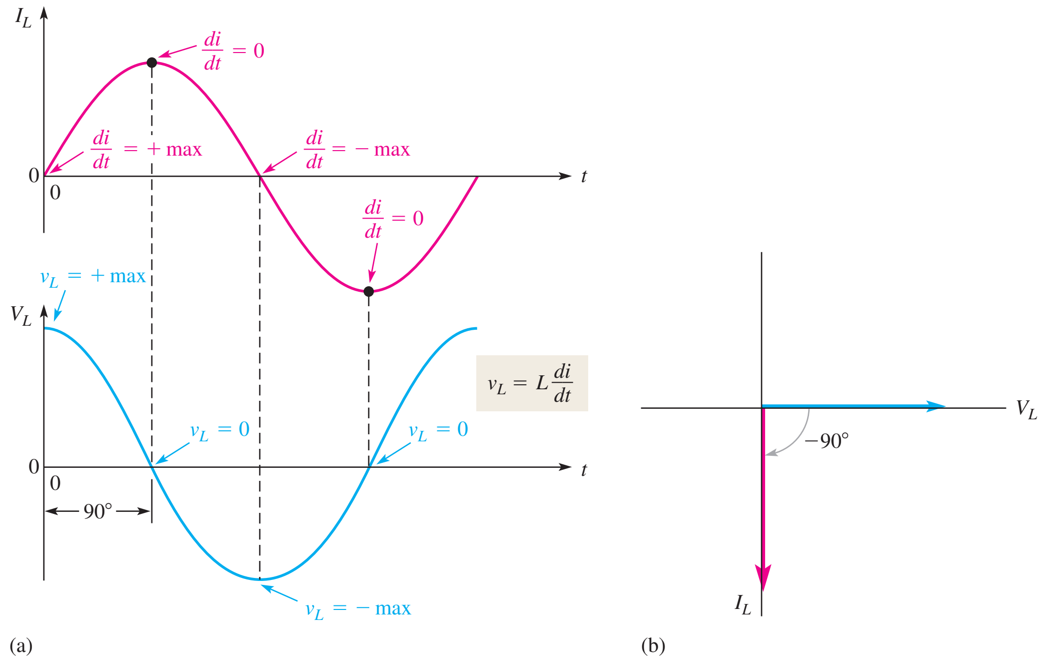
After completing this section, you should be able to

- **Analyze inductive ac circuits**
  - Explain why an inductor causes a phase shift between voltage and current
  - Define *inductive reactance*
  - Determine the value of inductive reactance in a given circuit
  - Discuss instantaneous, true, and reactive power in an inductor

Phase Relationship of Current and Voltage in an Inductor

From Equation 13–1, the formula for induced voltage, you can see that the faster the current through an inductor changes, the greater the induced voltage will be. For example, if the rate of change of current is zero, the voltage is zero [$v_{\text{ind}} = L(di/dt) = L(0) = 0$ V]. When di/dt is a positive-going maximum, v_{ind} is a positive maximum; when di/dt is a negative-going maximum, v_{ind} is a negative maximum.

When a sinusoidal current is in an inductor, the current can be expressed mathematically as $i(t) = I_p(\sin 2\pi ft)$. The current curve has the same shape as the mathematical sine curve as shown in Figure 13–32(a). The rate of change of the sine function is the cosine function, which leads the sine function by 90°. Because the induced voltage across the inductor is the rate of change of current, the voltage leads the current by 90° in an ideal inductor as shown in Figure 13–32(b).



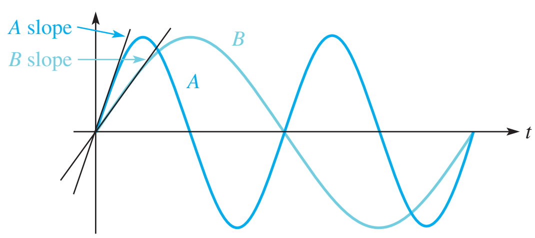
▲ **FIGURE 13–32**

Phase relation of V_L and I_L in an inductor. Current always lags the inductor voltage by 90°.

Inductive Reactance, X_L

Inductive reactance is the opposition to sinusoidal current, expressed in ohms. The symbol for inductive reactance is X_L.

To develop a formula for X_L, we use the relationship $v_{ind} = L(di/dt)$ and the curves in Figure 13–33. The rate of change of current is directly related to frequency. The faster the current changes, the higher the frequency. For example, you can see that in Figure 13–33, the slope of sine wave A at the zero crossings is steeper than that of sine wave B. Recall that the slope of a curve at a point indicates the rate of change at that point. Sine wave A has a higher frequency than sine wave B, as indicated by a greater maximum rate of change (di/dt is greater at the zero crossings).



▲ **FIGURE 13–33**

Slope indicates rate of change. Sine wave A has a greater rate of change at the zero crossing than B, and thus A has a higher frequency.

When frequency increases, di/dt increases, and thus v_{ind} increases. When frequency decreases, di/dt decreases, and thus v_{ind} decreases. The induced voltage is directly dependent on frequency.

$$\overset{\uparrow}{v_{ind}} = L(d\overset{\uparrow}{i}/dt) \quad \text{and} \quad \underset{\downarrow}{v_{ind}} = L(d\underset{\downarrow}{i}/dt)$$

An increase in induced voltage means more opposition (X_L is greater). Therefore, X_L is directly proportional to induced voltage and thus directly proportional to frequency.

X_L is proportional to f.

Now, if di/dt is constant and the inductance is varied, an increase in L produces an increase in v_{ind}, and a decrease in L produces a decrease in v_{ind}.

$$\overset{\uparrow}{v_{ind}} = \overset{\uparrow}{L}(di/dt) \quad \text{and} \quad \underset{\downarrow}{v_{ind}} = \underset{\downarrow}{L}(di/dt)$$

Again, an increase in v_{ind} means more opposition (greater X_L). Therefore, X_L is directly proportional to induced voltage and thus directly proportional to inductance. The inductive reactance is directly proportional to both f and L.

X_L is proportional to fL.

The formula (derived in Appendix B) for inductive reactance, X_L, is

$$X_L = 2\pi fL$$

Equation 13–12

Inductive reactance, X_L, is in ohms when f is in hertz and L is in henries. As with capacitive reactance, the 2π term is a constant factor in the equation, which comes from the relationship of a sine wave to rotational motion.

EXAMPLE 13–12

A sinusoidal voltage is applied to the circuit in Figure 13–34. The frequency is 10 kHz. Determine the inductive reactance.

► FIGURE 13–34

Solution Convert 10 kHz to 10×10^3 Hz and 5 mH to 5×10^{-3} H. Therefore, the inductive reactance is

$$X_L = 2\pi fL = 2\pi(10 \times 10^3\ \text{Hz})(5 \times 10^{-3}\ \text{H}) = \mathbf{314\ \Omega}$$

Related Problem What is X_L in Figure 13–34 if the frequency is increased to 35 kHz?

Reactance for Series Inductors

As given in Equation 13–4, the total inductance of series inductors is the sum of the individual inductances. Because reactance is directly proportional to the inductance, the total reactance of series inductors is the sum of the individual reactances.

$$X_{L(tot)} = X_{L1} + X_{L2} + X_{L3} + \cdots + X_{Ln}$$

Equation 13–13

Notice that Equation 13–13 has the same form as Equation 13–4. It also has the same form as the formulas for finding the total opposition to current such as the total resistance of series resistors or the total reactance of series capacitors. When combining the resistance or reactance of the same type of component in series (resistors, inductors, or capacitors), you simply add the individual oppositions to obtain the total.

Reactance for Parallel Inductors

In an ac circuit with parallel inductors, Equation 13–6 was given to find the total inductance. It stated that the total inductance is the reciprocal of the sum of the reciprocals of the inductors. Likewise, the total inductive reactance is the reciprocal of the sum of the reciprocals of the individual reactances.

Equation 13–14

$$X_{L(tot)} = \frac{1}{\dfrac{1}{X_{L1}} + \dfrac{1}{X_{L2}} + \dfrac{1}{X_{L3}} + \cdots + \dfrac{1}{X_{Ln}}}$$

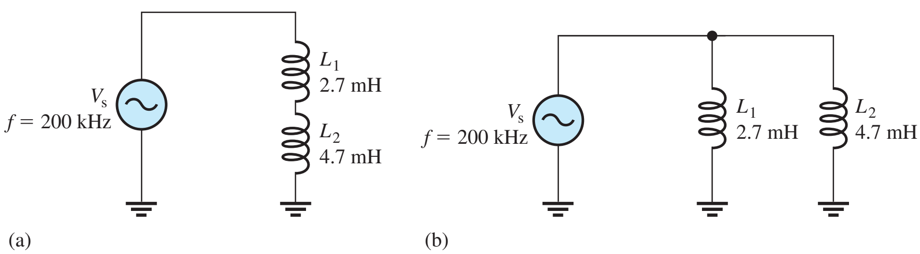
Notice that Equation 13–14 has the same form as Equation 13–6. It also has the same form as the formulas for finding the total resistance of parallel resistors or the total reactance of parallel capacitors. When combining the resistance or reactance of the same type of component in parallel (resistors, inductors, or capacitors), you take the reciprocal of the sum of the reciprocals to obtain the total opposition.

For two inductors in parallel, Equation 13–14 can be reduced to the product-over-sum form. Thus,

$$X_{L(tot)} = \frac{X_{L1}X_{L2}}{X_{L1} + X_{L2}}$$

EXAMPLE 13–13

What is the total inductive reactance of each circuit of Figure 13–35?



▲ **FIGURE 13–35**

Solution The reactances of the individual inductors are the same in both circuits. From Equation 13–12,

$$X_{L1} = 2\pi fL_1 = 2\pi(200\text{ kHz})(2.7\text{ mH}) = 3.39\text{ k}\Omega$$
$$X_{L2} = 2\pi fL_2 = 2\pi(200\text{ kHz})(4.7\text{ mH}) = 5.91\text{ k}\Omega$$

For the series inductors in Figure 13–35(a), the total reactance is the sum of X_{L1} and X_{L2}, as given in Equation 13–13.

$$X_{L(tot)} = X_{L1} + X_{L2} = 3.39\text{ k}\Omega + 5.91\text{ k}\Omega = \mathbf{9.30\text{ k}\Omega}$$

For the inductors in parallel in Figure 13–35(b), determine the total reactance by Equation 13–14 or from the product-over-sum rule using X_{L1} and X_{L2}.

$$X_{L(tot)} = \frac{X_{L1}X_{L2}}{X_{L1} + X_{L2}} = \frac{(3.39\text{ k}\Omega)(5.91\text{ k}\Omega)}{3.39\text{ k}\Omega + 5.91\text{ k}\Omega} = \mathbf{2.15\text{ k}\Omega}$$

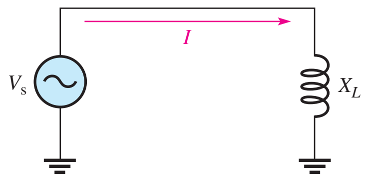
You can also obtain the total reactance for either series or parallel inductors by first finding the total inductance and then substituting in Equation 13–12 to find the total reactance.

Related Problem What is the total inductive reactance for each circuit in Figure 13–35 if $L_1 = 1$ mH and L_2 is unchanged?

Ohm's Law The reactance of an inductor is analogous to the resistance of a resistor. In fact, X_L, just like X_C and R, is expressed in ohms. Since inductive reactance is a form of opposition to current, Ohm's law applies to inductive circuits as well as to resistive circuits and capacitive circuits, and it is stated as follows for Figure 13–36.

$$I = \frac{V_S}{X_L}$$

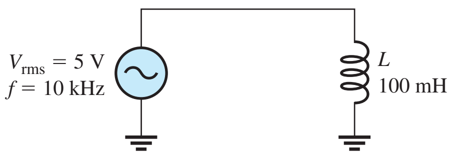
When applying Ohm's law in ac circuits, you must express both the current and the voltage in the same way, that is, both in rms, both in peak, and so on.



▲ FIGURE 13–36

EXAMPLE 13–14 Determine the rms current in Figure 13–37.

► FIGURE 13–37



Solution Convert 10 kHz to 10×10^3 Hz and 100 mH to 100×10^{-3} H. Then calculate X_L.

$$X_L = 2\pi fL = 2\pi(10 \times 10^3\text{ Hz})(100 \times 10^{-3}\text{ H}) = 6.28\text{ k}\Omega$$

Apply Ohm's law to determine the rms current.

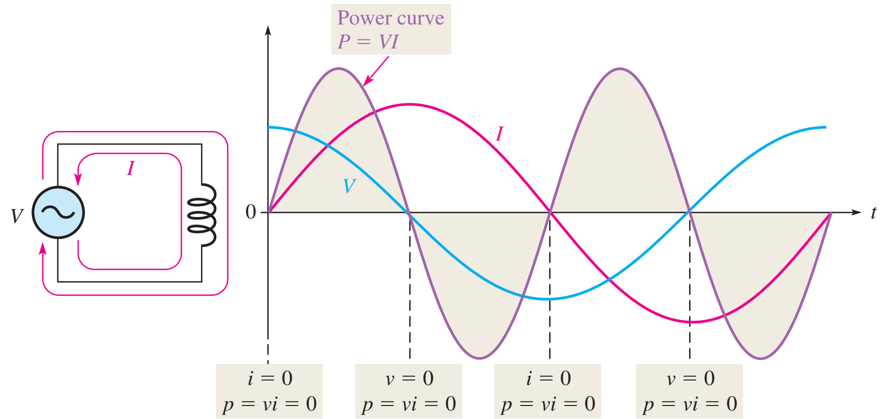
$$I_{rms} = \frac{V_{rms}}{X_L} = \frac{5\text{ V}}{6.28\text{ k}\Omega} = \mathbf{796\ \mu A}$$

Related Problem Determine the rms current in Figure 13–37 for the following values: $V_{rms} = 12$ V, $f = 4.9$ kHz, and $L = 680$ mH.

Use Multisim file E13-14 to verify the calculated results in this example and to confirm your calculation for the related problem.

Power in an Inductor

As discussed earlier, an inductor stores energy in its magnetic field when there is current in the inductor. An ideal inductor (assuming no winding resistance) does not dissipate energy; it only stores it. When an ac voltage is applied to an ideal inductor, energy is stored by the inductor during a portion of the cycle, then the stored energy is returned to the source during another portion of the cycle. No net energy is lost in an ideal inductor due to conversion to heat. Figure 13–38 shows the power curve that results from one cycle of inductor current and voltage.



▲ FIGURE 13–38

Power curve.

Instantaneous Power (p) The product of v and i gives instantaneous power. At points where v or i is zero, p is also zero. When both v and i are positive, p is also positive. When either v or i is positive and the other negative, p is negative. When both v and i are negative, p is positive. As you can see in Figure 13–38, the power follows a sinusoidal-shaped curve. Positive values of power indicate that energy is stored by the inductor. Negative values of power indicate that energy is returned from the inductor to the source. Note that the power fluctuates at a frequency twice that of the voltage or current as energy is alternately stored and returned to the source.

True Power (P_{true}) Ideally, all of the energy stored by an inductor during the positive portion of the power cycle is returned to the source during the negative portion. No net energy is lost due to conversion to heat in the inductor, so the true power is zero. Actually, because of winding resistance in a practical inductor, some power is always dissipated, and there is a very small amount of true power, which can normally be neglected.

Equation 13–15
$$P_{true} = (I_{rms})^2 R_W$$

Reactive Power (P_r) The rate at which an inductor stores or returns energy is called its **reactive power**, with the unit of VAR (volt-ampere reactive). The reactive power is a nonzero quantity because at any instant in time the inductor is actually taking energy from the source or returning energy to it. Reactive power does not represent an energy loss due to conversion to heat. The following formulas apply:

Equation 13–16
$$P_r = V_{rms} I_{rms}$$

Equation 13–17
$$P_r = \frac{V_{rms}^2}{X_L}$$

Equation 13–18
$$P_r = I_{rms}^2 X_L$$

EXAMPLE 13–15

A 10 V rms signal with a frequency of 1 kHz is applied to a 10 mH coil with a negligible winding resistance. Determine the reactive power (P_r).

Solution First, calculate the inductive reactance and current values.

$$X_L = 2\pi fL = 2\pi(1\text{ kHz})(10\text{ mH}) = 62.8\ \Omega$$

$$I = \frac{V_S}{X_L} = \frac{10\text{ V}}{62.8\ \Omega} = 159\text{ mA}$$

Then, use Equation 13–18.

$$P_r = I^2X_L = (159\text{ mA})^2(62.8\ \Omega) = \mathbf{1.59\ VAR}$$

Related Problem What happens to the reactive power if the frequency increases?

The Quality Factor (Q) of a Coil

The **quality factor (Q)** is the ratio of the reactive power in an inductor to the true power in the winding resistance of the coil or the resistance in series with the coil. It is a ratio of the power in L to the power in R_W. The quality factor is important in resonant circuits, which are studied in Chapter 17. A formula for Q is developed as follows:

$$Q = \frac{\text{reactive power}}{\text{true power}} = \frac{I^2X_L}{I^2R_W}$$

The current is the same in L and R_W; thus, the I^2 terms cancel, leaving

$$Q = \frac{X_L}{R_W}$$

Equation 13–19

When the resistance is just the winding resistance of the coil, the circuit Q and the coil Q are the same. Note that Q is a ratio of like units and, therefore, has no unit itself. The quality factor is also known as unloaded Q because it is defined with no load across the coil. Because X_L is frequency dependent, Q is also frequency dependent.

SECTION 13–5 CHECKUP

1. State the phase relationship between current and voltage in an inductor.
2. Calculate X_L for $f = 5$ kHz and $L = 100$ mH.
3. At what frequency is the reactance of a 50 μH inductor equal to 800 Ω?
4. Calculate the rms current in Figure 13–39.
5. An ideal 50 mH inductor is connected to a 12 V rms source. What is the true power? What is the reactive power at a frequency of 1 kHz?

► FIGURE 13–39

13–6 Inductor Applications

Inductors are not as versatile as capacitors and tend to be more limited in their application due, in part, to size, cost, and nonideal behavior (internal resistance, etc.). Also, for integrated circuits even small value inductors require a larger area than other components, which increases cost. Large inductors that require a core cannot be integrated. Two common applications for inductors are noise reduction applications and in tuned circuits. Another important application is in switching regulators, which are covered in Section 16–8.

After completing this section, you should be able to

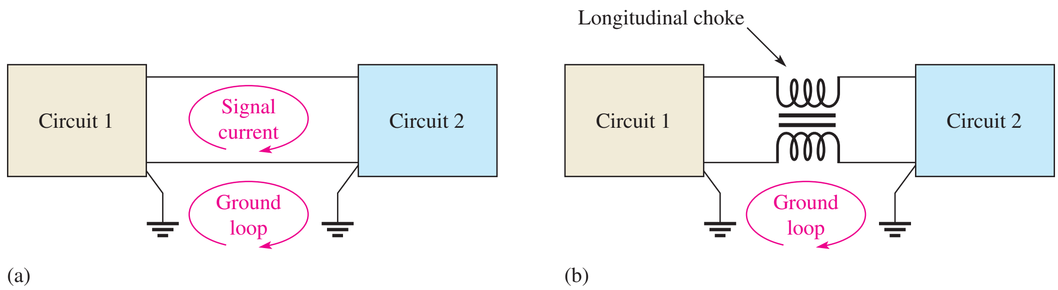
- **Discuss some inductor applications**
  - Discuss two ways in which noise enters a circuit
  - Describe the suppression of electromagnetic interference (EMI)
  - Explain how a ferrite bead is used
  - Discuss the basics of tuned circuits

Noise Suppression

One of the most important applications of inductors has to do with suppressing unwanted electrical noise. The inductors used in these applications are generally wound on a closed core to avoid having the inductor become a source of radiated noise itself. Two types of noise are conductive noise and radiated noise.

Conductive Noise Many systems have common conductive paths connecting different parts of the system, which can conduct high frequency noise from one part of the system to another. Consider the case of two circuits connected with common lines as shown in Figure 13–40(a). A path for high frequency noise exists though the common grounds, creating a condition known as a *ground loop*. Ground loops are particularly a problem in instrumentation systems, where a transducer may be located a distance from the recording system and noise current in the ground can affect the signal.

If the signal of interest changes slowly, a special inductor, called a *longitudinal choke*, can be installed in the signal line as shown in Figure 13–40(b). The longitudinal choke is a form of transformer (covered in Chapter 14) that acts as inductors in each signal line. The ground loop sees a high impedance path, thus reducing the



▲ **FIGURE 13–40**

Longitudinal choke to reduce low-frequency ground-loop current.

noise, while the low-frequency signal is coupled through the low impedance of the choke.

Switching circuits also tend to generate high-frequency noise (above 10 MHz) by virtue of the high-frequency components present. (Recall from Section 11–9 that a pulse waveform contains many high-frequency harmonics.) Certain types of power supplies use high-speed switching circuits that are a source of conductive and radiated noise.

Because an inductor's impedance increases with frequency, inductors are good for blocking electrical noise from these supplies, which should carry only dc. Inductors are frequently installed in the power supply lines to suppress this conductive noise, so that one circuit does not adversely affect another circuit. One or more capacitors may also be used in conjunction with the inductor to improve filtering action.

Radiated Noise Noise can also enter a circuit by way of an electromagnetic field. The noise source can be an adjacent circuit or a nearby power supply. There are several approaches to reducing the effects of radiated noise. Usually, the first step is to determine the cause of the noise and isolate it using shielding or filtering.

Inductors are widely employed in filters that are used to suppress radio-frequency noise. The inductor used for noise suppression must be carefully selected so as not to become a source of radiated noise itself. For high frequencies (>20 MHz), inductors wound on highly permeable toroidal cores are widely used, as they tend to keep the magnetic flux restricted to the core.

RF Chokes

Inductors used for the purpose of blocking very high frequencies are called *radio frequency (RF) chokes*. RF chokes are used for conductive or radiated noise. They are special inductors designed to block high frequencies from getting into or leaving parts of a system by providing a high impedance path for high frequencies. In general, the choke is placed in series with the line for which RF suppression is required. Depending on the frequency of the interference, different types of chokes are required. A common type of electromagnetic interference (EMI) filter wraps the signal line on a toroidal core several times. The toroidal configuration is desired because it contains the magnetic field so that the choke does not become a source of noise itself.

Another common type of RF choke is a ferrite bead such as the example in Figure 13–41. All wires have inductance, and the ferrite bead is a small ferromagnetic material that is strung onto the wire to increase its inductance. The impedance presented by the bead is a function of both the material and the frequency, as well as the size of the bead. It is an effective and inexpensive "choke" for high frequencies. Ferrite beads are common in high-frequency communication systems. Sometimes several are strung together in series to increase the effective inductance. In addition to beads that slip over wires, surface mount ferrite beads also are used in series on PCB traces to reject conducted high-frequency noise, similar to conventional chokes.

▲ **FIGURE 13–41**

Typical ferrite bead.

Tuned Circuits

Inductors are used in conjunction with capacitors to provide frequency selection in communications systems. These tuned circuits allow a narrow band of frequencies to be selected while all other frequencies are rejected. The tuners in your TV and radio receivers are based on this principle and permit you to select one channel or station out of the many that are available.

Frequency selectivity is based on the fact that the reactances of both capacitors and inductors depend on the frequency and on the interaction of these two components when connected in series or parallel. Since the capacitor and the inductor produce opposite phase shifts, their combined opposition to current can be used to obtain a desired response at a selected frequency. Tuned *RLC* circuits are covered in Chapter 17.

SECTION 13–6 CHECKUP

1. Name two types of unwanted noise.
2. What do the letters *EMI* stand for?
3. How is a ferrite bead used?

Application Activity

In this application, you will see how you can test coils for their unknown inductance values using a test setup consisting of a square-wave generator and an oscilloscope. Two coils for which the inductance values are not known are tested using simple laboratory instruments to determine the inductance values. The method is to place the coil in series with a resistor with a known value and measure the time constant. Knowing the time constant and the resistance value, the value of L can be calculated.

The method of determining the time constant is to apply a square wave to the circuit and measure the resulting voltage across the resistor. Each time the square-wave input voltage goes high, the inductor is energized and each time the square wave goes back to zero, the inductor is de-energized. The time it takes for the exponential resistor voltage to increase to approximately its final value equals five time constants. This operation is illustrated in Figure 13–42. To make sure that the winding resistance of the coil can be neglected, it must be measured and the value of the resistor used in the circuit must be selected to be considerably larger than the winding and source resistances.

The Winding Resistance

Assume that the winding resistance of the coil in Figure 13–43 has been measured with an ohmmeter and found to be 85 Ω. To make the winding and source resistances negligible for time constant measurement, a 10 kΩ series resistor is used in the circuit.

1. If 10 V dc is connected with the clip leads as shown, how much current is in the circuit after $t = 5\tau$?

Inductance of Coil 1

Refer to Figure 13–44. To measure the inductance of coil 1, a square-wave voltage is applied to the circuit. The amplitude of the square wave is adjusted to 10 V. The frequency is adjusted so that the inductor has time to fully energize during each square-wave pulse; the scope is set to view a complete energizing curve as shown.

2. Determine the approximate circuit time constant.
3. Calculate the inductance of coil 1.

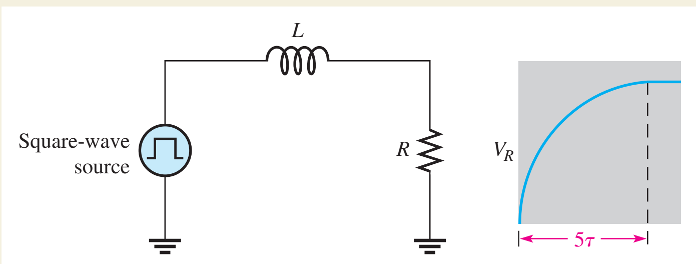
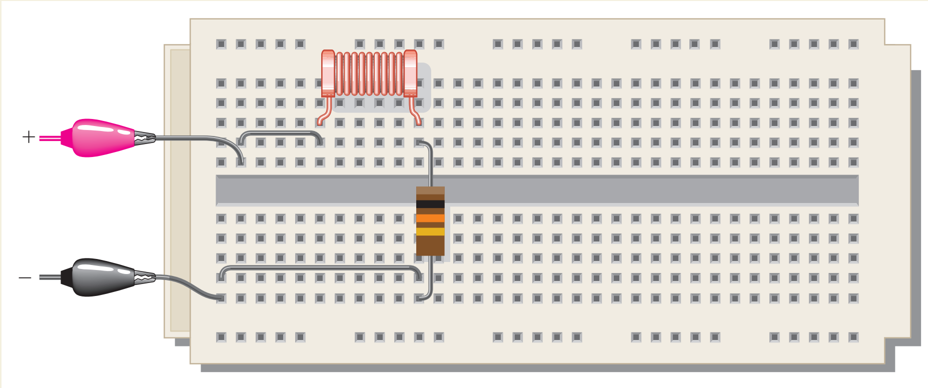


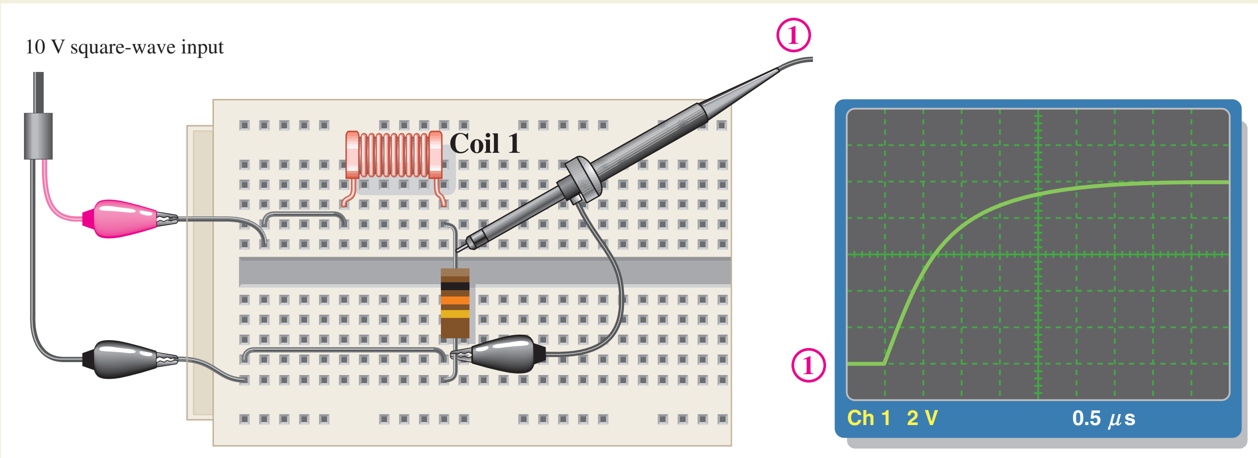
FIGURE 13–42

Circuit for time-constant measurement.



▲ **FIGURE 13–43**

Breadboard setup for measuring the time constant.



▲ **FIGURE 13–44**

Testing coil 1.

The Inductance of Coil 2

Refer to Figure 13–45 in which coil 2 replaces coil 1. To determine the inductance, a 10 V square wave is applied to the breadboarded circuit. The frequency of the square-wave is adjusted so that the inductor has time to fully energize during each square-wave pulse; the scope is set to view a complete energizing curve as shown.

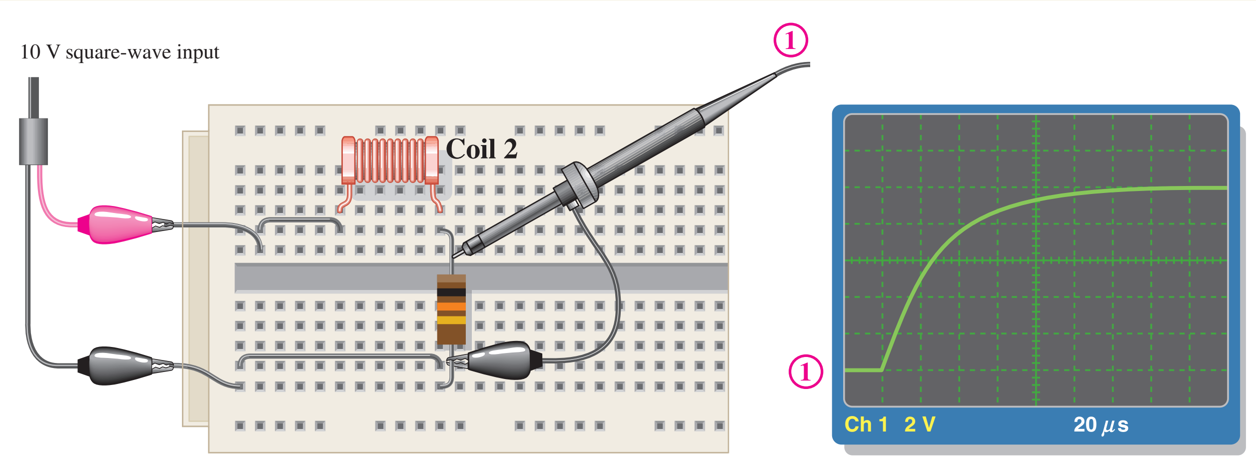
4. Determine the approximate circuit time constant.
5. Calculate the inductance of coil 2.
6. Discuss any difficulty in using this method.
7. Specify how you can use a sinusoidal input voltage instead of a square wave to determine inductance.

Review

8. What is the maximum square-wave frequency that can be used in Figure 13–44?
9. What is the maximum square-wave frequency that can be used in Figure 13–45?
10. What ha ppens if the frequency exceeds the maximum you determined in Questions 8 and 9? Explain how your measurements would be affected.

Multisim Analysis

Open your Multisim software. Connect the *RL* circuit using the value of the resistance shown and the value of the inductance determined in activity 3. Verify the time constant by measurement. Repeat for the inductance determined in activity 5.



▲ FIGURE 13–45

Testing coil 2.

SUMMARY

- Inductance is a measure of a coil's ability to establish an induced voltage as a result of a change in its current.
- An inductor opposes a change in its own current.
- Faraday's law states that relative motion between a magnetic field and a coil induces a voltage across the coil.
- The amount of induced voltage is directly proportional to the inductance and to the rate of change in current.
- Lenz's law states that the polarity of induced voltage is such that the resulting induced current is in a direction that opposes the change in the magnetic field that produced it.
- Energy is stored by an inductor in its magnetic field.
- One henry is the amount of inductance when current, changing at the rate of one ampere per second, induces one volt across the inductor.
- Inductance is directly proportional to the square of the number of turns, the permeability, and the cross-sectional area of the core. It is inversely proportional to the length of the core.
- The permeability of a core material is an indication of the ability of the material to establish a magnetic field.
- The time constant for a series *RL* circuit is the inductance divided by the resistance.
- In an *RL* circuit, the increasing or decreasing voltage and current in an inductor make a 63% change during each time-constant interval.
- Increasing and decreasing voltages and currents follow exponential curves.
- Inductors add in series.
- Total parallel inductance is less than that of the smallest inductor in parallel.
- Voltage leads current by 90° in an inductor.
- Inductive reactance, X_L, is directly proportional to frequency and inductance.
- The true power in an inductor is zero; that is, no energy is lost in an ideal inductor due to conversion to heat, only in its winding resistance.

KEY TERMS

Key terms and other bold terms in the chapter are defined in the end-of-book glossary.

Henry (H) The unit of inductance.

Induced voltage Voltage produced as a result of a changing magnetic field.

Inductance The property of an inductor whereby a change in current causes the inductor to produce a voltage that opposes the change in current.

Inductive reactance The opposition of an inductor to sinusoidal current. The unit is the ohm.

Inductor An electrical device formed by a coil of wire having the property of inductance; also known as *coil.*

Quality factor (*Q*) The ratio of reactive power to true power in an inductor.

***RL* time constant** A fixed time interval, set by the L and R values, that determines the time response of a circuit and is equal to L/R.

Winding The loops or turns of wire in an inductor.

FORMULAS

13–1	$v_{ind} = L\left(\frac{di}{dt}\right)$	Induced voltage
13–2	$W = \frac{1}{2}LI^2$	Energy stored by an inductor
13–3	$L = \frac{N^2\mu A}{l}$	Inductance in terms of physical parameters
13–4	$L_T = L_1 + L_2 + L_3 + \cdots + L_n$	Series inductance
13–5	$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n}$	Reciprocal of total parallel inductance
13–6	$L_T = \frac{1}{\left(\frac{1}{L_1}\right) + \left(\frac{1}{L_2}\right) + \left(\frac{1}{L_3}\right) + \cdots + \left(\frac{1}{L_n}\right)}$	Total parallel inductance
13–7	$\tau = \frac{L}{R}$	Time constant
13–8	$v = V_F + (V_i - V_F)e^{-Rt/L}$	Exponential voltage (general)
13–9	$i = I_F + (I_i - I_F)e^{-Rt/L}$	Exponential current (general)
13–10	$i = I_F(1 - e^{-Rt/L})$	Increasing exponential current beginning at zero
13–11	$i = I_ie^{-Rt/L}$	Decreasing exponential current ending at zero
13–12	$X_L = 2\pi fL$	Inductive reactance
13–13	$X_{L(tot)} = X_{L1} + X_{L2} + X_{L3} + \cdots + X_{Ln}$	Series inductive reactance
13–14	$X_{L(tot)} = \frac{1}{\frac{1}{X_{L1}} + \frac{1}{X_{L2}} + \frac{1}{X_{L3}} + \cdots + \frac{1}{X_{Ln}}}$	Parallel inductive reactance
13–15	$P_{true} = (I_{rms})^2R_W$	True power
13–16	$P_r = V_{rms}I_{rms}$	Reactive power
13–17	$P_r = \frac{V_{rms}^2}{X_L}$	Reactive power
13–18	$P_r = I_{rms}^2X_L$	Reactive power
13–19	$Q = \frac{X_L}{R_W}$	Quality factor

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. An inductor opposes any change in its current.
2. An inductance of 10 mH is equivalent to 1,000 μH.
3. Inductance is proportional to the square of the number of windings.
4. Inductors have no resistance.
5. Ideally, an inductor appears as a short to dc.
6. The amount of voltage induced in a coil is directly proportional to the rate of change of the magnetic field with respect to the coil.
7. Inductive reactance is inversely proportional to frequency.
8. The total reactance of series inductors is the sum of the individual reactances.
9. Voltage lags current in an inductor.
10. The unit of inductive reactance is the ohm.

SELF-TEST

Answers are at the end of the chapter.

1. An inductance of 0.05 μH is larger than
   (a) 0.0000005 H **(b)** 0.000005 H **(c)** 0.000000008 H **(d)** 0.00005 mH
2. An inductance of 0.33 mH is smaller than
   (a) 33 μH **(b)** 330 μH **(c)** 0.05 mH **(d)** 0.0005 H
3. When the current through an inductor increases, the amount of energy stored in the electromagnetic field
   (a) decreases **(b)** remains constant **(c)** increases **(d)** doubles
4. When the current through an inductor doubles, the stored energy
   (a) doubles **(b)** quadruples **(c)** is halved **(d)** does not change
5. The winding resistance of a coil can be decreased by
   (a) reducing the number of turns **(b)** using a larger wire
   (c) changing the core material **(d)** either answer (a) or (b)
6. The inductance of an iron-core coil increases if
   (a) the number of turns is increased **(b)** the iron core is removed
   (c) the length of the core is increased **(d)** larger wire is used
7. Four 10 mH inductors are in series. The total inductance is
   (a) 40 mH **(b)** 2.5 mH **(c)** 40,000 μH **(d)** answers (a) and (c)
8. A 1 mH, a 3.3 mH, and a 0.1 mH inductor are connected in parallel. The total inductance is
   (a) 4.4 mH **(b)** greater than 3.3 mH **(c)** less than 0.1 mH **(d)** answers (a) and (b)
9. An inductor, a resistor, and a switch are connected in series to a 12 V battery. At the instant the switch is closed, the inductor voltage is
   (a) 0 V **(b)** 12 V **(c)** 6 V **(d)** 4 V
10. A sinusoidal voltage is applied across an inductor. When the frequency of the voltage is increased, the current
    (a) decreases **(b)** increases **(c)** does not change **(d)** momentarily goes to zero
11. An inductor and a resistor are in series with a sinusoidal voltage source. The frequency is set so that the inductive reactance is equal to the resistance. If the frequency is increased, then
    (a) $V_R > V_L$ **(b)** $V_L < V_R$ **(c)** $V_L = V_R$ **(d)** $V_L > V_R$
12. An ohmmeter is connected across an inductor and the pointer indicates an infinite value. The inductor is
    (a) good **(b)** open **(c)** shorted **(d)** resistive

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 13–48.

1. The switch is in position 1. When it is thrown into position 2, the inductance between A and B
   (a) increases (b) decreases (c) stays the same
2. If the switch is moved from position 3 to position 4, the inductance between A and B
   (a) increases (b) decreases (c) stays the same

Refer to Figure 13–51.

3. If R were 10 kΩ instead of 1.0 kΩ and the switch is closed, the time it takes for the current to reach its maximum value
   (a) increases (b) decreases (c) stays the same
4. If L is decreased from 10 mH to 1 mH and the swtich is closed, the time constant
   (a) increases (b) decreases (c) stays the same
5. If the source voltage drops from +15 V to +10 V, the time constant
   (a) increases (b) decreases (c) stays the same

Refer to Figure 13–54.

6. If the frequency of the source voltage is increased, the total current
   (a) increases (b) decreases (c) stays the same
7. If L_2 opens, the current through L_1
   (a) increases (b) decreases (c) stays the same
8. If the frequency of the source voltage is decreased, the ratio of the values of the currents through L_2 and L_3
   (a) increases (b) decreases (c) stays the same

Refer to Figure 13–55.

9. If the frequency of the source voltage increases, the voltage across L_1
   (a) increases (b) decreases (c) stays the same
10. If L_3 opens, the voltage across L_2
    (a) increases (b) decreases (c) stays the same

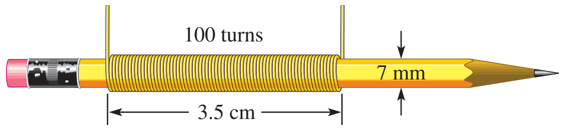
PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the book.

SECTION 13–1 The Basic Inductor

1. Convert the following to millihenries:
   (a) 1 H (b) 250 μH (c) 10 μH (d) 0.0005 H
2. Convert the following to microhenries:
   (a) 300 mH (b) 0.08 H (c) 5 mH (d) 0.00045 mH
3. What is the voltage across a coil when $di/dt = 10$ mA/μs and $L = 5\ \mu$H?
4. Fifty volts are induced across a 25 mH coil. At what rate is the current changing?
5. The current through a 100 mH coil is changing at a rate of 200 mA/s. How much voltage is induced across the coil?

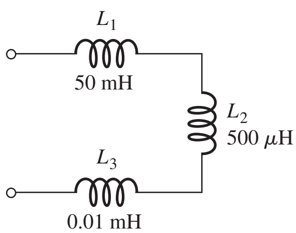
6. How many turns are required to produce 30 mH with a coil wound on a cylindrical core having a cross-sectional area of 10×10^{-5} m^2 and a length of 0.05 m? The core has a permeability of 1.2×10^{-6} H/m.
7. What amount of energy is stored in a 4.7 mH inductor when the current is 20 mA?
8. Compare the inductance of two inductors that are identical except that inductor 2 has twice the number of turns as inductor 1.
9. Compare the inductance of two inductors that are identical except that inductor 2 is wound on an iron coil (relative permeability = 150) and inductor 1 is wound on a low carbon steel core (relative permeability = 200).
10. A student wraps 100 turns of wire on a pencil that is 7 mm in diameter as shown in Figure 13–46. The pencil is a nonmagnetic core so has the same permeability as a vacuum ($4\pi \times 10^{-6}$ H/m). Determine the inductance of the coil that is formed.



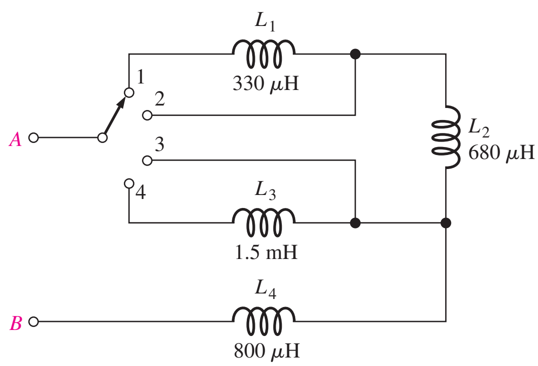
▲ FIGURE 13–46

SECTION 13–3 Series and Parallel Inductors

11. Five inductors are connected in series. The lowest value is 5 μH. If the value of each inductor is twice that of the preceding one, and if the inductors are connected in order of ascending values, what is the total inductance?
12. Suppose that you require a total inductance of 50 mH. You have available a 10 mH coil and a 22 mH coil. How much additional inductance do you need?
13. Determine the total inductance in Figure 13–47.

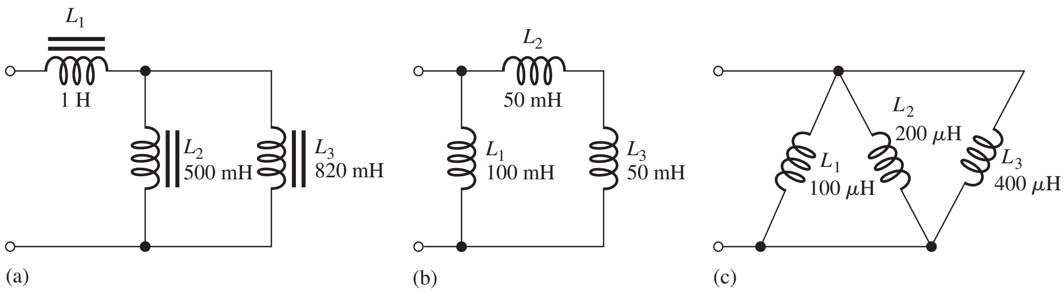


▲ FIGURE 13–47



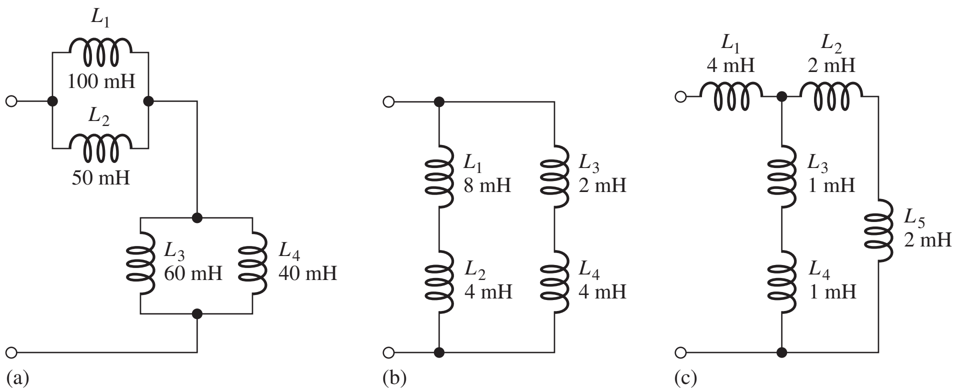
▲ FIGURE 13–48

14. What is the total inductance between points A and B for each switch position in Figure 13–48?
15. Determine the total parallel inductance for the following coils in parallel: 75 μH, 50 μH, 25 μH, and 15 μH.
16. You have a 12 mH inductor, and it is your smallest value. You need an inductance of 8 mH. What value can you use in parallel with the 12 mH to obtain 8 mH?
17. Determine the total inductance of each circuit in Figure 13–49.



▲ FIGURE 13–49

18. Determine the total inductance of each circuit in Figure 13–50.

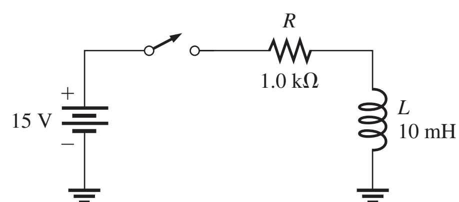


▲ FIGURE 13–50

SECTION 13–4 Inductors in DC Circuits

19. Determine the time constant for each of the following series RL combinations:
   (a) $R = 100\ \Omega$, $L = 100\ \mu\text{H}$ **(b)** $R = 4.7\ \text{k}\Omega$, $L = 10\ \text{mH}$
   (c) $R = 1.5\ \text{M}\Omega$, $L = 3\ \text{H}$
20. In a series RL circuit, determine how long it takes the current to build up to its full value for each of the following:
   (a) $R = 56\ \Omega$, $L = 50\ \mu\text{H}$ **(b)** $R = 3{,}300\ \Omega$, $L = 15\ \text{mH}$
   (c) $R = 22\ \text{k}\Omega$, $L = 100\ \text{mH}$
21. In the circuit of Figure 13–51, there is initially no current. Determine the inductor voltage at the following times after the switch is closed:
   (a) $10\ \mu\text{s}$ **(b)** $20\ \mu\text{s}$ **(c)** $30\ \mu\text{s}$ **(d)** $40\ \mu\text{s}$ **(e)** $50\ \mu\text{s}$
22. Determine the inductor current in Figure 13–51 at each of the times specified in Problem 21.

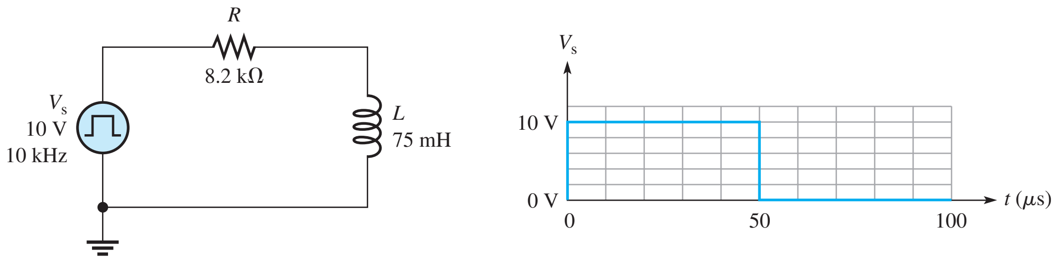
► FIGURE 13–51



23. Determine the time constant for the circuit in Figure 13–52.
*24. For the ideal inductor in Figure 13–52, calculate the current at each of the following times:
   (a) $10\ \mu\text{s}$ **(b)** $20\ \mu\text{s}$ **(c)** $30\ \mu\text{s}$

25. Repeat Problem 21 for the following times:

(a) 2 μs **(b)** 5 μs **(c)** 15 μs



▲ FIGURE 13–52

***26.** Repeat Problem 24 for the following times:

(a) 65 μs **(b)** 75 μs **(c)** 85 μs

27. In Figure 13–51, at what time after switch closure does the inductor voltage reach 5 V?

28. (a) What is the polarity of the induced voltage across the inductor in Figure 13–52 when the square wave is rising?

(b) What is the current just before the square wave drops to zero?

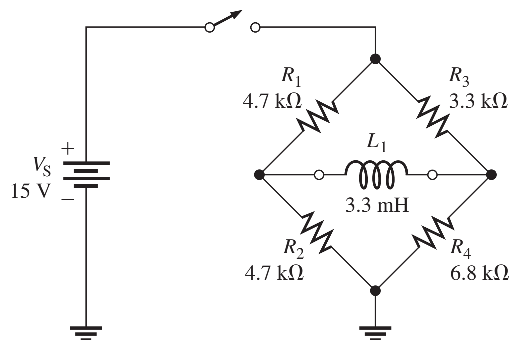
29. Determine the time constant for the circuit in Figure 13–53.

***30. (a)** What is the current in the inductor 1.0 μs after the switch closes in Figure 13–53?

(b) What is the current after 5τ have elapsed?

***31.** For the circuit in Figure 13–53, assume the switch has been closed for more than 5τ and is opened. What is the current in the inductor 1.0 μs after the switch is opened?

► FIGURE 13–53



SECTION 13–5 Inductors in AC Circuits

32. Find the total reactance for each circuit in Figure 13–49 when a voltage with a frequency of 5 kHz is applied across the terminals.

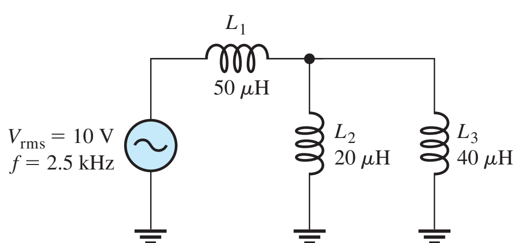
33. Find the total reactance for each circuit in Figure 13–50 when a 400 Hz voltage is applied.

34. Determine the total rms current in Figure 13–54. What are the currents through L_2 and L_3?

35. What frequency will produce 500 mA total rms current in each circuit of Figure 13–50 with an rms input voltage of 10 V?

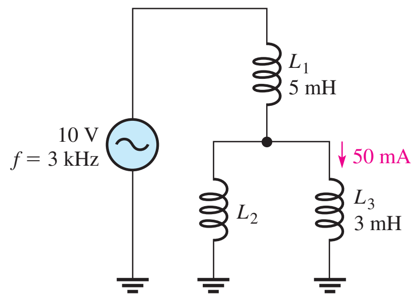
36. Determine the reactive power in Figure 13–54.

► FIGURE 13–54



37. Determine I_{L2} in Figure 13–55.

▶ FIGURE 13–55



38. Starting with Equation 13–12, show that the unit for inductive reactance is the ohm.

SECTION 13–6

39. Calculate the inductance of a ferrite bead if it has a reactance of 270 Ω at $f = 31.5$ MHz.

Multisim Troubleshooting and Analysis

These problems require Multisim.

40. Open file P13-40 and measure the voltage across each inductor.

41. Open file P13-41 and measure the voltage across each inductor.

42. Open file P13-42 and measure the current. Double the frequency and measure the current again. Reduce the original frequency by one-half and measure the current. Explain your observations.

43. Open file P13-43 and determine the fault if there is one.

44. Open file P13-44 and find the fault if there is one.

ANSWERS

SECTION CHECKUPS

SECTION 13–1 The Basic Inductor

1. Inductance depends on number of turns of wire, permeability, cross-sectional area, and core length.
2. $v_{ind} = 7.5$ mV
3. **(a)** L increases when N increases.
   (b) L decreases when core length increases.
   (c) L decreases when core cross-sectional area decreases.
   (d) L decreases when ferromagnetic core is replaced by air core.
4. All wire has some resistance, and because inductors are made from turns of wire, there is always resistance.
5. Adjacent turns in a coil act as plates of a capacitor.

SECTION 13–2 Types of Inductors

1. Two categories of inductors are fixed and variable.
2. **(a)** air core **(b)** iron core **(c)** variable with ferrite core

SECTION 13–3 Series and Parallel Inductors

1. Inductances are added in series.
2. $L_T = 2.60$ mH
3. $L_T = 5(100 \text{ mH}) = 500$ mH

4. The total parallel inductance is smaller than that of the smallest-value inductor in parallel.
5. True, calculation of parallel inductance is similar to parallel resistance.
6. **(a)** $L_T = 24\ \mu H$ **(b)** $L_T = 7.69$ mH

SECTION 13–4 Inductors in DC Circuits

1. $V_L = IR_W = 600$ mV
2. $i = 0$ V, $v_L = 20$ V
3. $v_L = 0$ V
4. $\tau = 500$ ns, $i_L = 3.93$ mA

SECTION 13–5 Inductors in AC Circuits

1. Voltage leads current by 90 degrees in an inductor.
2. $X_L = 2\pi fL = 3.14\ k\Omega$
3. $f = X_L/2\pi L = 2.55$ MHz
4. $I_{rms} = 15.9$ mA
5. $P_{true} = 0$ W; $P_r = 458$ mVAR

SECTION 13–6 Inductor Applications

1. Conductive and radiated
2. Electromagnetic interference
3. A ferrite bead is placed on a wire to increase its inductance, creating an RF choke.

RELATED PROBLEMS FOR EXAMPLES

13–1 25 mH
13–2 10.2 mH
13–3 0.5 Wb/s
13–4 150 μH
13–5 20.3 μH
13–6 227 ns
13–7 $I_F = 17.6$ mA, $\tau = 147$ ns
at 1τ: $i = 11.1$ mA; $t = 147$ ns
at 2τ: $i = 15.1$ mA; $t = 294$ ns
at 3τ: $i = 16.7$ mA; $t = 441$ ns
at 4τ: $i = 17.2$ mA; $t = 588$ ns
at 5τ: $i = 17.4$ mA; $t = 735$ ns
13–8 at 0.2 ms, $i = 0.218$ mA
at 0.8 ms, $i = 0.035$ mA
13–9 10 V
13–10 3.83 mA
13–11 6.69 mA
13–12 1.10 kΩ
13–13 **(a)** 7.18 kΩ; **(b)** 1.04 kΩ
13–14 573 mA
13–15 P_r decreases

TRUE/FALSE QUIZ

1. T **2.** F **3.** T **4.** F **5.** T
6. T **7.** F **8.** T **9.** F **10.** T

SELF-TEST

1. (c) **2.** (d) **3.** (c) **4.** (b) **5.** (d) **6.** (a)
7. (d) **8.** (c) **9.** (b) **10.** (a) **11.** (d) **12.** (b)

CIRCUIT DYNAMICS QUIZ

1. (a) **2.** (b) **3.** (b) **4.** (b) **5.** (b)
6. (b) **7.** (b) **8.** (c) **9.** (c) **10.** (a)