RC CIRCUITS

15

CHAPTER OUTLINE

CHAPTER OBJECTIVES

- Use complex numbers to express phasor quantities

PART 1: SERIES CIRCUITS

- Describe the relationship between current and voltage in a series *RC* circuit
- Determine the impedance of a series *RC* circuit
- Analyze a series *RC* circuit

PART 2: PARALLEL CIRCUITS

- Determine impedance and admittance in a parallel *RC* circuit
- Analyze a parallel *RC* circuit

PART 3: SERIES-PARALLEL CIRCUITS

- Analyze series-parallel *RC* circuits

PART 4: SPECIAL TOPICS

- Determine power in *RC* circuits
- Discuss some basic *RC* applications
- Troubleshoot *RC* circuits

KEY TERMS

- Complex plane
- Real number
- Imaginary number
- Rectangular form
- Polar form
- Impedance
- *RC* lag circuit
- *RC* lead circuit
- Capacitive susceptance (B_C)
- Admittance (Y)
- Apparent power (P_a)
- Power factor
- Filter
- Frequency response
- Cutoff frequency
- Bandwidth

APPLICATION ACTIVITY PREVIEW

The frequency response of the *RC* input circuit in an amplifier is similar to the one you worked with in Chapter 12 and is the subject of this chapter's application activity.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at
http://www.pearsonhighered.com/careersresources/

INTRODUCTION

An *RC* circuit contains both resistance and capacitance. In this chapter, basic series and parallel *RC* circuits and their responses to sinusoidal ac voltages are presented. Series-parallel combinations are also analyzed. True, reactive, and apparent power in *RC* circuits are discussed and some basic *RC* circuit applications are introduced. Applications of *RC* circuits include filters, amplifier coupling, oscillators, and wave-shaping circuits. Troubleshooting is also covered in this chapter.

Section 15–1 provides an introduction to complex numbers, an important tool for the analysis of ac circuits. The complex number system is a way to mathematically express a phasor quantity and allows phasor quantities to be added, subtracted, multiplied, and divided. You will use complex numbers throughout Chapters 15, 16, and 17.

COVERAGE OPTIONS

Following Section 15–1, The Complex Number System, this chapter and Chapters 16 and 17 are each divided into four parts: Series Circuits, Parallel Circuits, Series-Parallel Circuits, and Special Topics. This organization facilitates either of two options to the coverage of reactive circuits in Chapters 15, 16, and 17.

Option 1 Study the complex number system first; then study all *RC* circuit topics (Chapter 15), followed by all *RL* circuit topics (Chapter 16), and then all *RLC* circuit topics (Chapter 17). Using this approach, you simply cover Chapters 15, 16, and 17 in sequence.

Option 2 After studying the complex number system, study *series* reactive circuits. Then study *parallel* reactive circuits, followed by *series-parallel* reactive circuits and finally *special topics.* Using this approach, you cover Section 15–1; then Part 1: Series Circuits in Chapters 15, 16, and 17; then Part 2: Parallel Circuits in Chapters 15, 16, and 17; then Part 3: Series-Parallel Circuits in Chapters 15, 16, and 17. Finally, Part 4: Special Topics can be covered in each of the chapters.

15–1 THE COMPLEX NUMBER SYSTEM

Complex numbers allow mathematical operations with phasor quantities and are useful in the analysis of ac circuits. With the complex number system, you can add, subtract, multiply, and divide quantities that have both magnitude and angle, such as sine waves and other ac circuit quantities. Most scientific calculators can perform operations with complex numbers. Consult your user's manual for the exact procedure.

After completing this section, you should be able to

- **Use complex numbers to express phasor quantities**
  - Describe the complex plane
  - Represent a point on the complex plane
  - Discuss real and imaginary numbers
  - Express phasor quantities in both rectangular and polar forms
  - Convert between rectangular and polar forms
  - Do arithmetic operations with complex numbers

Positive and Negative Numbers

Positive numbers are represented by points to the right of the origin on the horizontal axis of a graph, and negative numbers are represented by points to the left of the origin, as illustrated in Figure 15–1(a). Also, positive numbers are represented by points on the vertical axis above the origin, and negative numbers are represented by points below the origin, as shown in Figure 15–1(b).

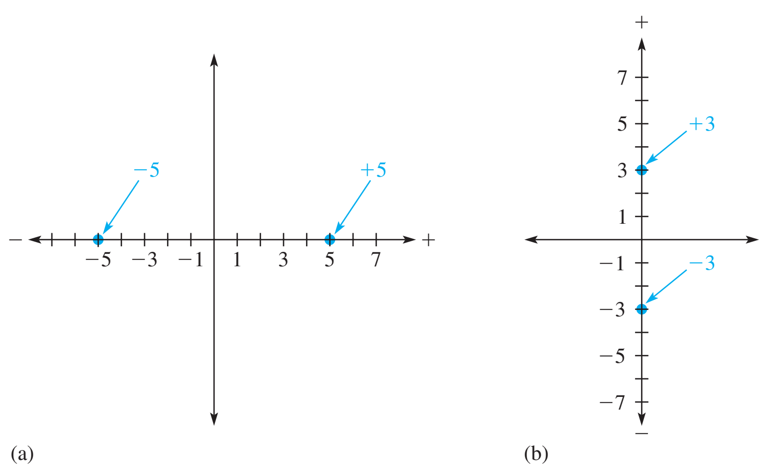
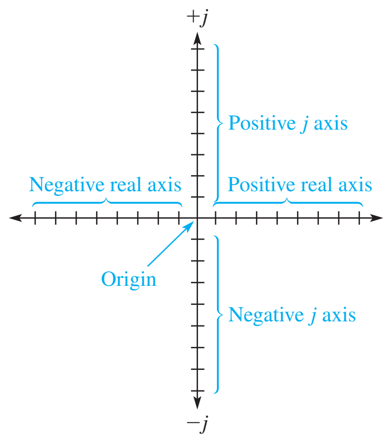


FIGURE 15–1

Graphic representation of positive and negative numbers.

The Complex Plane

To distinguish between values on the horizontal axis and values on the vertical axis, a **complex plane** is used. In the complex plane, the horizontal axis is called the *real axis,* and the vertical axis is called the *imaginary axis,* as shown in Figure 15–2. In electrical circuit work, a $\pm j$ prefix is used to designate numbers that lie on the imaginary axis in order to distinguish them from numbers lying on the real axis. This prefix is known

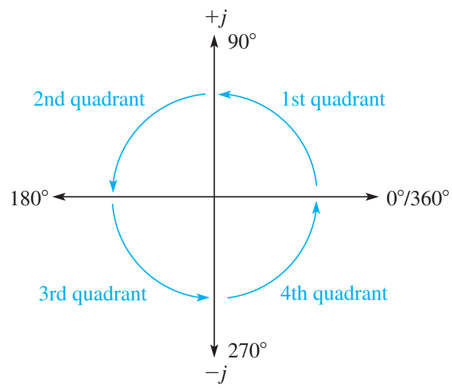


▶ FIGURE 15–2

The complex plane.

as the *j operator*. In mathematics, an *i* is used instead of a *j*, but in electric circuits, the *i* can be confused with instantaneous current, so *j* is used.

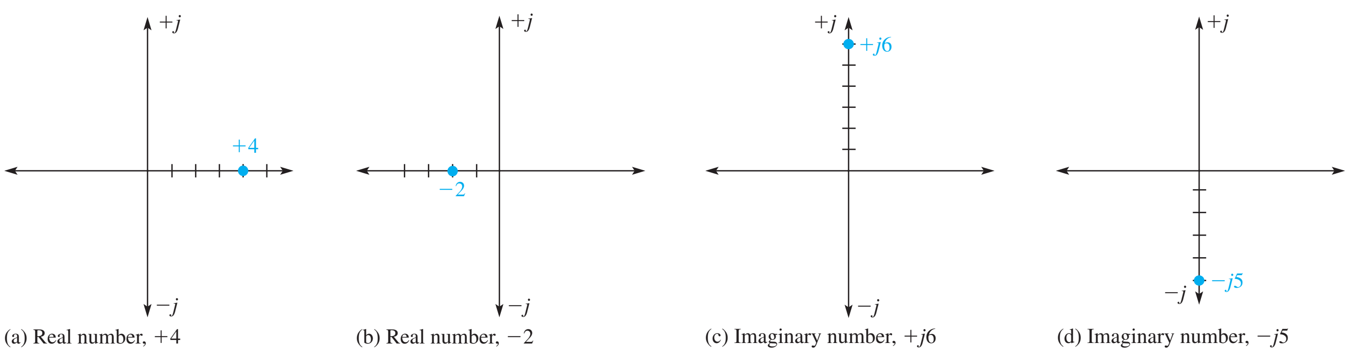
Angular Position on the Complex Plane Angular positions are represented on the complex plane, as shown in Figure 15–3. The positive real axis represents zero degrees. Proceeding counterclockwise, the $+j$ axis represents 90°, the negative real axis represents 180°, the $-j$ axis is the 270° point, and, after a full rotation of 360°, you are back to the positive real axis. Notice that the plane is divided into four quadrants.



▶ FIGURE 15–3

Angles on the complex plane.

Representing a Point on the Complex Plane A point located on the complex plane is classified as real, imaginary ($\pm j$), or a combination of the two. For example, a point located 4 units from the origin on the positive real axis is the positive **real number**, +4, as shown in Figure 15–4(a). A point 2 units from the origin on the



▲ FIGURE 15–4

Real and imaginary (*j*) numbers on the complex plane.

negative real axis is the negative real number, −2, as shown in part (b). A point on the +*j* axis 6 units from the origin, as shown in part (c), is the positive **imaginary number**, +*j*6. Finally, a point 5 units along the −*j* axis is the negative imaginary number, −*j*5, as shown in part (d).

When a point lies on neither axis but somewhere in one of the four quadrants, it is a complex number and is defined by its coordinates. For example, in Figure 15–5, the point located in the first quadrant has a real value of +4 and a *j* value of +*j*4 and is expressed as +4, +*j*4. The point located in the second quadrant has coordinates −3 and +*j*2. The point located in the third quadrant has coordinates −3 and −*j*5. The point located in the fourth quadrant has coordinates of +6 and −*j*4.

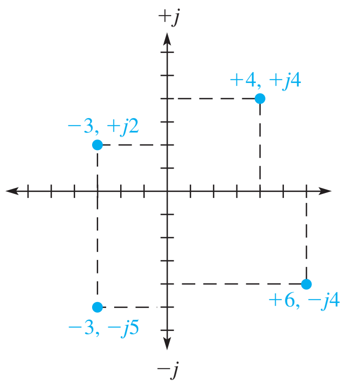


FIGURE 15–5

Coordinate points on the complex plane.

EXAMPLE 15–1

(a) Locate the following points on the complex plane: 7, *j*5; 5, −*j*2; −3.5, *j*1; and −5.5, −*j*6.5.

(b) Determine the coordinates for each point in Figure 15–6.

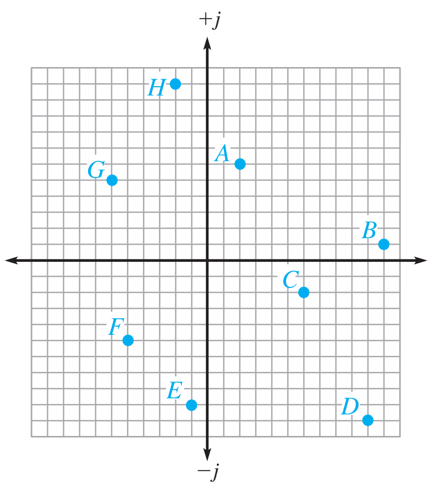
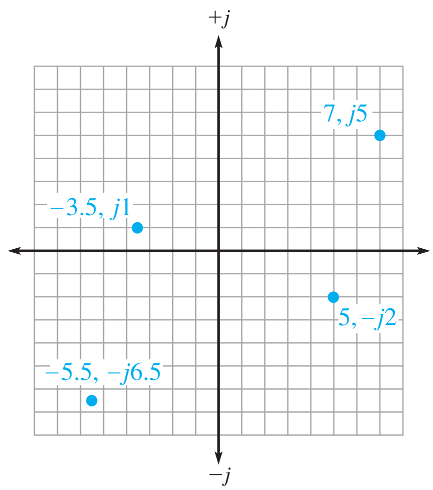


FIGURE 15–6

Solution **(a)** See Figure 15–7.



▲ **FIGURE 15–7**

(b) A: **2, j6** B: **11, j1** C: **6, −j2** D: **10, −j10**

E: **−1, −j9** F: **−5, −j5** G: **−6, j5** H: **−2, j11**

*Related Problem** In what quadrant is each of the following points located?

(a) $+2.5, +j1$ **(b)** $7, -j5$ **(c)** $-10, -j5$ **(d)** $-11, +j6.8$

*Answers are at the end of the chapter.

Value of *j*

If you multiply the positive real value of $+2$ by j, the result is $+j2$. This multiplication has effectively moved the $+2$ through a 90° angle to the $+j$ axis. Similarly, multiplying $+2$ by $-j$ rotates it $-90°$ to the $-j$ axis. Thus, j is considered a rotational operator.

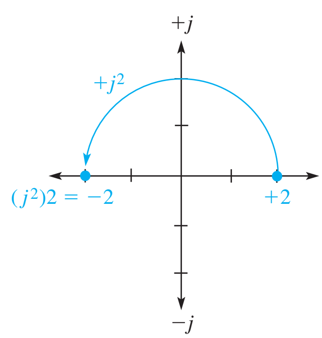
Mathematically, the j operator has a value of $\sqrt{-1}$. If $+j2$ is multiplied by j, you get

$$j^2 2 = (\sqrt{-1})(\sqrt{-1})(2) = (-1)(2) = -2$$

This calculation effectively places the value on the negative real axis. Therefore, multiplying a positive real number by j^2 converts it to a negative real number, which, in effect, is a rotation of 180° on the complex plane. This operation is illustrated in Figure 15–8.

▶ **FIGURE 15–8**

Effect of the j operator on location of a number on the complex plane.



Rectangular and Polar Forms

Rectangular and polar are two forms of complex numbers that are used to represent phasor quantities. Each has certain advantages when used in circuit analysis, depending on the particular application. A phasor quantity contains both *magnitude* and angular position or *phase*. In this text, italic letters such as V and I are used to represent magnitude only, and boldfaced nonitalic letters such as **V** and **I** are used to represent complete phasor quantities.

Rectangular Form A phasor quantity is represented in **rectangular form** by the algebraic sum of the real value (A) of the coordinate and the j value (B) of the coordinate, expressed in the following general form:

$$A + jB$$

Examples of phasor quantities are $1 + j2$, $5 - j3$, $-4 + j4$, and $-2 - j6$, which are shown on the complex plane in Figure 15–9. As you can see, the rectangular coordinates describe the phasor in terms of its values projected onto the real axis and the j axis. An "arrow" drawn from the origin to the coordinate point in the complex plane represents graphically the phasor quantity.

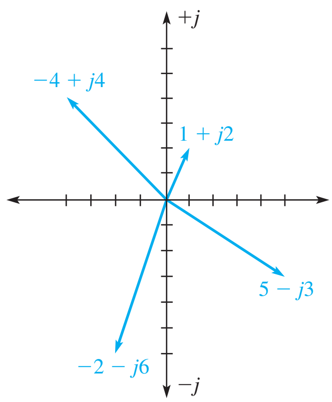


FIGURE 15–9

Examples of phasors specified by rectangular coordinates.

Polar Form Phasor quantities can also be expressed in **polar form**, which consists of the phasor magnitude (C) and the angular position relative to the positive real axis (θ), expressed in the following general form:

$$C\angle\pm\theta$$

Examples are $2\angle 45°$, $5\angle 120°$, $4\angle -110°$, and $8\angle -30°$. The first number is the magnitude, and the symbol $\angle$ precedes the value of the angle. Figure 15–10 shows these phasors on the complex plane. The length of the phasor, of course, represents the

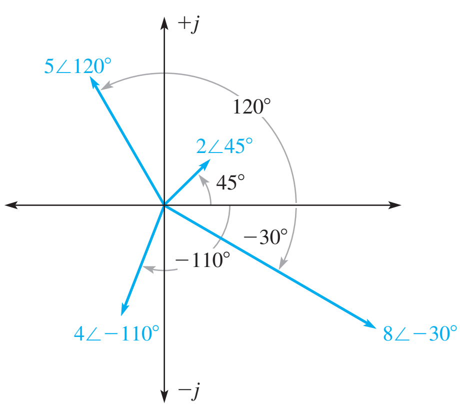
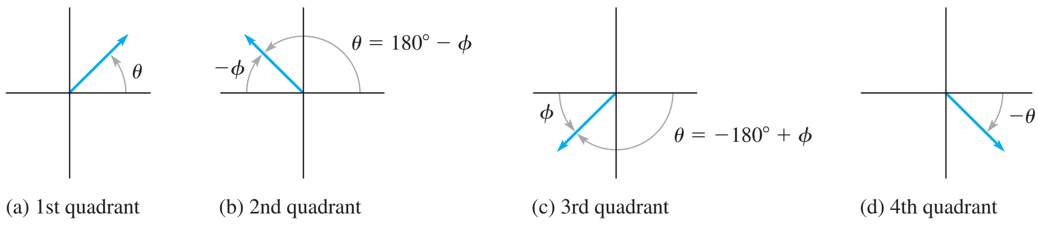


FIGURE 15–10

Examples of phasors specified by polar values.

magnitude of the quantity. Keep in mind that for every phasor expressed in polar form, there is also an equivalent expression in rectangular form.

Conversion from Rectangular to Polar Form A phasor can exist in any of the four quadrants of the complex plane, as indicated in Figure 15–11. The phase angle θ in each case is measured relative to the positive real axis (0°), and ϕ (phi) is the angle in the 2nd and 3rd quadrants relative to the negative real axis, as shown below:



▲ **FIGURE 15–11**

All possible phasor quadrant locations.

The first step to convert from rectangular form to polar form is to determine the magnitude of the phasor. A phasor can be visualized as forming a right triangle in the complex plane, as indicated in Figure 15–12, for each quadrant location. The horizontal side of the triangle is the real value, A, and the vertical side is the j value, B. The hypotenuse of the triangle is the length of the phasor, C, representing the magnitude, and can be expressed, using the Pythagorean theorem, as

Equation 15–1

$$C = \sqrt{A^2 + B^2}$$

Next, the angle θ indicated in parts (a) and (d) of Figure 15–12 is expressed as an inverse tangent function.

Equation 15–2

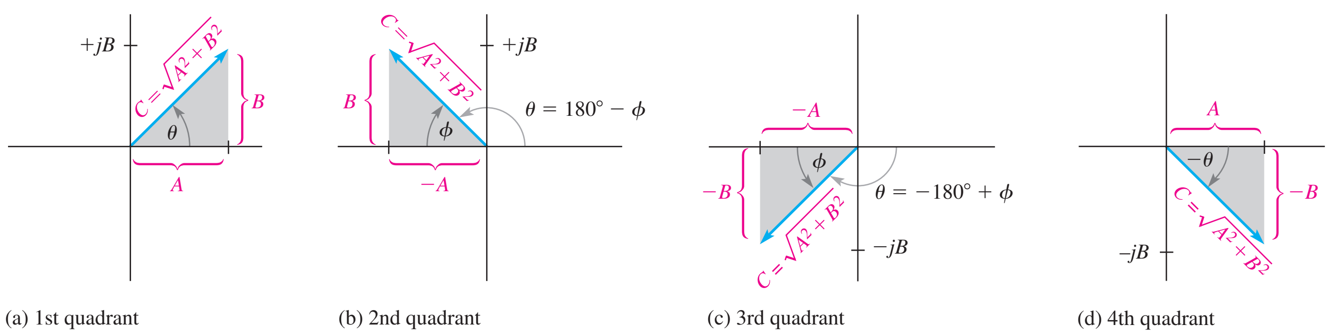
$$\theta = \tan^{-1}\left(\frac{\pm B}{A}\right)$$

The angle θ indicated in parts (b) and (c) of Figure 15–12 is

$$\theta = \pm 180° \mp \phi$$

which includes both conditions as indicated by the dual signs.

$$\theta = \pm 180° \mp \tan^{-1}\left(\frac{B}{A}\right)$$



▲ **FIGURE 15–12**

Right angle relationships in the complex plane.

In each case the appropriate signs must be used in the calculation.

The general formula for converting from rectangular to polar is

$$\pm A \pm jB = C\angle \pm\theta \qquad \text{Equation 15–3}$$

Example 15–2 illustrates the conversion procedure.

EXAMPLE 15–2

Convert the following complex numbers from rectangular form to polar form by determining the magnitude and angle:

(a) $8 + j6$ **(b)** $10 - j5$

Solution **(a)** The magnitude of the phasor represented by $8 + j6$ is

$$C = \sqrt{A^2 + B^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

Since the phasor is in the first quadrant, use Equation 15–2. The angle is

$$\theta = \tan^{-1}\left(\frac{\pm B}{A}\right) = \tan^{-1}\left(\frac{6}{8}\right) = 36.9^\circ$$

θ is the angle relative to the positive real axis. The polar form of $8 + j6$ is

$$C\angle\theta = \mathbf{10\angle 36.9^\circ}$$

(b) The magnitude of the phasor represented by $10 - j5$ is

$$C = \sqrt{10^2 + (-5)^2} = \sqrt{125} = 11.2$$

Since the phasor is in the fourth quadrant, use Equation 15–2. The angle is

$$\theta = \tan^{-1}\left(\frac{-5}{10}\right) = -26.6^\circ$$

θ is the angle relative to the positive real axis. The polar form of $10 - j5$ is

$$C\angle\theta = \mathbf{11.2\angle -26.6^\circ}$$

Related Problem Convert $18 + j23$ to polar form.

Conversion from Polar to Rectangular Form The polar form gives the magnitude and angle of a phasor quantity, as indicated in Figure 15–13.

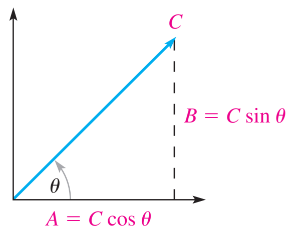


FIGURE 15–13

Polar components of a phasor.

To get the rectangular form, you must find sides A and B of the triangle, using the rules from trigonometry stated below:

$$A = C\cos\theta \qquad \text{Equation 15–4}$$

$$B = C\sin\theta \qquad \text{Equation 15–5}$$

The polar-to-rectangular conversion formula is

$$C\angle\theta = A + jB \qquad \text{Equation 15–6}$$

The following example demonstrates this conversion.

EXAMPLE 15–3

Convert the following polar quantities to rectangular form:

(a) $10\angle 30^\circ$ **(b)** $200\angle -45^\circ$

Solution **(a)** The real part of the phasor represented by $10\angle 30^\circ$ is

$$A = C\cos\theta = 10\cos 30^\circ = 10(0.866) = 8.66$$

The j part of this phasor is

$$jB = jC\sin\theta = j10\sin 30^\circ = j10(0.5) = j5$$

The rectangular form of $10\angle 30^\circ$ is

$$A + jB = \mathbf{8.66 + \mathit{j}5}$$

(b) The real part of the phasor represented by $200\angle -45^\circ$ is

$$A = 200\cos(-45^\circ) = 200(0.707) = 141$$

The j part is

$$jB = j200\sin(-45^\circ) = j200(-0.707) = -j141$$

The rectangular form of $200\angle -45^\circ$ is

$$A + jB = \mathbf{141 - \mathit{j}141}$$

Related Problem Convert $78\angle -26^\circ$ to rectangular form.

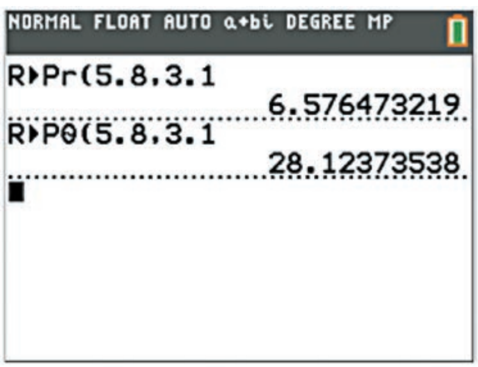
Scientific calculators can perform operations on complex numbers including converting from rectangular form to polar form. The following example shows how to enter complex numbers on the TI-84 Plus CE calculator and convert from rectangular form to polar form and vice versa. Consult with your owner's manual for your calculator.

EXAMPLE 15–4

(a) Using the TI-84 calculator, convert the complex number $5.8 + j3.1$ to polar form.

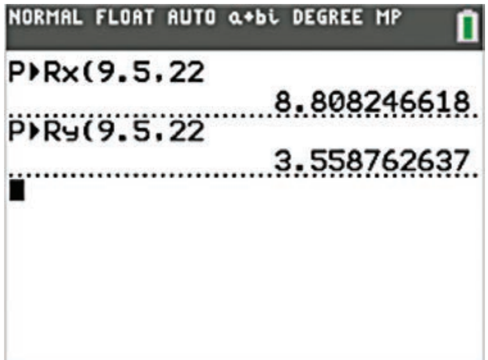
(b) Using the TI-84 calculator, convert the complex number $9.5\angle 22^\circ$ to rectangular form.

Solution **(a)** The simplest (but not the only) way to do conversions on the TI-84 is to use the angle menu. Select the angle menu by pressing. 2nd apps Select option 5 (press the 5 key) and enter the real and imaginary parts separated by a comma as shown in the following sequence: 5 . 8 , 3 . 1 enter. The magnitude is displayed (6.58). Repeat this process using option 6 and the angle (28.1°) is displayed. The following screen shows the result:



The angle (28.1°) can be shown in either radians or degrees depending on your selection in the third row of the mode menu. Thus, $5.8 + j3.1 = \mathbf{6.58\angle 28.1°}$

(b) Choose 2nd apps, option 7 and enter the magnitude and angle as shown in the following sequence: 9 . 5 , 2 2 enter. The real (x) part will be displayed (8.81). Repeat the procedure with option 8 and the imaginary (y) part will be displayed (3.56). The following screen shows the result: Thus, $9.5\angle 22° = \mathbf{8.81 + j3.56}$



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Related Problem Using TI-84 calculator, convert the complex number $7.8 - j12.1$ to polar form. (Use the (−) key to input the negative value.)

Mathematical Operations

Addition Complex numbers must be in rectangular form in order to add them. The rule is

Add the real parts of each complex number to get the real part of the sum. Then add the *j* parts of each complex number to get the *j* part of the sum.

EXAMPLE 15–5 Add the following sets of complex numbers:

(a) $8 + j5$ and $2 + j1$ **(b)** $20 - j10$ and $12 + j6$

Solution

(a) $(8 + j5) + (2 + j1) = (8 + 2) + j(5 + 1) = \mathbf{10 + j6}$

(b) $(20 - j10) + (12 + j6) = (20 + 12) + j(-10 + 6) = 32 + j(-4) = \mathbf{32 - j4}$

Related Problem Add $5 - j11$ and $-6 + j3$.

Subtraction As in addition, the numbers must be in rectangular form to be subtracted. The rule is

Subtract the real parts of the numbers to get the real part of the difference. Then subtract the *j* parts of the numbers to get the *j* part of the difference.

EXAMPLE 15–6 Perform the following subtractions:

(a) Subtract $1 + j2$ from $3 + j4$.

(b) Subtract $10 - j8$ from $15 + j15$.

Solution **(a)** $(3 + j4) - (1 + j2) = (3 - 1) + j(4 - 2) = \mathbf{2 + j2}$

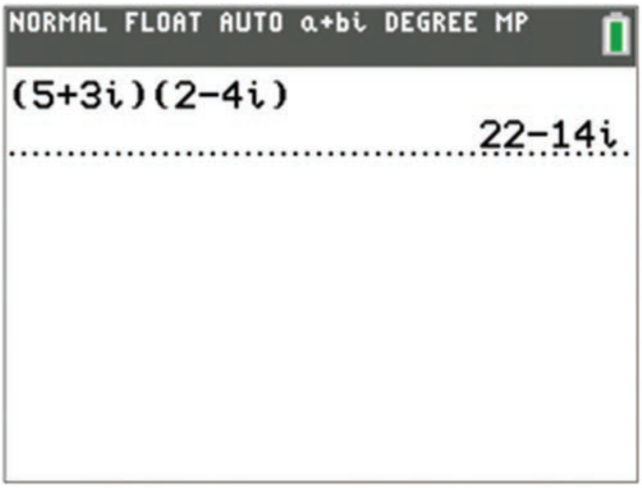
(b) $(15 + j15) - (10 - j8) = (15 - 10) + j[15 - (-8)] = \mathbf{5 + j23}$

Related Problem Subtract $3.5 - j4.5$ from $-10 - j9$.

Multiplication Multiplication of two complex numbers in rectangular form is accomplished by multiplying, in turn, each term in one number by both terms in the other number and then combining the resulting real terms and the resulting j terms (recall that $j \times j = -1$). You can start with the "FOIL" rule for (First, Outside, Inside, Last). As an example,

$$(5 + j3)(2 - j4) = 10 - j20 + j6 + 12 = 22 - j14$$

Many calculators can perform mathematical operations on complex numbers. For example, on the TI-84, the terms are entered as shown above but the calculator uses i instead of j to represent values on the y-axis. (The multiply operation is implied.) For the TI-84, the i is selected by pressing 2nd . . The result can be shown in either rectangular or polar notation depending on the user selection in the mode menu. The result is



If your calculator does not work directly with complex numbers, multiplication of two complex numbers is easier when both numbers are in polar form, so it is best to convert to polar form before multiplying. The rule is

Multiply the magnitudes, and add the angles algebraically.

EXAMPLE 15–7

Perform the following multiplications:

(a) $10\angle 45^\circ$ times $5\angle 20^\circ$ **(b)** $2\angle 60^\circ$ times $4\angle -30^\circ$

Solution **(a)** $(10\angle 45^\circ)(5\angle 20^\circ) = (10)(5)\angle(45^\circ + 20^\circ) = \mathbf{50\angle 65^\circ}$

(b) $(2\angle 60^\circ)(4\angle -30^\circ) = (2)(4)\angle[60^\circ + (-30^\circ)] = \mathbf{8\angle 30^\circ}$

Related Problem Multiply $50\angle 10^\circ$ times $30\angle -60^\circ$.

Division Division of two complex numbers in rectangular form is accomplished by multiplying both the numerator and the denominator by the **complex conjugate** of the denominator and then combining terms and simplifying. The complex conjugate of a number is found by changing the sign of the *j* term. As an example,

$$\frac{10 + j5}{2 + j4} = \frac{(10 + j5)(2 - j4)}{(2 + j4)(2 - j4)} = \frac{20 - j30 + 20}{4 + 16} = \frac{40 - j30}{20} = 2 - j1.5$$

Alternatively, the problem can be entered into a calculator that performs complex arithmetic and solved directly.

If your calculator cannot represent complex numbers, division is easier when the numbers are in polar form, so it is best to convert to polar form before dividing. The rule is

Divide the magnitude of the numerator by the magnitude of the denominator to get the magnitude of the quotient. Then subtract the denominator angle from the numerator angle to get the angle of the quotient.

EXAMPLE 15–8

Perform the following divisions:

(a) Divide $100\angle 50°$ by $25\angle 20°$.

(b) Divide $15\angle 10°$ by $3\angle -30°$.

Solution

(a) $$\frac{100\angle 50°}{25\angle 20°} = \left(\frac{100}{25}\right)\angle(50° - 20°) = \mathbf{4\angle 30°}$$

(b) $$\frac{15\angle 10°}{3\angle -30°} = \left(\frac{15}{3}\right)\angle[10° - (-30°)] = \mathbf{5\angle 40°}$$

Related Problem Divide $24\angle -30°$ by $6\angle 12°$.

SECTION 15–1 CHECKUP

Answers are at the end of the chapter.

1. Convert $2 + j2$ to polar form. In which quadrant does this phasor lie?
2. Convert $5\angle -45°$ to rectangular form. In which quadrant does this phasor lie?
3. Add $1 + j2$ and $3 - j1$.
4. Subtract $12 + j18$ from $15 + j25$.
5. Multiply $8\angle 45°$ times $2\angle 65°$.
6. Divide $30\angle 75°$ by $6\angle 60°$.

Part 1 Series Circuits

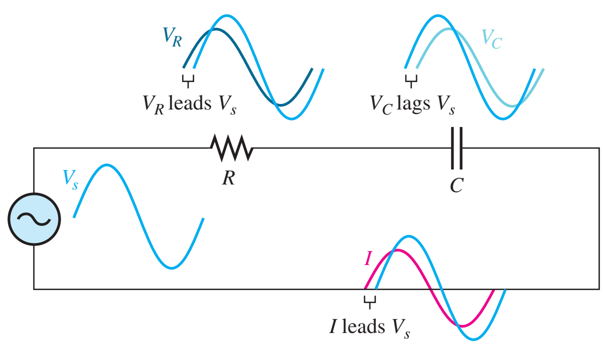
15–2 Sinusoidal Response of Series *RC* Circuits

When a sinusoidal voltage is applied to a series *RC* circuit, each resulting voltage drop and the current in the circuit are also sinusoidal and have the same frequency as the applied voltage. The capacitance causes a phase shift between the voltage and current that depends on the relative values of the resistance and the capacitive reactance.

After completing this section, you should be able to

- **Describe the relationship between current and voltage in a series *RC* circuit**
  - Discuss voltage and current waveforms
  - Discuss phase shift

As shown in Figure 15–14, the resistor voltage (V_R), the capacitor voltage (V_C), and the current (I) are all sine waves with the frequency of the source. Phase shifts are introduced because of the capacitance. The resistor voltage and current *lead* the source voltage, and the capacitor voltage *lags* the source voltage. The phase angle between the current and the capacitor voltage is always 90°. These generalized phase relationships are indicated in Figure 15–14.



▶ **FIGURE 15–14**

Illustration of sinusoidal response with general phase relationships of V_R, V_C, and I relative to the source voltage. V_R and I are in phase while V_R and V_C are 90° out of phase.

The amplitudes and the phase relationships of the voltages and current depend on the values of the resistance and the **capacitive reactance**. When a circuit is purely resistive, the phase angle between the applied (source) voltage and the total current is zero. When a circuit is purely capacitive, the phase angle between the applied voltage and the total current is 90°, with the current leading the voltage. When there is a combination of both resistance and capacitive reactance in a circuit, the phase angle between the applied voltage and the total current is somewhere between 0° and 90°, depending on the relative values of the resistance and the capacitive reactance.

SECTION 15–2 CHECKUP

1. A 60 Hz sinusoidal voltage is applied to an *RC* circuit. What is the frequency of the capacitor voltage? What is the frequency of the current?
2. What causes the phase shift between V_s and I in a series *RC* circuit?
3. When the resistance in a series *RC* circuit is greater than the capacitive reactance, is the phase angle between the applied voltage and the total current closer to 0° or to 90°?

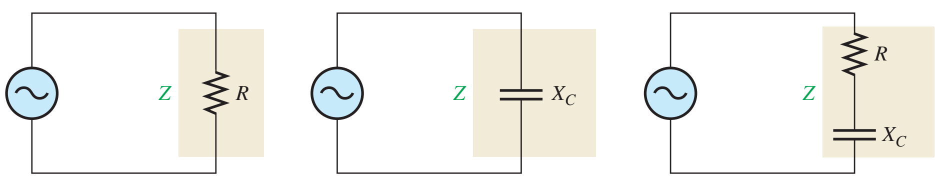
15–3 IMPEDANCE OF SERIES *RC* CIRCUITS

Impedance is the total opposition to sinusoidal current. Its unit is the ohm. The impedance of a series *RC* circuit consists of resistance and capacitive reactance. It causes a phase difference between the total current and the source voltage. Therefore, the impedance is a complex value that consists of a magnitude component and a phase angle component.

After completing this section, you should be able to

- **Determine the impedance of a series *RC* circuit**
  - Define *impedance*
  - Express capacitive reactance in complex form
  - Express total impedance in complex form
  - Draw an impedance triangle
  - Calculate impedance magnitude and the phase angle

In a purely resistive circuit, the impedance is simply equal to the total resistance. In a purely capacitive circuit, the impedance is equal to the total capacitive reactance. The impedance of a series *RC* circuit is determined by both the resistance and the capacitive reactance. These cases are illustrated in Figure 15–15. The magnitude of the impedance is symbolized by *Z*.



(a) $Z = R$ (b) $Z = X_C$ (c) Z includes both R and X_C

▲ **FIGURE 15–15**

Three cases of impedance.

Capacitive reactance is a phasor quantity and is expressed as a complex number in rectangular form as

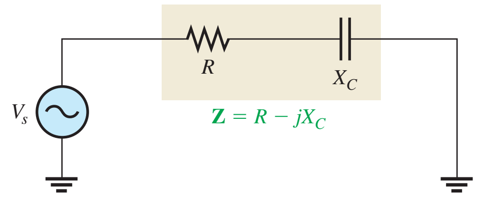
$$\mathbf{X}_C = -jX_C$$

where boldface $\mathbf{X}_C$ designates a phasor quantity (representing both magnitude and angle) whereas non-bold X_C represents just the magnitude.

In the series RC circuit of Figure 15–16, the total impedance is the phasor sum of R and $-jX_C$ and is expressed as

Equation 15–7

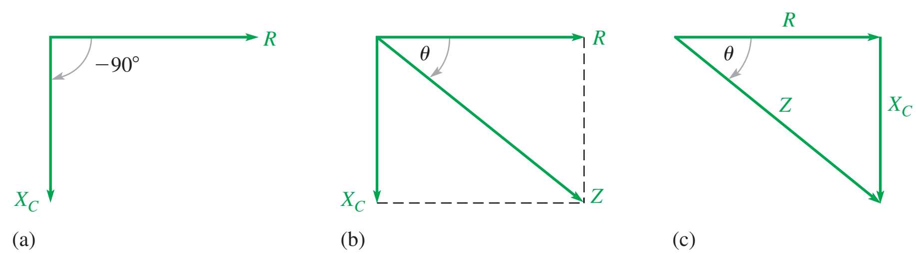
$$\mathbf{Z} = R - jX_C$$



▲ FIGURE 15–16

Impedance in a series RC circuit.

In ac analysis, both R and X_C are as shown in the phasor diagram of Figure 15–17(a), with X_C appearing at a $-90°$ angle with respect to R. This relationship comes from the fact that the capacitor voltage in a series RC circuit lags the current, and thus the resistor voltage, by 90°. Since $\mathbf{Z}$ is the phasor sum of R and $-jX_C$, its phasor representation is as shown in Figure 15–17(b). A repositioning of the phasors, as shown in part (c), forms a right triangle called the *impedance triangle.* The length of each phasor represents the magnitude in ohms, and the angle θ is the phase angle of the RC circuit and represents the phase difference between the applied voltage and the current.



▲ FIGURE 15–17

Development of the impedance triangle for a series RC circuit.

From right-angle trigonometry (Pythagorean theorem), the magnitude (length) of the impedance can be expressed in terms of the resistance and reactance as

$$Z = \sqrt{R^2 + X_C^2}$$

The italic letter Z represents the magnitude of the phasor quantity $\mathbf{Z}$ and is expressed in ohms.

The phase angle, θ, is expressed as

$$\theta = -\tan^{-1}\left(\frac{X_C}{R}\right)$$

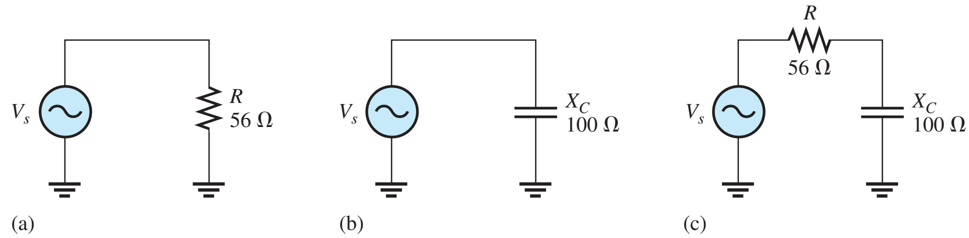
The symbol $\tan^{-1}$ stands for inverse tangent. You can find the $\tan^{-1}$ value on your calculator. Combining the magnitude and angle, the phasor expression for impedance in polar form is

Equation 15–8

$$\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

EXAMPLE 15–9

For each circuit in Figure 15–18, write the phasor expression for the impedance in both rectangular form and polar form.



▲ **FIGURE 15–18**

Solution For the circuit in Figure 15–18(a), the impedance is

$$\mathbf{Z} = R - j0 = R = \mathbf{56\ \Omega} \qquad \text{in rectangular form } (X_C = 0)$$

$$\mathbf{Z} = R\angle 0^\circ = \mathbf{56\angle 0^\circ\ \Omega} \qquad \text{in polar form}$$

The impedance is simply the resistance, and the phase angle is zero because pure resistance does not cause a phase shift between the voltage and current.

For the circuit in Figure 15–18(b), the impedance is

$$\mathbf{Z} = 0 - jX_C = \mathbf{-j100\ \Omega} \qquad \text{in rectangular form } (R = 0)$$

$$\mathbf{Z} = X_C\angle -90^\circ = \mathbf{100\angle -90^\circ\ \Omega} \qquad \text{in polar form}$$

The impedance is simply the capacitive reactance, and the phase angle is -90° because the capacitance causes the current to lead the voltage by 90°.

For the circuit in Figure 15–18(c), the impedance in rectangular form is

$$\mathbf{Z} = R - jX_C = \mathbf{56\ \Omega\ -\ j100\ \Omega}$$

The impedance in polar form is

$$\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

$$= \sqrt{(56\ \Omega)^2 + (100\ \Omega)^2}\angle -\tan^{-1}\left(\frac{100\ \Omega}{56\ \Omega}\right) = \mathbf{115\angle -60.8^\circ\ \Omega}$$

Note that the unit (Ω) is written after the magnitude and angle for the result shown; however, some people prefer to write the unit with the magnitude (i.e., 115 $\Omega\angle -60.8^\circ$). Either method is correct.

In this case, the impedance is the phasor sum of the resistance and the capacitive reactance. The phase angle is fixed by the relative values of X_C and R. Rectangular to polar conversion can be done on a scientific calculator (refer to your user's manual).

Related Problem Draw the impedance phasor diagram for the circuit in Figure 15–18(c).

SECTION 15–3 CHECKUP

1. The impedance of a certain *RC* circuit is 150 Ω − *j*220 Ω. What is the value of the resistance? The capacitive reactance?
2. A series *RC* circuit has a total resistance of 33 kΩ and a capacitive reactance of 50 kΩ. Write the phasor expression for the impedance in rectangular form.
3. For the circuit in Question 2, what is the magnitude of the impedance? What is the phase angle?

15–4 Analysis of Series *RC* Circuits

Ohm's law and Kirchhoff's voltage law are used in the analysis of series *RC* circuits to determine voltage, currents and impedance. Also, in this section *RC* lead and lag circuits are examined.

After completing this section, you should be able to

- **Analyze a series *RC* circuit**
  - Apply Ohm's law and Kirchhoff's voltage law to series *RC* circuits
  - Express the voltages and current as phasor quantities
  - Show how impedance and phase angle vary with frequency
  - Analyze the *RC* lag circuit
  - Analyze the *RC* lead circuit

Ohm's Law

The application of Ohm's law to series *RC* circuits involves the use of the phasor quantities of **Z**, **V**, and **I**. Although rectangular forms directly represent the real (resistive) and imaginary (reactive) components of electrical quantities, keep in mind that the use of boldface nonitalic letters indicates phasor quantities where both magnitude and angle are included. The three equivalent forms of Ohm's law are as follows:

Equation 15–9
$$\mathbf{V} = \mathbf{IZ}$$

Equation 15–10
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}}$$

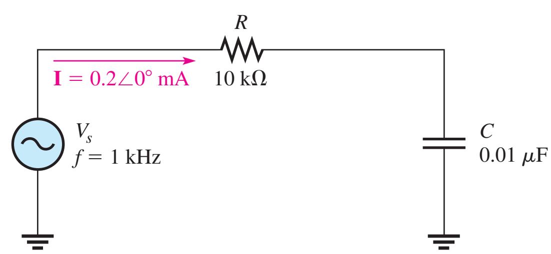
Equation 15–11
$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

If you are not using a calculator that can do complex arithmetic, multiplication and division are most easily accomplished with the polar forms. Since Ohm's law calculations involve multiplications and divisions, you should generally express the voltage, current, and impedance in polar form. The following two examples show the relationship between the source voltage and source current. In Example 15–10, the current is the reference and in Example 15–11, the voltage is the reference. Notice that the reference is drawn along the *x*-axis in both cases. Rectangular form directly represents the real (resistive) and imaginary (reactive) components of electrical quantities, so it is a starting point for writing the impedance in the following examples.

EXAMPLE 15–10

The current in Figure 15–19 is expressed in polar form as $\mathbf{I} = 0.2\angle 0° \text{ mA}$. Determine the source voltage expressed in polar form, and draw a phasor diagram showing the relation between source voltage and current.

► FIGURE 15–19



Solution The magnitude of the capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1{,}000\text{ Hz})(0.01\ \mu\text{F})} = 15.9\text{ k}\Omega$$

The total impedance in rectangular form is

$$\mathbf{Z} = R - jX_C = 10\text{ k}\Omega - j15.9\text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

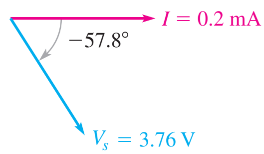
$$= \sqrt{(10\text{ k}\Omega)^2 + (15.9\text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{15.9\text{ k}\Omega}{10\text{ k}\Omega}\right) = 18.8\angle -57.8^\circ\text{ k}\Omega$$

Use Ohm's law to determine the source voltage.

$$\mathbf{V}_s = \mathbf{IZ} = (0.2\angle 0^\circ\text{ mA})(18.8\angle -57.8^\circ\text{ k}\Omega) = \mathbf{3.76\angle -57.8^\circ\ V}$$

The magnitude of the source voltage is 3.76 V at an angle of -57.8° with respect to the current; that is, the voltage lags the current by 57.8°, as shown in the phasor diagram of Figure 15–20.

▶ FIGURE 15–20

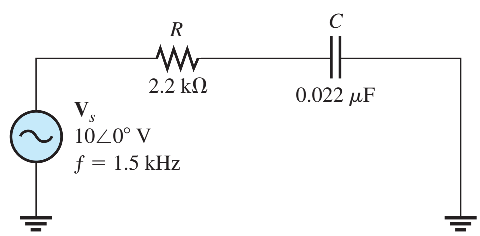


Related Problem Determine $\mathbf{V}_s$ in Figure 15–19 if $f = 2$ kHz and $\mathbf{I} = 0.2\angle 0^\circ$ A.

EXAMPLE 15–11

Determine the current in the circuit of Figure 15–21, and draw a phasor diagram showing the relation between source voltage and current.

▶ FIGURE 15–21



Solution The magnitude of the capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1.5\text{ kHz})(0.022\ \mu\text{F})} = 4.82\text{ k}\Omega$$

The total impedance in rectangular form is

$$\mathbf{Z} = R - jX_C = 2.2\text{ k}\Omega - j4.82\text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

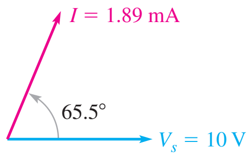
$$= \sqrt{(2.2\text{ k}\Omega)^2 + (4.82\text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{4.82\text{ k}\Omega}{2.2\text{ k}\Omega}\right) = 5.30\angle -65.5^\circ\text{ k}\Omega$$

Use Ohm's law to determine the current.

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{10\angle 0^\circ\text{ V}}{5.30\angle -65.5^\circ\text{ k}\Omega} = \mathbf{1.89\angle 65.5^\circ\text{ mA}}$$

The magnitude of the current is 1.89 mA. The positive phase angle of 65.5° indicates that the current leads the voltage by that amount, as shown in the phasor diagram of Figure 15–22.

▶ FIGURE 15–22



Related Problem Determine **I** in Figure 15–21 if the frequency is increased to 5 kHz.

MultiSim

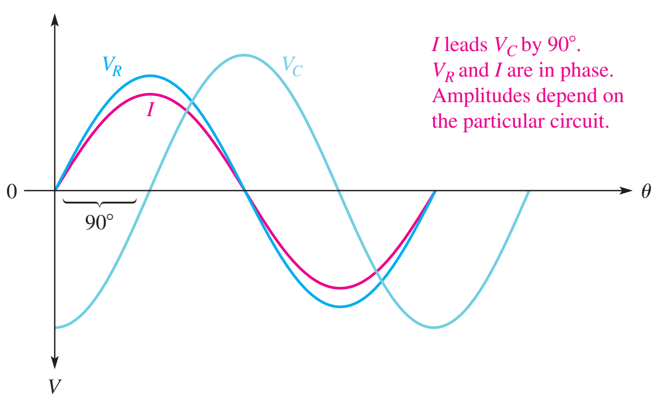
Use Multisim file E15-11 to verify the calculated results in this example and to confirm your calculation for the related problem.

Phase Relationships of Current and Voltages

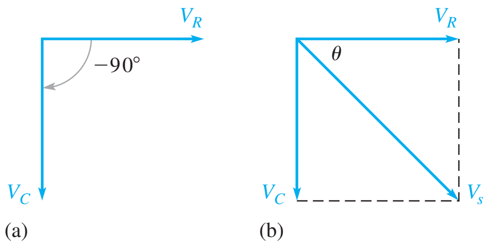
In a series RC circuit, the current is the same through both the resistor and the capacitor. Thus, the resistor voltage is in phase with the current, and the capacitor voltage lags the current by 90°. Therefore, there is a phase difference of 90° between the resistor voltage, V_R, and the capacitor voltage, V_C, as shown in the waveform diagram of Figure 15–23.

▶ FIGURE 15–23

Phase relation of voltages and current in a series RC circuit.



From Kirchhoff's voltage law, the sum of the voltage drops must equal the applied voltage. However, since V_R and V_C are not in phase with each other, they must be added as phasor quantities, with V_C lagging V_R by 90°, as shown in Figure 15–24(a).



▲ **FIGURE 15–24**

Voltage phasor diagram for a series *RC* circuit.

As shown in Figure 15–24(b), $\mathbf{V}_s$ is the phasor sum of V_R and V_C, as expressed in rectangular form in the following equation:

Equation 15–12

$$\mathbf{V}_s = V_R - jV_C$$

This equation can be expressed in polar form as

Equation 15–13

$$\mathbf{V}_s = \sqrt{V_R^2 + V_C^2}\angle -\tan^{-1}\left(\frac{V_C}{V_R}\right)$$

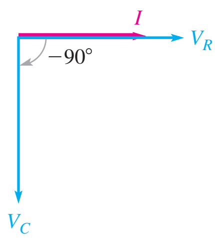
where the magnitude of the source voltage is

$$V_s = \sqrt{V_R^2 + V_C^2}$$

and the phase angle between the resistor voltage and the source voltage is

$$\theta = -\tan^{-1}\left(\frac{V_C}{V_R}\right)$$

Since the resistor voltage and the current are in phase, θ also represents the phase angle between the source voltage and the current. Figure 15–25 shows a complete voltage and current phasor diagram that represents the waveform diagram of Figure 15–23.



▲ **FIGURE 15–25**

Voltage and current phasor diagram for the waveforms in Figure 15–23.

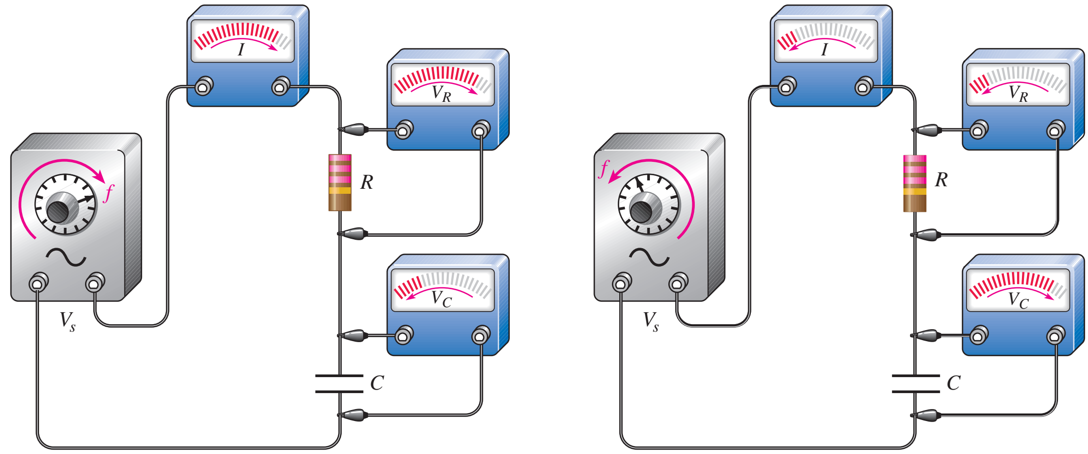
Variation of Impedance and Phase Angle with Frequency

As you know, capacitive reactance varies inversely with frequency. Since $Z = \sqrt{R^2 + X_C^2}$, you can see that when X_C increases, the entire term under the square root sign increases and thus the magnitude of the total impedance also increases; and when X_C decreases, the magnitude of the total impedance also decreases. Therefore, *in a series RC circuit, Z is inversely dependent on frequency.*

Figure 15–26 illustrates how the voltages and current in a series *RC* circuit vary as the frequency increases or decreases, with the source voltage held at a constant value. In part (a), as the frequency is increased, X_C decreases; so less voltage is dropped across the capacitor. Also, Z decreases as X_C decreases, causing the current to increase. An increase in the current causes more voltage across R.

In Figure 15–26(b), as the frequency is decreased, X_C increases; so more voltage is dropped across the capacitor. Also, Z increases as X_C increases, causing the current to decrease. A decrease in the current causes less voltage across R.

Changes in Z and X_C can be observed as shown in Figure 15–27. As the frequency increases, the voltage across Z remains constant because V_s is constant. Also, the voltage across C decreases. The increasing current indicates that Z is decreasing. It does so because of the inverse relationship stated in Ohm's law ($Z = V_Z/I$). The increasing current also indicates that X_C is decreasing ($X_C = V_C/I$). The decrease in V_C corresponds to the decrease in X_C.

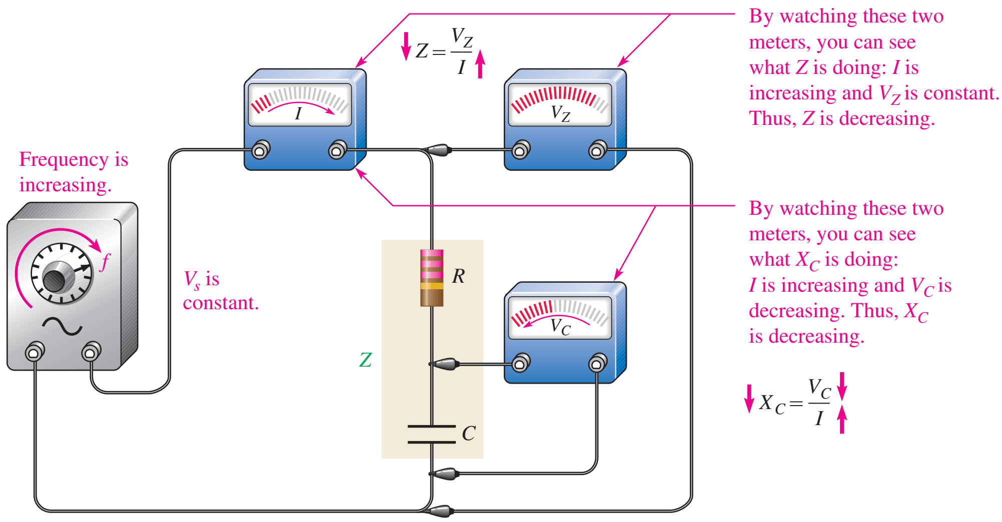


(a) As frequency is increased, Z decreases as X_C decreases, causing I and V_R to increase and V_C to decrease.

(b) As frequency is decreased, Z increases as X_C increases, causing I and V_R to decrease and V_C to increase.

▲ **FIGURE 15–26**

An illustration of how the variation of impedance affects the voltages and current as the source frequency is varied. The source voltage is held at a constant amplitude.

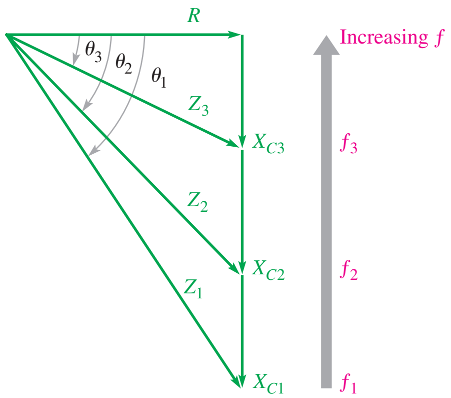


▲ **FIGURE 15–27**

An illustration of how Z and X_C change with frequency.

Since X_C is the factor that introduces the phase angle in a series *RC* circuit, a change in X_C produces a change in the phase angle. As the frequency is increased, X_C becomes smaller, and thus the phase angle decreases. As the frequency is decreased, X_C becomes larger, and thus the phase angle increases. The angle between V_s and V_R is the phase angle of the circuit because I is in phase with V_R. By measuring the phase of V_R, you are effectively measuring the phase of I. An oscilloscope is normally used to observe the phase angle by measuring the phase angle between V_s and one of the component voltages. You can also use the oscilloscope to observe a plot of the magnitude as a function of frequency by using a special function generator called a **sweep generator** to drive the circuit. In this case, the time base of the oscilloscope is converted to a frequency base.

Figure 15–28 uses the impedance triangle to illustrate the variations in X_C, Z, and θ as the frequency changes. Of course, R remains constant. The main point is that because X_C varies inversely with the frequency, so also do the magnitude of the total impedance and the phase angle. Example 15–12 illustrates this.



▲ FIGURE 15–28

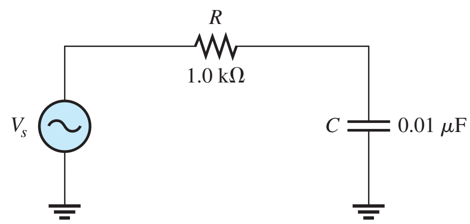
As the frequency increases, X_C decreases, Z decreases, and θ decreases. Each value of frequency can be visualized as forming a different impedance triangle.

EXAMPLE 15–12

For the series RC circuit in Figure 15–29, determine the magnitude of the total impedance and the phase angle for each of the following values of input frequency:

(a) 10 kHz **(b)** 20 kHz **(c)** 30 kHz

▶ FIGURE 15–29



Solution **(a)** For $f = 10$ kHz,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(10\text{ kHz})(0.01\ \mu\text{F})} = 1.59\text{ k}\Omega$$

$$\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

$$= \sqrt{(1.0\text{ k}\Omega)^2 + (1.59\text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{1.59\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = 1.88\angle -57.8°\text{ k}\Omega$$

Thus, $Z = \mathbf{1.88\ k\Omega}$ and $\theta = \mathbf{-57.8°}$.

(b) For $f = 20$ kHz,

$$X_C = \frac{1}{2\pi(20\text{ kHz})(0.01\ \mu\text{F})} = 796\ \Omega$$

$$\mathbf{Z} = \sqrt{(1.0\text{ k}\Omega)^2 + (796\ \Omega)^2}\angle -\tan^{-1}\left(\frac{796\ \Omega}{1.0\text{ k}\Omega}\right) = 1.28\angle -38.5°\text{ k}\Omega$$

Thus, $Z = \mathbf{1.28\ k\Omega}$ and $\theta = \mathbf{-38.5°}$.

(c) For $f = 30$ kHz,

$$X_C = \frac{1}{2\pi(30\text{ kHz})(0.01\ \mu\text{F})} = 531\ \Omega$$

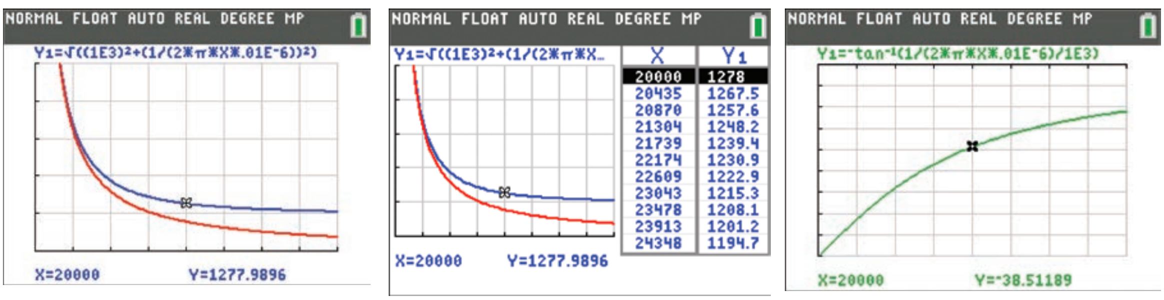
$$\mathbf{Z} = \sqrt{(1.0\text{ k}\Omega)^2 + (531\ \Omega)^2}\angle -\tan^{-1}\left(\frac{531\ \Omega}{1.0\text{ k}\Omega}\right) = 1.13\angle -28.0^\circ\text{ k}\Omega$$

Thus, $Z = \mathbf{1.13\ k\Omega}$ and $\theta = \mathbf{-28.0^\circ}$.

Notice that as the frequency increases, X_C, Z, and θ decrease.

Related Problem Find the magnitude of the total impedance and the phase angle in Figure 15–29 for $f = 1$ kHz.

A TI-84 Plus CE graphing calculator can be used to show how X_C, Z, and θ are dependent on frequency. The values from Example 15–12 are used to illustrate this idea. For X_C and Z, enter each equation separately and plot them both as shown in Figure 15–30(a). The red line represents the magnitude of X_C and the blue line represents the magnitude of Z. Using the trace function, you can read values for frequency and reactance or impedance. You can also view a table of values the calculator uses to draw the graph by selecting GRAPH-TABLE in the mode menu. (Set up table parameters using 2nd window.) Figure 15–30(b) shows the result. Notice that at low frequencies, X_C dominates the magnitude of the impedance and at high frequencies, X_C dominates the impedance. (Grid lines are at increments of $f = 5{,}000$ Hz and $Z = 1{,}000\ \Omega$.) You can also use the graphing calculator to view how phase angle is affected by frequency in this example. Figure 15–30(c) shows the result. Grid lines in (c) are at increments of $f = 5{,}000$ Hz and $\theta = 10^\circ$.



(a) Impedance (red) and reactance (blue) as a function of frequency

(b) Plotted with table data

(c) Phase angle as a function of frequency

▲ **FIGURE 15–30**

Graphs for Example 15–12.

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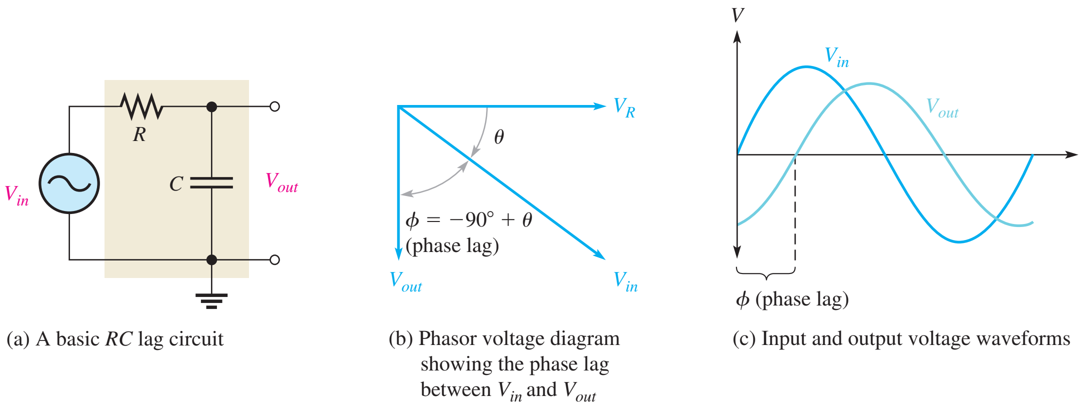
The *RC* Lag Circuit

An ***RC* lag circuit** is a phase shift circuit in which the output voltage lags the input voltage by a specified amount. Figure 15–31(a) shows a series *RC* circuit with the output voltage taken across the capacitor. The source voltage is the input, V_{in}. As you know, θ, the phase angle between the current and the input voltage, is also the phase angle between the resistor voltage and the input voltage because V_R and I are in phase with each other.

Since V_C lags V_R by 90°, the phase angle between the capacitor voltage and the input voltage is the difference between -90° and θ, as shown in Figure 15–31(b). The capacitor voltage is the output, and it lags the input, thus creating a basic lag circuit.

The input and output voltage waveforms of the lag circuit are shown in Figure 15–31(c). The amount of phase difference, designated ϕ, between the input



▲ FIGURE 15–31

RC lag circuit ($V_{out} = V_C$).

and the output is dependent on the relative sizes of the capacitive reactance and the resistance, as is the magnitude of the output voltage.

Phase Difference Between Input and Output As already established, θ is the phase angle between I and V_{in}. The angle between V_{out} and V_{in} is designated ϕ (phi) and is developed as follows.

The polar expressions for the input voltage and the current are $V_{in}\angle 0°$ and $I\angle\theta$, respectively. The output voltage in polar form is

$$\mathbf{V}_{out} = (I\angle\theta)(X_C\angle -90°) = IX_C\angle(-90° + \theta)$$

The preceding equation states that the output voltage is at an angle of $-90° + \theta$ with respect to the input voltage. Since $\theta = -\tan^{-1}(X_C/R)$, the angle ϕ between the input and output is

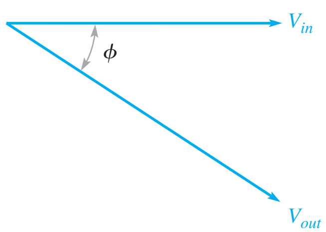
$$\phi = -90° + \tan^{-1}\left(\frac{X_C}{R}\right)$$

Equivalently, this angle can be expressed as

$$\phi = -\tan^{-1}\left(\frac{R}{X_C}\right)$$

Equation 15–14

This angle is always negative, indicating that the output voltage lags the input voltage, as shown in Figure 15–32.

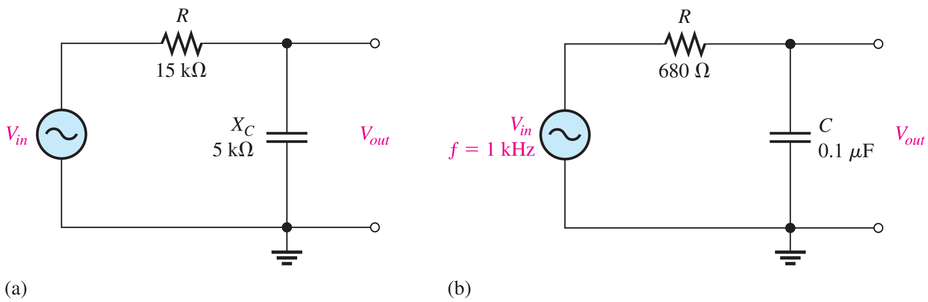


◄ FIGURE 15–32

Illustration of voltages in an *RC* lag circuit.

EXAMPLE 15–13

Determine the amount of phase lag from input to output in each lag circuit in Figure 15–33.



▲ FIGURE 15–33

Solution For the lag circuit in Figure 15–33(a),

$$\phi = -\tan^{-1}\left(\frac{R}{X_C}\right) = -\tan^{-1}\left(\frac{15\text{ k}\Omega}{5\text{ k}\Omega}\right) = \mathbf{-71.6^\circ}$$

The output lags the input by 71.6°.

For the lag circuit in Figure 15–33(b), first determine the capacitive reactance.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1\text{ kHz})(0.1\ \mu\text{F})} = 1.59\text{ k}\Omega$$

$$\phi = -\tan^{-1}\left(\frac{R}{X_C}\right) = -\tan^{-1}\left(\frac{680\ \Omega}{1.59\text{ k}\Omega}\right) = \mathbf{-23.1^\circ}$$

The output lags the input by 23.1°.

Related Problem In a lag circuit, what happens to the phase lag if the frequency increases?

Use Multisim files E15-13A, E15-13B, and E15-13C to verify the calculated results in this example and to confirm your calculation for the related problem.

Magnitude of the Output Voltage To evaluate the output voltage in terms of its magnitude, visualize the *RC* lag circuit as a voltage divider. A portion of the total input voltage is dropped across the resistor and a portion across the capacitor. Because the output voltage is the voltage across the capacitor, it can be calculated using either Ohm's law ($V_{out} = IX_C$) or the voltage divider formula.

Equation 15–15

$$V_{out} = \left(\frac{X_C}{\sqrt{R^2 + X_C^2}}\right)V_{in}$$

The phasor expression for the output voltage of an *RC* lag circuit is

$$\mathbf{V}_{out} = V_{out}\angle\phi$$

EXAMPLE 15–14

For the lag circuit in Figure 15–33(b) (Example 15–13), determine the output voltage in phasor form when the input voltage has an rms value of 10 V. Draw the input and output voltage waveforms showing the proper phase relationship. The capacitive reactance X_C (1.59 kΩ) and ϕ (−23.1°) were found in Example 15–13.

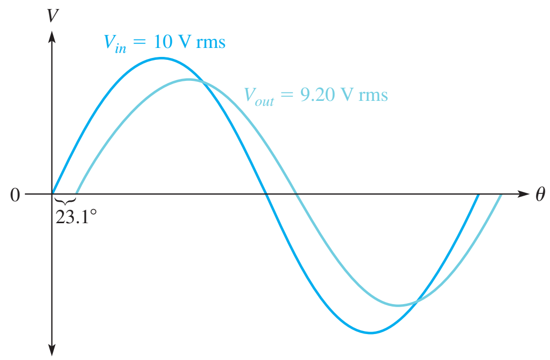
Solution The output voltage in phasor form is

$$\mathbf{V}_{out} = V_{out}\angle\phi = \left(\frac{X_C}{\sqrt{R^2 + X_C^2}}\right)V_{in}\angle\phi$$

$$= \left(\frac{1.59\text{ k}\Omega}{\sqrt{(680\ \Omega)^2 + (1.59\text{ k}\Omega)^2}}\right)10\angle-23.1^\circ\text{ V} = \mathbf{9.20\angle-23.1^\circ\text{ V rms}}$$

The waveforms are shown in Figure 15–34. Notice that the output voltage lags the input voltage by 23.1°.

▶ FIGURE 15–34

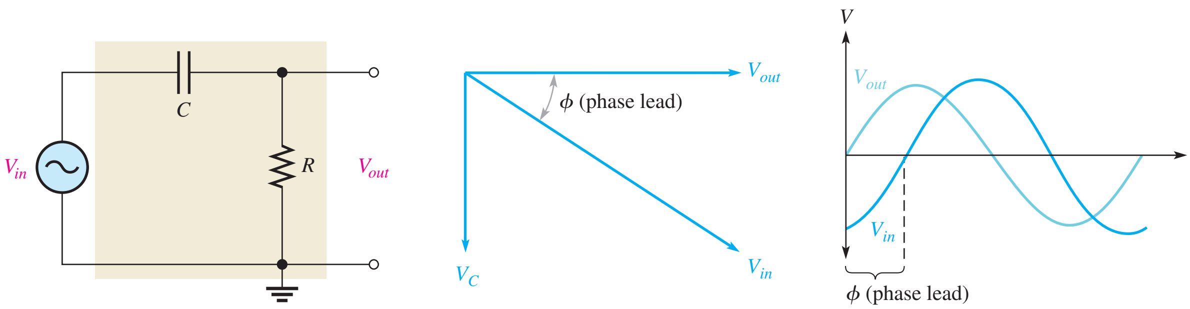


Related Problem In a lag circuit, what happens to the output voltage if the frequency increases?

Use Multisim files E15-14A and E15-14B to verify the calculated results in this example and to confirm your calculation for the related problem.

The *RC* Lead Circuit

An ***RC* lead circuit** is a phase shift circuit in which the output voltage leads the input voltage by a specified amount. When the output of a series *RC* circuit is taken across the resistor rather than across the capacitor, as shown in Figure 15–35(a), it becomes a lead circuit.



(a) A basic *RC* lead circuit

(b) Phasor voltage diagram showing the phase lead between V_{in} and V_{out}

(c) Input and output voltage waveforms

▲ FIGURE 15–35

RC lead circuit ($V_{out} = V_R$).

Phase Difference Between Input and Output In a series RC circuit, the current leads the input voltage. Also, as you know, the resistor voltage is in phase with the current. Since the output voltage is taken across the resistor, the output leads the input, as indicated by the phasor diagram in Figure 15–35(b). The waveforms are shown in Figure 15–35(c).

As in the lag circuit, the amount of phase difference between the input and output and the magnitude of the output voltage in the lead circuit are dependent on the relative values of the resistance and the capacitive reactance. When the input voltage is assigned a reference angle of 0°, the angle of the output voltage is the same as θ (the angle between total current and applied voltage) because the resistor voltage (output) and the current are in phase with each other. Therefore, since $\phi = \theta$ in this case, the expression is

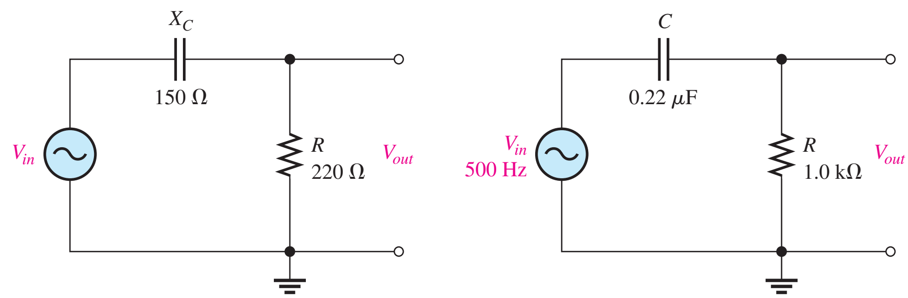
Equation 15–16

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

This angle is positive because the output leads the input.

EXAMPLE 15–15

Calculate the output phase angle for each circuit in Figure 15–36.



(a) (b)

▲ **FIGURE 15–36**

Solution For the lead circuit in Figure 15–36(a),

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{150\ \Omega}{220\ \Omega}\right) = \mathbf{34.3°}$$

The output leads the input by 34.3°.

For the lead circuit in Figure 15–36(b), first determine the capacitive reactance.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(500\ \text{Hz})(0.22\ \mu\text{F})} = 1.45\ \text{k}\Omega$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1.45\ \text{k}\Omega}{1.0\ \text{k}\Omega}\right) = \mathbf{55.4°}$$

The output leads the input by 55.4°.

Related Problem In a lead circuit, what happens to the phase lead if the frequency increases?

MultiSim

Use Multisim files E15-15A, E15-15B, and E15-15C to verify the calculated results in this example and to confirm your calculation for the related problem.

Magnitude of the Output Voltage Since the output voltage of an *RC* lead circuit is taken across the resistor, the magnitude can be calculated using either Ohm's law ($V_{out} = IR$) or the voltage-divider formula.

$$V_{out} = \left(\frac{R}{\sqrt{R^2 + X_C^2}}\right)V_{in}$$

Equation 15–17

The expression for the output voltage in phasor form is

$$\mathbf{V}_{out} = V_{out}\angle\phi$$

EXAMPLE 15–16

The input voltage in Figure 15–36(b) (Example 15–15) has an rms value of 10 V. Determine the phasor expression for the output voltage. Draw the waveform relationships for the input and output voltages showing peak values. The phase angle (55.4°) and X_C (1.45 kΩ) were found in Example 15–15.

Solution The phasor expression for the output voltage is

$$\mathbf{V}_{out} = V_{out}\angle\phi = \left(\frac{R}{\sqrt{R^2 + X_C^2}}\right)V_{in}\angle\phi$$

$$= \left(\frac{1.0\text{ k}\Omega}{1.76\text{ k}\Omega}\right)10\angle 55.4^\circ\text{ V} = \mathbf{5.69\angle 55.4^\circ\ V\ rms}$$

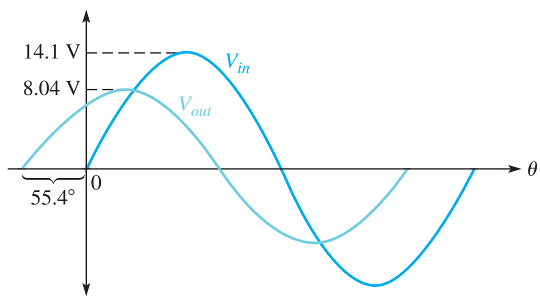
The peak value of the input voltage is

$$V_{in(p)} = 1.41V_{in(rms)} = 1.41(10\text{ V}) = 14.1\text{ V}$$

The peak value of the output voltage is

$$V_{out(p)} = 1.41V_{out(rms)} = 1.41(5.68\text{ V}) = 8.04\text{ V}$$

The waveforms are shown in Figure 15–37.



▲ FIGURE 15–37

Related Problem In a lead circuit, what happens to the output voltage if the frequency is reduced?

Use Multisim files E15-16A and E15-16B to verify the calculated results in this example and to confirm your calculation for the related problem.

SECTION 15–4 CHECKUP

1. In a certain series *RC* circuit, $V_R = 4\ V$, and $V_C = 6$ V. What is the magnitude of the source voltage?
2. In Question 1, what is the phase angle between the source voltage and the current?
3. What is the phase difference between the capacitor voltage and the resistor voltage in a series *RC* circuit?
4. When the frequency of the applied voltage in a series *RC* circuit is increased, what happens to the capacitive reactance? What happens to the magnitude of the total impedance? What happens to the phase angle?
5. A certain *RC* lag circuit consists of a 4.7 kΩ resistor and a 0.022 μF capacitor. Determine the phase shift between input and output at a frequency of 3 kHz.
6. An *RC* lead circuit has the same component values as the lag circuit in Question 5. What is the magnitude of the output voltage at 3 kHz when the input is 10 V rms?

OPTION 2 NOTE

Coverage of series reactive circuits continues in Chapter 16, Part 1, on page 725.

Part 2

PARALLEL CIRCUITS

15–5 IMPEDANCE AND ADMITTANCE OF PARALLEL *RC* CIRCUITS

In this section, you will learn how to determine the impedance and phase angle of a parallel *RC* circuit. The impedance consists of a magnitude component and a phase angle component. Also, capacitive susceptance and admittance of a parallel *RC* circuit are introduced.

After completing this section, you should be able to

- **Determine impedance and admittance in a parallel *RC* circuit**
  - Express total impedance in complex form
  - Define and calculate *conductance, capacitive susceptance,* and *admittance*

Figure 15–38 shows a basic parallel *RC* circuit connected to an ac voltage source.

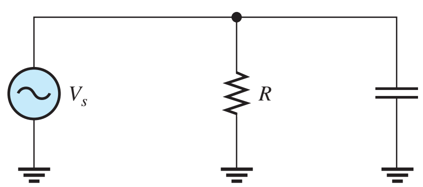


FIGURE 15–38

Basic parallel *RC* circuit.

The expression for the total impedance is developed as follows, using complex numbers. Since there are only two circuit components, *R* and *C*, the total impedance can be found from the product-over-sum rule.

$$\mathbf{Z} = \frac{(R\angle 0^\circ)(X_C\angle -90^\circ)}{R - jX_C}$$

By multiplying the magnitudes, adding the angles in the numerator, and converting the denominator to polar form, you get

$$\mathbf{Z} = \frac{RX_C\angle(0^\circ - 90^\circ)}{\sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)}$$

Now, by dividing the magnitude expression in the numerator by that in the denominator, and by subtracting the angle in the denominator from that in the numerator, you get

$$\mathbf{Z} = \left(\frac{RX_C}{\sqrt{R^2 + X_C^2}}\right)\angle\left(-90^\circ + \tan^{-1}\left(\frac{X_C}{R}\right)\right)$$

Equivalently, this expression can be written as

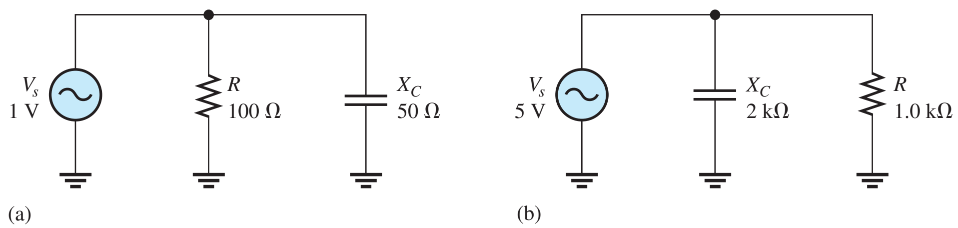
Equation 15–18

$$\mathbf{Z} = \frac{RX_C}{\sqrt{R^2 + X_C^2}}\angle -\tan^{-1}\left(\frac{R}{X_C}\right)$$

Notice that Equation 15–18 is simply the complex version of the product-over-sum rule.

EXAMPLE 15–17

For each circuit in Figure 15–39, determine the magnitude and phase angle of the total impedance.



▲ FIGURE 15–39

Solution For the circuit in Figure 15–39(a), the total impedance is

$$\mathbf{Z} = \left(\frac{RX_C}{\sqrt{R^2 + X_C^2}}\right)\angle -\tan^{-1}\left(\frac{R}{X_C}\right)$$

$$= \left(\frac{(100\ \Omega)(50\ \Omega)}{\sqrt{(100\ \Omega)^2 + (50\ \Omega)^2}}\right)\angle -\tan^{-1}\left(\frac{100\ \Omega}{50\ \Omega}\right) = \mathbf{44.7\angle -63.4^\circ\ \Omega}$$

Thus, $Z = 44.7\ \Omega$ and $\theta = -63.4°$.

For the circuit in Figure 15–39(b), the total impedance is

$$\mathbf{Z} = \left(\frac{(1.0\ \text{k}\Omega)(2\ \text{k}\Omega)}{\sqrt{(1.0\ \text{k}\Omega)^2 + (2\ \text{k}\Omega)^2}}\right)\angle -\tan^{-1}\left(\frac{1.0\ \text{k}\Omega}{2\ \text{k}\Omega}\right) = \mathbf{894\angle -26.6^\circ\ \Omega}$$

Thus, $Z = 894\ \Omega$ and $\theta = -26.6°$.

Related Problem Determine **Z** in Figure 15–39(a) if the frequency is doubled.

Conductance, Susceptance, and Admittance

Recall that **conductance,** G, is the reciprocal of resistance. The phasor expression for conductance is expressed as

$$\mathbf{G} = \frac{1}{R\angle 0^\circ} = G\angle 0^\circ$$

Two new terms are now introduced for use in parallel RC circuits. **Capacitive susceptance (B_C)** is the reciprocal of capacitive reactance. The phasor expression for capacitive susceptance is

$$\mathbf{B}_C = \frac{1}{X_C\angle -90^\circ} = B_C\angle 90^\circ = +jB_C$$

Admittance (Y) is the reciprocal of impedance. The phasor expression for admittance is

$$\mathbf{Y} = \frac{1}{Z\angle \pm\theta} = Y\angle \mp\theta$$

The unit of each of these terms is the siemens (S), which is the reciprocal of the ohm.

In working with parallel circuits, it is often easier to use conductance (G), capacitive susceptance (B_C), and admittance (Y) rather than resistance (R), capacitive reactance (X_C), and impedance (Z). In a parallel *RC* circuit, as shown in Figure 15–40, the total admittance is simply the phasor sum of the conductance and the capacitive susceptance.

$$\mathbf{Y} = G + jB_C$$

Equation 15–19

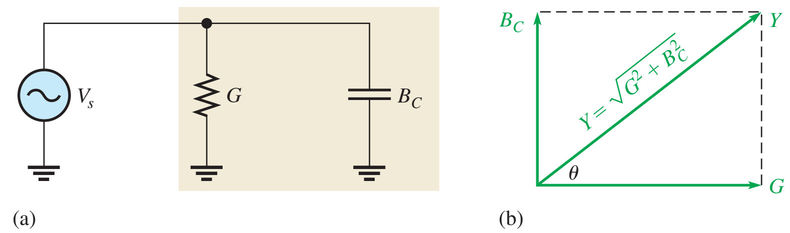


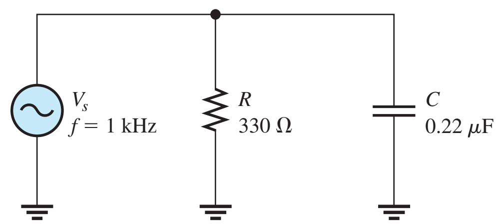
FIGURE 15–40

Admittance in a parallel *RC* circuit.

EXAMPLE 15–18

Determine the total admittance (**Y**) and then convert it to total impedance (**Z**) in Figure 15–41. Draw the admittance phasor diagram.

FIGURE 15–41



Solution From Figure 15–41, $R = 330\ \Omega$; thus $G = 1/R = 1/330\ \Omega = 3.03$ mS. The capacitive reactance is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(1{,}000\text{ Hz})(0.22\ \mu\text{F})} = 723\ \Omega$$

The capacitive susceptance magnitude is

$$B_C = \frac{1}{X_C} = \frac{1}{723\ \Omega} = 1.38\text{ mS}$$

The total admittance is

$$\mathbf{Y}_{tot} = G + jB_C = \mathbf{3.03\ mS + \mathit{j}1.38\ mS}$$

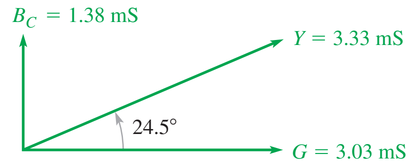
which can be expressed in polar form as

$$\mathbf{Y}_{tot} = \sqrt{G^2 + B_C^2}\angle\tan^{-1}\left(\frac{B_C}{G}\right)$$

$$= \sqrt{(3.03\text{ mS})^2 + (1.38\text{ mS})^2}\angle\tan^{-1}\left(\frac{1.38\text{ mS}}{3.03\text{ mS}}\right) = \mathbf{3.33\angle 24.5^\circ mS}$$

The admittance phasor diagram is shown in Figure 15–42.

▶ FIGURE 15–42



Convert total admittance to total impedance as follows:

$$\mathbf{Z}_{tot} = \frac{1}{\mathbf{Y}_{tot}} = \frac{1}{(3.33\angle 24.5^\circ \text{ mS})} = \mathbf{300\angle -24.5\ \Omega}$$

Related Problem Calculate the total admittance in Figure 15–41 if f is increased to 2.5 kHz.

SECTION 15–5 CHECKUP

1. Define *conductance, capacitive susceptance,* and *admittance.*
2. If $Z = 100\ \Omega$ what is the value of Y?
3. In a certain parallel RC circuit, $R = 47\ \text{k}\Omega$ and $X_C = 75\ \text{k}\Omega$ Determine Y.
4. In Question 3, what is Z?

15–6 ANALYSIS OF PARALLEL *RC* CIRCUITS

Ohm's law and Kirchhoff's current law are used in the analysis of *RC* circuits. Current and voltage relationships in a parallel *RC* circuit are examined.

After completing this section, you should be able to

- **Analyze a parallel *RC* circuit**
  - Apply Ohm's law and Kirchhoff's current law to parallel *RC* circuits
  - Express the voltages and currents as phasor quantities
  - Show how impedance and phase angle vary with frequency
  - Convert from a parallel circuit to an equivalent series circuit

For convenience in the analysis of parallel circuits, the Ohm's law formulas using impedance, previously stated, can be rewritten for admittance using the relation $Y = 1/Z$. Remember, the use of boldface nonitalic letters indicates phasor quantities.

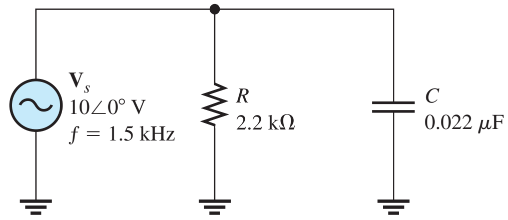
Equation 15–20
$$\mathbf{V} = \frac{\mathbf{I}}{\mathbf{Y}}$$

Equation 15–21
$$\mathbf{I} = \mathbf{VY}$$

Equation 15–22
$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}}$$

EXAMPLE 15–19

Determine the total current and phase angle in Figure 15–43. Draw a phasor diagram showing the relationship of $\mathbf{V}_s$ and $\mathbf{I}_{tot}$.



▲ **FIGURE 15–43**

Solution The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1.5\text{ kHz})(0.022\ \mu\text{F})} = 4.82\text{ k}\Omega$$

The capacitive susceptance magnitude is

$$B_C = \frac{1}{X_C} = \frac{1}{4.82\text{ k}\Omega} = 207\ \mu\text{S}$$

The conductance magnitude is

$$G = \frac{1}{R} = \frac{1}{2.2\text{ k}\Omega} = 455\ \mu\text{S}$$

The total admittance is

$$\mathbf{Y}_{tot} = G + jB_C = 455\ \mu\text{S} + j207\ \mu\text{S}$$

Converting to polar form yields

$$\mathbf{Y}_{tot} = \sqrt{G^2 + B_C^2}\angle\tan^{-1}\left(\frac{B_C}{G}\right)$$

$$= \sqrt{(455\ \mu\text{S})^2 + (207\ \mu\text{S})^2}\angle\tan^{-1}\left(\frac{207\ \mu\text{S}}{455\ \mu\text{S}}\right) = 500\angle 24.5°\ \mu\text{S}$$

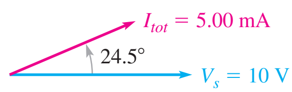
The phase angle is 24.5°.

Use Ohm's law to determine the total current.

$$\mathbf{I}_{tot} = \mathbf{V}_s\mathbf{Y}_{tot} = (10\angle 0°\text{ V})(500\angle 24.5°\ \mu\text{S}) = \mathbf{5.00\angle 24.5°\ mA}$$

The magnitude of the total current is 5.00 mA, and it leads the applied voltage by **24.5**°, as the phasor diagram in Figure 15–44 indicates.

► **FIGURE 15–44**

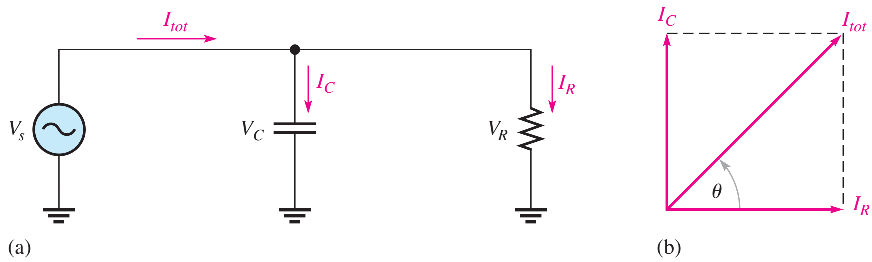


Related Problem What is the total current (in polar form) if f is doubled?

Use Multisim files E15-19A and E15-19B to verify the calculated results in this example and to confirm your calculation for the related problem.

Phase Relationships of Currents and Voltages

Figure 15–45(a) shows all the currents in a basic parallel *RC* circuit. The total current, I_{tot}, divides at the junction into the two branch currents, I_R and I_C. The applied voltage, V_s, appears across both the resistive and the capacitive branches, so V_s, V_R, and V_C are all in phase and of the same magnitude.



▲ **FIGURE 15–45**

Currents in a parallel *RC* circuit. The current directions shown in (a) are instantaneous and, of course, reverse when the source voltage reverses.

The current through the resistor is in phase with the voltage. The current through the capacitor leads the voltage, and thus the resistive current, by 90°. By Kirchhoff's current law, the total current is the phasor sum of the two branch currents, as shown by the phasor diagram in Figure 15–45(b). The total current is expressed as

Equation 15–23

$$\mathbf{I}_{tot} = I_R + jI_C$$

This equation can be expressed in polar form as

Equation 15–24

$$\mathbf{I}_{tot} = \sqrt{I_R^2 + I_C^2}\angle\tan^{-1}\left(\frac{I_C}{I_R}\right)$$

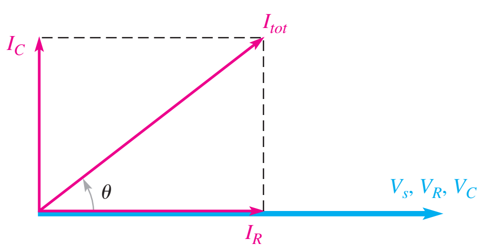
where the magnitude of the total current is

$$I_{tot} = \sqrt{I_R^2 + I_C^2}$$

and the phase angle between the resistor current and the total current is

$$\theta = \tan^{-1}\left(\frac{I_C}{I_R}\right)$$

Since the resistor current and the applied voltage are in phase, θ also represents the phase angle between the total current and the applied voltage. Figure 15–46 shows a complete current and voltage phasor diagram.



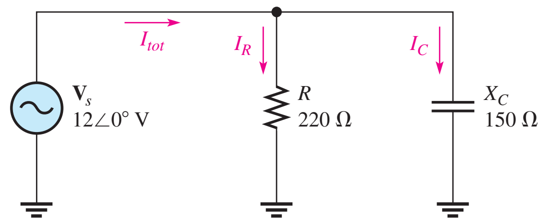
▲ **FIGURE 15–46**

Current and voltage phasor diagram for a parallel *RC* circuit (amplitudes depend on the particular circuit).

EXAMPLE 15–20

Determine the value of each current in Figure 15–47, and describe the phase relationship of each with the applied voltage. Draw the current phasor diagram.

▶ FIGURE 15–47



Solution The resistor current, the capacitor current, and the total current are expressed as follows:

$$\mathbf{I}_R = \frac{\mathbf{V}_s}{\mathbf{R}} = \frac{12\angle 0^\circ\ \text{V}}{220\angle 0^\circ\ \Omega} = \mathbf{54.6\angle 0^\circ\ mA}$$

$$\mathbf{I}_C = \frac{\mathbf{V}_s}{\mathbf{X}_C} = \frac{12\angle 0^\circ\ \text{V}}{150\angle -90^\circ\ \Omega} = \mathbf{80\angle 90^\circ\ mA}$$

$$\mathbf{I}_{tot} = I_R + jI_C = 54.6\ \text{mA} + j80\ \text{mA}$$

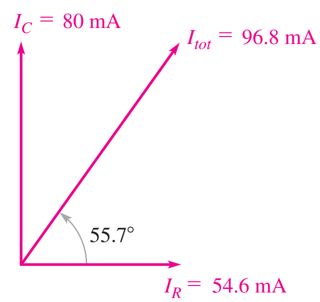
Converting $\mathbf{I}_{tot}$ to polar form yields

$$I_{tot} = \sqrt{I_R^2 + I_C^2}\angle\tan^{-1}\left(\frac{I_C}{I_R}\right)$$

$$= \sqrt{(54.6\ \text{mA})^2 + (80\ \text{mA})^2}\angle\tan^{-1}\left(\frac{80\ \text{mA}}{54.6\ \text{mA}}\right) = \mathbf{96.8\angle 55.7^\circ\ mA}$$

As the results show, the resistor current is 54.6 mA and is in phase with the voltage. The capacitor current is 80 mA and leads the voltage by 90°. The total current is 96.8 mA and leads the voltage by 55.7°. The phasor diagram in Figure 15–48 illustrates these relationships.

▶ FIGURE 15–48



Related Problem In a parallel circuit, $\mathbf{I}_R = 100\angle 0^\circ$ mA and $\mathbf{I}_C = 60\angle 90^\circ$ mA. Determine the total current.

Conversion from Parallel to Series Form

For every parallel *RC* circuit, there is an equivalent series *RC* circuit for a given frequency. Two circuits are considered equivalent when they both present an equal impedance at their terminals; that is, the magnitude of impedance and the phase angle are identical.

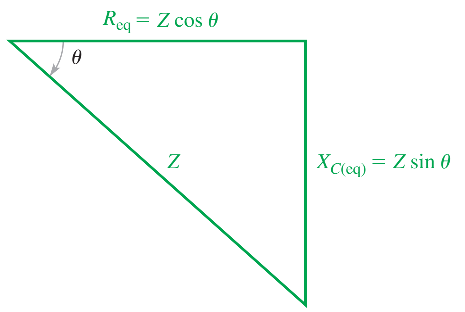
To obtain the equivalent series circuit for a given parallel RC circuit, first find the impedance and phase angle of the parallel circuit. Then use the values of Z and θ to construct an impedance triangle, shown in Figure 15–49. The vertical and horizontal sides of the triangle represent the equivalent series resistance and capacitive reactance as indicated. These values can be found using the following trigonometric relationships:

Equation 15–25
$$R_{eq} = Z\cos\theta$$

Equation 15–26
$$X_{C(eq)} = Z\sin\theta$$

► FIGURE 15–49

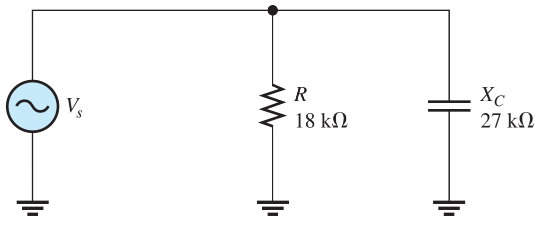
Impedance triangle for the series equivalent of a parallel RC circuit. Z and θ are the known values for the parallel circuit. R_{eq} and $X_{C(eq)}$ are the series equivalent values.



EXAMPLE 15–21

Convert the parallel circuit in Figure 15–50 to a series form.

► FIGURE 15–50



Solution First, find the admittance of the parallel circuit as follows:

$$G = \frac{1}{R} = \frac{1}{18\text{ k}\Omega} = 55.6\ \mu\text{S}$$

$$B_C = \frac{1}{X_C} = \frac{1}{27\text{ k}\Omega} = 37.0\ \mu\text{S}$$

$$\mathbf{Y} = G + jB_C = 55.6\ \mu\text{S} + j37.0\ \mu\text{S}$$

Converting to polar form yields

$$\mathbf{Y} = \sqrt{G^2 + B_C^2}\angle\tan^{-1}\left(\frac{B_C}{G}\right)$$

$$= \sqrt{(55.6\ \mu\text{S})^2 + (37.0\ \mu\text{S})^2}\angle\tan^{-1}\left(\frac{37.0\ \mu\text{S}}{55.6\ \mu\text{S}}\right) = 66.8\angle 33.7^\circ\ \mu\text{S}$$

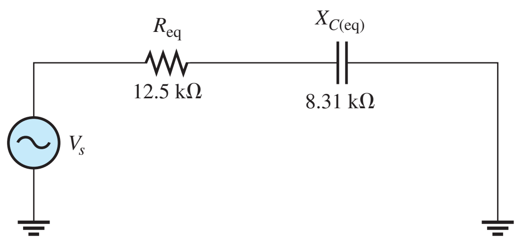
Then, the total impedance is

$$\mathbf{Z}_{tot} = \frac{1}{\mathbf{Y}} = \frac{1}{66.8\angle 33.7^\circ\ \mu\text{S}} = 15.0\angle -33.7^\circ\text{ k}\Omega$$

Converting to rectangular form yields

$$\mathbf{Z}_{tot} = Z\cos\theta - jZ\sin\theta = R_{\text{eq}} - jX_{C(\text{eq})}$$
$$= 15.0\text{ k}\Omega\cos(-33.6°) - j15.0\text{ k}\Omega\sin(-33.6°) = 12.5\text{ k}\Omega - j8.31\text{ k}\Omega$$

The equivalent series *RC* circuit is a 12.5 kΩ resistor in series with a capacitive reactance of 8.31 kΩ. This is shown in Figure 15–51.



▲ FIGURE 15–51

Related Problem The impedance of a parallel *RC* circuit is $\mathbf{Z} = 10\angle -26°$ kΩ. Convert to an equivalent series circuit.

SECTION 15–6 CHECKUP

1. The admittance of a parallel *RC* circuit is 3.50 mS, and the applied voltage is 6 V. What is the total current?
2. In a certain parallel *RC* circuit, the resistor current is 10 mA, and the capacitor current is 15 mA. Determine the magnitude and phase angle of the total current. This phase angle is measured with respect to what?
3. What is the phase angle between the capacitor current and the applied voltage in a parallel *RC* circuit?

OPTION 2 NOTE

Coverage of parallel reactive circuits continues in Chapter 16, Part 2, on page 738.

Part 3

Series-Parallel Circuits

15–7 Analysis of Series-Parallel *RC* Circuits

The concepts studied with respect to series and parallel circuits are used to analyze circuits with combinations of both series and parallel *R* and *C* components.

After completing this section, you should be able to

- **Analyze series-parallel *RC* circuits**
  - Determine total impedance
  - Calculate currents and voltages
  - Measure impedance and phase angle

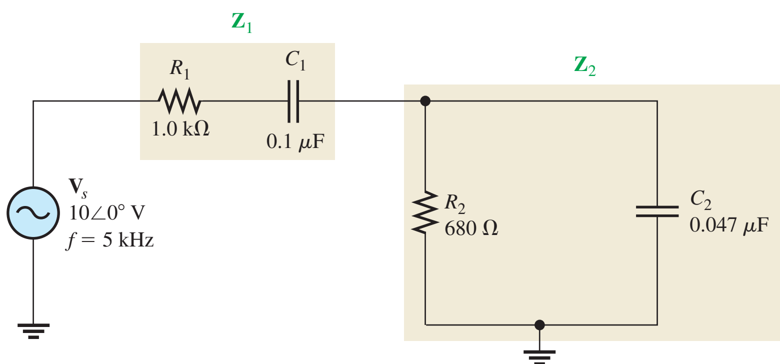
The impedance of series components is most easily expressed in rectangular form, and the impedance of parallel components is best found by using polar form. The steps for analyzing a circuit with a series and a parallel component are illustrated in Example 15–22. First express the impedance of the series part of the circuit in rectangular form and the impedance of the parallel part in polar form. Next, convert the impedance of the parallel part to rectangular form and add it to the impedance of the series part. Once you determine the rectangular form of the total impedance, you can convert it to polar form in order to see the magnitude and phase angle and to calculate the current.

EXAMPLE 15–22

In the series-parallel *RC* circuit of Figure 15–52, determine the following:

(a) total impedance **(b)** total current **(c)** phase angle by which I_{tot} leads V_s

► FIGURE 15–52



Solution **(a)** First, calculate the magnitudes of capacitive reactance.

$$X_{C1} = \frac{1}{2\pi fC} = \frac{1}{2\pi(5\text{ kHz})(0.1\ \mu\text{F})} = 318\ \Omega$$

$$X_{C2} = \frac{1}{2\pi fC} = \frac{1}{2\pi(5\text{ kHz})(0.047\ \mu\text{F})} = 677\ \Omega$$

One approach is to find the impedance of the series portion and the impedance of the parallel portion and combine them to get the total impedance. The impedance of the series combination of R_1 and C_1 is

$$\mathbf{Z}_1 = R_1 - jX_{C1} = 1.0\text{ k}\Omega - j318\ \Omega$$

To determine the impedance of the parallel portion, first determine the admittance of the parallel combination of R_2 and C_2.

$$G_2 = \frac{1}{R_2} = \frac{1}{680\ \Omega} = 1.47\text{ mS}$$

$$B_{C2} = \frac{1}{X_{C2}} = \frac{1}{677\ \Omega} = 1.48\text{ mS}$$

$$\mathbf{Y}_2 = G_2 + jB_{C2} = 1.47\text{ mS} + j1.48\text{ mS}$$

Converting to polar form yields

$$\mathbf{Y}_2 = \sqrt{G_2^2 + B_{C2}^2}\angle\tan^{-1}\left(\frac{B_{C2}}{G_2}\right)$$

$$= \sqrt{(1.47\text{ mS})^2 + (1.48\text{ mS})^2}\angle\tan^{-1}\left(\frac{1.48\text{ mS}}{1.47\text{ mS}}\right) = 2.08\angle 45.1^\circ\text{ mS}$$

Then, the impedance of the parallel portion is

$$\mathbf{Z}_2 = \frac{1}{\mathbf{Y}_2} = \frac{1}{2.08\angle 45.1^\circ\text{ mS}} = 480\angle -45.1^\circ\ \Omega$$

Converting to rectangular form yields

$$\mathbf{Z}_2 = Z_2\cos\theta - jZ_2\sin\theta$$

$$= (480\ \Omega)\cos(-45.1^\circ) - j(480\ \Omega)\sin(-45.1^\circ) = 339\ \Omega - j340\ \Omega$$

The series portion and the parallel portion are in series with each other. Combine $\mathbf{Z}_1$ and $\mathbf{Z}_2$ to get the total impedance.

$$\mathbf{Z}_{tot} = \mathbf{Z}_1 + \mathbf{Z}_2$$

$$= (1.0\text{ k}\Omega - j318\ \Omega) + (338\ \Omega - j340\ \Omega) = 1.34\text{ k}\Omega - j658\ \Omega$$

Expressing $\mathbf{Z}_{tot}$ in polar form yields

$$\mathbf{Z}_{tot} = \sqrt{Z_1^2 + Z_2^2}\angle -\tan^{-1}\left(\frac{Z_2}{Z_1}\right)$$

$$= \sqrt{(1.34\text{ k}\Omega)^2 + (640\ \Omega)^2}\angle -\tan^{-1}\left(\frac{658\ \Omega}{1.34\text{ k}\Omega}\right) = \mathbf{1.49\angle -26.2^\circ\ k\Omega}$$

(b) Use Ohm's law to determine the total current.

$$\mathbf{I}_{tot} = \frac{\mathbf{V}_s}{\mathbf{Z}_{tot}} = \frac{10\angle 0^\circ\text{ V}}{1.49\angle -26.2^\circ\text{ k}\Omega} = \mathbf{6.70\angle 26.2^\circ\ mA}$$

(c) The total current leads the applied voltage by **26.2°.**

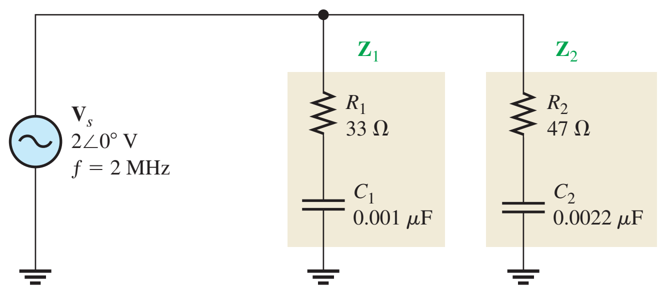
Related Problem Determine the voltages across $\mathbf{Z}_1$ and $\mathbf{Z}_2$ in Figure 15–52 and express in polar form.

Use Multisim files E15-22A, E15-22B, and E15-22C to verify the calculated results in the (b) part of this example and to confirm your calculation for the related problem.

Example 15–23 shows two sets of series components in parallel. The approach is to first express each branch impedance in rectangular form and then convert each of these impedances to polar form. Next, calculate each branch current using polar notation. Once you know the branch currents, you can find the total current by adding the two branch currents in rectangular form. In this particular case, the total impedance is not required.

EXAMPLE 15–23

Determine all currents in Figure 15–53. Draw a current phasor diagram.



▲ FIGURE 15–53

Solution First, calculate X_{C1} and X_{C2}.

$$X_{C1} = \frac{1}{2\pi fC} = \frac{1}{2\pi(2\text{ MHz})(0.001\ \mu\text{F})} = 79.6\ \Omega$$

$$X_{C2} = \frac{1}{2\pi fC} = \frac{1}{2\pi(2\text{ MHz})(0.0022\ \mu\text{F})} = 36.2\ \Omega$$

Next, determine the impedance of each of the two parallel branches.

$$\mathbf{Z}_1 = R_1 - jX_{C1} = 33\ \Omega - j79.6\ \Omega$$

$$\mathbf{Z}_2 = R_2 - jX_{C2} = 47\ \Omega - j36.2\ \Omega$$

Convert these impedances to polar form.

$$\mathbf{Z}_1 = \sqrt{R_1^2 + X_{C1}^2}\angle -\tan^{-1}\left(\frac{X_{C1}}{R_1}\right)$$

$$= \sqrt{(33\ \Omega)^2 + (79.6\ \Omega)^2}\angle -\tan^{-1}\left(\frac{79.6\ \Omega}{33\ \Omega}\right) = 86.2\angle -67.5°\ \Omega$$

$$\mathbf{Z}_2 = \sqrt{R_2^2 + X_{C2}^2}\angle -\tan^{-1}\left(\frac{X_{C2}}{R_2}\right)$$

$$= \sqrt{(47\ \Omega)^2 + (36.2\ \Omega)^2}\angle -\tan^{-1}\left(\frac{36.2\ \Omega}{47\ \Omega}\right) = 59.3\angle -37.6°\ \Omega$$

Calculate each branch current.

$$\mathbf{I}_1 = \frac{\mathbf{V}_s}{\mathbf{Z}_1} = \frac{2\angle 0^\circ \text{ V}}{86.2\angle -67.5^\circ\ \Omega} = \mathbf{23.2\angle 67.5^\circ\ mA}$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_s}{\mathbf{Z}_2} = \frac{2\angle 0^\circ \text{ V}}{59.3\angle -37.6^\circ\ \Omega} = \mathbf{33.7\angle 37.6^\circ\ mA}$$

To get the total current, express each branch current in rectangular form so that they can be added.

$$\mathbf{I}_1 = 8.89 \text{ mA} + j21.4 \text{ mA}$$
$$\mathbf{I}_2 = 26.7 \text{ mA} + j20.6 \text{ mA}$$

The total current is

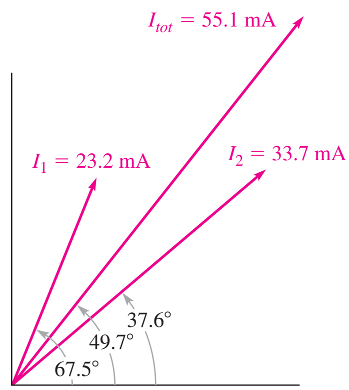
$$\begin{aligned}\mathbf{I}_{tot} &= \mathbf{I}_1 + \mathbf{I}_2\\ &= (8.89 \text{ mA} + j21.4 \text{ mA}) + (26.7 \text{ mA} + j20.6 \text{ mA}) = 35.6 \text{ mA} + j42.0 \text{ mA}\end{aligned}$$

Converting $\mathbf{I}_{tot}$ to polar form yields

$$\mathbf{I}_{tot} = \sqrt{(35.6 \text{ mA})^2 + (42.0 \text{ mA})^2}\angle\tan^{-1}\left(\frac{42.0\ \Omega}{35.6\ \Omega}\right) = \mathbf{55.1\angle 49.7^\circ\ mA}$$

The current phasor diagram is shown in Figure 15–54.

▶ FIGURE 15–54



Related Problem Determine the voltages across each component in Figure 15–53 and draw a voltage phasor diagram.

Use Multisim files E15-23A through E15-23G to verify the calculated results in this example and to confirm your calculations for the related problem.

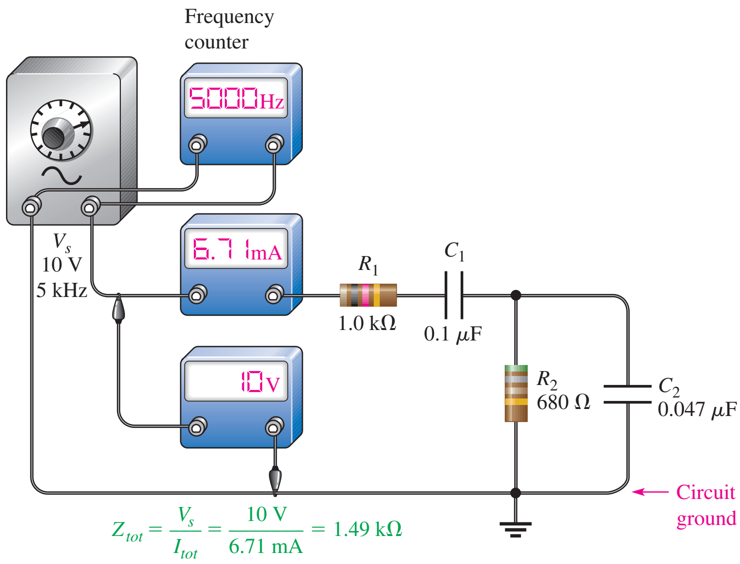
Measurement of Z_{tot}

Now, let's see how the value of Z_{tot} for the circuit in Example 15–22 can be determined by measurement. First, the total impedance is measured as outlined in the following steps and as illustrated in Figure 15–55 (other ways are also possible):

Step 1: Using a sine wave generator, set the source voltage to a known value (10 V) and the frequency to 5 kHz. If your generator is not accurate, then it is advisable to check the voltage with an ac voltmeter and the frequency with a frequency counter rather than relying on the marked values on the generator controls.

▶ FIGURE 15–55

Determining Z_{tot} by measurement of V_s and I_{tot}.



Step 2: Connect an ac ammeter as shown in Figure 15–55, and measure the total current. Alternatively, you can measure the voltage across R_1 with a voltmeter and calculate the current.

Step 3: Calculate the total impedance by using Ohm's law.

Measurement of Phase Angle, θ

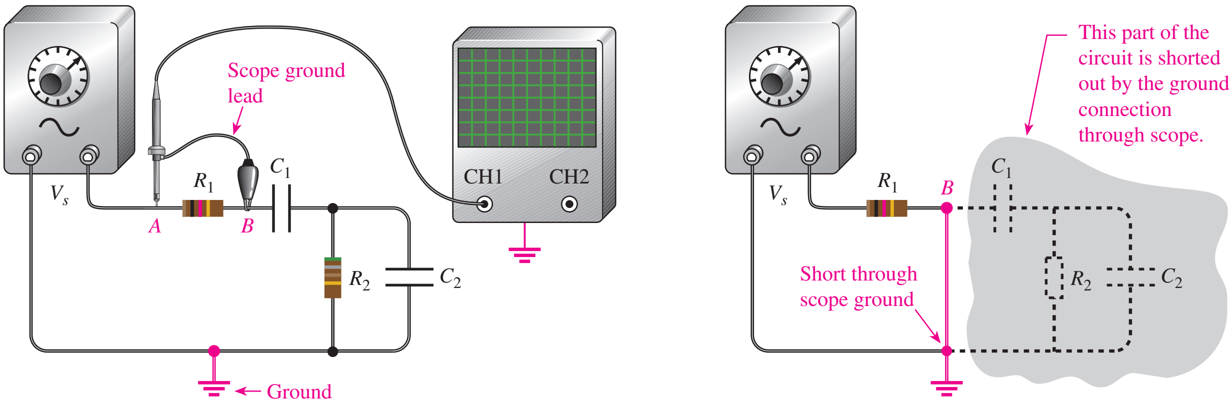
To measure the phase angle, the source voltage and the total current must be displayed on an oscilloscope screen in the proper time relationship. Two basic types of scope probes are available to measure the quantities with an oscilloscope: the voltage probe and the current probe. The current probe is a convenient device, but it is often not as readily available as a voltage probe. We will confine our phase measurement technique to the use of voltage probes in conjunction with the oscilloscope. Although there are special isolation methods, a typical oscilloscope voltage probe has two points that are connected to the circuit: the probe tip and the ground lead. Thus, all voltage measurements must be referenced to ground.

Since only voltage probes are to be used, the total current cannot be measured directly. However, for phase measurement, the voltage across R_1 is in phase with the total current and can be used to establish the phase angle of the current.

Before proceeding with the actual phase measurement, there is a problem with displaying V_{R1}. If the scope probe is connected across the resistor, as indicated in Figure 15–56(a), the ground lead of the scope will short point B to ground, thus bypassing the rest of the components and effectively removing them from the circuit electrically, as illustrated in Figure 15–56(b) (assuming that the scope is not isolated from power line ground).

To avoid this problem, you can switch the generator output terminals so that one end of R_1 is connected to the ground terminal, as shown in Figure 15–57(a). Now the scope can be connected across it to display V_{R1}, as indicated in Figure 15–57(b). The other probe is connected across the voltage source to display V_s as indicated. Now channel 1 of the scope has V_{R1} as an input, and channel 2 has V_s. The scope should be triggered from the source voltage (channel 2 in this case).

Before connecting the probes to the circuit, you should align the two horizontal lines (traces) so that they appear as a single line across the center of the scope screen. To do so, ground the probe tips and adjust the vertical position knobs to move the traces toward the center line of the screen until they are superimposed. This procedure

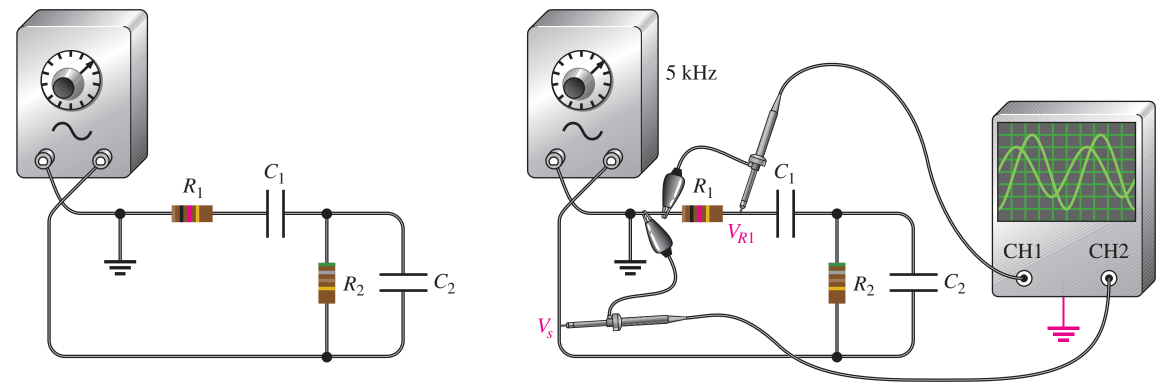


(a) Ground lead on scope probe grounds point *B*.

(b) The effect of grounding point *B* is to short out the rest of the circuit.

▲ **FIGURE 15–56**

Effects of measuring directly across a component when the instrument and the circuit are grounded.



(a) Ground repositioned so that one end of R_1 is grounded.

(b) The scope displays V_{R1} and V_s. V_{R1} represents the phase of the total current.

▲ **FIGURE 15–57**

Repositioning ground so that a direct voltage measurement can be made with respect to ground without shorting out part of the circuit.

ensures that both waveforms have the same zero crossing so that an accurate phase measurement can be made.

Once you have stabilized the waveforms on the scope screen, you can measure the period of the source voltage. Next, use the Volts/Div controls to adjust the amplitudes of the waveforms until they both appear to have the same amplitude. Now, spread the waveforms horizontally by using the Sec/Div control to expand the distance between them. This horizontal distance represents the time between the two waveforms. The number of divisions between the waveforms along any horizontal lines times the Sec/Div setting is equal to the time between them, Δt. Also, you can use the cursors to determine Δt if your oscilloscope has this feature.

Once you have determined the period, T, and the time between the waveforms, Δt, you can calculate the phase angle with the following equation:

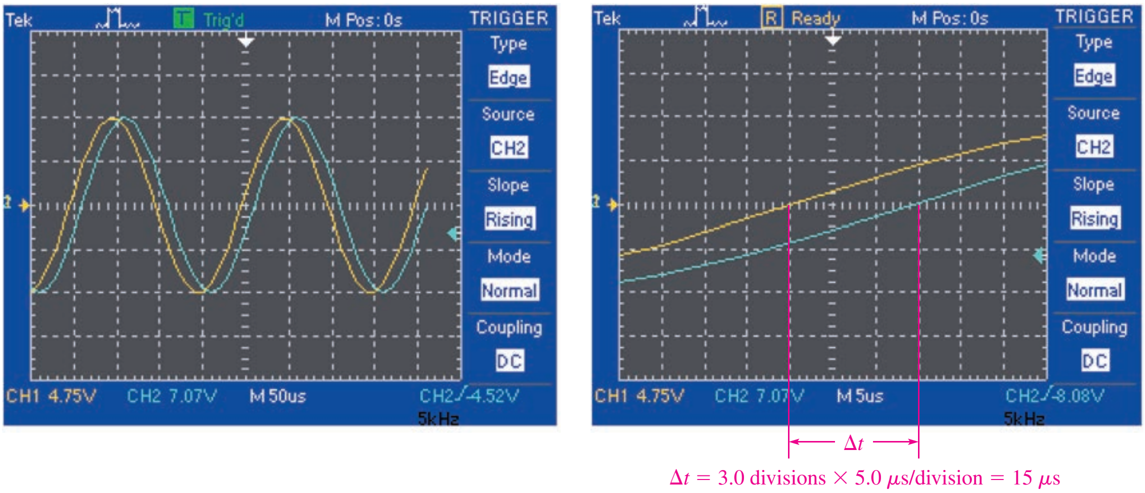
$$\theta = \left(\frac{\Delta t}{T}\right)360^\circ \qquad \text{Equation 15–27}$$

Figure 15–58 shows simulated screen displays for an oscilloscope in Multisim. In Figure 15–58(a), the waveforms are aligned and set to the same apparent amplitude by adjusting the fine Volts/Div control. The period of these waveforms is 200 μs. The Sec/Div control is adjusted to spread the waveforms out to read Δt more accurately. As shown in part (b), there are 3.0 divisions between the crossings on the center line. The Sec/Div control is set t 5.0 μs and there are 3.0 divisions between the waveforms.

$$\Delta t = 3.0 \text{ divisions} \times 5.0\ \mu\text{s/division} = 15\ \mu\text{s}$$

The phase angle is

$$\theta = \left(\frac{\Delta t}{T}\right)360^\circ = \left(\frac{15\ \mu\text{s}}{200\ \mu\text{s}}\right)360^\circ = 27^\circ$$



(a) (b)

▲ **FIGURE 15–58**

Determining the phase angle on the oscilloscope.

SECTION 15–7 CHECKUP

1. What is the equivalent series *RC* circuit for the series-parallel circuit in Figure 15–52?
2. What is the total impedance in polar form of the circuit in Figure 15–53?

OPTION 2 NOTE

Coverage of series-parallel reactive circuits continues in Chapter 16, Part 3, on page 747.

Special Topics

Part 4

15–8 Power in *RC* Circuits

In a purely resistive ac circuit, all of the energy delivered by the source is dissipated in the form of heat by the resistance. In a purely capacitive ac circuit, all of the energy delivered by the source is stored by the capacitor during a portion of the voltage cycle and then returned to the source during another portion of the cycle so that there is no net energy conversion to heat. When there is both resistance and capacitance, some of the energy is alternately stored and returned by the capacitance and some is dissipated by the resistance during each alteration. The amount of energy converted to heat is determined by the relative values of the resistance and the capacitive reactance.

After completing this section, you should be able to

- **Determine power in *RC* circuits**
  - Explain true and reactive power
  - Draw the power triangle
  - Define *power factor*
  - Explain apparent power
  - Calculate power in an *RC* circuit

When the resistance in a series *RC* circuit is greater than the capacitive reactance, more of the total energy delivered by the source is converted to heat by the resistance than is stored by the capacitor. Likewise, when the reactance is greater than the resistance, more of the total energy is stored and returned than is converted to heat.

The formulas for power in a resistor, sometimes called *true power* (P_{true}), and the power in a capacitor, called *reactive power* (P_r), are restated here. The unit of true power is the watt, and the unit of reactive power is the VAR (volt-ampere reactive).

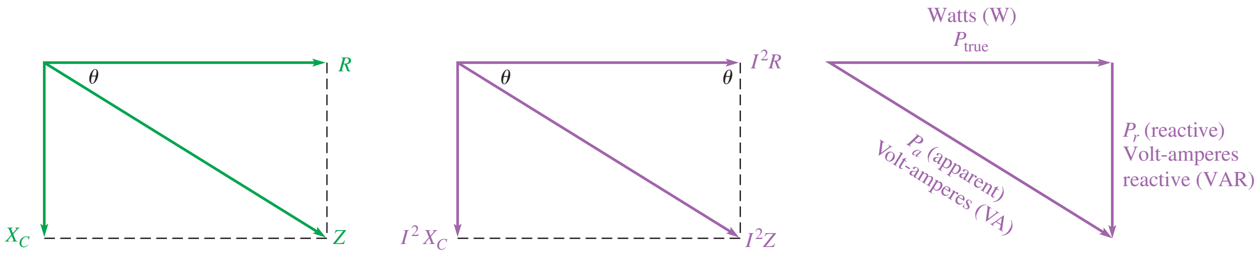
$$P_{true} = I^2R$$ **Equation 15–28**

$$P_r = I^2X_C$$ **Equation 15–29**

The rms value of current is used for these equations in order for the power to be in watts.

Power Triangle for *RC* Circuits

The generalized impedance phasor diagram for a series *RC* circuit is shown in Figure 15–59(a). A phasor relationship for the powers can also be represented by a similar diagram because the respective magnitudes of the powers, P_{true} and P_r, differ from R and X_C by a factor of I^2. This is shown in Figure 15–59(b).



(a) Impedance phasors (b) Power phasors (c) Power triangle

▲ FIGURE 15–59

Development of the power triangle for a series *RC* circuit.

The resultant power phasor, I^2Z, represents the **apparent power P_a**. At any instant in time P_a is the total power that appears to be transferred between the source and the *RC* circuit. The unit of apparent power is the volt-ampere, VA. The expression for apparent power is

Equation 15–30

$$P_a = I^2Z$$

The power phasor diagram in Figure 15–59(b) can be rearranged in the form of a right triangle, as shown in Figure 15–59(c). This is called the *power triangle.* Using the rules of trigonometry, P_{true} can be expressed as

$$P_{true} = P_a \cos\theta$$

Since P_a equals I^2Z or VI, the equation for the true power dissipation in an *RC* circuit can be written as

Equation 15–31

$$P_{true} = VI\cos\theta$$

where V is the applied voltage and I is the total current.

For the case of a purely resistive current, $\theta = 0°$ and $\cos 0° = 1$, so P_{true} equals VI. For the case of a purely capacitive circuit, $\theta = 90°$ and $\cos 90° = 0$, so P_{true} is zero. As you already know, there is no power dissipation in an ideal capacitor.

Power Factor

The term $\cos\theta$ is called the **power factor** and is stated as

Equation 15–32

$$PF = \cos\theta$$

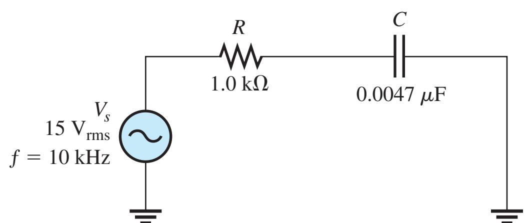
As the phase angle between applied voltage and total current increases, the power factor decreases, indicating an increasingly reactive circuit. The smaller the power factor, the smaller the power dissipation.

The power factor can vary from 0 for a purely reactive circuit to 1 for a purely resistive circuit. In an *RC* circuit, the power factor is referred to as a leading power factor because the current leads the voltage.

EXAMPLE 15–24

Determine the power factor and the true power in the circuit of Figure 15–60.

► FIGURE 15–60



Solution The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(10\text{ kHz})(0.0047\ \mu\text{F})} = 3.39\text{ k}\Omega$$

The total impedance of the circuit in rectangular form is

$$\mathbf{Z} = R - jX_C = 1.0\text{ k}\Omega - j3.39\text{ k}\Omega$$

Converting to polar form yields

$$\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$$

$$= \sqrt{(1.0\text{ k}\Omega)^2 + (3.39\text{ k}\Omega)^2}\angle -\tan^{-1}\left(\frac{3.39\text{ k}\Omega}{1.0\text{ k}\Omega}\right) = 3.53\angle -73.6^\circ\text{ k}\Omega$$

The angle associated with the impedance is θ, the angle between the applied voltage and the total current; therefore, the power factor is

$$PF = \cos\theta = \cos(-73.6^\circ) = \mathbf{0.283}$$

The current magnitude is

$$I = \frac{V_s}{Z} = \frac{15\text{ V}}{3.53\text{ k}\Omega} = 4.25\text{ mA}$$

The true power is

$$P_{\text{true}} = V_s I\cos\theta = (15\text{ V})(4.25\text{ mA})(0.283) = \mathbf{18.1\ mW}$$

Related Problem What is the power factor if f is reduced by half in Figure 15–60?

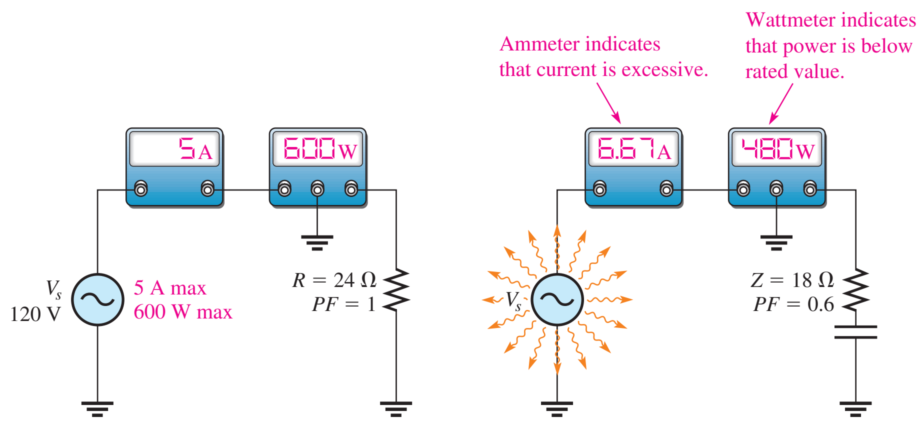
Significance of Apparent Power

As mentioned, apparent power is the power that appears to be transferred between the source and the load, and it consists of two components—a true power component and a reactive power component.

In all electrical and electronic systems, it is the true power that does work. The reactive power is simply shuttled back and forth between the source and load. Ideally, in terms of performing useful work, all of the power transferred to the load should be true power and none of it reactive power. However, in most practical situations the load has some reactance associated with it, and therefore you must deal with both power components.

In Chapter 14, the use of apparent power in relation to transformers was discussed. For any reactive load, there are two components of the total current: the resistive component and the reactive component. If you consider only the true power (watts) in a load, you are dealing with only a portion of the total current that the load demands from a source. In order to have a realistic picture of the actual current that a load will draw, you must consider apparent power (VA).

A source such as an ac generator can provide current to a load up to some maximum value. If the load draws more than this maximum value, the source can be damaged. Figure 15–61(a) shows a 120 V generator that can deliver a maximum current of 5 A to a load. Assume that the generator is rated at 600 W and is connected to a purely resistive load of 24 Ω (power factor of 1). The ammeter shows that the current is 5 A, and the wattmeter indicates that the power is 600 W. The generator has no problem under these conditions, although it is operating at maximum current and power.



(a) Generator operating at its limits with a resistive load.

(b) Generator is in danger of internal damage due to excess current, even though the wattmeter indicates that the power is below the maximum wattage rating.

▲ **FIGURE 15–61**

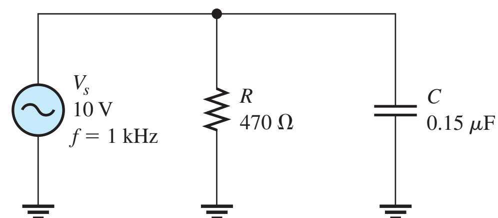
Wattage rating of a source is inappropriate when the load is reactive. The rating should be in VA rather than in watts.

Now, consider what happens if the load is changed to a reactive one with an impedance of 18 Ω and a power factor of 0.6, as indicated in Figure 15–61(b). The current is 120 V/18 Ω = 6.67 A, which *exceeds* the maximum. Even though the wattmeter reads 480 W, which is less than the power rating of the generator, the excessive current probably will cause damage. This illustration shows that a true power rating can be deceiving and is inappropriate for ac sources. The ac generator should be rated at 600 VA, a rating that manufacturers generally use, rather than 600 W.

EXAMPLE 15–25

For the circuit in Figure 15–62, find the true power, the reactive power, and the apparent power.

▶ **FIGURE 15–62**



Solution The capacitive reactance and currents through R and C are

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1{,}000\text{ Hz})(0.15\ \mu\text{F})} = 1{,}061\ \Omega$$

$$I_R = \frac{V_s}{R} = \frac{10\text{ V}}{470\ \Omega} = 21.3\text{ mA}$$

$$I_C = \frac{V_s}{X_C} = \frac{10\text{ V}}{1{,}061\ \Omega} = 9.43\text{ mA}$$

The true power is

$$P_{\text{true}} = I_R^2 R = (21.3\text{ mA})^2(470\ \Omega) = \mathbf{213\ mW}$$

The reactive power is

$$P_r = I_C^2 X_C = (9.43\text{ mA})^2(1{,}061\ \Omega) = \mathbf{94.3\ mVAR}$$

The apparent power is

$$P_a = \sqrt{P_{\text{true}}^2 + P_r^2} = \sqrt{(213\text{ mW})^2 + (94.3\text{ mVAR})^2} = \mathbf{233\ mVA}$$

Related Problem What is the true power in Figure 15–62 if the frequency is changed to 2 kHz?

SECTION 15–8 CHECKUP

1. To which component in an *RC* circuit is the power dissipation due?
2. The phase angle, θ, is 45°. What is the power factor?
3. A certain series *RC* circuit has the following parameter values: $R = 330\ \Omega$, $X_C = 460\ \Omega$, and $I = 2\text{ A}$. Determine the true power, the reactive power, and the apparent power.

15–9 BASIC APPLICATIONS

RC circuits are found in a variety of applications, often as part of a more complex circuit. Three applications are phase shift oscillators, frequency-selective circuits (filters), and ac coupling.

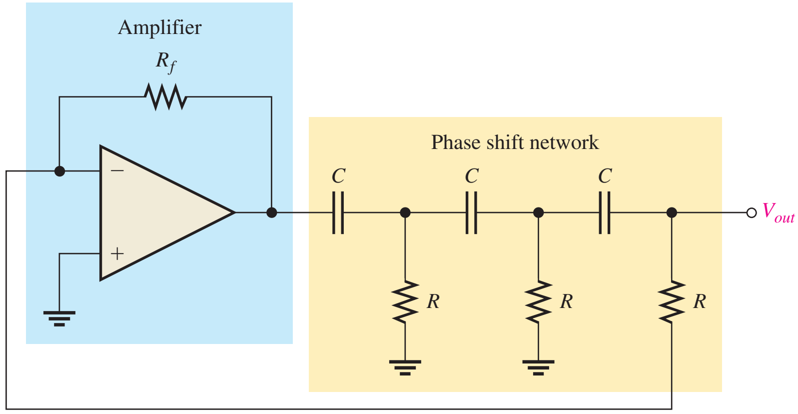
After completing this section, you should be able to

- **Discuss some basic *RC* applications**
  - Discuss how the *RC* circuit is used as an oscillator
  - Discuss how the *RC* circuit operates as a filter
  - Discuss ac coupling

The Phase Shift Oscillator

As you know, a series *RC* circuit will shift the phase of the output voltage by an amount that depends on the values of *R* and *C* and the frequency of the signal. This ability to shift phase depending on frequency is vital in certain feedback oscillator circuits. An **oscillator** is a circuit that generates a periodic waveform and is an important circuit for many electronic systems. You will study oscillators in a devices course, so the focus here is on the application of *RC* circuits for shifting phase. The requirement is that a fraction of the output of the oscillator is returned to the input (called "feedback") in the proper phase to reinforce the input and sustain oscillations. Generally, the requirement is to feed back the signal with a total of 180° of phase shift.

A single *RC* circuit is limited to phase shifts that are smaller than 90°. The basic *RC* lag circuit discussed in Section 15–4 can be "stacked" to form a complex *RC* network as shown in Figure 15–63, which shows a specific circuit called a phase-shift oscillator. The phase shift oscillator typically uses three equal-component *RC* circuits that produce the required 180° phase shift at a certain frequency, which will be the frequency at which the oscillator works. The output of the amplifier is phase shifted by the *RC* network and returned to the input of the amplifier, which provides sufficient gain to maintain oscillations.



▲ FIGURE 15–63

Phase shift oscillator.

The process of putting several *RC* circuits together results in a loading effect, so the overall phase shift is not the same as simply adding the phase shifts of the individual *RC* circuits. The detailed calculation for this circuit is shown in Appendix B. With equal components, the frequency at which a 180° phase shift occurs is given by the equation

Equation 15–33

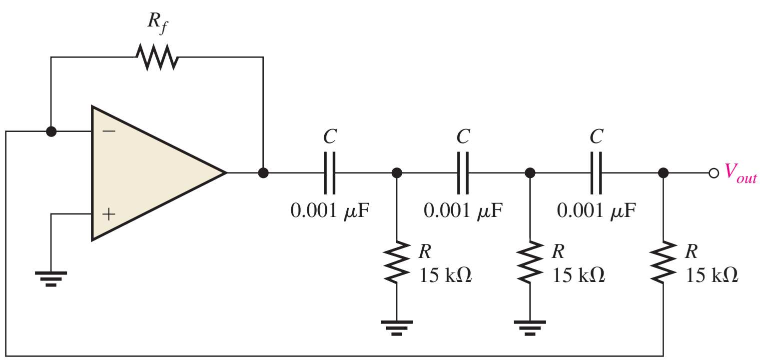
$$f_r = \frac{1}{2\pi\sqrt{6}\,RC}$$

It also turns out that the *RC* network attenuates (reduces) the signal from the amplifier by a factor of 29; the amplifier must make up for this attenuation by having a gain of −29 to sustain oscillation (the minus sign takes into account the phase shift).

EXAMPLE 15–26

In Figure 15–64, calculate the output frequency.

► FIGURE 15–64



Solution

$$f_r = \frac{1}{2\pi\sqrt{6}RC} = \frac{1}{2\pi\sqrt{6}\,(15\text{ k}\Omega)(0.001\ \mu\text{F})} = \mathbf{4.33\ kHz}$$

Related Problem If all of the capacitors are changed to 0.0027 μF, what is the oscillator frequency?

The *RC* Circuit as a Filter

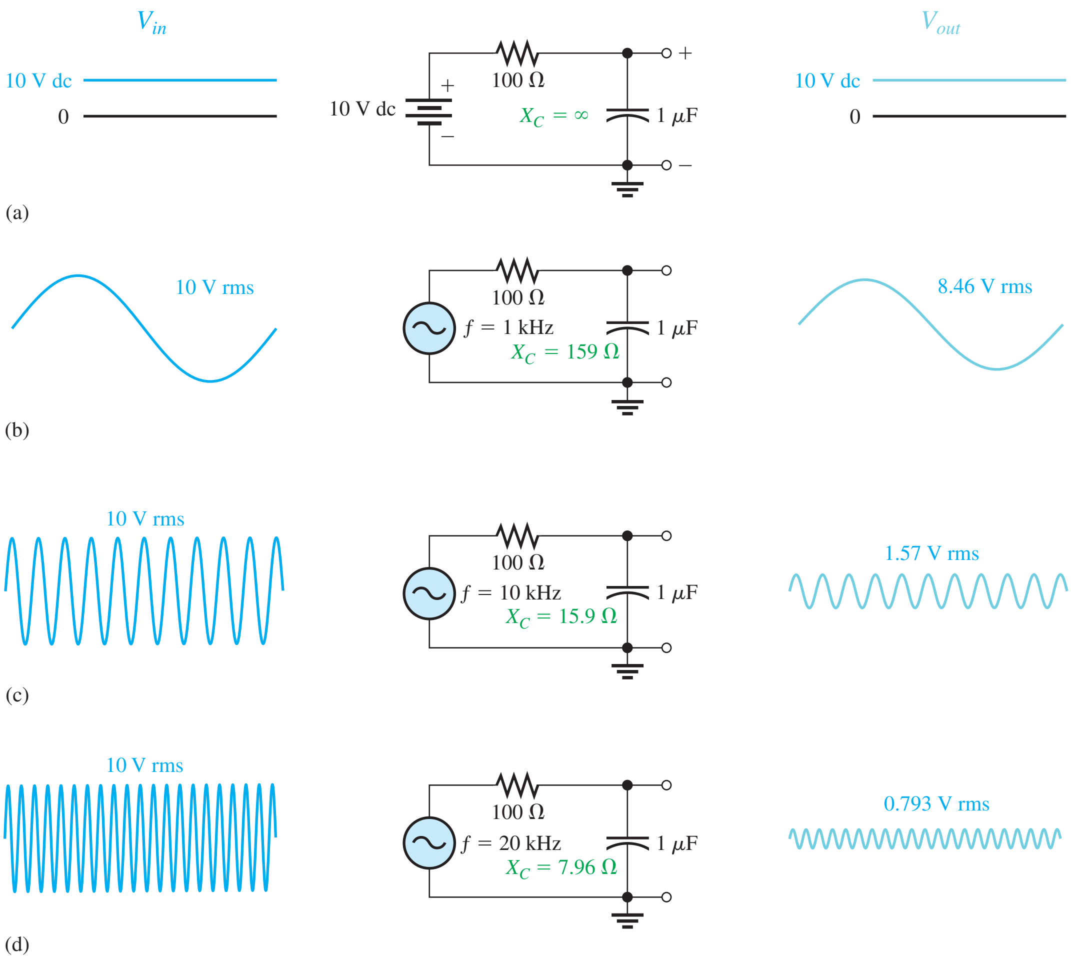
Filters are frequency-selective circuits that permit signals of certain frequencies to pass from the input to the output while blocking all others. That is, all frequencies but the selected ones are filtered out. Filters are covered in greater depth in Chapter 18 but are introduced here as an application example.

Series *RC* circuits exhibit a frequency-selective characteristic and therefore act as basic filters. There are two types. The first one that we examine, called a **low-pass filter**, is realized by taking the output across the capacitor, just as in a lag circuit. The second type, called a **high-pass filter**, is implemented by taking the output across the resistor, as in a lead circuit.

Low-Pass Filter You have already seen what happens to the output magnitude and phase angle in a lag circuit. In terms of its filtering action, we are interested primarily in the variation of the output magnitude with frequency.

Figure 15–65 shows the filtering action of a series *RC* circuit using specific values for illustration. In part (a) of the figure, the input is zero frequency (dc). Since the capacitor blocks constant direct current, the output voltage equals the full value of the input voltage because there is no voltage dropped across *R*. Therefore, the circuit passes all of the input voltage to the output (10 V in, 10 V out).

In Figure 15–65(b), the frequency of the input voltage has been increased to 1 kHz, causing the capacitive reactance to decrease to 159 Ω. For an input voltage of 10 V rms,



▲ FIGURE 15–65

Low-pass filtering action (phase shifts are not indicated).

the output voltage is approximately 8.46 V rms, which can be calculated using the voltage-divider approach or Ohm's law.

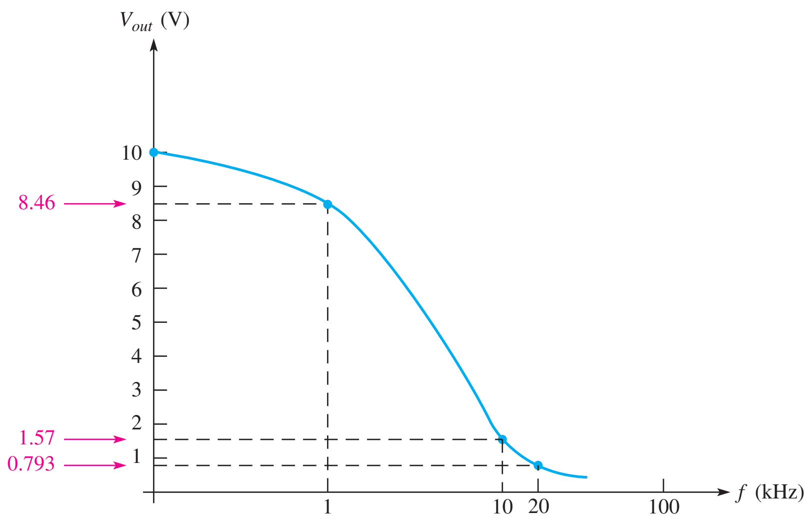
In Figure 15–65(c), the input frequency has been increased to 10 kHz, causing the capacitive reactance to decrease further to 15.9 Ω. For a constant input voltage of 10 V rms, the output voltage is now 1.57 V rms.

As the input frequency is increased further, the output voltage continues to decrease and approaches zero as the frequency becomes very high, as shown in Figure 15–65(d).

A description of the circuit action is as follows: As the frequency of the input increases, the capacitive reactance decreases. Because the resistance is constant and the capacitive reactance decreases, the voltage across the capacitor (output voltage) also decreases according to the voltage-divider principle. The input frequency can be increased until it reaches a value at which the reactance is so small compared to the resistance that the output voltage can be neglected because it is very small compared to the input voltage. At this value of frequency, the circuit is essentially completely blocking the input signal.

As shown in Figure 15–65, the circuit passes dc (zero frequency) completely. As the frequency of the input increases, less of the input voltage is passed through to the output; that is, the output voltage decreases as the frequency increases. It is apparent that the lower frequencies pass through the circuit much better than the higher frequencies. This *RC* circuit is therefore a very basic form of low-pass filter.

The **frequency response** of the low-pass filter circuit in Figure 15–65 is shown in Figure 15–66 with a graph of output voltage magnitude versus frequency. This graph, called a *response curve,* indicates that the output decreases as the frequency increases. Notice that the frequency scale is logarithmic, which is typically the way frequency responses are drawn for filters.



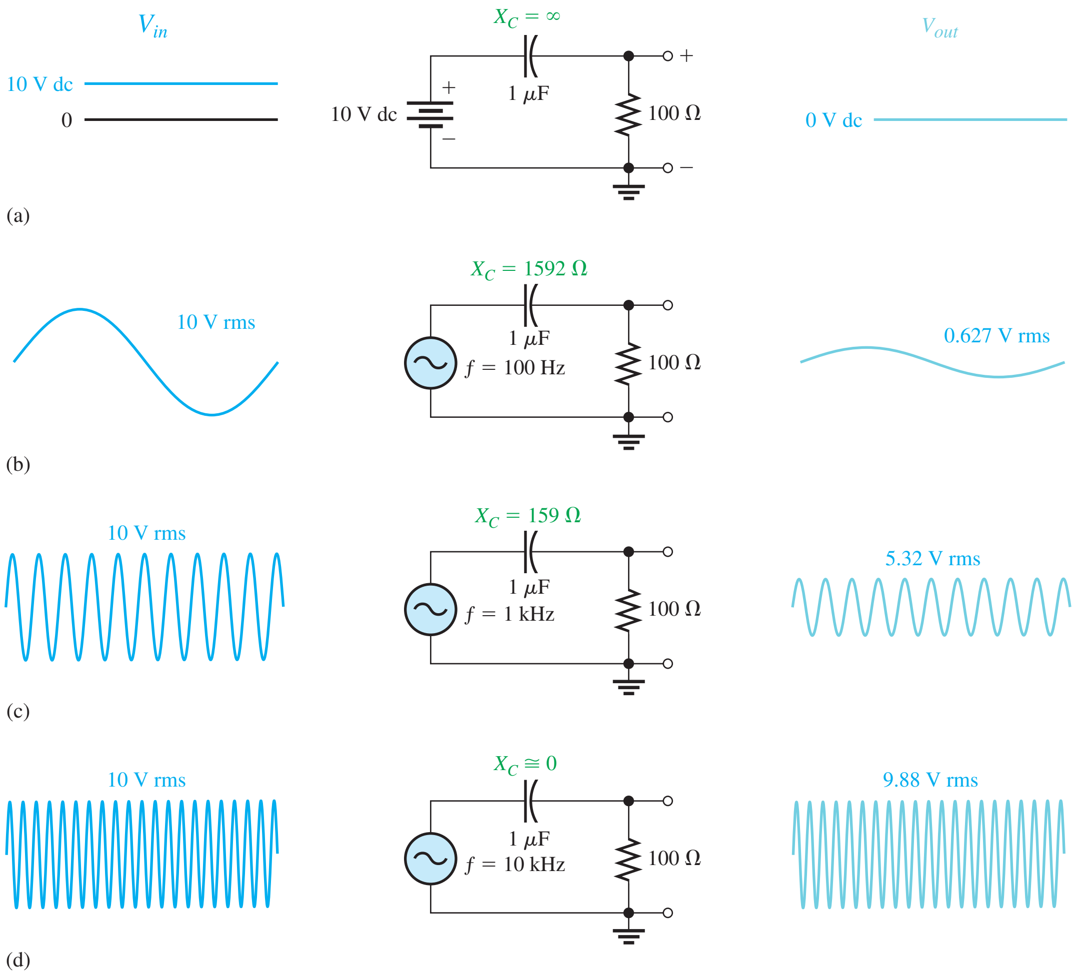
▲ **FIGURE 15–66**

Frequency response curve for the low-pass filter in Figure 15–65.

High-Pass Filter Figure 15–67 illustrates high-pass filtering action, where the output is taken across the resistor, just as in a lead circuit. When the input voltage is dc (zero frequency) in part (a), the output is zero volts because the capacitor blocks direct current; therefore, no voltage is developed across *R.*

In Figure 15–67(b), the frequency of the input signal has been increased to 100 Hz with an rms value of 10 V. The output voltage is 0.627 V rms. Thus, only a small percentage of the input voltage appears on the output at this frequency.

In Figure 15–67(c), the input frequency is increased further to 1 kHz, causing more voltage to be developed across the resistor because of the further decrease in the capacitive reactance. The output voltage at this frequency is 5.32 V rms. As you can see,



▲ **FIGURE 15–67**

High-pass filtering action (phase shifts are not indicated).

the output voltage increases as the frequency increases. A value of frequency is reached at which the reactance is negligible compared to the resistance, and most of the input voltage appears across the resistor, as shown in Figure 15–67(d).

As illustrated, this circuit tends to prevent lower frequencies from appearing on the output but allows higher frequencies to pass through from input to output. Therefore, this *RC* circuit is a basic form of high-pass filter.

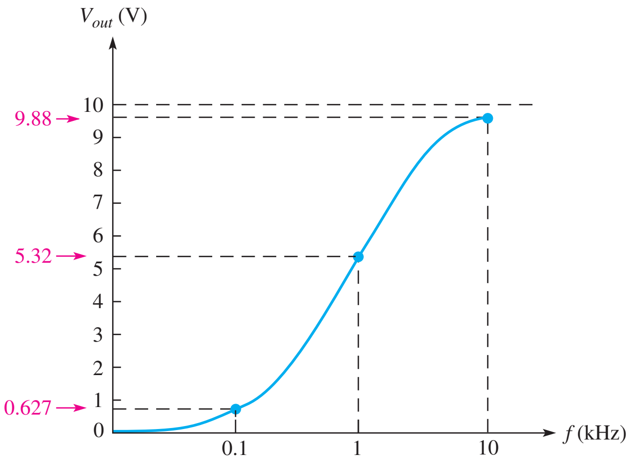
The frequency response of the high-pass filter circuit in Figure 15–67 is shown in Figure 15–68 with a graph of output voltage magnitude versus frequency. This response curve shows that the output increases as the frequency increases and then levels off and approaches the value of the input voltage.

The Cutoff Frequency and the Bandwidth of a Filter The frequency at which the capacitive reactance equals the resistance in a low-pass or high-pass *RC* filter is called the **cutoff frequency** and is designated f_c. This condition is expressed as $1/(2\pi f_c C) = R$. Solving for f_c results in the following formula:

$$f_c = \frac{1}{2\pi RC}$$

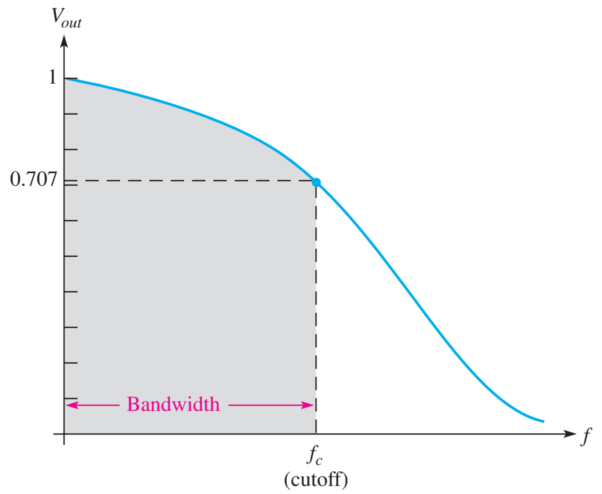
Equation 15–34

At f_c, the output voltage of the filter is 70.7% of its maximum value. It is standard practice to consider the cutoff frequency as the limit of a filter's performance in terms



▲ **FIGURE 15–68**

Frequency response curve for the high-pass filter in Figure 15–67.



▲ **FIGURE 15–69**

Normalized general response curve of a low-pass filter showing the cutoff frequency and the bandwidth.

of passing or rejecting frequencies. For example, in a high-pass filter, all frequencies above f_c are considered to be passed by the filter, and all those below f_c are considered to be rejected. The reverse is true for a low-pass filter.

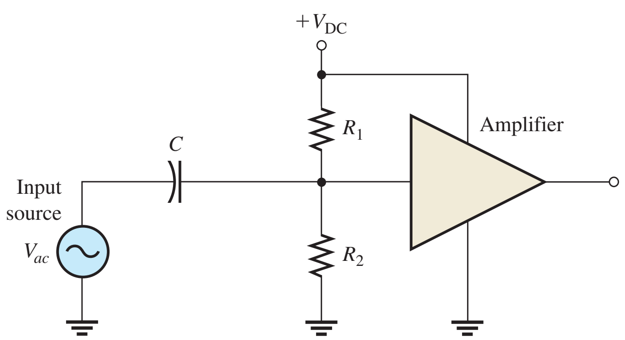
The range of frequencies that is considered to be passed by a filter is called the **bandwidth**. Figure 15–69 illustrates the bandwidth and the cutoff frequency for a low-pass filter.

Coupling an AC Signal into a DC Bias Circuit

Figure 15–70 shows an *RC* circuit that is used to create a dc voltage level with an ac voltage superimposed on it. This type of circuit is commonly found in amplifiers in which the dc voltage is required to **bias** the amplifier to the proper operating point and the signal voltage to be amplified is coupled through a capacitor and superimposed on the dc level. The capacitor prevents the low internal resistance of the signal source from affecting the dc bias voltage.

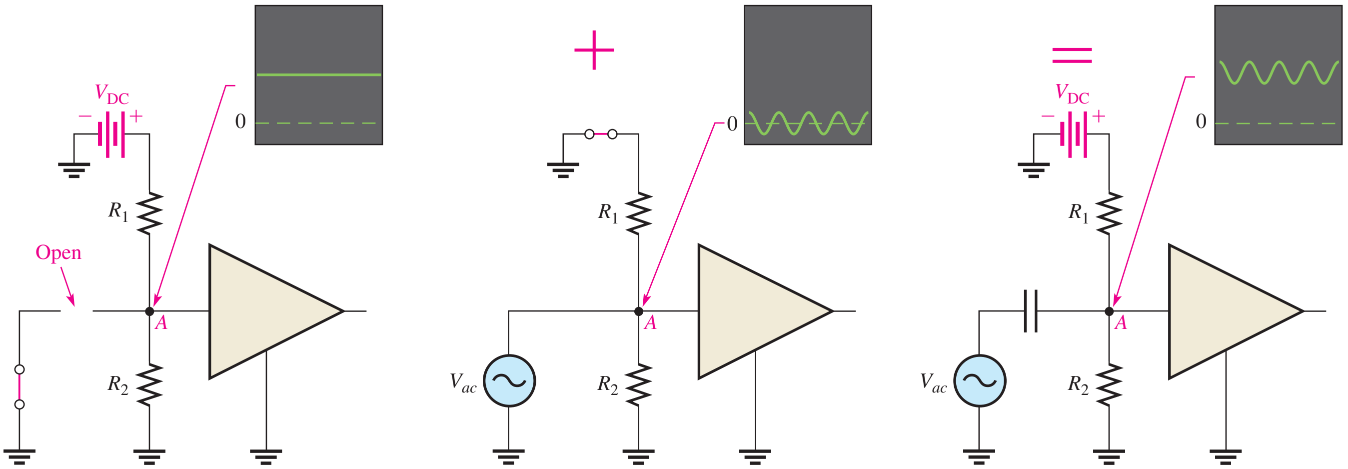
▶ **FIGURE 15–70**

Amplifier bias and signal-coupling circuit.



In this type of application, a relatively large value of capacitance is selected so that for the frequencies to be amplified, the reactance is very small compared to the resistance of the bias network. When the reactance is very small (ideally zero), there is practically no phase shift or signal voltage dropped across the capacitor. Therefore, all of the signal voltage passes from the source to the input of the amplifier.

Figure 15–71 illustrates the application of the superposition principle to the circuit in Figure 15–70. In part (a), the ac source has been effectively removed from the circuit and replaced with a short to represent its ideal internal resistance (actual generators typically have 50 Ω or 600 Ω of internal resistance). Since C is open to dc, the voltage at point A is determined by the voltage-divider action of R_1 and R_2 and the dc voltage source.



(a) dc equivalent: ac source replaced by short. C is open to dc. R_1 and R_2 act as dc voltage divider.

(b) ac equivalent: dc source is replaced by short. C is short to ac. All of V_{ac} is coupled to point A.

(c) dc + ac: Voltages are superimposed at point A.

▲ **FIGURE 15–71**

The superposition of dc and ac voltages in an *RC* bias and coupling circuit.

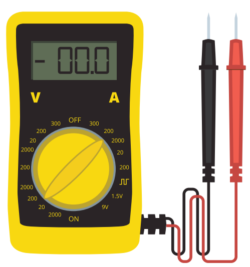
In Figure 15–71(b), the dc source has been effectively removed from the circuit and replaced with a short to represent its ideal internal resistance. Since C appears as a short at the frequency of the ac, the signal voltage is coupled directly to point A and appears across the parallel combination of R_1 and R_2.

Figure 15–71(c) illustrates that the combined effect of the superposition of the dc and the ac voltages results in the signal voltage "riding" on the dc level.

SECTION 15–9 CHECKUP

1. How much phase shift is produced by the *RC* circuit in a phase shift oscillator?
2. When an *RC* circuit is used as a low-pass filter, across which component is the output taken?

15–10 Troubleshooting

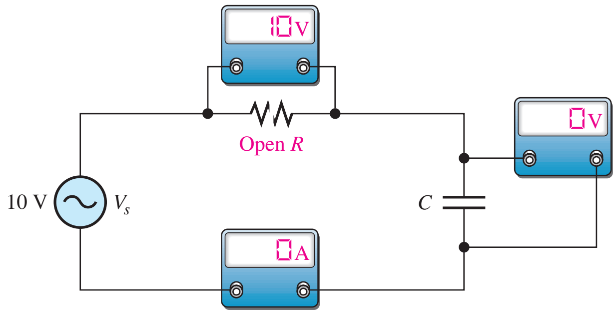


Typical component failures or degradation have an effect on the frequency response of basic *RC* circuits.

After completing this section, you should be able to

- **Troubleshoot *RC* circuits**
  - Find an open resistor or open capacitor
  - Find a shorted capacitor
  - Find a leaky capacitor

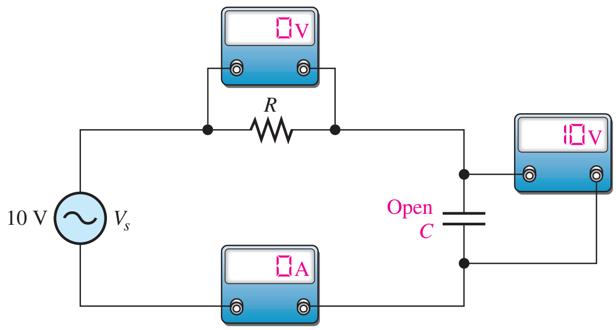
Effect of an Open Resistor It is easy to see how an open resistor affects the operation of a basic series *RC* circuit, as shown in Figure 15–72. Obviously, there is no path for current, so the capacitor voltage remains at zero; thus, the total voltage, V_s, appears across the open resistor.



▶ **FIGURE 15–72**

Effect of an open resistor.

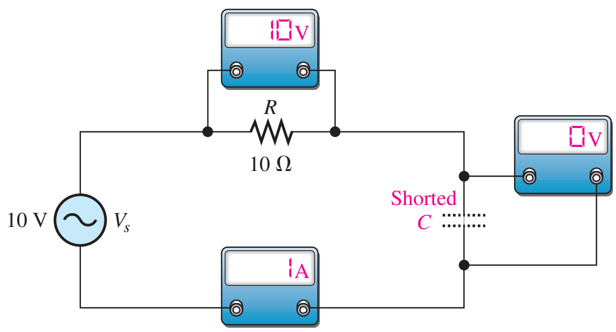
Effect of an Open Capacitor When the capacitor is open, there is no current; thus, the resistor voltage remains at zero. The total source voltage is across the open capacitor, as shown in Figure 15–73.



▶ **FIGURE 15–73**

Effect of an open capacitor.

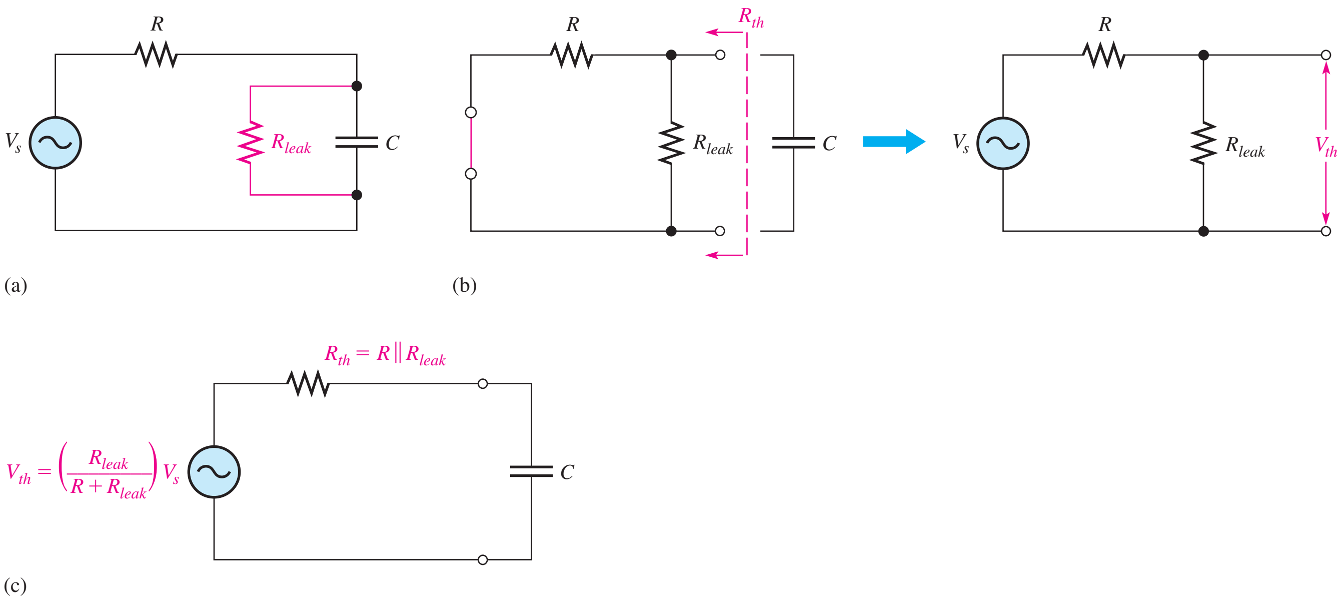
Effect of a Shorted Capacitor Capacitors rarely short, although SMD (surface-mount devices) multi-layer ceramic capacitors can crack, which over time can create a dc short due to dendritic growth between the internal electrodes (plates). In case of a short, the voltage across it is zero, the current equals V_s/R, and the total voltage appears across the resistor, as shown in Figure 15–74.



▶ **FIGURE 15–74**

Effect of a shorted capacitor.

Effect of a Leaky Capacitor When a large electrolytic capacitor exhibits a high leakage current, the leakage resistance effectively appears in parallel with the capacitor, as shown in Figure 15–75(a). When the leakage resistance is comparable in value to the circuit resistance, *R*, the circuit response is drastically affected. The circuit, looking from the capacitor toward the source, can be thevenized, as shown in Figure 15–75(b). The Thevenin equivalent resistance is *R* in parallel with R_{leak} (the source appears as a



▲ FIGURE 15–75

Effect of a leaky capacitor.

short), and the Thevenin equivalent voltage is determined by the voltage-divider action of R and R_{leak}.

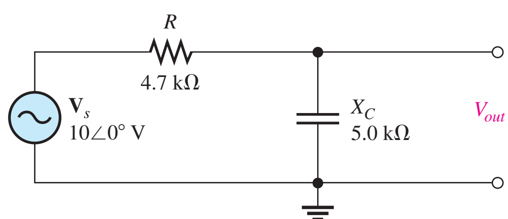
$$R_{th} = R \| R_{leak} = \frac{RR_{leak}}{R + R_{leak}}$$

$$V_{th} = \left(\frac{R_{leak}}{R + R_{leak}}\right) V_s$$

As you can see, the voltage across the capacitor is reduced since $V_{th} < V_s$. Also, the circuit time constant is reduced, and the current is increased. The Thevenin equivalent circuit is shown in Figure 15–75(c).

EXAMPLE 15–27

Assume that the capacitor in Figure 15–76 is degraded to a point where its leakage resistance is 10 kΩ. Determine the output voltage under the degraded condition.



▲ FIGURE 15–76

Solution The effective circuit resistance is

$$R_{th} = \frac{RR_{leak}}{R + R_{leak}} = \frac{(4.7\text{ k}\Omega)(10\text{ k}\Omega)}{14.7\text{ k}\Omega} = 3.20\text{ k}\Omega$$

To determine the output voltage, find the Thevenin equivalent voltage.

$$V_{th} = \left(\frac{R_{leak}}{R + R_{leak}}\right)V_s = \left(\frac{10\text{ k}\Omega}{14.7\text{ k}\Omega}\right)10\text{ V} = 6.80\text{ V}$$

Then,

$$V_{out} = \left(\frac{X_C}{\sqrt{R_{th}^2 + X_C^2}}\right)V_{th} = \left(\frac{5.0\text{ k}\Omega}{\sqrt{(3.2\text{ k}\Omega)^2 + (5.0\text{ k}\Omega)^2}}\right)6.80\text{ V} = \mathbf{5.73\text{ V}}$$

Related Problem What would the output voltage be if the capacitor were not leaky?

Other Troubleshooting Considerations

So far, you have learned about specific component failures and the associated voltage measurements. Many times, however, the failure of a circuit to work properly is not the result of a faulty component. A loose wire, a bad contact, or a poor solder joint can cause an open circuit. A short can be caused by a wire clipping or solder splash. Things as simple as not plugging in a power supply or a function generator happen more often than you might think. Wrong values in a circuit (such as an incorrect resistor value), the function generator set at the wrong frequency, or the wrong output connected to the circuit can cause improper operation.

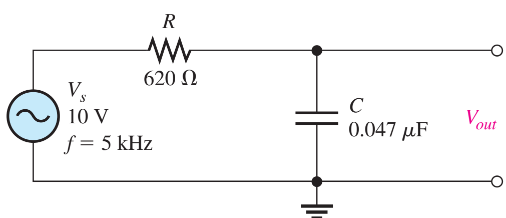
When you have problems with a circuit, always check to make sure that the instruments are properly connected to the circuits and to a power outlet. Also, look for obvious things such as a broken or loose contact, a connector that is not completely plugged in, or a piece of wire or a solder bridge that could be shorting something out.

The point is that you should consider all possibilities, not just faulty components, when a circuit is not working properly. The following example illustrates this approach with a simple circuit using the APM (analysis, planning, and measurement) method.

EXAMPLE 15–28

The circuit represented by the schematic in Figure 15–77 has no output voltage, which is the voltage across the capacitor. You expect to see about 7.4 V at the output. The circuit is physically constructed on a protoboard. Use your troubleshooting skills to find the problem.

► FIGURE 15–77



Solution Apply the APM method to this troubleshooting problem.

Analysis: First think of the possible causes for the circuit to have no output voltage.

1. There is no source voltage or the frequency is so high that the capacitive reactance is almost zero.

2. There is a short between the output terminals. Either the capacitor could be internally shorted, or there could be some physical short in the circuit.
3. There is an open between the source and the output. This would prevent current and thus cause the output voltage to be zero. The resistor could be open, or the conductive path could be open due to a broken or loose connecting wire or a bad protoboard contact.
4. There is an incorrect component value. The resistor could be so large that the current and, therefore, the output voltage are negligible. The capacitor could be so large that its reactance at the input frequency is near zero.

Planning: You decide to make some visual checks for problems such as the function generator power cord not plugged in or the frequency set at an incorrect value. Also, broken leads, shorted leads, as well as an incorrect resistor color code or capacitor label often can be found visually. If nothing is discovered after a visual check, then you will make voltage measurements to track down the cause of the problem. You decide to use a digital oscilloscope and a DMM to make the measurements.

Measurement: Assume that you find the function generator is plugged in and the frequency setting appears to be correct. Also, you find no visible opens or shorts during your visual check, and the component values are correct.

The first step in the measurement process is to check the voltage from the source with the scope. Assume a 10 V rms sine wave with a frequency of 5 kHz is observed at the circuit input as shown in Figure 15–78(a). The correct voltage is present, so *the first possible cause has been eliminated.*

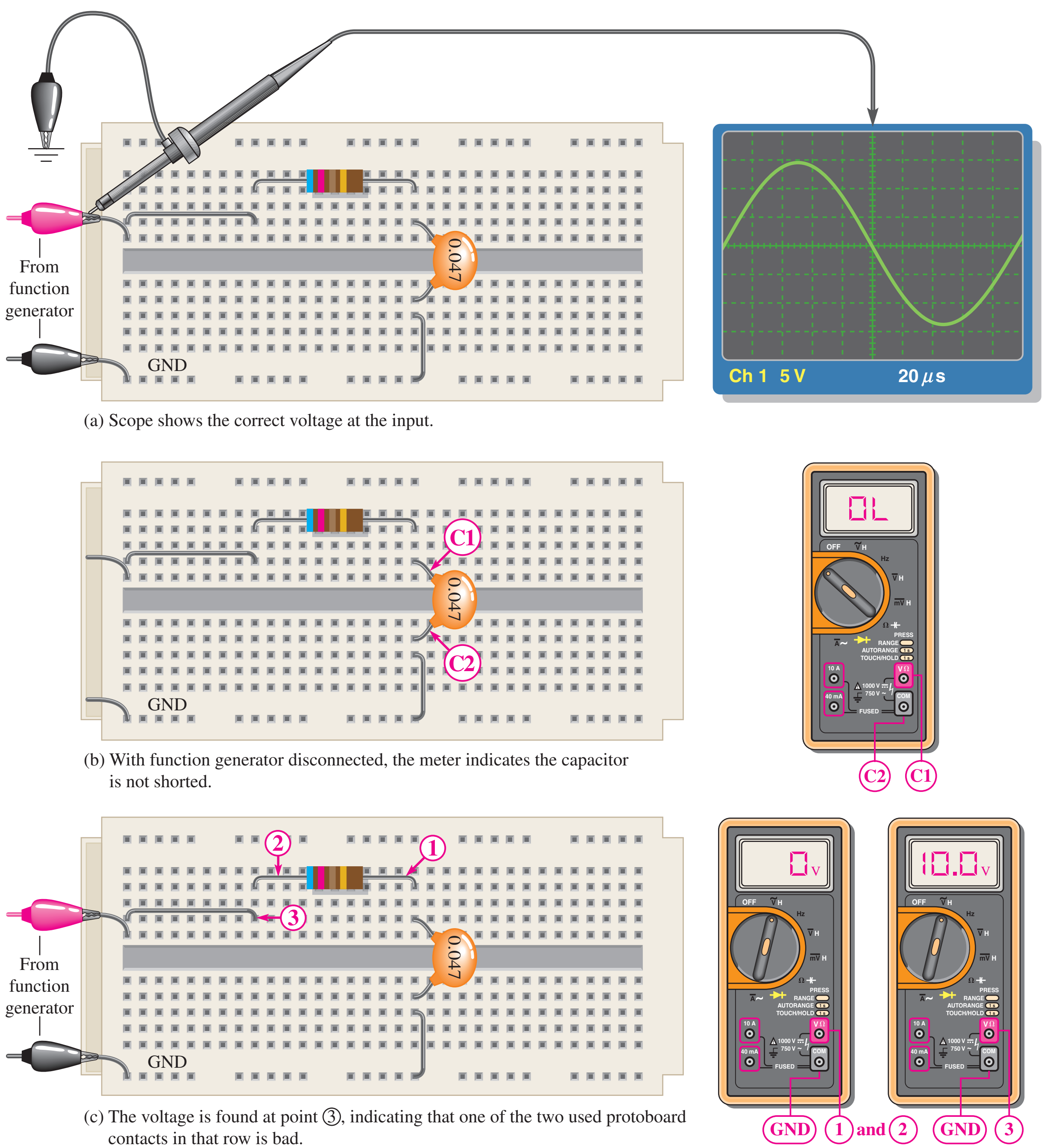
Next, check for a shorted capacitor by disconnecting the source and placing a DMM (set on the ohmmeter function) across the capacitor. If the capacitor is good, an open will be indicated by an OL (overload) in the meter display after a short charging time. Assume the ohmmeter indicates OL as shown in Figure 15–78(b). *The second possible cause has been eliminated.*

Since the voltage has been "lost" somewhere between the input and the output, you must now look for the voltage. Reconnect the source and measure the voltage across the resistor with the DMM (set on the voltmeter function) from one resistor lead to the other. The voltage across the resistor is zero. This means there is no current, which indicates an open somewhere in the circuit.

Now, begin tracing the circuit back toward the source looking for the voltage (you could also start from the source and work forward). You can use either the scope or the DMM but decide to use the multimeter with one lead connected to ground and the other used to probe the circuit. As shown in Figure 15–78(c), the voltage on the right lead of the resistor, point ①, reads zero. Since you already have measured zero voltage across the resistor, the voltage on the left resistor lead at point ② must be zero as the meter indicates. Next, moving the meter probe to point ③, you read 10 V. You have found the voltage! Since there is zero volts on the left resistor lead, and there is 10 V at point ③, one of the two contacts in the protoboard hole into which the wire leads are inserted is bad. It could be that the small contacts were pushed in too far and were bent or broken so that the circuit lead does not make contact.

Move either or both the resistor lead and the wire to another hole in the same row. Assume that when the resistor lead is moved to the hole just above, you have voltage at the output of the circuit (across the capacitor).

Related Problem Suppose you had measured 10 V across the resistor before the capacitor was checked. What would this have indicated?



(a) Scope shows the correct voltage at the input.

(b) With function generator disconnected, the meter indicates the capacitor is not shorted.

(c) The voltage is found at point (3), indicating that one of the two used protoboard contacts in that row is bad.

▲ FIGURE 15–78

SECTION 15–10 CHECKUP

1. Describe the effect of a leaky capacitor on the response of an *RC* circuit.
2. In a series *RC* circuit, if all of the applied voltage appears across the capacitor, what is the problem?
3. What faults can cause 0 V across a capacitor in a series *RC* circuit if the source is functioning properly?

Application Activity

In Chapter 12, you studied the capacitively coupled input to an amplifier with voltage-divider bias. In this application, the output voltage and phase lag of a similar amplifier's input circuit are examined to determine how they change with frequency. If too much voltage is dropped across the coupling capacitor, the overall performance of the amplifier is adversely affected.

As you learned in Chapter 12, the coupling capacitor (C_1) in Figure 15–79 passes the input signal voltage to the input of the amplifier (point A to point B) without affecting the dc level at point B produced by the resistive voltage divider (R_1 and R_2). If the input frequency is high enough so that the reactance of the coupling capacitor is negligibly small, essentially no ac signal voltage is dropped across the capacitor. As the signal frequency is reduced, the capacitive reactance increases and more of the signal voltage is dropped across the capacitor. This lowers the overall voltage gain of the amplifier and thus degrades its performance.

The amount of signal voltage that is coupled from the input source (point A) to the amplifier input (point B) is determined by the values of the capacitor and the dc bias resistors (assuming the amplifier has no loading effect) in Figure 15–79. These components actually form a high-pass RC filter, as shown in Figure 15–80. The voltage-divider bias resistors are effectively in parallel with each other as far as the ac source is concerned because the power supply has zero internal resistance. The lower end of R_2 goes to ground and

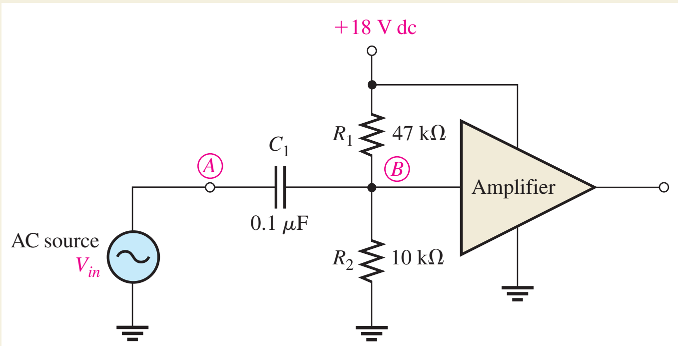


FIGURE 15–79

A capacitively coupled amplifier.

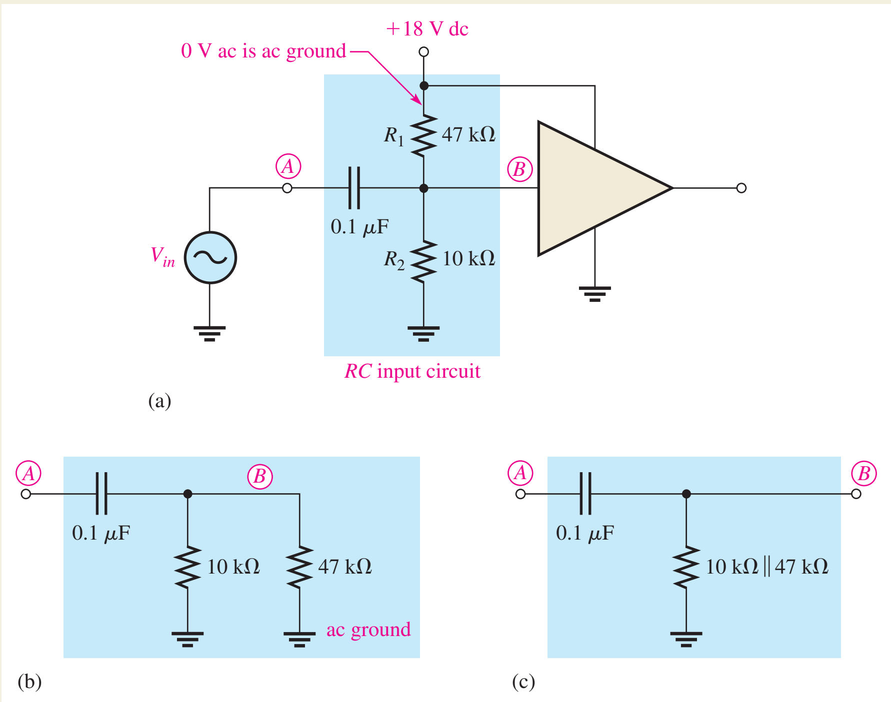
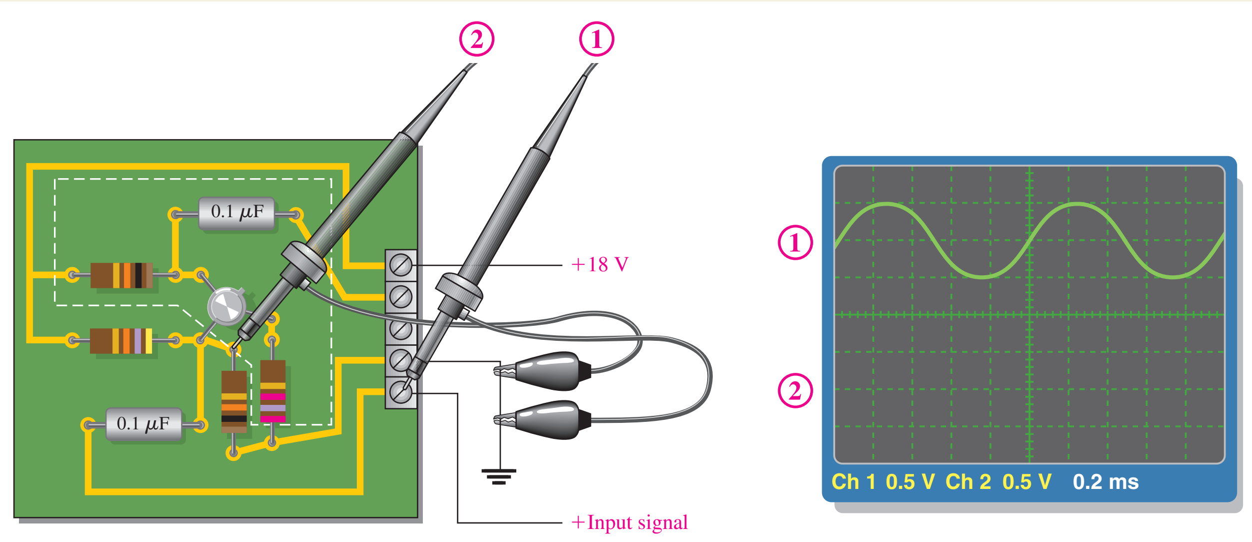


FIGURE 15–80

The RC input circuit acts effectively like a high-pass RC filter.



▲ FIGURE 15–81

Measuring the input circuit response at frequency f_1. Circled numbers relate scope inputs to the probes. The channel 1 waveform is shown.

the upper end of R_1 goes to the dc supply voltage as shown in Figure 15–80(a). Since there is no ac voltage at the +18 V dc terminal, the upper end of R_1 is at 0 V ac, which is referred to as *ac ground*. The development of the circuit into an effective high-pass *RC* filter is shown in parts (b) and (c).

The Amplifier Input Circuit

1. Determine the value of the equivalent resistance of the input circuit. Assume the amplifier (shown inside the white dashed lines in Figure 15–81) has no loading effect on the input circuit.

The Response at Frequency f_1

Refer to Figure 15–81. The input signal voltage is applied to the amplifier circuit board and displayed on channel 1 of the oscilloscope, and channel 2 is connected to a point on the circuit board.

2. Determine to what point on the circuit the channel 2 probe is connected, the frequency, and the voltage that should be displayed.

The Response at Frequency f_2

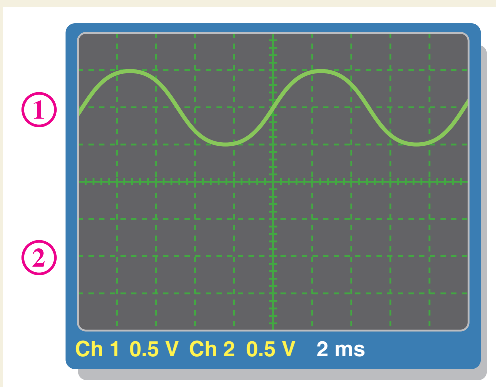
Refer to Figure 15–82 and the circuit board in Figure 15–81. The input signal voltage displayed on channel 1 of the oscilloscope is applied to the amplifier circuit board.

3. Determine the frequency and the voltage that should be displayed on channel 2.
4. State the difference between the channel 2 waveforms determined for f_1 and f_2. Explain the reason for the difference.

The Response at Frequency f_3

Refer to Figure 15–83 and the circuit board in Figure 15–81. The input signal voltage displayed on channel 1 of the oscilloscope is applied to the amplifier circuit board.

5. Determine the frequency and the voltage that should be displayed on channel 2.
6. State the difference between the channel 2 waveforms determined for f_2 and f_3. Explain the reason for the difference.

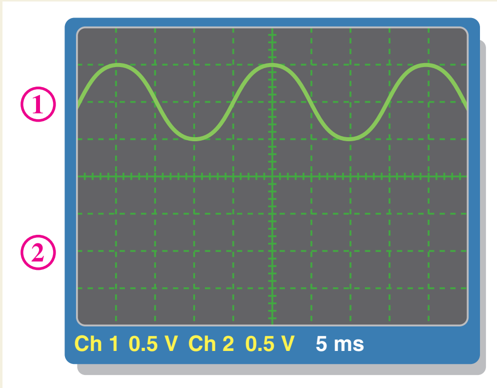


▲ FIGURE 15–82

Measuring the input circuit response at frequency f_2. The channel 1 waveform is shown.

▶ FIGURE 15–83

Measuring the input circuit response at frequency f_3. The channel 1 waveform is shown.



Response Curve for the Amplifier Input Circuit

7. Determine the frequency at which the signal voltage at point B in Figure 15–79 is 70.7% of its maximum value.
8. Plot the response curve using this voltage value and the values at frequencies f_1, f_2, and f_3.
9. How does this curve show that the input circuit acts as a high-pass filter?
10. What can you do to the circuit to lower the frequency at which the voltage is 70.7% of maximum without affecting the dc bias voltage?

Review

11. Explain the effect on the response of the amplifier input circuit of reducing the value of the coupling capacitor.
12. What is the voltage at point B in Figure 15–79 if the coupling capacitor opens when the ac input signal is 10 mV rms?
13. What is the voltage at point B in Figure 15–79 if resistor R_1 is open when the ac input signal is 10 mV rms?

Multisim Analysis

Using Multisim, connect the equivalent circuit in Figure 15–80(b).

- Apply an input signal voltage at the same frequency and amplitude as shown in Figure 15–81. Measure the voltage at point B with the oscilloscope and compare the result from Activity 2.
- Apply an input signal voltage at the same frequency and amplitude as shown in Figure 15–82. Measure the voltage at point B with the oscilloscope and compare the result from Activity 3.
- Apply an input signal voltage at the same frequency and amplitude as shown in Figure 15–83. Measure the voltage at point B with the oscilloscope and compare the result from Activity 5.

OPTION 2 NOTE

Coverage of special topics continues in Chapter 16, Part 4, on page 751.

SUMMARY

- A complex number consists of a real and imaginary part. Imaginary numbers are equal to a real number multiplied by the positive square root of -1.
- The rectangular form of a complex number consists of a real part and a j part of the form $A + jB$.
- The polar form of a complex number consists of a magnitude and an angle of the form $C\angle \pm\theta$.
- Complex numbers can be added, subtracted, multiplied, and divided.
- When a sinusoidal voltage is applied to an RC circuit, the current and all the voltage drops are also sine waves.
- Total current in a series or parallel RC circuit always leads the source voltage.
- The resistor voltage is always in phase with the current.
- The capacitor voltage always lags the current by 90°.
- In a lag circuit, the output voltage lags the input voltage in phase.

- In a lead circuit, the output voltage leads the input voltage.
- In an *RC* circuit, the impedance is determined by both the resistance and the capacitive reactance combined.
- Impedance is expressed in units of ohms.
- The circuit phase angle is the angle between the total current and the applied (source) voltage.
- The impedance of a series *RC* circuit varies inversely with frequency.
- The phase angle (θ) of a series *RC* circuit varies inversely with frequency.
- For each parallel *RC* circuit, there is an equivalent series circuit for any given frequency.
- For each series *RC* circuit, there is an equivalent parallel circuit for any given frequency.
- The impedance of a circuit can be determined by measuring the applied voltage and the total current and then applying Ohm's law.
- In an *RC* circuit, part of the power is resistive and part reactive.
- The phasor combination of resistive power (true power) and reactive power is called *apparent power.*
- Apparent power is expressed in volt-amperes (VA).
- The power factor (*PF*) indicates how much of the apparent power is true power.
- A power factor of 1 indicates a purely resistive circuit, and a power factor of 0 indicates a purely reactive circuit.
- A filter passes certain frequencies and rejects others.
- A phase shift oscillator uses an *RC* network to produce a 180° phase shift.

KEY TERMS

Key terms and other bold terms in the chapter are defined in the end-of-book glossary.

Admittance (Y) A measure of the ability of a reactive circuit to permit current; the reciprocal of impedance. The unit is the siemens (S).

Apparent power (P_a) The phasor combination of resistive power (true power) and reactive power. The unit is the volt-ampere (VA).

Bandwidth The range of frequencies that is considered to be passed by a filter.

Capacitive susceptance (B_C) The ability of a capacitor to permit current: the reciprocal of capacitive reactance. The unit is the siemens (S).

Complex plane An area consisting of four quadrants on which a quantity containing both magnitude and direction can be represented.

Cutoff frequency The frequency at which the output voltage of a filter is 70.7% of the maximum output voltage.

Filter A type of circuit that passes certain frequencies and rejects all others.

Frequency response In electric circuits, the variation in the output voltage (or current) over a specified range of frequencies.

Imaginary number A number that exists on the vertical axis of the complex plane; it consists of a real number multiplied by the positive square root of -1.

Impedance The total opposition to sinusoidal current expressed in ohms.

Polar form One form of a complex number made up of a magnitude and an angle.

Power factor The relationship between volt-amperes and true power or watts. Volt-amperes multiplied by the power factor equals true power.

***RC* lag circuit** A phase shift circuit in which the output voltage, taken across the capacitor, lags the input voltage by a specified angle.

***RC* lead circuit** A phase shift circuit in which the output voltage, taken across the resistor, leads the input voltage by a specified angle.

Real number A number that exists on the horizontal axis of the complex plane.

Rectangular form One form of a complex number made up of a real part and an imaginary part.

FORMULAS

Complex Numbers

15–1 $C = \sqrt{A^2 + B^2}$

15–2 $\theta = \tan^{-1}\left(\frac{\pm B}{A}\right)$

15–3 $\pm A \pm jB = C\angle \pm\theta$

15–4 $A = C\cos\theta$

15–5 $B = C\sin\theta$

15–6 $C\angle\theta = A + jB$

Series *RC* Circuits

15–7 $\mathbf{Z} = R - jX_C$

15–8 $\mathbf{Z} = \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\left(\frac{X_C}{R}\right)$

15–9 $\mathbf{V} = \mathbf{IZ}$

15–10 $\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}}$

15–11 $\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$

15–12 $\mathbf{V}_s = V_R - jV_C$

15–13 $\mathbf{V}_s = \sqrt{V_R^2 + V_C^2}\angle -\tan^{-1}\left(\frac{V_C}{V_R}\right)$

Lag Circuit

15–14 $\phi = -\tan^{-1}\left(\frac{R}{X_C}\right)$

15–15 $V_{out} = \left(\frac{X_C}{\sqrt{R^2 + X_C^2}}\right)V_{in}$

Lead Circuit

15–16 $\phi = \tan^{-1}\left(\frac{X_C}{R}\right)$

15–17 $V_{out} = \left(\frac{R}{\sqrt{R^2 + X_C^2}}\right)V_{in}$

Parallel *RC* Circuits

15–18 $\mathbf{Z} = \left(\frac{RX_C}{\sqrt{R^2 + X_C^2}}\right)\angle -\tan^{-1}\left(\frac{R}{X_C}\right)$

15–19 $\mathbf{Y} = G + jB_C$

15–20 $\mathbf{V} = \frac{\mathbf{I}}{\mathbf{Y}}$

15–21 $\mathbf{I} = \mathbf{VY}$

15–22 $\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}}$

15–23 $\mathbf{I}_{tot} = I_R + jI_C$

15–24 $\mathbf{I}_{tot} = \sqrt{I_R^2 + I_C^2}\angle\tan^{-1}\left(\frac{I_C}{I_R}\right)$

15–25 $R_{eq} = Z\cos\theta$

15–26 $X_{C(eq)} = Z \sin \theta$

15–27 $\theta = \left(\dfrac{\Delta t}{T}\right)360°$

Power in *RC* Circuits

15–28 $P_{true} = I^2R$

15–29 $P_r = I^2X_C$

15–30 $P_a = I^2Z$

15–31 $P_{true} = VI\cos\theta$

15–32 $PF = \cos\theta$

Applications

15–33 $f_r = \dfrac{1}{2\pi\sqrt{6}RC}$

15–34 $f_c = \dfrac{1}{2\pi RC}$

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The imaginary axis and real axis are parts of the complex plane.
2. Multiplying a number by j is equivalent to rotating the number by 90°.
3. Two forms of complex numbers are rectangular and circular.
4. You cannot multiply or divide complex numbers.
5. The impedance of a series *RC* circuit can be expressed as a complex quantity.
6. Phasors can be used to represent complex quantities.
7. Voltage leads the current in terms of phase in a series *RC* circuit.
8. The impedance in a reactive circuit is dependent on frequency.
9. In a series *RC* lag circuit, the output is taken across the resistor.
10. Admittance is the reciprocal of impedance.
11. In a parallel *RC* circuit, the total current and voltage are in phase.
12. Power factor is determined by the phase angle between voltage and current.

SELF-TEST

Answers are at the end of the chapter.

1. A positive angle of 20° is equivalent to a negative angle of
   (a) −160° **(b)** −340° **(c)** −70° **(d)** −20°
2. In the complex plane, the number $3 + j4$ is located in the
   (a) first quadrant **(b)** second quadrant **(c)** third quadrant **(d)** fourth quadrant
3. In the complex plane, $12 - j6$ is located in the
   (a) first quadrant **(b)** second quadrant **(c)** third quadrant **(d)** fourth quadrant
4. The complex number $5 + j5$ is equivalent to
   (a) $5\angle 45°$ **(b)** $25\angle 0°$ **(c)** $7.07\angle 45°$ **(d)** $7.07\angle 135°$
5. The complex number $35\angle 60°$ is equivalent to
   (a) $35 + j35$ **(b)** $35 + j60$ **(c)** $17.5 + j30.3$ **(d)** $30.3 + j17.5$
6. $(4 + j7) + (-2 + j9)$ is equal to
   (a) $2 + j16$ **(b)** $11 + j11$ **(c)** $-2 + j16$ **(d)** $2 - j2$
7. $(16 - j8) - (12 + j5)$ is equal to
   (a) $28 - j13$ **(b)** $4 - j13$ **(c)** $4 - j3$ **(d)** $-4 + j13$
8. $(5\angle 45°)(2\angle 20°)$ is equal to
   (a) $7\angle 65°$ **(b)** $10\angle 25°$ **(c)** $10\angle 65°$ **(d)** $7\angle 25°$

9. $(50\angle 10°)/(25\angle 30°)$ is equal to
   (a) $25\angle 40°$ **(b)** $2\angle 40°$ **(c)** $25\angle -20°$ **(d)** $2\angle -20°$
10. The complex conjugate of $2.5 - j5.0$ is
   (a) $5.59\angle -63.4°$ **(b)** $5.0 - j2.5$ **(c)** $j2.5 - 5.0$ **(d)** $2.5 + j5.0$
11. In a series *RC* circuit, the voltage across the resistance is
   (a) in phase with the source voltage **(b)** lagging the source voltage by 90°
   (c) in phase with the current **(d)** lagging the current by 90°
12. In a series *RC* circuit, the voltage across the capacitor is
   (a) in phase with the source voltage **(b)** in phase with the current
   (c) lagging the resistor voltage by 90° **(d)** lagging the source voltage by 90°
13. When the frequency of the voltage applied to a series *RC* circuit is increased, the impedance
   (a) increases **(b)** decreases
   (c) remains the same **(d)** doubles
14. When the frequency of the voltage applied to a series *RC* circuit is decreased, the phase angle
   (a) increases **(b)** decreases
   (c) remains the same **(d)** becomes erratic
15. In a series *RC* circuit when the frequency and the resistance are doubled, the impedance
   (a) doubles **(b)** is halved
   (c) is quadrupled **(d)** cannot be determined exactly without values
16. In a series *RC* circuit, 10 V rms is measured across the resistor and 10 V rms is also measured across the capacitor. The rms source voltage is
   (a) 20 V **(b)** 14.14 V **(c)** 28.28 V **(d)** 10 V
17. The voltages in Question 15 are measured at a certain frequency. To make the resistor voltage greater than the capacitor voltage, the frequency
   (a) must be increased **(b)** must be decreased
   (c) is held constant **(d)** has no effect
18. When $R = X_C$, the phase angle is
   (a) 0° **(b)** +90° **(c)** −90° **(d)** 45°
19. To decrease the phase angle below 45°, the following condition must exist:
   (a) $R = X_C$ **(b)** $R < X_C$ **(c)** $R > X_C$ **(d)** $R = 10X_C$
20. When the frequency of the source voltage is increased, the impedance of a parallel *RC* circuit
   (a) increases **(b)** decreases **(c)** does not change
21. In a parallel *RC* circuit, there is 1 A rms through the resistive branch and 1 A rms through the capacitive branch. The total rms current is
   (a) 1 A **(b)** 2 A **(c)** 2.28 A **(d)** 1.414 A
22. A power factor of 1 indicates that the circuit phase angle is
   (a) 90° **(b)** 45° **(c)** 180° **(d)** 0°
23. For a certain load, the true power is 100 W and the reactive power is 100 VAR. The apparent power is
   (a) 200 VA **(b)** 100 VA **(c)** 141.4 VA **(d)** 141.4 W
24. AC sources are normally rated in
   (a) watts **(b)** volt-amperes **(c)** volt-amperes reactive **(d)** none of these

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 15–87.

1. If *C* opens, the voltage across it
   (a) increases **(b)** decreases **(c)** stays the same
2. If *R* opens, the voltage across *C*
   (a) increases **(b)** decreases **(c)** stays the same

3. If the frequency is increased, the voltage across R
   (a) increases (b) decreases (c) stays the same

Refer to Figure 15–88.

4. If R_1 opens, the voltage across R_2
   (a) increases (b) decreases (c) stays the same
5. If C_2 is increased to 0.47 μF, the voltage across it
   (a) increases (b) decreases (c) stays the same

Refer to Figure 15–94.

6. If R becomes open, the voltage across the capacitor
   (a) increases (b) decreases (c) stays the same
7. If the source voltage increases, X_C
   (a) increases (b) decreases (c) stays the same

Refer to Figure 15–99.

8. If R_2 opens, the voltage from the top of R_2 to ground
   (a) increases (b) decreases (c) stays the same
9. If C_2 shorts out, the voltage across C_1
   (a) increases (b) decreases (c) stays the same
10. If the frequency of the source voltage is increased, the current through the resistors
    (a) increases (b) decreases (c) stays the same
11. If the frequency of the source voltage is decreased, the current through the capacitors
    (a) increases (b) decreases (c) stays the same

Refer to Figure 15–104.

12. If C_3 opens, the voltage from point B to ground
    (a) increases (b) decreases (c) stays the same
13. If C_2 opens, the voltage from point B to ground
    (a) increases (b) decreases (c) stays the same
14. If a short develops from point C to ground, the voltage from point A to ground
    (a) increases (b) decreases (c) stays the same
15. If capacitor C_3 opens, the voltage from B to D
    (a) increases (b) decreases (c) stays the same
16. If the source frequency increases, the voltage from point C to ground
    (a) increases (b) decreases (c) stays the same
17. If the source frequency increases, the current from the source
    (a) increases (b) decreases (c) stays the same
18. If R_2 shorts out, the voltage across C_1
    (a) increases (b) decreases (c) stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the book.

SECTION 15–1 The Complex Number System

1. What are the two characteristics of a quantity indicated by a complex number?
2. Locate the following numbers on the complex plane:
   (a) $+6$ (b) -2 (c) $+j3$ (d) $-j8$

3. Locate the points represented by each of the following coordinates on the complex plane:
   (a) $3, j5$ (b) $-7, j1$ (c) $-10, -j10$

*4. Determine the coordinates of each point having the same magnitude but located 180° away from each point in Problem 3.

*5. Determine the coordinates of each point having the same magnitude but located 90° away from those in Problem 3.

6. Points on the complex plane are described below. Express each point as a complex number in rectangular form:
   (a) 3 units to the right of the origin on the real axis, and up 5 units on the j axis.
   (b) 2 units to the left of the origin on the real axis, and 1.5 units up on the j axis.
   (c) 10 units to the left of the origin on the real axis, and down 14 units on the $-j$ axis.

7. What is the value of the hypotenuse of a right triangle whose sides are 10 and 15?

8. Convert each of the following rectangular numbers to polar form:
   (a) $40 - j40$ (b) $50 - j200$ (c) $35 - j20$ (d) $98 + j45$

9. Convert each of the following polar numbers to rectangular form:
   (a) $1000\angle -50°$ (b) $15\angle 160°$ (c) $25\angle -135°$ (d) $3\angle 180°$

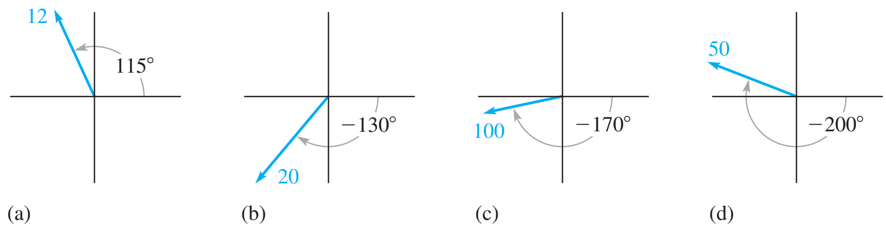
10. Express each of the following polar numbers using a negative angle to replace the positive angle:
   (a) $10\angle 120°$ (b) $32\angle 85°$ (c) $5\angle 310°$

11. Identify the quadrant in which each point in Problem 8 is located.

12. Identify the quadrant in which each point in Problem 10 is located.

13. Write the polar expressions using positive angles for each phasor in Figure 15–84.

▶ FIGURE 15–84



14. Add the following sets of complex numbers:
   (a) $9 + j3$ and $5 + j8$ (b) $3.5 - j4$ and $2.2 + j6$
   (c) $-18 + j23$ and $30 - j15$ (d) $12\angle 45°$ and $20\angle 32°$
   (e) $3.8\angle 75°$ and $1 + j1.8$ (f) $50 - j39$ and $60\angle -30°$

15. Perform the following subtractions:
   (a) $(2.5 + j1.2) - (1.4 + j0.5)$ (c) $(-45 - j23) - (36 + j12)$
   (b) $(8 - j4) - 3\angle 25°$ (d) $48\angle 135° - 33\angle -60°$

16. Multiply the following numbers:
   (a) $4.5\angle 48°$ and $3.2\angle 90°$ (b) $120\angle -220°$ and $95\angle 200°$
   (c) $-3\angle 150°$ and $4 - j3$ (d) $67 + j84$ and $102\angle 40°$
   (e) $15 - j10$ and $-25 - j30$ (f) $0.8 + j0.5$ and $1.2 - j1.5$

17. Perform the following divisions:
   (a) $\dfrac{8\angle 50°}{2.5\angle 39°}$ (b) $\dfrac{63\angle -91°}{9\angle 10°}$ (c) $\dfrac{28\angle 30°}{14 - j12}$ (d) $\dfrac{40 - j30}{16 + j8}$

18. Perform the following operations:
   (a) $\dfrac{2.5\angle 65° - 1.8\angle -23°}{1.2\angle 37°}$ (b) $\dfrac{(100\angle 15°)(85 - j150)}{25 + j45}$
   (c) $\dfrac{(250\angle 90° + 175\angle 75°)(50 - j100)}{(125 + j90)(35\angle 50°)}$ (d) $\dfrac{(1.5)^2(3.8)}{1.1} + j\left(\dfrac{8}{4} - j\dfrac{4}{2}\right)$

PART 1: SERIES CIRCUITS

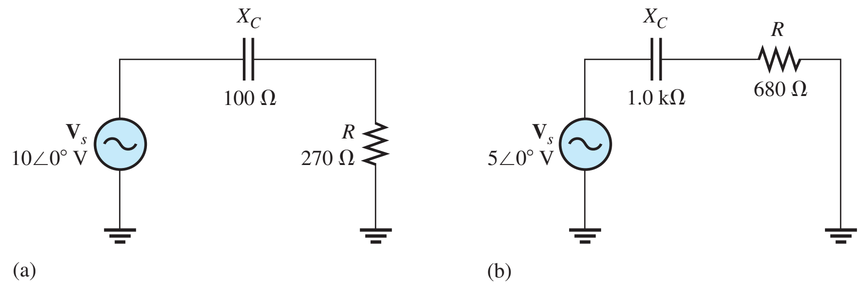
SECTION 15–2 Sinusoidal Response of Series *RC* Circuits

19. An 8 kHz sinusoidal voltage is applied to a series *RC* circuit. What is the frequency of the voltage across the resistor? Across the capacitor?

20. What is the wave shape of the current in the circuit of Problem 19?

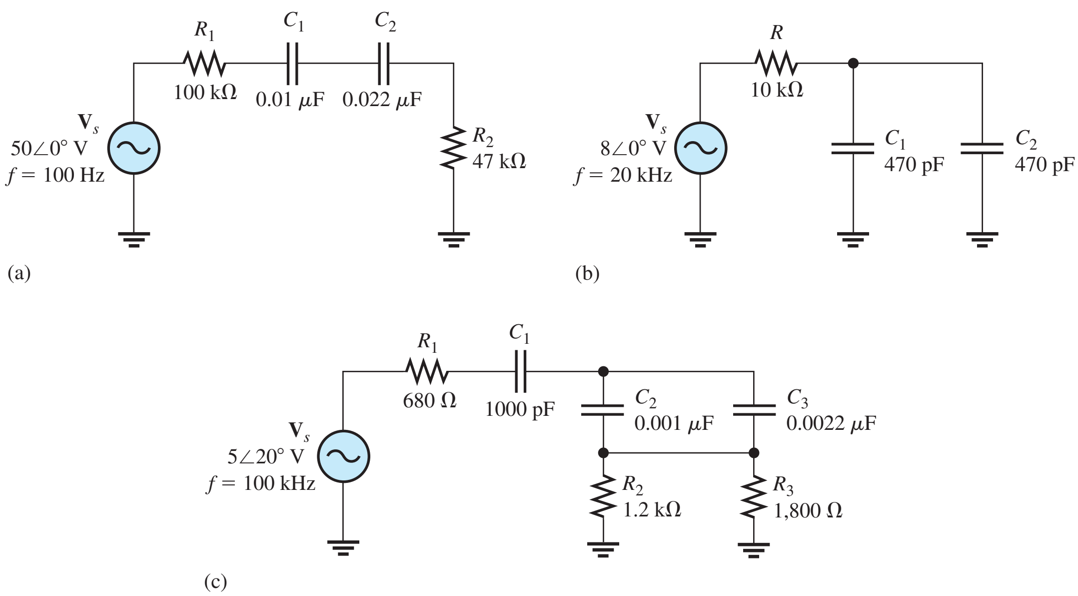
SECTION 15–3 Impedance of Series *RC* Circuits

21. Express the total impedance of each circuit in Figure 15–85 in both polar and rectangular forms.



▲ **FIGURE 15–85**

22. Determine the impedance magnitude and phase angle in each circuit in Figure 15–86.



▲ **FIGURE 15–86**

23. For the circuit of Figure 15–87, determine the impedance expressed in rectangular form for each of the following frequencies:

(a) 100 Hz **(b)** 500 Hz **(c)** 1 kHz **(d)** 2.5 kHz

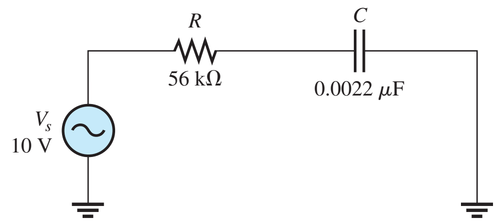
24. Repeat Problem 23 for $C = 0.0047\ \mu F$.

25. Determine the values of R and X_C in a series *RC* circuit for the following values of total impedance:

(a) $\mathbf{Z} = 33\ \Omega - j50\ \Omega$ **(b)** $\mathbf{Z} = 300\angle -25^\circ\ \Omega$

(c) $\mathbf{Z} = 1.8\angle -67.2^\circ\ k\Omega$ **(d)** $\mathbf{Z} = 789\angle -45^\circ\ \Omega$

▶ FIGURE 15–87



SECTION 15–4 Analysis of Series *RC* Circuits

26. Express the current in polar form for each circuit of Figure 15–85.

27. Express the current in each circuit of Figure 15–85 in rectangular form.

28. Calculate the total current in each circuit of Figure 15–86 and express in polar form.

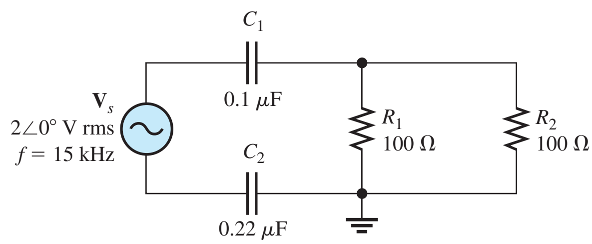
29. Express the current in each circuit of Figure 15–86 in rectangular form.

30. Determine the phase angle between the applied voltage and the current for each circuit in Figure 15–86.

31. Repeat Problem 30 for the circuit in Figure 15–87, using $f = 5$ kHz.

32. For the circuit in Figure 15–88, draw the phasor diagram showing all voltages and the total current. Indicate the phase angles.

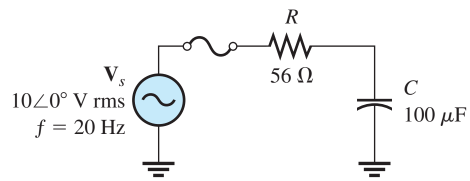
▶ FIGURE 15–88



33. For the circuit in Figure 15–89, determine the following in polar form:

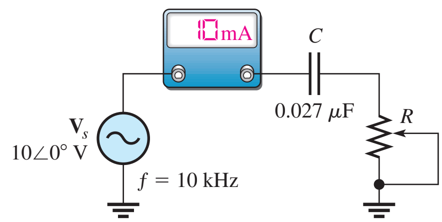
(a) $\mathbf{Z}$ **(b)** $\mathbf{I}_{tot}$ **(c)** $\mathbf{V}_R$ **(d)** $\mathbf{V}_C$

▶ FIGURE 15–89



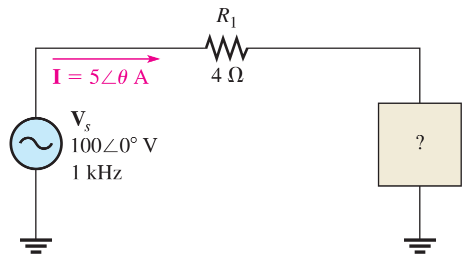
***34.** To what value must the rheostat be set in Figure 15–90 to make the total current 10 mA? What is the resulting angle?

▶ FIGURE 15–90



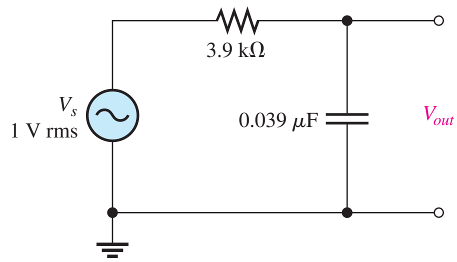
35. Determine the series element or elements that must be installed in the block of Figure 15–91 to meet the following requirements: $P_{true} = 400$ W and there is a leading power factor (I_{tot} leads V_s).

▶ FIGURE 15–91



36. For the lag circuit in Figure 15–92, determine the phase shift between the input voltage and the output voltage for each of the following frequencies:
(a) 1 Hz **(b)** 100 Hz **(c)** 1 kHz **(d)** 10 kHz

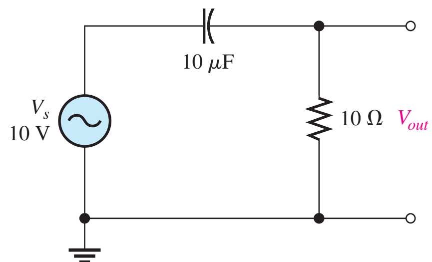
▶ FIGURE 15–92



37. The lag circuit in Figure 15–92 also acts as a low-pass filter. Draw a response curve for this circuit by plotting the output voltage versus frequency for 0 Hz to 10 kHz in 1 kHz increments.

38. Repeat Problem 36 for the lead circuit in Figure 15–93.

▶ FIGURE 15–93



39. Plot the frequency response curve of the output amplitude for the lead circuit in Figure 15–93 for a frequency range of 0 Hz to 10 kHz in 1 kHz increments.

40. Draw the voltage phasor diagram for the circuit in Figure 15–92 for a frequency of 5 kHz with $V_s = 1$ V rms.

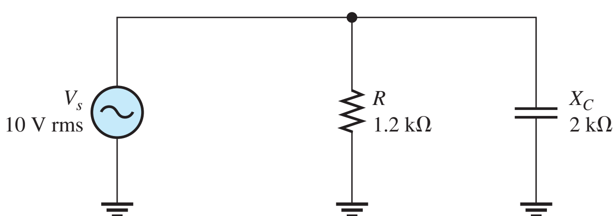
41. Repeat Problem 40 for the circuit in Figure 15–93. $V_s = 10$ V rms and $f = 1$ kHz.

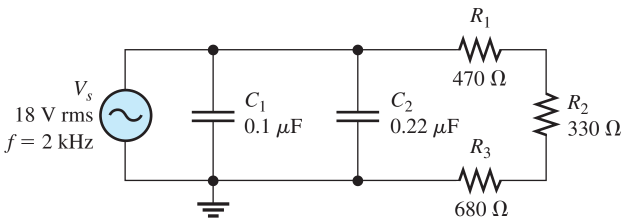
PART 2: PARALLEL CIRCUITS

SECTION 15–5 Impedance and Admittance of Parallel *RC* Circuits

42. Determine the impedance and express it in polar form for the circuit in Figure 15–94.

▶ FIGURE 15–94





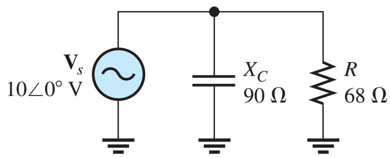
▲ FIGURE 15–95

43. Determine the impedance magnitude and phase angle in Figure 15–95.
44. Repeat Problem 43 for the following frequencies:
    (a) 1.5 kHz (b) 3 kHz (c) 5 kHz (d) 10 kHz

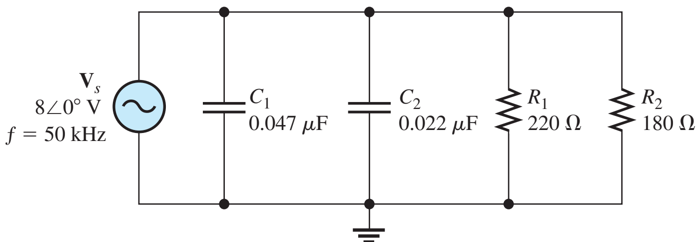
SECTION 15–6 Analysis of Parallel *RC* Circuits

45. For the circuit in Figure 15–96, find all the currents and voltages in polar form.

► FIGURE 15–96



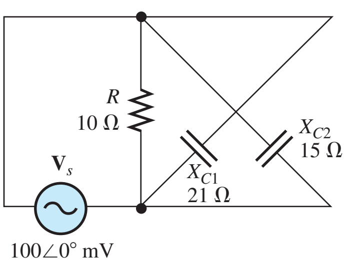
46. For the parallel circuit in Figure 15–97, find the magnitude of each branch current and the total current. What is the phase angle between the applied voltage and the total current?



▲ FIGURE 15–97

47. For the circuit in Figure 15–98, determine the following:
    (a) **Z** (b) $\mathbf{I}_R$ (c) $\mathbf{I}_{C(tot)}$ (d) $\mathbf{I}_{tot}$ (e) θ

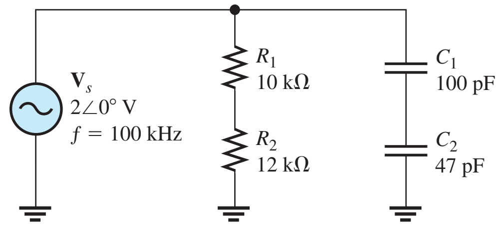
► FIGURE 15–98



48. Repeat Problem 47 for $R = 5.6\ \text{k}\Omega$, $C_1 = 0.047\ \mu\text{F}$, $C_2 = 0.022\ \mu\text{F}$, and $f = 500$ Hz.

*49. Convert the circuit in Figure 15–99 to an equivalent series form.

FIGURE 15–99



*50. Determine the value to which R_1 must be adjusted to get a phase angle of 30° between the source voltage and the total current in Figure 15–100.

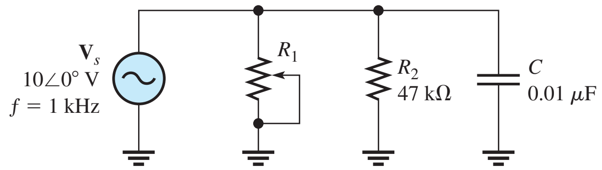


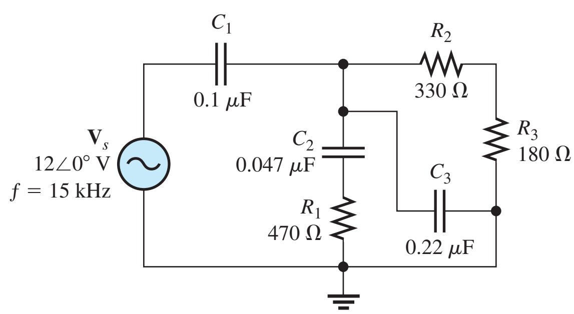
FIGURE 15–100

PART 3: SERIES-PARALLEL CIRCUITS

SECTION 15–7 Analysis of Series-Parallel *RC* Circuits

51. Determine the voltages in polar form across each element in Figure 15–101. Draw the voltage phasor diagram.

FIGURE 15–101



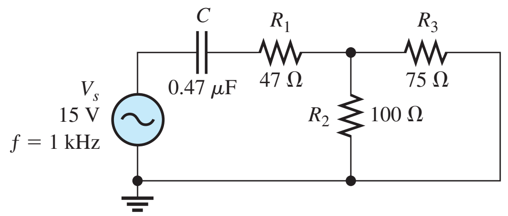
52. Is the circuit in Figure 15–101 predominantly resistive or predominantly capacitive?

53. Find the current through each branch and the total current in Figure 15–101. Express the currents in polar form. Draw the current phasor diagram.

54. For the circuit in Figure 15–102, determine the following:

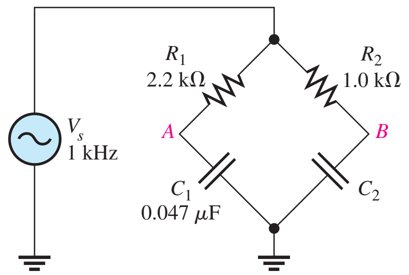
(a) $\mathbf{I}_{tot}$ **(b)** θ **(c)** $\mathbf{V}_{R1}$ **(d)** $\mathbf{V}_{R2}$ **(e)** $\mathbf{V}_{R3}$ **(f)** $\mathbf{V}_C$

FIGURE 15–102



*55. Determine the value of C_2 in Figure 15–103 when $V_A = V_B$.

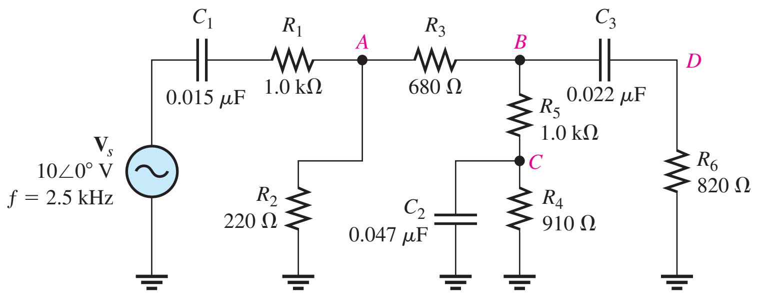
▶ FIGURE 15–103



*56. Determine the voltage and its phase angle at each point labeled in Figure 15–104.

*57. Find the current through each component in Figure 15–104.

*58. Draw the voltage and current phasor diagram for Figure 15–104.



▲ FIGURE 15–104

PART 4: SPECIAL TOPICS

SECTION 15–8 Power in *RC* Circuits

59. In a certain series *RC* circuit, the true power is 2 W, and the reactive power is 3.5 VAR. Determine the apparent power.

60. In Figure 15–89, what is the true power and the reactive power?

61. What is the power factor for the circuit of Figure 15–99?

62. Determine P_{true}, P_r, P_a, and *PF* for the circuit in Figure 15–102. Draw the power triangle.

*63. A single 240 V, 60 Hz source drives two loads. Load *A* has an impedance of 50 Ω and a power factor of 0.85. Load *B* has an impedance of 72 Ω and a power factor of 0.95.

(a) How much current does each load draw?

(b) What is the reactive power in each load?

(c) What is the true power in each load?

(d) What is the apparent power in each load?

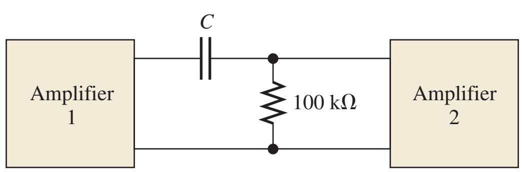
(e) Which load has more voltage drop along the lines connecting it to the source?

SECTION 15–9 Basic Applications

64. Calculate the frequency of oscillation for the circuit in Figure 15–63 if all *C*s are 0.0022 μF and all *R*s are 10 kΩ.

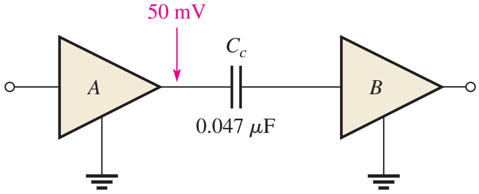
*65. What value of coupling capacitor is required in Figure 15–105 so that the signal voltage at the input of amplifier 2 is at least 70.7% of the signal voltage at the output of amplifier 1 when the frequency is 20 Hz?

▶ FIGURE 15–105



66. The rms value of the signal voltage out of amplifier A in Figure 15–106 is 50 mV. If the input resistance to amplifier B is 10 kΩ, how much of the signal is lost due to the coupling capacitor when the frequency is 3 kHz?

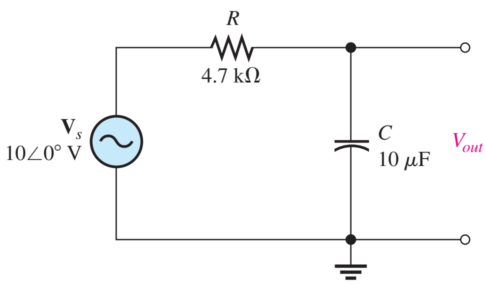
► FIGURE 15–106



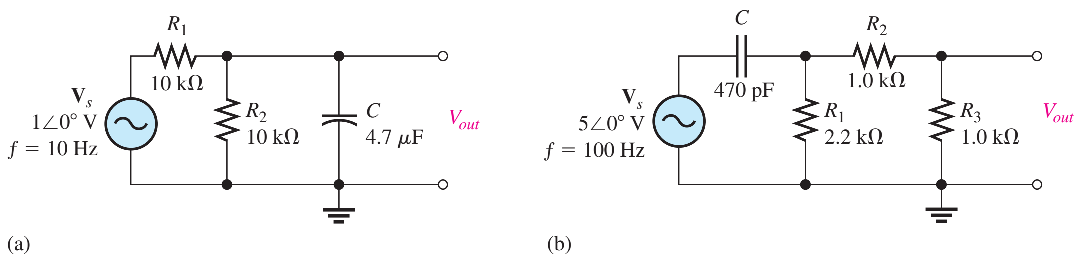
SECTION 15–10 Troubleshooting

67. Assume that the capacitor in Figure 15–107 is excessively leaky. Show how this degradation affects the output voltage and phase angle, assuming that the leakage resistance is 5 kΩ and the frequency is 10 Hz.

► FIGURE 15–107



***68.** Each of the capacitors in Figure 15–108 has developed a leakage resistance of 2 kΩ. Determine the output voltages under this condition for each circuit.



▲ FIGURE 15–108

69. Determine the output voltage for the circuit in Figure 15–108(a) for each of the following failure modes, and compare it to the correct output:

(a) R_1 open **(b)** R_2 open **(c)** C open **(d)** C shorted

70. Determine the output voltage for the circuit in Figure 15–108(b) for each of the following failure modes, and compare it to the correct output:

(a) C open **(b)** C shorted **(c)** R_1 open **(d)** R_2 open **(e)** R_3 open

Multisim Troubleshooting and Analysis

These problems require Multisim.

71. Open file P15-71 and determine if there is a fault. If so, find the fault.

72. Open file P15-72 and determine if there is a fault. If so, find the fault.

73. Open file P15-73 and determine if there is a fault. If so, find the fault.
74. Open file P15-74 and determine if there is a fault. If so, find the fault.
75. Open file P15-75 and determine if there is a fault. If so, find the fault.
76. Open file P15-76 and determine if there is a fault. If so, find the fault.
77. Open file P15-77 and determine the frequency response for the filter.
78. Open file P15-78 and determine the frequency response for the filter.

ANSWERS

SECTION CHECKUPS

SECTION 15–1 The Complex Number System

1. $2.83\angle 45°$; first
2. $3.54 - j3.54$, fourth
3. $4 + j1$
4. $3 + j7$
5. $16\angle 110°$
6. $5\angle 15°$

SECTION 15–2 Sinusoidal Response of Series *RC* Circuits

1. The voltage frequency is 60 Hz. The current frequency is 60 Hz.
2. The capacitive reactance causes the phase shift.
3. The phase angle is closer to 0°.

SECTION 15–3 Impedance of Series *RC* Circuits

1. $R = 150\ \Omega$; $X_C = 220\ \Omega$
2. $\mathbf{Z} = 33\ \text{k}\Omega - j50\ \text{k}\Omega$
3. $Z = \sqrt{R^2 + X_C^2} = 59.9\ \text{k}\Omega$; $\theta = -\tan^{-1}(X_C/R) = -56.6°$

SECTION 15–4 Analysis of Series *RC* Circuits

1. $V_s = \sqrt{V_R^2 + V_C^2} = 7.21\ \text{V}$
2. $\theta = -\tan^{-1}(X_C/R) = -56.3°$
3. $\theta = 90°$
4. When f increases, X_C decreases, Z decreases, and θ decreases.
5. $\phi = -90° + \tan^{-1}(X_C/R) = -62.8°$
6. $V_{out} = \left(R/\sqrt{R^2 + X_C^2}\right)V_{in} = 8.90\ \text{V rms}$

SECTION 15–5 Impedance and Admittance of Parallel *RC* Circuits

1. Conductance is the reciprocal of resistance, capacitive susceptance is the reciprocal of capacitive reactance, and admittance is the reciprocal of impedance.
2. $Y = 1/Z = 1/100\ \Omega = 10\ \text{mS}$
3. $\mathbf{Y} = 1/\mathbf{Z} = 25.1\angle 32.1°\ \mu\text{S}$
4. $\mathbf{Z} = 39.8\angle -32.1°\ \text{k}\Omega$

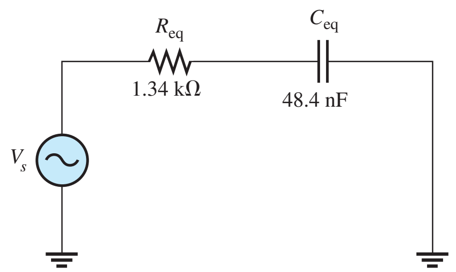
SECTION 15–6 Analysis of Parallel *RC* Circuits

1. $I_{tot} = V_s Y = 21.0\ \text{mA}$
2. $I_{tot} = \sqrt{I_R^2 + I_C^2} = 18.0\ \text{mA}$; $\theta = \tan^{-1}(I_C/I_R) = 56.3°$; θ is with respect to applied voltage.
3. $\theta = 90°$

SECTION 15–7 Analysis of Series-Parallel *RC* Circuits

1. See Figure 15–109.
2. $\mathbf{Z}_{tot} = \mathbf{V}_s/\mathbf{I}_{tot} = Z_{tot} = 36.3\,\Omega\angle -49.7°$

▶ FIGURE 15–109



SECTION 15–8 Power in *RC* Circuits

1. Power dissipation is due to resistance.
2. $PF = \cos\theta = 0.707$
3. $P_{true} = I^2R = 1.32\text{ kW}$; $P_r = I^2X_C = 1.84\text{ kVAR}$; $P_a = I^2Z = 2.27\text{ kVA}$

SECTION 15–9 Basic Applications

1. 180°
2. The output is across the capacitor.

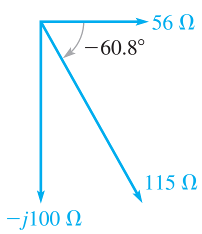
SECTION 15–10 Troubleshooting

1. The leakage resistance acts in parallel with C, which alters the circuit time constant.
2. The capacitor is open.
3. An open series resistor or the capacitor shorted will result in 0 V across the capacitor.

RELATED PROBLEMS FOR EXAMPLES

15–1	**(a)** 1st	**(b)** 4th	**(c)** 3rd	**(d)** 2nd
15–2	$29.2\angle 52.0°$			
15–3	$70.1 - j34.2$			
15–4	$14.4\angle -57.2°$			
15–5	$-1 - j8$			
15–6	$-13.5 - j4.5$			
15–7	$1{,}500\angle -50°$			
15–8	$4\angle -42°$			
15–9	See Figure 15–110.			

▶ FIGURE 15–110



15–10 $\mathbf{V}_s = 2.56\angle -38.5°\text{ V}$

15–11 $\mathbf{I} = 3.80\angle 33.3°\text{ mA}$

15–12 $Z = 16.0\ \text{k}\Omega, \theta = -86.4°$

15–13 The phase lag increases.

15–14 The output voltage decreases.

15–15 The phase lead decreases.

15–16 The output voltage decreases.

15–17 $\mathbf{Z} = 24.3\angle -76.0°\ \Omega$

15–18 $\mathbf{Y} = 4.60\angle 48.8°\ \text{mS}$

15–19 $\mathbf{I} = 6.15\angle 42.4°\ \text{mA}$

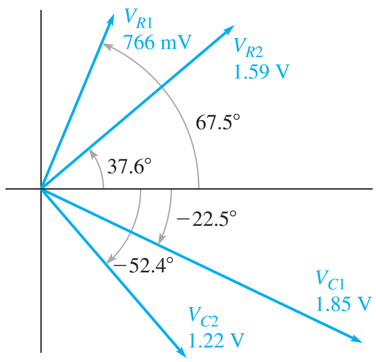
15–20 $\mathbf{I}_{tot} = 117\angle 31.0°\ \text{mA}$

15–21 $R_{\text{eq}} = 8.99\ \text{k}\Omega$, $X_{C(\text{eq})} = 4.38\ \text{k}\Omega$

15–22 $\mathbf{V}_1 = 7.04\angle 8.53°\ \text{V}$, $V_2 = 3.22\angle -18.9°\ \text{V}$

15–23 $\mathbf{V}_{R1} = 766\angle 67.5°\ \text{mV}$; $\mathbf{V}_{C1} = 1.85\angle -22.5°\ \text{V}$; $\mathbf{V}_{R2} = 1.59\angle 37.6°\ \text{V}$; $\mathbf{V}_{C2} = 1.22\angle -52.4°\ \text{V}$; See Figure 15–111.

► **FIGURE 15–111**



15–24 $PF = 0.146$

15–25 $P_{\text{true}} = 213\ \text{mW}$

15–26 1.60 kHz

15–27 $V_{out} = 7.29\ \text{V}$

15–28 Resistor open

TRUE/FALSE QUIZ

1. T	**2.** T	**3.** F	**4.** F	**5.** T	**6.** T
7. F	**8.** T	**9.** F	**10.** T	**11.** F	**12.** T

SELF-TEST

1. (b)	**2.** (a)	**3.** (d)	**4.** (c)	**5.** (c)	**6.** (a)	**7.** (b)	**8.** (c)
9. (d)	**10.** (d)	**11.** (c)	**12.** (b)	**13.** (b)	**14.** (a)	**15.** (d)	**16.** (b)
17. (a)	**18.** (d)	**19.** (c)	**20.** (b)	**21.** (d)	**22.** (d)	**23.** (c)	**24.** (b)

CIRCUIT DYNAMICS QUIZ

1. (a)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (a)
9. (a)	**10.** (c)	**11.** (b)	**12.** (a)	**13.** (a)	**14.** (b)	**15.** (a)	**16.** (b)
17. (a)	**18.** (a)						