CAPACITORS

12

CHAPTER OUTLINE

CHAPTER OBJECTIVES

- Describe the basic construction and characteristics of a capacitor
- Discuss various types of capacitors
- Analyze series capacitors
- Analyze parallel capacitors
- Analyze capacitive dc switching circuits
- Analyze capacitive ac circuits
- Discuss some capacitor applications
- Describe the operation of switched-capacitor circuits

KEY TERMS

- Capacitor
- Dielectric
- Farad (F)
- *RC* time constant
- Capacitive reactance
- Instantaneous power
- True power
- Reactive power
- VAR (volt-ampere reactive)
- Ripple voltage

APPLICATION ACTIVITY PREVIEW

In this application activity, you will see how a capacitor is used to couple signal voltages to and from an amplifier. You will also troubleshoot the circuit using oscilloscope waveforms.

VISIT THE COMPANION WEBSITE

Study aids for this chapter are available at http://www.pearsonhighered.com/careersresources/

INTRODUCTION

In previous chapters, the resistor has been the only passive electrical component that you have studied. Capacitors and inductors are other types of basic passive electrical components. You will study inductors in Chapter 13.

In this chapter, you will learn about the capacitor and its characteristics. The physical construction and electrical properties are examined, and the effects of connecting capacitors in series and in parallel are analyzed. How a capacitor works in both dc and ac circuits is an important part of this coverage and forms the basis for the study of reactive circuits in terms of both frequency response and time response.

The *capacitor* is an electrical device that can store electrical charge, thereby creating an electric field that, in turn, stores energy. The measure of the energy-storing ability of a capacitor is its *capacitance*. When a sinusoidal signal is applied to a capacitor, it reacts in a certain way and produces an opposition to current, which depends on the frequency of the applied signal. This opposition to current is called *capacitive reactance*.

12–1 THE BASIC CAPACITOR

A **capacitor** is a passive electrical component that stores electrical charge and has the property of capacitance.

After completing this section, you should be able to

- **Describe the basic construction and characteristics of a capacitor**
  - Explain how a capacitor stores charge
  - Define *capacitance* and state its unit
  - Explain how a capacitor stores energy
  - Discuss voltage rating and temperature coefficient
  - Explain capacitor leakage
  - Specify how the physical characteristics affect the capacitance

Basic Construction

In its simplest form, a capacitor is an electrical device that stores electrical charge and is constructed of two parallel conductive plates separated by an insulating material called the **dielectric**. Connecting leads are attached to the parallel plates. A basic capacitor is shown in Figure 12–1(a), and a schematic symbol is shown in part (b).

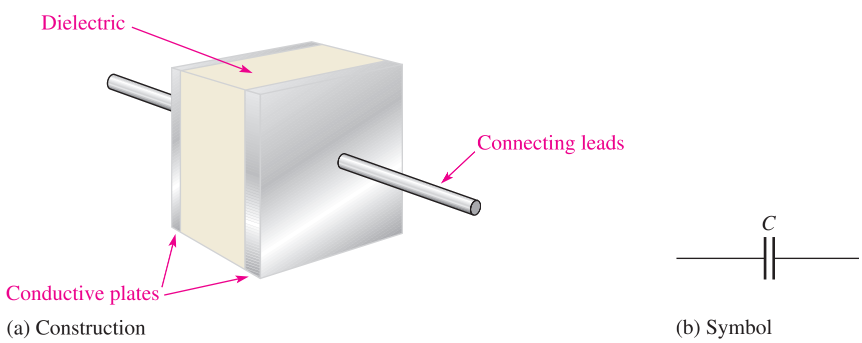


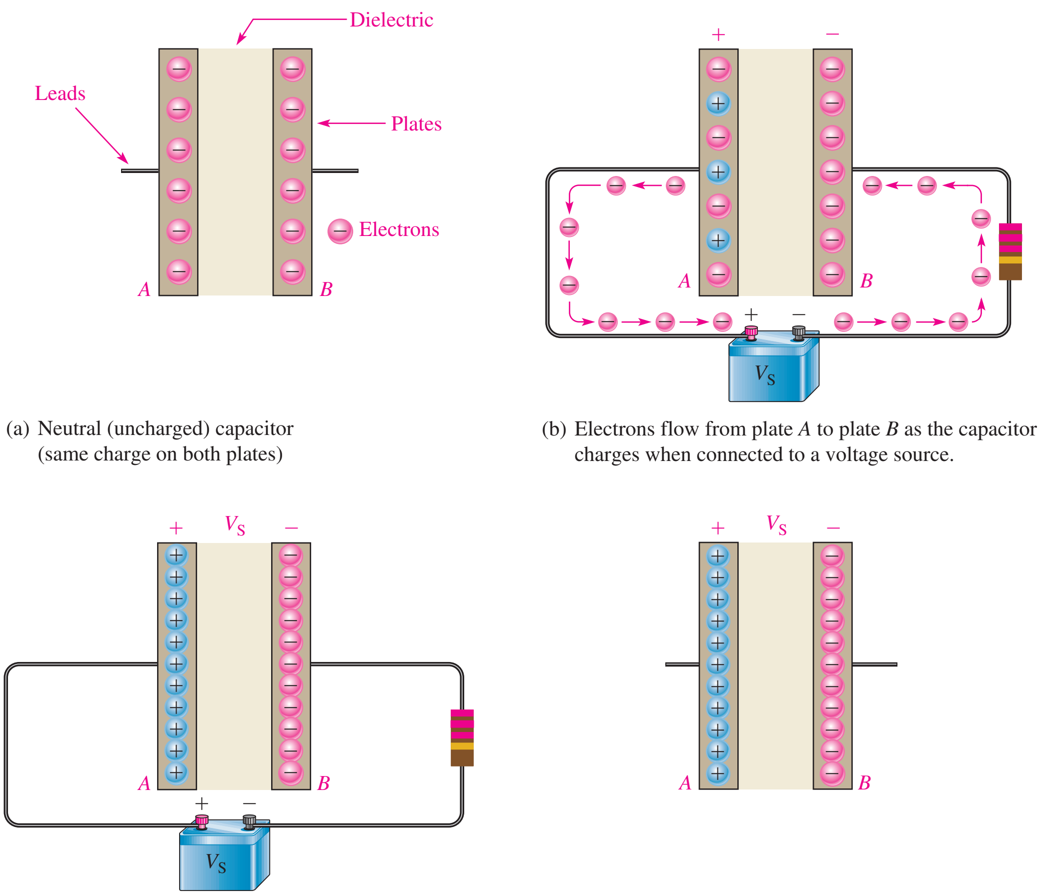
FIGURE 12–1
The basic capacitor.

How a Capacitor Stores Charge

In the neutral state, both plates of a capacitor have an equal number of free electrons, as indicated in Figure 12–2(a). When the capacitor is connected to a voltage source through a resistor, as shown in part (b), electrons (negative charge) are removed from plate *A*, and an equal number are deposited on plate *B*. As plate *A* loses electrons and plate *B* gains electrons, plate *A* becomes positive with respect to plate *B*. During this charging process, electrons flow only through the connecting leads. No electrons flow through the dielectric of the capacitor because it is an insulator; however, the dielectric becomes polarized. The movement of electrons ceases when the voltage across the capacitor equals the source voltage, as indicated in Figure 12–2(c). If the capacitor is disconnected from the source, it retains the stored charge for a long period of time (the length of time depends on the type of capacitor) and still has the voltage across it, as shown in Figure 12–2(d). A charged capacitor can act as a temporary battery.

SAFETY NOTE

Capacitors are capable of storing dangerous electrical charge for a long time after power has been turned off in a circuit. Be careful when touching or handling capacitors in or out of a circuit. If you touch the leads, you may be in for a shock as the capacitor discharges through you! It is usually good practice to discharge a capacitor using a shorting tool with an insulated grip of some sort before handling the capacitor.



(a) Neutral (uncharged) capacitor (same charge on both plates)

(b) Electrons flow from plate A to plate B as the capacitor charges when connected to a voltage source.

(c) After the capacitor charges to V_S, no electrons flow while connected to the voltage source.

(d) Ideally, the capacitor retains charge when disconnected from the voltage source.

▲ FIGURE 12–2

Illustration of a capacitor storing charge.

Capacitance

The amount of charge that a capacitor can store per unit of voltage across its plates is its capacitance, designated C. That is, **capacitance** is a measure of a capacitor's ability to store charge. The more charge per unit of voltage that a capacitor can store, the greater its capacitance, as expressed by the following formula:

Equation 12–1
$$C = \frac{Q}{V}$$

where C is capacitance, Q is charge, and V is voltage.

By rearranging the terms in Equation 12–1, you can obtain two other formulas.

Equation 12–2
$$Q = CV$$

Equation 12–3
$$V = \frac{Q}{C}$$

The Unit of Capacitance The farad (F) is the basic unit of capacitance. Recall that the coulomb (C) is the unit of electrical charge.

One farad is the amount of capacitance when one coulomb (C) of charge is stored with one volt across the plates.

Most capacitors that are used in electronics work have capacitance values that are specified in microfarads (μF) and picofarads (pF). A microfarad is one-millionth of a farad ($1\ \mu\text{F} = 1 \times 10^{-6}$ F), and a picofarad is one-trillionth of a farad

(1 pF = 1×10^{-12} F). These are the most common units for marking capacitors, but nF is commonly used in schematics and programs like Multisim and LTSpice. The unit mF is occasionally used but less so because it is so large. Conversions for farads, microfarads, and picofarads are given in Table 12–1.

TO CONVERT FROM	TO	MOVE THE DECIMAL POINT
Farads	Microfarads	6 places to right ($\times 10^{6}$)
Farads	Picofarads	12 places to right ($\times 10^{12}$)
Microfarads	Farads	6 places to left ($\times 10^{-6}$)
Microfarads	Picofarads	6 places to right ($\times 10^{6}$)
Picofarads	Farads	12 places to left ($\times 10^{-12}$)
Picofarads	Microfarads	6 places to left ($\times 10^{-6}$)

TABLE 12–1

Common metric conversions for capacitors.

EXAMPLE 12–1

(a) A certain capacitor stores 50 microcoulombs (50 μC) with 10 V across its plates. What is its capacitance in units of microfarads?

(b) A 2.2 μF capacitor has 100 V across its plates. How much charge does it store?

(c) Determine the voltage across a 0.68 μF capacitor that is storing 20 microcoulombs (20 μC) of charge.

Solution

(a) $C = \dfrac{Q}{V} = \dfrac{50\ \mu\text{C}}{10\ \text{V}} = \mathbf{5\ \mu F}$

(b) $Q = CV = (2.2\ \mu\text{F})(100\ \text{V}) = \mathbf{220\ \mu C}$

(c) $V = \dfrac{Q}{C} = \dfrac{20\ \mu\text{C}}{0.68\ \mu\text{F}} = \mathbf{29.4\ V}$

*Related Problem** Determine V if C = 5,000 pF and Q = 1.0 μC.

*Answers are at the end of the chapter.

EXAMPLE 12–2

Convert the following values to microfarads:

(a) 0.00001 F **(b)** 0.0047 F **(c)** 1,000 pF **(d)** 220 pF

Solution

(a) $0.00001\ \text{F} \times 10^{6}\ \mu\text{F/F} = \mathbf{10\ \mu F}$ **(b)** $0.0047\ \text{F} \times 10^{6}\ \mu\text{F/F} = \mathbf{4{,}700\ \mu F}$

(c) $1{,}000\ \text{pF} \times 10^{-6}\ \mu\text{F/pF} = \mathbf{0.001\ \mu F}$ **(d)** $220\ \text{pF} \times 10^{-6}\ \mu\text{F/pF} = \mathbf{0.00022\ \mu F}$

Related Problem Convert 47,000 pF to microfarads.

EXAMPLE 12–3

Convert the following values to picofarads:

(a) 0.1×10^{-8} F **(b)** 0.000022 F **(c)** 0.01 μF **(d)** 0.0047 μF

Solution

(a) $0.1 \times 10^{-8}\ \text{F} \times 10^{12}\ \text{pF/F} = \mathbf{1{,}000\ pF}$

(b) $0.000022\ \text{F} \times 10^{12}\ \text{pF/F} = \mathbf{22 \times 10^{6}\ pF}$

(c) $0.01\ \mu\text{F} \times 10^{6}\ \text{pF}/\mu\text{F} = \mathbf{10{,}000\ pF}$

(d) $0.0047\ \mu\text{F} \times 10^{6}\ \text{pF}/\mu\text{F} = \mathbf{4{,}700\ pF}$

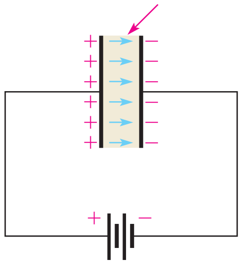
Related Problem Convert 100 μF to picofarads.

How a Capacitor Stores Energy

A capacitor stores energy in the form of an electric field that is established by the opposite charges stored on the two plates. The electric field is represented by lines of force between the positive and negative charges and is concentrated within the dielectric, as shown in Figure 12–3. Although current does not flow through the dielectric, molecules in the dielectric orient themselves with the electric field, creating a polarized region within the dielectric. The polarized molecules within the dielectric create an electric field that reduces the overall field within the dielectric. A good dielectric is one which is easily polarized.

FIGURE 12–3

The electric field stores energy in a capacitor.



The plates in Figure 12–3 have acquired a charge because they are connected to a battery. This creates an electric field between the plates, which stores energy. The energy stored in the electric field is directly related to the size of the capacitor and to the square of the voltage as given by the following equation for the energy stored:

Equation 12–4

$$W = \frac{1}{2}CV^2$$

When capacitance (C) is in farads and voltage (V) is in volts, the energy (W) is in joules.

Voltage Rating

Every capacitor has a limit on the amount of voltage that it can withstand across its plates. The voltage rating specifies the maximum dc voltage that can be applied without risk of damage to the device. If this maximum voltage, commonly called the *breakdown voltage* or *working voltage*, is exceeded, permanent damage to the capacitor can result.

You must consider both the capacitance and the voltage rating before you use a capacitor in a circuit application. The choice of capacitance value is based on particular circuit requirements. The voltage rating should always be above the maximum voltage expected in a particular application.

Dielectric Strength The breakdown voltage of a capacitor is determined by the **dielectric strength** of the dielectric material used. The dielectric strength is expressed in V/mil (1 mil $= 0.001$ in. $= 25.4 \times 10^{-6}$ m $= 25.4\ \mu$m $=$ 25.4 microns). Table 12–2 lists typical values for several materials. Exact values vary depending on the specific composition of the material.

TABLE 12–2

Some common dielectric materials and their dielectric strengths.

MATERIAL	DIELECTRIC STRENGTH (V/MIL)
Air	80
Oil	375
Ceramic	1,000
Paper (paraffined)	1,200
Teflon®	1,500
Mica	1,500
Glass	2,000

A capacitor's dielectric strength can best be explained by an example. Assume that a certain capacitor has a plate separation of 1 mil and that the dielectric material is ceramic. This particular capacitor can withstand a maximum voltage of 1,000 V because its dielectric strength is 1,000 V/mil. If the maximum voltage is exceeded, the dielectric may break down (i.e., puncture the dielectric) and conduct current, causing permanent damage to the capacitor. Similarly, if the ceramic capacitor has a plate separation of 2 mils, its breakdown voltage is 2,000 V.

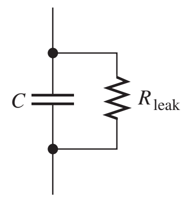
Temperature Coefficient

The **temperature coefficient** indicates the amount and direction of a change in capacitance value with temperature. A positive temperature coefficient means that the capacitance increases with an increase in temperature or decreases with a decrease in temperature. A negative coefficient means that the capacitance decreases with an increase in temperature or increases with a decrease in temperature.

Temperature coefficients are typically specified in parts per million per Celsius degree (ppm/°C). For example, a negative temperature coefficient of 150 ppm/°C for a 1 μF capacitor means that for every degree rise in temperature, the capacitance decreases by 150 pF (there are one million picofarads in one microfarad).

Leakage

No insulating material is perfect. The dielectric of any capacitor will conduct some very small amount of current. Thus, the charge on a capacitor will eventually leak off. Some types of capacitors, such as large electrolytic types, have higher leakages than others. An equivalent circuit for a nonideal capacitor is shown in Figure 12–4. The parallel resistor R_{leak} represents the extremely high resistance (several hundred kilohms or more) of the dielectric material through which there is leakage current.



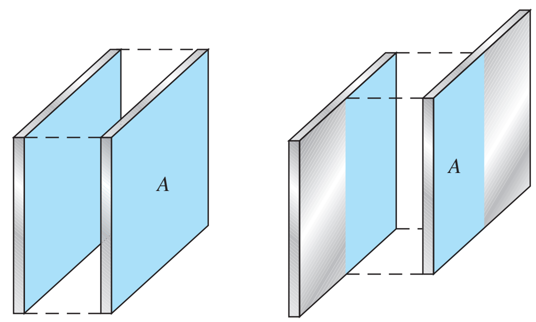
▲ FIGURE 12–4

Equivalent circuit for a nonideal capacitor.

Physical Characteristics of a Capacitor

The following parameters are important in establishing the capacitance and the voltage rating of a capacitor: plate area, plate separation, and dielectric constant.

Plate Area *Capacitance is directly proportional to the physical size of the plates as determined by the plate area, A.* A larger plate area produces more capacitance, and a smaller plate area produces less capacitance. Figure 12–5(a) shows that the plate area of a parallel plate capacitor is the area of one of the plates. If the plates are moved in relation to each other, as shown in Figure 12–5(b), the overlapping area determines the effective plate area. This variation in effective plate area is the basis for a certain type of variable capacitor.



(a) Full plate area: more capacitance

(b) Reduced plate area: less capacitance

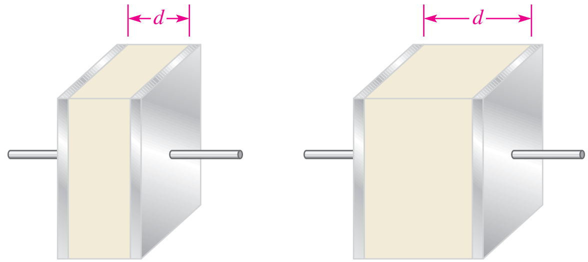
◄ FIGURE 12–5

Capacitance is directly proportional to plate area (A).

Plate Separation *Capacitance is inversely proportional to the distance between the plates.* The plate separation is designated d, as shown in Figure 12–6. A greater separation of the plates produces a smaller capacitance, as illustrated in the figure.

▶ **FIGURE 12–6**

Capacitance is inversely proportional to the distance between the plates.



(a) Plates closer together: more capacitance

(b) Plates further apart: less capacitance

As previously discussed, the breakdown voltage is directly proportional to the plate separation. The further the plates are separated, the greater the breakdown voltage.

Dielectric Constant As you know, the insulating material between the plates of a capacitor is called the *dielectric.* Dielectric materials tend to reduce the voltage between plates for a given charge and thus increase the capacitance. If the voltage is fixed, more charge can be stored due to the presence of a dielectric than can be stored without a dielectric. The measure of a material's ability to establish an electric field is called the **dielectric constant** or *relative permittivity*, symbolized by ε_r. (ε is the Greek letter epsilon.)

Capacitance is directly proportional to the dielectric constant. The dielectric constant of a vacuum is defined as 1 and that of air is very close to 1. These values are used as a reference, and all other materials have values of ε_r specified with respect to that of a vacuum or air. For example, a material with $\varepsilon_r = 8$ can result in a capacitance eight times greater than that of air with all other factors being equal.

Table 12–3 lists several common dielectric materials and a typical dielectric constant for each. The values can vary because it depends on the specific composition of the material.

▶ **TABLE 12–3**

Some common dielectric materials and their typical dielectric constants.

MATERIAL	TYPICAL ε_r VALUE
Air (vacuum)	1.0
Teflon®2.0	
Paper (paraffined)	2.5
Oil	4.0
Mica	5.0
Glass	7.5
Ceramic	1,200

The dielectric constant (relative permittivity) is dimensionless because it is a relative measure. It is a ratio of the absolute permittivity of a material, ε, to the absolute permittivity of a vacuum, ε_0, as expressed by the following formula:

Equation 12–5

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

The value of ε_0 is 8.85×10^{-12} F/m (farads per meter).

Formula You have seen how capacitance is directly related to plate area, A, and the dielectric constant, ε_r, and inversely related to plate separation, d. An exact formula for calculating the capacitance in terms of these three quantities is

Equation 12–6

$$C = \frac{A\varepsilon_r(8.85 \times 10^{-12}\ \text{F/m})}{d}$$

where A is in square meters (m^2), d is in meters (m), and C is in farads (F). Recall that the absolute permittivity of a vacuum, ε_0, is 8.85×10^{-12} F/m and that the absolute permittivity of a dielectric (ε), as derived from Equation (12–5), is

$$\varepsilon = \varepsilon_r(8.85 \times 10^{-12}\ \text{F/m})$$

EXAMPLE 12–4

Determine the capacitance of a parallel plate capacitor having a plate area of $0.01\ m^2$ and a plate separation of 0.5 mil (1.27×10^{-5} m). The dielectric is mica, which has a dielectric constant of 5.0.

Solution Use Equation 12–6.

$$C = \frac{A\varepsilon_r(8.85 \times 10^{-12}\ \text{F/m})}{d} = \frac{(0.01\ \text{m}^2)(5.0)(8.85 \times 10^{-12}\ \text{F/m})}{1.27 \times 10^{-5}\ \text{m}} = \mathbf{34.8\ nF}$$

Related Problem Determine C where $A = 3.6 \times 10^{-5}\ \text{m}^2$, $d = 1$ mil (2.54×10^{-5} m), and ceramic is the dielectric.

SECTION 12–1 CHECKUP
Answers are at the end of the chapter.

1. Define *capacitance*.
2. (a) How many microfarads are in one farad?
   (b) How many picofarads are in one farad?
   (c) How many picofarads are in one microfarad?
3. Convert 0.0015 μF to picofarads. To farads.
4. How much energy in joules is stored by a 0.01 μF capacitor with 15 V across its plates?
5. (a) When the plate area of a capacitor is increased, does the capacitance increase or decrease?
   (b) When the distance between the plates is increased, does the capacitance increase or decrease?
6. The plates of a ceramic capacitor are separated by 2 mils. What is the typical breakdown voltage?
7. A capacitor with a value of 2 μF at 25°C has a positive temperature coefficient of 50 ppm/°C. What is the capacitance value when the temperature increases to 125°C?

12–2 Types of Capacitors

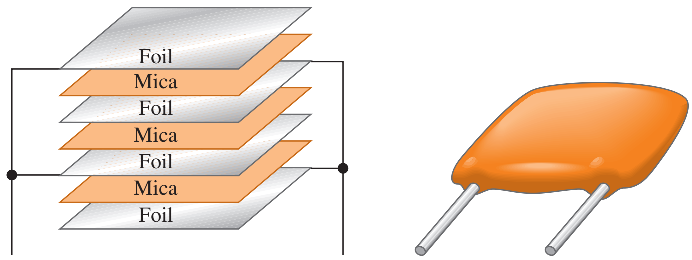
Capacitors normally are classified according to the type of dielectric material and whether they are polarized or nonpolarized. The most common types of dielectric materials are mica, ceramic, plastic-film, and electrolytic (aluminum oxide and tantalum oxide).

After completing this section, you should be able to

- **Discuss various types of capacitors**
  - Describe the characteristics of mica, ceramic, plastic-film, and electrolytic capacitors
  - Describe types of variable capacitors
  - Identify capacitor labeling
  - Discuss capacitance measurement

Fixed Capacitors

Mica Capacitors Two types of mica capacitors are stacked-foil and silver-mica. The basic construction of the stacked-foil type is shown in Figure 12–7. It consists of alternate layers of metal foil and thin sheets of mica. The metal foil forms the plate, with alternate foil sheets connected together to increase the plate area. More layers are used to increase the plate area, thus increasing the capacitance. The mica/foil stack is encapsulated in an insulating material such as Bakelite®, as shown in Figure 12–7(b). A silver-mica capacitor is formed in a similar way by stacking mica sheets with silver electrode material screened on them.



(a) Stacked layer arrangement

(b) Layers are pressed together and encapsulated

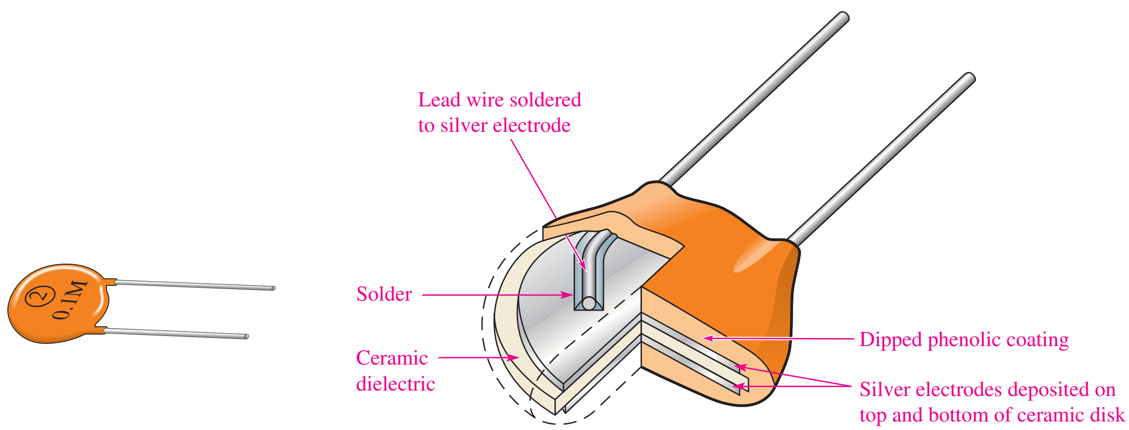
▲ **FIGURE 12–7**

Construction of a typical radial-lead mica capacitor.

Mica capacitors are available with capacitance values ranging from 1 pF to 0.1 μF and voltage ratings from 100 V dc to 2,500 V dc. Mica has a typical dielectric constant of 5.

Ceramic Capacitors Ceramic dielectrics provide very high dielectric constants (1,200 is typical). As a result, comparatively high capacitance values can be achieved in a small physical size. Ceramic capacitors are commonly available in a ceramic disk form, as shown in Figure 12–8, in a multilayer radial-lead configuration, as shown in Figure 12–9, or in a leadless ceramic chip, as shown in Figure 12–10, for surface mounting on printed circuit boards.

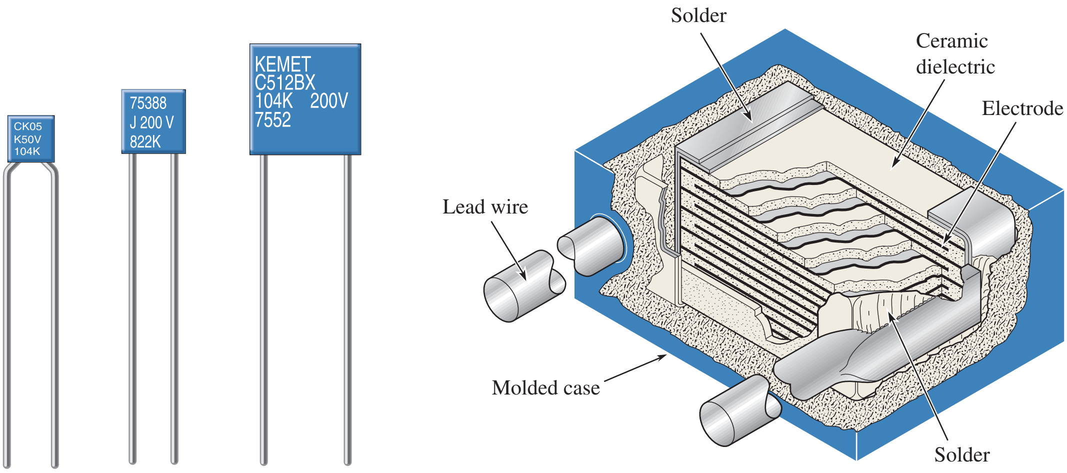
Ceramic capacitors typically are available in capacitance values ranging from 1 pF to 100 μF with voltage ratings up to 6 kV.



(a) (b)

▲ **FIGURE 12–8**

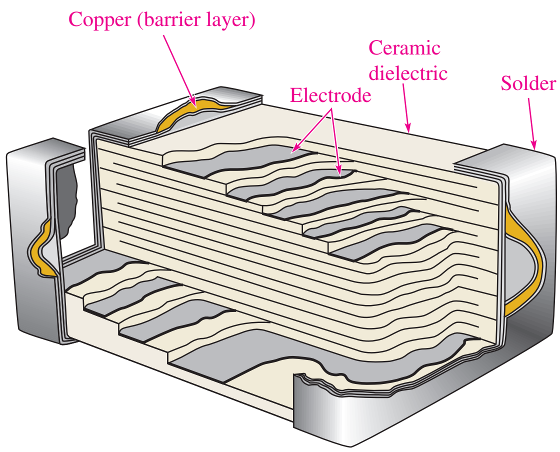
A ceramic disk capacitor and its basic construction.



(a) (b)

▲ **FIGURE 12–9**

(a) Typical ceramic capacitors. (b) Construction view.

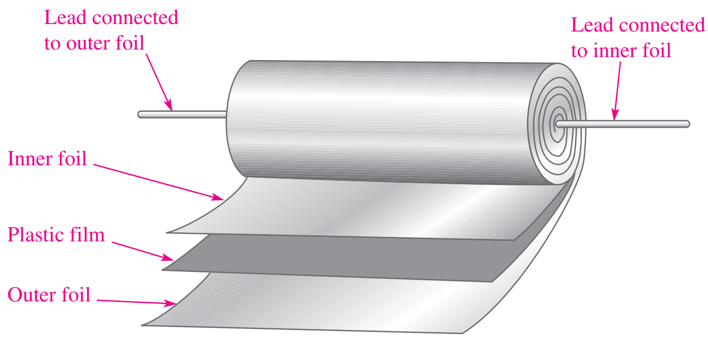


◀ **FIGURE 12–10**

Construction view of a typical ceramic chip capacitor used for surface mounting on printed circuit boards.

Plastic-Film Capacitors Common dielectric materials used in plastic-film capacitors include polycarbonate, propylene, polyester, polystyrene, polypropylene, and mylar. Some of these types have capacitance values up to 100 μF but most are less than 1 μF.

Figure 12–11 shows a common basic construction used in many plastic-film capacitors. A thin strip of plastic-film dielectric is sandwiched between two thin metal



▲ **FIGURE 12–11**

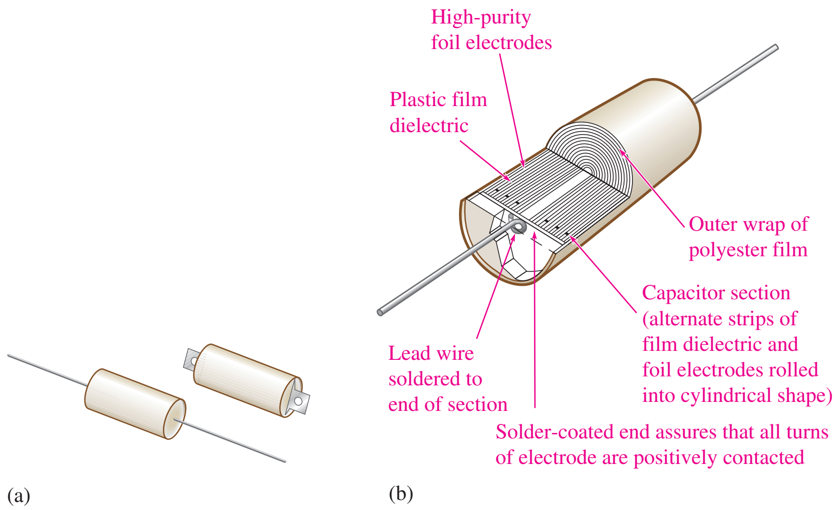
Basic construction of axial-lead tubular plastic-film dielectric capacitors.

TECH NOTE

Scientists are working with graphene, a carbon-based material that may be used for improved charge storage in rechargeable batteries and ultra-capacitors. The ability to store large amounts of charge is important in many applications from office copiers to improving the efficiency of electric and hybrid vehicles. The new technology may speed development of renewable energies such as wind and solar power, which need to be able to store large amounts of energy.

strips that act as plates. One lead is connected to the inner plate and one is connected to the outer plate as indicated. The strips are then rolled in a spiral configuration and encapsulated in a molded case. Thus, a large plate area can be packaged in a relatively small physical size, thereby achieving large capacitance values. Another method uses metal deposited directly on the film dielectric to form the plates.

Figure 12–12(a) shows typical plastic-film capacitors. Figure 12–12(b) shows a construction view for one type of plastic-film capacitor.



▲ **FIGURE 12–12**

(a) Typical capacitors. (b) Construction view of plastic-film capacitor.

SAFETY NOTE

Be extremely careful with electrolytic capacitors because it does make a difference which way an electrolytic capacitor is connected. Always observe the proper polarity. If a polarized capacitor is connected backwards, it may explode and cause injury.

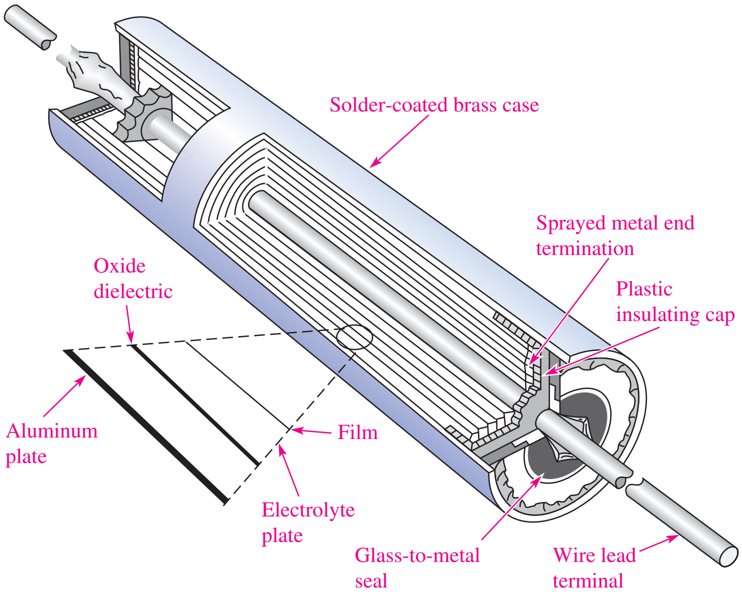
Electrolytic Capacitors Electrolytic capacitors are polarized so that one plate is positive and the other negative. These capacitors are used for capacitance values from 1 μF up to over 200,000 μF, but they have relatively low breakdown voltages (350 V is a typical maximum but some are rated for much lower voltages). Electrolytic capacitors typically have high leakage current. In this text, capacitors with values of 1 μF or greater are considered to be polarized.

In recent years, manufacturers have developed new electrolytic capacitors with much larger capacitance values; however, these new capacitors have lower voltage ratings than smaller-value capacitors and tend to be expensive. Super capacitors with capacitances of hundreds of farads are available. These capacitors are useful for battery backup and for applications like small motor starters that require a very large capacitance.

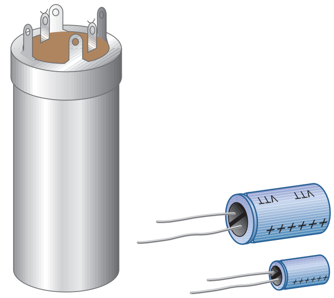
Electrolytic capacitors offer much higher capacitance values than mica or ceramic capacitors, but their voltage ratings are typically lower. Aluminum electrolytics are probably the most commonly used type. While other capacitors use two similar plates, the electrolytic consists of one plate of aluminum foil and another plate made of a conducting electrolyte applied to a material such as plastic film. These two "plates" are separated by a layer of aluminum oxide that forms on the surface of the aluminum plate. Figure 12–13(a) illustrates the basic construction of a typical aluminum electrolytic capacitor with axial leads. Other electrolytics with radial leads are shown in Figure 12–13(b); the symbol for an electrolytic capacitor is shown in part (c).

Tantalum electrolytics can be in either a tubular configuration similar to Figure 12–13 or "tear drop" shape as shown in Figure 12–14. In the tear drop configuration, the positive plate is actually a pellet of tantalum powder rather than a sheet of foil. Tantalum pentoxide forms the dielectric, and manganese dioxide forms the negative plate.

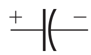
Because of the process used for the insulating oxide dielectric, the metallic (aluminum or tantalum) plate must be connected so that it is always positive with respect



(a) Construction view of an axial-lead electrolytic capacitor



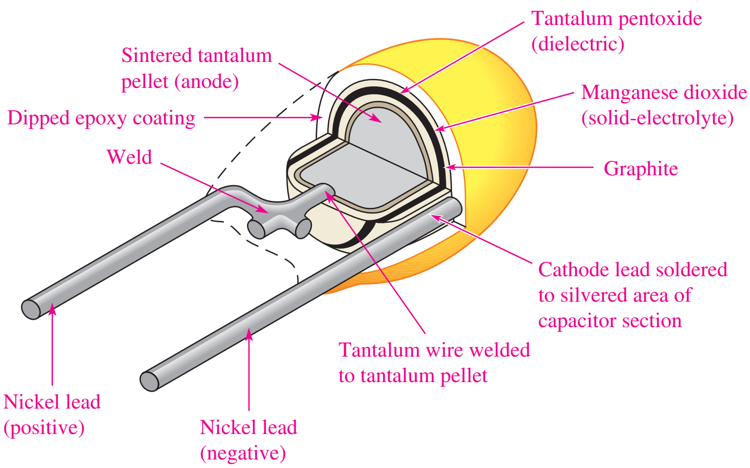
(b) Typical radial-lead electrolytics



(c) Symbol for an electrolytic capacitor. The straight plate is positive and the curved plate is negative, as indicated

▲ **FIGURE 12–13**

Examples of electrolytic capacitors.

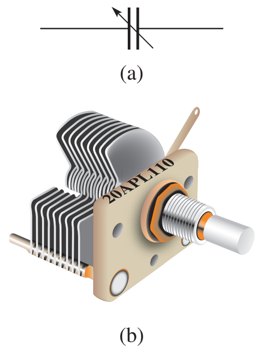


▲ **FIGURE 12–14**

Construction view of a typical "tear drop" shaped tantalum electrolytic capacitor.

to the electrolyte plate, and, thus all electrolytic capacitors are polarized. The metal plate (positive lead) is usually indicated by a plus sign or some other obvious marking and must always be connected in a dc circuit where the voltage across the capacitor does not change polarity regardless of any ac present. Reversal of the polarity of the voltage will usually result in complete destruction of the capacitor.

The problem of dielectric absorption occurs mostly in electrolytic capacitors when they do not completely discharge during use and retain a residual charge. Dielectric absorption is the spontaneous development of a capacitor voltage due to delayed dipole discharge within the dielectric when a capacitor is briefly discharged after being charged for a long time. Approximately 25% of defective capacitors exhibit this condition.



▲ FIGURE 12–15

(a) Schematic symbol for a variable capacitor. (b) A variable capacitor with a rotary control.

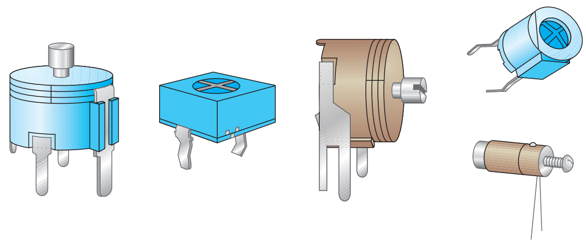
Variable Capacitors

Variable capacitors are used in a circuit when there is a need to adjust the capacitance value either manually or automatically. These capacitors are generally less than 300 pF but are available in larger values for specialized applications. The schematic symbol for a variable capacitor is shown in Figure 12–15(a). Figure 12–15(b) shows a variable capacitor with a rotary control in which capacitance is varied by increasing or decreasing the overlap of the capacitor plates.

Adjustable capacitors that normally have slotted screw-type adjustments and are used for very fine adjustments in a circuit are called **trimmers**. Ceramic or mica is a common dielectric in these types of capacitors, and the capacitance usually is changed by adjusting the plate separation. Generally, trimmer capacitors have values less than 100 pF. Figure 12–16 shows some typical devices.

► FIGURE 12–16

Examples of trimmer capacitors.



The **varactor** is a semiconductor device that exhibits a capacitance characteristic that is varied by changing the voltage across its terminals. This device usually is covered in detail in a course on electronic devices.

Capacitor Labeling

Capacitor values are indicated on the body of the capacitor either by typographical labels or by color codes. Typographical labels consist of letters and numbers that indicate various parameters such as capacitance, voltage rating, and tolerance.

Some capacitors carry no unit designation for capacitance. In these cases, the units are implied by the value indicated and are recognized by experience. For example, a ceramic capacitor marked .001 or .01 has units of microfarads because picofarad values that small are not available. As another example, a ceramic capacitor labeled 50 or 330 has units of picofarads because microfarad units that large normally are not available in this type. In some cases, a 3-digit designation is used. The first two digits are the first two digits of the capacitance value. The third digit is the number of zeros after the second digit. For example, 103 means 10,000 pF. In some instances, the units are labeled as pF or μF; sometimes the microfarad unit is labeled as MF or MFD.

A voltage rating appears on some types of capacitors with WV or WVDC and is omitted on others. When it is omitted, the voltage rating can be determined from information supplied by the manufacturer. The tolerance of the capacitor is usually labeled as a percentage, such as $\pm 10\%$. The temperature coefficient is indicated by a *parts per million* marking. This type of label consists of a P or N followed by a number. For example, N750 means a negative temperature coefficient of 750 ppm/°C, and P30 means a positive temperature coefficient of 30 ppm/°C. An NP0 designation means that the positive and negative coefficients are zero; thus the capacitance does not change with temperature. Certain types of capacitors are color coded. Refer to Appendix C for additional capacitor labeling and color code information.

▲ FIGURE 12–17

A typical autoranging capacitance meter. (Courtesy of B + K Precision)

Capacitance Measurement

A capacitance meter such as the one shown in Figure 12–17 can be used to check the value of a capacitor. Also, many DMMs provide a capacitance measurement feature. No matter what meter you use, it is important to discharge the capacitor before attempting to measure its value. Most capacitors change value over a period of time, some more than others. Ceramic capacitors, for example, often exhibit a 10% to 15% change in value during the first year. Electrolytic capacitors are particularly subject to value change due to drying of the electrolytic solution. Capacitor aging is a principle reason that circuit designers derate capacitors in their designs. In other cases, capacitors may be labeled incorrectly or the wrong value may have been installed in the circuit. Although a value change represents less than 25% of defective capacitors, a value check can quickly eliminate this as a source of trouble when troubleshooting a circuit.

For the B and K Precision 890C capacitance meter (shown), capacitors up to 50,000 μF can be measured. Some capacitance meters can also be used to check for leakage current in capacitors. In order to check for leakage, a sufficient voltage must be applied across the capacitor to simulate operating conditions. This is automatically done by the test instrument. Over 40% of all defective capacitors have excessive leakage current and electrolytics are particularly susceptible to this problem.

SECTION 12–2 CHECKUP

1. Name one common way capacitors can be classified.
2. What is the difference between a fixed and a variable capacitor?
3. What type of capacitor is polarized?
4. What precautions must be taken when installing a polarized capacitor in a circuit?

12–3 Series Capacitors

The total capacitance of a series connection of capacitors is less than the individual capacitance of any of the capacitors. Capacitors in series divide voltage across them in proportion to their capacitance.

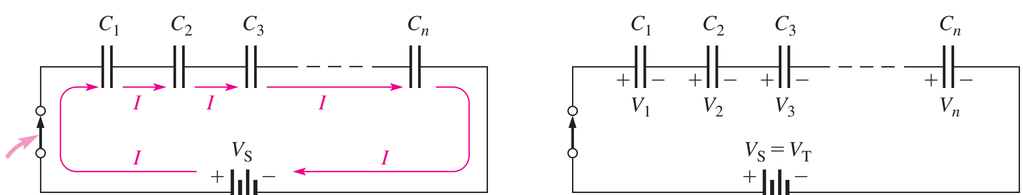
After completing this section, you should be able to

- **Analyze series capacitors**
  - Determine total capacitance
  - Determine capacitor voltages

Total Capacitance

When capacitors are connected in series, the total capacitance is less than the smallest capacitance value because the effective plate separation increases. The calculation of total series capacitance is analogous to the calculation of total resistance of parallel resistors (Chapter 6).

Consider the generalized circuit in Figure 12–18(a), which has n capacitors in series with a voltage source and a switch. When the switch is closed, the capacitors charge as



(a) Charging current is same for each capacitor, $I = Q/t$.

(b) All capacitors store same amount of charge and $V = Q/C$.

▲ **FIGURE 12–18**

A series capacitive circuit.

current is established through the circuit. Since this is a series circuit, the current must be the same at all points, as illustrated. Since current is the rate of flow of charge, the amount of charge stored by each capacitor is equal to the total charge, expressed as

Equation 12–7

$$Q_T = Q_1 = Q_2 = Q_3 = \cdots = Q_n$$

Next, according to Kirchhoff's voltage law, the sum of the voltages across the charged capacitors must equal the total voltage, V_T, as shown in Figure 12–18(b). This is expressed in equation form as

$$V_T = V_1 + V_2 + V_3 + \cdots + V_n$$

From Equation 12–3, $V = Q/C$. When this relationship is substituted into each term of the voltage equation, the following result is obtained:

$$\frac{Q_T}{C_T} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3} + \cdots + \frac{Q_n}{C_n}$$

Since the charges on all the capacitors are equal, the Q terms can be factored and canceled, resulting in

Equation 12–8

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$

Taking the reciprocal of both sides of Equation 12–8 yields the following general formula for total series capacitance:

Equation 12–9

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}}$$

Remember,

The total series capacitance is always less than the smallest capacitance.

Two Capacitors in Series When only two capacitors are in series, a special form of Equation 12–8 can be used.

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{C_1 + C_2}{C_1 C_2}$$

Taking the reciprocal of the left and right terms gives the formula for total capacitance of two capacitors in series.

Equation 12–10

$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$

Notice that this product-over-sum rule for *series* capacitors is analogous to the product-over-sum rule for *parallel* resistors.

Capacitors of Equal Value in Series This special case is another in which a formula can be developed from Equation 12–8. When all capacitor values are the same and equal to C, the formula is

$$\frac{1}{C_T} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} + \cdots + \frac{1}{C}$$

Adding all the terms on the right yields

$$\frac{1}{C_T} = \frac{n}{C}$$

where n is the number of equal-value capacitors. Taking the reciprocal of both sides yields

Equation 12–11

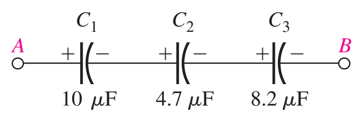
$$C_T = \frac{C}{n}$$

The capacitance value of the equal capacitors divided by the number of equal series capacitors gives the total capacitance. Notice that Equation 12–11 is analogous to finding the total value of n equal value resistors in parallel.

EXAMPLE 12–5

Determine the total capacitance between points A and B in Figure 12–19.

▶ FIGURE 12–19



Solution Use Equation 12–9.

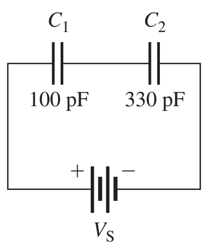
$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{1}{\frac{1}{10\ \mu F} + \frac{1}{4.7\ \mu F} + \frac{1}{8.2\ \mu F}} = \mathbf{2.30\ \mu F}$$

Related Problem If a 4.7 μF capacitor is connected in series with the three existing capacitors in Figure 12–19, what is C_T?

EXAMPLE 12–6

Find the total capacitance, C_T, in Figure 12–20.

▶ FIGURE 12–20



Solution From Equation 12–10,

$$C_T = \frac{C_1 C_2}{C_1 + C_2} = \frac{(100\ pF)(330\ pF)}{430\ pF} = \mathbf{76.7\ pF}$$

You can also use Equation 12–9.

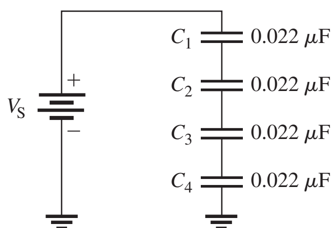
$$C_T = \frac{1}{\frac{1}{100\ pF} + \frac{1}{330\ pF}} = \mathbf{76.7\ pF}$$

Related Problem Determine C_T if $C_1 = 470$ pF and $C_2 = 680$ pF in Figure 12–20.

EXAMPLE 12–7

Determine C_T for the series capacitors in Figure 12–21.

▶ FIGURE 12–21



Solution Since $C_1 = C_2 = C_3 = C_4 = C$, use Equation 12–11,

$$C_T = \frac{C}{n} = \frac{0.022\ \mu F}{4} = \mathbf{5.50\ nF}$$

Related Problem Determine C_T if the capacitor values in Figure 12–21 are doubled.

Capacitor Voltages

A series connection of charged capacitors acts as a voltage divider. The voltage across each capacitor in series is inversely proportional to its capacitance value, as shown by the formula $V = Q/C$. You can determine the voltage across any individual capacitor in series with the following formula:

Equation 12–12

$$V_x = \left(\frac{C_T}{C_x}\right)V_T$$

where C_x is any capacitor in series (such as C_1, C_2, or C_3), V_x is the voltage across C_x, and V_T is the total voltage across the capacitors. The derivation is as follows: Since the charge on any capacitor in series is the same as the total charge ($Q_x = Q_T$), and since $Q_x = V_xC_x$ and $Q_T = V_TC_T$,

$$V_xC_x = V_TC_T$$

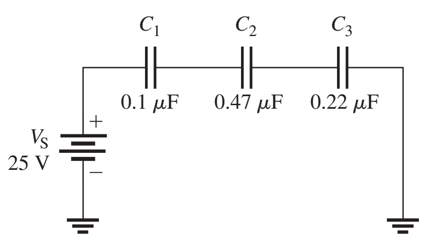
Solving for V_x yields

$$V_x = \frac{C_TV_T}{C_x}$$

The largest-value capacitor in a series connection will have the smallest voltage across it. The smallest-value capacitor will have the largest voltage across it.

EXAMPLE 12–8 Find the voltage across each capacitor in Figure 12–22.

► FIGURE 12–22



Solution Calculate the total capacitance.

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}} = \frac{1}{\frac{1}{0.1\ \mu F} + \frac{1}{0.47\ \mu F} + \frac{1}{0.22\ \mu F}} = \mathbf{60.0\ nF}$$

From Figure 12–22, $V_S = V_T = 25$ V. Therefore, use Equation 12–12 to calculate the voltage across each capacitor.

$$V_1 = \left(\frac{C_T}{C_1}\right)V_T = \left(\frac{0.06\ \mu\text{F}}{0.1\ \mu\text{F}}\right)25\text{ V} = \mathbf{15.0\ V}$$

$$V_2 = \left(\frac{C_T}{C_2}\right)V_T = \left(\frac{0.06\ \mu\text{F}}{0.47\ \mu\text{F}}\right)25\text{ V} = \mathbf{3.19\ V}$$

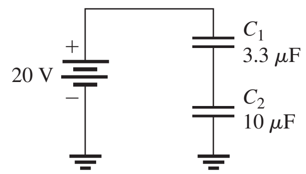
$$V_3 = \left(\frac{C_T}{C_3}\right)V_T = \left(\frac{0.06\ \mu\text{F}}{0.22\ \mu\text{F}}\right)25\text{ V} = \mathbf{6.82\ V}$$

Related Problem Another 0.47 μF capacitor is connected in series with the existing capacitor in Figure 12–22. Determine the voltage across the new capacitor, assuming all the capacitors are initially uncharged.

SECTION 12–3 CHECKUP

1. Is the total capacitance of a series connection less than or greater than the value of the smallest capacitor?
2. The following capacitors are in series: 100 pF, 220 pF, and 560 pF. What is the total capacitance?
3. A 0.01 μF and a 0.015 μF capacitor are in series. Determine the total capacitance.
4. Five 100 pF capacitors are connected in series. What is C_T?
5. Determine the voltage across C_1 in Figure 12–23.

► FIGURE 12–23



12–4 PARALLEL CAPACITORS

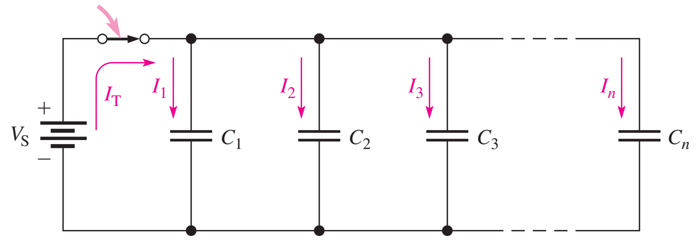
Capacitances add when capacitors are connected in parallel.

After completing this section, you should be able to

- **Analyze parallel capacitors**
  - Determine total capacitance

When capacitors are connected in parallel, the total capacitance is the sum of the individual capacitances because the effective plate area increases. The calculation of total parallel capacitance is analogous to the calculation of total series resistance (Chapter 5).

Consider what happens when the switch in Figure 12–24 is closed. The total charging current from the source divides at the junction of the parallel branches. There is a separate charging current through each branch so that a different charge can be stored by each capacitor. By Kirchhoff's current law, the sum of all of the charging



▲ **FIGURE 12–24**

Capacitors in parallel.

currents is equal to the total current. Therefore, the sum of the charges on the capacitors is equal to the total charge. Also, the voltages across all of the parallel branches are equal. These observations are used to develop a formula for total parallel capacitance as follows for the general case of n capacitors in parallel.

Equation 12–13

$$Q_T = Q_1 + Q_2 + Q_3 + \cdots + Q_n$$

From Equation 12–2, $Q = CV$. When this relationship is substituted into each term of Equation 12–13, the following result is obtained:

$$C_TV_T = C_1V_1 + C_2V_2 + C_3V_3 + \cdots + C_nV_n$$

Since $V_T = V_1 = V_2 = V_3 = \cdots = V_n$, the voltages can be factored and canceled, giving

Equation 12–14

$$C_T = C_1 + C_2 + C_3 + \cdots + C_n$$

Equation 12–14 is the general formula for total parallel capacitance where n is the number of capacitors. Remember,

The total parallel capacitance is the sum of all the capacitors in parallel.

For the special case when all of the capacitors have the same value, C, multiply the value by the number (n) of capacitors in parallel.

Equation 12–15

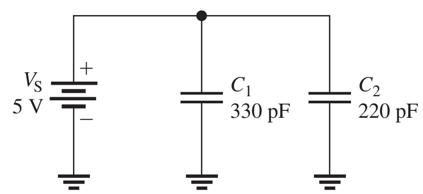
$$C_T = nC$$

Notice that Equations 12–14 and 12–15, for parallel capacitors, are analogous to finding the resistance of series resistors.

EXAMPLE 12–9

What is the total capacitance in Figure 12–25? What is the voltage across each capacitor?

► **FIGURE 12–25**



Solution The total capacitance is

$$C_T = C_1 + C_2 = 330 \text{ pF} + 220 \text{ pF} = \mathbf{550\ pF}$$

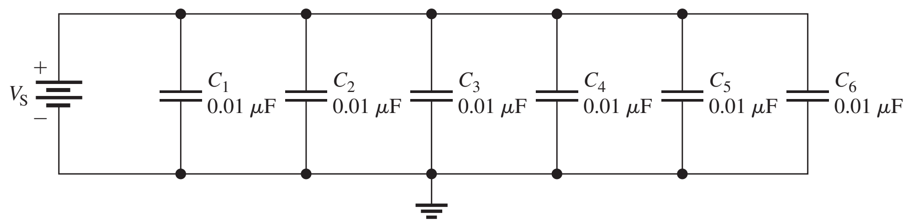
The voltage across each capacitor in parallel is equal to the source voltage.

$$V_S = V_1 = V_2 = \mathbf{5\ V}$$

Related Problem What is C_T if a 100 pF capacitor is connected in parallel with C_1 and C_2 in Figure 12–25?

EXAMPLE 12–10

Determine C_T in Figure 12–26.



▲ FIGURE 12–26

Solution There are six equal-value capacitors in parallel, so $n = 6$.

$$C_T = nC = (6)(0.01\ \mu F) = \mathbf{0.06\ \mu F}$$

Related Problem If three more 0.01 μF capacitors are connected in parallel in Figure 12–26, what is the total capacitance?

SECTION 12–4 CHECKUP

1. How is total parallel capacitance determined?
2. In a certain application, you need 0.05 μF. The only values available are 0.01 μF, which are available in large quantities. How can you get the total capacitance that you need?
3. The following capacitors are in parallel: 10 pF, 56 pF, 33 pF, and 68 pF. What is C_T?

12–5 CAPACITORS IN DC CIRCUITS

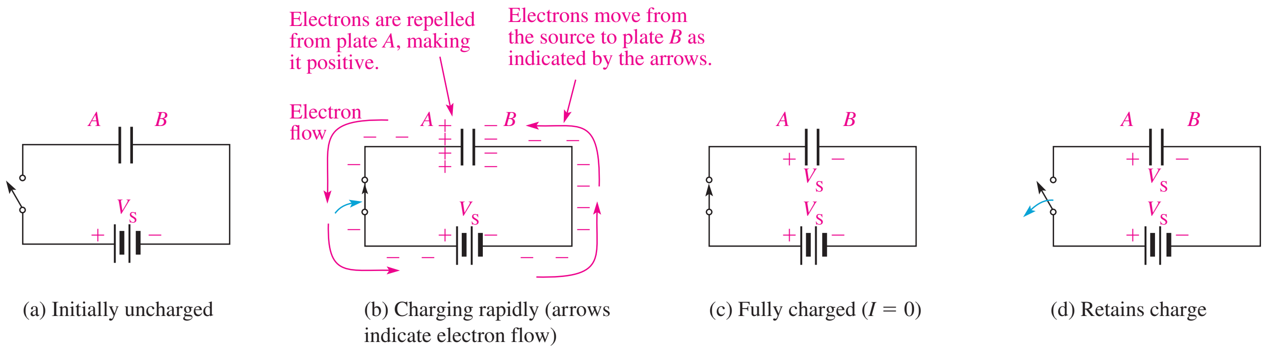
A capacitor will charge up when it is connected to a dc voltage source. The buildup of charge across the plates occurs in a predictable manner that is dependent on the capacitance and the resistance in a circuit.

After completing this section, you should be able to

- **Analyze capacitive dc switching circuits**
  - Describe the charging and discharging of a capacitor
  - Define *RC time constant*
  - Relate the time constant to charging and discharging of a capacitor
  - Write equations for the charging and discharging curves
  - Explain why a capacitor blocks dc

Charging a Capacitor

A capacitor will charge when it is connected to a dc voltage source, as shown in Figure 12–27. The capacitor in part (a) of the figure is uncharged; that is, plate *A* and plate *B* have equal numbers of free electrons and positive metallic ions. There is no net charge on either plate. When the switch is closed, as shown in part (b), the source



▲ FIGURE 12–27

Charging a capacitor.

SAFETY NOTE

Large capacitors used in high voltage circuits can store a lethal amount of energy long after power is removed. Even close proximity can cause a painful high-voltage arc to result in an electric shock. Normally, capacitors are discharged through an insulated resistor that is rated for the current and power that it will dissipate. A shorting wire or bar is NOT recommended for large capacitors or high voltages as it can damage the capacitor. Commercially made capacitor discharge tools are available in these cases but need to be rated for the specific voltage and capacitance.

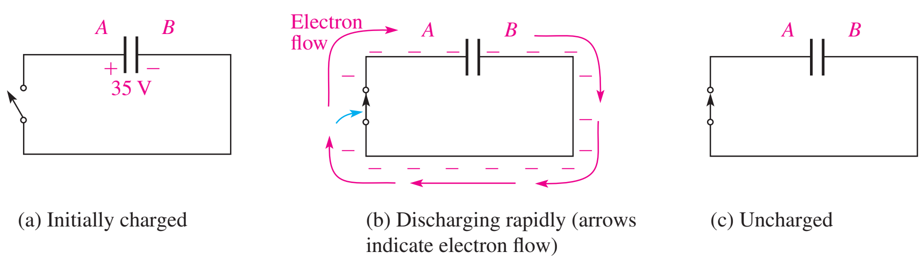
moves electrons to plate *B* causing it to have a negative charge. The excess of electrons on plate *B* repels electrons from plate *A*, leaving it with a net positive charge consisting of the positive metal ions. The electrons from plate *A* return to the source as indicated by the arrows. This process is very fast and the voltage across the plates very soon is equal to the applied voltage, V_S, but opposite in polarity, as shown in part (c). When the capacitor is fully charged, there is no current.

A capacitor blocks constant dc.

When the charged capacitor is disconnected from the source, as shown in Figure 12–27(d), it remains charged for long periods of time, depending on its leakage resistance, and can cause severe electrical shock. The charge on an electrolytic capacitor generally leaks off more rapidly than in other types of capacitors.

Discharging a Capacitor

When a wire is connected across a charged capacitor, as shown in Figure 12–28, the capacitor will discharge. (This is not recommended for large capacitors or high voltages—see Safety Note.) In this particular case, a very low resistance path (the wire) is connected across the capacitor with a switch. Before the switch is closed, the capacitor is charged to 35 V, as indicated in part (a). When the switch is closed, as shown in part (b), the excess electrons on plate *B* move through the circuit to plate *A* and very quickly discharge the capacitor. As a result of the electrons moving through the low resistance of the wire, the energy stored by the capacitor is dissipated in the wire. At this time, the voltage across the capacitor is zero, and the capacitor is completely discharged, as shown in part (c).



▲ FIGURE 12–28

Discharging a charged capacitor.

Current and Voltage During Charging and Discharging

Notice in Figures 12–27 and 12–28 that the direction of electron flow during discharge is opposite to that during charging. It is important to understand that ideally *there is no current through the dielectric of the capacitor during charging or discharging* *because the dielectric is an insulating material.* There is current from one plate to the other only through the external circuit.

Figure 12–29(a) shows a capacitor connected in series with a resistor and a switch to a dc voltage source. Initially, the switch is open and the capacitor is uncharged with zero volts across its plates. At the instant the switch is closed, the current jumps to its maximum value and the capacitor begins to charge. The current is maximum initially because the capacitor has zero volts across it and, therefore, effectively acts as a short. From Kirchhoff's voltage law, the source voltage is across R and the initial current is $I = V_S/R$. As time passes and the capacitor charges, the current decreases and the voltage across the capacitor (V_C) increases. The resistor voltage is proportional to the current during this charging period.

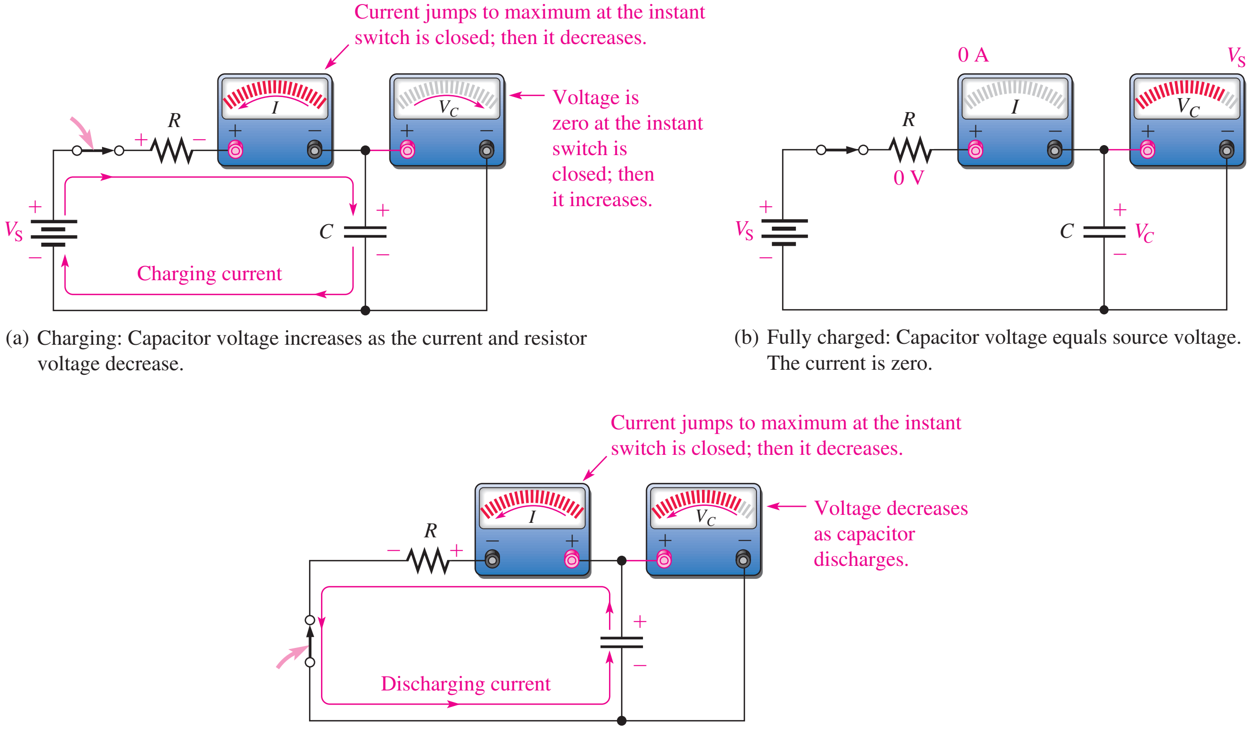
After a certain period of time, the capacitor reaches full charge. At this point, the current is zero and the capacitor voltage is equal to the dc source voltage, as shown in Figure 12–29(b). If the switch were opened now, the capacitor would retain its full charge (neglecting any leakage).

In Figure 12–29(c), the voltage source has been removed. When the switch is closed, the capacitor begins to discharge. The initial current is maximum and is given by V_C/R but in a direction opposite to its direction during charging. As time passes, the current and capacitor voltage decrease. The resistor voltage is always proportional to the current. When the capacitor has fully discharged, the current and the capacitor voltage are zero.

Remember the following rules about capacitors in dc circuits:

1. A capacitor appears as an *open* to constant voltage.
2. A capacitor appears as a *short* to an instantaneous change in voltage.

Now let's examine in more detail how the voltage and current change with time in a capacitive circuit.



(a) Charging: Capacitor voltage increases as the current and resistor voltage decrease.

(b) Fully charged: Capacitor voltage equals source voltage. The current is zero.

(c) Discharging: Capacitor voltage, resistor voltage, and the current decrease from their initial maximum values. Note that the discharge current is opposite to the charge current.

▲ **FIGURE 12–29**

Current and voltage in a charging and discharging capacitor.

The *RC* Time Constant

In a practical situation, there cannot be capacitance without some resistance in a circuit. It may simply be the small resistance of a wire, a Thevenin source resistance, or it may be a physical resistor. Because of this, the charging and discharging characteristics of a capacitor must always be considered with the associated resistance. The resistance introduces the element of *time* in the charging and discharging of a capacitor.

When a capacitor charges or discharges through a resistance, a certain time is required for the capacitor to charge fully or discharge fully. The voltage across a capacitor cannot change instantaneously because a finite time is required to move charge from one point to another. The time constant of a series *RC* circuit determines the rate at which the capacitor charges or discharges.

The *RC* time constant is a fixed time interval that equals the product of the resistance and the capacitance in a series *RC* circuit.

The time constant is expressed in seconds when resistance is in ohms and capacitance is in farads. It is symbolized by τ (Greek letter tau), and the formula is

Equation 12–16

$$\tau = RC$$

Recall that $I = Q/t$. The current depends on the amount of charge moved in a given time. When the resistance is increased, the charging current is reduced, thus increasing the charging time of the capacitor. When the capacitance is increased, the amount of charge increases; thus, for the same current, more time is required to charge the capacitor.

It is useful to show that the unit for resistance multiplied by the unit for capacitance produces the unit for time. This is a common check to verify that an equation is consistent called dimensional analysis. (It does not prove that an equation is correct, but it can indicate a problem.) By Ohm's law, $R = \dfrac{V}{I}$, so the unit for resistance can be written as $\dfrac{\text{volt}}{\text{ampere}}$. Capacitance was defined as $C = \dfrac{Q}{V}$ (Equation 12–1), which has units of $\dfrac{\text{coulomb}}{\text{volt}}$. Analyzing the units for *RC* then gives

$$RC = \left(\frac{\cancel{\text{volt}}}{\text{ampere}}\right)\left(\frac{\text{coulomb}}{\cancel{\text{volt}}}\right) = \frac{\text{coulomb}}{\text{ampere}} = \frac{\cancel{\text{coulomb}}}{\dfrac{\cancel{\text{coulomb}}}{\text{second}}} = \text{second.}$$

EXAMPLE 12–11

A series *RC* circuit has a resistance of 1.0 MΩ and a capacitance of 4.7 μF. What is the time constant?

Solution

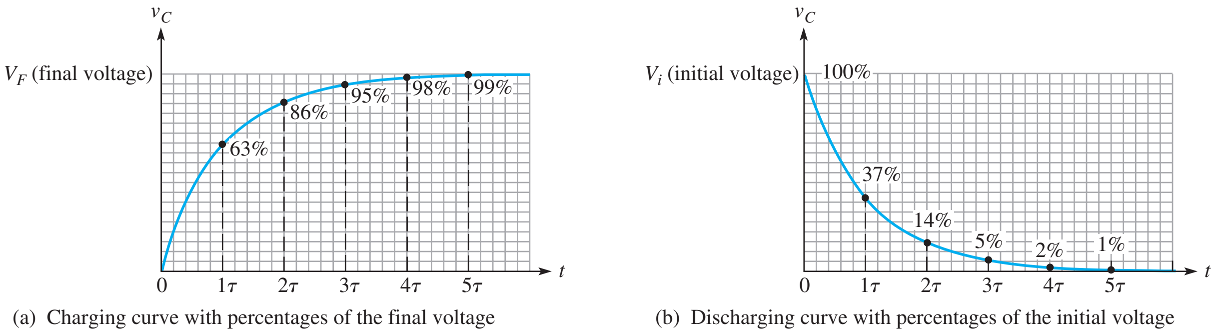
$$\tau = RC = (1.0 \times 10^6\ \Omega)(4.7 \times 10^{-6}\ \text{F}) = \mathbf{4.7\ s}$$

Related Problem A series *RC* circuit has a 270 kΩ resistor and a 3,300 pF capacitor. What is the time constant?

When a capacitor is charging or discharging between two voltage levels, the charge on the capacitor changes by approximately 63% of the difference in the levels in one time constant. An uncharged capacitor charges to 63% of its fully charged voltage in one time constant. When a capacitor is discharging, its voltage drops to approximately 100% − 63% = 37% of its initial value in one time constant, which is a 63% change.

The Charging and Discharging Curves

A capacitor charges and discharges following a nonlinear curve, as shown in Figure 12–30. In these graphs, the approximate percentage of full charge is shown at each time-constant interval. This type of curve follows a precise mathematical formula and is



▲ FIGURE 12–30

Exponential voltage curves for the charging and discharging of an *RC* circuit.

called an *exponential curve.* The charging curve is an increasing exponential, and the discharging curve is a decreasing exponential. It takes five time constants to change the voltage by 99% (considered 100%). This five time-constant interval is generally accepted as the time to fully charge or discharge a capacitor and is called the *transient time.*

General Formula The general expressions for either increasing or decreasing exponential curves are given in the following equations for both instantaneous voltage and instantaneous current.

$$v = V_F + (V_i - V_F)e^{-t/\tau}$$ **Equation 12–17**

$$i = I_F + (I_i - I_F)e^{-t/\tau}$$ **Equation 12–18**

where V_F and I_F are the final values of voltage and current, and V_i and I_i are the initial values of voltage and current. The lowercase italic letters v and i are the instantaneous values of the capacitor voltage and current at time t, and e is the base of natural logarithms. The e^x key on a calculator makes it easy to work with this exponential term.

Charging from Zero The formula for the special case in which an increasing exponential voltage curve begins at zero ($V_i = 0$), as shown in Figure 12–30(a), is given in Equation 12–19. It is developed as follows, starting with the general formula, Equation 12–17.

$$v = V_F + (V_i - V_F)e^{-t/\tau} = V_F + (0 - V_F)e^{-t/RC} = V_F - V_F e^{-t/RC}$$

Factoring out V_F, you have

$$v = V_F(1 - e^{-t/RC})$$ **Equation 12–19**

Using Equation 12–19, you can calculate the value of the charging voltage of a capacitor at any instant of time if it is initially uncharged. You can calculate an increasing current by substituting i for v and I_F for V_F in Equation 12–19.

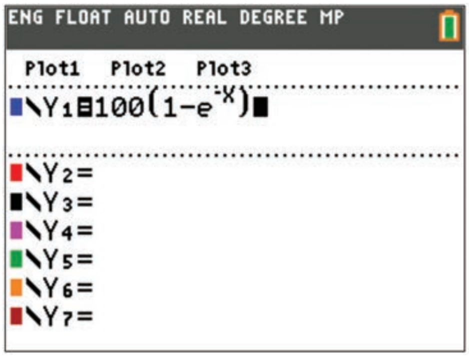
EXAMPLE 12–12

Graph the general formula exponential charging curve for an *RC* circuit using the TI-84 graphing calculator.

Solution In Equation 12–19, the dependent variable is v (instantaneous voltage) and the independent variable is t (time). On the calculator, y will represent v and x will represent t. Let $V_F = 100$ V; it can then represent 100% in reading the graph. Let $RC = 1$, so that the time axis will represent τ.

Press (y=) and enter the equation. Since $RC = 1$, it is not necessary to enter it as you are dividing by 1. Figure 12–31 shows the screen after entering the equation:

▶ **FIGURE 12–31**



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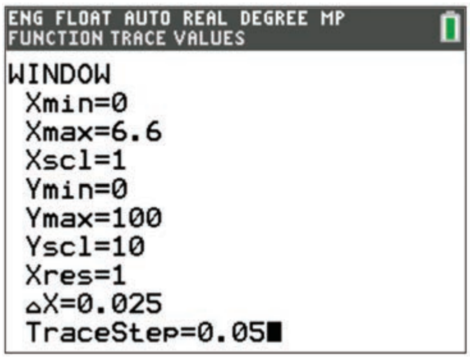
Before graphing the result, you need to set up the scale for the graph. Press 2nd zoom and choose RectGC (white letters on a black background). To show a grid, choose GridLine.

Next, press window and set up minimum and maximum values for the x and y coordinates, spacing between ticks on each axis and other parameters. Figure 12–32 shows suggested parameter settings.

Press graph and the graph is displayed as shown in Figure 12–33.

▶ **FIGURE 12–32**



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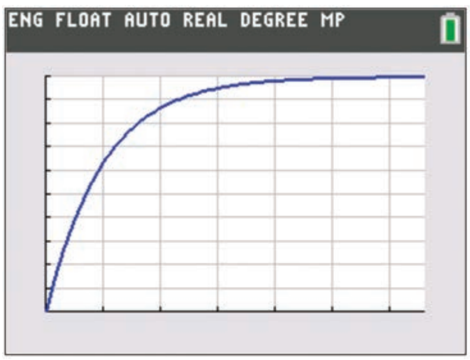
▶ **FIGURE 12–33**



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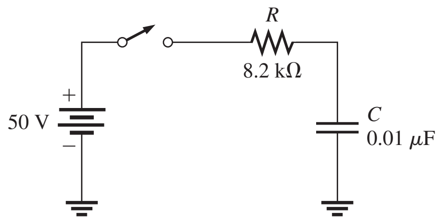
Related Problem You can use the trace key to view the specific values used to create the graph. Because 100 V was selected as the maximum value, you can read y values in percentage.

Describe the changes that need to be made to view the discharge curve in Figure 12–30(b) on the calculator.

EXAMPLE 12–13

In Figure 12–34, determine the capacitor voltage 50 μs after the switch is closed if the capacitor is initially uncharged. Draw the charging curve.

▶ **FIGURE 12–34**



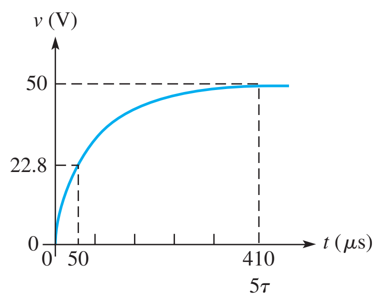
Solution The time constant is $RC = (8.2\text{ k}\Omega)(0.01\ \mu\text{F}) = 82\ \mu\text{s}$. The voltage to which the capacitor will fully charge is 50 V (this is V_F). The initial voltage is zero. Notice that 50 μs is less than one time constant; so the capacitor will charge less than 63% of the full voltage in that time.

$$v_C = V_F(1 - e^{-t/RC}) = (50\text{ V})(1 - e^{-50\ \mu\text{s}/82\ \mu\text{s}})$$

$$= (50\text{ V})(1 - e^{-0.61}) = (50\text{ V})(1 - 0.543) = \mathbf{22.8\ V}$$

The charging curve for the capacitor is shown in Figure 12–35.

▶ **FIGURE 12–35**



You can determine an exponential function on your calculator by using the e^x key and entering the value of the exponent of e.

You can also calculate the curve with a graphing calculator. The equation and sample graph are shown in Figure 12–36. The grid lines are at 10 V (y-axis) and 50 ms (x-axis).

▶ **FIGURE 12–36**

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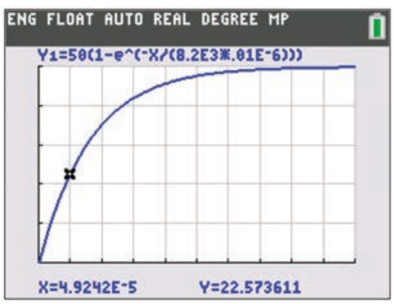


Related Problem Determine the capacitor voltage 15 μs after switch closure in Figure 12–34.

MultiSim

Use Multisim file E12–13 to verify the calculated results in this example and to confirm your calculation for the related problem.

Discharging to Zero The formula for the special case in which a decreasing exponential voltage curve ends at zero ($V_F = 0$), as shown in Figure 12–30(b), is derived from the general formula as follows:

$$v = V_F + (V_i - V_F)e^{-t/\tau} = 0 + (V_i - 0)e^{-t/RC}$$

This reduces to

Equation 12–20

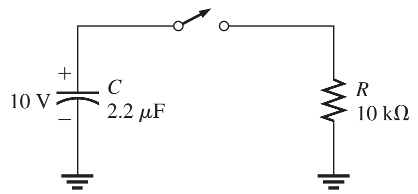
$$v = V_i e^{-t/RC}$$

where V_i is the voltage at the beginning of the discharge. You can use this formula to calculate the discharging voltage at any instant, as Example 12–14 illustrates.

EXAMPLE 12–14

Determine the capacitor voltage in Figure 12–37 at a point in time 6 ms after the switch is closed. Draw the discharging curve.

▶ **FIGURE 12–37**

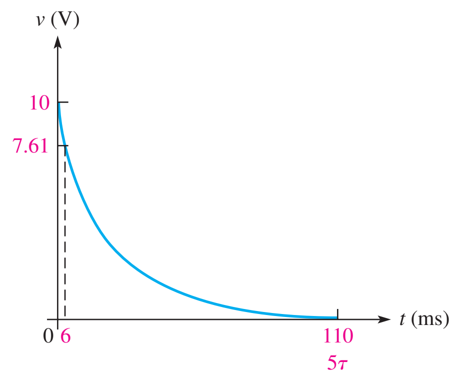


Solution The discharge time constant is $RC = (10\text{ k}\Omega)(2.2\ \mu\text{F}) = 22$ ms. The initial capacitor voltage is 10 V. Notice that 6 ms is less than one time constant, so the capacitor will discharge less than 63%. Therefore, it will have a voltage greater than 37% of the initial voltage at 6 ms.

$$v_C = V_i e^{-t/RC} = (10\text{ V})e^{-6\text{ ms}/22\text{ ms}} = (10\text{ V})e^{-0.27} = (10\text{ V})(0.761) = \mathbf{7.61\ V}$$

The discharging curve for the capacitor is shown in Figure 12–38.

▶ **FIGURE 12–38**



Related Problem In Figure 12–37, change R to 2.2 kΩ and determine the capacitor voltage 1 ms after the switch is closed.

Graphical Method Using Universal Exponential Curves The universal curves in Figure 12–39 provide a graphic solution of the charge and discharge of capacitors. Example 12–15 illustrates this graphical method.

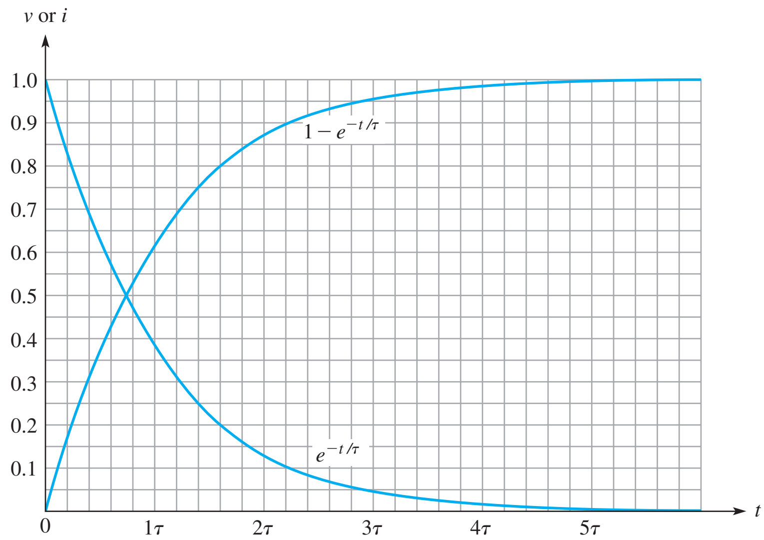


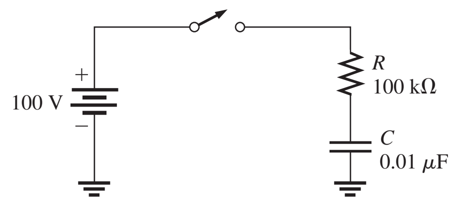
FIGURE 12–39

Normalized universal exponential curves.

EXAMPLE 12–15

How long will it take the initially uncharged capacitor in Figure 12–40 to charge to 75 V? What is the capacitor voltage 2 ms after the switch is closed? Use the normalized universal exponential curves in Figure 12–39 to determine the answers.

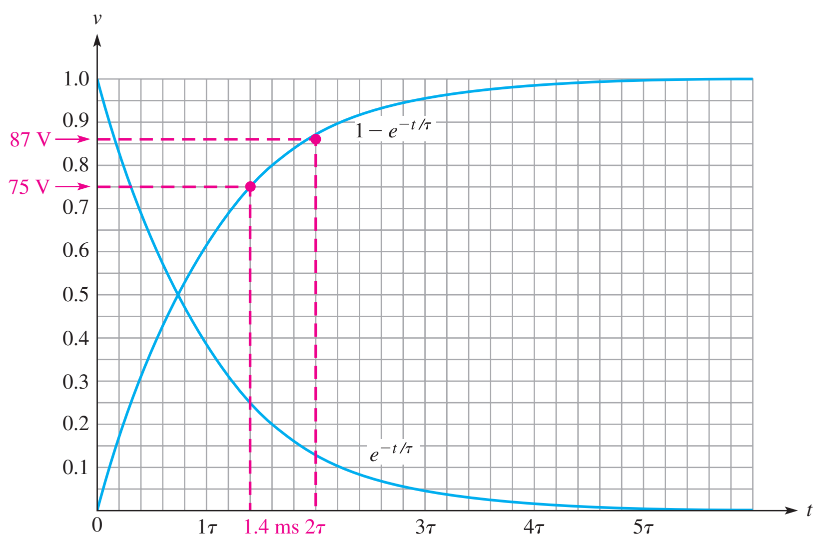
FIGURE 12–40



Solution The full charge voltage is 100 V, which is at the 100% level (1.0) on the normalized vertical scale of the graph. The value 75 V is 75% of maximum, or 0.75 on the graph. You can see that this value occurs at 1.4 time constants. In this circuit, one time constant is $RC = (100\text{ k}\Omega)(0.01\ \mu\text{F}) = 1\text{ ms}$. Therefore, the capacitor voltage reaches 75 V at 1.4 ms after the switch is closed.

On the universal exponential curve, you see that the capacitor is at approximately 87 V (0.87 on the vertical axis) in 2 ms, which is 2 time constants. These graphic solutions are shown in Figure 12–41.

FIGURE 12–41



Related Problem Using the normalized universal exponential curves, determine how long it will take the capacitor in Figure 12–40 to charge to 50 V? What is the capacitor voltage 3 ms after switch closure?

MultiSim

Use Multisim file E12–15 to verify the calculated results in this example and to confirm your calculation for the related problem. Use a square wave to replace the dc voltage source and the switch.

Time-Constant Percentage Tables The percentages of full charge or discharge at each time-constant interval can be calculated using the exponential formulas, or they can be extracted from the universal exponential curves. The results are summarized in Tables 12–4 and 12–5.

TABLE 12–4

Percentage of final charge after each charging time-constant interval.

NUMBER OF TIME CONSTANTS	APPROXIMATE % OF FINAL CHARGE
1	63.2
2	86.5
3	95.0
4	98.2
5	99.3 (considered 100%)

TABLE 12–5

Percentage of initial charge after each discharging time-constant interval.

NUMBER OF TIME CONSTANTS	APPROXIMATE % OF INITIAL CHARGE
1	36.8
2	13.5
3	5.0
4	1.8
5	0.7 (considered 0)

Solving for Time

Occasionally, it is necessary to determine how long it will take a capacitor to charge or discharge to a specified voltage. Equations 12–17 and 12–19 can be solved for t if v is specified. The natural logarithm (abbreviated ln) of $e^{-t/RC}$ is the exponent $-t/RC$. Therefore, taking the natural logarithm of both sides of the equation allows you to solve for time. This procedure is done as follows for the decreasing exponential formula when $V_F = 0$ (Equation 12–20).

$$v = V_i e^{-t/RC}$$

$$\frac{v}{V_i} = e^{-t/RC}$$

$$\ln\left(\frac{v}{V_i}\right) = \ln e^{-t/RC}$$

$$\ln\left(\frac{v}{V_i}\right) = \frac{-t}{RC}$$

Equation 12–21

$$t = -RC\ln\left(\frac{v}{V_i}\right)$$

The same procedure can be used for the increasing exponential formula in Equation 12–19 as follows:

$$v = V_F(1 - e^{-t/RC})$$

$$\frac{v}{V_F} = 1 - e^{-t/RC}$$

$$1 - \frac{v}{V_F} = e^{-t/RC}$$

$$\ln\left(1 - \frac{v}{V_F}\right) = \ln e^{-t/RC}$$

$$\ln\left(1 - \frac{v}{V_F}\right) = \frac{-t}{RC}$$

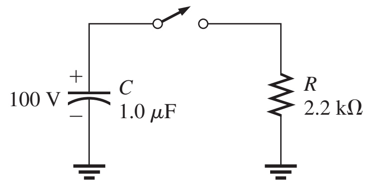
$$t = -RC\ln\left(1 - \frac{v}{V_F}\right)$$

Equation 12–22

EXAMPLE 12–16

In Figure 12–42, how long will it take the capacitor to discharge to 25 V when the switch is closed?

► **FIGURE 12–42**



Solution Use Equation 12–21 to find the discharge time.

$$t = -RC\ln\left(\frac{v}{V_i}\right) = -(2.2\text{ k}\Omega)(1.0\ \mu\text{F})\ln\left(\frac{25\text{ V}}{100\text{ V}}\right)$$

$$= -(2.2\text{ ms})\ln(0.25) = -(2.2\text{ ms})(-1.39) = \mathbf{3.05\ ms}$$

You can determine ln(0.25) with your calculator by using the LN key.

Related Problem How long will it take the capacitor in Figure 12–42 to discharge to 50 V?

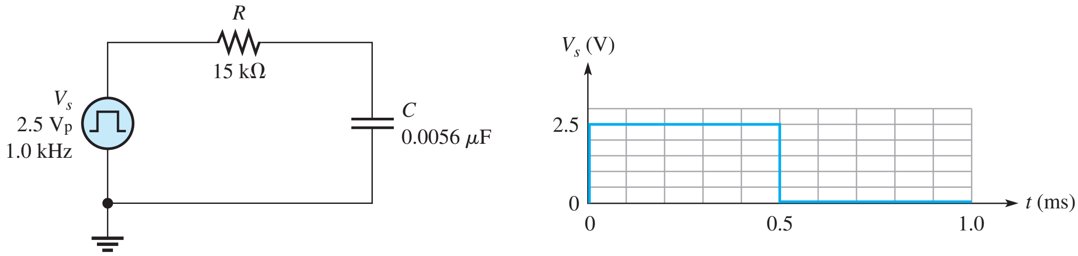
Response to a Square Wave

A common case that illustrates the rising and falling exponential occurs when an *RC* circuit is driven with a square wave that has a long period compared to the time constant. The square wave provides on and off action but, unlike a single switch, it provides a discharge path back through the generator when the wave drops back to zero.

When the square wave rises, the voltage across the capacitor rises exponentially toward the maximum value of the square wave in a time that depends on the time constant. When the square wave returns to the zero level, the capacitor voltage decreases exponentially, again depending on the time constant. The Thevenin resistance of the generator is part of the *RC* time constant; however, it can be ignored if it is small compared to *R*. Example 12–17 shows the waveforms for the case where the period is long compared to the time constant; other cases will be covered in detail in Chapter 20.

EXAMPLE 12–17

In Figure 12–43, calculate the voltage across the capacitor every 0.1 ms for one complete period of the input. Then sketch the capacitor waveform. Assume the Thevenin resistance of the generator is negligible.



▲ FIGURE 12–43

Solution

$$\tau = RC = (15\ \text{k}\Omega)(0.0056\ \mu\text{F}) = 0.084\ \text{ms}$$

The period of the square wave is 1 ms, which is approximately 12τ. This means that 6τ will elapse after each change of the pulse, allowing the capacitor to fully charge and fully discharge.

For the increasing exponential,

$$v = V_F(1 - e^{-t/RC}) = V_F(1 - e^{-t/\tau})$$

At 0.1 ms: $v = 2.5\ \text{V}(1 - e^{-0.1\ \text{ms}/0.084\ \text{ms}}) = 1.74\ \text{V}$

At 0.2 ms: $v = 2.5\ \text{V}(1 - e^{-0.2\ \text{ms}/0.084\ \text{ms}}) = 2.27\ \text{V}$

At 0.3 ms: $v = 2.5\ \text{V}(1 - e^{-0.3\ \text{ms}/0.084\ \text{ms}}) = 2.43\ \text{V}$

At 0.4 ms: $v = 2.5\ \text{V}(1 - e^{-0.4\ \text{ms}/0.084\ \text{ms}}) = 2.48\ \text{V}$

At 0.5 ms: $v = 2.5\ \text{V}(1 - e^{-0.5\ \text{ms}/0.084\ \text{ms}}) = 2.49\ \text{V}$

For the decreasing exponential,

$$v = V_i(e^{-t/RC}) = V_i(e^{-t/\tau})$$

In the equation, time is shown from the point when the change occurs (subtracting 0.5 ms from the actual time). For example, at 0.6 ms, $t = 0.6\ \text{ms} - 0.5\ \text{ms} = 0.1\ \text{ms}$.

At 0.6 ms: $v = 2.5\ \text{V}(e^{-0.1\ \text{ms}/0.084\ \text{ms}}) = 0.760\ \text{V}$

At 0.7 ms: $v = 2.5\ \text{V}(e^{-0.2\ \text{ms}/0.084\ \text{ms}}) = 0.231\ \text{V}$

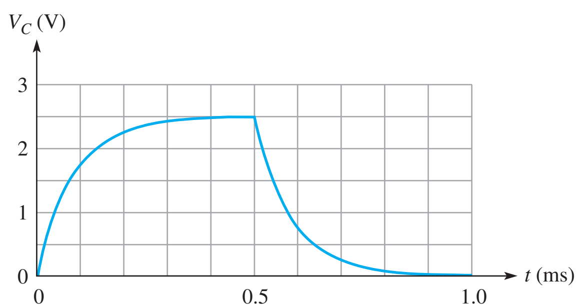
At 0.8 ms: $v = 2.5\ \text{V}(e^{-0.3\ \text{ms}/0.084\ \text{ms}}) = 0.070\ \text{V}$

At 0.9 ms: $v = 2.5\ \text{V}(e^{-0.4\ \text{ms}/0.084\ \text{ms}}) = 0.021\ \text{V}$

At 1.0 ms: $v = 2.5\ \text{V}(e^{-0.5\ \text{ms}/0.084\ \text{ms}}) = 0.007\ \text{V}$

Figure 12–44 is a plot of these results.

► FIGURE 12–44



Related Problem What is the capacitor voltage at 0.65 ms?

SECTION 12–5 CHECKUP

1. Determine the time constant when $R = 1.2\,\text{k}\Omega$ and $C = 1{,}000\,\text{pF}$.
2. If the circuit mentioned in Question 1 is charged with a 5 V source, how long will it take the capacitor to reach 4.0 V?
3. For a certain circuit, $\tau = 1\,\text{ms}$. If it is charged with a 10 V battery, what will the capacitor voltage be at each of the following times: 2 ms, 3 ms, 4 ms, and 5 ms?
4. A capacitor is charged to 100 V. If it is discharged through a resistor, what is the capacitor voltage at one time constant?

12–6 CAPACITORS IN AC CIRCUITS

As you know, a capacitor blocks dc. A capacitor passes ac but with an amount of opposition, called capacitive reactance, that depends on the frequency of the ac.

After completing this section, you should be able to

- **Analyze capacitive ac circuits**
    - Explain why a capacitor causes a phase shift between voltage and current
    - Define *capacitive reactance*
    - Determine the value of capacitive reactance in a given circuit
    - Discuss instantaneous, true, and reactive power in a capacitor

To explain fully how capacitors work in ac circuits, the concept of the derivative must be introduced. *The derivative of a time-varying quantity is the instantaneous rate of change of that quantity.*

Recall that current is the rate of flow of charge (electrons). Therefore, instantaneous current, i, can be expressed as the instantaneous rate of change of charge, q, with respect to time, t.

$$i = \frac{dq}{dt}$$

Equation 12–23

The term dq/dt is the derivative of q with respect to time and represents the instantaneous rate of change of q. Also, in terms of instantaneous quantities, $q = Cv$. Therefore, from a basic rule of differential calculus, the derivative of q with respect to time is $dq/dt = C(dv/dt)$. Since $i = dq/dt$, we get the following relationship:

$$i = C\left(\frac{dv}{dt}\right)$$

Equation 12–24

This formula states

> **The instantaneous capacitor current is equal to the capacitance times the instantaneous rate of change of the voltage across the capacitor.**

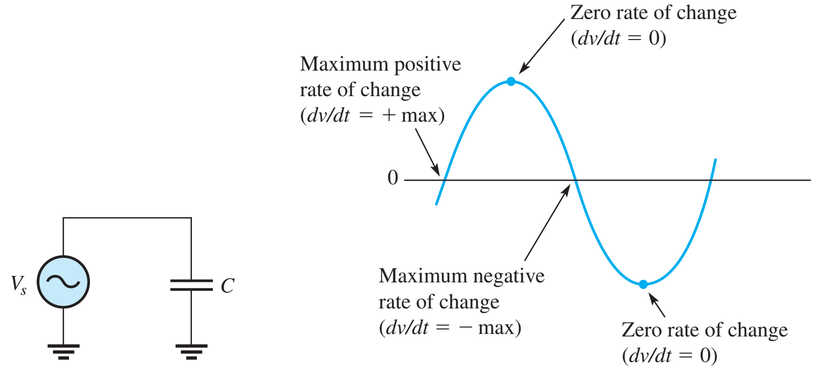
The faster the voltage across a capacitor changes, the greater the current.

Phase Relationship of Current and Voltage in a Capacitor

Consider what happens when a sinusoidal voltage is applied across a capacitor, as shown in Figure 12–45(a). The voltage waveform has a maximum rate of change ($dv/dt = \text{max}$) at the zero crossings and a zero rate of change ($dv/dt = 0$) at the peaks, as indicated in Figure 12–45(b).

The phase relationship between the current and the voltage occurs when the capacitor can be established from Equation 12–24. When $dv/dt = 0$, i is also zero because $i = C(dv/dt) = C(0) = 0$. When dv/dt is a positive-going maximum, i is a positive maximum; when dv/dt is a negative-going maximum, i is a negative maximum.

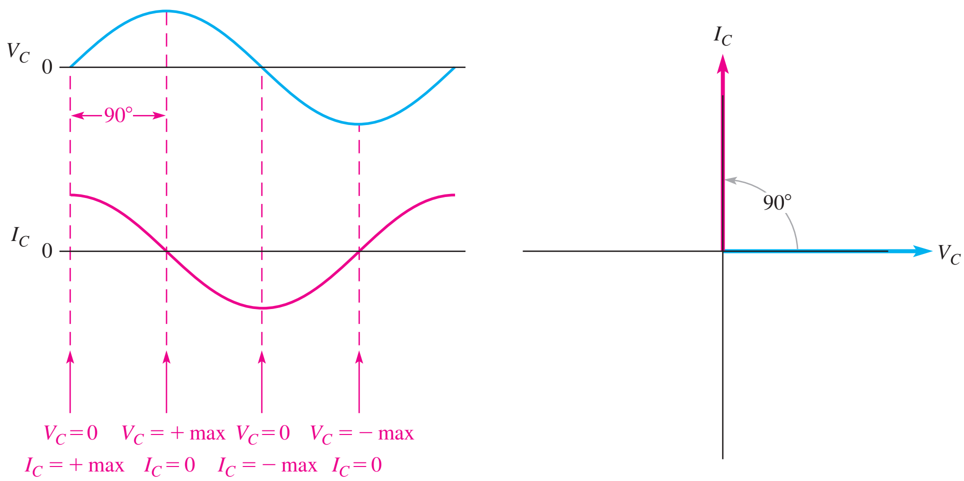
When a sinusoidal voltage is applied to a capacitor, the voltage across the capacitor can be expressed mathematically as $v(t) = V_p(\sin 2\pi ft)$. The voltage curve has the same shape as the mathematical sine curve as shown in Figure 12–46(a). The rate of change of the sine function is the cosine function, which leads the sine function by 90°. Because the current is the rate of change of voltage, current in an ideal capacitor leads voltage by 90° as shown in Figure 12–46(b).



(a) Circuit (b) The rates of change of the sine wave

▲ **FIGURE 12–45**

A sine wave applied to a capacitor.



(a) Waveforms (b) Phasor diagram

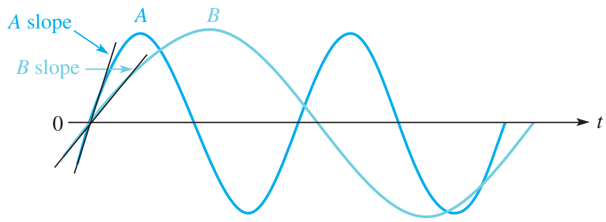
▲ **FIGURE 12–46**

Phase relation of V_C and I_C in a capacitor. Current always leads the capacitor voltage by 90°.

Capacitive Reactance, X_C

Capacitive reactance is the opposition to sinusoidal current, expressed in ohms. The symbol for capacitive reactance is X_C.

To develop a formula for X_C, we use the relationship $i = C(dv/dt)$ and the curves in Figure 12–47. The rate of change of voltage is directly related to frequency. The faster the voltage changes, the higher the frequency. For example, you can see that in Figure 12–47 the slope of sine wave A at the zero crossings is steeper than that of



▲ **FIGURE 12–47**

The higher frequency waveform (*A*) has a greater slope at its zero crossings, corresponding to a higher rate of change.

sine wave *B*. The slope of a curve at a point indicates the rate of change at that point. Sine wave *A* has a higher frequency than sine wave *B*, as indicated by a greater maximum rate of change (dv/dt is greater at the zero crossings).

When frequency increases, dv/dt increases, and thus i increases. When frequency decreases, dv/dt decreases, and thus i decreases.

$$\overset{\uparrow}{i} = C(\overset{\uparrow}{dv/dt}) \quad \text{and} \quad \underset{\downarrow}{i} = C(\underset{\downarrow}{dv/dt})$$

An increase in i means that there is less opposition to current (X_C is less), and a decrease in i means a greater opposition to current (X_C is greater). Therefore, X_C is inversely proportional to i and thus inversely proportional to frequency.

X_C is inversely proportional to f, shown as $\dfrac{1}{f}$.

From the same relationship $i = C(dv/dt)$, you can see that if dv/dt is constant and C is varied, an increase in C produces an increase in i, and a decrease in C produces a decrease in i.

$$\overset{\uparrow}{i} = \overset{\uparrow}{C}(dv/dt) \quad \text{and} \quad \underset{\downarrow}{i} = \underset{\downarrow}{C}(dv/dt)$$

Again, an increase in i means less opposition (X_C is less), and a decrease in i means greater opposition (X_C is greater). Therefore, X_C is inversely proportional to i and thus inversely proportional to capacitance.

The capacitive reactance is inversely proportional to both f and C.

X_C is inversely proportional to fC, shown as $\dfrac{1}{fC}$.

Thus far, we have determined a proportional relationship between X_C and $1/fC$. Equation 12–25 is the complete formula for calculating X_C. The derivation is given in Appendix B.

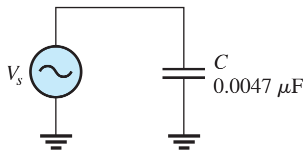
$$X_C = \frac{1}{2\pi fC}$$ **Equation 12–25**

Capacitive reactance, X_C, is in ohms when f is in hertz and C is in farads. Notice that 2π appears in the denominator as a constant of proportionality. This term is derived from the relationship of a sine wave to rotational motion.

EXAMPLE 12–18

A sinusoidal voltage is applied to a capacitor, as shown in Figure 12–48. The frequency of the sine wave is 1 kHz. Determine the capacitive reactance.

▶ FIGURE 12–48



Solution

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1 \times 10^3\ \text{Hz})(0.0047 \times 10^{-6}\ \text{F})} = \mathbf{33.9\ k\Omega}$$

Related Problem Determine the frequency required to make the capacitive reactance in Figure 12–48 equal to 10 kΩ.

Use Multisim file E12–18 to verify the calculated results in this example and to confirm your calculation for the related problem.

Reactance for Series Capacitors

When capacitors are in series in an ac circuit, the total capacitance is smaller than the smallest individual capacitance. Because the total capacitance is smaller, the total capacitive reactance (opposition to current) must be larger than any individual capacitive reactance. With series capacitors, the total capacitive reactance ($X_{C(tot)}$) is the sum of the individual reactances.

Equation 12–26

$$X_{C(tot)} = X_{C1} + X_{C2} + X_{C3} + \cdots + X_{Cn}$$

Compare this formula with Equation 5–1 for finding the total resistance of series resistors. In both cases, you simply add the individual oppositions.

Reactance for Parallel Capacitors

In ac circuits with parallel capacitors, the total capacitance is the sum of the capacitances. Recall that the capacitive reactance is inversely proportional to the capacitance. Because the total parallel capacitance is larger than any individual capacitances, the total capacitive reactance must be smaller than the reactance of any individual capacitor. With parallel capacitors, the total reactance is found by

Equation 12–27

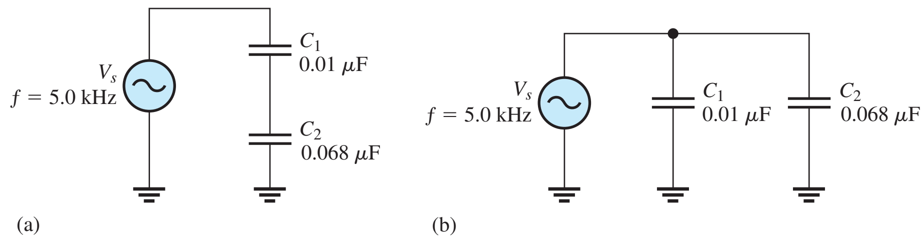
$$X_{C(tot)} = \frac{1}{\dfrac{1}{X_{C1}} + \dfrac{1}{X_{C2}} + \dfrac{1}{X_{C3}} + \cdots + \dfrac{1}{X_{Cn}}}$$

Compare this formula with Equation 6–2 for parallel resistors. As in the case of parallel resistors, the total opposition (resistance or reactance) is the reciprocal of the sum of the reciprocals of the individual oppositions.

For two capacitors in parallel, Equation 12–27 can be reduced to the product-over-sum form. This is useful because, for most practical circuits, more than two capacitors in parallel is not common.

$$X_{C(tot)} = \frac{X_{C1}X_{C2}}{X_{C1} + X_{C2}}$$

EXAMPLE 12–19 What is the total capacitive reactance of each circuit in Figure 12–49?



▲ FIGURE 12–49

Solution The reactances of the individual capacitors are the same in both circuits.

$$X_{C1} = \frac{1}{2\pi f C_1} = \frac{1}{2\pi(5.0\text{ kHz})(0.01\ \mu\text{F})} = 3.18\text{ k}\Omega$$

$$X_{C2} = \frac{1}{2\pi f C_2} = \frac{1}{2\pi(5.0\text{ kHz})(0.068\ \mu\text{F})} = 468\ \Omega$$

The Series Circuit: For the capacitors in series in Figure 12–49(a), the total reactance is the sum of X_{C1} and X_{C2}, as given in Equation 12–26.

$$X_{C(tot)} = X_{C1} + X_{C2} = 3.18\text{ k}\Omega + 468\ \Omega = \mathbf{3.65\text{ k}\Omega}$$

Alternatively, you can obtain the total series reactance by first finding the total capacitance using Equation 12–10. Then calculate the total reactance.

$$C_{tot} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(0.01\ \mu\text{F})(0.068\ \mu\text{F})}{0.01\ \mu\text{F} + 0.068\ \mu\text{F}} = 0.0087\ \mu\text{F}$$

$$X_{C(tot)} = \frac{1}{2\pi f C_{tot}} = \frac{1}{2\pi(5.0\text{ kHz})(0.0087\ \mu\text{F})} = \mathbf{3.65\text{ k}\Omega}$$

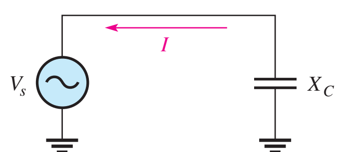
The Parallel Circuit: For the capacitors in parallel in Figure 12–49(b), determine the total reactance from the product-over-sum rule using X_{C1} and X_{C2}.

$$X_{C(tot)} = \frac{X_{C1}X_{C2}}{X_{C1} + X_{C2}} = \frac{(3.18\text{ k}\Omega)(468\ \Omega)}{3.18\text{ k}\Omega + 468\ \Omega} = \mathbf{408\ \Omega}$$

Related Problem Determine the total parallel capacitive reactance by first finding the total parallel capacitance.

Ohm's Law The reactance of a capacitor is analogous to the resistance of a resistor. In fact, both are expressed in ohms. Since both R and X_C are forms of opposition to current. Ohm's law applies to capacitive circuits as well as to resistive circuits; and it is stated as follows for Figure 12–50:

$$I = \frac{V_s}{X_C}$$

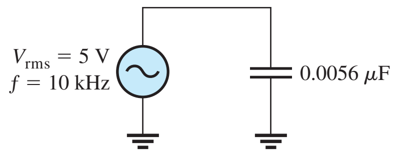


▲ FIGURE 12–50

When applying Ohm's law in ac circuits, you must express both the current and the voltage in the same way, that is, both in rms, both in peak, and so on.

EXAMPLE 12–20

Determine the rms current in Figure 12–51.



▲ **FIGURE 12–51**

Solution First, determine the capacitive reactance.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(10 \times 10^3\ \text{Hz})(0.0056 \times 10^{-6}\ \text{F})} = 2.84\ \text{k}\Omega$$

Then apply Ohm's law.

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{5\ \text{V}}{2.84\ \text{k}\Omega} = \mathbf{1.76\ mA}$$

Related Problem Change the frequency in Figure 12–51 to 25 kHz and determine the rms current.

Use Multisim file E12–20 to verify the calculated results in this example and to confirm your calculation for the related problem.

Capacitive Voltage Divider

In ac circuits, capacitors can be used in applications that require a voltage divider. (Some oscillator circuits use this method to tap off a fraction of the output.) The voltage across a series capacitor was given as Equation 12–12, which is repeated here ($V_T = V_s$).

$$V_x = \left(\frac{C_T}{C_x}\right)V_s$$

A resistive voltage divider is expressed in terms of a resistance ratio, which is a ratio of oppositions. You can think of the capacitive voltage divider by applying this idea from a resistive divider, but using reactance in place of resistance. The equation for the voltage across a capacitor in a capacitive voltage divider can be written as

Equation 12–28

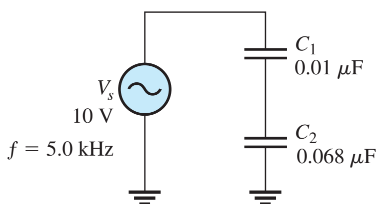
$$V_x = \left(\frac{X_{Cx}}{X_{C(tot)}}\right)V_s$$

where X_{Cx} is the reactance of capacitor C_x, $X_{C(tot)}$ is the total capacitive reactance, and V_x is the voltage across capacitor C_x. Either Equation 12–12 or Equation 12–28 can be used to find the voltage from a divider, as illustrated in Example 12–21.

EXAMPLE 12–21

What is the voltage across C_2 in the circuit of Figure 12–52?

► **FIGURE 12–52**



Solution The reactance of the individual capacitors and the total reactance were determined in Example 12–19. Substituting into Equation 12–28.

$$V_2 = \left(\frac{X_{C2}}{X_{C(tot)}}\right)V_s = \left(\frac{468\ \Omega}{3.65\ \text{k}\Omega}\right)10\ \text{V} = \mathbf{1.28\ V}$$

Notice that the voltage across the larger capacitor is the smaller fraction of the total.

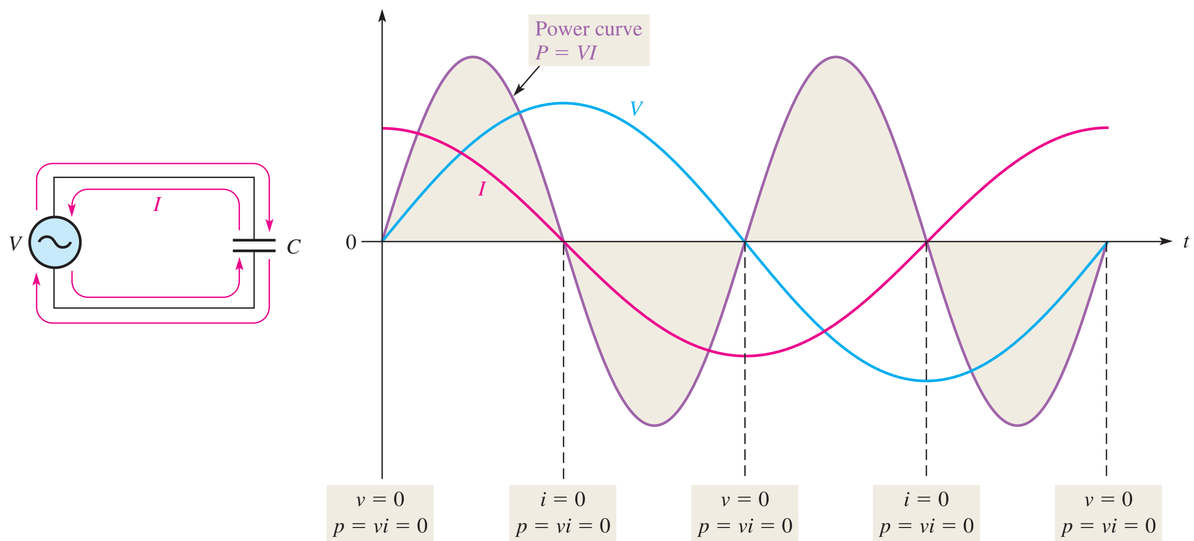
You can obtain the same result from Equation 12–12.

$$V_2 = \left(\frac{C_T}{C_2}\right)V_s = \left(\frac{0.0087\ \mu\text{F}}{0.068\ \mu\text{F}}\right)10\ \text{V} = \mathbf{1.28\ V}$$

Related Problem Use Equation 12–28 to determine the voltage across C_1.

Power in a Capacitor

As discussed earlier in this chapter, a charged capacitor stores energy in the electric field within the dielectric. An ideal capacitor does not dissipate energy; it only stores it temporarily. When an ac voltage is applied to a capacitor, energy is stored by the capacitor during a portion of the voltage cycle; then the stored energy is returned to the source during another portion of the cycle. Ideally, there is no net energy loss. Figure 12–53 shows the power curve that results from one cycle of capacitor voltage and current.



▲ FIGURE 12–53

Power curve.

Instantaneous Power (p) The product of v and i gives **instantaneous power**. At points where v or i is zero, p is also zero. When both v and i are positive, p is also positive. When either v or i is positive and the other is negative, p is negative. When both v and i are negative, p is positive. As you can see, the power follows a sinusoidal-shaped curve. Positive values of power indicate that energy is stored by the capacitor. Negative values of power indicate that energy is returned from the capacitor to the source. Note that the power alternates at a frequency twice that of the voltage or current as energy is alternately stored and returned to the source.

True Power (P_{true}) Ideally, all of the energy stored by a capacitor during the positive portion of the power cycle is returned to the source during the negative portion. No net energy is lost due to conversion to heat in the capacitor, so the **true power** is zero. Actually, because of leakage and foil resistance in a practical capacitor, a small percentage of the total power is dissipated in the form of true power.

Reactive Power (P_r) The rate at which a capacitor stores or returns energy is called its **reactive power**. The reactive power is a nonzero quantity, because at any instant in time, the capacitor is actually taking energy from the source or returning energy to it. Reactive power does not represent an energy loss. The following formulas apply:

Equation 12–29
$$P_r = V_{rms}I_{rms}$$

Equation 12–30
$$P_r = \frac{V_{rms}^2}{X_C}$$

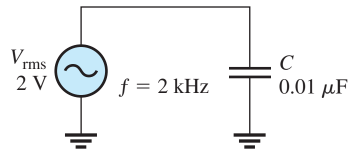
Equation 12–31
$$P_r = I_{rms}^2 X_C$$

Notice that these equations are of the same form as those introduced in Chapter 4 for power in a resistor. The voltage and current are expressed in rms. The unit of reactive power is **VAR (volt-ampere reactive)**.

EXAMPLE 12–22

Determine the true power and the reactive power in Figure 12–54.

▶ FIGURE 12–54



Solution The true power, P_{true}, is *always* **zero** for an ideal capacitor. The reactive power is determined by first finding the value for the capacitive reactance and then using Equation 12–30.

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(2 \times 10^3\text{ Hz})(0.01 \times 10^{-6}\text{ F})} = 7.96\text{ k}\Omega$$

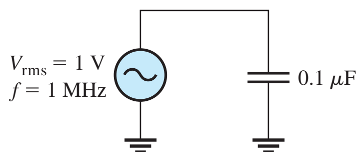
$$P_r = \frac{V_{rms}^2}{X_C} = \frac{(2\text{ V})^2}{7.96\text{ k}\Omega} = 503 \times 10^{-6}\text{ VAR} = \mathbf{503\ \mu VAR}$$

Related Problem If the frequency is doubled in Figure 12–54, what are the true power and the reactive power?

SECTION 12–6 CHECKUP

1. State the phase relationship between current and voltage in a capacitor.
2. Calculate X_C for f = 5 kHz and C = 50 pF.
3. At what frequency is the reactance of a 0.1 μF capacitor equal to 2 kΩ?
4. Calculate the rms current in Figure 12–55.
5. A 1 μF capacitor is connected to an ac voltage source of 12 V rms. What is the true power?
6. In Question 5, determine the reactive power at a frequency of 500 Hz.

▶ FIGURE 12–55



12–7 Capacitor Applications

Capacitors are widely used in many electrical and electronic applications.

After completing this section, you should be able to

- **Discuss some capacitor applications**
  - Describe a power supply filter
  - Explain the purpose of coupling and bypass capacitors
  - Discuss the basics of capacitors applied to tuned circuits, timing circuits, and computer memories

If you pick up any circuit board, open any power supply, or look inside any piece of electronic equipment, chances are you will find capacitors of one type or another. These components are used for a variety of reasons in both dc and ac applications.

Electrical Storage

One of the most basic applications of a capacitor is as a backup voltage source for low-power circuits such as certain types of semiconductor memories in computers. This particular application requires a very high capacitance value and negligible leakage.

The storage capacitor is connected between the dc power supply input to the circuit and ground. When the circuit is operating from its normal power supply, the capacitor remains fully charged to the dc power supply voltage. If the normal power source is disrupted, effectively removing the power supply from the circuit, the storage capacitor temporarily becomes the power source for the circuit.

A capacitor provides voltage and current to a circuit as long as its charge remains sufficient. As current is drawn by the circuit, charge is removed from the capacitor and the voltage decreases. For this reason, the storage capacitor can only be used as a temporary power source. The length of time that a capacitor can provide sufficient power to the circuit depends on the capacitance and the amount of current drawn by the circuit. The smaller the current and the higher the capacitance, the longer the time a capacitor can provide power to a circuit.

Power Supply Filtering

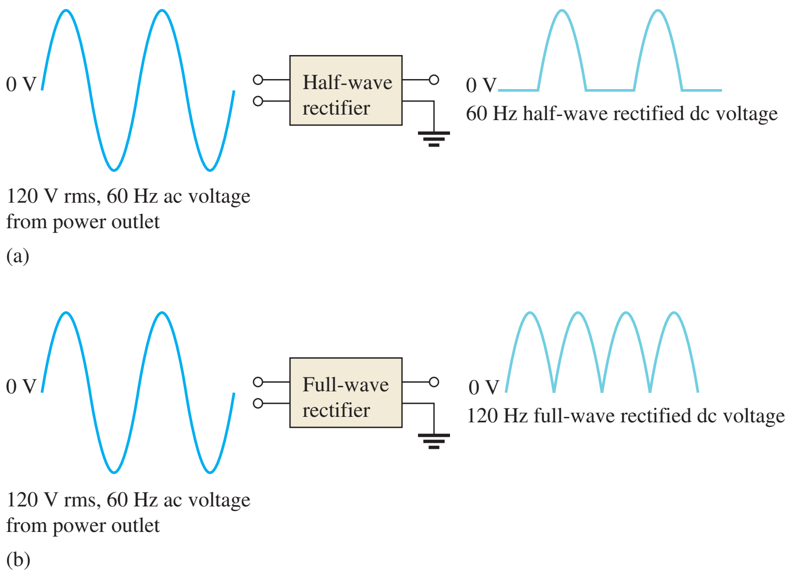
A basic dc power supply consists of a circuit known as a **rectifier** followed by a **filter**. The rectifier converts the 120 V, 60 Hz sinusoidal voltage available at a standard outlet to a pulsating dc voltage that can be either a half-wave rectified voltage or a full-wave rectified voltage, depending on the type of rectifier circuit. As shown in Figure 12–56(a), a half-wave rectifier removes each negative half-cycle of the sinusoidal voltage. As shown in Figure 12–56(b), a full-wave rectifier actually reverses the polarity of the negative portion of each cycle. Both half-wave and full-wave rectified voltages are dc because, even though they are changing, they do not alternate polarity.

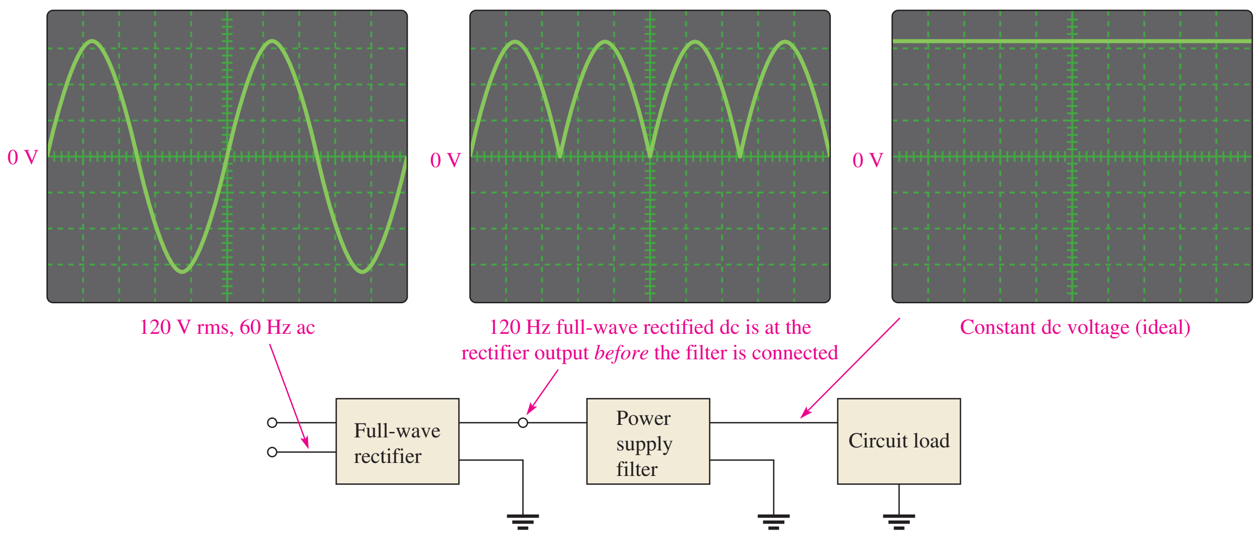
To be useful for powering electronic circuits, the rectified voltage must be changed to constant dc voltage because all circuits require constant power. The filter nearly eliminates the fluctuations in the rectified voltage and ideally provides a smooth constant-value dc voltage to the load that is the electronic circuit, as indicated in Figure 12–57.

The Capacitor as a Power Supply Filter Capacitors are used as filters in dc power supplies because of their ability to store electrical charge. Figure 12–58(a) shows a dc power supply with a full-wave rectifier and a capacitor filter. The operation can be described from a charging and discharging point of view as follows. Assume the capacitor is initially uncharged. When the power supply is first turned on and the first

▶ FIGURE 12–56

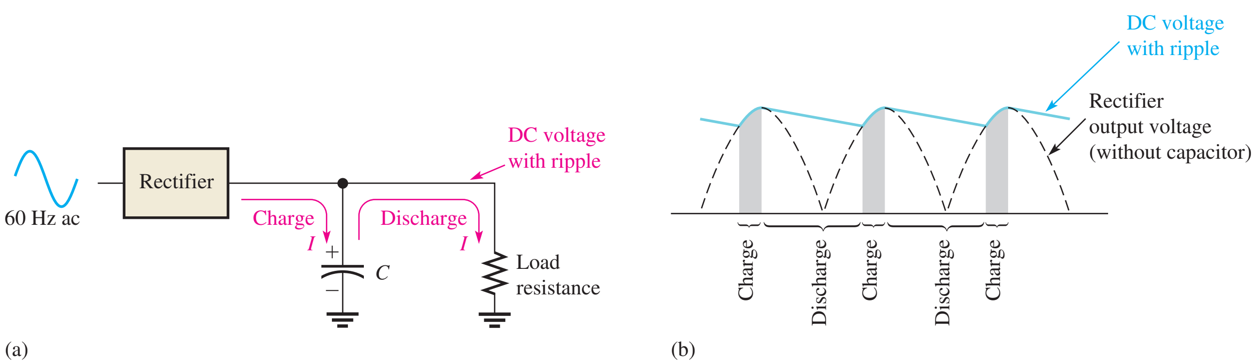
Half-wave and full-wave rectifier operation.





▲ FIGURE 12–57

Basic waveforms showing the operation of a dc power supply.



▲ FIGURE 12–58

Basic operation of a power supply filter capacitor.

cycle of the rectified voltage occurs, the capacitor will quickly charge through the low forward resistance of the rectifier. The capacitor voltage will follow the rectified voltage curve up to the peak of the rectified voltage. As the rectified voltage passes the peak and begins to decrease, the capacitor will begin to discharge very slowly through the high resistance of the load circuit, as indicated in Figure 12–58(b). The amount of discharge is typically very small and is exaggerated in the figure for purposes of illustration. The next cycle of the rectified voltage will recharge the capacitor back to the peak value by replenishing the small amount of charge lost since the previous peak. This pattern of a small amount of charging and discharging continues as long as the power is on.

A rectifier is designed so that it allows current only in the direction to charge the capacitor. The capacitor will not discharge back through the rectifier but will only discharge a small amount through the relatively high resistance of the load. The small fluctuation in voltage due to the charging and discharging of the capacitor is called the **ripple voltage**. A good dc power supply has a very small amount of ripple on its dc output. The discharge time constant of a power supply filter capacitor depends on its capacitance and the resistance of the load; consequently, the higher the capacitance value, the longer the discharge time and therefore, the smaller the ripple voltage.

DC Blocking and AC Coupling

Capacitors are commonly used to block the constant dc voltage in one part of a circuit from getting to another part. As an example of this, a capacitor is connected between two stages of an amplifier to prevent the dc voltage at the output of stage 1 from affecting the dc voltage at the input of stage 2, as illustrated in Figure 12–59. Assume that, for proper operation, the output of stage 1 has a zero dc voltage and the input to stage 2 has a 3 V dc voltage. The capacitor prevents the 3 V dc at stage 2 from getting to the stage 1 output and affecting its zero value, and vice versa.

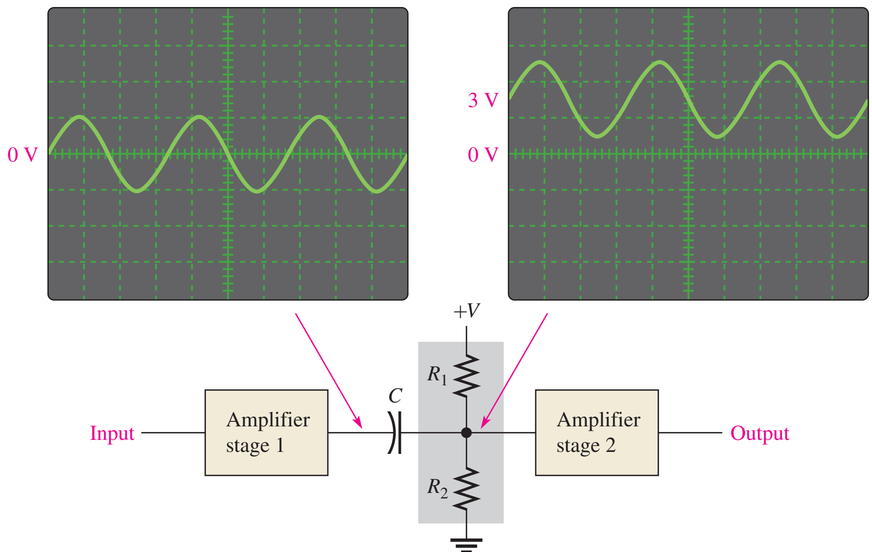


FIGURE 12–59
An application of a capacitor to block dc and couple ac in an amplifier.

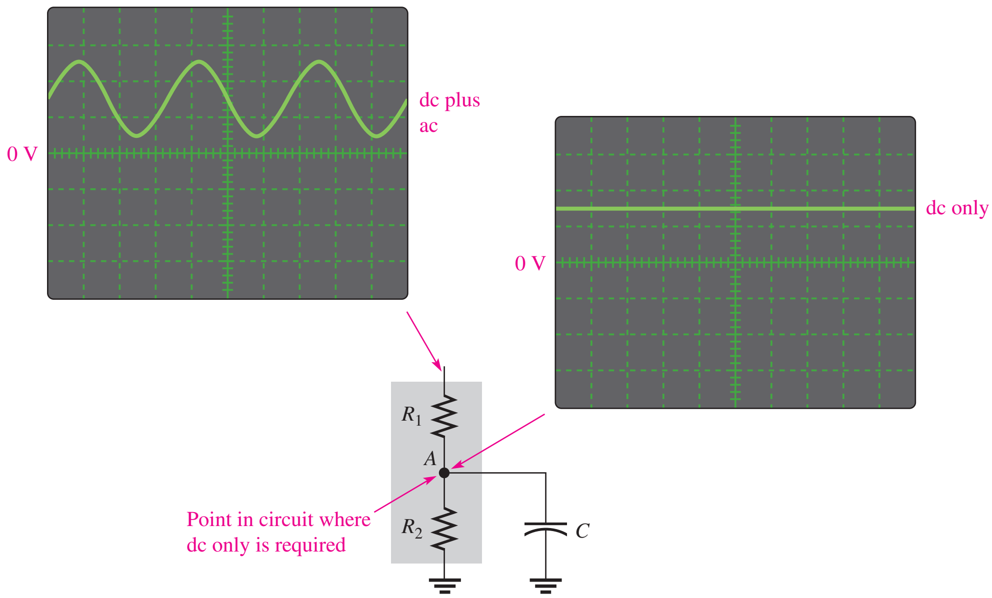
If a sinusoidal signal voltage is applied to the input to stage 1, the signal voltage is increased (amplified) and appears on the output of stage 1, as shown in Figure 12–59. The amplified signal voltage is then coupled through the capacitor to the input of stage 2 where it is superimposed on the 3 V dc level and then again amplified by stage 2. In order for the signal voltage to be passed through the capacitor without being reduced, the capacitor must be large enough so that its reactance at the frequency of the signal voltage is negligible. In this type of application, the capacitor is known as a *coupling* *capacitor*, which ideally appears as an open to dc and as a short to ac. As the signal frequency is reduced, the capacitive reactance increases and, at some point, the capacitive reactance becomes large enough to cause a significant reduction in ac voltage between stage 1 and stage 2.

Power Line Decoupling

Capacitors connected from the dc supply voltage line to ground are used on circuit boards to decouple unwanted voltage transients or spikes that occur on the dc supply voltage because of fast switching digital circuits. A voltage transient contains high frequencies that may affect the operation of the circuits. These transients are shorted to ground through the very low reactance of the decoupling capacitors. Several decoupling capacitors are often used at various points along the supply voltage line on a circuit board.

Bypassing

Another capacitor application is to bypass an ac voltage around a resistor in a circuit without affecting the dc voltage across the resistor. In amplifier circuits, for example, dc voltages called *bias voltages* are required at various points. For the amplifier to operate properly, certain bias voltages must remain constant and, therefore, any ac voltages must be removed. A sufficiently large capacitor connected from a bias point to ground provides a low reactance path to ground for ac voltages, leaving the constant dc bias voltage at the given point. At lower frequencies, the bypass capacitor becomes less effective because of its increased reactance. This bypass application is illustrated in Figure 12–60.



▲ **FIGURE 12–60**

Example of the operation of a bypass capacitor. Point *A* is at ac ground due to the low reactance path through the capacitor.

Signal Filters

Capacitors are essential to the operation of a class of circuits called *filters* that are used for selecting one ac signal with a certain specified frequency from a wide range of signals with many different frequencies or for selecting a certain band of frequencies and eliminating all others. A common example of this application is in radio and television receivers where it is necessary to select the signal transmitted from a given station and eliminate or filter out the signals transmitted from all the other stations in the area.

When you tune your radio or TV, you are actually changing the capacitance in the tuner circuit (which is a type of filter) so that only the signal from the station or channel you want passes through to the receiver circuitry. Capacitors are used in conjunction with resistors, inductors (covered in the next chapter), and other components in these types of filters. The topic of filters will be covered in Chapter 18.

The main characteristic of a filter is its frequency selectivity, which is based on the fact that the reactance of a capacitor depends on frequency ($X_C = 1/2\pi fC$).

Timing Circuits

Another important area in which capacitors are used is in timing circuits that generate specified time delays or produce waveforms with specific characteristics. Recall that the time constant of a circuit with resistance and capacitance can be controlled by selecting appropriate values for R and C. The charging time of a capacitor can be used as a basic time delay in various types of circuits; however, capacitors tend to have more tolerance variation than other components, so RC timing circuits are not used when timing is critical. An example application is the circuit that controls the turn indicators on your car where the light flashes on and off at regular intervals.

Computer Memories

Dynamic memories in computers use very tiny capacitors as the basic storage element for binary information, which consists of two binary digits, 1 and 0. A charged capacitor can represent a stored 1 and a discharged capacitor can represent a stored 0. Patterns of 1s and 0s that make up binary data are stored in a memory that consists of an array of capacitors with associated circuitry. You will study this topic in a computer or digital fundamentals course.

SECTION 12–7 CHECKUP

1. Explain how half-wave or full-wave rectified dc voltages are smoothed out by a filter capacitor.
2. Explain the purpose of a coupling capacitor.
3. How large must a coupling capacitor be?
4. Explain the purpose of a decoupling capacitor.
5. Discuss how the relationship of frequency and capacitive reactance is important in frequency-selective circuits such as signal filters.
6. What characteristic of a capacitor is most important in time-delay applications?

12–8 SWITCHED-CAPACITOR CIRCUITS

Another important application of capacitors is in programmable analog arrays, which are implemented in integrated circuit (IC) form. Switched capacitors are used to implement various types of programmable analog circuits in which capacitors take the place of resistors. Example applications include analog to digital converters and analog memory arrays. Capacitors can be implemented on an IC chip more easily than a resistor can, and they offer other advantages such as zero power dissipation. When a resistance is required in a circuit, the switched capacitor can be made to emulate a resistor. Using switched-capacitor emulation, resistor values can be readily changed by reprogramming and accurate and stable resistance values can be achieved.

After completing this section, you should be able to

- **Describe the operation of switched-capacitor circuits**
  - Explain how switched-capacitor circuits emulate resistors

Recall that current is defined in terms of charge Q and time t as

$$I = \frac{Q}{t}$$

This formula states that current is the rate at which charge flows through a circuit. Also recall that the basic definition of charge in terms of capacitance and voltage is

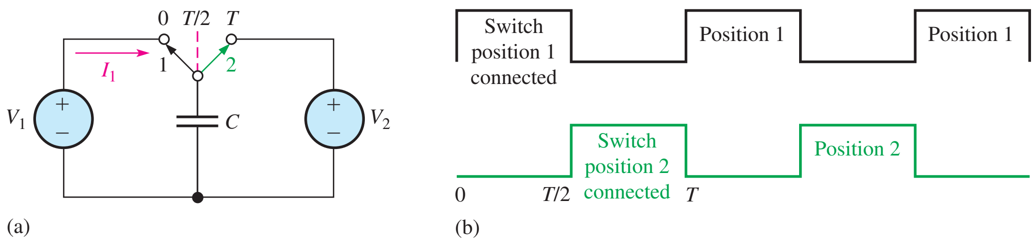
$$Q = CV$$

Substituting CV for Q, the current can be expressed as

$$I = \frac{CV}{t}$$

Basic Operation

A general model of a switched-capacitor circuit is shown in Figure 12–61. It consists of a capacitor, two arbitrary voltage sources (V_1 and V_2), and a two-pole switch. In actual circuits, the switch is implemented with transistors. Let's examine this circuit for a specified period of time, T, which is then repeated. Assume that V_1 and V_2 are constant during the time period T. Of particular interest is the average current I_1 from the source V_1 during the period of time, T.



▲ FIGURE 12–61

Basic operation of a switched-capacitor circuit. The voltage source symbol represents a time-varying symbol.

During the first half of the time period T, the switch is in position 1, as indicated in Figure 12–61. Therefore, there is a current I_1 due to V_1 that is charging the capacitor during the interval from $t = 0$ to $t = T/2$. During the second half of the time period, the switch is in position 2, as indicated, and there is no current from V_1; therefore, the average current from the source V_1 over the time period T is

$$I_{1(avg)} = \frac{Q_{1(T/2)} - Q_{1(0)}}{T}$$

$Q_{1(0)}$ is the charge at $t = 0$ and $Q_{1(T/2)}$ is the charge at $t = T/2$. So, $Q_{1(T/2)} - Q_{1(0)}$ is the net charge transferred while the switch is in position 1.

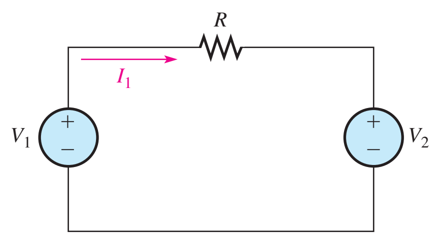
The capacitor voltage at $T/2$ is equal to V_1, and the capacitor voltage at 0 or T is equal to V_2. Using the formula $Q = CV$ and substituting into the previous equation, you obtain

$$I_{1(avg)} = \frac{CV_{1(T/2)} - CV_{2(0)}}{T} = \frac{C(V_{1(T/2)} - V_{2(0)})}{T}$$

Since V_1 and V_2 are assumed to be constant during T, the average current can be expressed as

Equation 12–32

$$I_{1(avg)} = \frac{C(V_1 - V_2)}{T}$$



◀ FIGURE 12–62

Resistive circuit.

Figure 12–62 shows an equivalent circuit with a resistor instead of the capacitor and switches.

Applying Ohm's law to the resistive circuit, the current is

$$I_1 = \frac{V_1 - V_2}{R}$$

Setting $I_{1(avg)}$ in the switched-capacitor circuit equal to current in the resistive circuit, you have

$$\frac{C(V_1 - V_2)}{T} = \frac{V_1 - V_2}{R}$$

Canceling the $V_1 - V_2$ terms and solving for R gives the equivalent resistance.

$$R = \frac{T}{C}$$

Equation 12–33

This important result shows that a switched-capacitor circuit can emulate a resistor with a value determined by the time T and the capacitance C. Remember that the switch is in each position for one-half of the time period T and that T can be varied by varying the frequency at which the switches are operated. In a programmable analog device, the switching frequency is a programmable parameter for each emulated resistor and can be set to achieve a precise resistor value. Since $T = 1/f$, the resistance in terms of frequency is

$$R = \frac{1}{fC}$$

Equation 12–34

EXAMPLE 12–23

The input resistor, R, of an amplifier circuit is to be replaced with a switched-capacitor circuit. Assume that the switched capacitor value is 1,000 pF. You want the switched capacitor to emulate a 10 kΩ resistor. Determine the frequency at which the capacitor must be switched.

Solution Using the formula $R = T/C$,

$$T = RC = (10\text{ k}\Omega)(1{,}000\text{ pF}) = 10\ \mu\text{s}$$

This means that the switch must be operated at a frequency of

$$f = \frac{1}{T} = \frac{1}{10\ \mu\text{s}} = \mathbf{100\ kHz}$$

The duty cycle is 50% so that the switch is in each position half of the period.

Related Problem At what frequency must the 1,000 pF capacitor be switched to emulate a 5.6 kΩ resistor?

SECTION 12–8 CHECKUP

1. How does a switched capacitor emulate a resistor?
2. What factors determine the resistance value that a given switched-capacitor circuit can emulate?
3. In a practical implementation, what devices are used for switches?

Application Activity

Capacitors are used in certain types of amplifiers to couple the ac signal while blocking the dc voltage. Capacitors are used in many other applications, but in this application, you will focus on the coupling capacitors in an amplifier circuit. This topic was introduced in Section 12–7. A knowledge of amplifier circuits is not necessary for this assignment.

All amplifier circuits contain transistors that require dc voltages to establish proper operating conditions for amplifying ac signals. These dc voltages are referred to as bias voltages. As indicated in Figure 12–63(a), a common type of dc bias circuit used in amplifiers is the voltage divider formed by R_1 and R_2, which sets up the proper dc voltage at the input to the amplifier.

When an ac signal voltage is applied to the amplifier, the input coupling capacitor, C_1, prevents the internal resistance of the ac source from changing the dc bias voltage. Without the capacitor, the internal source resistance would appear in parallel with R_2 and drastically change the value of the dc voltage.

The coupling capacitance is chosen so that its reactance (X_C) at the frequency of the ac signal is very small compared to the bias resistor values. The coupling capacitance therefore efficiently couples the ac signal from the source to the input of the amplifier. On the source side of the input coupling capacitor there is only ac but on the amplifier side there is ac plus dc (the signal voltage is riding on the dc bias voltage set by the voltage divider), as indicated in Figure 12–63(a). Capacitor C_2 is the output coupling capacitor, which couples the amplified ac signal to another amplifier stage that would be connected to the output.

You will check three amplifier boards like the one in Figure 12–63(b) for the proper input voltages using an oscilloscope. If the voltages are incorrect, you will determine the most likely fault. For all measurements, assume the amplifier has no dc loading effect on the voltage-divider bias circuit.

The Printed Circuit Board and the Schematic

1. Check the printed circuit board in Figure 12–63(b) to make sure it agrees with the amplifier schematic in part (a).

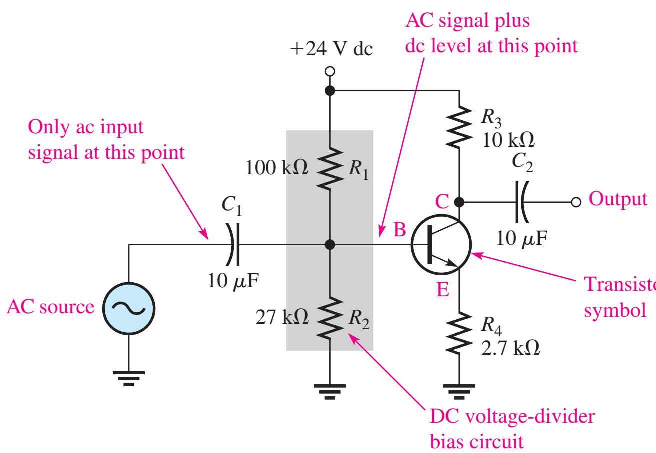
Testing Board 1

The oscilloscope probe is connected from channel 1 to the board as shown in Figure 12–64. The input signal from a sinusoidal voltage source is connected to the board and set to a frequency of 5 kHz with an amplitude of 1 V rms.

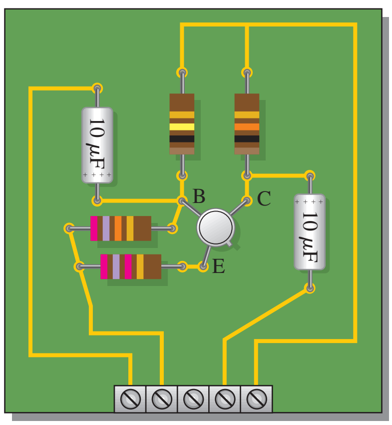
2. Determine if the voltage and frequency displayed on the scope are correct. If the scope measurement is incorrect, specify the most likely fault in the circuit.

Testing Board 2

The oscilloscope probe is connected from channel 1 to board 2 the same as was shown in Figure 12–64 for board 1. The input signal from the sinusoidal voltage source is the same as it was for board 1.



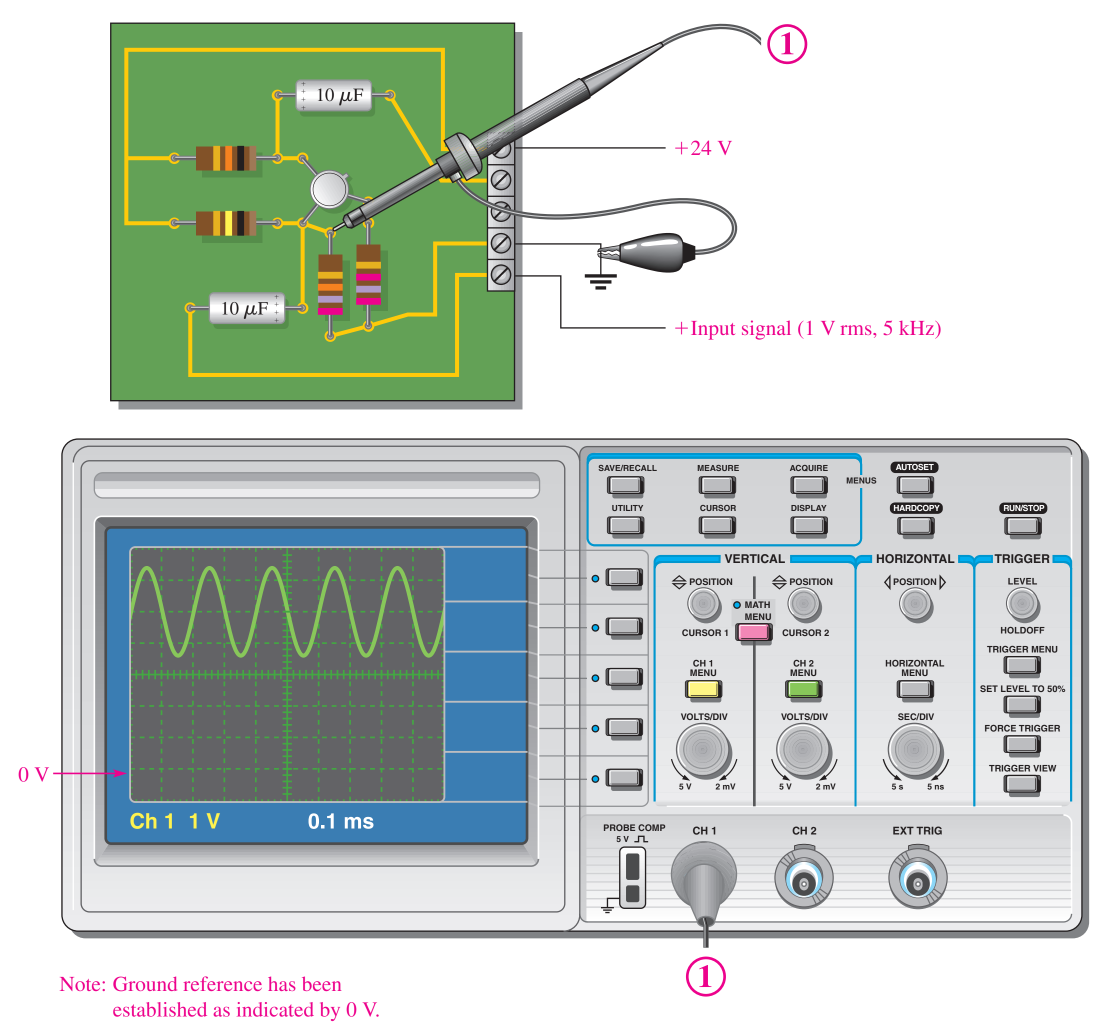
(a) Amplifier schematic



(b) Amplifier board

▲ FIGURE 12–63

A capacitively coupled amplifier.



▲ FIGURE 12–64

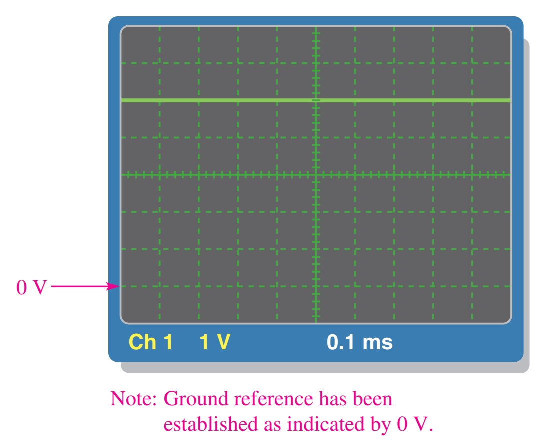
Testing board 1.

3. Determine if the scope display in Figure 12–65 is correct. If the scope measurement is incorrect, specify the most likely fault in the circuit.

Testing Board 3

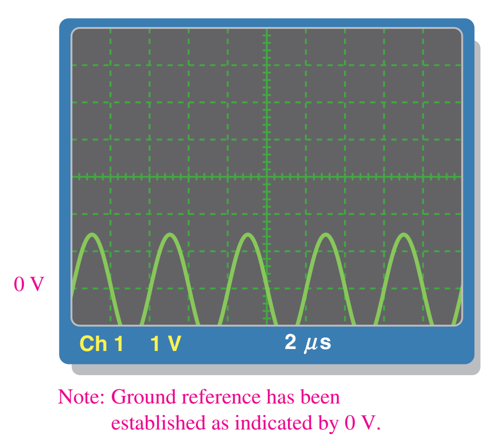
The oscilloscope probe is connected from channel 1 to board 3 the same as was shown in Figure 12–64 for board 1. The input signal from the sinusoidal voltage source is the same as before.

4. Determine if the scope display in Figure 12–66 is correct. If the scope measurement is incorrect, specify the most likely fault in the circuit.



▲ FIGURE 12–65

Testing board 2.



▲ **FIGURE 12–66**

Testing board 3.

Review

5. Explain why the input coupling capacitor is necessary when connecting an ac source to the amplifier.
6. Capacitor C_2 in Figure 12–63 is an output coupling capacitor. Generally, what would you expect to measure at the point in the circuit labelled C and at the output of the circuit when an ac input signal is applied to the amplifier?

SUMMARY

- A capacitor is composed of two parallel conducting plates separated by an insulating material called the *dielectric*.
- A capacitor stores energy in the electric field between the plates.
- One farad is the amount of capacitance when one coulomb of charge is stored with one volt across the plates.
- Capacitance is directly proportional to the plate area and inversely proportional to the plate separation.
- The dielectric constant is an indication of the ability of a material to establish an electric field.
- The dielectric strength is one factor that determines the breakdown voltage of a capacitor.
- A capacitor blocks constant dc.
- The time constant for a series *RC* circuit is the resistance times the capacitance.
- In an *RC* circuit, the voltage and current in a charging or discharging capacitor make a 63% change during each time-constant interval.
- Five time constants are required for a capacitor to charge fully or to discharge fully. This is called the *transient time.*
- Charging and discharging follow exponential curves.
- Total series capacitance is less than that of the smallest capacitor in series.
- Capacitance adds in parallel.
- Current leads voltage by 90° in a capacitor.
- Capacitive reactance, X_C, is inversely proportional to frequency and capacitance.
- The total capacitive reactance of series capacitors is the sum of the individual reactances.
- The total capacitive reactance of parallel capacitors is the reciprocal of the sum of the reciprocals of the individual reactances.
- The true power in an ideal capacitor is zero; that is, no energy is lost in an ideal capacitor due to conversion to heat.
- A switched capacitor circuit is used in certain integrated circuits to emulate a resistor.

KEY TERMS

Key terms and other bold terms in the chapter are defined in the end-of-book glossary.

Capacitive reactance The opposition of a capacitor to sinusoidal current. The unit is ohm.

Capacitor An electrical device consisting of two conductive plates separated by an insulating material and possessing the property of capacitance.

Dielectric The insulating material between the plates of a capacitor.

Farad (F) The unit of capacitance.

Instantaneous power (p) The value of power in a circuit at any given instant of time.

***RC* time constant** A fixed time interval set by R and C values that determines the time response of a series RC circuit. It equals the product of the resistance and the capacitance.

Reactive power (P_r) The rate at which energy is alternately stored and returned to the source by a capacitor. The unit is the VAR.

Ripple voltage The small fluctuation in voltage due to the charging and discharging of a capacitor.

True power (P_{true}) The power that is dissipated in a circuit, usually in the form of heat.

VAR (volt-ampere reactive) The unit of reactive power.

FORMULAS

12–1	$C = \dfrac{Q}{V}$	Capacitance in terms of charge and voltage
12–2	$Q = CV$	Charge in terms of capacitance and voltage
12–3	$V = \dfrac{Q}{C}$	Voltage in terms of charge and capacitance
12–4	$W = \dfrac{1}{2}CV^2$	Energy stored by a capacitor
12–5	$\varepsilon_r = \dfrac{\varepsilon}{\varepsilon_0}$	Dielectric constant (relative permittivity)
12–6	$C = \dfrac{A\varepsilon_r(8.85 \times 10^{-12}\ \text{F/m})}{d}$	Capacitance in terms of physical parameters
12–7	$Q_T = Q_1 = Q_2 = Q_3 = \cdots = Q_n$	Total charge of series capacitors (general)
12–8	$\dfrac{1}{C_T} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \cdots + \dfrac{1}{C_n}$	Reciprocal of total series capacitance (general)
12–9	$C_T = \dfrac{1}{\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3} + \cdots + \dfrac{1}{C_n}}$	Total series capacitance (general)
12–10	$C_T = \dfrac{C_1C_2}{C_1 + C_2}$	Total capacitance of two capacitors in series
12–11	$C_T = \dfrac{C}{n}$	Total capacitance of equal-value capacitors in series
12–12	$V_x = \left(\dfrac{C_T}{C_x}\right)V_T$	Capacitor voltage for capacitors in series
12–13	$Q_T = Q_1 + Q_2 + Q_3 + \cdots + Q_n$	Total charge of parallel capacitors (general)
12–14	$C_T = C_1 + C_2 + C_3 + \cdots + C_n$	Total parallel capacitance (general)
12–15	$C_T = nC$	Total capacitance of equal-value capacitors in parallel
12–16	$\tau = RC$	Time constant
12–17	$v = V_F + (V_i - V_F)e^{-t/\tau}$	Exponential voltage (general)
12–18	$i = I_F + (I_i - I_F)e^{-t/\tau}$	Exponential current (general)
12–19	$v = V_F(1 - e^{-t/RC})$	Increasing exponential voltage beginning at zero
12–20	$v = V_ie^{-t/RC}$	Decreasing exponential voltage ending at zero

12–21	$t = -RC\ln\left(\frac{v}{V_i}\right)$	Time on decreasing exponential ($V_F = 0$)
12–22	$t = -RC\ln\left(1 - \frac{v}{V_F}\right)$	Time on increasing exponential ($V_i = 0$)
12–23	$i = \frac{dq}{dt}$	Instantaneous current using charge derivative
12–24	$i = C\left(\frac{dv}{dt}\right)$	Instantaneous capacitor current using voltage derivative
12–25	$X_C = \frac{1}{2\pi fC}$	Capacitive reactance
12–26	$X_{C(tot)} = X_{C1} + X_{C2} + X_{C3} + \cdots + X_{Cn}$	Total series reactance
12–27	$X_{C(tot)} = \frac{1}{\frac{1}{X_{C1}} + \frac{1}{X_{C2}} + \frac{1}{X_{C3}} + \cdots + \frac{1}{X_{Cn}}}$	Total parallel reactance
12–28	$V_x = \left(\frac{X_{Cx}}{X_{C(tot)}}\right)V_s$	Capacitive voltage divider
12–29	$P_r = V_{rms}I_{rms}$	Reactive power in a capacitor
12–30	$P_r = \frac{V_{rms}^2}{X_C}$	Reactive power in a capacitor
12–31	$P_r = I_{rms}^2X_C$	Reactive power in a capacitor
12–32	$I_{1(avg)} = \frac{C(V_1 - V_2)}{T}$	Switched-capacitor average current
12–33	$R = \frac{T}{C}$	Switched-capacitor equivalent resistance
12–34	$R = \frac{1}{fC}$	Switched-capacitor equivalent resistance

TRUE/FALSE QUIZ

Answers are at the end of the chapter.

1. The area of the plates of a capacitor is proportional to the capacitance.
2. A capacitance of 1,200 pF is the same as 1.2 μF.
3. When two capacitors are in series with a voltage source, the smaller capacitor will have the larger voltage.
4. When two capacitors are in series, the total capacitance is less than the smallest one.
5. When two capacitors are in parallel with a voltage source, the smaller capacitor will have the larger voltage.
6. A capacitor appears as an open to a constant dc.
7. A capacitor appears as a short to an instantaneous change in voltage.
8. When a capacitor is charging or discharging between two levels, the charge on the capacitor changes by 63% of the difference in one time constant.
9. Capacitive reactance is proportional to the applied frequency.
10. The total reactance of series capacitors is the product-over-sum of the individual reactances.
11. Voltage leads current in a capacitor.
12. The unit of reactive power is the VAR.

SELF-TEST

Answers are at the end of the chapter.

1. Which of the following accurately describes a capacitor?
   (a) The plates are conductive.
   (b) The dielectric is an insulator between the plates.

(c) There is constant direct current (dc) through a fully charged capacitor.
(d) A practical capacitor stores charge indefinitely when disconnected from the source.
(e) none of the above answers
(f) all of the above answers
(g) only answers (a) and (b)

2. Which one of the following statements is true?
(a) There is current through the dielectric of a charging capacitor.
(b) When a capacitor is connected to a dc voltage source, it will charge to the value of the source.
(c) An ideal capacitor can be discharged by disconnecting it from the voltage source.

3. A capacitance of 0.01 mF is larger than
(a) 0.00001 F **(b)** 100,000 pF **(c)** 1,000 pF **(d)** all of these answers

4. A capacitance of 1,000 pF is smaller than
(a) 0.01 μF **(b)** 0.001 μF **(c)** 0.00000001 F **(d)** both (a) and (c)

5. When the voltage across a capacitor is increased, the stored charge
(a) increases **(b)** decreases **(c)** remains constant **(d)** fluctuates

6. When the voltage across a capacitor is doubled, the stored charge
(a) stays the same **(b)** is halved **(c)** increases by four **(d)** doubles

7. The voltage rating of a capacitor is increased by
(a) increasing the plate separation **(b)** decreasing the plate separation
(c) increasing the plate area **(d)** answers (b) and (c)

8. The capacitance value is increased by
(a) decreasing the plate area **(b)** increasing the plate separation
(c) decreasing the plate separation **(d)** increasing the plate area
(e) answers (a) and (b) **(f)** answers (c) and (d)

9. A 1 μF, a 2.2 μF, and a 0.047 μF capacitor are connected in series. The total capacitance is less than
(a) 1 μF **(b)** 2.2 μF **(c)** 0.047 μF **(d)** 0.001 μF

10. Four 0.022 μF capacitors are in parallel. The total capacitance is
(a) 0.022 μF **(b)** 0.088 μF **(c)** 0.011 μF **(d)** 0.044 μF

11. An uncharged capacitor and a resistor are connected in series with a switch and a 12 V battery. At the instant the switch is closed, the voltage across the capacitor is
(a) 12 V **(b)** 6 V **(c)** 24 V **(d)** 0 V

12. In Question 11, the voltage across the capacitor when it is fully charged is
(a) 12 V **(b)** 6 V **(c)** 24 V **(d)** −6 V

13. In Question 11, the capacitor will reach full charge in a time equal to approximately
(a) RC **(b)** $5RC$ **(c)** $12RC$ **(d)** cannot be predicted

14. A sinusoidal voltage is applied across a capacitor. When the frequency of the voltage is increased, the current
(a) increases **(b)** decreases **(c)** remains constant **(d)** ceases

15. A capacitor and a resistor are connected in series to a sine wave generator. The frequency is set so that the capacitive reactance is equal to the resistance and, thus, an equal amount of voltage appears across each component. If the frequency is decreased,
(a) $V_R > V_C$ **(b)** $V_C > V_R$ **(c)** $V_R = V_C$

16. Two equal-value capacitors are connected in series. At any frequency, the total reactance is
(a) one-half the reactance of each capacitor
(b) equal to the reactance of each capacitor
(c) twice the reactance of each capacitor

17. Switched-capacitor circuits are used to
    (a) increase capacitance (b) emulate inductance
    (c) emulate resistance (d) generate sine wave voltages

CIRCUIT DYNAMICS QUIZ

Answers are at the end of the chapter.

Refer to Figure 12–74

1. If the capacitors are initially uncharged and the switch is thrown to the closed position, the charge on C_1
   (a) increases (b) decreases (c) stays the same
2. If C_4 is shorted with the switch closed, the charge on C_1
   (a) increases (b) decreases (c) stays the same
3. If the switch is closed and C_2 fails open, the charge on C_1
   (a) increases (b) decreases (c) says the same

Refer to Figure 12–75

4. Assume the switch is closed and C is allowed to fully charge. When the switch is opened, the voltage across C
   (a) increases (b) decreases (c) stays the same
5. If C has failed open when the switch is closed, the voltage across C
   (a) increases (b) decreases (c) stays the same

Refer to Figure 12–78

6. If the switch is closed allowing the capacitor to charge and then the switch is opened, the voltage across the capacitor
   (a) increases (b) decreases (c) stays the same
7. If R_2 opens, the time it takes the capacitor to fully charge
   (a) increases (b) decreases (c) stays the same
8. If R_4 opens, the maximum voltage to which the capacitor can charge
   (a) increases (b) decreases (c) stays the same
9. If V_S is reduced, the time required for the capacitor to fully change
   (a) increases (b) decreases (c) stays the same

Refer to Figure 12–81(b)

10. If the frequency of the ac source is increased, the total current
    (a) increases (b) decreases (c) stays the same
11. If C_1 opens, the current through C_2
    (a) increases (b) decreases (c) stays the same
12. If the value of C_2 is changed to 1 μF, the current through it
    (a) increases (b) decreases (c) stays the same

PROBLEMS

More difficult problems are indicated by an asterisk (*).
Answers to odd-numbered problems are at the end of the book.

SECTION 12–1 The Basic Capacitor

1. (a) Find the capacitance when $Q = 50\ \mu$C and $V = 10$ V.
   (b) Find the charge when $C = 0.001\ \mu$F and $V = 1$ kV.
   (c) Find the voltage when $Q = 2$ mC and $C = 200\ \mu$F.

2. Convert the following values from microfarads to picofarads:
   (a) 0.1 μF (b) 0.0025 μF (c) 4.7 μF
3. Convert the following values from picofarads to microfarads:
   (a) 1,000 pF (b) 3,500 pF (c) 250 pF
4. Convert the following values from farads to microfarads:
   (a) 0.0000001 F (b) 0.0022 F (c) 0.0000000015 F
5. How much energy is stored in a 1,000 μF capacitor that is charged to 500 V?
6. What size capacitor is capable of storing 10 mJ of energy with 100 V across its plates?
7. Calculate the absolute permittivity, ε, for each of the following materials. Refer to Table 12–3 for ε_r values.
   (a) air (b) oil (c) glass (d) Teflon®
8. A mica capacitor has square plates that are 3.8 cm on a side and separated by 2.5 mils. What is the capacitance?
9. A capacitor has square plates that are 1.5 cm on a side with a mica dielectric that is 0.2 mm thick. Calculate the capacitance.

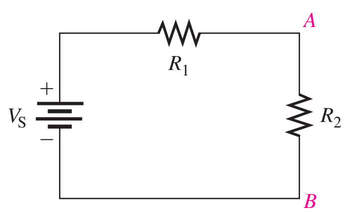
*10. A student wants to construct a 1 F capacitor out of two square plates for a science fair project. He plans to use a paper dielectric ($\varepsilon_r = 2.5$) that is 8×10^{-5} m thick. The science fair is to be held in the Astrodome. Will his capacitor fit in the Astrodome? What would be the size of the plates if it could be constructed?

11. A student decides to construct a capacitor using two conducting plates 30 cm on a side. He separates the plates with a paper dielectric ($\varepsilon_r = 2.5$) that is 8×10^{-5} m thick. What is the capacitance of his capacitor?
12. At ambient temperature (25°C), a certain capacitor is specified to be 1,000 pF. It has a negative temperature coefficient of 200 ppm/°C. What is its capacitance at 75°C?
13. A 0.001 μF capacitor has a positive temperature coefficient of 500 ppm/°C. How much change in capacitance will a 25°C increase in temperature cause?

SECTION 12–2 Types of Capacitors

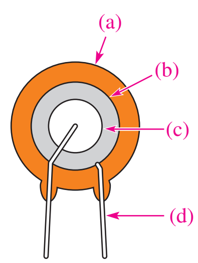
14. In the construction of a stacked-foil mica capacitor, how is the plate area increased?
15. Of mica or ceramic, which type of capacitor has the highest dielectric constant?
16. Show how to connect an electrolytic capacitor across R_2 between points A and B in Figure 12–67.

► FIGURE 12–67

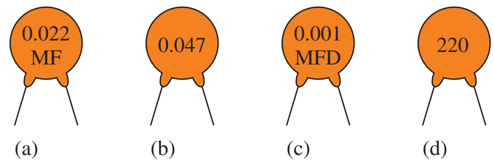


17. Name two types of electrolytic capacitors. How do electrolytics differ from other capacitors?
18. Identify the parts of the ceramic disk capacitor shown in the cutaway view of Figure 12–68.

► FIGURE 12–68



▶ FIGURE 12–69



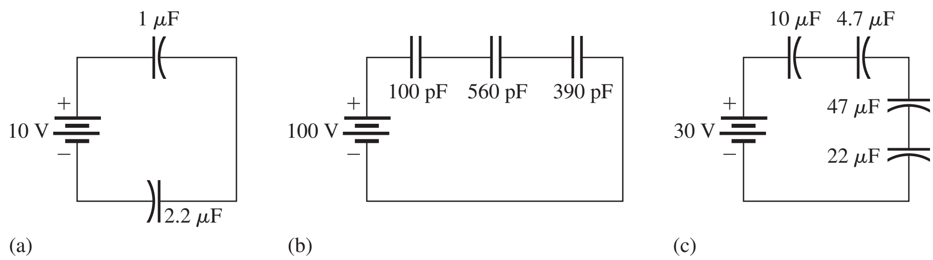
19. Determine the value of the ceramic disk capacitors in Figure 12–69.

SECTION 12–3 Series Capacitors

20. Five 1,000 pF capacitors are in series. What is the total capacitance?

21. Find the total capacitance for each circuit in Figure 12–70.

22. For each circuit in Figure 12–70, determine the voltage across each capacitor.

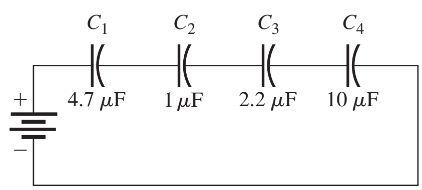


▲ FIGURE 12–70

23. Two series capacitors (one 1 μF, the other of unknown value) are charged from a 12 V source. The 1 μF capacitor is charged to 8 V and the other to 4 V. What is the value of the unknown capacitor?

24. The total charge stored by the series capacitors in Figure 12–71 is 10 μC. Determine the voltage across each of the capacitors.

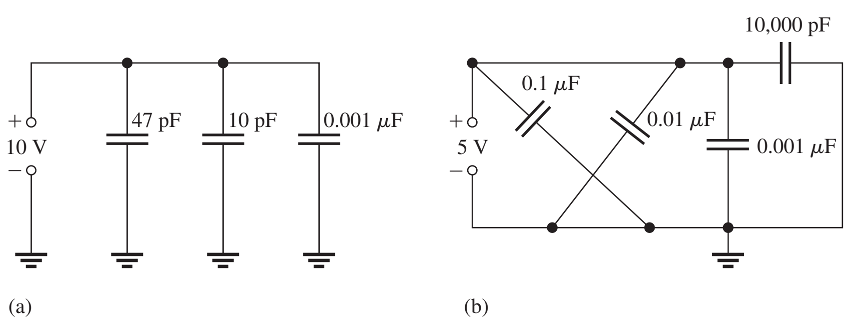
▶ FIGURE 12–71



SECTION 12–4 Parallel Capacitors

25. Determine C_T for each circuit in Figure 12–72.

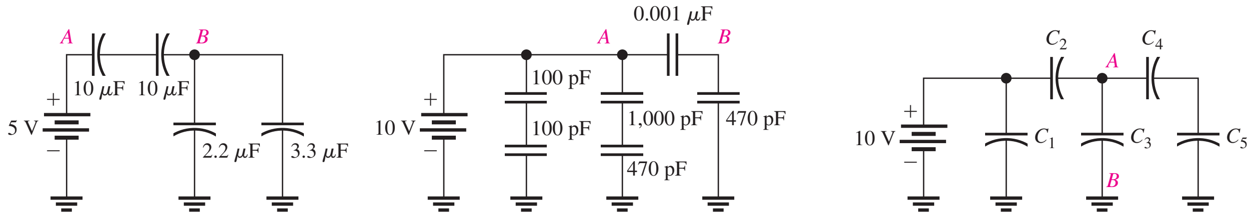
26. What is the charge on each capacitor in Figure 12–72?



▲ FIGURE 12–72

27. Determine C_T for each circuit in Figure 12–73.

28. What is the voltage between nodes A and B in each circuit in Figure 12–73?



(a) (b) (c) $C = 1\ \mu$F for each capacitor

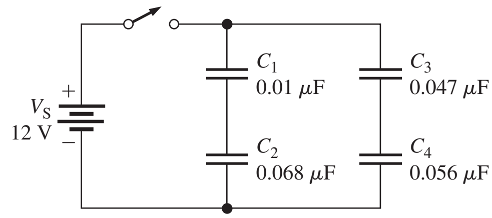
▲ **FIGURE 12–73**

***29.** Initially, the capacitors in the circuit in Figure 12–74 are uncharged.

(a) After the switch is closed, what total charge is supplied by the source?

(b) What is the voltage across each capacitor?

► **FIGURE 12–74**



SECTION 12–5 Capacitors in DC Circuits

30. Determine the time constant for each of the following series RC combinations:

(a) $R = 100\ \Omega$, $C = 1\ \mu\text{F}$ **(b)** $R = 10\ \text{M}\Omega$, $C = 47\ \text{pF}$

(c) $R = 4.7\ \text{k}\Omega$, $C = 0.0047\ \mu\text{F}$ **(d)** $R = 1.5\ \text{M}\Omega$, $C = 0.01\ \mu\text{F}$

31. Determine how long it takes the capacitor to reach full charge for each of the following combinations:

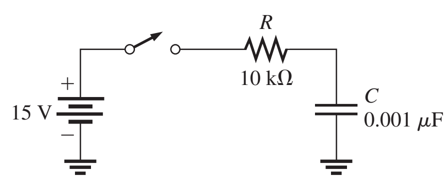
(a) $R = 56\ \Omega$, $C = 47\ \mu\text{F}$ **(b)** $R = 3{,}300\ \Omega$, $C = 0.015\ \mu\text{F}$

(c) $R = 22\ \text{k}\Omega$, $C = 100\ \text{pF}$ **(d)** $R = 56\ \text{k}\Omega$, $C = 1{,}000\ \text{pF}$

32. In the circuit of Figure 12–75, the capacitor is initially uncharged. Determine the capacitor voltage at the following times after the switch is closed:

(a) 10 μs **(b)** 20 μs **(c)** 30 μs **(d)** 40 μs **(e)** 50 μs

► **FIGURE 12–75**



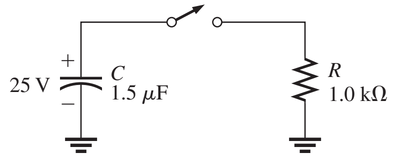
33. In Figure 12–76, the capacitor is charged to 25 V. When the switch is closed, what is the capacitor voltage after the following times?

(a) 1.5 ms **(b)** 4.5 ms **(c)** 6 ms **(d)** 7.5 ms

34. Repeat Problem 32 for the following time intervals:

(a) 2 μs **(b)** 5 μs **(c)** 15 μs

▶ FIGURE 12–76

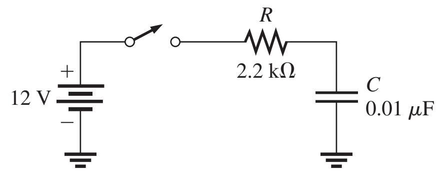


35. Repeat Problem 33 for the following times:

(a) 0.5 ms **(b)** 1 ms **(c)** 2 ms

***36.** Derive the formula for finding the time at any point on an increasing exponential voltage curve. Use this formula to find the time at which the voltage in Figure 12–77 reaches 6 V after switch closure.

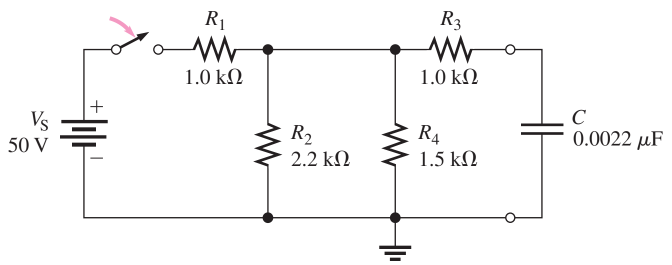
▶ FIGURE 12–77



37. How long does it take C to charge to 8 V in Figure 12–75?

38. How long does it take C to discharge to 3 V in Figure 12–76?

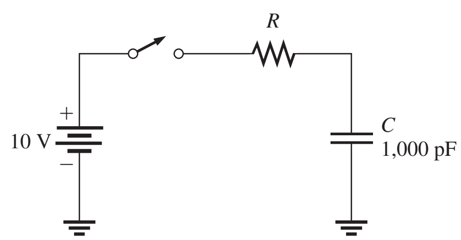
39. Determine the time constant for the circuit in Figure 12–78.



▲ FIGURE 12–78

***40.** In Figure 12–79, the capacitor is initially uncharged. At $t = 10\ \mu s$ after the switch is closed, the instantaneous capacitor voltage is 7.2 V. Determine the value of R.

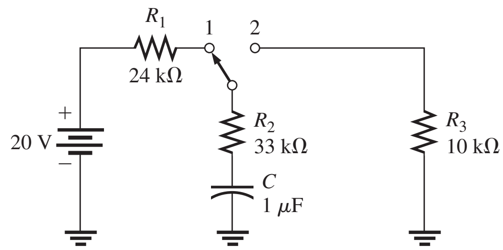
▶ FIGURE 12–79



***41. (a)** The capacitor in Figure 12–80 is uncharged when the switch is thrown into position 1. The switch remains in position 1 for 10 ms and is then thrown into position 2, where it remains indefinitely. Draw the complete waveform for the capacitor voltage.

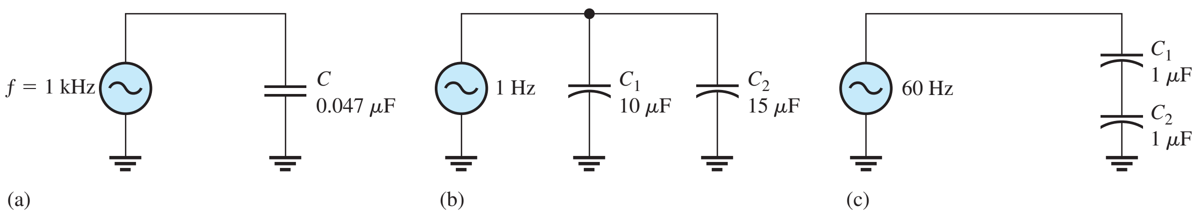
(b) If the switch is thrown back to position 1 after 5 ms in position 2, and then left in position 1, how would the waveform appear?

▶ FIGURE 12–80



SECTION 12–6 Capacitors in AC Circuits

42. What is the value of the total capacitive reactance in each circuit in Figure 12–81?



▲ FIGURE 12–81

43. In Figure 12–73, each dc voltage source is replaced by a 10 V rms, 2 kHz ac source. Determine the total reactance in each case.

44. In each circuit of Figure 12–81, what frequency is required to produce an X_C of 100 Ω? An X_C of 1 kΩ?

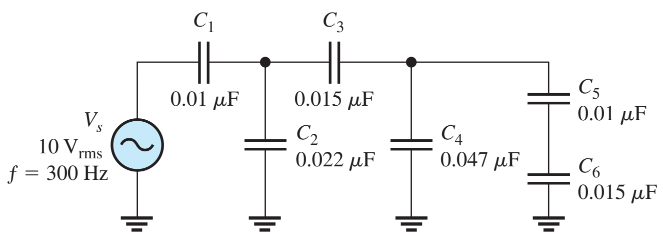
45. A sinusoidal voltage of 20 V rms produces an rms current of 100 mA when connected to a certain capacitor. What is the reactance?

46. A 10 kHz voltage is applied to a 0.0047 μF capacitor, and 1 mA of rms current is measured. What is the value of the voltage?

47. Determine the true power and the reactive power in Problem 46.

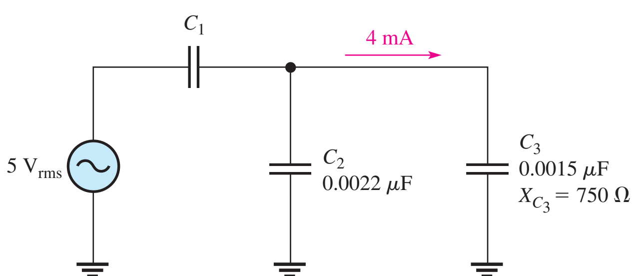
***48.** Determine the ac voltage across each capacitor and the current in each branch of the circuit in Figure 12–82.

▶ FIGURE 12–82



49. Find the value of C_1 in Figure 12–83.

▶ FIGURE 12–83



*50. If C_4 in Figure 12–82 opened, determine the voltages that would be measured across the other capacitors.

51. Starting with Equation 12–25, show that the unit for capacitive reactance is ohm.

SECTION 12–7 Capacitor Applications

52. Ideally, what should the reactance of a bypass capacitor be in order to eliminate a 10 kHz ac voltage at a given point in an amplifier circuit?

53. If another capacitor is connected in parallel with the existing capacitor in the power supply filter of Figure 12–58, how is the ripple voltage affected?

SECTION 12–8 Switched-Capacitor Circuits

54. In a switched-capacitor circuit, the 100 pF capacitor is switched at a frequency of 8 kHz. What resistor value is emulated?

55. The capacitor in a switched-capacitor circuit has a value of 2,200 pF and is switched with a waveform having a period of 10 μs. Determine the value of the resistor that it emulates.

Multisim Troubleshooting and Analysis

These problems require Multisim.

56. Open file P12–56 and measure the voltage across each capacitor.

57. Open file P12–57 and measure the current. Decrease the frequency by one-half and measure the current again. Double the original frequency and measure the current again. Explain your observations.

58. Open file P12–58 and find the open capacitor if there is one.

59. Open file P12–59 and find the shorted capacitor if there is one.

ANSWERS

SECTION CHECKUPS

SECTION 12–1 The Basic Capacitor

1. Capacitance is the ability (capacity) to store electrical charge.
2. **(a)** 1,000,000 μF in 1 F **(b)** 1×10^{12} pF in 1 F **(c)** 1,000,000 pF in 1 μF
3. 0.0015 μF = 1500 pF; 0.0015 μF = 0.0000000015 F
4. $W = \frac{1}{2}CV^2 = 1.125\ \mu\text{J}$
5. **(a)** C increases. **(b)** C decreases.
6. (1,000 V/mil) (2 mil) = 2 kV
7. $C = 2.01\ \mu\text{F}$

SECTION 12–2 Types of Capacitors

1. Capacitors can be classified by the dielectric material.
2. The capacitance value of a fixed capacitor cannot be changed; the capacitance value of a variable capacitor can be changed.
3. Electrolytic capacitors are polarized.
4. When connecting a polarized capacitor, make sure the voltage rating is sufficient. Connect the positive end to the positive side of the circuit.

SECTION 12–3 Series Capacitors

1. Series C_T is less than smallest C.
2. $C_T = 61.2$ pF
3. $C_T = 0.006\ \mu\text{F}$
4. $C_T = 20$ pF
5. $V_{C1} = 15.0$ V

SECTION 12–4 Parallel Capacitors

1. The values of the individual capacitors are added in parallel.
2. Achieve C_T by using five 0.01 μF capacitors in parallel.
3. $C_T = 167$ pF

SECTION 12–5 Capacitors in DC Circuits

1. $\tau = RC = 1.2\ \mu$s
2. 1.93 μs
3. $v_{2ms} = 8.65$ V; $v_{3ms} = 9.50$ V; $v_{4ms} = 9.82$ V; $v_{5ms} = 9.93$ V
4. $v_C = 36.8$ V

SECTION 12–6 Capacitors in AC Circuits

1. Current leads voltage by 90° in a capacitor.
2. $X_C = 1/2\pi fC = 637\ \text{k}\Omega$
3. $f = 1/2\pi X_C C = 796$ Hz
4. $I_{rms} = 628$ mA
5. $P_{true} = 0$ W
6. $P_r = 452$ mVAR

SECTION 12–7 Capacitor Applications

1. Once the capacitor charges to the peak voltage, it discharges very little before the next peak, thus smoothing the rectified voltage.
2. A coupling capacitor allows ac to pass from one point to another, but blocks constant dc.
3. A coupling capacitor must be large enough to have a negligible reactance at the frequency that is to be passed without opposition.
4. A decoupling capacitor shorts power line voltage transients to ground.
5. X_C is inversely proportional to frequency and so is the filter's ability to pass ac signals.
6. The charging time

SECTION 12–8 Switched-Capacitor Circuits

1. By moving the same amount of charge corresponding to the current in the equivalent resistance
2. Switching frequency and capacitance value
3. Transistors

RELATED PROBLEMS FOR EXAMPLES

12–1 200 kV
12–2 0.047 μF
12–3 100×10^6 pF
12–4 62.7 pF
12–5 1.54 μF
12–6 278 pF
12–7 0.011 μF
12–8 2.83 V
12–9 650 pF
12–10 0.09 μF
12–11 891 μs
12–12 8.36 V

12–13 8.13 V
12–14 ≈0.74 ms; 95 V
12–15 1.52 ms
12–16 0.419 V
12–17 3.39 kHz
12–18 (a) 1.83 kΩ (b) 204 Ω
12–19 4.40 mA
12–20 8.72 V
12–21 0 W; 1.01 mVAR
12–22 179 kHz

TRUE/FALSE QUIZ

1. T **2.** F **3.** T **4.** T **5.** F **6.** T
7. T **8.** T **9.** F **10.** F **11.** F **12.** T

SELF-TEST

1. (g) **2.** (b) **3.** (c) **4.** (d) **5.** (a) **6.** (d) **7.** (a)
8. (f) **9.** (c) **10.** (b) **11.** (d) **12.** (a) **13.** (b) **14.** (a)
15. (b) **16.** (c) **17.** (c)

CIRCUIT DYNAMICS QUIZ

1. (a) **2.** (c) **3.** (c) **4.** (c) **5.** (a) **6.** (b)
7. (a) **8.** (a) **9.** (c) **10.** (a) **11.** (c) **12.** (b)