MTCS-103(T)- Parallel Processing

Assignment-3

Monte Carlo Case Studies

Temperature at a Point Inside a 2-D Plate

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Problem Title: Steady-State Temperature Estimation in a 2-D Plate using Monte Carlo Simulation

Problem Description:

We wish to determine the steady-state temperature at a certain interior spot within a two-dimensional plate formed of a homogeneous material with insulated top and bottom bounds and set temperature values defined along its edges. Laplace's equation governs the distribution of interior temperature:

$\nabla^2 T = 0$

where T stands for a point's temperature and ∇^2 is the Laplacian operator.

Input:

- A 2-D plate that is represented as a grid or mesh, with each point on the grid denoting a specific place on the plate.
- Boundaries that define the temperatures at the plate's margins.
- The location's coordinates on the plate at which a temperature estimate is required.
- A convergence threshold ε , that specifies the temperature estimate's allowable margin of error.

Output:

The steady state temperature that is estimated at a specified point within the plate.

Goal:

By randomly moving around the grid of the plate, use a Monte Carlo simulation strategy to iteratively estimate the temperature at the specified interior point of the plate. As temperatures are added, the algorithm should continue random walks until the estimated temperature converges inside the specified threshold.

Constraints:

- It is expected that the plate is comprised of a thin, uniform material.
- The edges' temperatures are fixed, and the top and bottom bounds are insulated.
- It is expected that the distribution of temperatures within the plate can be adequately described by Laplace's equation, $\nabla^2 T = 0$.
- When the average temperature value converges, the process stops, indicating that the estimate is accurate enough.
- Recursion may be used in the method to investigate nearby spots.
- The quantity of random walks and the convergence criterion affect how accurate the temperature estimate is.

The Method that is used in the textbook is as follows:

1) Discretization of the Plate:

The 2-D plate must be discretized in the initial stage. In order to do this, the continuous plate must be divided into a grid of discrete points. Each location's temperature is represented by a number. We may use numerical methods and work with discrete data thanks to this grid.

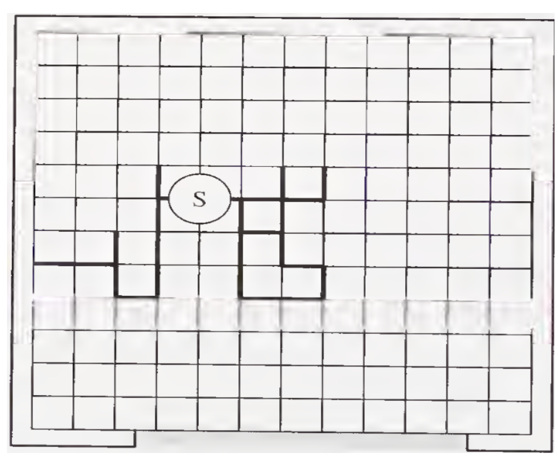
2) Laplace’s Equation:

Laplace’s equation, $\nabla^2 T = 0$, governs this problem, where T is the temperature at any point within the plate. His equation tells that the Laplacian of the temperature field is zero, which indicates that the temperature at a point is the average of the temperatures at its neighboring points. The fundamental mathematical description for heat distribution within the plate is proved by the above formulation.

3) Monte Carlo Simulation:

The Monte Carlo simulation serves as the main methodology in this strategy. Each step indicates a move to a north, south, east, or west neighboring point during a random walk within the grid. This is how the simulation is run:

i) A random neighbor is chosen to begin at the target point S, and the temperature at that neighbor is added to an accumulator.
ii) Once one of the plate's edges is reached, the random walk resumes. The temperature at the edge is then added to the accumulator as a result.
iii) The procedure is repeated at random, with the accumulator adding up the temperatures and the walker picking random neighbors.
iv) The current estimated temperature at point S is derived as the average temperature of the points encountered thus far (averaging the accumulator and the average edge temperature). The average edge temperature encountered throughout all random walks is tracked.
v) Until the average temperature value converges, the algorithm repeats this process. When comparing successive estimates, convergence is assessed by determining whether the difference is less than a set threshold.



4) Recursion:

There is a recursive process involved in the Monte Carlo simulation. The algorithm repeatedly takes arbitrary steps while investigating nearby sites and gathering temperatures. Through the use of several pathways and nearby spots on the plate, the recursion enables the estimation of the temperature at point S.

5) Convergence Criteria:

A key component of this strategy is the algorithm's convergence. When the average temperature value converges, it comes to an end, indicating that the temperature estimate at point S has stabilized. Typically, convergence is described in terms of a predetermined threshold, which, when reached, indicates that the guess is accurate enough.

Alternative Parallelization Techniques:

1. Parallelization of Random Walks:

The computing task is distributed among several concurrent threads or processes using this technique. Concurrent execution is made possible by the independent execution of a subset of random walks by each of these threads or processes. By utilizing the multi-core capabilities of contemporary processors or distributed computing clusters, this method greatly accelerates the convergence of the Monte Carlo simulation. It is especially helpful for managing big and complex simulations since it makes the best use of resources and takes the shortest amount of time to compute, giving immediate insights into temperature distributions and facilitating sensitivity analysis and optimization in the face of uncertainty.

2. Domain Decomposition:

The 2-D plate is systematically divided into smaller, non-overlapping subdomains using this technique, and each subdomain is subsequently assigned to a different processing unit. The estimation of the temperature within each processing unit's or computational entity's assigned subdomain enables simultaneous simulation execution across various processing units. The assigned subdomains can be processed concurrently, accelerating the Monte Carlo simulation's overall convergence. This method, which is essential for load balancing and effective resource use, makes sure that the computational workload is divided evenly throughout the designated subdomains.

3. Parallel Monte Carlo Path Generation:

In this method, many Monte Carlo routes are simultaneously generated, each one representing a distinct trajectory of random walks and temperature predictions. Concurrent route generation offers significant benefits, especially in situations where multiple temperature estimates are necessary, such as when analyzing temperature profiles at various locations on the plate or running numerous simulations with various initial conditions. The time needed for simulations is significantly decreased by running these paths simultaneously because more effectively using computational resources. The Monte Carlo approach is highly suited for handling complex problems across a range of scientific, technical, and financial applications because to Parallel Monte Carlo Path Generation, which not only speeds up temperature estimations but also improves the method's scalability and adaptability.