DELTA Pg No.
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Unit ⇒ 1

Signal :- Signal is an electric or electromagnetic current carrying data, that can be transmitted or received.

- Mathmatically represented as a function of an independent variable e.g. density, depth, etc. Therefore, a signal is a physical quantity that varies with time, space, or any other independent variable by which information can be conveyed. Here independent variable is time

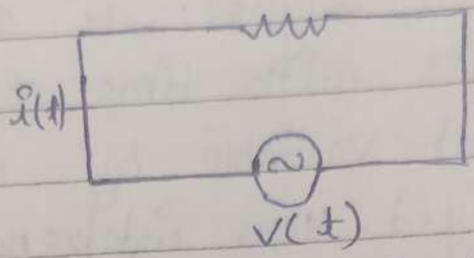
System :- A system is any physical set of components or a function of several devices that takes a signal in input, and produces a signal as output

Energy Signal :- A signal is said to be an energy signal if and only if its total energy E is finite i.e $0<E<\infty$
For an energy signal, the average power $P=0$
The nonperiodic signal are the example of energy signal.

Power signal :- A signal is said to be a power signal if its average power P is finite, i.e $0<P<\infty$. For a power signal the total energy $E=\infty$. The periodic signal are the example of power signal.

Continuous time Case

In electric circuit, the signal may represent current or voltage. Consider a voltage $v(t)$ is applied across a resistance R & $i(t)$ is the current flowing through it as shown in the fig



Power in the resistance R is given by

$$P(t) = v(t) \cdot i(t) \quad -- (1)$$

By Ohm's law

$$P(t) = v(t)\frac{v(t)}{R} = \frac{v^2(t)}{R} \quad - (2)$$

Also

$$P(t) = i(t)R \cdot i(t) = i^2(t)R \quad - (3)$$

When the values of the resistance $R = 1\,\Omega$ then the power dissipated in it is known as normalised power.

Normalised power $= p(t) = v^2(t) = i^2(t)$ - (4)

If $v(t)$ or $i(t)$ is denoted by a continuous-time signal $x(t)$, then the instantaneous power is equal to the square of the amplitude of the signal

$$p(t) = |x(t)|^2 \quad - (5)$$

Therefore the average power or normalised power of a continuous time signal $x(t)$ is given by,

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-(T/2)}^{(T/2)} |x(t)|^2 \, dt \text{ , watts} \quad - (6)$$

The total energy or normalised energy of a continuous time signal is defined as

$$E = \lim_{T \to \infty} \int_{-(T/2)}^{(T/2)} |x(t)|^2 \, dt \text{ Joules}$$

Discrete Time Case :-

For the discrete time signal $x(n)$, the integrals are replaced by summations. Hence, the total energy of the discrete time signal $x(n)$ is defined as

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

The average power of a discrete time signal $x(t)$ is defined as

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

imp points

* Both energy & power signal are mutually exclusive, i.e, no signal can be both power & energy signal.
* A signal is neither energy nor power signal if both energy & power of the signal are equal to infinity.
* All practical signal have finite energy thus they are energy signals
* A signal whose amplitude is constant over infinite duration is a power signal
* The energy of a signal is not affected by the time shifting & time inversion It is only affected by the time scaling
* Generally periodic signal are power signal.
  ——— aperiodic ——— energy signal.
* Power of energy signal $=0$
* Energy of power signal $=\infty$

Class I

Classification of Signal :— Total finite energy $E_\infty < \infty$ & zero at Avg point.

energy signal $P_\infty = \lim\limits_{T\to\infty} \frac{E_\infty}{2T} = 0$

Class 2 signal :— $P_\infty > 0$ the $E_\infty = \infty$

Ex $x[n] = 4$ $P_\infty = 16$

Class 3 signal :- P_∞ E_∞

$x(t) = t$ → independent

dependent

Transformation of the independent variable:-

1) **Shifting** :- The signal can be delayed $(x(t-T))$ or advanced $(x(t+T))$ by incrementing or decrementing the independent variable. The shape of the graph remains same only shifted on the time axis.

2) **Scaling** :- The signal can be compressed $(x(at), a>1)$ or expanded $(x(t/a), a>1$ or $x(at), 1>a>0)$.

Here the shape/behaviour of the graph of the signal changes as the fundamental time period changes. In compression the time period ⇓ & in expansion the time period ⇑.

3) **Reversal** :- also called folding as the graph is folded about Y-axis or T if given $x(T-t)$

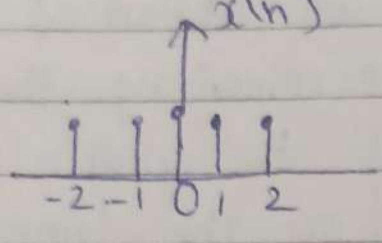
Even Signal :- A signal which is symmetrical about the vertical axis or time origin is known as even signal or even function. Therefore, the even signals are also called the symmetrical signal. E.g. Cosine wave

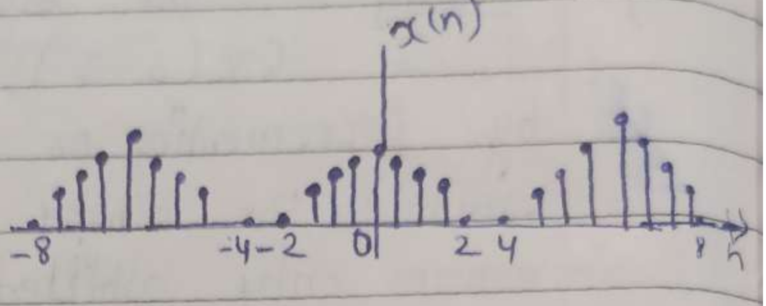
Discrete-time Even Signal

A discrete-time signal $x(n)$ is said to be even signal or symmetrical signal if it satisfies the condition,

$$x(n) = x(-n) \; ; \; \text{for } -\infty < n < \infty$$

$$x_e[n] = \frac{1}{2}\left[x[n] + x[-n]\right]$$





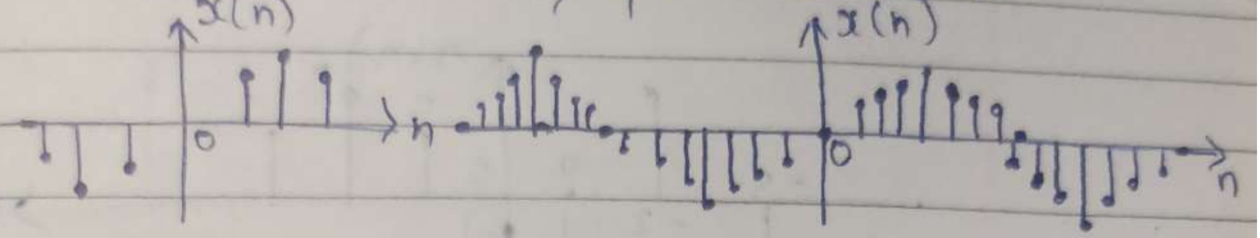
Properties

- The even signal are symmetrical about the vertical axis.
- The value of an even signal at time (t) is same as at time $(-t)$.
- The even signal is identical with its reflection about the origin.
- Area under the even signal is two time of its one side area.

<u>Odd Signal</u> :- A signal that is anti-symmetrical about the vertical axis is known as odd signal or odd function. Therefore, the odd signal are also called antisymmetric signal.
eg Sine wave

Discreate - time odd signal :- A discrete time signal x(n) is said to be an add signal or antisymmetric signal if it satisfies the following condition.

$x(-n) = -x(n)$; for $-\infty < n < \infty$



Properties

The odd signal is antisymmetric about the orgin

The value of add signal at time (t) is negative of its value at time (-t) for all t, i.e, $-\infty < t < \infty$.

The odd signal must necessarily be zero at time $t = 0$ to hold $x(0) = -x(0)$.

Area under the odd signal is always zero

$x(t) = \sin 2t$	$x(t) = \cos 5t$
$x(-t) = -\sin 2t$	$x(-t) = \cos 5t$
$-x(t) = -\sin 2t$	$-x(t) = -\cos 5t$
$x(t) \neq x(-t)$ but $x(-t) = -x(t)$	$x(t) = x(-t)$ & $x(-t) \neq -x(t)$
This is odd signal	This is even signal

Representation of Discrete Time signals :-

Graphical Representation

Functional

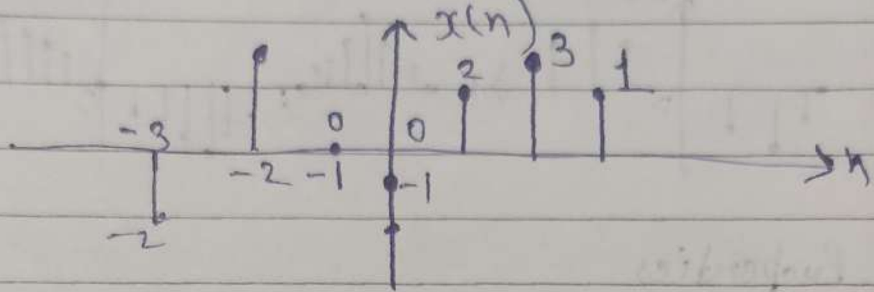
Tubular

Sequence

 Sign.

1) Graphical :- Consider a discrete time signal $x(n)$ with values :-

$x(-3) = -2,\ x(-2) = 3,\ (x-1) = 0,\ x(0) = -1$
$x(1) = 2,\ x(2) = 3,\ x(3) = 1$



2) Functional :- In the functional representation of discrete time signal, the magnitude of the signal is written aginst the values of n. Therefore, the above discrete time signal $x(n)$ can be represented using functional rep.

$$x(n) = \begin{cases} -3 & x = -3 \\ 0 & x = -2 \\ 2 & x = -1 \\ 3 & x = 0 \\ 1 & x = 1 \\ 5 & x = 2 \end{cases}$$

$$x_n \begin{cases} 2n & x \geq 0 \\ 0 & x < 0 \end{cases}$$

3) Tubular representation :- In this the sampling instant n & the magnitude of the discrete time

n	-3	-2	-1	0	1	2	3
$x(n)$	-2	3	0	-1	2	3	1

Sequence representation :- The discrete time signal $x(n)$ can be represented in the sequence representation

$x(n) = \{-2, 3, 0, -1, 2, 3, 1 \uparrow\}$

Here, the arrow mark denotes the term corresponding to $n = 0$. When no arrow is indicated in the sequence representation of a discrete time signal sequence then the first term of the sequence corresponds to $n = 0$

$x[n] = \{ - - - - -3, 0, 2, 3, 1, 5 - - \}$

Sum & Product of Discrete Time Sequences:-

Sum $= \{C_n\} = \{a_n\} + \{b_n\} \rightarrow C_n = a_n + b_n$

Product $= \{C_n\} = \{a_n\}\{b_n\} \rightarrow C_n = a_n b_n$

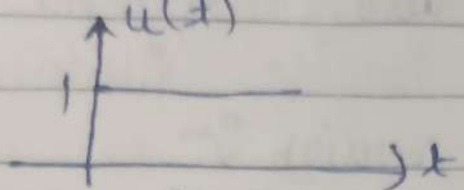
Constant $= \{C_n\} = K\{a_n\} \rightarrow C_n = Ka_n$

Defination :- The signal which are defined only at discrete instants of time are known as discrete time signal. The D.T.S are represented by $x(n)$ where n is the independent variable in domain.

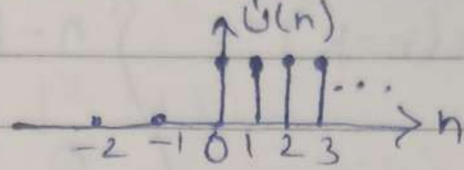
Standard signal :- These signal are those signal which are used to represent another signal

Unit Step Signal :- The step signal or step function is that type of standard signal which exists only for the positive time & it is zero for negative time. In other words a signal $x(t)$ is said to be step signal if and only if it exists for $t \geq 0$ & zero for $t < 0$. The step signal is an imp signal used for analysis of many system.
If a step signal has unity magnitude then it is known as unit step signal or unit step function. It is denoted by $u(t)$.
The step signal is equivalent to applying a signal to a system whose magnitude suddenly changes & remains constant forever after application. If we want to obtain a signal which start at $t=0$, so that it may have a value of zero for $t<0$, then we only need to multiply the given signal with the unit step signal

Continuous :- $u(t) = \{1$ for $t \geq 0$, 0 for $t < 0$



Discrete $u(n) = \{1$ for $n \geq 0$, 0 for $n < 0$



Shifted version $\Rightarrow$ $u(n-k)$

$u_n = \{1, 1, 1, 1, 1 - - \}$
for $k = 0$

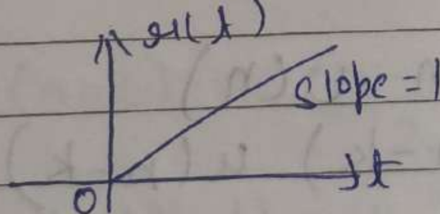
$u_n = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$

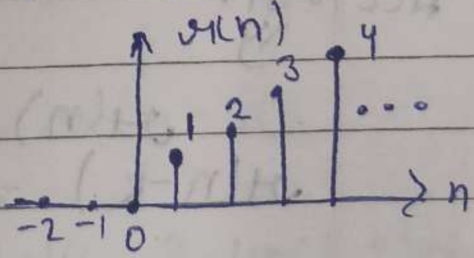
when $k = -2$

$\{0\,0\,1, 1, 1, 1\}$

Unit ramp signal :- A ramp function or ramp signal is a type of standard signal which starts at $t = 0$ & ↑ linearly with time. The unit ramp function has unit slop.

Continuous :- $r(t) = \{1$ for $t \geq 0$, 0 for $t < 0$
also $r(t) = t\,u(t)$





$r(n) = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$

discrete :- $r(n) = \{ n$ for $n \geq 0$ 0 for $n<0$
also $r(n) = n\,u(n)$

shifted version :-

$$r(n-k) = \begin{cases} n-k & n \geq k \\ 0 & n < k \end{cases}$$

Relationship b/w unit ramp & unit Step

The unit ramp signal can be obtained by integrating the unit step signal with respect to time. In other words, a unit step signal can be obtained by differentiating the unit ramp signal.

unit step

$u(t) = \{ 1$ for $t \geq 0$ 0 for $t<0$

unit ramp =

$r(t) = \int u(t)\,dt = \int dt = t$; for $t \geq 0$

Also $u(t) = \frac{d}{dt} r(t)$

Acc to sir

$r(n) = n\,u(n)$

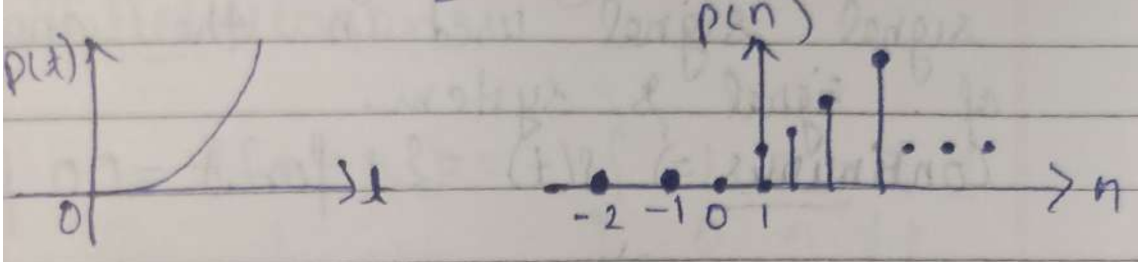
$r(n-k) = (n-k)\,u(n-k)$

Unit Parabolic Signal :- When a signal gives the Constant acceleration distinction of actual input signal such a signal is known as parabolic signal or parabolic function. It is also known as unit acceleration signal. The unit parabolic signal start at $t=0$

Continuous $P(t) = \begin{cases} \frac{t^2}{2} & \text{for } t \geq 0 \\ 0 & \text{for } t<0 \end{cases}$

Also

$$P(t) = \frac{t^2}{2} u(t)$$



Discrete $P(n) = \begin{cases} \frac{n^2}{2} & \text{for } n \geq 0 \\ 0 & \text{for } n<0 \end{cases}$

$$P(n) = \frac{n^2}{2} u(n)$$

$$P(x) = \begin{cases} \frac{x^2}{2} & x \geq 0 \\ 0 & x \leq 0 \end{cases}$$

shifted version

$$p(n-k) = \begin{cases} \frac{(n-k^2)}{2} & n \geq k \\ & n<k \end{cases}$$

unit shifted fun

$$P(n-k) = \frac{(n-k)^2}{2} \cdot u(n-k$$

Relationship b/w Unit Parabolic & unit step

$$P(t) = \iint u(t)\,dt\,dt = \iint 1\,dt\,dt = \int t\,dt = \frac{t^2}{2} \text{ fo } t \geq 0$$

or

$$u(t) = \frac{d^2}{dt^2} P(t)$$

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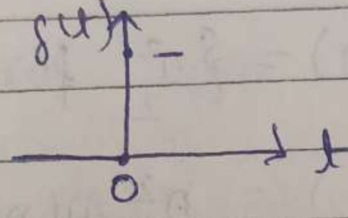
Unit Impulse Signal :- An ideal impulse signal is a signal that is zero everywhere but at the origin ($t=0$), it is infinitely high. Although, the area of the impulse is finite. The unit impulse signal is the most widely used standard signal signal used in the analysis of signal & system.

Continuous ⇒ $\delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{for } t \neq 0 \end{cases}$



$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$

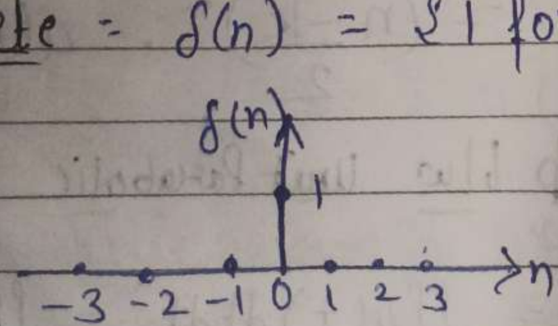
Sampling ⇒ $\int_{-\infty}^{\infty} x(t)\,\delta(t)\,dt = x(0)$

Shifting ⇒ $\int_{-\infty}^{\infty} x(t)\,\delta(t-t_0)\,dt = x(t_0)$

Scaling ⇒ $\delta(at) = \frac{1}{|a|}\,\delta(t)$

Product : $x(t)\delta(t) = x(0)\delta(t) = x(0)$;
$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$

Discrete = $\delta(n) = \begin{cases} 1 & \text{for } n = 0 \\ 0 & \text{for } n \neq 0 \end{cases}$



Properties

1) $\delta(n) = u(n) - u(n-1)$

2) $x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$

$\delta(n-k) = \begin{cases} 1 & \text{for } n = k \\ 0 & \text{for } n \neq k \end{cases}$

$\sum_{n=-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$

$$\int_{-\infty}^{\infty} \delta(t)\,dt = u(t)$$

and

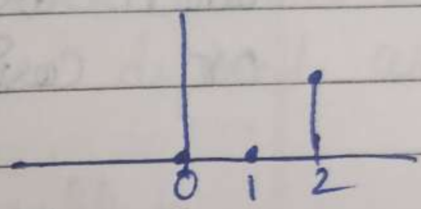
$$\delta(t) = \frac{d}{dt}u(t)$$

$$\begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$

Shifted version

$$\delta(n-K) = \begin{cases} 1 & n = K \\ 0 & n \neq K \end{cases}$$

$K = 2$



Properties of discreat time unit sample sequence

$$u(n) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$\delta(n) = u(n) - u(n-1)$$

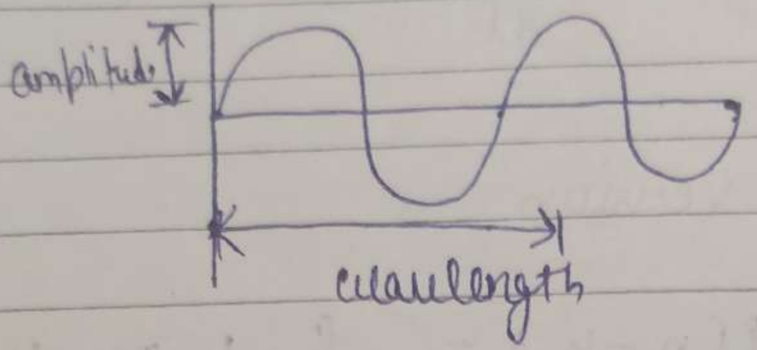
$$\delta(n-K) = \begin{cases} 1 & n = K \\ 0 & n \neq K \end{cases}$$

$$x(n) = \sum_{k=-\infty}^{\infty} x(k)\,\delta(n-k)$$

$$\sum_{-\infty}^{\infty} x(n)\,\delta(n-n_0) = x(n_0)$$

Sinusoidal Sequence :- A sinusoidal wave signal is a tyke of Continuous wave that has a smooth & repetitive oscillation.

It is based on the sine & cosine trigonometric function, which describes the curve of the wave



Sinusoidal signal are periodic funt^{n} which are based on the sine or cosine fun. from trignometry

$$x(t) = A\cos(\omega_0 t + \phi)$$
$$x(t) = A\cos(2\pi f_0 t + \phi)$$

Real exponential Signal :- An exponential signal or exponential fun. is a fun. that literally represents an exponential increasing or decreasing series.

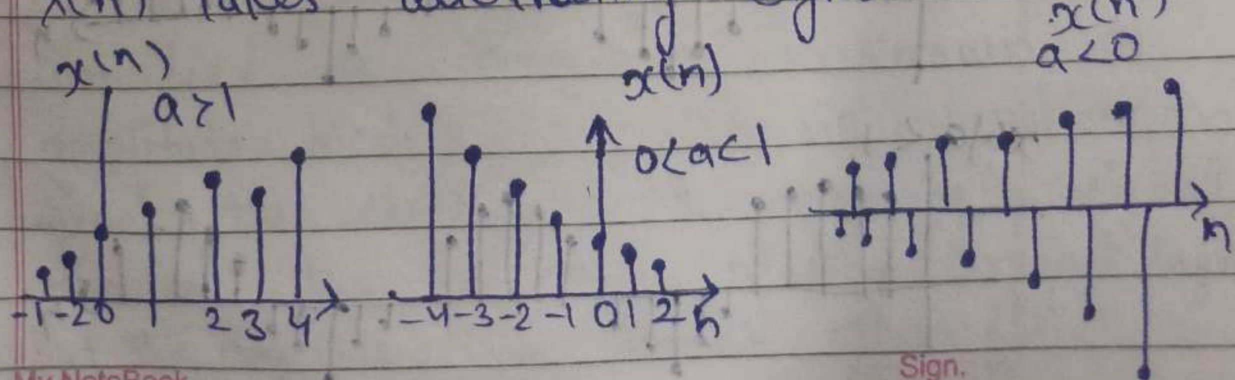
Discrete - Time Real Exponential signal :-

A real exponential signal which is define at discrete instants of time is called a discrete time real exponential signal. A discrete - time real exponential sequence is defined as:-

$$x(n) = a^n \text{ for all } n$$

Depending upon the value of a dis. time real exponetial signal may be of following type.

* When $a > 1$, the exponential sequence $x(n)$ grows exponentially.
* When $0 < a < 1$, the exponential signal $x(n)$ decays exponentially.
* When $a < 0$, the exponential sequence $x(n)$ takes alternating signs.



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Complex Exponential Signal :-

A exponential signal whose samples are complex numbers (i.e with real & imaginary parts) is known as a Complex exp. signal

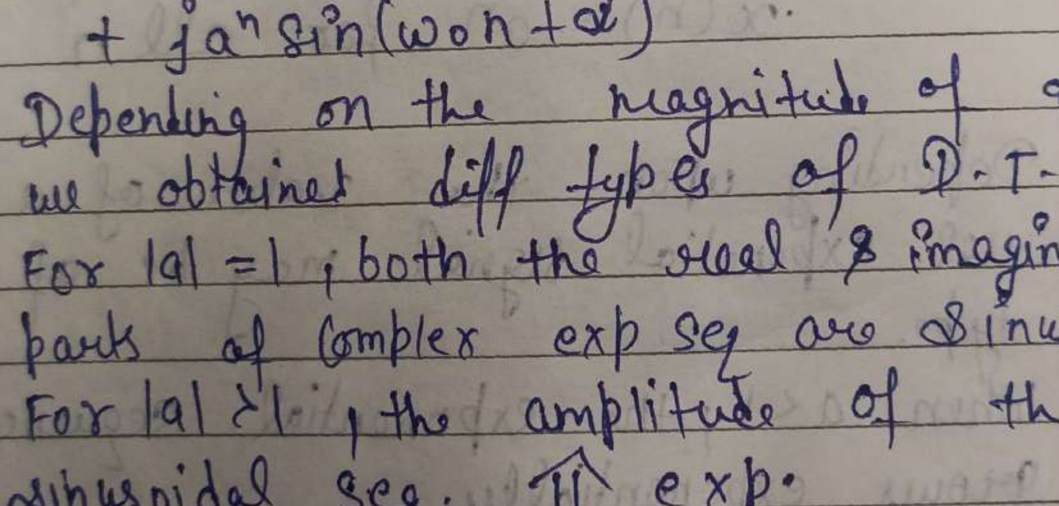
Dis-Time Complex Exp Sequence :-

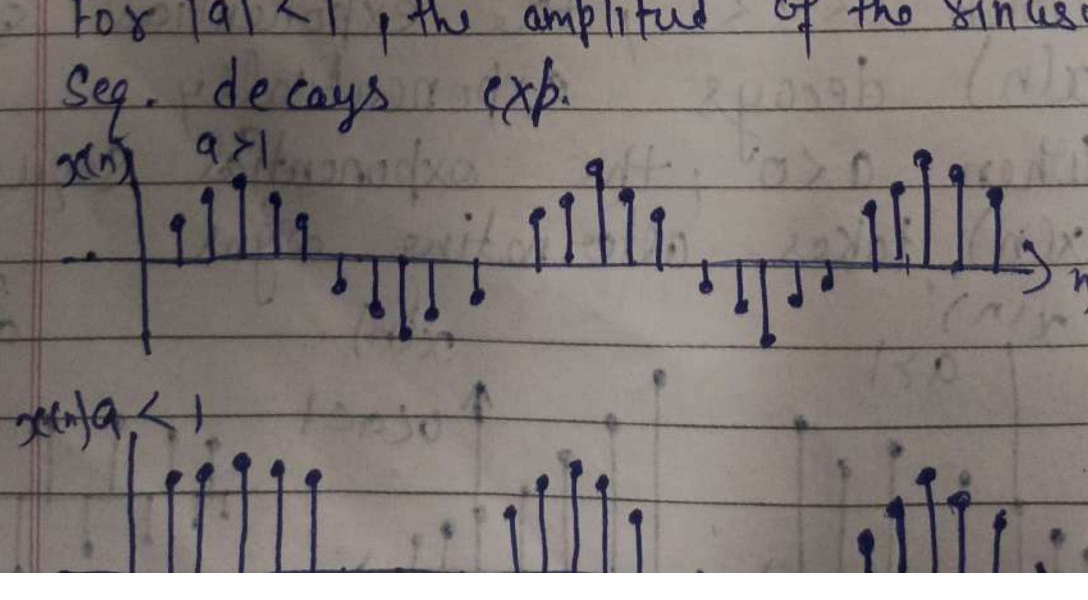
A Complex exp signal which is defined at dis instant of time is known as dis. TCES.

$x(n) = a^n e^{j(\omega_0 n + \alpha)} = a^n \cos(\omega_0 n + \alpha) + j a^n \sin(\omega_0 n + \alpha)$

Depending on the magnitude of a we obtained diff types of D.T.C.E.S

* For $|a| = 1$, both the real & imaginary parts of complex exp seq are sinusoids
* For $|a| > 1$, the amplitude of the sinusoidal seq. ↑ exp.
* For $|a| < 1$, the amplitude of the sinusoidal seq. decays exp.





Basic operation of Sequence

* Time Shifting
* Time reversal
* Time Scaling

→ Transform the value of n.

* Amplitude Scaling
* Signal addiction
* Signal multiplication

— In this we will ⇑ or ⇓ the amplitude

Amplitude Scaling :- The process of rescaling the amplitude of a signal i.e the amplitude of the signal is either amplified or attenuated is known as amplitude scaling. In the a.s operation on signal, the shape of the resulting signal remains the as that of the orginal signal but the amplitude is altered ⇑ or ⇓

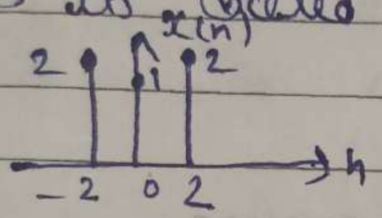
Amplitude Scaling of a dis-Time Signal

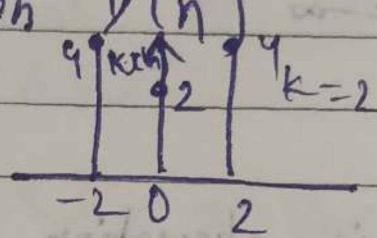
The amplitude scaling of a discrete time sequence $x(n)$ is defined as

$$y(n) = K\,x(n)$$

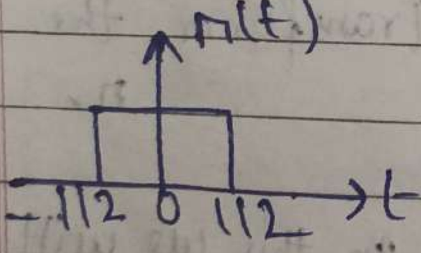
Where, K is constant. If $K>1$, the scaling is called amplification of the signal, while if $K<1$, the scaling is called attenuation of the signal.

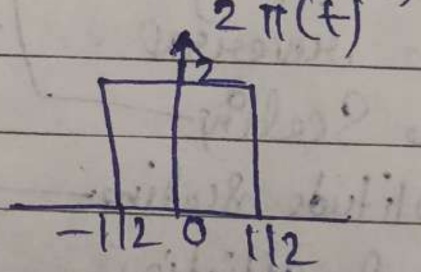
An arbitrary dis time sequence $x(n)$ & its scaled version $y(n)$



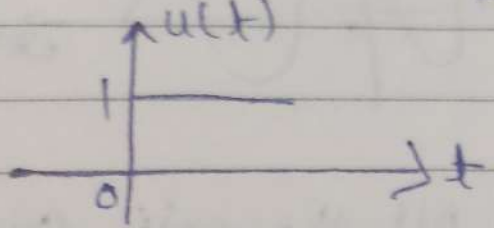


Q $x(t) = \Pi(t)$; $y(t) = 2\Pi(t)$



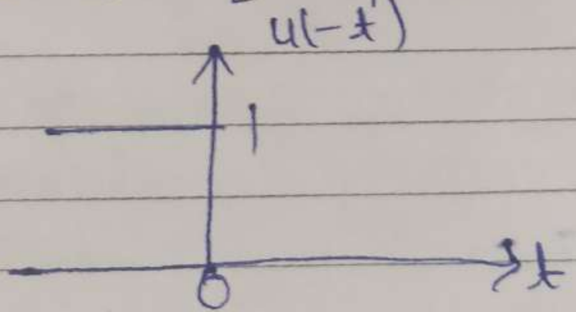


Causal Signal :- A continuous time signal $x(t) = 0$ for $t<0$. Therefore a causal signal does not exist for (-ve) time. The unit step signal $u(t)$ is an e.g. Similarly, a dis time seq. $x(n)$ is called the causal seq. if the seq $x(n) = 0$ for $n<0$

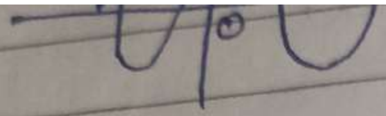


Anti - Causal Signal :-

A continuous - time signal $x(t)$ is called the anti - causal signal if $x(t) = 0$ for $t>0$. Hence an anti - causal signal does not exist for (+ve) time. The time reversed unit signal $u(-t)$ is an (e.g) of A.C.S
Similarly, a dis.T.S $x(n)$ is said to be anti - causal seq. if the seq. $x(n) = 0$ for $t>0$

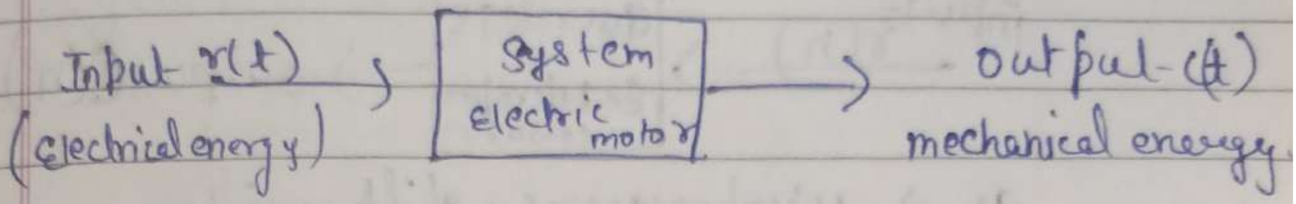


Non - Causal Signal :- A signal which is not causal is called the non - causal. Hence, by the definition a signal that exists for (+ve) as well



<u>Note</u> :- All the anti-causal signals are non-causal but the converse is not true.

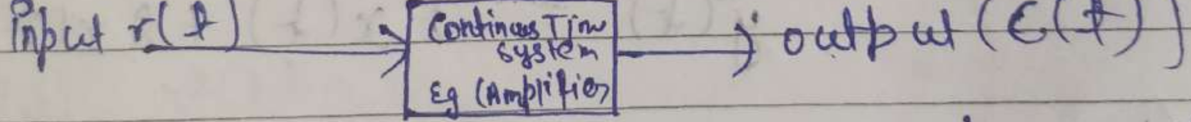
System ⇒ A system is defined as a physical device that can produce an output or de response for the given input.



electric motor, generator, etc are e.g of system

Classification of system

Continuous-Time system :- A system which transforms a continuous-time input signal into a continous time output signal is called the Continous time system. also it is defined for every instant of time



Relation of input & output is $c(t) = T[r(t)]$
e.g ⇒ Amplifiers, integrators, differentiators & filter circuit

Discrete-Time System :- A system which processes the discrete time input signals & produces discrete output signal is know as DT system

It is defined only at the discrete instant of time.

Relation of output & input $c(n) = T[r(n)]$

input $r(n)$ → [Discrete TS, Ex Micropro cessor] → output $C(n)$

eg ⇒ Microprocessors, shift register

<u>Static system</u> :– Some systems have feedback and some do not. Those, which do not have feedback system, their output depends only upon the present values of the input. Past value of the data is not present at that time. These type of system is known as static. It does not depend upon future value. It is also known as system with memoryless.

$$y(t) = x(t)$$
$$y(t) = t\,x(t) + 2x(t)$$

eg
$$y(n) = x_n \cos(\omega_0 x)$$
$$x[0] \cos(0)$$

<u>Dynamic System</u> :– A system is called dynamic if output of system dependent on past or future values of input at any instant of time. It is also known system

System with memory.

$$y(t) = x(t+1)$$
$$y(t) = t\,x(t) = x(t-1)$$

eg $y(t) = x(3t)$

Put $t = 1$

$$y(1) = x(3)$$

eg $y(t) = 5x(t)$

$$y(1) = 5x(1)$$

<u>Linear</u> Syste<u>m</u> :- A L.S is a system that satisfies the superposition principle, which states that the response of linear system to any input is the sum of the response to each individual input.

$$H[a_1x_1(t) + a_2x_2(t)] = a_1 H[x_1(t)] + a_2 H[x_2(t)]$$
$$\because H[a_1x_1(t) + a_2x_2(t)] = a_1y_1(t) + a_2y_2(t)$$

e.g $(t) = x^2(t)$

$y_1(t) = H[x_1(t)] = x_1^2(t)$

$y_2(t) = H[x_2(t)] = x_2^2(t)$

$H[a_1x_1(t) + a_2x_2(t)] = [a_1x_1(t) + a_2x_2(t)]^2$

This is eg of non linear.

<u>Non linear System</u> :- A non linear system is a system that does not satisfy the superposition principle which states that the response of a

linear system to any input is the sum of the responses to each individual input.

Time variant :- A time - varying system is a system whose behavior changes over time. This means that the output of the system at a given time will depend not only on the current input to the system, but also on the previous input & the passage of time. This will either stable or unstable convolution is dependent.

$$y(n,t) \neq y(n-t)$$

where $y(n,t) = T[x(n-t)]$ = input change

$y(n-t)$ = output change

$$y(n) = x(-n)$$

$$y(n,t) = T[x(n-t)] = x(-n-t)$$

$$y(n-t) = x(-(n-t)) = x(-n+t)$$

$y(n,t) \neq y(n-t)$. Time variant

Time invariant :- A time - invariant system is a system whose behavior does not change over time. This means that the output of the system at a given time will depend only on the current input to the system & not on the previous input or the passage of time. Convolution is independent

This is generally unstable.

$$y(n,t) = y(n-t)$$

e.g $y(T) = \sin[x(T)]$

Stable system :- A stable system satisfies the BIBO (bounded input for Bounded output) condition. Here, bounded means finite in amplitude. For a stable system output should be bounded or finite, for finite or bounded input, at every instant of time

Note For a bounded signal, amplitude is finite

$$y(t) = x(t) + 10$$
$$x(t) = 2, y(t) = 12$$

Unstable :- Unstable system do not satisfy the BIBO condition. Therefore, for a bounded input, we cannot expect a bounded output in case of unstable

e.g $y(t) = t\,x(t)$

Causal System :- A system whose output or response at any time instant (t) depends only on the present and past values of the input but not on the future values of the input is called the Causal system eg $y(n) = 2x(t) + 3x(t-3)$

For present value $t = 1$

$y(1) = 2x(1) + 3x(1-2)$

<u>Non-Causal System</u> :- A system whose output or response at any instant (t) depends upon future values of the input is called non-causal system also called anticipative system.

e.g $y(n) = 2x(t) + 3x(t-3) + 6x(t+3)$

$y(1) = 2x(1) + 3x(-2) + 6x(4)$

Response of LTI system to Complex exponential

LTI ⇒ linear time invariant

linear combination 2 basic property of signal :-

1. The response of LTI system to each signal
2. Set of basic signal that can be used to construct broad & useful signal.

CT ⇒ $e^{st} \rightarrow H(s)\, e^{st}$ (complex signal; input)

DT ⇒ $z^n \rightarrow H(z)\, z^n$

A signal for which system output is constant time the input is refered to eigen function.

~~Cons~~ Amplitude ⇒ Eigen value

they create a response is eigen function

CT = input signal $x(t) \rightarrow$ LTI system $h(t)$ impulse response (system response)

$$y(t) = \int_{-\infty}^{+\infty} h(z)\, x(t-z)\, dz = \int_{-\infty}^{+\infty} h(z)\, e^{s(t-z)}\, dz$$

 Sign.

$$z^n \sum_{-\infty} h[k] z^{-k}$$

3 $y[n] = z^n H(z)$

function को Decompose into eigen fun

$$x(t) = a_1 e^{s_1 t} + a_2 e^{s_2 t} + a_3 e^{s_2 t}$$

eigen value ... eigen fun

eigen property

respon of fun w.r.t to eigen fun

$$\left[\begin{array}{l} a_1 e^{s_1 t} \rightarrow a_1 H_1(s_1) e^{s_1 t} \\ a_2 e^{s_2 t} \rightarrow a_2 H_2(s_2) e^{s_2 t} \\ a_3 e^{s_3 t} \rightarrow a_3 H_3(s_3) e^{s_3 t} \end{array}\right.$$

constant ... complex variable

$$y(t) = a_1 H_1(s_1) e^{s_1 t} + e_2 H_2(s_2) e^{s_2 t} + a_3 H_3(s_3) e^{s_3 t}$$

C.T $\quad x(t) = \sum_K a_K e^{s_K t}$

D.T $\quad x(n) = \sum_K a_K Z_K^n$

ex $y[n] = \sum_{K} a_K H(z_K) z_K^n$

linear rep of Harmonic relate complex exponetial

$$x(t) = x(t+T)$$

basic periodic s — Angular frequency

$x(t) = \cos \omega_0 t$

$x(t) = e^{j\omega t}$

$$\boxed{T = \frac{2\pi}{\omega_0}}$$

A fourier series is an expansion of a periodic function $f(x)$ in term of an infinite sum of sines and Cosines. Fourier Series makes use of the orthogonality relationships of the sine and Cosine function.

Convolution operation :-

$$y(n) = \sum_{-\infty}^{+\infty} x(k)\, h(n-k) = \sum_{-\infty}^{\infty} h(n)\, x(n-k)$$

Property

Commutative $y(n) = x(n) * h(n)$ or $= h(n) * x(n)$

Associative

$$[x(n) + h_1(n)] * h_2(n) = x(n) * [h_1(n) + h_2(n)]$$

Distrutive

Imp Shifting :- $x(n) * h(n) = y(n)$ then $x(n-k) * h(n-m) = y(n-k-m)$

$$x(n) * \delta(n) = x(n)$$

Convolution always perfored with finite fun.

Size of Convolution

$$x(n) = n_1$$
$$h(n) = n_2$$
$$y(n) = n + n_2 - 1$$

Starting index of $y(n)$ = Sum of starting index of $x(n)$ and $h(n)$

$y(n)$ output $n = n_1 + n_2$
$x(n)$ input n_1
$h(n)$ impulse n_2

length is define L_y

$$L_y = L_x + L_h - 1$$

4 method by which Convolution performed

Linear convolution using Graphical method
Linear convolution using Tubular method
Linear convolution —— Matrix
Linear —— —— Tubular array.

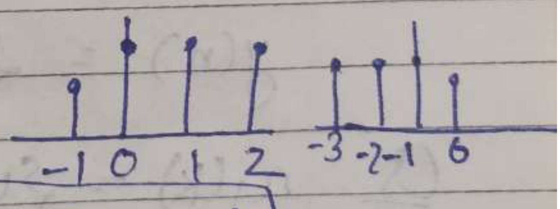
How to obtain $h(n-k)$

fold (hk) about $k=0$ to obtain $h(n-k)$ then shift $h(k)$ to n

$n \to +ve$ shift ~~left~~ right
$n \to -ve \Rightarrow$ ~~right~~ left

Q $x(n) = \{\overset{0}{4}, 2, 1, \overset{3}{3}\}$ -1 5

$h(n) = \{\underset{-1}{1}, \underset{\uparrow}{2}, 2, \underset{2}{1}\}$



$$\boxed{y(n) = \sum_{-\infty}^{\infty} x(k)\, h(n-k)}$$

$x = -1$ $y(-1) = \sum_{-\infty}^{\infty} x(k)\, h(-1-k) = 4 \cdot 1 = 4$

$x = 0$ $y(0) = \sum_{-\infty}^{\infty} x(k)\, h(-k) = 4 \cdot 2 + 1 \cdot 2 = 10$

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$n=1 \quad y(1) = \sum_{-\infty}^{\infty} x(k)\, h(1-k)$

$4+\cancel{2}4+\cancel{1}2+3 = 13 \qquad \cancel{4\cdot 2} + \cancel{1}\,2$
$\qquad\qquad\qquad\qquad\qquad 4 \cdot 2$

$n=2 = y(2) = \sum_{-\infty}^{\infty} x(k)\, h(2-k)$

$= 13$

$n=3 = y(3) = \sum_{-\infty}^{\infty} x(k)\, h(3-k)$

$= 10$

$n=4 = y(4) = \sum_{-\infty}^{\infty} x(k)\, h(4-k)$

$= 7$

$n=5 = y(5) = \sum_{-\infty}^{\infty} x[k]\, h(5-k)$

$= 3$

$y(n) = \{4, 10, 13, 13, 10, 7, 3\}$

Q $x_1(k) = \{4, 2, 1, 3\}$
$x_2(k) = \{1, \underset{\uparrow}{2}, 2, 1\}$

K	-4	-3	-2	-1	0	1	2	3	4	5	6	7
$x_1(k)$	-	-	-	-	4	2	1	3	-	-	-	-
$h_2(k)$	-	-	-	1	2	2	1	-	-	-	-	-
$h(-k)$	-	-	1	2	2	1	-	-	-	-	-	-
$n=-1$, $h(-1-k)$	-	1	2	2	1	-	-	-	-	-	-	-

$n=0,\ h(-k)$ - - 1 2 2 1 - - - - - - -

$n=1,\ h(1-k)$ - - - 1 2 2 1 - - - - - -

$n=2,\ h(2-k)$ - - - - 1 2 2 1 - - - -

$n=3\ h(3-k)$ - - - - - 1 2 2 1 - - -

$n=4\ h(4-k)$ - - - - - - 1 2 2 1 - -

$n=5\ h(5-k)$ - - - - - - - 1 2 2 1 - -

TABULAR METHOD :-

$x(n) = \{x_1, x_2, x_3, x_4\}$ $h(n) = \{h_1, h_2, h_3, h_4\}$

$h(n) = \{h_1, h_2, h_3, h_4\}$

	x_1	x_2	x_3	x_4
h_1	x_1h_1	x_2h_1	x_3h_1	x_4h_1
h_2	x_1h_2	x_2h_2	x_3h_2	x_4h_2
h_3	x_1h_3	x_2h_3	x_3h_3	x_4h_3
h_4	x_1h_4	x_2h_4	x_3h_4	x_4h_4

Q 4, 3, 1, 2   1, 2, 2, 1

x	1	2	2	1
4	4	8	8	4
3	3	6	6	3
1	1	2	2	1
2	2	4	4	2

$h(n) = 1, 2, 2, 1$

$x(n) = (4, 3, 1, 2)$

(2)

	4	3	1	2
1	4	3	1	2
2	8	6	2	4
2	8	6	2	4
1	4	3	1	2

$$A_{11}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 1 & 2 & 2 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow 4+0+0+0$

$8+3 = 11$

$8+6+(2) = 15$

$4+6+2+2 \Rightarrow 14$

(3)

	4	2	1	3
1	4	2	1	3
2	8	4	2	6
2	8	4	2	6
1	4	2	1	3

h(n)

Linear Convolution Using Matrices

$$\underset{H}{\begin{bmatrix} h(0) & 0 & \cdots & 0 \\ h(1) & h(0) & \cdots & 0 \\ \vdots \\ h(N_2-1) & \cdots & & 0 \\ 0 & (N_2-1) & \cdots \\ 0 & 0 & h(N_2-1) \\ 0 & 0 & h(N_2-1) \end{bmatrix}} \underset{x}{\begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ \vdots \\ \vdots \\ x(N_1-1) \end{bmatrix}} = \underset{y}{\begin{bmatrix} y(0) \\ \vdots \\ y(N_1+N_2-1) \end{bmatrix}}$$

$$\underset{H}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 2 & 2 & 1 & 0 \\ 1 & 2 & 2 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{x}{\begin{bmatrix} 4 \\ 2 \\ 1 \\ 3 \end{bmatrix}} = \underset{y}{\begin{bmatrix} 4 \\ 10 \\ 13 \\ 13 \\ 10 \\ 7 \\ 3 \end{bmatrix}}$$

$h(n) * x(n) = y(n)$

$y(n) * x(n) = y(n)$

DELTA Pg No. Date / /

4. Q $x(n) * y(n) =$

$$x(n) = \begin{cases} 2 & n = -2, 0, 1 \\ 3 & n = -1 \\ 0 & \text{elsewhere} \end{cases}$$

3. 1. 2

$$h(n) = \delta(n) - 2\delta(n-1) + 3\delta(n-2) + \delta(n-3)$$

= 4

$x(n) = \{2, 3, 2, 2\}$

$h(n) = \{1, -2, 3, 1\}$

Tubular method

	2	3	2	2
1	2	3	2	2
-2	-4	-6	-4	-4
3	6	9	6	6
1	2	3	2	2

$6 + (-6) + 2$

$8 \quad 10 + (-4) + 2$

$5 \quad 9 + (-4)$

2, -1, 2, 8, 5, 8, 2

Matrices method

$$\underset{H}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -2 & 1 & 0 & 0 \\ 3 & -2 & 1 & 0 \\ 1 & 3 & -2 & 1 \\ 0 & 1 & -2 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \times \underset{Y}{\begin{bmatrix} 2 \\ 3 \\ 2 \\ 2 \end{bmatrix}} = \underset{Y}{\begin{bmatrix} 2 \\ +3 \\ \\ \\ \end{bmatrix}}$$

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$x(n) = 3\delta(n+1) - 2\delta(n) + \delta(n-1) + 4\delta(n-2)$
$h(n) = 2\delta(n-1) + 5\delta(n-2) + 3\delta(n-3$

Trigonometric form of Fourier Series :-

Let $x(t)$ = A signal is a periodic signal with period $T = 2\pi/\omega_0$. Then sum of line sinusoidal is also periodic provided that sequencies are integral multiple of fundamental frequinues.

$x(t) = a_0 + a_1 \cos\omega_0 t + a_2 \cos 2\omega_0 t + \ldots a_n \cos\omega_0 t$
$+ b_1 \sin \cos t + b_2 \sin \cos t + \ldots b_n \sin \cos t$

i.e $x(t) = a_0 + \sum_{n=1}^{K} a_n \cos n \cos t + \sum_{n=1}^{K} b_n \sin n\omega_0 t$

for a signal to be periodic $x(t) = x(t+T)$

$x(t+T) = a_0 + \sum_{n=1}^{K} a_n \cos n\omega_0 (t+T) + \sum_{n=1}^{K} b_n \sin n\omega$

$(t+T) = a_0 + \sum_{n=1}^{K} a_n \cos \omega_0 n\left(t + \frac{2\pi}{\omega_0}\right) + \sum_{n=1}^{K} b_n \sin \omega \left(\frac{t+2}{\omega}\right.$

$a_0 + \sum_{n=1}^{K} a_n \cos(\omega_0 n t + 2\pi x) + \sum_{n=1}^{K} b_n \sin(\omega_0 t + 2\pi$

$a_0 + \sum_{n=1}^{K} a_n \cos n\omega_0 t + \sum_{n=1}^{K} b_n \sin(\omega_0 n t)$

$= x(t)$

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Evaluation of Fourier constant

The constant $a_0, a_1 - - - a_n, b_1 - - b_n$ are called as Fourier coefficient

$$\int_{t_0}^{t_0+T} x(t)\,dt = a_0 \int_{t_0}^{t_0+T} dt + \int_{t_0}^{t_0+T} \left[a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right] dt$$

$$a_0 T + \sum_{n=1}^{\infty} a_n \underbrace{\int_{t_0}^{t_0+T} \cos n\omega_0 t\,dt}_{0} + \sum_{n=1}^{\infty} b_n \underbrace{\int_{t_0}^{t_0+T} \sin n\omega_0 t\,dt}_{0}$$

$$\int_{t_0}^{t_0+T} x(t)\,dt = a_0 T$$

$$\boxed{a_0 = \frac{1}{T} \int_{t_0}^{t_0+T} x(t)\,dt}$$

$$\begin{cases} 0 & \text{for } m \neq n \\ \pi/2 & \text{for } m = n \neq 0 \end{cases} = \int_{t_0}^{t+T} \cos m\omega_0 t\,dt$$

$$\begin{matrix} 0 & \text{for } m \neq n \\ \pi/2 & \text{for } m = n \neq 0 \end{matrix} = \int_{b}^{t_0+T} \sin n\omega_0 t \sin m\omega_0 t\,dt =$$

$$\int_{t_0}^{t_0+T} \sin n\omega_0 t \cos m\omega_0 t\,dt = 0 \text{ for all } m \text{ an } n$$

$$\int_{t_0}^{t_0+T} x(t) \cos m\omega_0 t \, dt$$

$$= a_0 \int_{t_0}^{t_0+T} \underbrace{\cos m\omega_0 t \, dt}_{=0} + \sum_{n=1}^{\infty} a_n \int_{t_0}^{t_0+T} \cos n\omega_0 t \cos m\omega_0 t \, dt + \sum_{n=1}^{\infty} b_n$$

$$\int_{t_0}^{t_0+T} \underbrace{\sin n\omega_0 t \cos m\omega_0 t \, dt}_{=0}$$

$$\int_{t_0}^{t_0+T} x(t) \cos m\omega_0 t = a_m \, T/2$$

$$a_m = \frac{2}{T} \int_{t_0}^{t_0+T} x(t) \cos m\omega_0 t \, dt$$

$$a_n = \frac{2}{T} \int_{t_0}^{t_0+T} x(t) \cos n\omega_0 t \, dt$$

$$\boxed{b_n = \frac{2}{T} \int_{t_0}^{t_0+T} x(t) \sin n\omega_0 t \, dt}$$

Cosine representation of Fourier series

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$= a_0 + \sum_{n=1}^{\infty} \sqrt{a_n^2 + b_n^2} \left(\frac{a_n}{\sqrt{a_n^2 + b_n^2}} \cos n\omega_0 t + \frac{b_n}{\sqrt{a_n^2 + b_n^2}} \sin \right.$$

Substitute the value of $A_0 = a_0$, ~~A~~

$A_n = \sqrt{a_n^2 + b_n^2}$, $\cos\theta = \frac{a_n}{\sqrt{a_n^2+b_n^2}}$ and $\sin\theta = \frac{b_n}{\sqrt{a_n^2+b_n^2}}$

$$\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

$$x(t) = A_0 + \sum_{n=1}^{\infty} A_n [\cos\theta_n \cos n\omega_0 t - \sin\theta_n \sin n\omega_0 t$$

$$A_0 + \sum_{n=1}^{\infty} A_n [\cos(n\omega_0 t + \theta_n)]$$

WAVE SYMMETRY

Even Symmetry ⇒ $b_n = 0$, a_n and a_0

Odd Symmetry ⇒ rotation symmetry $a_n = a_0 = 0$ & b_n

Half wave symmetry ⇒ $a_n = 0$, only odd Harmonics exist

Quarter wave symmetry ⇒ $a_0 = 0$ a_n & b_n exist only for odd values of n.

if $x(t)$ is even $x(t) = x(-t)$ $x_e(t) = \frac{1}{2}$

if $x(t)$ is odd $x(t) = -x(-t)$ $x_o(t) = \frac{1}{2}$

$$x(t) = x_e(t) + x_o(t)$$

$$\int_{T/2}^{T/2} x_e(t)\,dt = 2\int_{0}^{T/2} x(t)\,dt$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t) \cos n w_0 t \, dt$$

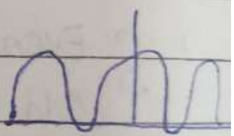
$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t) \sin w_0 n t \, dt$$

$$x(t) = x_o(t) + x_o(t)$$

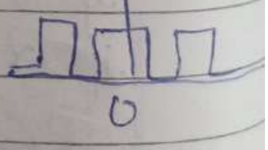
$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x_e(t) \cos w_0 n t \, dt + \int_{-T/2}^{T/2} x_o(t) \sin n \cos n t \, d$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x_e(t) \cos w_0 n t \, dt = \frac{2}{T}\left[2\int_{0}^{T/2} x_o(t) \cos n w_0 t\right.$$

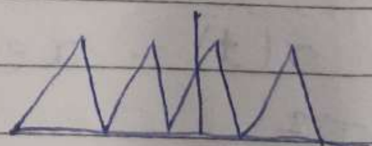
$$a_n = \frac{4}{T}\int_{0}^{T/2} x(t) \cos n w_0 t \, dt$$



$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t) dt = \frac{1}{T}\int x_e(t) dt = \frac{1}{T} \times 2\int_{0}^{T/2} x_e(t) \, dt$$



$$a_0 = \frac{2}{T}\int_{0}^{T/2} x(t) \, dt$$



Exponential Fourier Series

$x(t) \rightarrow \{ e^{jn\omega t}$, where $n = 0, \pm 1, \pm 2 \ldots \}$

$t_0 \rightarrow t_0 + T$ $\qquad T = \frac{2\pi}{\omega_0}$

Euler's Identity

$$A_n \cos(n\omega_0 t + \theta_n) = A_n \left[\frac{e^{j(n\omega_0 t + \theta_n)} + e^{-j(n\omega_0 t + \theta_n)}}{2} \right]$$

$$x(t) = A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2} \left[e^{j(n\omega_0 t + \theta_n)} + e^{-j(n\omega_0 t + \theta_n)} \right]$$

$$A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2} \left[e^{jn\omega_0 t} e^{j\theta_n} + e^{-jn\omega_0 t} e^{-j\theta_n} \right] = A_0 \sum_{n=1}^{\infty} \left(\cancel{\frac{A_n}{2} e^{-j\theta_n}} \right) e^{-jn\omega_0}$$

$$A_0 + \sum_{n=1}^{\infty} \left(\frac{A_n}{2} e^{jn\omega_0 t} e^{j\theta_n} \right) + \left(\sum_{n=1}^{\infty} \frac{A_n}{2} e^{-jn\omega_0 t} e^{-j\theta_n} \right)$$

$$A_0 + \sum_{n=1}^{\infty} \left(\frac{A_n}{2} e^{j\theta_n} \right) e^{jn\omega_0 t} + \sum_{n=1}^{\infty} \left(\frac{A_n}{2} e^{j(-\theta_n)} \right) e^{-jn\omega_0}$$

$$A_0 + \sum_{n=1}^{\infty} \left(\frac{A_n}{2} e^{j\theta_n} \right) e^{jn\omega_0 t} + \sum_{K=-1}^{-\infty} \left(\frac{A_n}{2} e^{j(\theta_n)} \right) e^{jK\omega_0 t}$$

$A_n = A_K$

$(-\theta_n) = \theta_K \qquad n > 0$

$\qquad\qquad\qquad K < 0$

Let us define

$$c_0 = A_0, \quad c_n = \frac{A_n}{2} e^{j\theta_n}, \quad n > 0$$

$$x(t) = A_0 + \sum_{n=1}^{\infty} \frac{A_n}{2} e^{j\theta_n} e^{j\omega_0 n t} + \sum_{n=-1}^{-\infty} \frac{A_n}{2} e^{j\theta_n} e^{j\omega_0 n t}$$

$$\boxed{x(t) = A_0 + \frac{A_n}{2} e^{j\theta_n} \sum_{-\infty}^{\infty} e^{j\omega_0 n t}}$$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} c_n e^{j\omega_0 n t}}$$

Determination of Fourier Coefficient

$$t_0 \to t_0 + T$$

$$e^{-jk\omega_0 t}$$

$$\int_{t_0}^{t_0+T} x(t) e^{-jk\omega_0 t} dt = \int_{t_0}^{t_0+T} \left(\sum_{n=-\infty}^{\infty} e^{jn\omega_0 t} \right) e^{-jk\omega_0 t}$$

$$\sum_{n=-\infty}^{\infty} c_n \int_{t_0}^{t_0+T} e^{jn\omega_0 t} e^{-jk\omega_0 t} dt$$

We know that

$$\int_{t_0}^{t_0+T} e^{jn\omega_0 t} e^{-jk\omega_0 t} dt = \begin{cases} 0 & k \neq n \\ 1 & k = n \end{cases}$$

$$t_0 + T \cdot c_n$$

$$c_k = \frac{1}{T} \int_{t_0} x(t) e^{-jk\omega_0 t} dt$$

$$C_n = \frac{1}{T}\int_{t_0} x(t)\, dt$$

~~Transformer series to~~ .

Trigometric fourier series from Expno F.S

$$x(t) = C_0 + \sum_{n=-\infty}^{\infty} C_n e^{jw_0nt} = C_0 + \sum_{n=-\infty}^{-1} C_n e^{jw_0nt} + \sum_{n=1}^{\infty} C_n e^{jw_0nt}$$

$$C_0 + \sum_{n=1}^{\infty} C_{-n} e^{-jw_0nt} + C_n e^{jnw_0t}$$

$$C_0 + \sum_{n=1}^{\infty} \left[C_{-n}\left(\cos nw_0t - j\sin nw_0t\right) + C_n\left(\cos nw_0t + j\sin nw_0t\right)\right]$$

$$C_0 + \sum_{n=1}^{\infty} (C_n + C_{-n})\cos nw_0t + j(C_n - C_{-n})\sin nw_0t$$

$$\boxed{\begin{aligned} a_0 &= C_0 \\ a_n &= C_n + C_{-n} \\ b_n &= j(C_n - C_{-n}) \end{aligned}}$$

$$a_0 + \sum_{n=1}^{\infty} a_n \cos nw_0t + \sum_{n=1}^{\infty} b_n \sin w_0nt$$

Fourier spectrum $\Rightarrow$

$$x(t) = \underset{T \to \infty}{Lt} \; x_T(t)$$

$$x_T(t) = \sum_{n=-\infty}^{\infty} c_n \, e^{i\omega_0 n t}$$

where $c_n = \dfrac{1}{T} \displaystyle\int_{-T/2}^{T/2} x_T(t) \, e^{-j\omega_0 n t} \, dt$ $\quad \omega_0 = 2\pi/T$

$$Tc_n = \int_{-T/2}^{T/2} x_T(t) \, e^{-j\omega_0 n t} \, dt$$

let $n\omega_0 = \omega$ at $T \to \infty$

As $T \to \infty$ we have $\omega_0 = 2\pi/T \to 0$

$$Tc_n = \underset{T \to \infty}{Lt} \int_{-T/2}^{T/2} x_T(t) \, e^{i\omega t} \, dt = \int_{-\infty}^{\infty} \left(\underset{T \to \infty}{Lt} \, x_T(t) \right) e^{-i\omega t}$$

(fourier transformer) $\quad \boxed{\displaystyle\int_{-\infty}^{\infty} x(t) \cdot e^{-j\omega t} \, dt = Y(}$

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$$X(\omega) = FT(x(t))$$
$$x(t) = F^{-1}(X(\omega))$$

$$x_T(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$

$$= \sum_{n=-\infty}^{\infty} \frac{X(\omega)}{T} e^{jn\omega_0 t}$$

$$\sum_{n=-\infty}^{\infty} \frac{x(\omega)}{2\pi} e^{j\omega_0 n t} \omega_0$$

$$x(t) = \lim_{T\to\infty} x_T(t)$$

$$\lim \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} X(\omega) e^{jn\omega_0 t} \omega_0$$

As $T \to \infty$, $\omega_0 = 2\pi/T$ becomes infinitesimally small $\to d\omega$

$$\boxed{x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega) e^{j\omega t}\, d\omega}$$

Fourier transformer pair (inverse)

$$\delta(t) = \begin{cases} 1 & t = 0 \\ 0 & \text{Otherwise} \end{cases}$$

$$X(\omega) = X_R(\omega) + j X_I(\omega)$$

Mag $\Rightarrow$ $\sqrt{X_R^2(\omega) + X_I^2(\omega)}$

phase $(X(\omega))$ $\tan^{-1} X_I(\omega)$

Dirichlet's condition

1 $x(t)$ is absolutely integratable $-\infty, \infty$

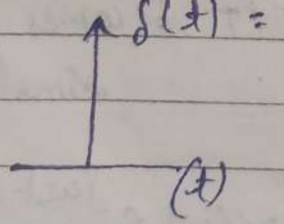
$$\int_{-\infty}^{\infty} x(t)\, dt < \infty$$

2 $x(t)$ has a finite no of discontinuity

3 Finite no of maxi & min with every finite time

Fourier trans of delta

$$\int_{-\infty}^{\infty} \delta(t) e^{-j\omega t} dt = 1 \cdot e^{-j\omega t}\Big|_{t=0} = 1$$



$$x(t) = e^{-at} u(t)$$

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

$$\int_0^\infty e^{-at} e^{-j\omega t} dt = \int_0^\infty e^{-t(a+j\omega)} dt$$

$$\left[\frac{e^{-t(a+j\omega)}}{-(a+j\omega)} \right]_0^\infty \frac{e^{-\infty} - e^0}{-(a+j\omega)} = \frac{1}{(a+j\omega)}$$

<u>Fourier Transform of periodic signal</u> :-

$x(t)$ is a periodic signal with time period T.

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

FT of $x(t)$

$$X(\omega) = F[x(t)] = F\left[\sum_{n=-\infty}^{\infty} C_n e^{j\omega_0 nt}\right]$$

$$= \sum_{n=-\infty}^{\infty} C_n F\left[e^{j\omega_0 nt}\right]$$

freq using a shifting theorem we have

$$F[1 \cdot e^{j\omega_0 nt}] = F[1] | \omega = \omega - n\omega_0$$

Bilateral transform

damping factor

Angular freq

$$X(\omega) = 2\pi \sum_{n=-\infty}^{\infty} C_n \, \delta(\omega - n\omega_0)$$

where C_n's are fourier coefficients

* $$C_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jn\omega_0 t}\, dt$$

complex freq

* $$L[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

Laplace transform

inverse of Laplace

$$L[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)\, e^{st}\, dt$$

Region of Con

[ ]