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Unit - 1

Sets, Relation and function

Sets :- Sets are represented as a collection of well-defined objects or elements and it does not change from person to person.

Objectives of Sets

1. Be able to perform the set operation of union, intersection, complement and diff.
2) To understand the order in which to perform set operation.
3) Know how to apply De Morgan's law in set theory.

Size of set :- The no. of elements in the set

* Three methods to describe sets :-

1. $\{0, 1, 2, 3, 4, 5\}$

2. $\{x : x$ is an integer and $x > -112$ & $x < 1912\}$

3. $\{x \in Z : x > -112$ & $x < 1912\}$

N → natural no. 1, 2, 3 Z integer -1, -2, ϕ empty set
Q → rational num : 1.5, $\frac{2}{5}$

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Types

1) Null or empty set
$\to \{\}$
denoted as ϕ

2) Singleton set. (only 1 element)
$\to \{1\}$

3) Subset
eg - $A = \{1,2,3,4\}$
$B = \{1,2\}$
$B \subseteq A$

4 Power set
An element set has 2^n is know as power set

eg $\{1,2\}$
$P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

Equality of sets

$\Rightarrow$ Let A and B be sets then, $A=B$ or A is equal to B if both A and B have the same element.

Finite and INFINITE SET

Finite sets are sets which has finite no. of element

Infinite set are set which has infinite or not finite num. of elements

$\Rightarrow$ Let A & B be sets A is subset of B, written as $A \subseteq B$.

$\Rightarrow$ Let A be a set.
then, $A \subseteq A$
$\phi \subseteq A$

$\Rightarrow$ $A = B$ if & only if $A \subseteq B$ & $B \subseteq A$

Q1 This problem concerns the following six sets
$A = \{0,2,4,6\}$
$B = \{1,3,5\}$
$C = \{0,1,2,3,4,5,6,7\}$
$D = \phi$
$E = N$
$F = \{\{0,2,4,6\}\}$

a) What sets are called subsets of A? D
b) — — — — — — B? D
c) — — — — — — C? A, B, D
d) — — — — — — D? —
e) — — — — — — E? A, B, C, D
f) — — — — — — F? A

OPERATIONS :-

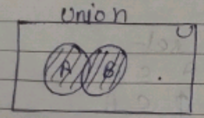
1. Union
2. Intersection

$A = \{1,2,3,7\}$
$B = \{1,5,6,7\}$

$U = \{1,2,3,4,5,6,7,8,9,10\}$

1) $A \cup B = \{1,2,3,5,6,7\}$

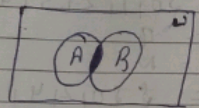
union



$\Rightarrow$ A

2) $A \cap B = \{1,7\}$

intersection

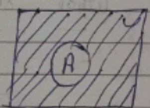


3) Complement

$\rightarrow \bar{A}$

$\bar{A} = U - A$

$= \{4,5,6,8,9,10\}$

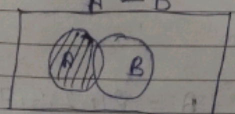


4) Difference

$\Rightarrow$ A-B (element present in A but not in B)

$\rightarrow$ B-A (— — B — — A)

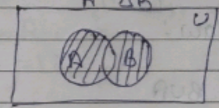
A — B



5) Symmetric Diff

$\Rightarrow$ $A \Delta B = (A-B) \cup (B-A) = (A \cup B) - (A \cap B)$

A ΔB



6) Cartesian product

$\Rightarrow$ $\Rightarrow$ $A \times B$ $A = \{1,2,3,4\}$ $B = \{4,6\}$

$A \times B = \{(1,4),(2,4)(3,4),(4,4),(1,6),(2,6)(3,6),(4,6)\}$

Q2 If $A = \{1,4\}$ $B = \{4,5\}$ $C = \{5,6\}$

find $(A \times B) \cap (A \times C)$

Ans $A \times B = \{(1,4),(4,4),(1,5),(4,5)\}$

$A \times C = \{(1,5),(4,5),(1,6),(4,6)\}$

$(A \times B) \cap (A \times C) = \{(1,5),(4,5)\}$

Laws

① Idemptent law :-

$A \cup A = A$

$A \cap A = A$

2 Associative law:

$(A \cup B) \cup C = A \cup (B \cup C)$
$(A \cap B) \cap C = A \cap (B \cap C)$

3 Commutative law:

$A \cup B = B \cup A$
$A \cap B = B \cap A$

4 Distributive law:

$A \cup (B \cap C) = (A \cup B) \cap (B \cup C)$
$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5 De Morgan law

$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

Q
$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{1, 2, 3, 4\}$
$B = \{1, 4, 5, 6\}$

$\bar{A} = U - A = \{5, 6, 7, 8, 9, 10\}$
$\bar{B} = U - B = \{2, 3, 7, 8, 9, 10\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$
$\overline{(A \cup B)} = \{7, 8, 9, 10\}$
$\bar{A} \cap \bar{B} = \{7, 8, 9, 10\}$
$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

6) Identity law

$A \cap \phi = \phi$
$A \cap U_n = A$

$A \cup \phi = A$
$A \cup U_n = U_n$

$\}$ U_n is universal set.

7) Complement law

$A \cup A' = U$ $\quad$ $A \cap A' = \phi$
$U' = \phi$ $\quad$ $\phi' = U$

8) Absorption law

$A \cup (A \cap B) = A$
$A \cap (A \cup B) = A$

9) Involution law

$\bar{\bar{A}} = A$

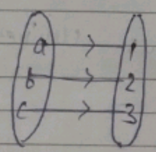
Counting principle

① Function :- A function assign exactly one element of one set to each element of other sets

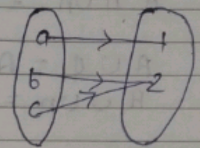
② A function is a rule that assigns each input exactly one output

Types of function :

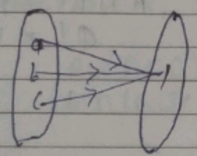
One - One



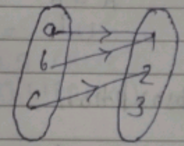
onto function



Many one function



Into function



Domain :- The set of all possible values which qualify as input to a function or it can ~~also~~ is called domain of function.

Range :- The set of all the outputs of a function is known as the range of the function

Codomain :- In relation & functions, the codomain is the set of all possible outcomes of the given relation or function. Sometimes, the codomain is also equal to the range of the functi hence, the range is the subset of the codomain

Potential ∃ No of elements

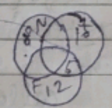
Q In a survey of 60 people it was found that 25 read Newsweek Magazine
26 read time
26 read fortun
26 read time
26 read time
9 read both newsweek & fortune
11 read both newsweek & time
8 read both time & fortune
3 read all three magazin

$|N| = 25 \quad |T| = 26 \quad |F| = 26 \quad |N \cap F| = 9$
$|N \cap T| = 11 \quad |T \cap F| = 8 \quad (N \cap T \cap F) = 3$

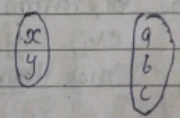
$|N \cup T \cup F| = |N| + |T| + |F| - |N \cap F| - (N \cap T) - |T \cap F| + |N \cap T \cap F|$

$25 + 26 + 26 - 9 - 11 - 8 + 3 = 52$

fill the venn diagram



NOTE If no map is onto then we have more potentias than A



If atleast one map is onto then potential of b is less then or equal to B

BOOLEAN FUNCTION :-

A fun $f: A^n \to A$ is called Boolean exp

Boolean exp

$A = \{x_1, x_2 - - x_n\}$ & three operation $+, \cdot, '$ —

$(x_1 + \bar{x}_2 \cdot \bar{x}_3) + (x_2 \cdot x_1)$ $\vee$ joint

or $\wedge$ Mid

$(x_1 \vee \bar{x}_2 \wedge \bar{x}_3) \vee (x_2 \wedge x_1)$

$f = A^x \to A$

$A^2 \to A$

$(x_1, x_2) \to (x_1 \vee x_2)$

TYES of BOOLEAN FUNCTION

POS (Product of Sum)	SOP (Sum of product)
$(\bar{x}_1 + x_2 + x_3) \cdot (\bar{x}_1 + \bar{x}_2 + x_3) \cdot (x_1 + x_2 + x_3)$	$(\bar{x}_1 \cdot x_2 \cdot x_3) + (\bar{x}_1 \cdot \bar{x}_2 \cdot \bar{x}_3) + (x_1 \cdot x_2 \cdot x_3)$
Max term :	Mini term
That is called CNF ⇒ Conjuctive normal form	This is called DNF ⇒ disjunctive normal form.

CNF ⇒ A boolean expression is said to be CNF if it is join/meet of max term

DNF :- A boolean expression is said to be DNF if it is join of min terms

formulas

① $a + a' = 1$
$a \vee a' = 1$

② $a \cdot a' = 0$
$a \wedge a' = 0$

③ $a + 0 = a$
$a \vee 0 = a$

④ $a \cdot 1 = a$
$a \wedge 1 = a$

⑤ $\cdot 1$
$\wedge 1$

⑥ $+ 0$
$\vee 0$

$f = x \vee y'$

$(x \wedge 1) \vee (1 \wedge y')$

$x \wedge (y \vee y') \vee (x \vee x') \wedge y')$

By distributive law

$(x \wedge y) \vee (x \wedge y') \vee (x \wedge y') \vee (x' \wedge y')$

Q Let $f(\bar{x} \wedge z) \vee y$ write of in DNF or Min term normal form

x	y	z	$\bar{x}$	$\bar{x} \wedge z$	f	
0	0	0	1	0	0	m_1
0	0	1	1	1	1	m_2
0	1	0	1	0	1	m_3
0	1	1	1	1	1	m_4
1	0	0	0	0	0	m_5
1	0	1	0	0	0	m_6
1	1	0	0	0	1	m_7
1	1	1	0	0	1	m_8

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$m_2 + m_3 + m_4 + m_7 + m_8$

$(\bar{x} \wedge \bar{y} \wedge z) \vee (\bar{x} \wedge y \wedge \bar{z}) \vee (\bar{x} \wedge y \wedge z) \vee (x \wedge y \wedge \bar{z}) \vee (x \wedge y \wedge z)$

Q1 $(x \vee \bar{y}) \wedge z$ write into CNF or Maxterm

x	y	z	$\bar{y}$	$x \vee \bar{y}$	$(x \vee \bar{y}) \wedge z$
0	0	0	1	1	0
0	0	1	1	1	1
0	1	0	0	0	0
0	1	1	0	0	0
1	0	0	1	0	0
1	0	1	1	0	0
1	1	0	0	1	0
1	1	1	0	1	1

$m_1 \cdot m_3 \cdot m_4 \cdot m_5 \cdot m_7$

$(x \vee y \vee z) \wedge (x \vee \bar{y} \vee z) \wedge (x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee z) \wedge (\bar{x} \vee \bar{y} \vee z)$

$x \vee 0 = x$

$x \vee (y \wedge y')$

Q2 $x \wedge (y' \vee z)$ into CNF in two variables

x	y	z	$y' \vee z$	$x \wedge (y' \vee z)$	y'
0	0	0	1	0	1
0	0	1	0	0	1
0	1	0	0	0	0
0	1	1	1	0	0
1	0	0	1	1	1
1	0	1	0	0	1
1	1	0	0	0	0
1	1	1	1	1	0

$m_1 + m_2 + m_3 + m_4 + m_6 + m_7$

$(x \vee y \vee z) \wedge (x \vee y \vee \bar{z}) \wedge (x \vee \bar{y} \vee z) \wedge (x \vee \bar{y} \vee \bar{z}) \wedge (\bar{x} \vee y \vee \bar{z}) \wedge (\bar{x} \vee \bar{y} \vee z)$

Q3 State & Prove De-morgan's law.

$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

$\overline{A \cap B} = \bar{A} \cup \bar{B}$

1)

$(A \cup B)' = A' \cap B'$	$(A \cap B)' = A' \cup B'$
$x \in (A \cup B)'$	$x \in (A \cap B)'$
$x \notin (A \cup B)$	$x \notin (A \cap B)$
$x \notin A$ & $x \notin B$	$x \notin A$ and $x \notin B$
$x \in A'$ & $x \in B'$	~~$x \notin A'$ and $x \notin$~~
$x \in = A' \cap B'$	$x \in A'$ and $x \in B'$
	$x \in = A' \cup B'$

Hence proved

Q let $f = x \vee y'$ write f in DNF

$f = x + y'$ in three variables x, y, z

$f = x \vee y'$

$= (x \wedge 1) \wedge 1 \quad \vee (1 \wedge y') \wedge 1$

$= (x \wedge (y \vee y')) \wedge (z \vee z') \vee ((x \vee x') \wedge y') \wedge (z \vee z')$

$= [(x \wedge y) \vee (x \wedge y')] \wedge (z \vee z') \vee [(x \wedge y') \vee (x' \wedge y')] \wedge (z \vee z')$

$= [(x \wedge y) \wedge (z \vee z')] \vee [(x \wedge y') \wedge (z \vee z')] \vee$

$[(x \wedge y') \wedge (z \vee z')] \vee [(x' \wedge y) \wedge (z \vee z')]$

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$[(x \wedge y) \wedge z) \vee (x \wedge y \wedge z')] \vee [(x \wedge y') \wedge z] \vee (x \wedge y' \wedge z')$
$\vee ((x \wedge y') \wedge z) \vee (x \wedge y \wedge z') \vee ((x' \wedge y' \wedge z)) \vee (x' \wedge y' \wedge z')$

Q Let $f = x \wedge y'$ write f in CNF in two variables x, y

$f = x \wedge y'$
$= (x \vee 0) \wedge (0 \vee y')$
$= (x \vee (y \wedge y')) \wedge ((x \wedge x') \vee y')$
$= x \vee y \wedge (x \vee y') \wedge (x \vee y') \wedge (x' \vee y')$

DNF ⇒ introduce 1
CNF ⇒ introduce 0

Relation

$A = \{1, 2\}$ $B = \{1, 2, 3\}$
$R_1 = \{\phi, (1,1)\ (1,3)\}$
$R_2 = \{(1,1)\ (1,3)\ (2,2)\}$

$A = \{1, 2\}$
$B = \{a, b, c\}$
$A \times B = \{(1,a)\ (1,b)$
$(1,c)\ (2,a)\ (2,b)\ (2,c)\}$

$R_{max} = A \times B$ Total no of
$R_{min} = \{\phi\}$ relation create $m \times n$ $\boxed{R \subseteq A \times B}$
$R \Rightarrow 2$

Binary relation R from A to B is a set of ordered pair, first a element is from set A & second element is from set B.

Q $A = \{1, 2, 3\}$ $B = \{4, 5\}$ then three binary relation
A to B on A
B to A on B

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If Set $A = \{1, 2, 3, 4\}$ $B = \{1, 3, 5\}$
$R = \{(x, y)\ (y < x\}$

A to B x, y
$(1,1), (1,3), (1,5), (2,1), (2,3), (2,5), (3,1)\ (3,3)\ (3,5)$
Domain of a Binary Relⁿ $R = (x, y) | y + 1 = 2x^2$ is set N find its range.

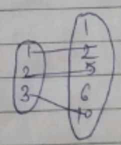
POSET (Partial ordering Relation)
POSET $[A, R]$ (Reflexive, Anti symmetric Transitive)

A relation R is said to antisymmetric if $(x, y) \in R$ then $(y, x) \in R$ only if $(x = y) \vee (x, y) \in A$

A relation R is said to transitive if xRy & yRz then $(xRz)\ \forall\ x, y, z \in A$.
R_1
$R_2 = \{(1,1)(2,2)(3,3), (1,2), (2,3), (1,3)\}$

Equivalence Relⁿ → Reflexive symmetric, transitive
$R_1 = (1,1)\ (2,2)\ (3,3)$
$R_2 = (1,1)\ (2,2)\ (3,3)\ (1,2)\ (2,1)$
$R_3 = (1,1)\ (2,2)\ (3,3)\ (3,2)\ (1,3)$

Relation



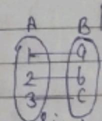
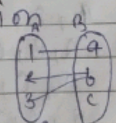


fig 1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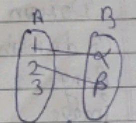


fig 3

⇒ This is example of function

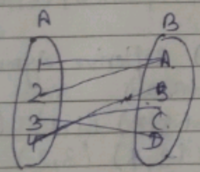
* Function

Let A and B are two non empty set a function from $A \to B$ is a set of ordered pair of $f \subseteq A \times B$ with the property that each element 'a' in set a their is a unique element in set B.

$(a, b) \in F$

Q let $A = \{1, 2, 3, 4\}$
$b = \{A, B, C, D\}$

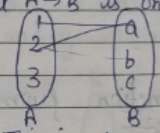
let function $\{(1,a)(2,a)(3,d),(4,c)\}$ find domain of function f or codomain of fun f & image or range



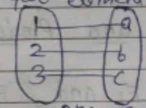
Domain ⇒ 1, 2, 3, 4
Codomain ⇒ A, B, C, D
Range ⇒ A, C, D
image ⇒ A, C, D
Preimage ⇒ 1, 2, 3, 4

Types of Function :-

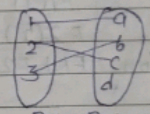
① One to One (Injective function) :- A fun f from set $A \to B$ is one to one if no two element in A are



This is not one - one



one - one

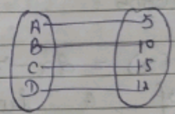


one - one

$x = y$
$x < y$
$x \leq y$

maped to same element in B.

Onto function = (Surjective function)

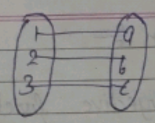


$|A| = |B|$
$|A| < |B|$
$|A| \leq |B|$

A function f $A \to B$ is said to Be onto or surjective if and only if for every element 'b' belongs 'B' then an element 'a' belongs 'A' with $f(a) = b$ in other words f is onto if range is equal to codomain

Bijective function :- A function is said to be bijective (bijenction) if f is both onto & one to one

$|A| = |B|$



Sum and Product form

Q Let F_1 and F_2 be function from $R \to R$ such that $F1(x) = x^2$
$F2(x) = x - x^2$ What are the function $F1 + F2$ and $F1F2$

$F1+F2 = x^2 + (x - x^2$

$2x^3 - x^2$

$F1F2 = x^3 - x^4$

Inverse of a function

Q let $f : R \xrightarrow{\text{to}} R$ $f(x) = 2x - 3$ then find inverse of a function

$$\boxed{\begin{array}{l} y = f(x) \\ x = f^{-1}(y) \end{array}}$$

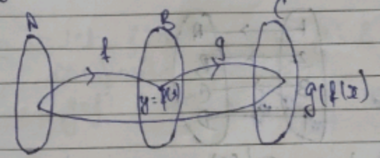
$y = 2x - 3$

$2x = y + 3$

$x = \frac{y+3}{2}$

$f^{-1}(y) = \frac{y+3}{2}$

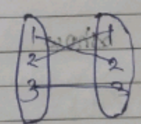
(iv) Composition function :- If $f : A \to B$ and $g : B \to C$ of the two function then function $gof : A \to C$ where $g(f(x))$ for all $x \in A$ is the composition function of f and g.



iQ If fun $R \to R$ with respect $f(x) = x^2+1$
$g(x) = 2x+$ then find $g(f)$ $gof : R \to R$

Q Let $A = \{1,2,3\}$ Define function $A \to A$ such that $f = \{(1,2),(2,1),(3,3)\}$ find $f^{-1}, f^2 \& f^3$

Ans



$f(1) = 2$
$f(2) = 1$
$f(3) = 3$

$f^{-1} = \{(2,1), (1,2), (3,3)\}$

$f^2 = fof(1) = f(f(1))$ $f(2) = 1$

$fof(2) = f(f(2))$ $f(1) = 2$

$fof(3) = f(f(3))$ $f(3) = 3$

$f^2 = \{(1,1)(2,2)(3,3)\}$

$f^3 = \{(1,2)(2,1)(3,3)\}$

* Q2 if $f(x)$ be then function from the $\{1,2,3,4\}$ To the set $\{A,B,C,D\}$ with $f(1) = A$, $f(2) = B$ $f(3) = C$, $f(4) = D$. then determine $f^{-1}(A)$ $f^{-1}(B)$, $f^{-1}(C)$ & $f^{-1}(D)$.

$$\begin{array}{ccc} 1 & \to & A \\ 2 & \to & B \\ 3 & \to & C \\ 4 & \to & D \end{array}$$

$f^{-1}(A) = 1$ $\quad f^{-1}(B) = 2$ $\quad f^{-1}(C) = 3$ $\quad f^{-1}(D) = 4$

Q3 if f function $A \to B$ & $G: B \to C$ If is one to one & onto function

Q4 let $f(x) = 3x - 12$ $\quad g(x) = \sqrt{x}$ & then find domain & range

* $f^2 = f \circ f$

Q5 for any set A & B show that $(A \cap B) \cup (A - B) = A$

Q6 Obtain the CNF and DNF of the given boolean function $(x+y)z$

x	y	z	$x+y$	$(x+y)z$
0	0	0	0	0
0	0	1		0
0	1	0	0	0
0	1	1	1	:
1	0	0	0	
1	0	1	1	
1	1	0	0	
1	1	1	0	

Q If $f(x) = 2x$ & $g(x) = 1$ then find $f \circ g$ if $x = 1$

Q If $f(x) = 2x + 1$ & $g(x) = -x^2$ then find $g(f)$ $x = 2$

Q $f(x) = x$ $\quad g(x) = 2x$ $\quad h(x) = 3x$ then composition of this function

<u>Theorem</u> :- let $f: A \to B$ and $g: B \to C$ be the composite function

$g \circ f: A \to C$

$\Rightarrow$ one to one

$\Rightarrow$ onto

$\Rightarrow (g \circ f)^{-1} = f^{-1} \circ g^{-1}$

* Let F be a function from the set of integer such that $f(x) = x + 1$ if it is invertibles if

$f(x) = x + 1$

$x = y$

$f(x) = x + 1 = f(y) = y + 1$

$x + 1 = y + 1$

$x = y$

$f(x) = x + 1$

$y = y + 1$

$x = y - 1$

$y - x + x \quad y - 1 + 1$

$f(x) = y$

Let f be the function from $\{A, B, C\}$ to the set $\{1, 2, 3\}$ such that $f(a) = 2$, $f(b) = 3$ & $f(c) = 1$ if it is invertible then what is inverse.

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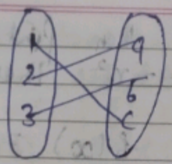

$f^{-1}(1) = c$

$f^{-1}(2) = a$

$f^{-1}(3) = b$

Unit - 2

Principles of Mathematical Induction (PMI)

* Let $P(n)$ be a statement involving the natural number n.
* To prove that $P(n)$ is true for all natural num. $n \geq 1$

1) Verify $P(n)$ for $n=1$ i.e ($P(n)$ is true for $n=1$
2) Suppose the result $P(n)$ is also true for $n=K \geq 1$
3) Using (i) & (ii) Prove that $P(K+1)$ is true.

Q1 Prove that by induction method.

$$1+2+3+ \dots n = \frac{n(n+1)}{2} \quad \forall n \in N$$

(1) $n=1$

L.H.S $=1$

R.H.S $= \frac{n(n+1)}{2} = \frac{1(1+1)}{2} = \frac{2}{2} = 1$

L.H.S = R.H.S

① is true for $n=1$

2) $n=K$

Assume ① is true for $n=K$

$1+2+3 - - K = \frac{K(K+1)}{2}$ — (ii)

3) Now $K+1$

L.H.S $= 1+2+3+ \dots K+K+1$

$\frac{k(k+1)}{2} + k+1$

$(k+1)\left(\frac{k}{2}+1\right)$

$k+1 \left(\frac{k+2}{2}\right)$ R.H.S $(k+1)(k+2)$

Hence $P(k+1)$ is true for n.

Q1 Prove that by induction method.

$1^2+2^2+3^2 - - n^2 = \frac{n(n+1)(2n+1)}{6}$

Ans 1) L.H.S $= 1^2 = 1$

R.H.S $= \frac{n(n+1)(2n+1)}{6} = \frac{1(1+1)(2(1)+1)}{6}$ - ①

$\frac{6}{6} = 1$

2) $n = k$

$1^2+2^2+3^2 - - - k^2 = \frac{k(k+1)(2k+1)}{6}$ - ②

3) Now $k+1$

$1^2+2^2+3^2 - - - - k^2+(k+1)^2$

$\frac{k(k+1)(2k+1)}{6} + (k+1)^2$ from eq ②

$k+1\left(\frac{k(2k+1)+(k+1)}{6}\right)$ - ③

Now Put $k+1$ in k

$\frac{(k+1)(k+2)(2k+3)}{6}$

L.H.S = R.H.S

Q3 Prove that by induction method

$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + - - - \frac{1}{\sqrt{n}} \geq \sqrt{n}$

① $n=1$

L.H.S $\frac{1}{\sqrt{1}} = 1$

R.H.S $= \sqrt{n} = \sqrt{1} = 1$

L.H.S = R.H.S

P(1) is true for 1

② $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + - - - \frac{1}{\sqrt{k}} \geq k$ - ②

Assume that ① is true for $n=k$

3) $n = k+1$

L.H.S of ① $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + - - - \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}}$ - ③

$\sqrt{k} + \frac{1}{\sqrt{k+1}} \Rightarrow k + \frac{1}{\sqrt{k+1} + \sqrt{k}}$

$\sqrt{k} + \frac{1}{(\sqrt{k+1} + \sqrt{k})} \times \frac{\sqrt{k+1} - \sqrt{k}}{\sqrt{k+1} - \sqrt{k}}$

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$\sqrt{K} + \frac{\sqrt{K+1} - \sqrt{K}}{K+1-K}$

$\sqrt{K} + \sqrt{K+1} - \sqrt{K}$

$\sqrt{K+1}$ L.H.S

Q4 Prove by induction method

$1+3+5+ - - 2n-1 = n^2$

1) $n=1$ R.H.S $= n^2$ - ①

L.H.S $=1$ Put 1 in eq 1

$n^2 = 1$

L.H.S = R.H.S

2) $1+3+5+ - - - 2K-1 = K^2$

3) Now K+1

$1+3+5+ \dots 2K-1 + 2K+1 = 2K+1+K^2$

$1+3+5+ \dots + 2(K+1)-1 = (K+1)^2$

$1+3+5+ - - 2K+2$

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Recursive function / defination

1. Recursive defination of an arithmatic Sequence

$a_n = a + nd$

$a_n = a_{n-1} + d$ $a_0 = a$

2. Recursive defination of an G.P

$x_n = a r^n$

$x_n = r x_{n-1}$

3. Fibnacci Series : 0, 1, 1, 2, 3, 5, 8, 13, 21

$f_0 = 0$

$f_1 = 1$

Recursive :-

Sometimes it is possible to defined an object in term of smaller version of its

Recursively defined fun :-

Their are certain arrgument called base fun/values for which a fun does not preffered to itself.

* Each time the fun does not preffered itself to itself the argument all the fun must be closer to the base value

Q $4, 6, 8, 10$

$f(n) = 2n + 2$

$f(0) = 2$
$f(1) = 4$
$f(2) = 6$
$f(3) = 8$
$f(4) = 10$

Q Determine the recursive formula for the $4, 8, 16, 32, 64, 128$ - - - -

Ans
$f(1) = 4 = a_1$
$f(2) = 8 = a_2$
$f(3) = 16 = a_3$

$f(n) = n = a_n$

$f(n) = f(n) \; f(n-1)$
$a_1 = 4$
$a_n = (2) \; a_{n-1}$

Q Suppose that f is defined by recursively
$f(0) = 3$

$f(0) = 3$
$f(n+1) = 2f(n) + 3$

find $f(1)$, $f(2)$, $f(3)$ & $f(4)$...

Solⁿ
$f(0+1) = 2f(0) + 3$
$f(1) = 2 \times 3 + 3 = 6 + 3 = 9$
$f(1+1) = 2f(1) + 3$
$f(2) = 2 \times 9 + 3 = 21$

$f(2+1) = 2f(2) + 3$
$f(3) = 2 \times 21 + 3 = 45$

How to write recursive formula for Gemotric sequence follow the given step.

Step 1 :-

Firstly, you need to ensure that the given series is in G.P or not

Step 2 :-

You need to find the common ratio of the given series.

Step 3 :-

Formulate, the recursive formula using the first term & create the formula by using the previous term.

G.P series $\Rightarrow$ $X_n = a r^n$
$X_n = r \times n - 1$

Division Algorithm

let a & b are two integer there exists q, r uniqueness

$a = bq + r$ ← Remainder, Quiotent, integer

Q $a = -262 \quad b = 3$

Soln

$a = bq + r$

$262 = 3(87) + 1 \qquad a = -262$

$\underbrace{-262}_{a} = \underbrace{3}_{b}(\underbrace{-87}_{q}) - \underbrace{1}_{r}$

$-262 = 3(-87) - 3 + 3 - 1$

$-262 = 3(-87-1) + 3 - 1$

$3(-88) + 2$

Remainder = 2

Quotient = -88

Q2 $a = -4461 \quad b = 16$ find the value of q and r by division algorithm

Soln

$a = bq + r$

$4461 = 16(278) + 13$

Multiply by (-1)

$-4461 = 16(-278) - 13$

Add & subtract (16)

$-4461 = 16(-278) - 16 + 16 - 13$

$-4461 = 16(-278-1) + 3$

$-4461 = 16(-279) + 3$

Remainder = -279

Quotient = 3

Q3 Use division algorithm to find Quotient & remainder -100 is divided 13

$100 = 13(7) + 9$

$-100 = 13(-7) - 9$

$-100 = 13(-7) - 13 + 13 - 9$

$-100 = 13(-7-1) + 4$

$-100 = 13(-8) + 4$

Quotient = -8

Remainder = 4.

NOTE ⇒

* When the divisior & divident both are given in negative number the quotient & remainder is always negative.

* The Quotient (q) can be (+ve) or (-ve) & the remainder is always non negative

Conclusion :- Describe how to find the integers quotient & remainder such that $a = bq + r$

* Show that our choice (r) is satisfied $0 \le r < b$ this condition.

* Stablised the uniqness of q & r.

Q Find $b \% a$ by division algorithm $b = -823 \quad a = 16$

$823 = 16(51) + 7$

$-823 = 16(-51) - 7$

$-823 = 16(-51) - 16 + 16 - 7$

$-823 = 16(-51-1) + 9_{r}$

$-823 = 16(-52) + 9 \rightarrow r$
$\quad q$

Q 4 $\quad a = -12 \quad b = -132$
Ans

Euclidean Algorithm :

let 'a' & 'b' & $d = \gcd(a, b)$
$d = ax + by$

$\gcd(540, 168) = \gcd(168, 36) = \gcd(36, 24) = \gcd(24, 12)$
$= 12$

$a = 540 \qquad b = 168$

$12 = 540x + 168y$

(1) $540 = 3(168) + 36$ or $36 = 540 - 3(168)$
(2) $168 = 4(36) + 24$ or $168 = 168 - 4(36)$
(3) $36 = 1(24) + 12$ or $12 = 36 - 1(24)$ - (3)

from eq (3) we can say that 12 is linear combination of (36, 24)

$12 = 36 - 1[168 - 4(36)]$ - (4)
$12 = (36) - 1(168) + 4(36)$
$12 = 5(36 - 1(168)$
use eqn (1) & (4)

$12 = 5[540 - 3(168)] - 1(16)$
$5(540) = 15(168) - 1(168)$

* Euclidean Algo or Euclid's Algo
* for computing the Greatest Common divisor
* ask HCF

The fundamental Theorem of Arithmatic

let a and b are two integers are said to be relatively prime if gcd of $(a,b) = 1$

Every integer $n \geq 1$ can be expressed as uniquely as the product of primes. This is Knouial form.

$n = P_1^m \cdot P_2^m \cdot P_3^m - - - - P_K^m.$

Q
$a = 2^4 . 3^3 . 7 . 11 . 13$
$b = 2^3 . 3^2 . 5^2 . 11 . 17$
Find $d = gcd(a,b)$
$M = LCM(a,b)$

Ans Step1 :- Find the prime no which are common in both

Step2 : Take smallest power
$d = 2^3 \times 3^2 \times 11^1 = 792$

L.C.M (a,b)

Step1 :- Take all prime num.
$2 \times 3 \times 7 \times 11 \times 5 \times 13 \times 17$

Step2 :- Take bigest power

$2^4 \times 3^3 \times 5^2 \times 7 \times 11 \times 13 \times 17 =$

Basic counting Principle

Principle of inclusion - Enclusion
$n(A \cup B) = n(A) + n(B) \underset{add}{} - n(A \cap B)$ sub

$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Q1 Find the no. of M.M student at colleges taking atlest one of the languages French, German, Russian. 65 students study french 45 student study German 42 study Russian

Solⁿ
French	n(A)	65
German	n(B)	45
Russian	n(C)	42
French & German	$n(A \cap B)$	20
French & Russian	$n(A \cap C)$	25
German & Russian	$n(B \cap C)$	15
study all of them	$n(A \cap B \cap C)$	8

$65 + 45 + 42 - 20 - 15 - 25 + 8 = 100$

R.

Pigeon hole Principle

If n+1 or more objects are placed into n boxes then their is atleast one box containing two or more objects

Q If six colors are used to paint 37 holes / house. So that atleast 7 hole of that will be same color

Ans $\frac{37}{6} = 6.2$ approx

So 7 is coming hence proved.

Sh Generalise pigeon hole principle
If n pigeon hole are occupied by Kn+1 or more pigeons then atleast one pigeon hole is occupied by K+1 or pigeons

Q Find the minimum num of students in a college to be sure that for of them are born in same month.

Ans Kn+1   n = 12   K+1 = 4
K· 36 + 1 = 37   K = 3

Q A box contain Ten blue balls, 20 red balls, 8 green balls, 15 Yellow balls & 25 white balls. How many balls must be choose to ensure that we have 12 balls of same colour.

n = 5   K+1 = 12
K = 11

Kn+1 -
5(11)+1 = 56.

Q If n pigeons are assigned to m pigeons holes show that atleast same pigeon hole contain among 2 pigeons also show that atleast 2 of 30 people. Their are born in people which are same month

Ans $\frac{13}{12} = 1.5$

approx ans 1.5 so 2 is ans

Q Show that if the we alot 26 rooms to the students in P.G Hostel from the room num. from 1 to 50 both enclusive atleast two allotments

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are consuctively numbers.

$$\frac{50}{26} = 1.9$$

The ans is 2

Introduction to Recurrence Relations

It is an eqn that recursively define a sequence where the next term is the function of previous term.

$S = \{2, 2^2, 2^3 \ldots 2^n\}$

$S = \{S_n\} = 2^n \rightarrow$ Sequence

$S_n = 2S_{n-1} \quad n \geq 2 \quad S_1 = 2$

$S_n = 2S_{n-1} \quad n \geq 2$ is Recurrence relation with initial condition $S_1 = 2$

2 = $S = \{3, 7, 11, 15 \ldots\}$

$S_n = S_{n-1} + 4 \quad n \geq 2$

$S_1 = 3$

3 = $\{1, 1, 2, 3, 5, 8, 13 \ldots\}$

$S_n = S_{n-1} + S_{n-2} \quad n \geq 3$

$S_1 = S_2 = 1$

Linear Recurrence Relation with constant Cofficient

$$c_0 a_n + c_1 a_{n-1} + c_2 a_{n-2} + c_3 a_{n-3} \ldots c_k a_{n-k} = \underbrace{F(n)}_{\substack{\text{Constant} \\ 0}}$$

c_0, c_1, c_2 & c_3 are constant coffiecient

Q1, a) $2a_n + 3a_{n-1} = 3$

b) $a_n - 7a_{n-1} + 12a_{n-2} = n - 4n$

c) $a_n = a_{n-1} + a_{n-2}$

2) a) $a_n = a_{n-1} \cdot a_{n-2}$

$a_n \cdot a_{n-1} \cdot a_{n-2} = n^2$

These two eqn is not Recursion relation

Order :- It is diff b/w highest & lowest subscript a_n.

Recurrence Relation	Order
$2a_n + 3a_{n-1} = 3$	1
$a_n + 7a_{n-1} + 2a_{n-2} = 0$	2
$a_n + a_{n-1} + a_{n-2} + a_{n-3} = 0$	3
$a_n = 2a_{n-1} + 1$	1

Degree :- Degree is define as highest power of A_n

Degree of Recurrence Relation is always 1.

Method to Solving Recurrence Relation :-

The method of characterstic Roots

- Homogeneous Recurrence Relation $f(n) = 0$
- Non homogeneous Recurrence relation

Step 1

$$a_n = r^n$$
$$a_{n-1} = r^{n-1}$$
$$a_{n-2} = r^{n-2}$$

Step 2

Find the eqn in term of R. which is called characterstics eqn or axillary eqn.

$$r^n + r^{n-1} + r^{n-2}$$

Step 3

Solve the characterstics eqn & find characterstics root

1) If the characterstics root are $r = r_1, r_2, r_3$ (distinct)

general soln

$$a_n = b_1 r_1^n + b_2 r_2^n + b_3 r_3^n$$

2) $r = r_1, r_1$

$$a_n = (b_1 + n b_2) r_1^n$$

3) $r = r_1, r_1, r_2, r_3$

$$a_n = (b_1 + n b_2) r_1^n + b_3 r_2^n + b_4 r_3^n$$

4) $r = r_1, r_1, r_1, r_2$

$$a_n = (b_1 + n b_2 + n^2 b_3) r_1^n + b_4 r_2^n$$

Q Solve $a_n = a_{n-1} + 2a_{n-2}$, $n \geq 2$ with initial condition $a_0 = n$ $a_1 = 1$

$$r^n - r^{n-1} + 2r^{n-2} = 0$$

$$r^n \left[1 - \frac{1}{r} + \frac{2}{r^2}\right] = 0$$

$$r^2 - r + 2 = 0$$
$$(-1, 2)$$

$$a_n = b_1 2^n + b_2 (-1)^n \quad -③$$

$a_0 = 0$

$a_1 = 1$

$a_0 = b_1 2^0 + b_2 (-1)$ $\qquad$ $a_1 = b_1 2^1 + b_2 (-1)^1$

$0 = b_1 + b_2 \; -①$ $\qquad$ $1 = 2b_1 - b_2 \; -②$

$b_1 = \frac{1}{3}$ $\quad$ $b_2 = \frac{-1}{3}$

$b_1 = b_2$

$$a_n = \frac{1}{3} 2^n + \left(-\frac{1}{3}\right) (-1)^n$$

Q Solve $a_n - 8a_{n-1} + 21 a_{n-2} - 18 a_{n-3} = 0$

$$r^3 - 8r^2 + 21r - 18 = 0$$
$$r = 2, 3, 3$$

$$a_n = b_1 (2^n) + (b_2 + n b_3) 3^n.$$

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Q3 Solve $a_n = 4(a_{n-1} + a_{n-2})$ with initial condition $a_0 = a_1 = 1$

$a_n - 4a_{n-1} - 4a_{n-2}$

$r^2 - 4r - 4$

$\boxed{r = 2,2}$

$a_n = (b_1 + nb_2)2^n$

$a_0 = 1$

$a_1 = 1$

Q4 $a_n = -a_{n-1} + 4a_{n-2} + 4a_{n-2} + 4a_{n-3}$

with $a_0 = 8$, $a_1 = 6$, $a_2 = 26$

$a_n + a_{n-1} - 4a_{n-2} - 4a_{n-3} = 0$

$r^3 + r^2 - 4r - 4 = 0$

$r = 2, -1, -2$

$b_1 2^n + b_2(-1)^n + b_3(-2)^n$

$a_0 = b_1(-1)^0 + b_2(2)^0 + b_3(-2)^0$

$8 = b_1 + b_2 + b_3$ $\quad b_1 = 1$

$6 = -b + 2b_2 - 2b_3$ $\quad b_2 = 5$

$26 = 4b_1 + b_2 + 4b_3$ $\quad b_3 = 2$

Q $a_n = 5a_{n-1} - 6a_{n-2}$ where $a_0 = 1$, $a_1 = 4$

$r^2 - 5r + 6 = 0$

$r = 2, 3$

$0 = b_1 2^n + b_2 3^n$

$1 = b_1 + b_2$ $\quad b_1 = -1$

$4 = 2b_1 + 3b_2$ $\quad b_2 = 2$

Q Solve the linear recurrence relation by method of characteristics root.

$a_n = 10a_{n-1} - 25a_{n-2}$

where $a_0 = 3$, $a_1 = 17$

$r^2 - 10r + 25$

$r = 5, 5$

$a_n = (b_1 + nb_2)r_1^n$

$a_n = (b_1 + nb_2)5^n$

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$3 = b_1 + n(b_2)5$

$3 = b_1$

$a_1 = (b_1 + 1b_2)5^1$

$17 = 5b_1 + 5b_2$

$17 = 15 + 5b_2$

$\frac{2}{5} = b_2$

$0.4 = b_2$

Q $a_n = a_{n-1} + n$ where $a_1 = 4$

$a_0 - a.$

Q Solve the recurrence relation $a_n = 7a_{n-1} - 10a_{n-2}$ with initial condition $a_0 = 2$, $a_1 = 3$

Ans

$r^2 - 7r + 10$

$r = 5, 2$

*

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Unit ⇒ 5

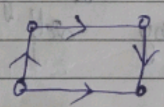
Graph theory

Graph :- The graph is a mathmatical structure consisting of two sets vertices and edges where V and E both are non empty set of vertices and edges.

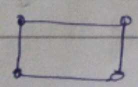
Trival graph
The graph consisting only one vertex and no edge..

Null graph :- A graph which consisting n no. of vertices but no edges.

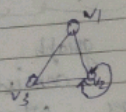
Directed graph :- A graph consist the direction of edges then it is called directed graph



Undirected Graph :- A graph which is not directed is known as undirected graph.

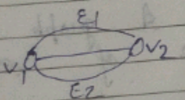


Self loop in graph :- If edge having the same vertex as both its end vertices is called self loop.



Proper edge :- An edge which is not self loop is called proper edge.

Multi edge :- A collection of two or more edges having identically end point



Simple graph :- A graph does not contain any self and multi edge.

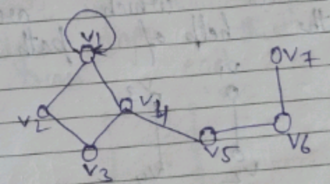
Multi graph :- A graph does not contain any self loop but contain multi edge

Pseudo graph :- A graph contain both self loop and multi edge is called Pseudo graph.

Incidence Graph :- Adjanancy Graph

let E_k be an edge joining two vertices v_i and v_j then E_k is said to be incidence of v_i and v_j.
Two vertices are said to adjacent if there exist an edge joining this vertices.

Degree of vertices :- No of outcoming & incoming edge.
The degree of vertex v in graph written as $d(v)$ = no of edges which are incident on v with self loop twice.



Isolated vertex :- A vertex having zero edge is called Isolated vertex.

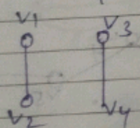
Pendent graph :- A vertex having one edge is known as called pendent graph

Finite :- A graph with a finite num. of vertices as well as edges is called finite graph.

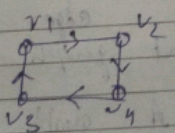
Infinite graph :- A graph with a infinite num of vertices as well as edges is known as Infinite graph.

Connected graph :- A graph will be known as connected graph if they have two vertices that are connected with the help of edge and path.

disconnected graph :- A graph will be known as disconnected graph if it contain two vertices which are disconnected with the help of path and edge.



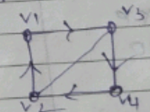
Cycle graph :- A graph will be known as cyclic graph if it completes a cycle which means they have direction



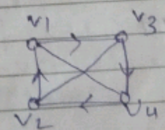
Complete graph :- A graph will be known as complete graph if each pair of vertices is connected with the help of exactly one edge. A simple graph will be a complete graph if there are n num. of vertices which are exactly one edge b/w each pair of vertices.

Total num of edges with n vertices $\frac{n(n-1)}{2}$

Planer Graph :- The graph will be known as planer graph if it is drawn in single plane and the two edges of this graph do not cross each other



Non planer :- A graph will be known as Non planer if it is not drawn in single plane and two edges of this graph must be cross each other.



Euler's Formula

A complete graph of 5 vertex is non-planer graph.

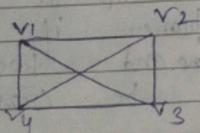
$R = E - V + 2$

R = No. of region
E = No of edges
V = No of vertex

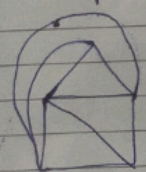
Σ degree $(v) = 2e$

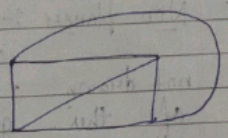
$G = V = 20$
degree = 3
$20 \times 3 = 60$
$60 = 2 \times e$
$e = 30$
edges

$r = E - V + 2$
$30 - 20 + 2$
$10 + 2$
$r = 12$
outer region

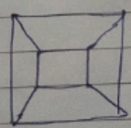


Non-planner





Plan



Let G be a Connected planar simple graph with e edge and V vertices and r no. of region then the euler formula be $R = E - V + 2$

1. Basic step $P(1)$ = ~~tree~~ true
2. Induction step let given step true for k
3. verify by proving the result for $k+1$

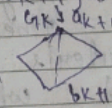
1) $n = 1$
$R_1 = E_1 - V_1 + 2$
$1 - 2 + 2$
$= 1$
$P(1)$ is true

2) $R_K = E_K - V_K + 2$ is true for G_K
$\Rightarrow$ Assume the eqn is true for $n = K$

3) Verification for $n = k+1$

Let (a_{k+1}, b_{k+1}) be the edge that is added to G_K

Case 1 $\Rightarrow$ G_{K+1}

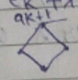


Both the vertices are in G_K

$r_{K+1} = r_{K+1}$ | $e_{K+1} = e_{K+1}$ | $V_{K+1} = V_K$ $\because$ both vertices are in G_K

$r_K + \cancel{r} = e_K + \cancel{r} - V_{K+1} + 2 = r_K = e_K - V_K + 2$

Case 2



$r_{K+1} = r_K$
$V_{K+1} = V_K + 1$
$e_{K+1} = e_{K+1}$

$r_{K+1} = e_{K+1} - V_{K+1} + 2$
$r_K = e_K + r - [V_K + r] + 2$
$r_K = e_K - V_K + 2$ True

Q2 If there are 15 vertices each of ~~ab~~ degree 2 then how many region does representation of this graph.

$15 \times 2 = 30$

Q3 Let G be a graph with 3 region and 3 edges No of vertices.

$R = E - V + 2$

$3 = 3 - V + 2$

$3 = 5 - V$

$V = 2$

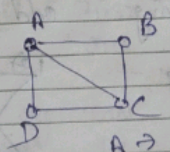
<u>Note</u> ⇒ If all vertices of any connected graph have an even degree then this type of graph will euler's graph. In other words we can say that euler's graph is a type of connected graph which euler's circuit

<u>Euler's Path</u> :- We can also called euler's path or euler's walk & eular tra.

2) If there is connected graph with the tra it means all edges of graph

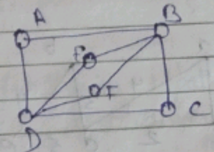
that type of path known as Euler's path

2) If there is connected graph which have walk that passes through each of every edge of the graph only ones then this type of path called Euler's path.



A → B → C → D → A

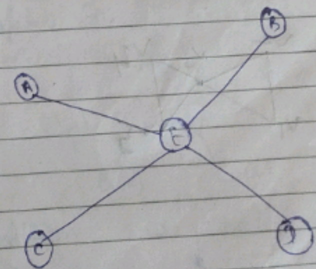
Eg Graph of 6 node the determine graph contain Euler's path.



B → E → D → A

B E D F B C D A B

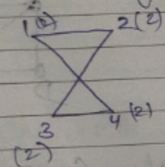
In the following Image we have graph with 5 nodes whether it contain eular path



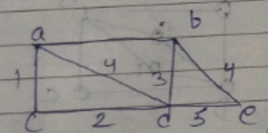
Euler's Graph

If a connected graph is Euler's graph if it has atmost two $\sim$ degree vertices
maxi In Euler circuit each vertex is of even degree. Euler's path maximum two vertex having odd degree

Euler circuit eg:

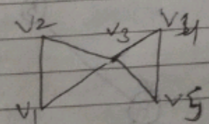


Euler's Path



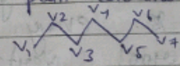
$deg(a) = 3$
$deg(b) = 3$
$deg(c) = 2$
$deg(d) = 4$
$deg(e) = 2$

Euler's Circuit



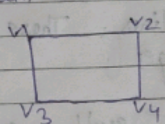
Hamiltonian Path   Hamiltonian Circuit

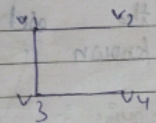
Hamiltanian Path :- A path contains each vertex exactly ones. eg



Hamiltanian Circuit First and last vertex are same.

On removing any one edge from a hamiltanian circuit then we are left with hamiltanian path

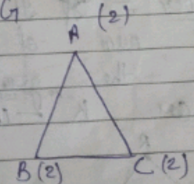




1, length of hamitanian path in a connected graph of n vertices is $(n-1)$ edges

2, let g be a Graph of n vertices the G has a hamilhanian path if for only two vertices 'u' and 'v' of graph G

3,



$deg(u) + deg(v) \geq n$
$2 + 2 \geq 3$
$4 \geq 3$

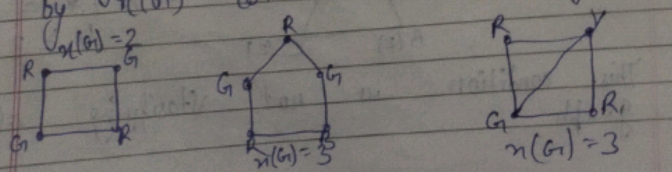
This condition is not statifying all graph

Hamiltanian Graph :- The graph is know as Hamiltanian graph if their is closed walk in a connected graph which passes each & every vertex exactly one. The root vertex are starting vertex. The hamiltanian walk must not repeat any edge. If their is a connected graph which contains a hamiltanian circuit.

The vertex of a graph is a set of point which are inter connected with the set of lines. These lines are known as edges

Graph coloring :- Coloring of graph constitute coloring vertices edges or regions of the graph. Coloring all the vertices of the graph is the property that no adjecent vertices have same color. Always try to color minimum num.

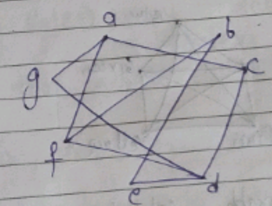
Chromatic number :- It is defined as least num. of colors needed by coloring for the graph which is denoted by $\chi(G)$ or K^{n} chromatic graph



Every bipartite type graph is two colorable.

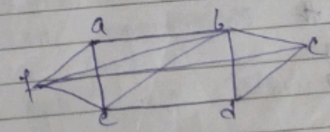
Bipartite graph :- G is Bipartite graph if vertex set B can be partation into two disjoint non empty sets S_1 and S_2 such that every edge in the graph connected a vertex in S_1 and a vertex in S_2. So that no edge in G connects either two vertices in S1 or two vertices in S_2.

eg



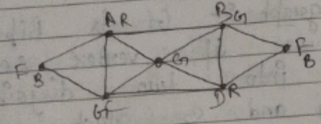
$V = \{a, b, c, d, e, f, g\}$
$S_1 = \{a, b, d\}$
$S_2 = \{c, e, f, g\}$

This is Bipartite graph



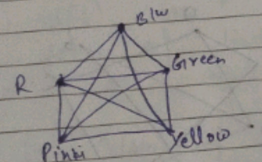
It is not a B.G because it vertex can't be partation in two sets such that edges can't connect two vertices from same subset

What is the chromatic num of following graph.



Chromatic $\chi(G) = 3$

~~Q.~~ Every planar graph with n vertices can be colored using atmost 5 colors



Applications of Graph coloring

Assignment
Scheduling the task
Suedco game
Prepare the time table
Conflict the sol^n
~~Ex~~ Clustring
data mining
Map Coloring

Graph coloring

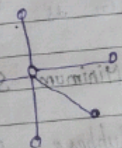
* 5 color theorem Proof

Every Planar graph with n vertices can be colored using atmost 5 color.
Prove 5 color theorem by Principal mathmatical induction.

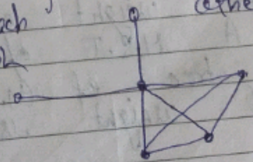
1) The basic step is $P(n \le 5)$ graph can be colored
Lecma Rule ⇒ Every planar graph contains a vertex with degree less than equall to 5. $\deg(v) \le 5$

2) It is true for $K = 4$ ⇒ Induction step

3) Verify for $K+1$.



Case 2 Assume that their is direct edge b/w V_1 and V_2. If we connect all the edges in given graph we found 1 and 2 edge are intersed each other. which are no planar graph

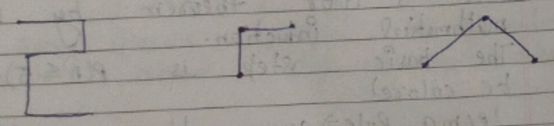


* Hence proved
$v \leq 5$ is true for planar graph

Tree Graphs

A graph (t) is called a tree graph if it is connected and no cycle

e.g



Theorem :— Let G be a graph with $n \geq 1$ vertices then the following are equivalent.

1) G is tree.
2) G is cycle free and has edges $n-1$.
3) G is connected with edges $n-1$.

Minimum Spanning tree :→ ~~G is cycle free~~

Suppose G is connected waited graph that is each edge of (G) is assign non negative num. called weight of edge. When the spanning tree of G is assign a total weight obtained by added the weight of the edges in tree. A M.S.T of G is a spanning tree of G whose total weight is as small as possible

Prim's Algorithm ⇒

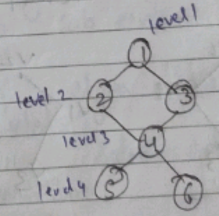
1) Arranged the edges of G in the order of decreasing weight.
2) Proceding sequencly and delete each edge that does not disconnect the graph untill $n-1$ edges are remain

Kruskal's Algorithm ⇒ Arrange the edges of G in order of ↑ weight

2) Starting only with the vertex G and proceding sequencly at each edge which does not result in the cycle with $n-1$ edges are added.

Graph Traversal

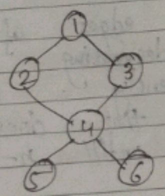
BFS



1	2	3	4	7	5	6	visited
1	2	3	4	7	5	6	explore

Queue works on BFS
Back tracking is not possible

DFS

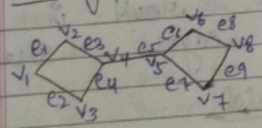


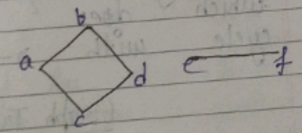
3
4
2
1

[1, 2, 4, 5, 6, 3, 7]

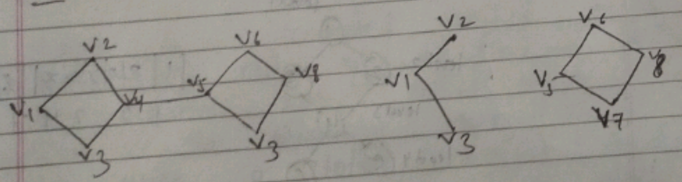
Connected Graph Component

Cut edge :-

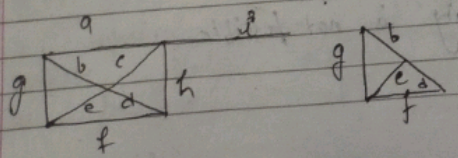




Cut Vertex (articulation Point / cut point)



Cut set



Connected Component :- A maximal connected sub graph of (G) is a component of graph G.

Cut edge :- An edge of e of graph G is said to be cut edge or a bridge of G if G-e is disconnected.

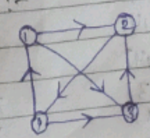
Cut Vertex :- A vertex V of a graph G is said to be cut vertex of G if G-V is disconnected.

Cut set :- The set of all minimum num. of edges of G whose removal diconnects ~~of~~ a Graph G is called a cut set G. The cut set S of a graph satisfy the following condition

1) S is a sub set of the edge set E of G.
2) Removal of edges from a connected graph G disconnected.
3) No proper sub set of G satisfy the condition.

4) ~~8~~ Directly connected graph types :-

Strongly connected graph :-



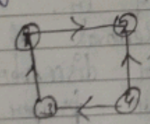
Pair of vertices	forward path	Reverse ver.	Backward path
(1,2)	1-2	(2,1)	2-4-1
(1,3)	1-3	(3,1)	3-2-4-1
(1,4)	1-2-4	(4,1)	4-1
(2,3)	2-4-3	(3,2)	3-2
(2,4)	2-4	(4,2)	4-1-2
(3,4)	3-2-4	(4,3)	4-3

A directed graph is called strongly connected if any/all pair of vertices of the graph both the vertices of the pair are reachable from one another.

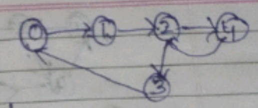
Connected or weakly connected

A directed graph is called connected or weakly connected if it is connected as an undirected graph in which each directed edge is converted to an undirected edge or graph.

Unilaterally Connected



Pair	forward	Backward
(1,2)	1-2	x
(1,3)	x	3-1
(2,4)	x	4-2
(3,4)	x	4-3
(2,3)	x	3-1-2
(1,4)	x	4-3-1

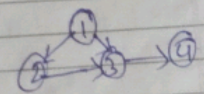


Pair	forward	backward
(0,1)	1-0	
(0,3)	0-1-2-3	~~0-3~~
(1,2)	1-2	
(2,4)	2-4	
(4,2)		~~4-2~~
(2,3)	2-3	
(3,0)	3-0	
(1,3)	1-2-3	
(2,0)	2-3-0	
(0,2)	0-1-2	
(0,4)	0-1-2-4	

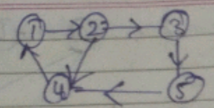
0 → 1 → 2 → 3

unilateral

(0,1)	0-1
(1,2)	1-2
(2,3)	2-3
(0,2)	0-1-2
(0,3)	0-1-2-3
(1,3)	1-2-3



(1,2)	1-2
(2,3)	2-3
(3,4)	3-4
(1,3)	1-3
(1,4)	1-3-4



Strongly

(1,2)	1-2
(1,3)	1-2-3
(3,5)	3-5
(5,4)	5-4
(4,1)	4-1
(2,4)	2-4
(2,3)	2-3
(5,1)	5-4-1

As → Close
Sg. → Associative
M → identity
Gp → Inverse

$*$ ⇒ binary operation

$(z^+, +)$ all +ve
$(z^-, -)$ all -ve

Algebraic structure

1. $N = \{1, 2, 3, 4 \ldots \infty\}$ natural num.
2. $Z = \{$ Set of all integer $\}$, $\{\ldots 2 \; -1 \; 0 \; 1 \; 2\}$
3. $Q = \{$ Set of all Rational no $\}$ $Q^* \Rightarrow$ set of rational no $- 0$ $\{\frac{2}{3}, \frac{1}{5}, \ldots\}$
4. $R = \{$ Set of real numbers $\}$
5. $C = \{$ Set of Complex numbers $\}$ $(a+ib)$

Algebraic structure for binary operation

Set (a,b) are not empty set then set {S} is called algebraic str. w.r.t binary operation $*$ if $(a*b) \in S$ for all $\forall (a,b) \in S$ where $*$ is a closer operation on set S.

Properties of Algebraic structure :-

1) Closer property :-
$(a*b)$ belongs S for all $a, b \in S$

2) Associativity :- $a*(b*c) = (a*b)*c$.

3) Commutative :- $a*b = b*a$

4) Identity property :- $a*e = a$, $e*a = a$
e.g. $2+0 = 2$ $0+2 = 2$
$2 \times 1 = 2$ $1 \times 2 = 2$

5) Inverses :- $a * a^{-1} = e$

Semigroup and Monoid

An algebraic stru. $(S, *)$ is called a Semigroup if it follows the associative property.

$$a * (b * c) = (a * b) * c$$

Monoid

Algebraic stru. (closer)

↓

Semi group (Associative)

↓

Monoid

A semigroup $(S, *)$ is called monoid if there exist an element $e \in S$ such that $a * e = e * a = a$

Q let A is a set of $\{1, 2, 3, 4 \ldots \infty\}$ and binary operation for multiplication defined by $A + B = AB$ for all $A, B \in A$ which of the following is true. Semigroup, Monoid or algebraic stru.

$(N, *)$

P

Group and Abelian group

A monoid $(S, *)$ with identity element e is called a group if each element $a \in S$ their exist an element $b \in S$ such that $a * b = b * a = e$ where b is the inverse of a and it is denoted by a^{-1}. $\boxed{a^{-1} = b}$.

A group $(G, *)$ is said to be abelian group if $a * b = b * a$ and $\forall a, b \in G$

Prove that set $G \{1, -1, i, -i\}$ is an abelian group w.r.t multiplication operation

Ans 1, Closer Property

×	1	-1	i	-i
1	1	-1	i	-i
-1	-1	1	-i	i
i	1	-i	-1	1
-i	-1	i	1	-i

{composition table}

All the entries in the table belongs to the set G So it is the closer property

2, Associative property :- $a * (b * c) = (a * b) * c$

Yes associative

if a is the generator of G then a^{-1} is also a G.

Subgroup

* A non empty sub set (H) of a Group (G) is a sub group of G if under binary operation of G. We use the notation H is the sub group of G. To indicate H is the subgroup of G also if H is a proper sub group then it is denoted by $H < G$

* For a subgroup (H) of group of (G) if first condition is $H \neq \phi$

* if $a, k \in H$, $ak \in H$

* if $a \in H$ then $a^{-1} \in H$

e.g. integer Z is a sub group of rational no. under addition operation

Note* G is the sub group of itself at $\{E\}$ is the Identity element also sub group of G then these are called trivial sub group.

* Sub group will have all the properties of a group.

* A subgroup $\{H\}$ of the group G is the normal sub group if $G^{-1}Hg = H$ for all $g \in G$.

* If H and K are the sub group of group G then H intersection of K $(H \cap K)$ sub grp of G

* If H and K are sub group of group G then $H \cup K$ may or may not be a sub group.

*

ORDER OF A GROUP

The order of group G is the no. of elements present in grp that is cardunitity. order of element $a \in G$ is the smallest positive and such that $a^n = E$ where E denotes the identity element and a^n denotes the product of n copies of a if no such an exit then a is said to infinite order all elements of finite grp have finite order.

Algebraic Structure with two Binary operation

1 Ring :- A ring is an algebraic structure with two binary operation $(R, +, \cdot)$.

e.g $(Z, +, \cdot)$

Condition 1

Prove $(Z, +, \cdot)$ is ring or not

$(Z, +)$ Abelian group
$(Z, \cdot)$ Semi group

Types of ring

Commutative ring ⇒ A ring $(R, +, \cdot)$ is called ~~of~~ Commutative ring if $(R, \cdot)$ is commutative also

Ring with unity ⇒ A ring $(R, +, \cdot)$ is called Ring with unity if $(R, \cdot)$ identity element.

Ring without zero division ⇒ $a \neq 0, b \neq 0$ & $ab \neq 0$

Ring with zero division ⇒ $a \neq 0, b \neq 0, ab = 0$

Integral domain ⇒ A commutative ring $(R, +, \cdot)$ is called integral domain if it is without zero divisior, or commutative ring

Field :- A commutative $(R, +, \cdot)$ is called field if every field domain has multiplicative inverse

$$2^{-1} = \frac{1}{2}$$

Every finite integral domain is field.

Boolean Algebra

$(B, \wedge, \vee, 0, 1)$

Properties

1 - Commutative properties
2 - Distributive ~~law~~
3 Identity law
4 - Complemented law
5 Idempotend law
6 - Associative law
7 Identity law
8 - Distributive law

Boolean Ring ⇒ A ring $(R, +, \cdot)$ is called a ring if all the elements are Idempotent ($\forall \in R$).
Ring

Q[Imp] Every boolean ring is commutative. Prove that.

Let R be a Boolean ring $x^2 = x \; \forall x \in R$.

Proof: R is Commutative

i.e. $ab = ba \; \forall ab \in R$

Consider - $(a+a)^2 = a+a \; \forall a \in R$

$a + a \in R$ — (1)

is $(a+a)(a+a) = a+a$

$a^2 + a^2 + a^2 + a^2 = a+a$ by eq (1)

$\cancel{a} + \cancel{a} + a + a = \cancel{a} + \cancel{a}$ by eq 1

$a + a = 0$

$\boxed{a = -a}$ — (2) $\forall a \in R$ by cancellation law of addition

Consider $(a+b)^2 = a+b \; \forall a, b \in R \quad a+b \in R$ by (1)

$(a+b)(a+b) = a+b$

$a^2 + ab + ba + b^2 = a + b$

$\cancel{a} + ab + ba + \cancel{b} = \cancel{a} + \cancel{b}$

$ab + ba = 0$

$ab = -ba$

$ab = -ba$

$ab = b(-a)$

$\boxed{ab = ba}$ — (3) $\forall a, b \in R$ using 1

Sub Algebra ⇒ Consider a boolean algebra $(B, *, +, 0, 1)$ $A \subseteq B$ then $(A, *, +, 0, 1)$ is called sub algebra or sub boolean algebra of B if A itself a boolean algebra that is A contains the elements (0, 1) & is close under multiplication, addition & complementation method also and & or operation

Duality theorem

OR ⇔ AND

NOT ⇔ NOT

XOR ⇔ XNOR

NAND ⇔ NOR

0 ⇔ 1

$\bar{A}B + A\bar{B} =$

Self dual function

$f(x, y, z) = (xy + yz + zx)$

$(x+y)(y+z)(z+x)$

$xy + xz + y + yz$

$y(x + 1 + z) + xz$

$y + yz$

$(y + xz)(z + x)$

$yz + xy + xz + xz$

$yz + xy + xz$

Self dual functions with variable :-

A function is said to be self dual if & only if its dual is equivalent to given function.

e.g $f(x,y,z) = (xy + yz + zx)$

1) find total no. of combination = $A, \bar{A}$
2) find total no of Boolean funct = $0, 1, A, \bar{A}$
3) Total no of self dual function → 2

A	f_1	f_2	f_3	f_4
0	0	1	0	1
1	0	1	1	0

For 2 variable

A	B	f_1	f_2	f_3	f_4
0	0	0	1	0	1
0	1	0	1	1	0
1	0	0	1	0	1
1	1	0	1	1	0

0 → 1 $\bar{A} + \bar{B}^{-1}$

1 → 0 $\bar{A} + B$

$A \to A$, $A \to \bar{A}$, $B \to B$, $\bar{B} \to \bar{B}$ → $A + \bar{B}$, $\bar{A} + B$

$A + \bar{B}$, $\bar{A} + B$: $A\bar{B}$, $\bar{A}B$ → $A\bar{B} + \bar{A}B$

AB, $\bar{A}\bar{B}$ → $AB + \bar{A}\bar{B}$

Q

$A \cdot (A+B)\, A$ = $A + (AB) = A$

$\overline{AB} = \bar{A} + \bar{B}$ = $\overline{A+B} = \bar{A} \cdot \bar{B}$

$(A+C)(A+B) = AB + AC$ = $AC\, AB$ = $A+B \cdot A+C$

$A+B = AB + \bar{A}B + A\bar{B}$ = AB = $A+B \cdot A+\bar{B} \cdot \bar{A}+B$

$AB + A + AB = 0$ $A+B \cdot A \cdot A+B = 1$

Propositional logic

And / conjection / join	$\wedge$	$P \wedge q$
OR / disjunction / meet	$\vee$	$P \vee q$
Negation	$\sim$	$\sim P$
Equivalent	$\leftrightarrow$	$P \leftrightarrow q$
Conditional if then	$\Rightarrow$	$P \Rightarrow q$
Bidirectional if & only if (iff)	$\Leftrightarrow$	$P \Leftrightarrow q$
NAND (NOT + AND)	$\uparrow$	$P \uparrow q$
NOR (NOT + OR)	$\downarrow$	$P \downarrow q$
XOR	$\oplus$	$P \oplus q$

P	q	$P \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

Tautologies, Contradiction, Contigency
T F T, F

Q1 $\{(P \vee \sim Q) \wedge (\sim P \vee \sim Q)\} \vee Q$

Ans

P	Q	~P	~Q	P∨~Q	~P∨~Q	P∨~Q∧~P∨~Q
T	T	F	F	T	F	F
T	F	F	T	T	T	T
F	T	T	F	F	T	F
F	F	T	T	T	T	T

∨Q
T
T
T
T

Q2 Determine wheather the following Propositions is Contraction or a tautology

$(p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee q) \wedge (\sim p \vee \sim q)$

∨ = OR

p	q	~p	~Q	p∨q	p∨~q	~p∨q	~p∨~q
T	T	F	F	T	T	T	F
T	F	F	T	T	T	F	T
F	T	T	F	T	F	T	T
F	F	T	T	F	T	T	T

Q3 Determine wheather the following is tautology or Contraction

$Q \vee (P \wedge \sim Q) \vee (\sim P \vee \sim Q)$

p	q	~p	~Q	P∧Q	P∨~Q	P∧~Q	~P∨~Q
T	T	F	F	F	T	F	F
T	F	F	T	F	T	T	T
F	T	T	F	F	T	F	T
F	F	T	T	F	F	F	T

Q4 Prove that proposition $p \vee \sim(q \wedge r)$ and $(p \vee \sim q) \vee \sim r$ is equivalent.

p	q	r	~q	~r	p∨~q	(p∨~q)∨~r
T	T	T	F	F	T	T
T	T	F	F	T	T	T
T	F	T	T	F	T	T
T	F	F	T	T	T	T
F	T	T	F	F	F	F
F	T	F	F	T	F	T
F	F	T	T	F	T	T
F	F	F	T	T	T	T

p	q	r	q∧r	~P	—
T	T	T	T	F	T
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	T	T
F	T	T	T	F	F
F	T	F	F	T	T
F	F	T	F	T	T
F	F	F	F	T	T

Hence proved

7. ~(p∨q), ~p∧~q

p	q	(p∨q)	~(p∨q)
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

p	~p	q	~q	~p∧~q
T	F	T	F	F
T	F	F	T	F
F	T	T	F	F
F	T	F	T	T

Hence proved ~(p∨q), ~p∧~q

(Conditional) Implication / if-then A→B

A	B	A→B
T	T	T
T	F	F
F	T	T
F	F	T

(Biconditional) if and only if A⇔B

A	B	A⇔B
T	T	T
T	F	F
F	T	F
F	F	T

Q [(A→B)∧A]→B tautology Prove

A	B	A→B	A→B∧A	[(A→B∧A]→B

Q Show that (A∨B)∧[(¬A)∧(¬B)]

A	B	~A	~B	A∨B	A∨B∧~A	A∨B∧~A∧B
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	F
F	F	T	T	F	F	F

Q ((A∨B)∧(¬A)→B)

A	B	~A	~B	A∨B	A∨B∧~A	~A
T	T	F	F	T	F	T
T	F	F	T	T	F	
F	T	T	F	T	T	T
F	F	T	T	F	F	

Q× Show that the proposition (p∧q) & ¬p∨¬q are logically equivalent

p	q	p∧q
T	T	F
T	F	F
F	T	F
F	F	F

p	q	~p	~q	~p∨~q
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

$\sim p(p \wedge q) \vee (\sim p \wedge q) = \sim p$

p	q	~p	~q	p∧q	p∧q	(~p∧q)	(p∧q)∨(~p∧q)	~p
T	T	F	F	T		F	T	F
T	F	F	T	T		F	T	F
F	T	T	F	F		F	F	T
F	F	T	T	F		T	T	T

Not equall to

Q Is $(p \to q) \to [(p \to q) \to q]$

p	q	p→q	(p→q)→q	(p→q)→[(p→q)→q]
T	T	T	T	T
T	F	F	T	T
F	T	T	T	T
F	F	T	F	F

Q $(\neg [p \to q) \leftrightarrow (q \to p)])$

Ans

p	q	p→q	(q→p)	[(p→q)↔(q→p)]	~
T	T	T	T	T	F
T	F	F	T	F	T
F	T	T	F	F	T
F	F	T	T	T	F

Q5

$(p \wedge q) \to r \, (p \to r) \wedge (q \to r)$

p	q	r	p∧q	p∧q	(p∧q)→r	p→r	q→r	(p→r)∧(q→r)
T	T	T	T		T	T	T	T
T	T	F	T		T	F	F	F
T	F	T	F		F	T	T	T
T	F	F	F		T	F	T	F
F	T	T	F		F	T	T	T
F	T	F	F		T	T	F	F
F	F	T	F		F	T	T	T
F	F	F	F		T	T	T	T

(p∧q)→r(p→r)∧(q→r)
T
F
T
T
T
T
T
T