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**Course: B. Tech Computer Science and Engineering (AI/MI)**  
**Subject: Engineering Mathematics,**  
**Subject Code: ETMT-109**  
**Semester: I**

**Time: 03 Hours****Max Marks: 70****Instructions to the Students:**

1. This Question paper consists of two Sections. All sections are compulsory.
2. Section A comprises 10 questions of short answer type. All questions are compulsory. Each question carries 02 marks.
3. Section B comprises 8 long answer type questions out of which students must attempt any 5. Each question carries 10 marks.
4. Do not write anything on the question paper.

Q.No.	SECTION -A (SHORT ANSWER TYPE QUESTIONS)	Marks
1. a.	Find the principal and general value of $\log(-1 + i)$ .	(2)
b.	If $\sin(\alpha + i\beta) = x + iy$ , prove that $x^2 \operatorname{cosec}^2 \alpha - y^2 \sec^2 \alpha = 1$ .	(2)
c.	Test the convergence of the following series: $\frac{1}{1 + \sqrt{2}} + \frac{2}{1 + 2\sqrt{3}} + \frac{3}{1 + 3\sqrt{4}} + \dots$	(2)
d.	Test the convergence of the infinite series $\sum u_n$ whose $n^{\text{th}}$ term is given by $u_n = \frac{n!}{n^n}$	(2)
e.	If $y = a \cos(\log x) + b \sin(\log x)$ , show that $x^2 y_{n+2} + (2n + 1)xy_{n+1} + (n^2 + 1)y_n = 0.$	(2)
f.	Find the radius of curvature of the curve $\sqrt{x} + \sqrt{y} = 1$ at the point $(\frac{1}{4}, \frac{1}{4})$ .	(2)
g.	For what value of "b" the rank of the matrix $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ b & 13 & 10 \end{bmatrix}$ is 2?	(2)
h.	Show that the system of equations $\begin{aligned} x + y + z &= -3, \\ 3x + y - 2z &= -2, \\ 2x + 4y + 7z &= 7 \end{aligned}$ are not consistent.	(2)



i. Solve  $x^2y \, dx - (x^3 + y^3)dy = 0$  (2)

j. Solve  $(D^2 - 3D + 2)y = \cosh x$  (2)

### SECTION -B (LONG ANSWER TYPE QUESTIONS)

2. If  $\tan(\theta + i\phi) = \tan\alpha + i \sec\alpha$ , then prove that (10)

$$e^{2\phi} = \pm \cot\left(\frac{\alpha}{2}\right) \text{ and } 2\theta = n\pi + \frac{\pi}{2} + \alpha.$$

3. Test the convergence of the following series: (10)

$$\sum \frac{n! x^n}{3.5.7 \dots (2n+1)}.$$

4. Define absolute and conditionally convergent series. Examine the convergence and absolute convergence of the series  $\sum \frac{(-1)^{n+1} n}{n^2+1}$ . (10)

5. If  $y = [x + \sqrt{1+x^2}]^m$ , using Leibnitz theorem, find  $(y_n)_0$ . (10)

6. Find all the asymptotes of the curve  $y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 - 1 = 0$ . (10)

7. Find the perimeter of the cardioid  $r = a(1 + \cos\theta)$ . (10)

8. Find the eigenvalues and eigenvectors of the matrix (10)

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix},$$

and also verify Cayley-Hamilton theorem.

9. Solve  $(x^2D^2 - xD - 3)y = x^2 \log x$ . (10)

==END OF PAPER==