Recursion: Python

Call Stack Construction in While Loop

A call stack with execution contexts can be constructed using a `while` loop, a `list` to represent the call stack and a `dictionary` to represent the execution contexts. This is useful to mimic the role of a call stack inside a recursive function.

Modeling Recursion as Call Stack

One can model recursion as a `call stack` with `execution contexts` using a `while` loop and a Python `list`. When the `base case` is reached, print out the call stack `list` in a LIFO (last in first out) manner until the call stack is empty.

Using another `while` loop, iterate through the call stack `list`. Pop the last item off the list and add it to a variable to store the accumulative result.

Print the result.

```
def countdown(value):
  call_stack = []
  while value > 0 :
    call_stack.append({"input":value})
    print("Call Stack:",call_stack)
    value -= 1
  print("Base Case Reached")
  while len(call_stack) != 0:
    print("Popping {} from call 
stack".format(call_stack.pop()))
    print("Call Stack:",call_stack)
countdown(4)
'''
Call Stack: [{'input': 4}]
Call Stack: [{'input': 4}, {'input': 3}]
Call Stack: [{'input': 4}, {'input': 3}, 
{'input': 2}]
Call Stack: [{'input': 4}, {'input': 3}, 
{'input': 2}, {'input': 1}]
Base Case Reached
Popping {'input': 1} from call stack
Call Stack: [{'input': 4}, {'input': 3}, 
{'input': 2}]
Popping {'input': 2} from call stack
Call Stack: [{'input': 4}, {'input': 3}]
Popping {'input': 3} from call stack
Call Stack: [{'input': 4}]
Popping {'input': 4} from call stack
Call Stack: []
'''
```

Recursion in Python

In Python, a recursive function accepts an argument and includes a condition to check whether it matches the base case. A recursive function has:

- Base Case - a condition that evaluates the current input to stop the recursion from continuing.
- Recursive Step - one or more calls to the recursive function to bring the input closer to the base case.

```
def countdown(value):
  if value <= 0:   #base case
    print("done")
  else:
    print(value)
    countdown(value-1)  #recursive case
```

Stack Overflow Error in Recursive Function

A recursive function that is called with an input that requires too many iterations will cause the call stack to get too large, resulting in a stack overflow error. In these cases, it is more appropriate to use an iterative solution. A recursive solution is only suited for a problem that does not exceed a certain number of recursive calls.

For example, `myfunction()` below throws a stack overflow error when an input of 1000 is used.

```
def myfunction(n):
  if n == 0:
    return n
  else:
    return myfunction(n-1)

myfunction(1000)  #results in stack overflow
error
```

Recursion and Nested Lists

A nested list can be traversed and flattened using a recursive function. The base case evaluates an element in the list. If it is not another list, the single element is appended to a flat list. The recursive step calls the recursive function with the nested list element as input.

```
def flatten(mylist):
  flatlist = []
  for element in mylist:
    if type(element) == list:
      flatlist += flatten(element)
    else:
      flatlist += element
  return flatlist

print(flatten(['a', ['b', ['c', ['d']], 'e'],
'f']))
# returns ['a', 'b', 'c', 'd', 'e', 'f']
```

Fibonacci Sequence

A Fibonacci sequence is a mathematical series of numbers such that each number is the sum of the two preceding numbers, starting from 0 and 1.

```
Fibonacci sequence: 0, 1, 1, 2, 3, 5, 8, 13,
21, ...
```

Fibonacci Recursion

Computing the value of a Fibonacci number can be implemented using recursion. Given an input of index N, the recursive function has two base cases – when the index is zero or 1. The recursive function returns the sum of the index minus 1 and the index minus 2.

The Big-O runtime of the Fibonacci function is O(2^N).

```
def fibonacci(n):
  if n <= 1:
    return n
  else:
    return fibonacci(n-1) + fibonacci(n-2)
```

Binary Search Tree

In Python, a binary search tree is a recursive data structure that makes sorted lists easier to search. Binary search trees:

- Reference two children at most per tree node.
- The “left” child of the tree must contain a value lesser than its parent.
- The “right” child of the tree must contain a value greater than it’s parent.

```
     5
    / \
   /   \
  3     8
 / \   / \
2   4 7   9
```

Build a Binary Search Tree

To build a binary search tree as a recursive algorithm do the following:

```
BASE CASE:
If the list is empty, return "No Child"
to show that there is no node.

RECURSIVE STEP:
1. Find the middle index of the list.
2. Create a tree node with the value of
the middle index.
3. Assign the tree node's left child to
a recursive call with the left half of
list as input.
4. Assign the tree node's right child
to a recursive call with the right half
of list as input.
5. Return the tree node.
```

```
def build_bst(my_list):
  if len(my_list) == 0:
    return "No Child"

  middle_index = len(my_list) // 2
  middle_value = my_list[middle_index]

  print("Middle index: {0}".format(middle_index))
  print("Middle value: {0}".format(middle_value))

  tree_node = {"data": middle_value}
  tree_node["left_child"] = build_bst(my_list[ : middle_index])
  tree_node["right_child"] = build_bst(my_list[middle_index + 1 : ])

  return tree_node

sorted_list = [12, 13, 14, 15, 16]
binary_search_tree = build_bst(sorted_list)
print(binary_search_tree)
```

Iterative Function for Factorials

To compute the factorial of a number, multiply all the numbers sequentially from 1 to the number.
An example of an iterative function to compute a factorial is given below.

```
def factorial(n):
  answer = 1
  while n != 0:
    answer *= n
    n -= 1
  return answer
```

Fibonacci Iterative Function

A Fibonacci sequence is made up adding two previous numbers beginning with 0 and 1. For example:

```
0, 1, 1, 2, 3, 5, 8, 13, ...
```

A function to compute the value of an index in the Fibonacci sequence, `fibonacci(index)` can be written as an iterative function.

```
def fibonacci(n):
  if n < 0:
    raise ValueError("Input 0 or greater only!")
  fiblist = [0, 1]
  for i in range(2,n+1):
    fiblist.append(fiblist[i-1] + fiblist[i-2])
  return fiblist[n]
```

Sum Digits with Recursion

Summing the digits of a number can be done recursively. For example:

```
552 = 5 + 5 + 2 = 12
```

```
def sum_digits(n):
  if n <= 9:
    return n
  last_digit = n % 10
  return sum_digits(n // 10) + last_digit

sum_digits(552) #returns 12
```

Recursively Find Minimum in List

We can use recursion to find the element with the minimum value in a list, as shown in the code below.

```
def find_min(my_list):
  if len(my_list) == 0:
    return None
  if len(my_list) == 1:
    return my_list[0]
  #compare the first 2 elements
  temp = my_list[0] if my_list[0] < my_list[1] else my_list[1]
  my_list[1] = temp
  return find_min(my_list[1:])

print(find_min([]) == None)
print(find_min([42, 17, 2, -1, 67]) == -1)
```

Palindrome in Recursion

A palindrome is a word that can be read the same both ways - forward and backward. For example, abba is a palindrome and abc is not.
The solution to determine if a word is a palindrome can be implemented as a recursive function.

```
def is_palindrome(str):
  if len(str) < 2:
    return True
  if str[0] != str[-1]:
    return False
  return is_palindrome(str[1:-1])
```

Recursive Multiplication

The multiplication of two numbers can be solved recursively as follows:

```
Base case: Check for any number that is
equal to zero.
Recursive step: Return the first number
plus a recursive call of the first
number and the second number minus one.
```

```
def multiplication(num1, num2):
  if num1 == 0 or num2 == 0:
    return 0
  return num1 + multiplication(num1, num2 - 1)
```

Recursive Depth of Binary Search Tree

A binary search tree is a data structure that builds a sorted input list into two subtrees. The left child of the subtree contains a value that is less than the root of the tree. The right child of the subtree contains a value that is greater than the root of the tree.
A recursive function can be written to determine the depth of this tree.

```
def depth(tree):
  if not tree:
    return 0
  left_depth = depth(tree["left_child"])
  right_depth = depth(tree["right_child"])
  return max(left_depth, right_depth) + 1
```