A Spatial Searching Method for Planning Under Time-Dependent Constraints for Eco-Driving in Signalized Traffic Intersection [1]

Nitesh Kumar

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1 Introduction

Traffic signals are an indispensable part of our day-to-day lives. For individual vehicles, they are also a primary cause of energy consumption because of idling, braking, and acceleration related to the traffic signals. [2] shows that 50.36 % of the fuel and 68.5 % of the of the travel time is consumed at intersections, while the intersection accounts for only 28.9 % of the total testing distance. For the fuel consumed at the intersection, 78.4 % is due to stopping and idling at the red light, and the subsequent acceleration.This problem opens up lots of opportunity for energy efficient motion planning at intersections.

This problem is well studied employing various numerical and analytical methods to prescribe optimal vehicle motion in terms of speed and acceleration while adhering to dynamic constraints and traffic rules. Previous work includes the use of Pontryagin's minimum principle (PMP) in [3] with both deterministic and stochastic traffic and in [4] signals information to optimize engine torque, brake force, and gearshift with known signal information. The solvability of these methods heavily depends on the convexity of objective and constraint functions. Alternatively, researchers have explored numerical methods for optimal or near-optimal solutions in discrete time and state space. Dynamic Programming (DP) has been applied to search for optimal motion solutions [5], with a focus on deterministic traffic signals. However, a significant source of uncertainty in motion planning at intersections is the dynamic nature of traffic signals, leading to the introduction of probabilistic models. One approach [6] involves statistical averaging of red and green light durations using a uniform distribution to define conditional probability. [7] DP methods with final point constraints have been leveraged to generate optimal motion planning with a spatial trajectory formulation [8], introducing stochastic variables to capture feasible passing times. Integrating eco-driving control with an [9] adaptive cruise controller has been explored to enhance safety by avoiding collisions and traffic violations.

Inspired from the above work, [1] made two key observation on the eco-driving problem formulation: (i) Optimization methods suffer from the curse of dimensionality, so any approach to reduce dimensionality will be desirable; (ii) the component of SPaT that has the highest uncertainty is only the phase offset (as considered in other studies such as [7]), but this uncertainty is removed as soon as the first signal change is observed, which can be detected by several types of sensors already available in the current production vehicle. Based on these observation [1] proposed an optimal eco-driving solution.

[1] considered a single vehicle at a single traffic intersection with known SPaT, but unknown phase offset. Similar to [7], a spatial state formulation is applied to their proposed solution. Key topics discussed by [1] are:

- Sampled Space Optimal Planning (SSOP): Leveraging the spatial formulation, and by imposing a simple constraint of non-negative velocity, and using piece-wise constant accelerations between samples, we can remove time as an independent state of the system during optimization, helping to reduce the system dimensionality.We detail the consequent reduction in computational complexity

- Stochastic State Space Planning (SSSP): Use of a MarkovDec ision Process (MDP) to model the belief of the traffic signal, and explicit consideration of the transition of the system from stochastic to deterministic in the optimization solution.

- A data-driven motion planning, which enables table lookup for control commands given initial velocity and signal in both SSOP and SSSP algorithms to reduce the online computation.

In this letter, the solution approach for SSOP and SSSP as suggested by [1] is reiterated, while the optimization technique used here is value iteration instead of the Depth First search method used in [1]. This letter is organized in the following manner. Section 2 describes the mathematical formulation of the problem satement. Section 3 details the proposed planning under full and partial knowledge of traffic signals. Section 4 presents the simulation results including the trajectory profiles of different algorithms and fuel cost statics, as well as the associated runtime costs, and the conclusion is drawn in Section 5.

2 Problem Formulation

The classical kinematic model for the vehicle is considered here, treating the acceleration as a controllable input to the vehicle. This treatment can be readily extended to include additional dynamics, such as the powertrain dynamics, by mapping applied torque to the acceleration.First, we illustrate the two possible formulations - namely spatial and temporal, in a continuous system setting: x and v represent the distance and velocity of the vehicle at a given time t. Now state dynamics can be parametrized as $x(\theta)$,$y(\theta)$,$t(\theta)$, where θ is the parametric value.

2.1 Temporal Formulation

In the temporal case, Δt will be used as a parametric value and the discretized state transition can be written as follows:

$$x_{k+1} = x_k + v_k \Delta t + \frac{1}{2} u_k \Delta t^2 \tag{1}$$

$$v_{k+1} = v_k + u_k \Delta t \tag{2}$$

However, we can see that our system transition would need the knowledge of two states, x_k and y_k. However, if we try to formalize our problem in spatial form, it has certain advantages in terms of state transition.

2.2 Spatial Formulation

In the spatial representation of dynamics, Δx will be used as a parametric value and the discretized state transition can be written as follows:

$$v_k = \sqrt{v_{k-1}^2 + 2u_k \Delta x} \tag{3}$$

$$\Delta t_k = \begin{cases} \frac{\sqrt{v_k^2 + 2u_k \Delta x} - v_k}{u_k} & \text{if } u_k \neq 0, \\ \frac{\Delta x}{v_k} & \text{if } u_k = 0 \end{cases} \tag{4}$$

The primary advantage of this representation is that it allows specification of position-based constraints, such as at a traffic intersection. It is to be noted that v_k can be obtained from (3) and Δt_k in (4) doesn't depend on Δt_{k-1} but only on v_k. Effectively, system dynamics depends only on one state v_k.

2.3 Traffic signal Dynamics

A common representation of traffic signal dynamics is the fixed green, yellow and red SPaT $\{T_g, T_y, T_r\}$ with offset ϕ_0. We define the set of traffic signals as $L = \{G, Y, R\}$, and the residual time of the traffic period by

$$t_{res} = (t + \phi_0) mod (T_g + T_y + T_r) \tag{5}$$

then the traffic light function $l : T \to L$ that maps time to traffic lights is given by

$$l(t) = \begin{cases} G & 0 \leq t_{res} \leq T_g, \\ Y & T_g < t_{res} \leq T_g + T_y, \\ R & T_g + T_y < t_{res} \leq T_g + T_y + T_r, \end{cases} \tag{6}$$

With the above, we can capture the constraint that the vehicle should stop at the traffic light as below:

$$(x_k = L) \wedge (l(t_k) = R) \to v_k = 0 \tag{7}$$

2.4 Fuel Consumption

In this letter the VT-CPFM-1 model form [10] is applied for fuel cost calculation, which is

$$FC(u(t), v(t)) = \begin{cases} \alpha_0 + \alpha_1 P(t) + \alpha_2 P(t)^2 & P >= 0 \\ \alpha_0 & P < 0 \end{cases} \tag{8}$$

Here $FC(u(t), v(t))$ is the instantaneous fuel consumption rate (Liter/s) at time t with velocity $v(t)$ and acceleration $u(t)$, $P(t)$ is the power of the vehicle defined in VT-CPFM-1 model [10].

For a Toyota Camry 2016 [11]:$\alpha_0 = 6.289e - 04, \alpha_1 = 2.676e - 05, \alpha 2 = 1e - 06$, with fuel density $748.9kg/m^3$. The fuel cost $c(v_k, u_k)$ associated with the movement from x_k to x_{k+1}, starting from v_k under the constant acceleration u_k can be obtained by integrating $FC(u(t), v(t))$ as below:

$$c(v_k, u_k) = \int_{\Delta t=0}^{\Delta t_k} FC(u_k, v_k + u_k \Delta t) d\Delta t \tag{9}$$

2.5 Eco Driving at Traffic Signal-Problem Statement

The primary objective of eco-driving is to navigate the traffic signal such that the fuel consumed is minimized, while traffic rules are not violated. In our sampled discrete spatial formulation, the problem to address can be expressed mathematically as the development of a control strategy, i.e. a sequence of constant acceleration commands $u = u_0, \ldots, u_m$, such that expectation of the total fuel costs are minimized:

$$\begin{aligned} & \min_u \sum_{k=0}^{m-1} E[c(v_k, u_k)] \\ & \text{s.t. } v_k = \sqrt{v_{k-1}^2 + 2u_k \Delta x}; \\ & u_k \in [u_{min}, u_{max}]; \\ & \text{if } l(t_{m-1}) = R, v_{m-1} = 0; \text{ else } v_{m-1} \in \mathcal{V} - \{0\}; \end{aligned} \tag{10}$$

3 Solution Approach

This section outlines the solution approach for the problem formulated in Section 2, considering two scenarios: (i) when both the Signal Phase and Timing (SPaT) information, denoted as T_g, T_y, T_r, and the initial offset ϕ_0 are known to the controller; (ii) when only the SPaT information is known, and the offset ϕ_0 is uncertain. The solutions for these scenarios are discussed separately, with the solution for partial knowledge building upon the approach for full knowledge.

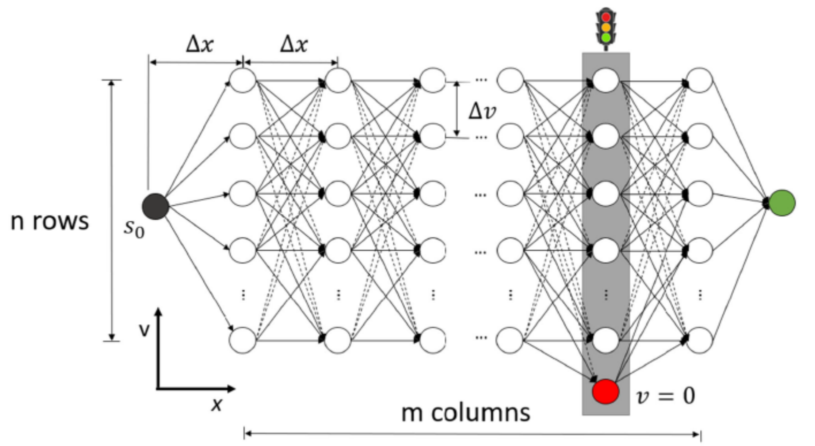


Figure 1: Graph representation of planning with traffic signal. Black vertex is the initial state s_0, k^{th} column of vertices represents the states of the same position $x_k = k\Delta x$ but different velocities $v \in \mathcal{V}$. The second right column includes the states at intersection, and the red vertex is the only possible state of zero velocity due to traffic signal, with other states with zero velocity omitted in this graph. The green vertex is the termination of the problem which has no actual value. The solid edges are valid state transitions, and dashed edges between vertices are inapplicable state transitions. [1]

3.1 SSOP:Plan under Full Traffic Signal Knowledge

In contrast to traditional shortest path algorithms like Dijkstra's and Depth First Search employed in [1], the proposed approach utilizes a value iteration-based method to determine the optimal set of accelerations that minimize fuel costs under fully known traffic signal dynamics. This method yields a lookup table of optimal actions for various starting and ending velocities, considering all possible initial offsets (ϕ_0). An additional advantage is the ease with which this approach can be extended to address the Stochastic State Space Planning (SSSP) problem by framing it as a Markov Decision Process (MDP), incorporating all potential traffic signal scenarios at each spatial instance.

Applying Bellman's optimality principle, the overall problem is divided into two distinct subproblems: (i) Eco-driving from the starting point to the traffic signal intersection, and (ii) Eco-driving from the traffic signal point to the destination. Each subproblem is independently solved, generating two lookup tables of optimal actions. The latter subproblem is computationally more straightforward, involving backpropagation from the destination to the traffic signal intersection. Since there are no traffic signal constraints beyond the traffic signal, the iterative process at each Δx interval requires only the information about the velocity (v_k) to determine the optimal acceleration (u_k) leading to minimal fuel cost. The state equations for this subproblem are defined accordingly.

$$s_{k+1} = f(s_k, u_k) = \begin{bmatrix} x_{k+1} \\ v_{k+1} \\ t_{k+1} \end{bmatrix} = \begin{bmatrix} x_k + \Delta x \\ \sqrt{v_k^2 + 2u_k \Delta x} \\ \frac{\sqrt{v_k^2+2u_k\Delta x} - v_k}{u_k} \end{bmatrix} \tag{11}$$

The immediate cost $C(s_k, u_k)$ at each Δx is a function of the state s_k and input u_k, where s_k is a function of Δx and v_k. Consequently, in every interval of Δx during value iteration, the iteration process is limited to the state space of v_k and the action space of u_k to minimize the total cost.

To address the (i) problem, additional information is required to adhere to traffic signal constraints. Introducing a new variable, t_{res} (6), into the state space becomes essential. This variable captures the information regarding the traffic signal at each spatial interval Δx. The updated state space is reformulated as follows:

$$s_{k+1} = f(s_k, u_k) = \begin{bmatrix} x_{k+1} \\ v_{k+1} \\ t_{k+1} \\ t_{res}^{k+1} \end{bmatrix} = \begin{bmatrix} x_k + \Delta x \\ \sqrt{v_k^2 + 2u_k \Delta x} \\ \frac{\sqrt{v_k^2+2u_k\Delta x} - v_k}{u_k} \\ (t_{res}^k + \Delta t_k)\%(T_g + T_y + T_r) \end{bmatrix} \tag{12}$$

Utilizing the information on traffic signal dynamics in (6), it is possible to estimate the state of the traffic signal at Δx by considering t_{res}^k. If $(x_k = L) \wedge (l(t_k) = R)$, the immediate cost is set to infinite; otherwise, it can be estimated using the formulation in 8. In this scenario, iteration is required across the state space of (v_k, t_{res}^k) to determine the optimal input u_k that minimizes the overall cost. While this increases the complexity of the optimization problem, it results in a lookup table with solutions for all possible ϕ_0 at the starting position. This stands in contrast to the DFS method, which provides the shortest path for a specific ϕ_0 at a given time. Additionally, this formulation proves advantageous when tackling the SSSP problem. The key benefit of employing value iteration over DFS lies in the ability to divide the problem into two subproblems, with only one of them suffering from the curse of dimensionality. In return, value iteration produces a lookup table that covers all phase offset values of the traffic signal.

3.2 SSSP: Plan Under Partial Traffic Signal Knowledge

The methodology outlined in the preceding section relies on knowledge of the timing offset ϕ_0, which is equivalent to knowing in advance when the traffic signal will change. By leveraging GPS localization [12], it becomes feasible to obtain signal information T_g, T_y, T_r. This section extends our approach to a stochastic scenario where the timing information for the traffic signal is partially unknown until the first detection of a light change. We assume that the SPaT information T_g, T_y, T_r is known, but the phase offset ϕ_0 is unknown. In SSSP, there is no information regarding the phase offset value of the traffic signal. However, at any given instant, we can observe the traffic signal without

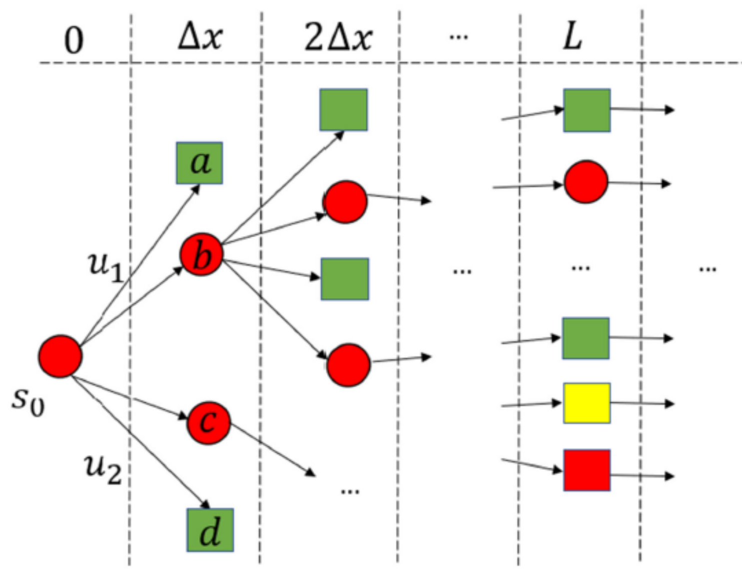


Figure 2: A tree representation of MDP. States are sorted in the columns by position s. Color of a state represents the signal. States has 2 shapes: states with round shape are in S_1, states with square shape are in S_2, intersection locates at $x = L$. [1]

uncertainty. Once a change is observed, we can estimate the phase offset using partial knowledge of traffic signal dynamics. In [1], a solution is proposed where the problem is treated as stochastic until it observes the first change in the light. After that, the problem switches back to SSOP (Figure3).

In our proposed approach, we introduce a novel strategy that eliminates the need to revert to SSOP after observing a signal change. Instead, we enhance the state representation by incorporating information on how the signal has been observed. The longer the signal has been observed, the higher the probability that the signal will change in the next spatial interval Δx. Additionally, we introduce a binary decision variable D_k in the state space, indicating whether we have full knowledge of the traffic signal or if there is still uncertainty in traffic signal dynamics. Two additional variables, t_{obs} and l, are introduced to capture the duration of observing the same signal and the color of the observed signal, respectively. Once a change is observed in the variable l, the signal's t_{obs} can be treated as the phase of the signal l.

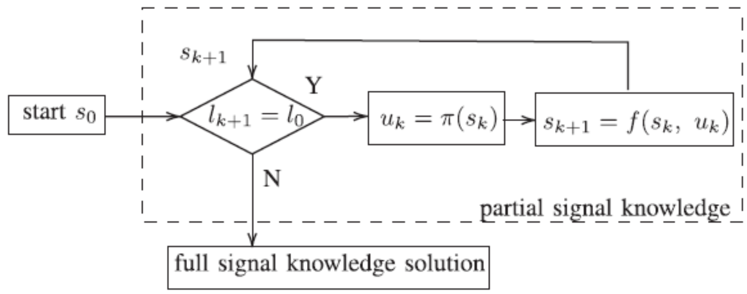


Figure 3: The temporal logic diagram for planning under partial traffic light knowledge [1]

$$s_k = \begin{bmatrix} x_k \\ v_k \\ t_k \\ t^k_{obs} \\ D_k \end{bmatrix} \tag{13}$$

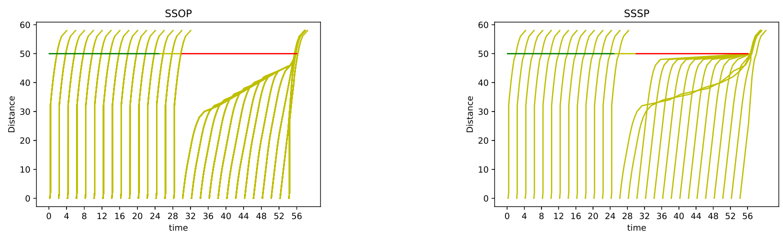
In the proposed approach, the system accounts for the stochastic nature of traffic signal dynamics at the beginning of the problem. This acknowledgment reflects the inherent uncertainty regarding when the traffic signal will change. As the vehicle progresses in space and time, continuously observing the traffic signal, the system gains more information. The duration of observing the same signal, denoted as t_{obs}, becomes a crucial factor.

The probability of the next signal state, l_{k+1}, given the current state l_k and the entire system state s_k, is expressed as $P(l_{k+1}|l_k, s_k)$.

$$P(l_{k+1} = l_k, s_k) = \begin{cases} \frac{T_{l_k} - t^k_{obs} - \Delta t_k}{T_{l_k} - t^k_{obs}} & \Delta t_k < T_{l_k} - t^k_{obs} \\ 0 & else \end{cases} \tag{14}$$
$$P(l_{k+1}! = l_k, s_k) = 1 - P(l_{k+1} = l_k, s_k)$$

This probability calculation considers the time the system has been observing the current signal state and the color of the observed signal. The longer the observation duration (t_{obs}), the higher the probability that the signal will change in the next spatial interval Δx. This probabilistic reasoning allows the system to adapt to the evolving certainty about the traffic signal dynamics.

In summary, the system starts with a stochastic viewpoint, reflecting the initial uncertainty, and gradually transitions to a more deterministic perspective as it continuously observes and learns from the traffic signal behavior over time and space.



(a) SSOP

(b) SSSP

Figure 4: Plots for SSOP and SSSP for different phase offset ϕ_0

4 Simulation Results

The simulation scenario consists of one vehicle passing through an intersection with a traffic signal located at L=50 m, SPaTs $\{T_g, T_y, T_r\}$ tested $\{25, 5, 26\}$. The initial position of the vehicle is $x = 0$, and initial velocity considered is $v_0 = 5m/s$. In SSOP and SSSP, other parameters include $\Delta x = 10m$, $\Delta v = 1m/s$, $v_{min} = 0m/s$, and $v_{max} = 22m/s \approx 50mph$; $u_{min} = -5m/s^2$ and $u_{max} = 8m/s^2$.

Fig 4a represents the results of SSOP algorithm for different values of initial ϕ_0. In this simulation ϕ_0 varies from $[0\ 56]sec$ at a interval of $2sec$. Time at $x = 0$ represents the ϕ_0. The simulation results indicate that as the initial phase offset increases, our vehicle gradually decelerates, ensuring that it arrives at the traffic light just as it turns green. This strategy appears to be optimal, considering that the total cost comprises idling cost and costs associated with acceleration and velocity. It is evident that the idling cost component will persist until the traffic light transitions back to green. Consequently, rather than accelerating, expending fuel, and subsequently waiting at the traffic signal, the more efficient approach is to decelerate. This helps conserve energy that would otherwise be wasted in power consumption due to acceleration and velocity.

While in case of SSSP Fig 4b,the green signal offset appears similar in both scenarios. However, when the initial phase offset falls within the yellow light region, SSSP considers both possibilities – the light turning red or remaining yellow. Consequently, it takes more conservative actions compared to the SSOP algorithm, which operates with complete knowledge of the traffic signal. As the traffic signal shifts from yellow to red, SSSP adapts its behavior to emulate SSOP, optimizing actions to align with the green signal. On the other hand, when the initial offset is in the red region, SSSP accounts for the potential transition to a green signal. This leads to more optimistic and assertive steps compared to SSOP, which, despite having full knowledge of traffic signal dynamics, decelerates when approaching the signal, anticipating a return to green.

These results highlight the effectiveness of both SSOP and SSSP algorithms in optimizing fuel consumption and adherence to traffic rules in dynamic traffic signal scenarios.

5 Conclusion

In conclusion, the presented research investigates an optimal eco-driving solution for vehicles at signalized traffic intersections under time-dependent constraints. The study leverages a spatial searching method and introduces two key approaches: Sampled Space Optimal Planning (SSOP) and Stochastic State Space Planning (SSSP).

The SSOP algorithm demonstrates its effectiveness by optimizing vehicle acceleration and velocity while considering traffic signal dynamics. Simulation results reveal that, as the initial phase offset increases, the vehicle strategically decelerates to reach the traffic light just as it turns green. This approach minimizes fuel consumption by avoiding unnecessary acceleration and waiting at the traffic signal.

On the other hand, the SSSP algorithm introduces a stochastic element to address partial knowledge of the traffic signal, specifically the unknown initial phase offset. The algorithm considers both conservative and optimistic steps based on the uncertainty in signal transitions. The simulation results confirm that SSSP adapts its behavior depending on the initial phase offset, demonstrating a balance between conservativeness and optimism.

The comparison between SSOP and SSSP shows that SSSP, with its consideration of partial knowledge, tends to take more conservative actions initially, resembling a cautious approach until the first signal change is observed. Once the change is detected, SSSP adjusts its behavior, resembling SSOP and optimizing actions for the green signal.

The proposed methodology and simulation results provide valuable insights into the trade-offs between conservativeness and optimism in eco-driving strategies at signalized intersections. The research contributes to the ongoing efforts to develop energy-efficient motion planning solutions that account for dynamic traffic signal conditions. Future work may explore real-world implementations and consider additional factors such as traffic density and communication between vehicles and infrastructure for further optimization.

Some other avenues that uses analytical approach can be explored are like MPC and LQR based approach to solve this problem. Only limitation to that is both LQR and MPC require a quadratic cost function. However, cost function is considered here in non-quadratic. If we can come up quadratic cost function approximation to the cost function used in this problem, then this problem can solved much faster

Exploring alternative analytical approaches, such as Model Predictive Control (MPC) and Linear Quadratic Regulator (LQR), could offer additional avenues for solving the eco-driving problem at signalized intersections. It's worth noting that both LQR and MPC traditionally operate with quadratic cost functions. However, a potential limitation in this context is that the cost function employed here is non-quadratic in nature.

One potential avenue for further investigation lies in developing a quadratic approximation for the non-quadratic cost function used in this problem. By achieving a suitable quadratic approximation, it may become possible to apply faster and more efficient optimization techniques, such as LQR and MPC, to enhance the speed of solving the eco-driving problem while incorporating the specific constraints and dynamics involved in signalized traffic intersections.

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