IE 532 - Analysis of Network Data

Project Report

Edge Disjoint Minimum Cost Multicommodity Flow Network

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Abstract:

Edge-disjoint minimum cost multi commodity flow problem is an interesting extension of minimum cost MCNF problem where every edge in the network path can be traversed once. This extension has widescale application especially in the transportation of radioactive material shipments.

Given a directed graph G, that represents a Road Network, k sets of random nodes suppliers and terminals and k commodities, our main objective is to find the Edge-disjoint minimum cost multi commodity flow in the network.

Introduction

Finding optimal flow configurations in transport networks is an important problem in many real-world applications. For instance, routing data packets in communication networks, or passengers in urban transportation networks, requires multicommodity approaches where mass of different types interacts in a shared infrastructure, contributing to minimize one unique cost.

Multicommodity network flow (MCNF) is an important and complicated problem in Operations Research which is recognized and used extensively in transportation and communication networks. MCNF is described over a network there is a requirement to transport/transfer more than 2 commodities from specific supply points to destination points while respecting the capacity of the network.

Though MCNF may look like a combination of multiple single commodity flow, it is actually highly complex than solving individual single commodity problems because of the interaction between commodities. While it is complex, MCNF problem can be formulated as a linear programming model and can be solved in polynomial time

A small example of a multicommodity problem is shown below from a paper published *On solving Multi-Commodity Flow Problems: An experimental evaluation* by Weibin Dai et al.(2015)

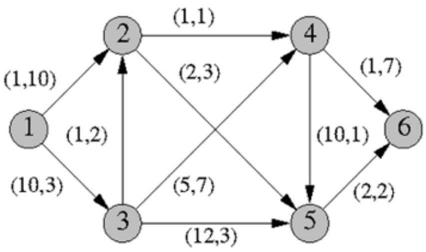


Figure 1: A small example

Table 2: Commodities of the example

Commodity k	$O(k)$	$D(k)$	Travel demand d^k
1	1	4	1
2	1	5	3
3	1	6	2

Table 3: Solution I of a small example

Edges	Commodity 1	Commodity 2	Commodity 3
(1,2)	0	3	1
(1,3)	1	0	1
(3,2)	0	0	0
(2,4)	0	0	1
(2,5)	0	3	0
(3,4)	1	0	1
(3,5)	0	0	0
(4,5)	0	0	0
(4,6)	0	0	2
(5,6)	0	0	0

Table 4: Solution II of a small example

Commodity	Path	Flow
Commodity 1	$1 \to 3 \to 4$	1
Commodity 2	$1 \to 2 \to 5$	3
Commodity 3	$1 \to 2 \to 4 \to 6$	1
	$1 \to 3 \to 4 \to 6$	1

A minimum cost multicommodity problem deals with solving the multicommodity demand and supply constraints with a minimum cost. With a single pair of origin and destination a minimum cost problem can be solved with considerable ease and as the number of commodity increases, the complexity goes up.

An **edge disjoint multicommodity network flow problem** is another extension to MCNF where network edges can only be traversed once during the flow. This problem occurs in many contexts where different items share the same resource, e.g., communication networks, transportation, and scheduling problems

One of the most important applications of edge disjoint MCNF is in the transportation of radioactive material, and nuclear wastes. The major impact will be the increase in the radiation level on a particular route as the same route will be used again and again which will lead to a considerable increase of radiations on the same area. As the intensity of radiations play a major role on its impact on human beings and nature, avoiding the same route will more than once will solve this issue or at least decrease the risks sue to these harmful radiations. In this project, we will be focusing on edge disjoint, multicommodity, minimum cost network flow of road network in West Virginia.

Problem Statement

The objective of this project is to formulate a linear program for solving an edge disjoint MCNF problem. The selected problem involves sending shipments carrying radioactive materials from multiple origin points to the corresponding destinations. The objective is to find edge disjoint paths while solving the MCNF at the minimum cost possible. Road network of West Virginia is the graph network used for this purpose.

Formulation

To solve MCFP, two necessary constraints must be considered. The first one is the travel demand constraint. It means that all the commodities need to be transported to their destinations. The other one is the edge capacity constraint. It means that the flows on each edge cannot exceed the flow capacity. Since we are dealing with edge disjoint flow, capacities of all edges are 1. This means that each edge can carry only one unit of flow. The first constraint is essentially the sum of a set of single-commodity flow problems. However, the second constraint needs to consider all the commodities together and it causes the interaction between commodities.

G(V,E) is a directed network. Here V and E are the set of nodes and edges and they have a size of n and m, respectively. For each edge ij, there is a cost cij and capacity uij. We need to transport K kinds of commodities from their origin nodes to destination nodes. s and t are used to represent origin and destination node of commodity k. In addition, d^k is the travel demand of commodity k. We need to find an optimal flow assignment with minimum cost, satisfying the travel demand and edge disjoint constraints for this problem. Thus, the MCFN problem can be formulated as following:

$$\min Z^* (x) = \sum_{k \in \mathbf{K}} \sum_{(i,j) \in A}^{n} c_{ij}\, x_{ij}^k \qquad \text{Objective Function}$$

$$\sum_{(i,j) \in \mathbf{A}} x_{ij}^k - \sum_{(i,j) \in \mathbf{A}} x_{ji}^k = b_i^k \qquad \forall i \in \mathbf{N}, \quad \forall k \in \mathbf{K}. \ \textbf{–Constraint 1}$$

$$\sum_{(i,j) \in \mathbf{A}} x_{ij}^k \leq u_{ij} \ \forall\, (i.j) \in \mathbf{A} \qquad \textbf{-Constraint 2}$$

$$x_{ij}^k = \{0,1\} \quad i \in \mathbf{A}, \forall\, k \in \mathbf{K} \qquad \textbf{- Variable restriction}$$

where, $b_i^k = d^k \ if\ i = s_k$,
$b_i^k = -d^k \ if\ i = t_k$,and
$b_i^k = 0 \ if\ i \in \mathbf{N} \{s_k , t_k\}$

The objective function aims to minimize the total transportation cost. Constraint 1 is the supply/demand constraint. Constraint 2 is the capacity constraint . Capacity of every edge is set to 1 making this an edge disjoint problem and variable restriction 3 decides whether flow of commodity k happens on edge i,j.

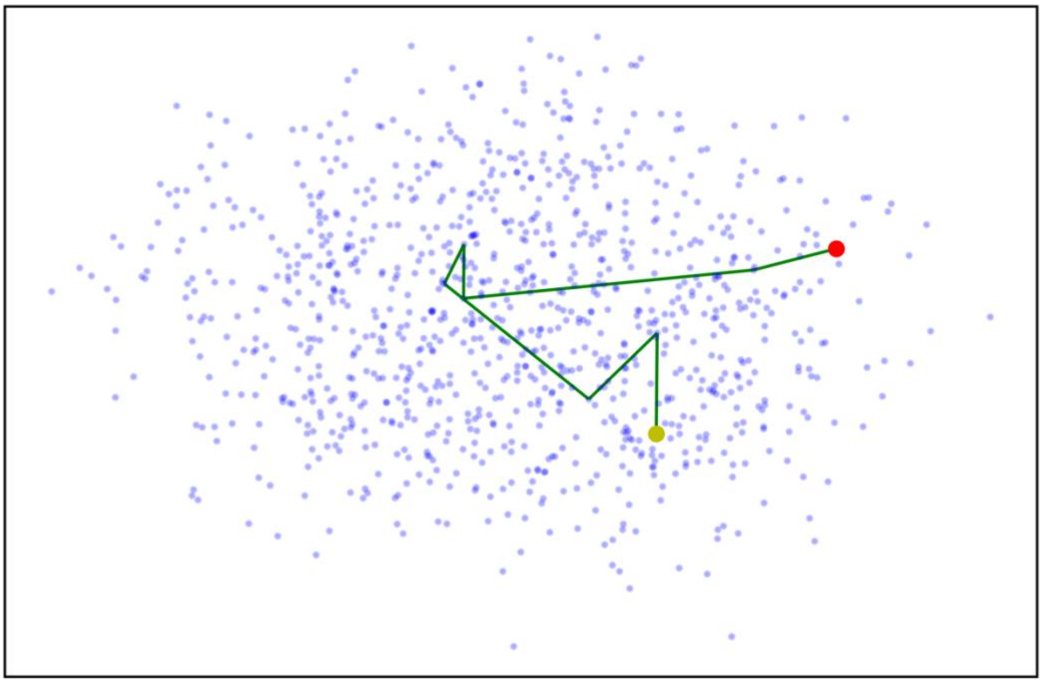
Methodology & Output

This project is done with Gurobi and NetworkX packages on python. Road network node data is collected as a txt file and a Graph is created with NetworkX. Gurobi optimizer is used to optimize the flow to get an edge disjoint MCNF. The formulation used in Gurobi is same as shown in earlier section.

We test the network for single commodity,2 commodity, 3 commodity and 4 commodity minimum cost flows with edge disjoint constraint.

Single Commodity Network Flow

Before optimizing a multicommodity network flows, we will have a look at the easy, single commodity single shipment network flow problem. We can solve this problem with just the NetworkX code of min_cut or max_flow. But we will try here, to apply the formulation developed above in the Gurobi optimizer and find the flow: The network for a single commodity min cost flow is shown below:

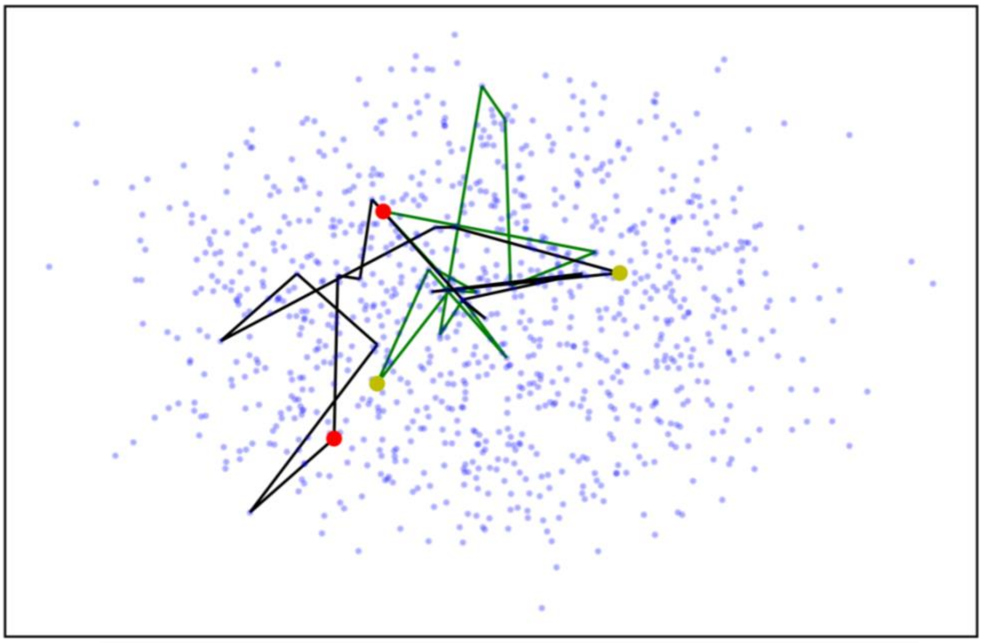


Minimum cost flow for a single commodity.

2 Commodity MCNF

2 origin and 2 destination nodes are selected at random from the network. Each origin sends 2 shipments to a fixed destination. As discussed earlier, the objective is to find a minimum flow path where, the shipments reach the destinations without using the same edge multiple times (edge disjoint constraint).

The formulation provided above was implemented with Gurobi package. A graph was plotted showing the path of the 2 commodities separately.

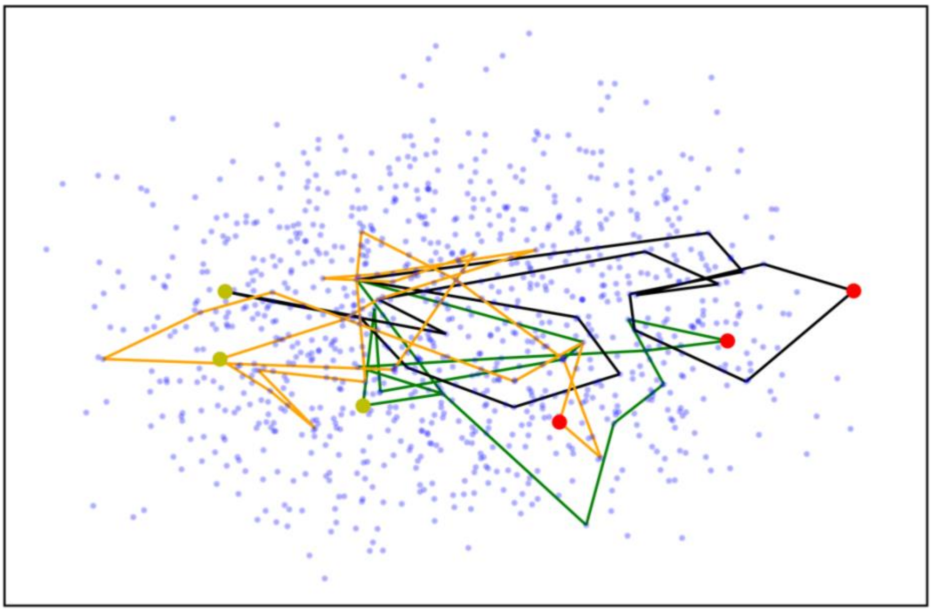


Edge Disjoint minimum cost flow for 2 commodities.

The red nodes indicate the destination point and yellow nodes indicate origin point.

3 Commodity MCNF

Like the above case, the optimization is done, this time with 3 commodities with 3 origins sending 3 shipments each to 3 destination point.



Red – Destination points Yellow- Origins

Edge Disjoint minimum cost flow for 3 commodities

4 Commodity MCNF

The optimization of a 4 commodity network flow gave a non-feasible solution which means that edge disjoint flow of 4 commodities (4 origins and 4 destinations) with 2 shipments each is not possible in the road network under study.

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Explored 1 nodes (4122 simplex iterations) in 0.24 seconds (0.28 work units)
Thread count was 8 (of 8 available processors)

Solution count 0

Model is infeasible
Best objective -, best bound -, gap -
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Infeasibility Error

Comparison of outputs

As we saw in the output graphs, the network grew bigger as the number of commodities increased. This, of course, was obvious as more edges become unavailable for the added shipments because of the edge disjoint constraint forcing them to take longer routes to reach the destination.

In the 3rd iteration when number of commodities was set to 4, there was no feasible edge disjoint MCNF in the network causing one or more of the shipments to not find an untraversed edge to reach the destination.

Conclusion

Edge disjoint MCNF is an extension to the regular MCNF that makes the problem *NP*-Hard. The network problem given was successfully optimized by Gurobi Optimizer using the formulation developed for min cost MCNF by costraining capacity value of edges to 1. Infeasibility at k+1- level of the number of commodities explain the network can handle only upto k commodities without violating edge disjoint constraint. In the case under study, the network could only handle 3 commodities at a time.

Edge disjoint MCNF approach is a powerful operational research approach to tackle and solve many complicated problems, especially in transportation and telecommunication contexts. The application of this can be extended to many other areas like production networks also. It is a powerful operational research approach to tackle and solve many complicated problems, especially in transportation and telecommunication contexts.

References –

 _ The multicommodity network flow problem: state of the art classification, applications, and solution methods *by* Khodakaram Salimifard & Sara Bigharaz (2022)

 _ On solving Multi-Commodity Flow Problems: An experimental evaluation *by* Weibin Daia,b,c, Jun Zhanga,b,c, Xiaoqian Suna,b,c,* (2015)

 _ Multicommodity Flows and Edge-Disjoint Paths by Bernhard Korte & Jens Vygen (2018)