MATHEMATICS

SECTION-A

Question numbers 1 to 6 carry 1 mark each.

1. Write the value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$.

2. If A=$\begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$ and B=$\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ and BA=(bij),find b_{21}+$b_{32}i$.

3. Write the number of all the possible matrices of order 2×3 with each entry 1 or 2.

4. Write the coordinates of the point which is the reflection of the point (α, β, γ) in the XZ-plane.

5. Find the position vector of the point which divides the join of points with position vectors $\vec{a} + 3\vec{b} and \vec{a} - \vec{b}$ in the ratio 1:3.

6. If $|\vec{a}|$=4,$|\vec{b}|$=3 and $\vec{a}.\vec{b}$=$6\sqrt{3}$,then find the value of $|\vec{a} \times \vec{b}|$.

SECTION-B

Question numbers 7 to 19 carry 4 marks each.

7. Solve for $x : tan^{-1}\left(\frac{2-x}{2+x}\right)$=$\left(\frac{1}{2}\right) tan^{-1}\left(\frac{x}{2}\right)$,$x$>0.

   OR

   Prove that $2sin^{-1}\left(\frac{3}{5}\right) - tan^{-1}\left(\frac{17}{31}\right) = \left(\frac{\pi}{4}\right)$.

8. On her birthday Seema decided to donate some money to chldren of an rophanage home.If there was 8 children less,everyone would have go ₹10 more.However,if there were 16 children more,every one would have got₹10 less.Using matrix method,find the numbers of children and the amount distributed by Seema.What values are refleced by Seema's decision?

9. If x=e^{cos2t} and y=e^{sin2t},prove that $\frac{dy}{dx}$=$\frac{-y}{x}$ $\frac{logx}{logy}$.
   Verify mean value theorem for the function $f(x) = 2sinx + sin2x$ on $[o, x]$.

10. Find the eqation of the tangent line to the curve y=$\sqrt{5}x - 3$−5,when parallel to the line $4x - 2y = 5$=0.

11. Show that the function f given by fx=$\begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & if x \neq 0 \\ \text{-1}, & if x = 0 \end{cases}$

is discontinuous at x=0.

12. Evaluate: $\int_1^5$ $\{|x-1|+|x-2|+|x-3|\}$dx.

OR

Evaluate: $\int_0^{\pi} \frac{xsinx}{1+3cos^2x}dx$.

13. Find: $\int \frac{2x+1}{(x^2)(x^2)}$dx.

14. Find $\int(3x+5)\sqrt{5x+4x-2x^2}$dx.

15. x$\frac{dy}{dx}+y-x+xycotx=0$;$x \neq 0$.

16. Solve the different equation:$(x^2+3xy+y^2)dx$ $-x^2dy=0$ given that $y=0$,when $x=1$.

17. Find the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$ if $\vec{a}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{b}=3\hat{i}+\hat{j}-2\hat{k}$,and hence find a vector perpendicular to both $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$.

18. Show that the line $\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}$ and $\frac{x-4}{2}=\frac{y}{0}=\frac{z+1}{3}$ intersect.Find their point of intersection.

19. A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, given that at least one girl must be there in the committee.

OR

A random variable X has the following probability distribution:

X	0	1	2	3	4	5	6
P(X)	c	2c	2c	3c	c^2	$2c^2$	$7c^2$+c

Find the value of C and also calculate mean of the distribution.

SECTION C

Question numbers 20 to 26 carry 6 marks each.

20. Show that the relation R defined by (a,b)R(c,d)$\Rightarrow a+d=b+c$ on the $A \times A$,where $A=\{1,2,3,...10\}$ is an equivalence class $[(3,4)]; a,b,c,d \epsilon A$.

21. Solve for x: $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix}$=0,
using properties of determinants.

OR

Using elementary row operation find the inverse of matrix X A= $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$
and hence solve the following system of equations $3x - 3y + 4z = 21, 2x - 3y + 4z = 20, -y + z = 5$.

22. Show that height of the cylinder of greatest volume which can be in scribed in aright circular cone of height h and semi-vertical angle α is one third that of and greatest volume of cylinder is $\frac{4}{27}\pi \tan^2 \alpha$.

OR

Find the intervals in which the function $f(x) = \frac{4sinx}{2+cosx} - x; 0 \leqslant x \leqslant 2\pi$ is strictly increasing or strictly decreasing.

23. Using integration,find the area of the triangle formed by inegative x-axis and tangent and normal to the circle $x^2 + y^2 = 9$ at $(-1, 2\sqrt{2})$.

24. Find the coordinates of the foot of perpendicular distance frm the point $P(4, 3, 2)$ to the plane $x + 2y + 3z = 2$.Also find the image of P in the plane.

25. A, B and C throw a pair of dice in that order alternately till one of them gets a total of 9 and wins the game. Find their respective probabilities of winning, if A starts first.

26. A company manufactures two types of cardigans type A and type B. It costs ₹360 to make a type A cardigan and ₹120 to make a type B cardigan. The company can make at most 300 cardigans and spend at most ₹$72,000$ a day. The number of cardigans of type B cannot exceed the number of cardigans of type A by more than ₹200. The company makes a profit of ₹100 for each cardigan of type A and ₹50 for every cardigan of type B.

    Formulate this problem as a linear programming problem to maximise the profit to the company. Solve it graphically and find maximum profit.