MATHEMATICS

QUESTIONS

1. Write the value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$.

2. If $\mathbf{A} = \begin{pmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{pmatrix}$and $\mathbf{B}$=$\begin{pmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{pmatrix}$ and $BA = (bij)$,find b_{21}+$b_{32}i$.

3. Write the number of all the possible matrices of order 2×3 with each entry 1 or 2.

4. Write the coordinates of the point which is the reflection of the point (α, β, γ) in the XZ-plane.

5. Find the position vector of the point which divides the join of points with position vectors $\mathbf{a} + 3\mathbf{b} and \mathbf{a} - \mathbf{b}$ in the ratio .

6. If $|\mathbf{a}| = 4$, $|\mathbf{b}| = 3$ and $\mathbf{a} \cdot \mathbf{b} = 6\sqrt{3}$,then find the value of $|\mathbf{a} \times \mathbf{b}|$.

7. Solve for

$$x : \tan^{-1}\left(\frac{2-x}{2+x}\right) = \left(\frac{1}{2}\right)\tan^{-1}\left(\frac{x}{2}\right), x{>}0 \quad (1)$$

.

8. Prove that

$$2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \left(\frac{\pi}{4}\right) \quad (2)$$

.

9. On her birthday Seema decided to donate some money to chldren of an rophanage home.If there was 8 children less,everyone would have go ₹10 more.However,if there were 16 children more,every one would have got₹10 less.Using matrix method,find the numbers of children and the amount distributed by Seema.What values are refleced by Seema's decision?

10. If

$$x = e^{\cos 2t} and y = e^{\sin 2t} \tag{3}$$

,prove that

$$\frac{dy}{dx} = \frac{-y}{x} \frac{logx}{logy} \tag{4}$$

. Verify mean value theorem for the function

$$f(x) = 2sinx + sin2x \tag{5}$$

.

11. Find the eqation of the tangent line to the curve

$$y = \sqrt{5}x - 3–5 \tag{6}$$

,when parallel to the line

$$4x - 2y = 5 = 0 \tag{7}$$

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12. Show that the function f given by $fx = \begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & \text{i}fx \neq 0 \\ -1, & \text{i}fx = 0 \end{cases}$ is discontinuous .

13. Evaluate:

$$\int_1^5 \{|x-1| + |x-2| + |x-3|\}dx \tag{8}$$

.

14. Evaluate:

$$\int_0^\pi \frac{xsinx}{1+3cos^2x}dx \tag{9}$$

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15. Find:

$$\int \frac{2x+1}{\left(x^2\right)\left(x^2\right)} dx \tag{10}$$

.

16. Find

$$\int \left(3x+5\right)\sqrt{5x+4x-2x^2} dx \tag{11}$$

.

17. x

$$\frac{dy}{dx} + y - x + xycotx = 0; x \neq 0 \tag{12}$$

.

18. Solve the different equation:

$$\left(x^2+3xy+y^2\right) dx - x^2 dy = 0 \tag{13}$$

given that

$$\mathrm{y} = 0, whenx = 1 \tag{14}$$

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19. Find the angle between the vectors

$$\mathbf{a}+\mathbf{b}\mathrm{a}nd\mathbf{a}-\mathbf{b}\mathrm{i}f\mathbf{a} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}\mathrm{and}\mathbf{b} = 3\hat{\imath} + \hat{\jmath} - 2\hat{k} \tag{15}$$

,and hence find a vector perpendicular to both

$$\mathbf{a}+\mathbf{b}\mathrm{a}nd\mathbf{a}-\mathbf{b} \tag{16}$$

.

20. Show that the line

$$\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} \tag{17}$$

and

$$\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3} \tag{18}$$

intersect.Find their point of intersection.

21. A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, given that at least one girl must be there in the committee.

22. Show that the relation R defined by

$$(a, b)\, R\, (c, d) \Rightarrow a + d = b + c \tag{19}$$

on the $A \times A$,where

$$A = \{1, 2, 3, ...10\} \tag{20}$$

is an equivalence class $\big((3, 4)\big)\,; a, b, c, d \epsilon A$.

23. Solve for x: $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$, using properties of determinants.

24. Using elementary row operation find the inverse of matrix X $\mathbf{A}$ $\begin{pmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{pmatrix}$

and hence solve the following system of equations

$$3x - 3y + 4z = 21, 2x - 3y + 4z = 20, -y + z = 5 \tag{21}$$

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25. Show that height of the cylinder of greatest volume which can be in scribed in aright circular cone of height h and semi-vertical angle α is one third that of and greatest volume of cylinder is

$$\frac{4}{27}\pi \tan^2 \alpha \tag{22}$$

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26. Find the intervals in which the function

$$f(x) = \frac{4sinx}{2 + cosx} - x; 0 \leq x \leq 2\pi \tag{23}$$

is strictly increasing or strictly decreasing.

27. Using integration,find the area of the triangle formed by inegative x-axis and tangent and normal to the circle

$$x^2 + y^2 = 9\text{a}t\left(-1, 2\sqrt{2}\right) \tag{24}$$

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28. Find the coordinates of the foot of perpendicular distance frm the point P $(4, 3, 2)$ to the plane

$$x + 2y + 3z = 2 \tag{25}$$

.Also find the image of P in the plane.

29. A, B and C throw a pair of dice in that order alternately till one of them gets a total of 9 and wins the game. Find their respective probabilities of winning, if A starts first.

30. A company manufactures two types of cardigans type A and type B. It costs ₹360 to make a type A cardigan and ₹120 to make a type B cardigan. The company can make at most 300 cardigans and spend at most ₹$72,000$ a day. The number of cardigans of type B cannot exceed the number of cardigans of type A by more than ₹200. The company makes a profit of ₹100 for each cardigan of type A and ₹50 for every cardigan of type B .

    Formulate this problem as a linear programming problem to maximise the profit to the company. Solve it graphically and find maximum profit.