# MATHEMATICS

# QUESTIONS

1. Write the value of $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$.

2. If $\mathbf{A} = \begin{bmatrix} 1 & -2 & 3 \\ -4 & 2 & 5 \end{bmatrix}$and $\mathbf{B}$=$\begin{bmatrix} 2 & 3 \\ 4 & 5 \\ 2 & 1 \end{bmatrix}$ and BA=$(bij)$,find $b_{21}$+$b_{32}i$.

3. Write the number of all the possible matrices of order $2 \times 3$ with each entry 1 or 2.

4. Write the coordinates of the point which is the reflection of the point $(\alpha, \beta, \gamma)$ in the XZ-plane.

5. Find the position vector of the point which divides the join of points with position vectors $\vec{a} + 3\vec{b} and \vec{a} - \vec{b}$ in the ratio 1 : 3.

6. If $\left|\vec{a}\right| = 4, \left|\vec{b}\right| = 3$ and $\vec{a} \cdot \vec{b} = 6\sqrt{3}$,then find the value of $\left|\vec{a} \times \vec{b}\right|$.

7. Solve for $x$ : $tan^{-1}\left(\frac{2-x}{2+x}\right)$=$\left(\frac{1}{2}\right) tan^{-1}\left(\frac{x}{2}\right)$,$x$>0.

8. Prove that $2sin^{-1}\left(\frac{3}{5}\right) - tan^{-1}\left(\frac{17}{31}\right) = \left(\frac{\pi}{4}\right)$.

9. On her birthday Seema decided to donate some money to chldren of an rophanage home.If there was 8 children less,everyone would have go ₹10 more.However,if there were 16 children more,every one would have got₹10 less.Using matrix method,find the numbers of children and the amount distributed by Seema.What values are refleced by Seema's decision?

10. If $x = e^{cos2t}$ and $y = e^{sin2t}$,prove that $\frac{dy}{dx}$=$\frac{-y}{x}$ $\frac{logx}{logy}$.
    Verify mean value theorem for the function $f(x) = 2sinx + sin2x$ on $[o, x]$.

11. Find the eqation of the tangent line to the curve $y = \sqrt{5}x - 3$−5,when parallel to the line $4x - 2y = 5 = 0$.

12. Show that the function $f$ given by fx=$\begin{cases} \frac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & if x \neq 0 \\ \text{-1}, & if x = 0 \end{cases}$

is discontinuous at $x = 0$.

13. Evaluate: $\int_1^5 \{|x-1| + |x-2| + |x-3|\}$dx.

14. Evaluate: $\int_0^{\pi} \frac{xsinx}{1+3cos^2x} dx$.

15. Find: $\int \frac{2x+1}{(x^2)(x^2)}$dx.

16. Find $\int (3x+5)\ \sqrt{5x+4x-2x^2}$dx.

17. x$\frac{dy}{dx} + y - x + xycotx = 0; x \neq 0$.

18. Solve the different equation:$\left(x^2 + 3xy + y^2\right) dx - x^2 dy = 0$ given that $y =$ 0,when $x = 1$.

19. Find the angle between the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ if $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$,and hence find a vector perpendicular to both $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$.

20. Show that the line $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0}$ and $\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3}$ intersect.Find their point of intersection.

21. A committee of 4 students is selected at random from a group consisting of 7 boys and 4 girls. Find the probability that there are exactly 2 boys in the committee, given that at least one girl must be there in the committee.

22. Show that the relation R defined by $(a,b)$R$(c,d) \Rightarrow a + d = b + c$ on the $A \times A$,where $A = \{1, 2, 3, ...10\}$ is an equivalence class $\left[(3,4)\right]; a, b, c, d \epsilon A$.

23. Solve for x: $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0,$

using properties of determinants.

24. Using elementary row operation find the inverse of matrix X **A** $\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$ and hence solve the following system of equations $3x - 3y + 4z = 21, 2x - 3y + 4z = 20, -y + z = 5$.

25. Show that height of the cylinder of greatest volume which can be in scribed in aright circular cone of height h and semi-vertical angle $\alpha$ is one third that of and greatest volume of cylinder is $\frac{4}{27}\pi \tan^2 \alpha$.

26. Find the intervals in which the function $f(x) = \frac{4sinx}{2+cosx} - x; 0 \leq x \leq 2\pi$ is strictly increasing or strictly decreasing.

27. Using integration,find the area of the triangle formed by inegative x-axis and tangent and normal to the circle $x^2 + y^2 = 9$ at $\left(-1, 2\sqrt{2}\right)$.

28. Find the coordinates of the foot of perpendicular distance frm the point P $(4, 3, 2)$ to the plane $x + 2y + 3z = 2$.Also find the image of P in the plane.

29. $A$, B and C throw a pair of dice in that order alternately till one of them gets a total of 9 and wins the game. Find their respective probabilities of winning, if A starts first.

30. A company manufactures two types of cardigans type A and type B. It costs ₹360 to make a type A cardigan and ₹120 to make a type B cardigan. The company can make at most 300 cardigans and spend at most ₹72, 000 a day. The number of cardigans of type B cannot exceed the number of cardigans of type A by more than ₹200. The company makes a profit of ₹100 for each cardigan of type A and ₹50 for every cardigan of type B.

    Formulate this problem as a linear programming problem to maximise the profit to the company. Solve it graphically and find maximum profit.