Question-2

Given matrix $= \begin{bmatrix} 0 & 0 & 1 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \end{bmatrix}$

Rotational Matrix R in 2d $R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

For Eigenvalue $\lambda = 1$ $Rx = \lambda x$

Eigenvalue Equation

$$\begin{bmatrix} 0 & 0 & 1 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 1 \begin{bmatrix} x_4 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 1/-\sqrt{2} & 1/\sqrt{2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 1 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x_3 \\ x_1 + x_2/\sqrt{2} \\ -x_1 + x_2/\sqrt{2} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Simplifying the above Equations

$$x_3 = x_1 \;;\; \frac{x_1 + x_2}{\sqrt{2}} = x_2 \;;\; -\frac{x_1 + x_2}{\sqrt{2}} = x_3$$

$\Rightarrow x_3 = x_1$

$x_1 = (\sqrt{2} - 1)x_2$

Let $x_1 = x_3 = k$ then $x_2 = \frac{k}{\sqrt{2}-1}$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k \\ k/\sqrt{2}-1 \\ k \end{bmatrix}$$

Normalizing $X^T X = 1$

$$\Rightarrow [x_1, x_2, x_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \left[k, \frac{k}{\sqrt{2}-1}, k\right] \begin{bmatrix} k \\ k/\sqrt{2}-1 \\ k \end{bmatrix} = 1$$

$$k^2 + \frac{k^2}{(\sqrt{2}-1)^2} + k^2 = 1$$

$$k^2 = \left(\frac{1}{17}\right)\left(5-2\sqrt{2}\right) \Rightarrow k = \sqrt{\left(\frac{1}{17}\right)\left(5-2\sqrt{2}\right)}$$

Hence unit vector of Rotation is

$$\vec{x} = \sqrt{\frac{17}{5-2\sqrt{2}}}\left(\hat{i} + \hat{j} + \hat{k}\right)$$

The unit vector for this vector is

$$k = \frac{\bar{x}}{|X|} = \frac{1}{\sqrt{3}}\left(\hat{i} + \hat{j} + \hat{k}\right)$$

$\cos\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ therefore $\boxed{\theta = 45^\circ \text{ counter clockwise}}$