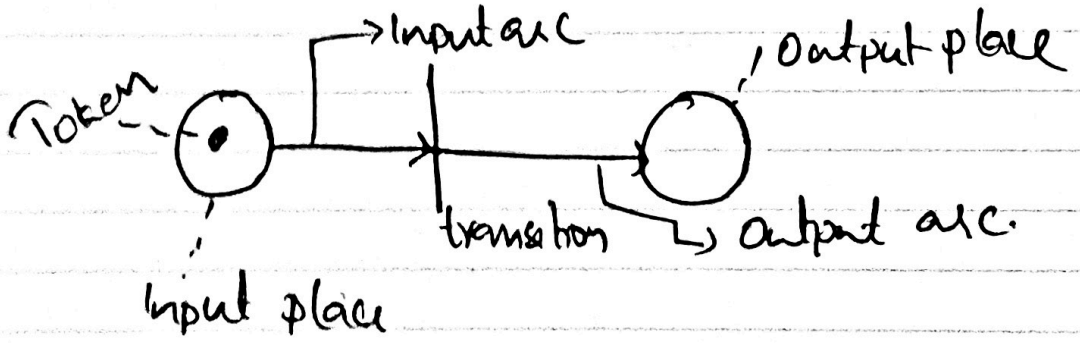
Continues time Markov Chain

Stoclastic Petri Nets

- Formal and graphical appealing for modelling systems concurrently.

* Petri Net → Bipaite directed graph with two nodes → places and transitions
* Arcs exist between places and transitions But no arcs is present between two places or two transitions.
* places represent conditions within the system and are graphically denoted by circles.
* Transitions represents events occuring in the system that change in the conditions and are denoted graphically as bars.

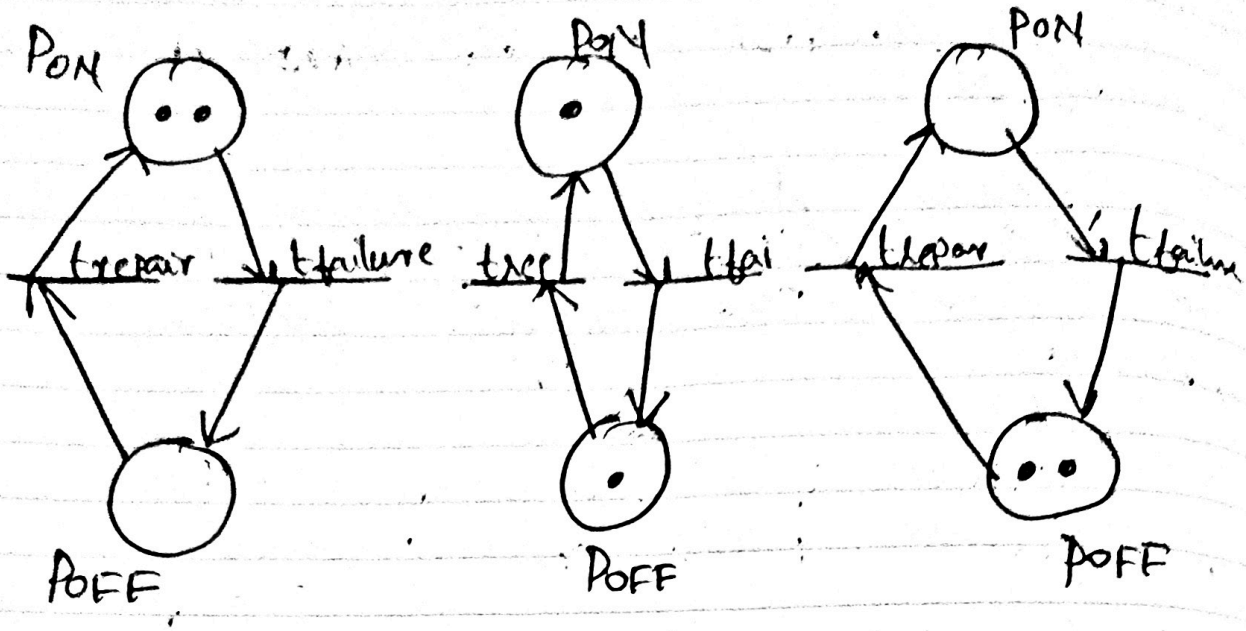


Definitions of components of petri nets.

* Input places : The set of places that are connected to the transition through input arcs.
* Input arcs : Arcs drawn from place to transitions. The represent the conditions that need to be satisfied for the events.
* Output place : Set of places to which output arcs exist from transition.
* Output arcs : directed arcs drawn from transition to places.
* Tokens : The dots associated with places. A place containing tokens indicate that condition is active.

Example - 1:

Enabling and Firing of transitions:



So, In the above example, we have 2 places, PON and POFF. Then two transitions Trepair and Tfailure.

So we will see each condition one by one. In the initial condition PON has two tokens. As a result Tfailure transition will be enabled. At the same time POFF has no tokens and therefore transition Trepair will not be enabled.

When Tfailure is enable, the required no. of tokens will be remov

from T_{ON} and required no of tokens will be deposited on T_{OFF}. So here P_{ON} already had one to two tokens and one will be removed and one token will be deposited in T_{OFF}.

Now T_{ON} and T_{OFF} has one tokens each. This condition will enable both T_{Repair} and $T_{Failure}$.

Now If $T_{Failure}$ fires, then one token will be removed and from P_{ON} and One token will be deposited in P_{OFF}. Now P_{ON} has 0 tokens and P_{OFF} has 2 tokens.

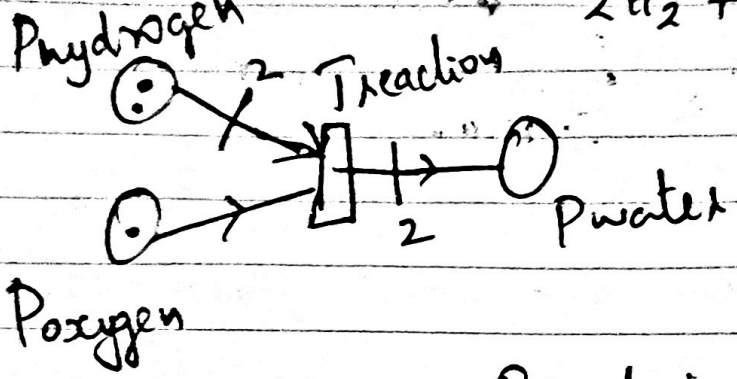
If T_{Repair} was fired instead of $T_{Failure}$ then one token will be removed from P_{OFF} and One token will be deposited in P_{ON}. Now P_{ON} has 2 tokens and P_{OFF} has 0 tokens.

When P_{OFF} has two tokens and P_{ON} has 0 token, $T_{Repaire}$

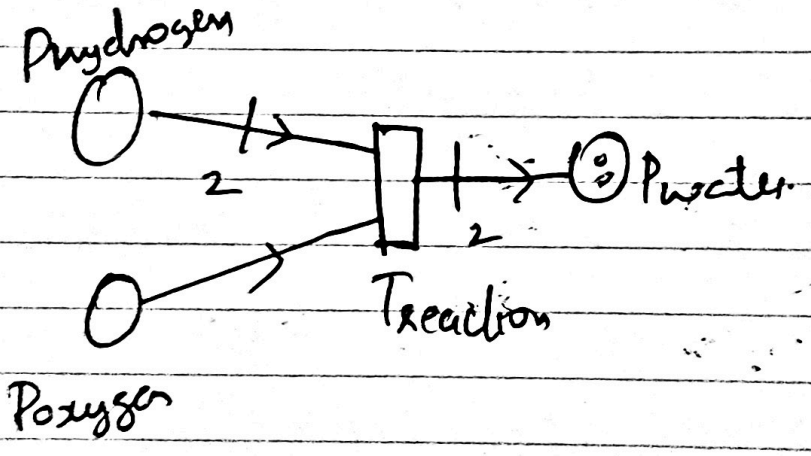
will be enabled and fired. Therefore one token will be removed from T_{OFF} and one token will be deposited in T_{ON}.

Example - 2

$$2H_2 + O_2 \rightleftharpoons 2H_2O$$



On firing

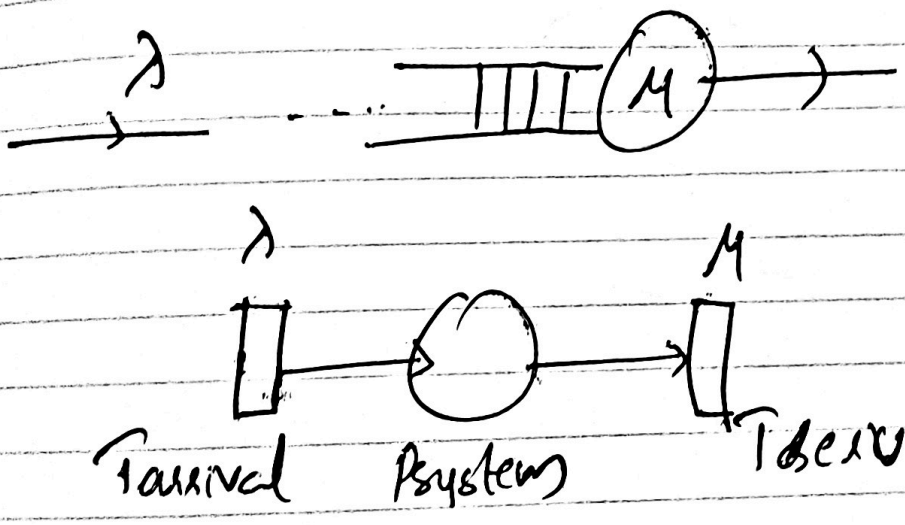


We have 3 places, Phydrogen, Poxygen and Pwater and transition Treaction.

when the transition reaction is enabled and fired, because of the multiplicity 2 in the input arc from hydrogen two tokens will be removed and because of multiplicity 1 on the input arc from Oxygen one token will be removed.

The output arc from transition reaction to output place Water contain 2 multiplicity and so two tokens will be deposited in water.

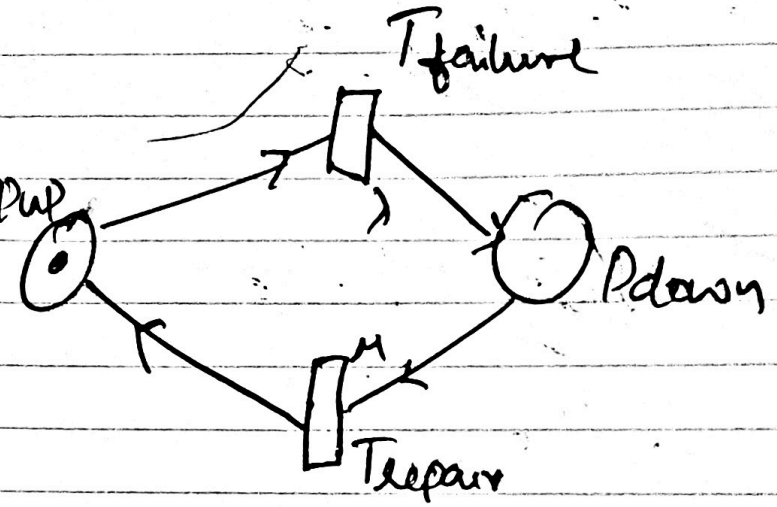
Example – 3



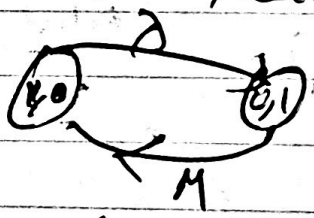
The M/M/1 model describe a single server queue with Poison arrival rate (λ) and exponential service time rate (μ). It can be modeled as a Petri net with one place (Psystem)

and two time transition (arrival, service) using tokens. The no. of tokens in p represents queue length. The reachability graph shows state transitions and resembles the system's Markov chain.

Example - 4



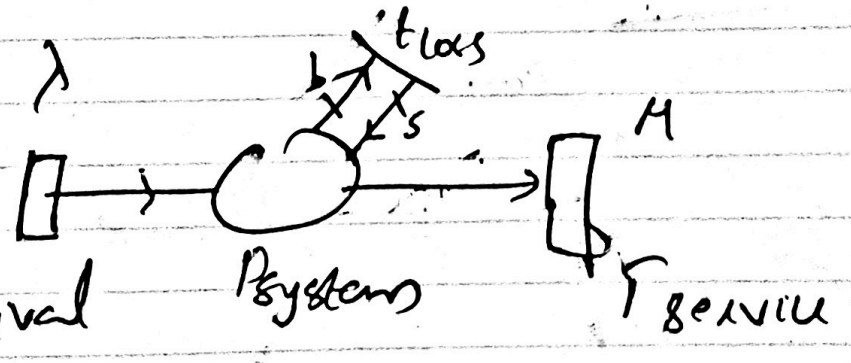
Reachability graph.



This example uses a two state stochastic Petri net to model a repairable processor system with exponential failure (λ) and repair (μ)

Up and down places represent the processor state. Tokens track the system state: One token in "up" for a working system and one in "Down" for a failed system. Transition model failure and repair, moving tokens between places. The reachability graph shows state transition and is equivalent to the systems markov chain.

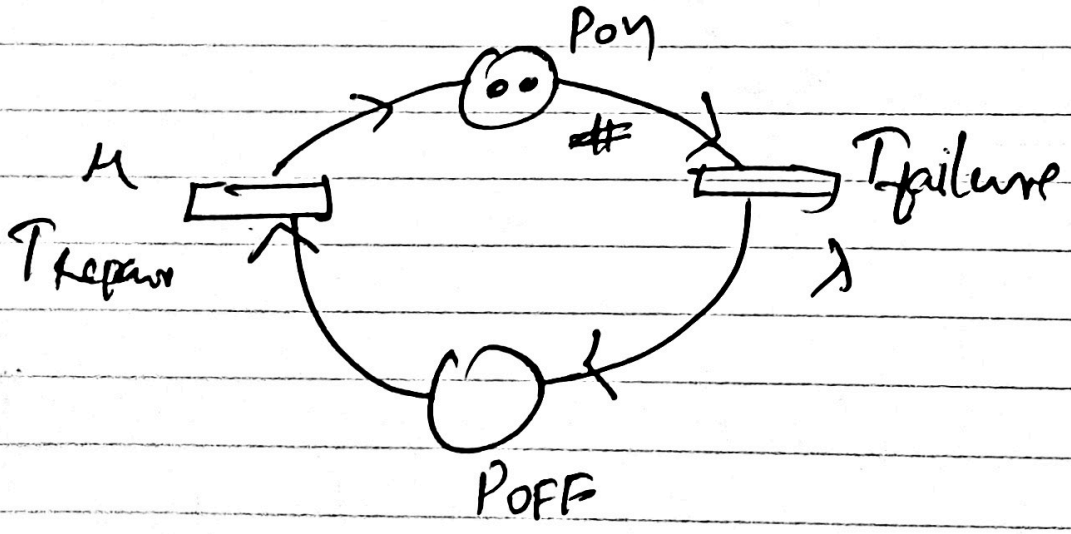
Example 5: M/M/1/5.



An M/M/1/5 queueing system limits the number of jobs in the system to 5. We can build a GSPN based on the M/M/1 model but

but with an additional instantaneous transition ("t loss") to enforce the capacity limit (when 6 tokens are in the system (P system), "t loss" removes and immediately adds back 5 tokens, preventing more than 5 jobs. The reachability graph shows 0-5 possible tokens, and transitions fire based on arrival (λ) and service (μ) rates.

Example -6



Two places (P_{ON}, P_{OFF}) and two transitions (failur, Repair) model components failure and Repairs (exponential distribution with rates λ and μ).

Tokens Represent Components State (on / off).

Reachability graph Shows marking (tokens in each place and transition rate (λ, 2λ, μ).

Reward Rate indicate System availability (1 for working System, 0 for failure).