

Home work

1) $M/M/1:2$ model.

$$\lambda = 8/\text{hr}, \quad \mu = 9/\text{hr}, \quad N = 2$$

$$P_0 = \frac{1-\rho}{1-\rho^{N+1}}; \quad \text{where } \rho = \frac{\lambda}{\mu}$$

$$P_0 = \frac{1-8/9}{1-(8/9)^{N+1}} = \frac{1-0.8888}{1-(0.8888)^3}$$

$$= \frac{0.1112}{0.2979} = \underline{\underline{0.3732}}$$

$$P_1 = \rho P_0 = \frac{8}{9} \times 0.3732 = 0.3317$$

$$P(\text{no queue}) = P_0 + P_1 = 0.7049$$

$$P_2 = \rho^2 P_0 = (0.8888)^2 \times 0.3732$$

$$= 0.7899 \times 0.3732$$

$$= 0.2948$$

Probability of not joining the system

$$= 1 - 0.2948 = \underline{\underline{0.7052}}$$

Effective arrival rate

$$\lambda_e = \lambda \times 0.7052$$

$$= 8 \times 0.7052$$

$$= \underline{\underline{5.6416}}$$

$$L_s = \frac{\rho [1 + N \rho^{(N+1)} - (N+1) \rho^N]}{(1-\rho) (1-\rho^{N+1})}$$

$$= \frac{0.8888 (1 + 2 \times (0.8888)^3 - 3 \times (0.8888)^2)}{(1-0.8888) (1-0.8888^3)}$$

$$= \frac{\cancel{0.8888} (\cancel{1 + 1.4042} - \cancel{5.3328})}{\cancel{0.1112} \times \cancel{0.2978}}$$

$$= 0.888 (\underline{1 + 1.404} - 2.3656)$$

$$0.1112 \times 0.2978$$

$$= \frac{0.888 \times 0.0384}{0.0331} = \underline{\underline{1.019}}$$

$$\omega_s = \frac{L_s}{\lambda_e} = \frac{1.019}{5.641} = \underline{\underline{0.1806}}$$

$$\omega_L = \omega_s - \frac{1}{\mu}$$