1) Calculate the Variances and the covariance of Auto and Home deductibles for the following insurance company.

Home

Auto	**0**	**1000**	**2000**	**5000**	
250	0.04	0.06	0.05	0.03	0.18
500	0.07	0.10	0.20	0.10	0.47
1000	0.02	0.03	0.15	0.15	0.35
Col Total	0.13	0.19	0.40	0.28	1.00

Solution: (Calculations in the attached file: Question 1.xlsx)

Variance(Auto)	**81850**
Variance(Home)	**3077900**
Covariance(Auto,Home)	**151800**

2) Calculate the covariance of the Auto and Home deductibles for the following company.

Auto	**0**	**1000**	**2000**	**5000**	**Row Total**
250	0.02	0.04	0.08	0.06	0.20
500	0.05	0.10	0.20	0.15	0.50
1000	0.03	0.06	0.12	0.09	0.30
Col. Total	0.10	0.20	0.40	0.30	1.00

Solution: (Calculations in the attached file: Question 2.xlsx)

Covariance(Auto,Home)	**0**

3)Show that Var(x) is the same as $E(X^2)-(E(x))^2$

Solution:

$$
\begin{aligned}
Var(X) &= E[(X-\mu)^2] \\
&= \sum (x-\mu)^2 * p(x) \\
&= \sum (x^2 - (2*x*\mu) + \mu^2)*p(x) \\
&= \sum x^2 p(x) - 2\mu \sum x*p(x) + (\mu^2 \sum p(x)) \\
&= E(X^2) - 2\mu^2 + \mu^2 \\
&= E[X^2] - \mu^2 \\
&= E[X^2] - (E[X])^2
\end{aligned}
$$

Hence Proved.

4)Show that $E(x+y)= E(x)+ E(y)$

Solution:

$E(X+Y)= \sum_{x,y} ((x+y)*P(X=x, Y=y))$

$= \sum_{x,y} (x*P(X=x, Y=y)) + \sum_{x,y} (y*P(X=x, Y=y))$

$= \sum x * \sum_y P(X=x, Y=y) + \sum y * \sum_x P(X=x, Y=y)$

Using total probability, $\sum_y P(X=x, Y=y) = P(X=x)$ and $\sum_x P(X=x, Y=y) = P(Y=y)$

$= \sum_x x*P(X=x) + \sum_y y*P(Y=y)$

$E[X]+E[Y]$

Hence Proved

5) Show that Covar(x,y)=0 if x and y are independent

$Covar(X,Y)= E(XY) - \mu(x)\ \mu(y)$

If x and y are independent, $E(XY)=E(X)*E(Y)$

$= E(X)*E(Y) - \mu(x)\ \mu(y)$

$= \mu(x)\ \mu(y)- \mu(x)\ \mu(y)$

$=0$

Hence Proved

6) Show that Var(x+y)=Var(x)+Var(y) if x and y are independent.

$Var(X+Y)=E[(X+Y)^2] - E(X+Y)^2$

$= E[X^2+2XY+Y^2] - (\mu(x) +\mu(y))^2$

$= E[X^2+2XY+Y^2] - (\mu(x)^2 +2\ \mu(x)\ \mu(y)+\mu(y)^2)$

$= E[X^2]- \mu(x)^2 + E[Y^2]- \mu(y)^2 +2(E[XY]- \mu(x)\ \mu(y))$

$=Var(X) +Var(Y) +2Covar(X,Y)$

We proved earlier that Covar(x,y)=0 if x and y are independent

$=Var(X)+Var(Y)$

Hence Proved.