Reminders and Topics

- **Project 9 due this Friday 4pm**

- This lecture:
  - **Depth-First Search in Graphs**
  - **Breadth-First Search in Graphs**
  - **Search Applications**

Graph Search / Traversal

- Graph search / traversal is a fundamental operation:
  - Is there a path from vertex X to vertex Y? If so, what's the shortest path?
  - Is this a connected graph? If not, how many connected sub-graphs are there?
  - As you will see later, many interesting problems can be formulated as graph search problem
- For binary trees, we learned three types of traversals: in-order, pre-order, post-order.
- For graphs, generally two types: DFS, BFS

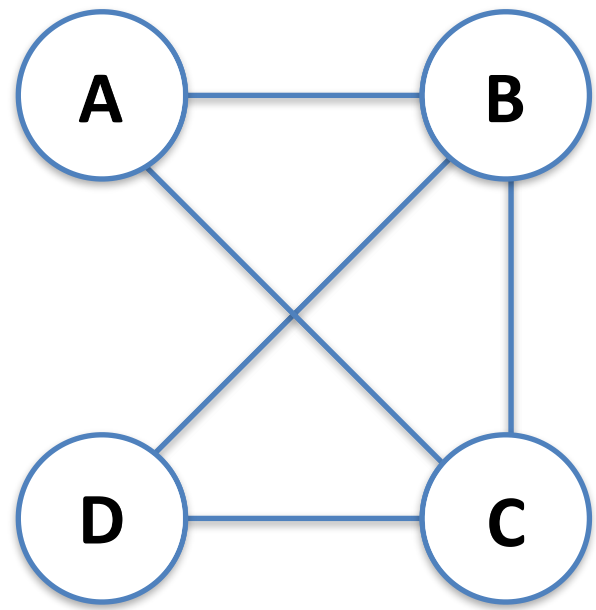
DFS and BFS

- **Depth-First Search**: start at a vertex, follows its edges to visit the deepest point, then moving up.
    - Go as far away from the starting vertex as possible (depth), before moving back.
    - Use a **Stack** to track where to go next.
- **Breadth-First Search**: traverse vertices in ‘levels’ — starting from a vertex, visit all its immediate neighbors, then neighbors of neighbors, and so on.
    - Stay as close to the starting vertex as possible (breadth), before moving to the next level.
    - Use a **Queue**.

Depth-First Search (DFS)

Example: start from vertex A. Visit all vertices connected to A. Initial version of pseudo-code:

```
push A to stack
while(stack not empty) {
  v = stack.pop();
  print/visit v
  push v’s neighbors
  to stack
}
```

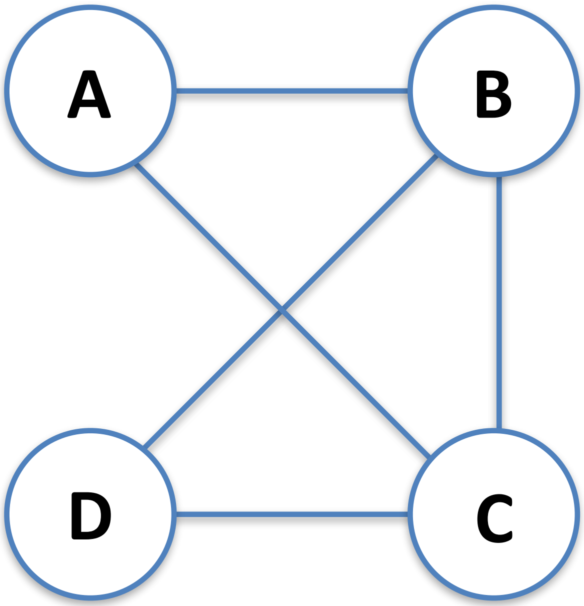


Depth-First Search

Example: start from vertex A. Visit all vertices connected to A. Initial version of pseudo-code:

Stack status:

```
A          // push A
C B        // pop A, and push
              A's neighbors
C D C A    // pop B, and push
              B's neighbors
C D C C B
```



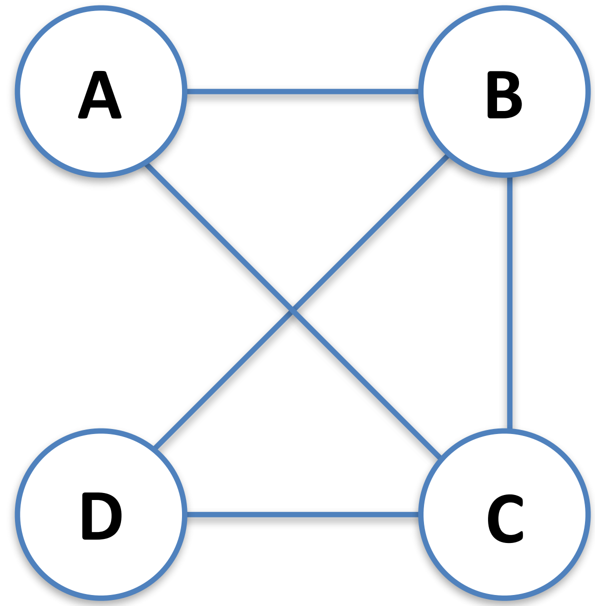
Wait a minute, there is a problem!

How do we fix it?

Depth-First Search

Correct version:

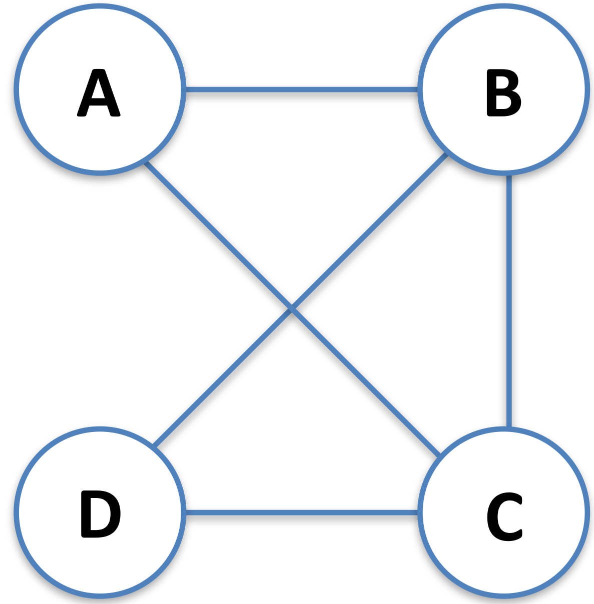
```
push A to stack
while(stack not empty) {
  v = stack.pop();
  print/visit v
  mark v as visited
  iterate through v's neighbors  {
    if a neighbor is not marked yet
      push it to stack
  }
}
```



Depth-First Search

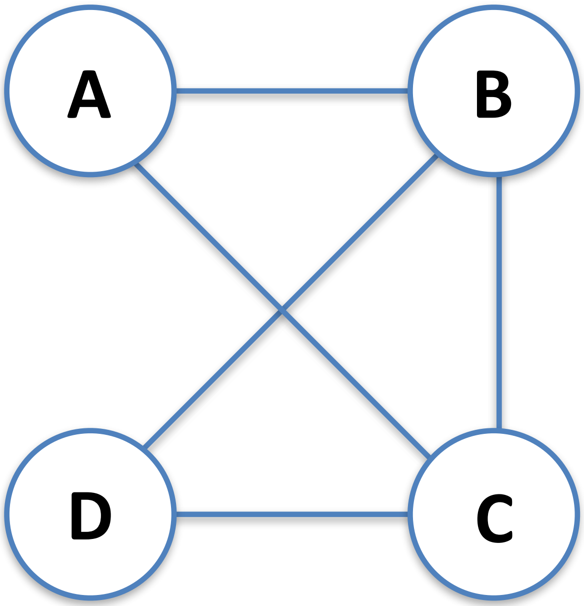
Stack status:

```
A          // push A
C B        // pop, mark A, push
              unvisited neighbors
C D C      // pop, mark B, push
              unvisited neighbors
C D D      // pop, mark C, push
              unvisited neighbors
C D        // pop, mark D, no
              unvisited neighbors
```



Something still doesn't seem ideal here. Why?

Depth-First Search

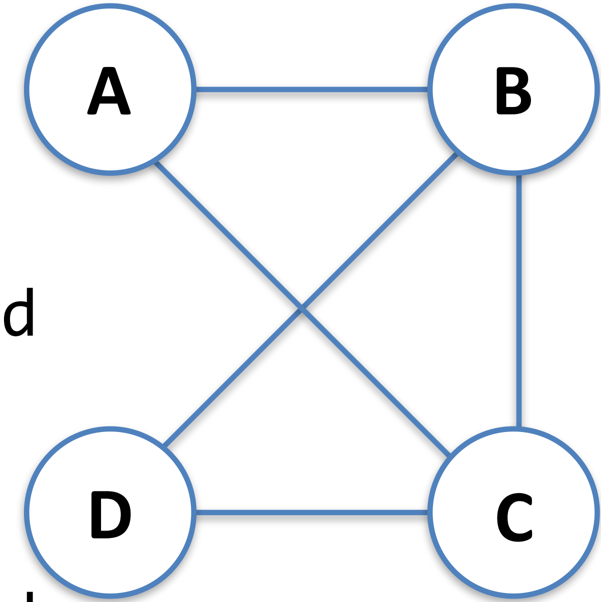


Efficient version:

```
mark A and print A
push A to stack
while(stack not empty) {
  v = stack.peek();
  n = getNextUnvisitedNeighbor(v);
  if none found, pop stack;
  else {
    mark n, print n, push n to stack
  }
}
```

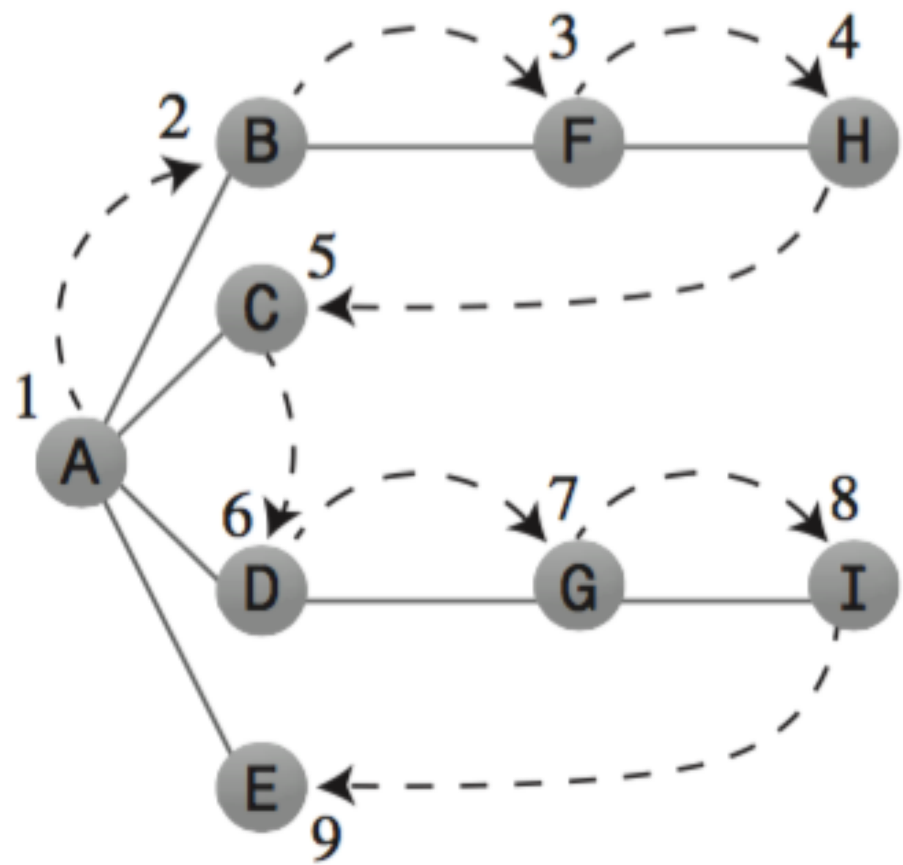
Stack status:

```
A        // push A
A B      // get A's next unvisited
            neighbor B, mark it
            push it to stack
A B C    // get B's next unvisited
            neighbor C, mark it, push it
A B C D  // get C's next unvisited neighbor
            D, mark it, push it.
A B C    // D has no unvisited neighbor, pop
A B      // C has no unvisited neighbor, pop
A        // pop
(empty)  // the end
```



Depth-First Search

Another Example: start from vertex A. Visit all vertices connected to A using DFS.



DFS Implementation Details

Assume the graph data structures stores:

- Vertices in an array called `vertexList[]`
- Each vertex has a field called `wasVisited`
- Adjacency matrix in a 2D array called `adjMat[][]`
- The number of vertices in `nVerts`

```
class Graph {
  private Vertex vertexList[];
  private int adjMat[][];
  private int nVerts;
  … …
```

DFS Implementation Details

```
// returns the next unvisited neighbor of v
public int getNextUnvisitedNeighbor (int v) {
  for (int j=0; j<nVerts; j++) {
    if( adjMat[v][j] == 1 &&
        vertexList[j].wasVisited == false ) {
      return j;
    }
    return -1;    // none found
}
```

DFS Implementation Details

```
// dfs from start vertex (0 by default)
public void dfs (int start) {
  vertexList[start].wasVisited = true;   // mark it
  print(start);  stack.push(start);
  while(!stack.isEmpty()) {
    int b = getNextUnvisitedNeighbor(stack.peek());
    if (b==-1)  stack.pop(); // no unvisited neighbor
    else {
      vertexList[b].wasVisited = true;
      print(b);  stack.push(b);
    }
  }
  // clear wasVisited marks
}
```

Finding a Path using DFS

```
// dfs from start vertex to end vertex
public boolean hasPath (int start, int end) {
  vertexList[start].wasVisited = true;   // mark it
  stack.push(start);
  int b = -1;
  while(!stack.isEmpty()) {
    b = getNextUnvisitedNeighbor(stack.peek());
    if (b==end) break;
    if (b==-1)  stack.pop(); // no unvisited neighbor
    else {
      vertexList[b].wasVisited=true;   stack.push(b);
    }
  }
  if(b==end) return true;
  else return false;
}
```

So how do I print out the path??

Vertices on the path are all stored in the stack!

Clicker Question #1

Given the adjacency matrix, if we perform DFS starting from vertex 2, in what order will the other vertices be visited?

(a) 4, 0, 3, 1

(b) 3, 0, 1, 4

(c) 3, 4, 0, 1

(d) 3, 4, 1, 0

(e) 3, 0, 4, 1

	0	*1*	*2*	*3*	*4*
0	0	0	0	1	1
1	0	0	0	1	0
2	0	0	0	1	1
3	1	1	1	0	1
4	1	0	1	1	0

Answer on next slide

Clicker Question #1

Given the adjacency matrix, if we perform DFS starting from vertex 2, in what order will the other vertices be visited?

(a) 4, 0, 3, 1

(b) 3, 0, 1, 4

(c) 3, 4, 0, 1

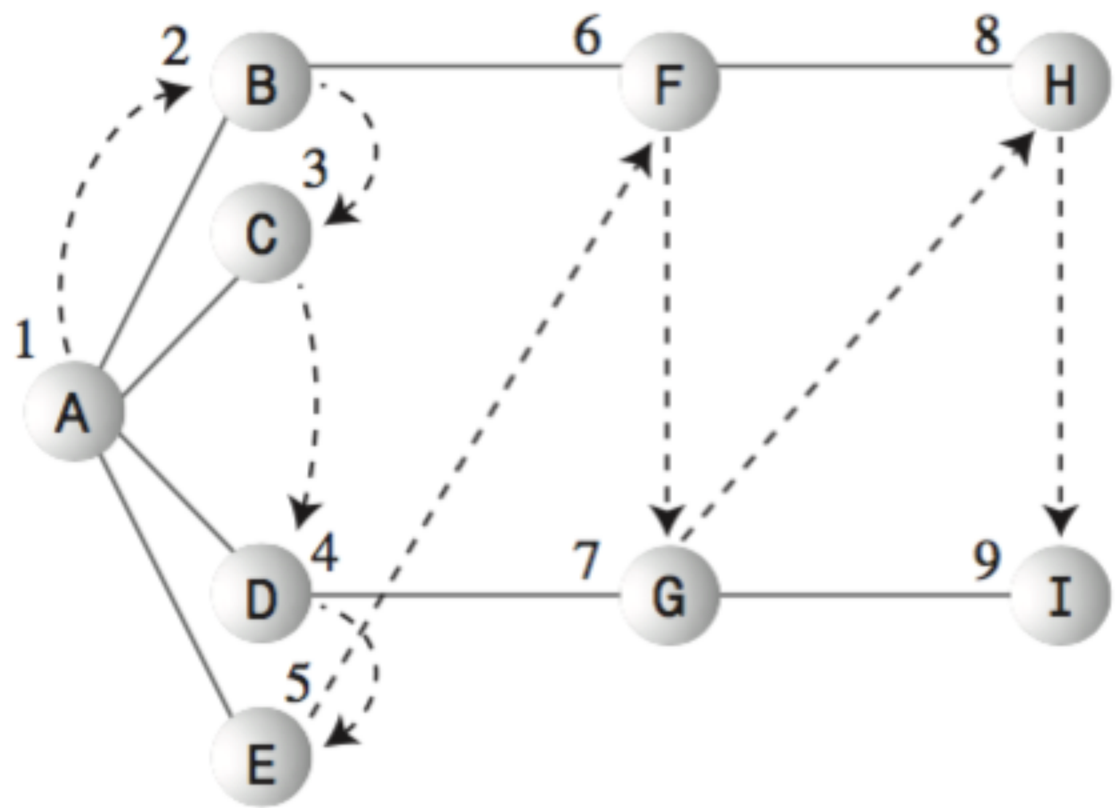
(d) 3, 4, 1, 0

(e) 3, 0, 4, 1

	0	*1*	*2*	*3*	*4*
0	0	0	0	1	1
1	0	0	0	1	0
2	0	0	0	1	1
3	1	1	1	0	1
4	1	0	1	1	0

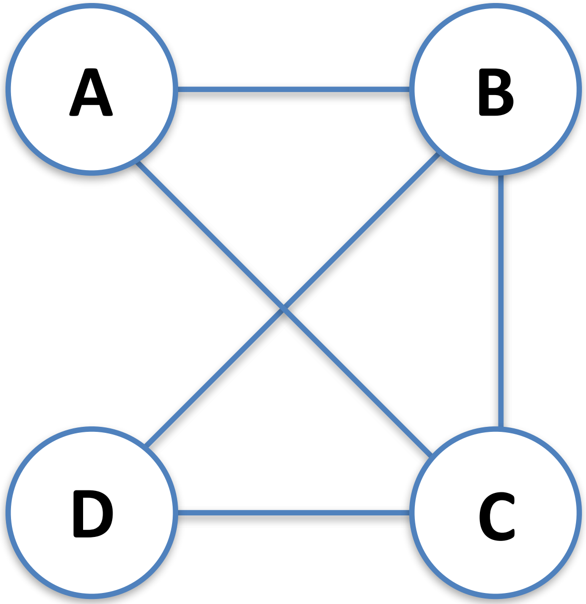
Breadth-First Search (BFS)

Example: start from vertex A. Visit all vertices connected to A using BFS.



Breadth-First Search (BFS)

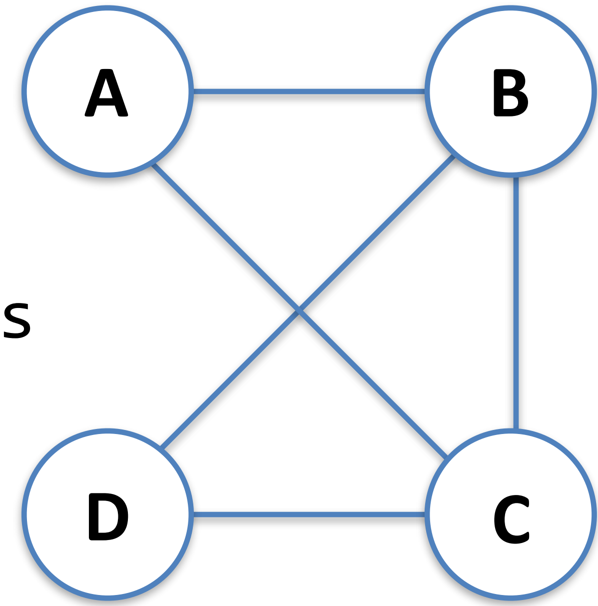
Pseudo-Code:




```
mark A and print A
enqueue A to queue
while(queue not empty) {
   v = queue.dequeue();
   while((n=getNextUnvisitedNeighbor(v)) != -1) {
      mark n, print n, enqueue n
   }
}
```

Queue status:

```
A          // enqueue A
B C        // dequeue A, add all A’s
              unvisited neighbors,
              visit them and,
              mark them as visited
C D        // dequeue B, add all B’s
              unvisited neighbors,
              visit and mark them
D          // dequeue C,
              C has no unvisited neighbors
(empty)    // the end
```



BFS Implementation Details

```
// bfs from start vertex (0 by default)
public void bfs (int start) {
  vertexList[start].wasVisited = true;   // mark it
  print(start);  queue.enqueue(start);
  int b;
  while(!queue.isEmpty()) {
    int v = queue.dequeue();
    while((b=getNextUnvisitedNeighbor(v)) != -1) {
      vertexList[b].wasVisited = true;
      print(b);  queue.enqueue(b);
    }
  }
  // clear wasVisited marks
}
```

Clicker Question #2

Given the adjacency matrix, if we perform BFS starting from vertex 2, in what order will the other vertices be visited?

(a) 4, 0, 3, 1

(b) 3, 0, 1, 4

(c) 3, 4, 0, 1

(d) 3, 4, 1, 0

(e) 3, 0, 4, 1

	0	*1*	*2*	*3*	*4*
0	0	0	0	1	1
1	0	0	0	1	0
2	0	0	0	1	1
3	1	1	1	0	1
4	1	0	1	1	0

Answer on next slide

Clicker Question #2

Given the adjacency matrix, if we perform BFS starting from vertex 2, in what order will the other vertices be visited?

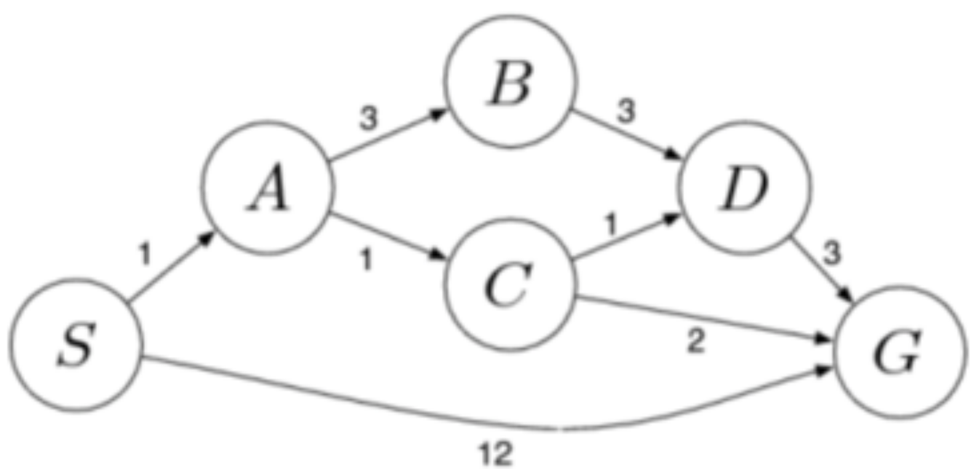
```
(a) 4, 0, 3, 1

(b) 3, 0, 1, 4

(c) 3, 4, 0, 1

(d) 3, 4, 1, 0

(e) 3, 0, 4, 1
```

	0	*1*	*2*	*3*	*4*
0	0	0	0	1	1
1	0	0	0	1	0
2	0	0	0	1	1
3	1	1	1	0	1
4	1	0	1	1	0

Search in Weighted Graphs

- In weighted graphs, we generally care about the shortest path from vertex X to Y in terms of the path weight (sum of weights on the path).

- Although we can still perform DFS and BFS in weighted graphs, they are often not so useful as they don't account for edge weight. For example, perform **BFS** on the following graph to find a path from S to G, what would you get? Is it the shortest path?



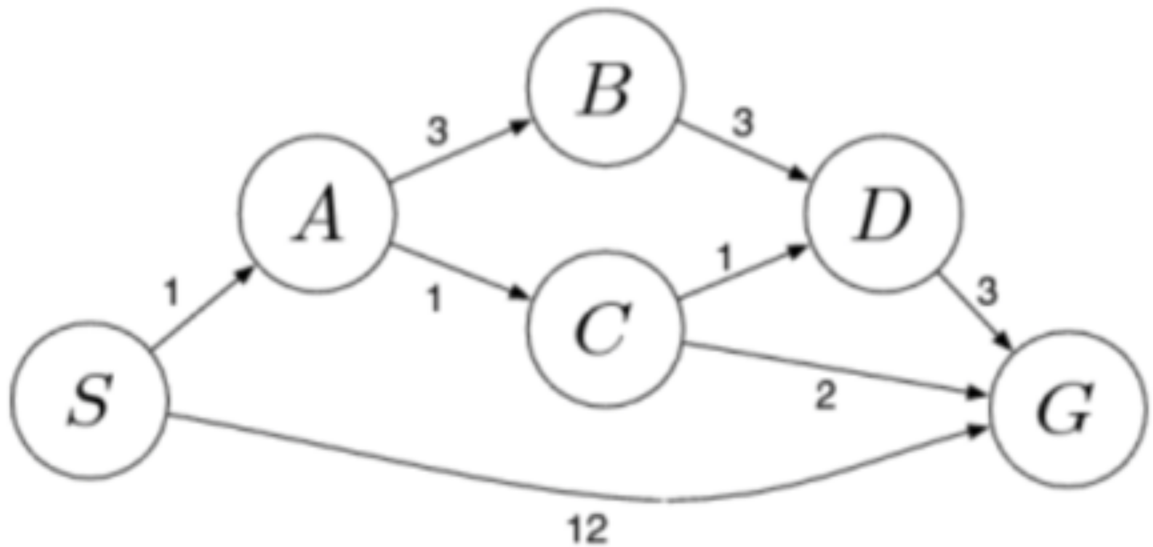
Search in Weighted Graphs

- BFS gives you a path with minimum number of edges, but not necessarily the shortest path (in terms of path weight).
- Analogy: BFS in a air flight graph can give you an itinerary with minimum number of flight segments, but not necessarily the minimum total price!
- **Shortest-Path Algorithm**: turns out we can modify BFS slightly to implement shortest-path algorithm.
    - The trick here is to use a Priority Queue instead of the standard FIFO Queue.
    - Shortest-path algorithm is a rich topic. Here we will only cover a flavor of it. You will learn more in upper-level classes.

Uniform Cost Search (UCS)

```
Insert the starting vertex into the queue
While the queue is not empty
  Dequeue the highest priority (for us, it’s
   shortest distance) vertex v from the queue
  if the path is ending in the end vertex
      print the path and exit
  else
      enqueue all unmarked neighbors of the
      v, with the cumulative distance as
      priority
```

Uniform Cost Search (UCS)



S is start, G is Goal
Initialization: { [S , 0] }
Iteration1: { [S->A , 1] , [S->G , 12] }
Iteration2: { [S->A->C , 2] , [S->A->B , 4] , [S->G , 12] }
Iteration3: { [S->A->C->D , 3] , [S->A->B , 4] , [S->A->C->G , 4] , [S->G , 12] }
Iteration4: { [S->A->B , 4] , [S->A->C->G , 4] , [S->A->C->D->G , 6] , [S->G , 12] }
Iteration5: { [S->A->C->G , 4] , [S->A->C->D->G , 6] , [S->A->B->D , 7] , [S->G , 12] }
Iteration6 gives the final output as S->A->C->G.

Clicker Question #3

Suppose we perform a UCS on this directed graph starting at vertex a, with b as the goal. When we first put b on the priority queue, what is its distance?

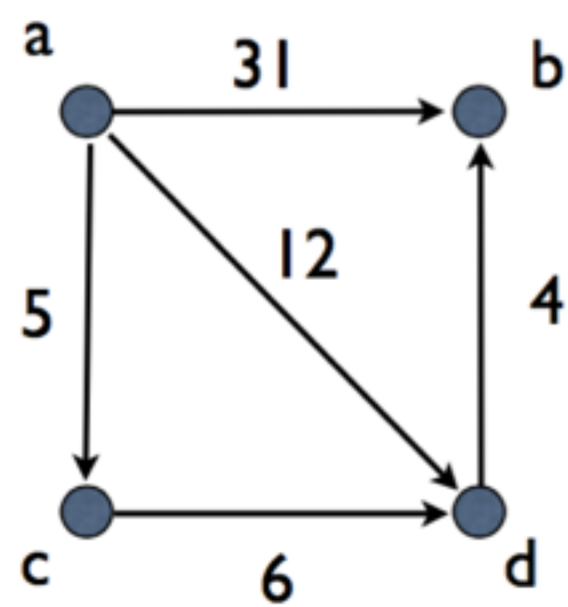
(a) 0

(b) 4

(c) 15

(d) 16

(e) 31



Answer on next slide

Clicker Question #3

Suppose we perform a UCS on this directed graph starting at vertex a, with b as the goal. When we first put b on the priority queue, what is its distance?

(a) 0

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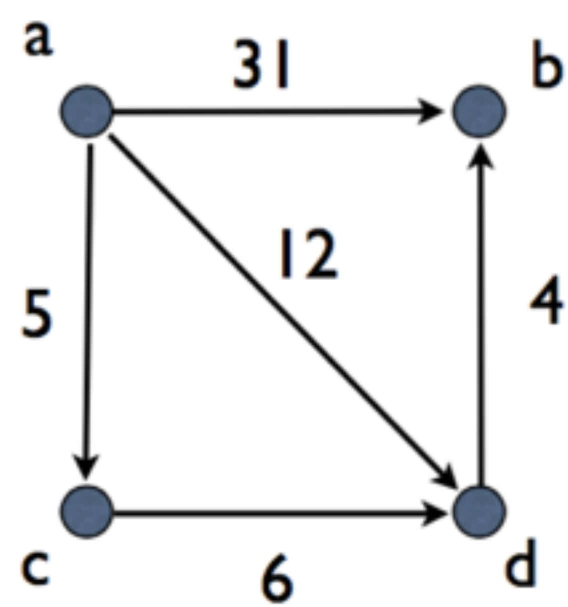
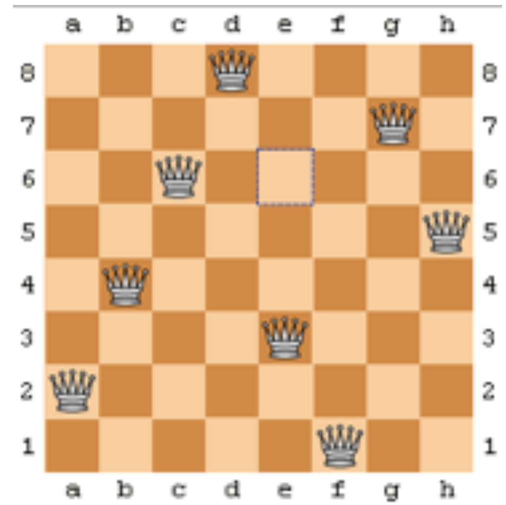
(e) 31



Graph Search Applications

- A lot of interesting computational problems can be viewed as graph search problems, where each vertex represents a state and each edge represents a valid move from one state to another state. Examples:

    - [Eight Queens Puzzle]()
    - [Knight's Tour]()
    - Sudoku
    - Maze
    - Flood Fill



Graph Search Applications

- A generally useful algorithm is called **backtracking**. It incrementally builds the candidates to the solutions, and abandons a partial solution **s** (i.e. backtracks) as soon as **s** is found to be impossible to form a valid complete solution.
- We can use a **stack** to store the current partial solution (e.g. a subset of eight queens, or a partial path in a maze). The stack allows us to go back (backtrack) to a previous state, and proceed to build the next partial solution.

Graph Search Applications

- For example, for the **Four Queens** Problem:
    - **We know that each row holds 1 and only 1 queen.**
    - Place the first queen at (1, 1), push it to stack.
    - Place the second queen at (2, 1). Is there a conflict with the first queen? If so, move the second queen to (2, 2), (2, 3) and so on until there is no conflict. Push it to stack.
    - Place the third queen at (3, i) where i is from 1 to 4, until there is no conflict with the previous queens. Push it to stack.
    - If you can’t find a valid spot for a queen (say, the third queen), pop the stack (thus you are back to working on the second queen), search for the **next** valid spot for it and push to stack.