

Time Complexity Method #1

Ex: ①

for ($i=0; i < n; i++$) n

Statement: n

$\{$ n $\}$ $\frac{1}{n}$

$O(n)$

* if i is incrementing.

② for ($i=n; i > 0; i--$) n+1

desn't
Reqd

stmt; n

Simple ex: if you go to

1 to 10 ($O(n)$) 10 to 1

If it'll take 10 steps only.

So no matter of code if it's

incrementing or decreasing

n+1 to 10 (com) 10 to 1

* here ($n+1$) of for loop

~~doesn't~~ doesn't effect the T/C

③ for ($i=1$; $i \leq n$; $i = i + 1$)
{ }

(for (i=1; i<n; i++)) Stmt → $\frac{n}{2}$
{ } $\frac{n}{2}$
 $O(n)$

$$f(n) = \frac{n}{2} \rightarrow O(n)$$

Even if the $f(n) = \frac{n}{2}$ still $\rightarrow O(n)$

Ex-②

for ($i=0$; $i < n$; $i++$) → $n+1$

{ }

for ($j=0$; $j < n$; $j++$) $n \times (n+1)$

{ } { }

stmt: $\rightarrow n \times n$

{ }

$f(n) = O(n^2)$

→ If we add all nested loops, we get $O(n^2)$

(Hence) $O(n^2)$ is the complexity

Ex. ③

for ($i=0$; $i < n$; $i++$)

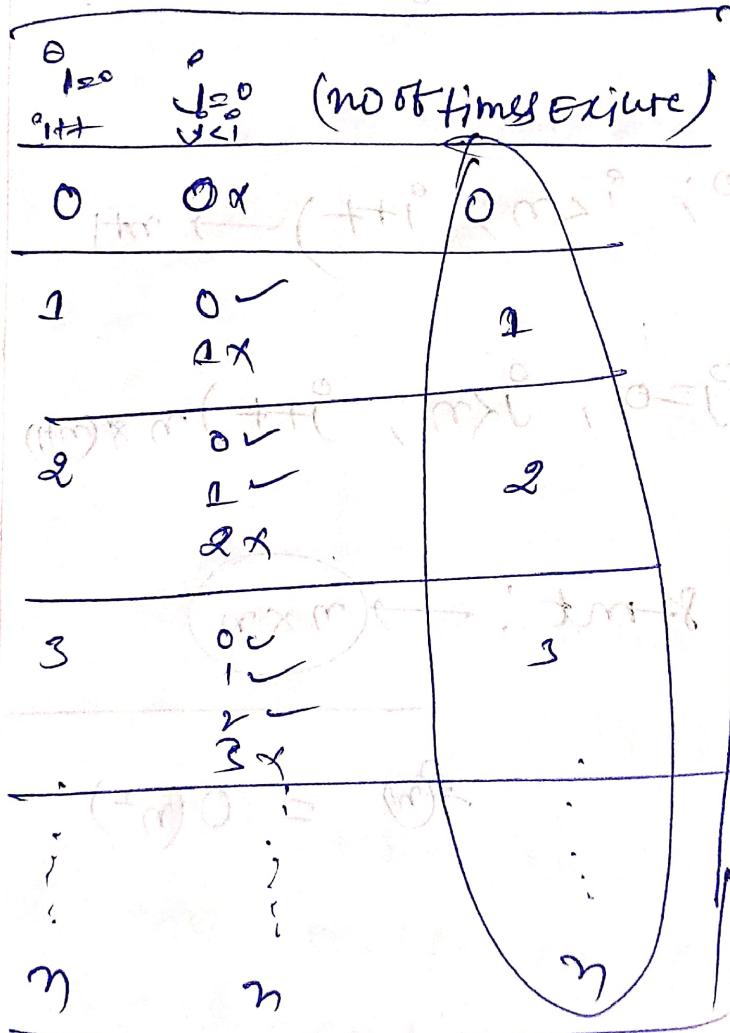
{
 for ($j=0$; $j < i$; $j++$)

{

 stmt;

{

($i=0$) \leftarrow $i=i+1$ }
 \downarrow



$$f(n) = \frac{n(n+1)}{2}$$

$$f(n) \underset{\text{2nd row}}{\approx} \frac{3^2 + 1}{2}$$

degree n^2

$$f(n) = O(n^2)$$

How many no. of times it's executed
 $1+2+3+4+\dots+n = \frac{n(n+1)}{2}$

Expt ④

$p = 0$

for ($i=1$; $p \leq n$; $i++$)

$$p = p + i;$$

}

Assume $p > n$

then only it'll stop

- 1 $P = p + 1$
1 $0 \leftarrow$ initially
1 $0 + 1$
2 $0 + 1 + 2$
3 $0 + 1 + 2 + 3 + \dots$
4 $1 + 2 + 3 + 4$
⋮
K $1 + 2 + 3 + 4 + \dots + K$

\Rightarrow Assume $p > n$

its stop

$$\boxed{P = \frac{k(k+1)}{2}}$$

$$\frac{k(k+1)}{2} > n$$

$$\boxed{K = \sqrt{n}}$$

$$K = K^2$$

$$K^2 > n$$

$$K > \sqrt{n}$$

$$\boxed{K = \sqrt{n}}$$

Time Complexity #2

Ex: ⑤

for ($i=1$; $i < n$; $i = i * 2$)

{ Stmt;

Stmt;

{

Assume $i > 2n$

$i = 2^k$

$$\frac{i = i * 2}{1}$$

$$1 \times 2 = 2$$

$$2 \times 2 = 2^2$$

$$2^2 \times 2 = 2^3$$

$$2^3 \times 2 = 2^4$$

$$\vdots$$

$$2^K = n$$

$$2^k = n$$

$$K = \log_2 n$$

$$T/C = O(\log_2 n)$$

$$T/C = O(\log_2 n)$$

From this we can observe that where the iterable down variable $i = \text{multiples of } 2$ then T/C would be $O(\log_2 n)$

Ex: ⑥

for ($i=1$; $i < n$; $i = i + 2$)

{ Stmt;

{

$$i = 1 \times 2 \times 2 \times 2 \times 2 \dots = n$$

$$2^k = n$$

$$k = \log_2 n$$

T/C

⑥

for ($i=1$; $i < n$; $i++$)

{ Stmt;

{

$$\underbrace{i + 1 + 2 + \dots + n}_{K = n}$$

$$O(n) T/C$$

⑤ for ($i=1$; $i < n$; $i = i * 2$)

{ Stmt;

$\rightarrow (\log n) T/C.$

* \rightarrow log n may give decimal values

float value

$$n = 8$$

$$i$$

$$i = 1$$

$n=8$	$(i=1, i < n; i = i * 2)$	$i=1$	$i < n$	T/F	$i = i * 2$	Out
		1	1 < 8	T	2	2
①		2	2 < 8	T	4	4
		4	4 < 8	T	8	8
		8	8 < 8	F	X	exit
						End

if $n=10$

$i=$ initially	$i < n$	T/F	$i = i * 2$	i
1	1 < 10	T	2	2
2	2 < 10	T	4	4
④ 4	4 < 10	T	8	8
8	8 < 10	T	16	16
16	16 < 10	F	X	X

$$\log_2 8 = 3$$

$$\log_2 2^3 = 3 \log_2 2 = 3$$

$$\log_2 10 \approx 3.3$$

if "1" Give depth value for 10

logn seal value

ex: ⑥

for ($i=n$; $i>1$,
 $i=i/2$)

{

stmt;

{

(more)

i starts from

n
 $n/2$

$n/2^2$
 $n/2^3$

$\frac{n}{2^K}$

assume

$i \leq 1$

$\frac{n}{2^K} \leq 1$

$\frac{n}{2^K} = 1$

$n = 2^K$

$K = \log_2 n$

T/C $O(\log_2 n)$

ex: ⑦

for ($R_{20}; i \neq k; i \neq j$)
 {
 stmt;
 }

 {
 stmt;
 }

So basically the loop

Runs till the $i \neq k$

Remember do here

Other words, don't matter

Required just

the middle one with

is complex

that is

$i \neq k$

if this becomes

$i \neq j$ then it will
stop terminate

$i \neq m$

$i^2 = n$ not feasible
 $i = \sqrt{n} \rightarrow O(\sqrt{n})$

T/C

All T/C formula's in short :-

Time/Complexity

for ($i=0$; $i < n$; $i++$) = $O(n)$

for ($i=0$; $i < n$; $i = i + 2$) = $O(n/2) = O(n)$

for ($i=n$; $i \geq 1$; $i--$) = $O(n)$

for ($i=1$, $i < n$; $i = i * 2$) = $O(\log_2 n)$

for ($i=1$, $i < n$; $i = i * 3$) = $O(\log_3 n)$

for ($i=n$, $i \geq 1$; $i = i / 2$) = $O(\log_2 n)$

nested loop

$n \leftarrow$ for ($i=0$; $i < n$; $i++$) \rightarrow innermost

~~making 2²~~ for ($j=0$, $j < n$; $j = j * 2$) $\rightarrow O(n \log_2 n)$

$n \leftarrow \log n$ same
 $\log(n)$

$2 \log n + n$
 $O(\log n)$

ex: ⑧

for ($i=0; i < n; i++$)
 {
 stmt; $\rightarrow O(n)$
 }

for ($j=0; j < n; j++$)

{
 stmt; $\rightarrow O(n)$
 }

$$f(n) = 2n$$

$$A(n) = O(n)$$

ex: ⑨ for ($i=0; i < n; i++$) \xrightarrow{n}

{
 for ($j=1; j < n; j = j*2$)
 {
 stmt $\rightarrow n \times \log n$
 }

$$n \times \log n$$

$$n \times \log n$$

$$2 \log n \times n$$

$$T/c = O(n \log n)$$

ex: ⑩

$$P \geq 0$$

for ($i=1; i < n;$
 $i = i*2$)

{
 ~~stmt~~ $i++$; $\rightarrow \log(n)$

for ($j=1; j < P;$
 $j = j*2$)

{
 stmt $\rightarrow \log(P)$

$$\log P = \log n$$

$$\cdot \log P$$

$$O(\log \log n)$$