Reg No. [illegible]
Name: Nithish G

DA - 2

1) Consider the following relation defined on the set of real numbers R

$R_1 = \{(a,b) : a|b\}$ (divides relation);

$R_2 = \{(a,b) : a \geq b\}$;

$R_3 = \{(a,b) : a = -b\}$;

$R_4 = \{(a,b) : a \neq b\}$;

$R_5 = \{(a,b) : b = a^2\}$;

find reflexive, symmetric, antisymmetric and transitive.

Soln:

i) $R_1 = \{(a,b) : a/b\}$

Reflexive

w.r.t $0 \in R$, but not 0/0 divides

$\therefore (R_1)$ is not reflexive

Symmetric

let $a, b \in R$ a/b but b/a in general

$\therefore (R_1)$ is not symmetric

Antisymmetric:

let $a, b \in R$ such that a/b & b/a only when $a = b$

$\therefore (R_1)$ is antisymmetric

Transitive

$a, b, c \in R$ such that a/b & b/c then w.r.t a/c

$\therefore (R_1)$ is transitive

ii) $R_2 = \{(a,b): a \geq b\}$

Reflexive	Symmetric	Antisymmetric	Transitive
$a \in R$ $(a,a) \in R, a \geq b$ ∴ $(R,2)$ is reflexive	$a,b \in R$ $a \geq b$ but $b \not\geq a$ in general ∴ $(R,2)$ is not symmetric	$a,b \in R$ $a \geq b$ & $b \geq a$ only when $a = b$ ∴ $(R,2)$ is antisymmetric	$a,b,c \in R$ such that $a \geq b$ & $b \geq c$ then $a \geq c$, ∴ It is transitive.

∴ $R_2 = \{(a,b): a \geq b\}$ is Partial ordering

iii) $R_3 = \{(a,b): a \neq b\}$

Reflexive	Symmetric	asymmetric	reflexive
$a \in R$ $(a,a) \in R$ but $a \neq a$ ∴ $(R,3)$ is not reflixive	let $a,b \in R$ $a \neq b$ & $b \neq a$ ∴ $(R,3)$ is Symmetric	$a,b \in R$ $a \neq b, b \neq a$ & $a = b$ but here $a \neq b$ so $(R,3)$ is not asymmetric	$a,b,c \in R$ $a \neq b$ & $b \neq c$ & $a \neq c$ ∴ $(R,3)$ is transitive

$(R,3) = \{(a,b): a \neq b\}$ is neither partial ordering nor equivalence relation.

iv) $R_4 = \{(a,b) : a = -b\}$

Reflexive	Symmetric	Antisymmetric	Transitive
$a \in R$	$a, b \in R$	$a, b \in R$	Let $a, b, c \in R$
$(a,a) \in R$ but	$a \neq b$	$a = -b$	$a = -b$, $b = -c$
$a \neq -a$ is	but $b \neq -a$	& $b = -a$	then $a = -c$
not reflexive	in general	$\Rightarrow a = -b$	$\therefore (R,4)$ is
	$(R,4)$ is not	$\therefore (R,4)$ is	transitive.
	Symmetric	antisymmetric	

$\therefore$ It is neither partial ordering nor equivalence relation.

v) $R_5 = \{(a,b) \cdot b = a^2\}$

Reflexive	Symmetric	asymmetric	transitive
$a \in R$	$a, b \in R$	$a, b \in R$	$a, b, c \in R$
(a,a) but $a = a^2$	$b = a^2$	$b = a^2$	$b = a^2$, $a = c^2$ then
in general	$b^2 = a$	$b^2 = a$	$b = c^2$
$\therefore (R,5)$ is not ~~transit~~ reflexive	$(R,5)$ is not Symmetric	$(R,5)$ is antisymmetric	$(R,5)$ is transitive

$\therefore$ It is neither partial ordering nor equivalence relation.

Tabulation:

sets	Reflexive	Symmetric	antisymmetric	Transitive
$R_1 = \{(a,b): a>1\}$	✗	✗	✓	✓
$R_2 = \{(a,b): a \geq b\}$	✓	✗	✓	✓
$R_3 = \{(a,b): a \neq b\}$	✗	✓	✗	✓
$R_4 \{(a,b): a = -b\}$	✗	✗	✓	✓
$R_5 = \{(a,b): b = a^2\}$	✗	✗	✓	✓

2)

Prove that R is equivalence relation, if R is relation on the set of positive integers such that $(a,b) \in R$ if and only if a^2+b is even.

Soln:

let X = +ve integers

$R = \{(a,b) \mid a^2+b \text{ is even}\}$

(i) Reflexive property:

For any $a \in X$, we see that a^2+b is a^2+a is even. $a=1$

$a^2+a =$ even

$1+1 = 2$, So it is reflexive

ii) Symmetric property:

$a, b \in X$ we see that $(a,b) \in R$

a^2+b is even

b^2+a is even $(b,a) \in R$

So it is symmetric.

iii) Transitive relation:

Suppose that $a, b, c \in x$ such that

$(a,b) \in R$ & $(b,c) \in R$

$\Rightarrow a^2+b$ is even & b^2+c is even

$\Rightarrow a^2+b + b^2+c$ is also even

$\Rightarrow a^2+c$ is even $\Rightarrow (a,c) \in R$

This is equivalent relation.

3) Let A be the set of factors of positive integer m & let $\leq$ be the relation divides i.e.

$\leq = \{(x,y) \mid x, y \in A$ x divides y$\}$

a) Show that $(A, \leq)$ is a poset.

b) Draw hase diagram for i) m=30,

ii) m=45, iii) m=100

Soln:

a) Let $A = \{a, b, c\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\},$
$\{b,c\}, \{a,b,c\}\}$

Now $(P(A), \leq)$ is a lattice

Claim: $(P(A), \leq)$ is a poset.

i) Reflexive:

W.K.T for subset $a \in P(A)$, $a \leq a$

So $(P(A), \leq)$ is reflexive

ii) Antisymmetric:

Let $A, B, \in P(A)$ such that $A \leq B$ and

$B \leq A \Rightarrow \boxed{A = B}$

So $(P(A), \leq)$ is antisymmetric.

iii) transitive:

Let $A, B, C \in \ell(A)$ such that $A \leq B$ & $B \leq C$

$\Rightarrow A \leq C$

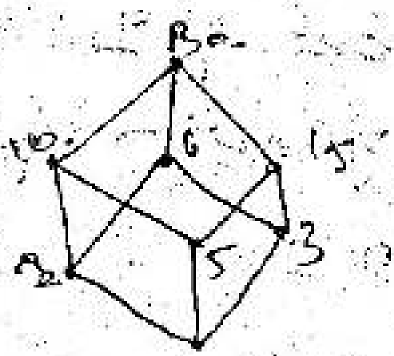
So $(\ell(A), \leq)$ is transitive.

From i), ii), iii) $(\ell(A), \leq)$ is a poset.

b) Hasse diagram.

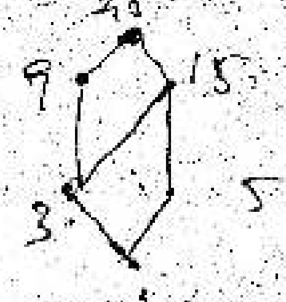
i) $m = 30$

$A_m = A_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$



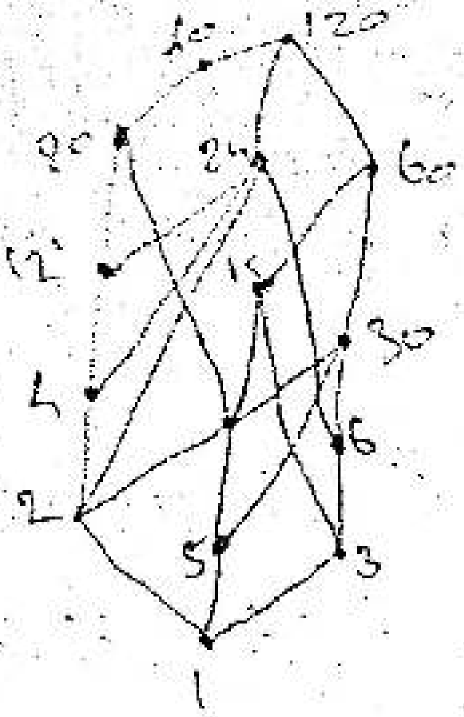
ii) $A_m = 45$

$A_m = \{1, 3, 5, 9, 15, 45\}$ $\hookrightarrow$



iii) $m = 120$

$A_m = \{1, 2, 3, \cdots 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120\}$.



(i) Determine which of the following are function! If so, investigate whether they are one to one or onto or both:

Let Z be the set of Integers. Let $f: Z \to Z$ be a function defined as follows for all $x \in Z$:

a) $f(x) = 1/x$; b) $f(x) = x-1$; c) $f(x) = \lceil \frac{x}{2} \rceil$

Soln:

a) the function of f is not a function. Because it is not define at $x = 2, 3, \ldots$ and also $x = -1, -2 \ldots$

iv) $f: Z \to Z$, $f(x) = x-1$ the function defined here is valid, for all integers. Since we get unique answer belongs to $Z = x$ which is also 'Z'.

a) claim: f is one to one.

$$\text{let } f(x) = f(y)$$

$$x-1 = y-1$$

$$x = y-1+1$$

$$\boxed{x = y} \text{ so } f \text{ is one to one.}$$

* f is not onto clearly since not every z ie +ve z the co-domain can be written in the form $x-1$ where $x \in Z$.

v) $f: Z \to Z$ $f(x) = \lceil x/2 \rceil$ the function is defined here is valid, for all Z since we get unique answer belongs to $Z = x$ which is also a 'Z'.

a) Claim: f is one to One.

let $f(x) = f(y)$

$$\lceil x/2 \rceil = \lceil y/2 \rceil \boxed{x = y}$$

so it is one to one

ii) f is Not onto clearly since not every +ve integer in the co domain can be written in the form x^2 where $x \in Z^+$

5) let 'Z' be the set of integers let $f: Z \times Z \to Z$ be function defined as follows for all $x, y \in Z$. show that the following function are onto but not one to one as

$f(x, y) = x + y$: b) $f(x, y) = x, y$

Soln:

a) $f(x, y) = x + y$

this is a valid function since for any 2 integers x, y, we can find the value $x + y$ which is also a unique integer.

i) verify:

f is 1 to 1"

$f(x_1, y_1) = f(x_2, y_2)$

$x_1 + y_1 = x_2 + y_2$

$x_1 - x_2 = y_2 - y_1$

ii) verify: "f is onto"

for every integer $z \in Z$, we can always find x, y such that $z = x + y$

eg a) $4 = 2 + 2$

b) $8 = 4 + 4$

etc --

So f is onto

on solving this equation we get several combination for x's & y's which a

so f is not one-to-one.

bc $f(x,y) = x \cdot y$

This is a valid function since for any integers x, y, we can find the value $x \cdot y$ which is also Z

*) verify :- f is 1 to 1

$f(x_1,y_1) = f(x_2,y_2)$

$x_1y_1 = x_2y_2$

$x_1/x_2 = y_2/y_1$

on solving this equation we get several combination for x's & y's which are

*) verify :- f is onto.

for every integer $z \in Z$ we can always find x, y such that $z = xy$

eg> $4 = 4 \cdot 1$

$8 = 4 \cdot 2$

So f' is onto

6.Q If $f(x) = x^2$, $g(x) = (x+1)$ and $h(x) = 2x$, find the following composition of function.

a) fog; b) $fogoh$; c) hof d) $hogof$

Sol

a) $fog = f(g(x)) = f(x+1) = x^2+1$

b) $fogoh = f(g(h x)) = f(g(2x)) = f(2(x+1))$

$\Rightarrow f(2x+2) = 2x^2+2$

c) $hof = h(f(x)) = h(x^2) = (2x)^2$

$= 4x^2$

d) $hogof = h(g(f x)) = h(g(x^2))$

$\Rightarrow h(g(x+1)^2) = h(x^2+2x+1)$

$\Rightarrow (2x)^2 + 2(2x) + 1 = 4x^2+4x+1$

7) How many permutation of the letters ABCDEFG contains:

a) A string BCD? b) A string BA & GF,

c) the string ABC & DE? d) The string CBA & ...

Soln

a) the letter BCD must appear always together as one block so, these three letters must be converted as one.

no. of string possible = 6! = 720

b) The letter BA and GF must appear always together as one block. So, these GF & BA must each counted as one.

no. of string possible = 6! = 720

c) letter ABC & DE must appear always together as one block so

have three letters and two [illegible]
[illegible] contained [illegible]

no. of [illegible]

[illegible] must appear
[illegible]
[illegible]

[illegible]

no. of strings [illegible]

Suppose that a department contains
10 men & 15 women. How many ways
are there to form a committee with
six members if it must have more women
than men?

How many ways are there to [illegible]
[illegible] 12 Disc [illegible] of a [illegible]
having [illegible]

a) To find the ways to select a committe with six members, if it must have more women than men, we can use sum & product rule.

no. of ways $= (1 + 15) \times (2 + 4) = 6 \times 6 = 36$

b) to find no. of ways to select first 3 prize winners of a competition having 15 competitor. we can use Product rule.

no. of ways $= 15 \times 14 \times 13 \Rightarrow 2730$

c) Two bit string can be made with length 8 which start with 1 bit an end with 00.

$\Rightarrow 1\times2\times2\times2\times2\times2\times1\times1$

$\Rightarrow 2^5$

$\Rightarrow 32$

...hat is ~~the~~ minimum number of student

...

...

...ince we are required to find the

... no of student. 90 is sufficient to

... all 50 states with 29 students,

... one more student

$N = ?$

$\left\lceil \frac{N}{50} \right\rceil = 30$ In this case suppose that $\left\lceil \frac{N}{k} \right\rceil = r$

...tor $N = k \times (r-1)+1$

$N = (50 \times 29)+1$

$N = 1450+1 \Rightarrow \boxed{N = 1451}$

A) Let us lable the name of 6 people as A, B, C, D, E and F. Consider the Person A. Then A will have friends or enemies among the remaining 5 people. By Pigeonhole Principle 'A' will have atleast $\lceil 5/2 \rceil = 3$ friends or e[illegible]

Without loss of generality, suppose fix that 'A' has 3 friends say B, C, D.
Among the persons B, C, D, if any two of them are friends say B and C then persons 'A', 'B' and C form the group of three mutual friends. Otherwise 'B', C and D form the group of three mutual enemies.

Hence the result.

(10) $A_1 = \left\lfloor \frac{500}{2} \right\rfloor = 250$ $\quad |A| = 250$

$A_2 = \left\lfloor \frac{500}{3} \right\rfloor = 166$

$A_3 = \frac{500}{5} = 100$ $\quad \therefore |A_2| = 334$

$\therefore |A_3| = 400$

$A_4 = \left\lfloor \frac{500}{7} \right\rfloor = 71$ $\quad \therefore |A_4| = 500 - 71 = 429$

divisible by 6, 10, 14, 15, 21, 35.

$|A_1 \cap A_2| = \left\lfloor \frac{500}{6} \right\rfloor = 83$ $\quad \therefore |A_1 \cap A_2| = 500 - 83 = 417$

$|A_1 \cap A_3| = \left\lfloor \frac{500}{10} \right\rfloor = 50$ $\quad \therefore |A_1 \cap A_3| = 450$

$|A_1 \cap A_4| = \left\lfloor \frac{500}{14} \right\rfloor = 35$ $\quad \therefore |A_1 \cap A_4| = 465$

$|A_2 \cap A_3| = \left\lfloor \frac{500}{15} \right\rfloor = 33$ $\quad \therefore |A_2 A_3| = 500 - 33 = 467$

$|A_2 \cap A_4| = \left\lfloor \frac{500}{21} \right\rfloor = 23$

$|A_3 \cap A_4| = \left\lfloor \frac{500}{35} \right\rfloor = 14$ $\quad \therefore |A_2 \cap A_4| = 500 - 23 = 477$

$|A_1 \cap A_2 \cap A_3| = \left\lfloor \frac{500}{30} \right\rfloor = 16$

$|A_2 \cap A_3 \cap A_4| = \left\lfloor \frac{500}{105} \right\rfloor = 4$