1

Reg. No.: 19BCS0012

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Assignment - 1

1) List the members of these sets.

a) $\{x \mid x$ is a real number such that $x^2=1\}$

Ans) There are two real numbers whose square equal 1 i.e. 1 and -1 so $\{1,-1\}$

b) $\{x \mid x$ is positive integer less than $12\}$

Ans)

x is positive integer so

$b=\{1,2,3,4,5,6,7,8,9,10,11\}$

c) $\{x \mid x$ is square of an integer and $x<100\}$

$c=\{1,2,3,4,5,6,7,8,9\}$

d) $\{x \mid x$ is integer such that $x^2=2$

It's a null set $\{\}$; no integers whose square is equal to 2

2) Let $A = \{a, b, c\}$, $B = \{x, y\}$, $C = \{0, 1\}$

i) $A \times B \times C$

$\Rightarrow$

i) $A \times B \times C = \{(a, x, 0), (a, x, 1), (a, y, 1)$

$(a, y, 0), (a, y, 1), (b, x, 0),$

$(b, x, 1), (b, y, 0), (b, y, 1),$

$(c, x, 0) (c, x, 1) (c, y, 0),$

$(c, y, 1)$

ii) $C \times B \times A = \{(0, x, a), (0, x, b), (0, x, c), (0, y, a),$

$(0, y, b), (0, y, c), (1, x, a), (1, x, b),$

$(1, x, c), (1, y, a), (1, y, b), (1, y, c)\}$

iii) $B \times B \times A = \{(x, x, a), (x, x, b), (x, x, c),$

$(x, y, a), (x, y, b), (x, y, c),$

$(y, x, a), (y, x, a), (y, x, c)$

$(y, y, a), (y, y, b), (y, y, c)\}$

3) Define the union, intersection, difference and complement operations on sets.

If $X = \{0 \ldots 10\}$ $A = \{0,2,4,6,8,10\}$

$B = \{0,1,2,3,4,5,6\}$ $C = \{4,5,6,7,8,9,10\}$

a) $A \cup B \cup C = \{0,1,2,3,4,5,6,7,8,9,10\}$

b) $A \cap B \cap C = \{4,6\}$

c) $(A \cup B) \cap C = \{4,5,6,8,10\}$

d) $(A \cap B) \cup C$

$A \cap B \Rightarrow \{0,2,4,6\} \cup C$

$(A \cap B) \cup C = \{0,2,4,5,6,7,8,9,10\}$

e) $(A - B) - C$

$\Rightarrow \{8,10\} - C \Rightarrow \{\phi\}$

f) $\overline{A \cap C}$ $\{4,6,8,10\}$

$\Rightarrow$ ~~$\{1,2,3,4,5,6$~~ $7,8,9$

$\overline{A \cap C} \Rightarrow \{1,2,3,7,9\}$

4) $A_i = \{1, 2, 3, \dots, i\}$, $i = 1, 2, 3, \dots$ and $B_i = \{-2, -1, 0, 1, 2, 3, \dots, i\}$.

$i = 1, 2, \dots$

find: $\bigcup_{i=1}^{n} A_i$

$$\bigcup_{i=1}^{n} A_i = A_n = \{1, 2, 3, n\}$$

ii) $\bigcap_{i=1}^{n} A_i$

$$\bigcap_{i=1}^{n} A_i = A_1 = \{1\}$$

iii) $\bigcup_{i=1}^{n} B_i$

$$\bigcup_{i=1}^{n} B_i = B_n = \{-2, -1, 0, 1, 2 \dots n\}$$

iv) $\bigcap_{i=1}^{n} B_i$

$$\bigcap_{i=1}^{n} B_i = \{\dots -2, -1, 0, 1\}$$

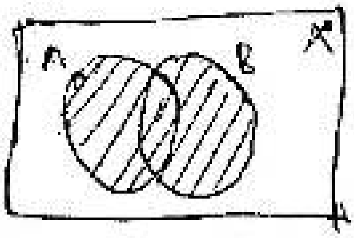
3) Let A and B any sets with universal set X. Using venn diagram prove the De-morgan's law of the set A and B

$(A \cup B)' = A' \cap B'$

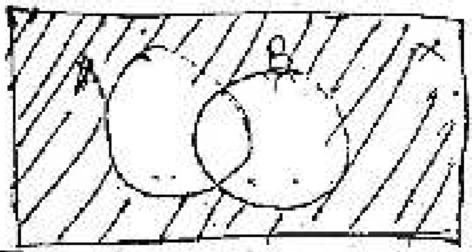
$(A \cap B)' = A' \cup B'$

L.H.S

$A \cup B$

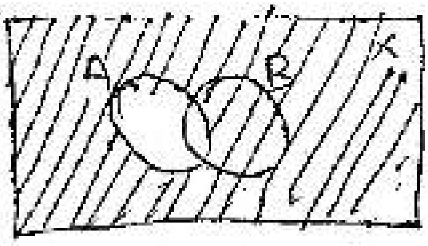


$(A \cup B)'$

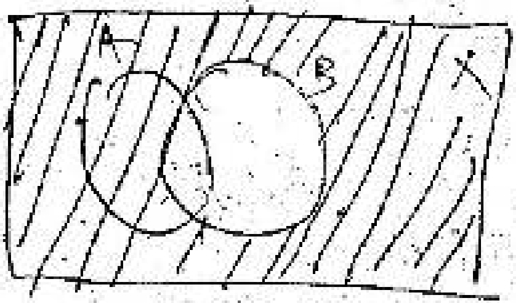


R.H.S

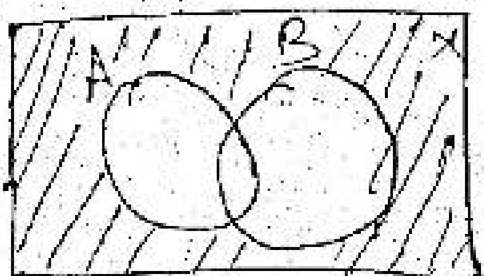
A'



B'



$A' \cap B'$



$\therefore (A \cup B)' = A' \cap B'$

i) Prove the following absorption laws.

a) $A \cup (A \cap B) = A$

$A = \{0, 1, 2, 3, 4\}$

$B = \{2, 3, 4, 5, 6\}$

$A \cap B = \{3, 4\}$

$A \cup (A \cap B) = \{0, 1, 2, 3, 4\}$

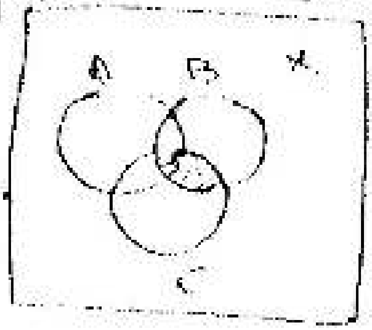
$\boxed{A \cup (A \cap B) = A}$

b) $A \cap (A \cup B) = A$

$A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$

$A \cap (A \cup B) = \{0, 1, 2, 3, 4\}$

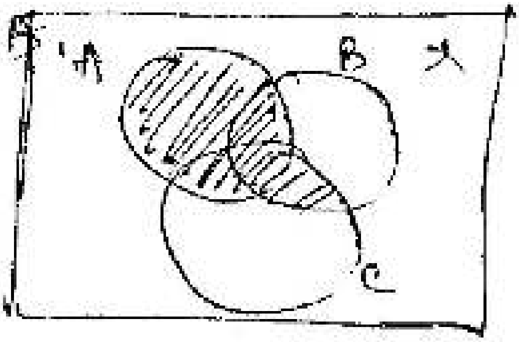
R.H.S $\quad A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$



$B\cap C$

$(A\cup B)\cap(A\cup C)$

$\therefore A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$

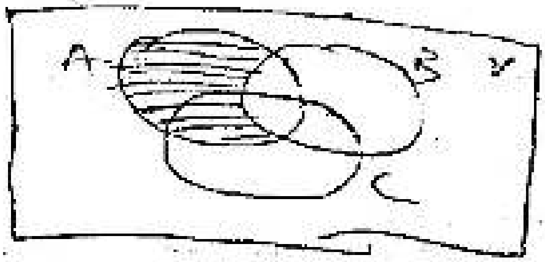


$A\cup(B\cap C)$

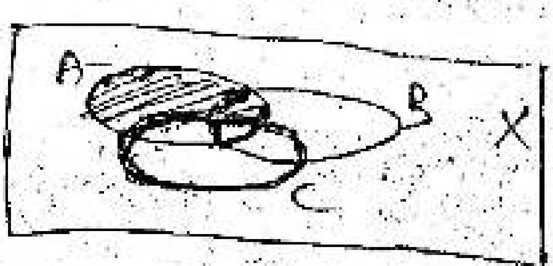
iii) $(A-B)-C = (A-C)-(B-C)$

L.H.S

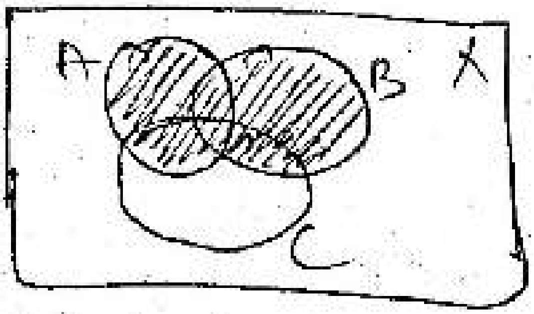
$(A-B)$



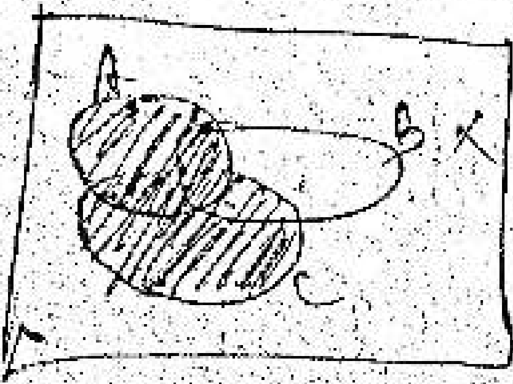
$(A-B)-C$



R.H.S

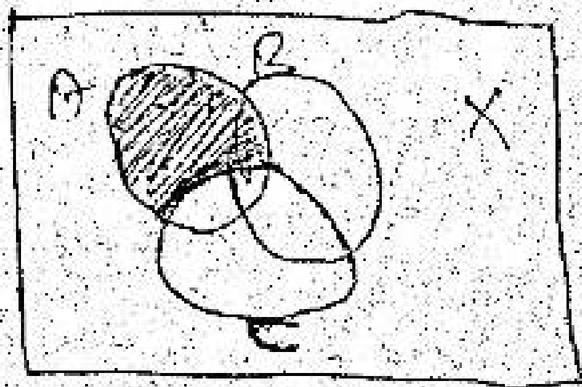


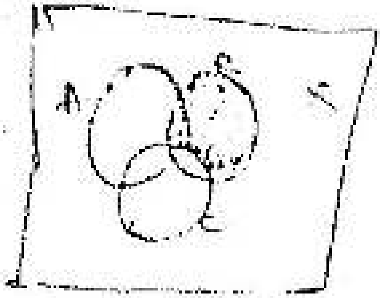
$A\cup B$



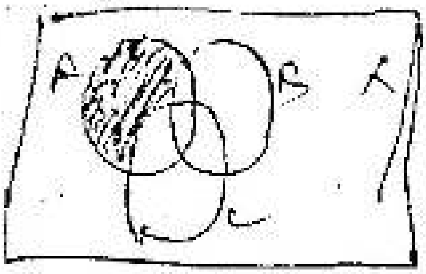
$A\cup C$

R.H.S





$(B-C)$



$(A-C)-(B-C)$

$\therefore (A-B)-C =$

$(A-C)-(B-C)$

Q. Determine:

$P(S)$, $S = \{\alpha, \beta, \gamma, \delta\}$;

$P(S) = \{\phi, \{\alpha\}, \{\beta\}, \{\gamma\},$

$\{\delta\}, \{\alpha, \beta\}, \{\alpha, \gamma\}, \{\alpha, \delta\}, \{\beta, \gamma\}, \{\beta, \delta\},$

$\{\gamma, \alpha\}, \{\gamma, \beta\}, \{\gamma, \delta\}, \{\delta, \alpha\}, \{\delta, \beta\},$

$\{\delta, \gamma\}\}, \{\alpha, \beta, \gamma, \delta\}\}$

Q) using the mathematical induction prove.

i) $1^3+2^3+3^3+ \text{———} + n^3 = \frac{n^2(n+1)^2}{4}$

we use the method of induction to prove the above result:-

let $P(n) = 1^3+2^3+3^3+\cdot\cdot+n^3 = \frac{n^2(n+1)^2}{4}$

Verify P(1)

$P(1) = \frac{1^2(1+1)^2}{4} \Rightarrow 4/4 = 1$

P(1) is true.

Inductive step:

a) Assume P(K) is true

i.e. $P(K) = 1^3+2^3+\cdot\cdot+K^3 = \frac{K^2(K+1)^2}{4}$

$P(K+1) = 1^3+2^3+\cdot\cdot+K^3+(K+1)^3$

$= P(K)+(K+1)^3$

$\Rightarrow \frac{K^2(K+1)^2}{4}+(K+1)^3$

$\Rightarrow \frac{K^2(K+1)^2+4(K+1)^3}{4}$

$$\Rightarrow (k+1)^2\left[\frac{k^2+4(k+1)}{4}\right]$$

$$\Rightarrow (k+1)^2\left[\frac{k^2+4k+4}{4}\right]$$

$$\Rightarrow \frac{(k+1)^2\,(k+2)^2}{4}$$

Hence $p(k+1)$ is true.

$P(n)=\frac{n^2(n+1)^2}{4}$ is true.

ii) $a+ar+ar^2+\ldots+ar^n=\frac{ar^{n+1}-a}{r-1}$

we can use the method of induction to prove the above result.

i) Basic step:-

i) $n=0$

$$P(0)=\frac{a(r)^{0+1}-a}{r-1}=\frac{a(\cancel{r-1})}{(\cancel{r-1})}=a$$

$P(0)$ is true

ii) $n=1$

$P(1) = a_1(1+r)$

$\Rightarrow \frac{a(r^2-1)}{r-1} = \frac{a(r+1)(r-1)}{r-1}$

$\Rightarrow a(r+1) = a + ar \therefore P(1)$ is true.

2nd Induction step.

a) Assume that the result is true for $P(K)$

$P(K) = a + ar + ar^2 + \dots + ar^K = \frac{ar^{K+1} - a}{r-1}$

b) Prove the result for $P(K+1)$.

$P(K+1) = a + ar + ar^2 + \dots + ar^K + ar^{K+1}$

$\Rightarrow P(K) + ar^{K+1}$

$\Rightarrow \frac{ar^{K+1} - a}{r-1} + ar^{K+1}$

$\Rightarrow \frac{ar^{K+1} - a + (r-1)(ar^{K+1})}{r-1}$

$\Rightarrow \frac{ar^{K+1} - a + r \cdot ar^{K+1} - ar^{K+1}}{r-1}$

$P(K+1) \Rightarrow \frac{ar^{K+2} - a}{r-1}$

10) Using the mathematical

a) $\sum_{j=1}^{n} j^2 = \frac{n(n+1)(2n+1)}{6}$

we use the method of induction to prove the above result.

$P(1) = \frac{1(1+1)(2+1)}{6} \Rightarrow 2 \cdot 3/6 = 1$

$\therefore$ So $P(1)$ is true.

a) Assume $P(K)$ is true

i.e: $P(K) = 1^2 + 2^2 + \cdots + K^2 = \frac{K(K+1)(2K+1)}{6}$

b) Prove the result for $P(K+1)$

$P(K+1) = 1^2 + 2^2 + 3^2 + \cdots + K^2 + (K+1)^2$

$\Rightarrow P(K) + (K+1)^2$

$\Rightarrow \frac{K(K+1)(2K+1)}{6} + (K+1)^2$

$\Rightarrow \frac{K(K+1)(2K+1) + 6(K+1)^2}{6}$

$\Rightarrow \frac{K+1[K(2K+1) + 6(K+1)]}{6}$

$\Rightarrow (K+1)(2K^2 + K + 6K + 6)$

$$\Rightarrow \frac{(k+1)(2k^2+7k+6)}{6}$$

$$\Rightarrow \frac{(k+1)(2k^2+3k+4k+6)}{6}$$

$$\Rightarrow \frac{(k+1)(k+2)(2k+3)}{6} \Rightarrow .$$

$$P(k+1) = \frac{(k+1)(k+2)(2k+3)}{6}$$

Hence $P(k+1)$ is true.

therfor $P(n) = \frac{n(n+1)(2n+1)}{6}$ is true

ii) $\sum_{j=1}^{n}(2j-1)=n^2$

we can use the method of induction to prove the above result.

$P(n) = 1+3+5+ \cdots + (2n-1) = n^2$

Basic step:

$P(1) \cdot 2(1)-1=1$

So $P(1)$ is true.

2.) Inductive step

Assume $P(k)$ is true

$P(k) = 1+3+5+ \cdots + (2k-1) = k^2$

b) We have to prove the result for $P(K+1)$.

$P(K+1) = 1 + 3 + 5 + \dots + (2K-1) + (2K+1)$

$\Rightarrow P(K) + 2K + 1$

$\Rightarrow K^2 + 2K + 1 \Rightarrow (K+1)^2$

Hence $P(K+1)$ is true

Therefore $P(n) = n^2$ is true.