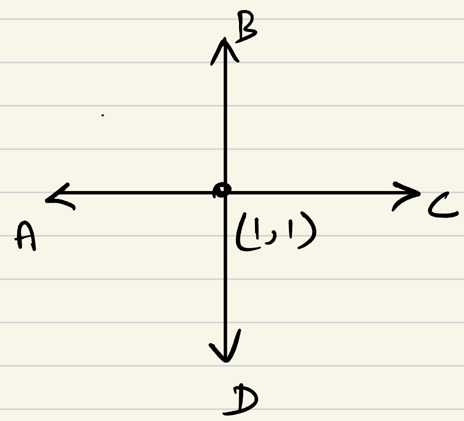
Slope

Given



A, B, C, D → values taken with respect to surface with 0° slope

To determine,

turns of nut in each direction

P, Q → offset at y (front, back)

↓

offset at x (left, right)

Plane equation

$$z(x, y) = px + qy + r$$

$p \rightarrow$ slope in x

$q \rightarrow$ slope in y

$r \rightarrow$ offset by z

So, goal is to get

$\Rightarrow$ For each known point of (x_i, y_i, z_i)

↳ It gives plane eq

$\downarrow$

$$z_i \approx px_i + qy_i + r$$

So, for 4 data point we have to find $W = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$

In matrix

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \\ x_4 & y_4 & 1 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}$$

$$\underset{A}{} \qquad \underset{W}{} \qquad \underset{Z}{}$$

A W Z

So, $A \cdot W = Z$

Since we have many unknowns Least square method,

For $A \cdot W - Z = 0$

$\Rightarrow \min |A \cdot W - Z|^2 = 0$

To find min,

Gradient of fn is found

$$\Rightarrow \frac{d}{dw}(A \cdot w - z)^2 = 0$$

$$\Rightarrow A^T A w - A^T z = 0$$

(by standard normal eq)

To find w,

$$A^T A w = A^T z$$

$$w = (A^T A)^{-1} (A^T z)$$

x, y → Reference point

↓

Taken as (1, 1)

$$A = \begin{bmatrix} x & 0 & 1 \\ 0 & y & 1 \\ -x & 0 & 1 \\ 0 & -y & 1 \end{bmatrix} \qquad z = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$$

$a, b, c, d \rightarrow$ Value from UI (encoder)

For $(A^T A) = \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 2y^2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$$A^T z = \begin{bmatrix} x(a-c) \\ y(b-d) \\ a+b+c+d \end{bmatrix}$$

$$W = (A^T A)^{-1} A^T z = \begin{bmatrix} \frac{a-c}{2x} \\ \frac{b-d}{2y} \\ \frac{a+b+c+d}{4} \end{bmatrix}$$

so final equation

$$z(x,y) = \left(\frac{a-c}{2x}\right)x + \left(\frac{b-d}{2y}\right)y + \left(\frac{a+b+c+d}{4}\right)$$