Promys India Problems Dataset
Solutions

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3) Given:

i) 1/1 in S
ii) a/b in S, a/b in low terms (HCF is 1), then b/2a is in S
iii) if a/b and c/d in S, then a+c/b+d is in S
What are the numbers in S and which are not?

Solution:

1/1 in S = a/b

then ,
b/2a = ½.$_{1}$ = ½ (let's assume and denote as c/d)

Then,

$a/b = 1/1$, $c/d = 1/2$

$a+c/b+d = 1+1/1+2 = 2/3$

- fraction a/b is in **lowest terms** and satisfies $a,b >= 1$
- The set S consists of all positive rational numbers a/b where a and b are coprime integers, $a>0$, and $b>0$.
- If we assume $b/2a = 3/4$, then the result will be another Rational number a/b.

That is **S = {a/b a,b $\in$ Z^+ , greatest common divisor(a,b) = 1}.**

4) Given:

$n>=2$
$X1 = 1$
$X2 = 2$
Eqn = $X_{(n-1)} . X_{(n)} . X_{(n+1)} = 1$

Solution:

For finding X3 (from applying n=3 in equation),

$X1.X2.X3 = 1$ (Both X1 and X2 given)
$1*2*X3 = 1$
$X3 = 1/2$

Now for X4 (from applying n= in equation),

$X2 . X3 . X4 = 1$ (Both X2 and X3 we know)
$2*½*X4 = 1$
$X4 = 1$

Now for X4 (from applying n= in equation),

$X3 . X4 . X5 = 1$ (Both X3 and X4 we know)
$½*1*X5 = 1$
$X5 = 2$

Then The sequence would be:
1 , 2 , ½ , 1 , 2 , ½ ,..

Actually the sequence would Repeat and The start value or term would be '½'.

½ , 1 , 2 , ½ , 1 , 2 ,..

Then finding for Y sequences,

Given:

Equation = $y_{n-1} . y_{n+1} + y_n$

$y1 = 1$
$y2 = 2$
$n>=2$

Solution:

For finding $y3$ (from applying $n=2$ in equation),

$y1.y3+y2 = 1$ (Both $y1$ and $y2$ given)
$y3+2 = 1$
$y3 = -1$

Now for $y4$ (from applying $n=3$ in equation),

$y2 . y4 + y3 = 1$ (Both $y2$ and $y3$ we know)
$2y_4 + (-1) = 1$
$y4 = 1$

Now for $y5$ (from applying $n=4$ in equation),

$y3 . y4 + y5 = 1$ (Both $y3$ and $y4$ we know)
$-1*y5+1 = 1$
$y5 = 0$

Probably $y6$ will be 1,

Then the Sequence would be,

$y1, y2 , y3 , y4…$

$= 1 , -1 , 0 , 1 , -1 , 0 , 1 , -1 , 0$ ……….

Actually the sequence would Repeat and The start value or term would be ‘0’.

0 , 1 , -1 , 0 , 1 , -1 ,.....

5) Given:

3*3 grid box, with numbers written from 1 to 9 accordingly.
Product of rows written
Product of Columns written

1	2	3	Product = 6
4	5	6	Product = 120
7	8	9	Product = 504
Product = 28	Product = 80	Product = 162	

Arrange them in an order of getting the products of each row and column same (Product(rows) = Product(columns))

Solutions:

- It would be complex to find the arrangement(s) where we can get (Product(rows) = Product(columns)). Because as per the No.of ways of arrangements we can do in 3*3 grid is 9! (9 factorial) which is = 3,62,880 (No.of Possible arrangements in total)

- To find that arrangement of getting (Product(rows) = Product(columns)) we could be going through arranging all these arrangements which is a big and complex (might be impossible) no. of ways to do.
- Same follows for the 5*5 (25!) and 11*11 (121!) grid as well, as the value of 'n' increases the arrangement becomes more and more complex and impossible to arrange to.
- Hence, As the value of 'n' increases the possibility becomes impossible further

7) Given:

Equilateral Triangle with 2 vertices have integer coordinates while the third vertex is between 1/10 distance of a point with integer coordinates.

What about within 1/100? Within 1/n?

Solution:

- In 1/10, it is difficult to find as the integer value is not near with between 1/10 and lesser
- In 1/100 , it is possible to get integer values as follows
- My assumption - Between 1 - 10 There is less possibility where the numbers are also less. Whereas in 1-100 the possibility is more as there are more numbers in between.
- Hence the value of 'n' is increased We would likely have a better chance of getting the third vertex between them.

8) Given:

3*3 grid with letters arranged inside with repetition.
Drawing lines through the bottom left that hits the letters
For example the set of letter retrieved from doing this is (for left) {A,B,C}.
For right {A,F,H}

G	H	I
D	E	F
A	B	C

*And letters repeating in further grids

Do these lines pass through more than three different Letters?
Can you get the same set of letters From two different lines?
Find four sets of Letters that you can get from drawing lines in this grid?

What about these in a 5*5 grid?
What about 6*6 grid?
What about these in 7*7 grid?

Solution:

- No, There is only n*n (3*3 , 5*5, 7*7 , 6*6) grids and further the letters are repeated. So, The lines do not pass through more than 'n' letters (for all 3*3 , 5*5 , 7*7 , 6*6 grids)
- Mathematically, The slopes of each line are different/unique so they pass in different directions. Hence we won't get the same set of letters from Two different lines hence their slopes are different.
- For 3*3 the sets are {A,D,G} , {A,H,F} , {A,E,I} , {A,B,C}
- For 5*5 the sets are {A,B,C,D,E} , {A,G,M,S,Y} , {A,F,K,P,U} , {A,H,N,T}
- For 6*6 the sets are {A, B, C, D, E, F}, {A, G, M, S, Y, AE}, {A, H, O, V, AC, AJ}, {A, I, Q, W}
- For 7*7 the sets are {A, B, C, D, E, F, G}, {A, H, O, V, AC, AJ, AQ}, {A, I, Q, Y, AG, AO}, {A, J, R, Z}

2) Given:

Hare's first jump: The rubber band stretches to 1 km = 1,00,000 cm 1km=100,000cm. Flea jumps 1 cm 1cm forward.

The rubber band stretches to 2 km = 2,00,000 Cm 2km=200,000cm. Flea jumps 1cm, but its position on the rubber band is now scaled by the new total length

After n n jumps, the rubber band has stretched to

n km = 100000n cm

If the flea was previously at position x x, its new position x' x ′ is scaled by the ratio of the old length to the new length: x'

X’ = x.Old Length / New length

Solution:

1st Jump:

1,00,000 cm and flea jumps 1 cm

2nd Jump:

2,00,000 cm and the flea jumps one more 1 cm

So , using the derived Formula

X’ = 1* 1,00,000 / 2,00,000

X’ = 0.5 cm

The flea jumps 1 cm so

0.5+1 = 1.5

X’ = 1.5 * 2,00,000 / 3,00,000

X’ = 1 cm

And the flea jumps 1 cm so its,

1+1 = 2

Hence the sequence would be $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \ldots\ldots$

- Hence,
  Yes, the flea will eventually catch the hare, but it will take an incredibly long time.
- This is because,
- The flea makes infinite progress due to the divergence of the harmonic series.
- The hare keeps moving forward, but the rubber band stretches proportionally, allowing the flea to "scale up" its progress over time.

Sources: (Faculty , Chatgpt , Google Search , Math forums [like, mathforums.com])