ES-331 Probability and Random Processes

Image Restoration

Assignment-2

Introduction

Image can be corrupted due to various reasons like noise, distortion, exposure, defocus blur, movement blur etc. Distortion occurs in real life while transmitting an image through a medium or while taking an image. This leads to some loss of information. Thus we want to restore the information. It seems that blurring is an irreversible operation and that the information is lost, especially in case of a big blur radius where everything the image seems to be smoothed very much. However, in most cases we know the properties of the medium from which the image is corrupted and the image can be restored. In this project, we consider the case of defocus blur and additive gaussian noise.

The Blurring process

Let f be the original image, let h be be the blurring function, let g be the resulting blurred image and let n be the noise function. Then,

$$g(x,y) = f(x,y) * h(x,y) + n(x,y)$$

Where $*$ denotes convolution.

The process of applying the blur function is convolution, i.e., some path of the image convoles into a pixel of the blurred image. If no noise is added to the system then we can restore the image by simply reversing the convolution process i.e. by using deconvolution.

In this project we use the gaussian blur function.

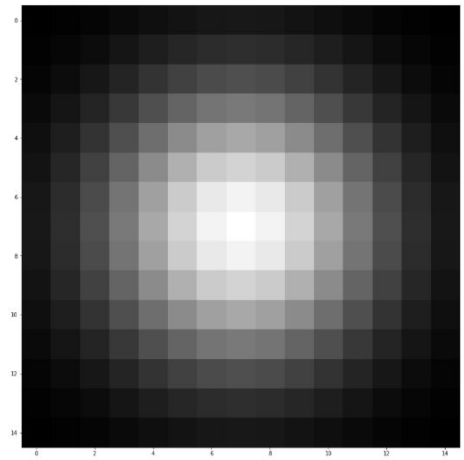


Fig1: Gaussian Blur Kernel

When we apply this to an image, it becomes blurred.

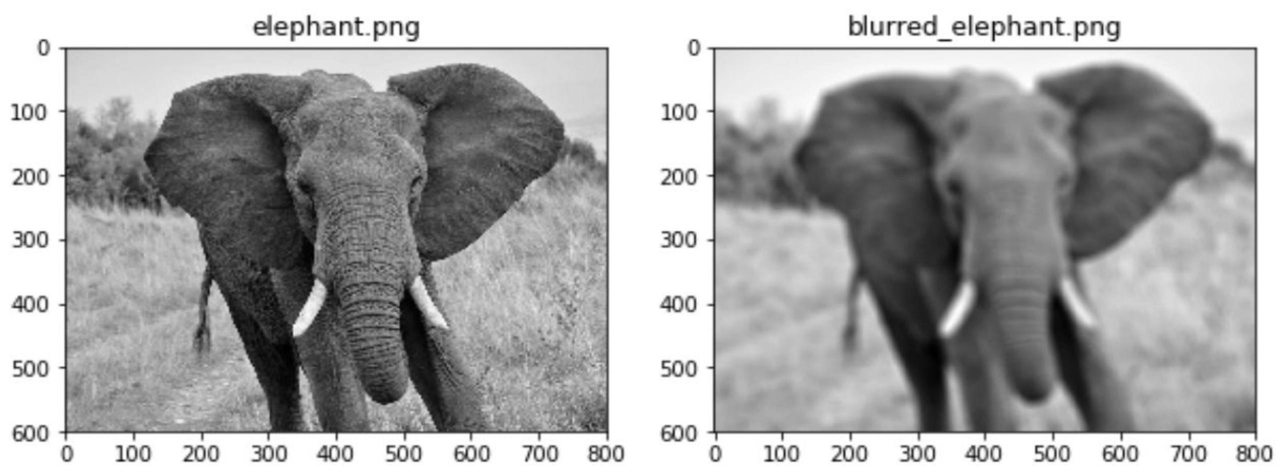


Fig2: Original and blurred image

The Noise process

There can be many reasons for noise in a model such as thermal vibration, magnetic influence, temperature, etc. Noise can be considered as a random process which is usually modelled using Gaussian additive noise. The noise is additive because it is added to noise that might be intrinsic in the system. It is defined by two parameters: mean and variance.

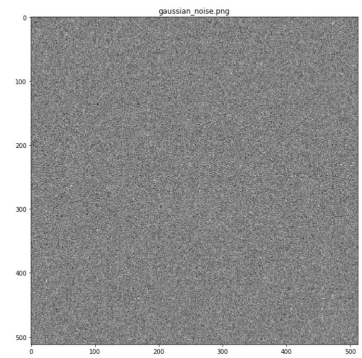


Fig3: Gaussian Noise

When we add this noise to the elephant we get the following:

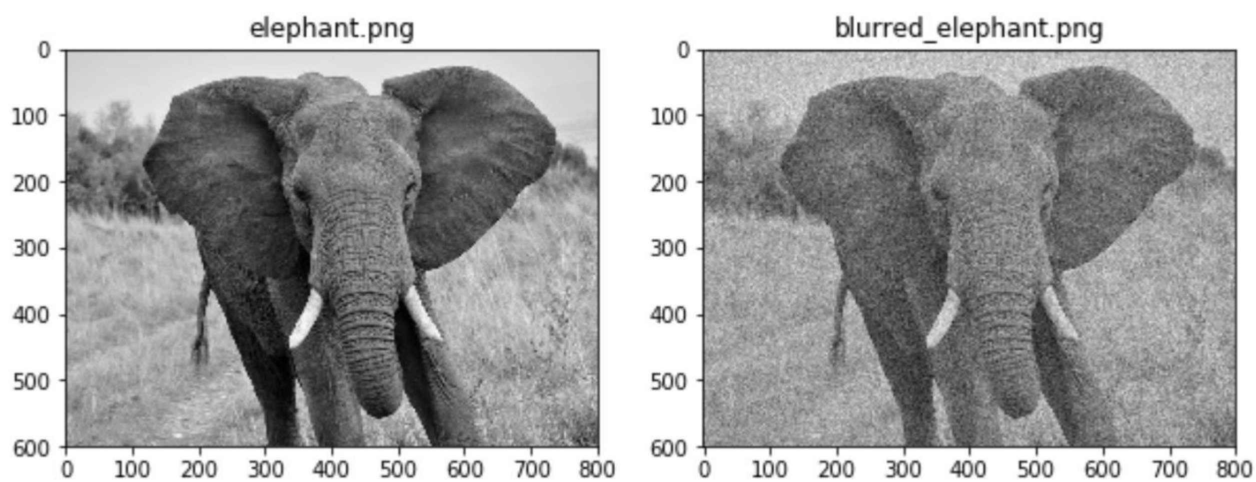


Fig4: Original and Noisy Image

Types of filter

In this project we have used only grayscale images. The process for colored images is the same. Each step has to be applied to the R, B and G channels separately.

Inverse Filter

We know that

$$g(x,y) = f(x,y) * h(x,y) + n(x,y)$$

Applying Fourier transform we get:

$$G(u,v) = F(u,v)\,H(u,v) + N(u,v)$$

Thus, $G(u,v)/H(u,v) = F(u,v) + N(u,v)/H(u,v)$

Thus, $F'(u,v) = F(u,v) + N(u,v)/H(u,v)$.

Thus, the original image can be approximated by taking the inverse fourier transform of $F'(u,v)$

As we can see from the equation that this approximation does not eliminate the noise present in the sample. In case of no blur the image is restored.

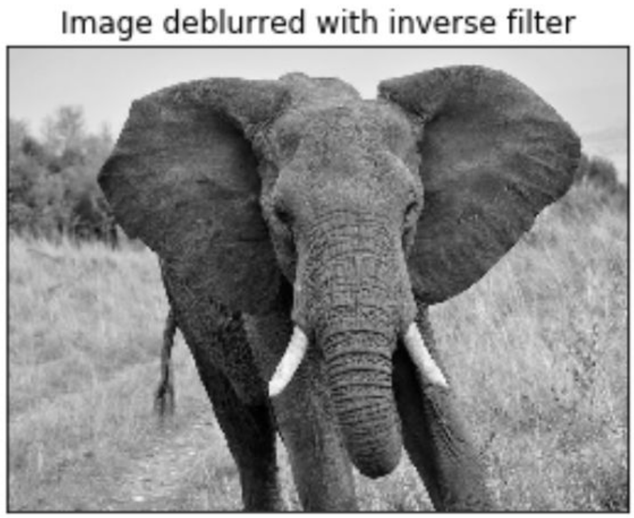


Fig4: Image restored with inverse filter (no noise)

However, in case of noise the image is not restored.

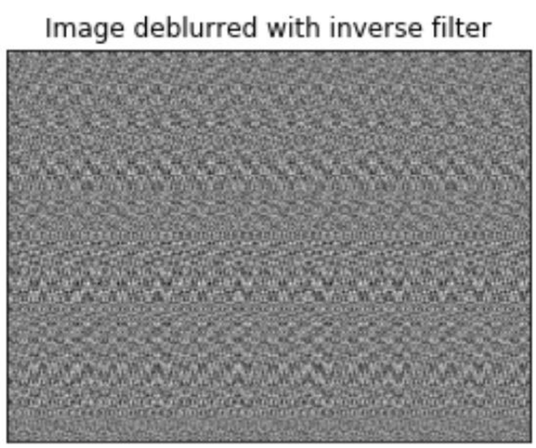


Fig5: Inverse filter(with noise)

Wiener Filter

This filter takes into account the noise present in the system. It considers the image and the noise as random processes and finds the image such that the mean square error is minimal between the original and the final image. In this, the image is approximated as

$$F'(u,v) = \left(\frac{|H(u,v)|^2}{H(u,v)(|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v))} \right) G(u,v)$$

The the image is obtained by taking the inverse fourier transform of $F'(u,v)$

Here S_η and S_f denote the energy spectrum of noise and the source image respectively. Usually we do not know these values, thus it is considered as a constant k and can be found experimentally.

$$F'(u,v) = \left(\frac{|H(u,v)|^2}{H(u,v)(|H(u,v)|^2 + k)} \right) G(u,v)$$

Fig6: Image restored with wiener filter (no noise)

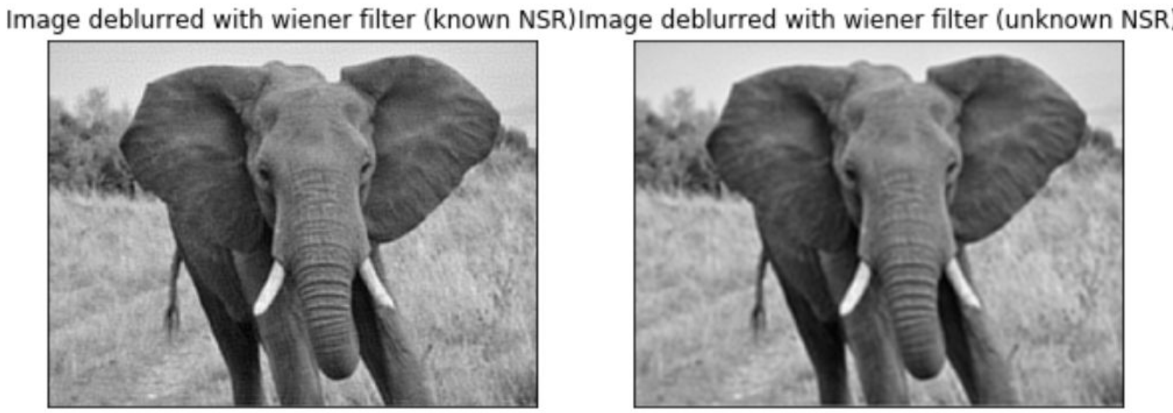


Fig7: Image restored with wiener filter with known and unknown k (NSR) (with noise)

Richardson Lucy Method

Unlike the previous two methods this is a non linear method. This method is iterative and its recurrence relation is as follows:

$$f'_{k+1}(x,y) = f'_k + (g(x,y) - f'_k * h(x,y)) * h(-x,-y)$$

In this method we do not use Fourier Transform and calculate the image in the spatial domain only.

When we run this image with blur and no noise we get

Fig8: Restored image with Richardson Lucy method(no noise)

When we run this image with blur and noise we get

Fig9: Restored image with Richardson Lucy method(with noise)

Metric for evaluation:

We use MSE and PSNR as a metric for evaluation of the images obtained after restoration.

MSE (Mean Square Error) is given as follows:

$$MSE(f'(x,y),\ f(x,y)) = \frac{1}{N}\Sigma(f'(x,y)_i - f(x,y)_i)^2$$

PSNR (Peak Signal to Noise Ratio) is given as follows:

$$PSNR(f'(x,y),\ f(x,y) = 20log_{10}(MAX(f(x,y))) - 10log(MSE(f'(x,y),\ f(x,y)))$$

Here, $MAX(f(x,y) = 256$.

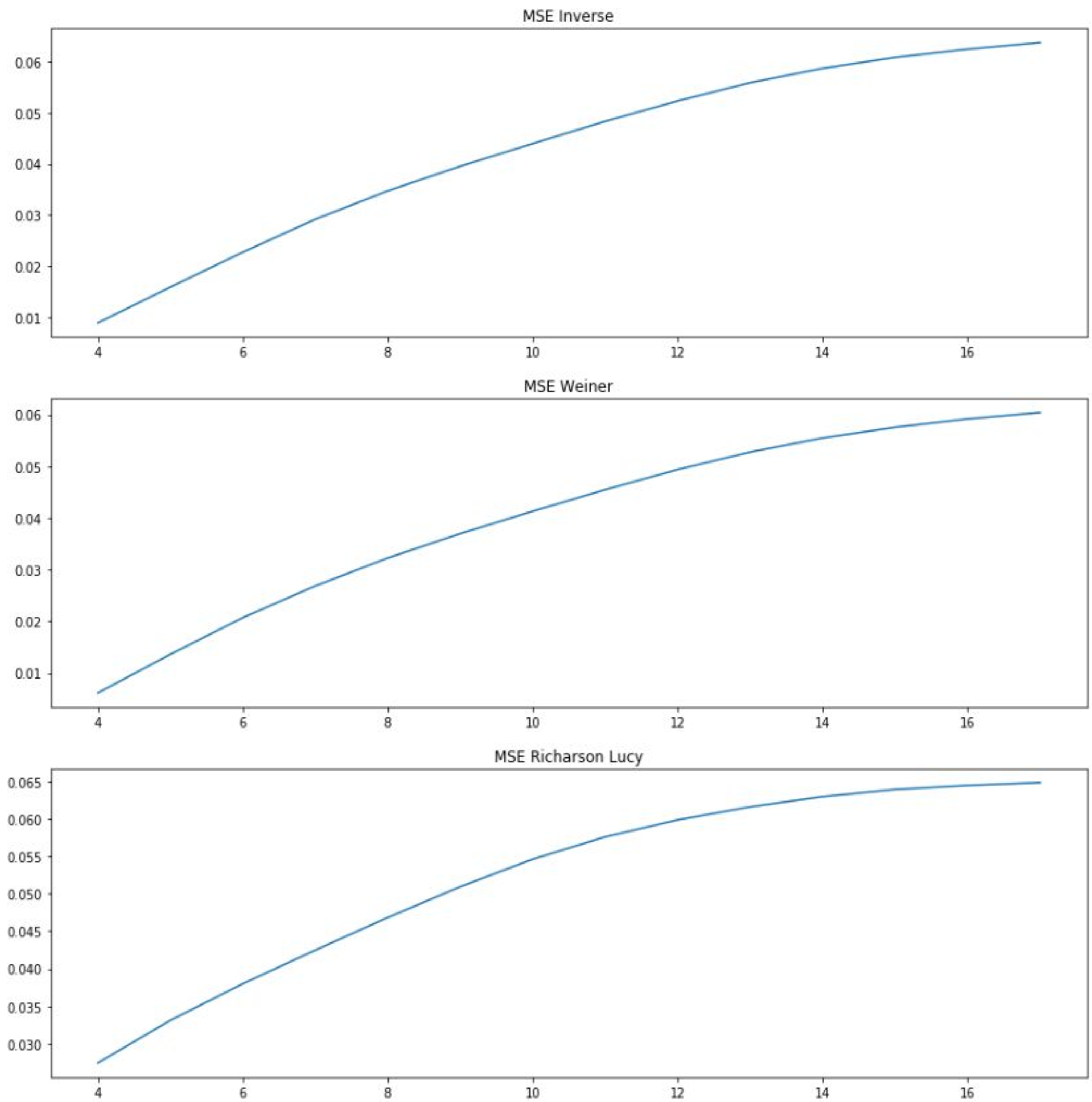
Below is the table for elephant.jpg:

Method	**MSE (no noise)**	**MSE (with noise)**	**PSNR (no noise)**	**PSNR (with noise)**
None	0.01226	0.01226	67.2803	67.2803
Inverse	1.1105e-20	123894373.4276	247.7098	-33.6555
Wiener	0.00177	• 0.0026104 (known NSR) • 0.003571 (unknown NSR)	75.6948	• 72.6324 (known NSR) • 72.6324 (unknown NSR)
Richardson Lucy	0.01501	0.01521	66.4017	66.3394

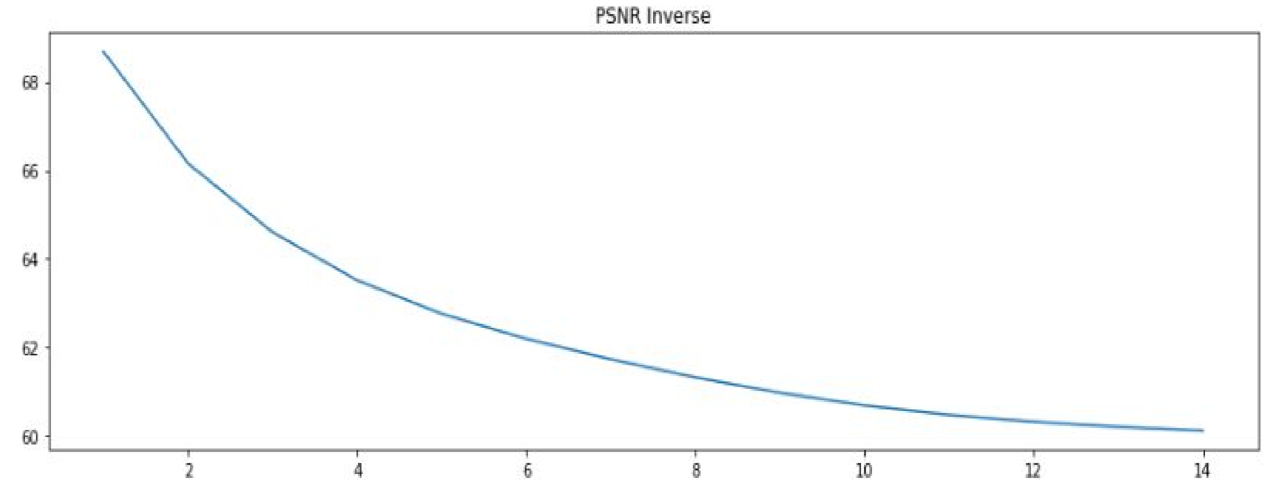
We see that MSE decreases for Inverse filter(no noise) and Wiener filter as compared to corrupted image. However, it is more for Richardson Lucy method. Also PSNR increases for inverse (no noise) and wiener filter but it decreases for Richardson Lucy method. For inverse filter (with noise) we get very high MSE and PSNR is negative. PSNR can not be negative but we it negative because value at some places becomes >256 after taking inverse fourier transform. For Richardson Lucy method we see that the images are visually of good quality but the metrics give us bad results. Thus, these metrics are not perfect.

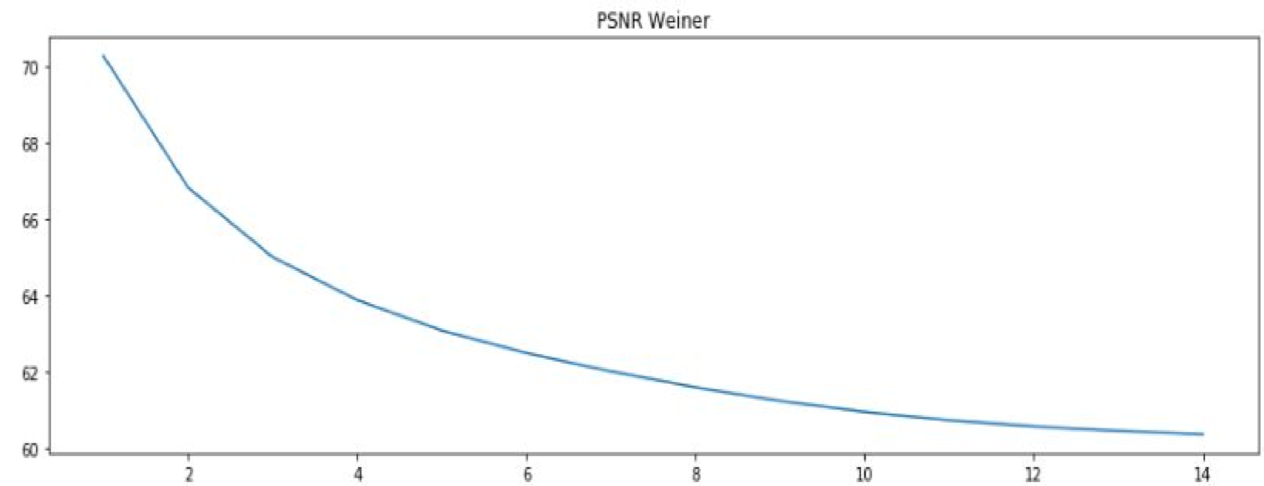
Graphs:

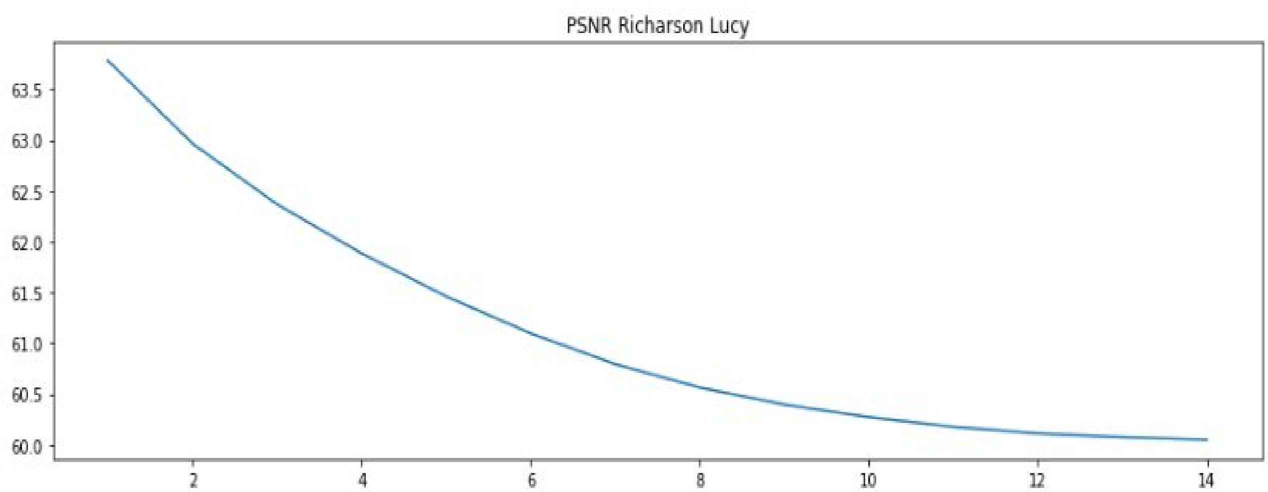
When we change the the blur for images with no noise the MSE varies as follows:



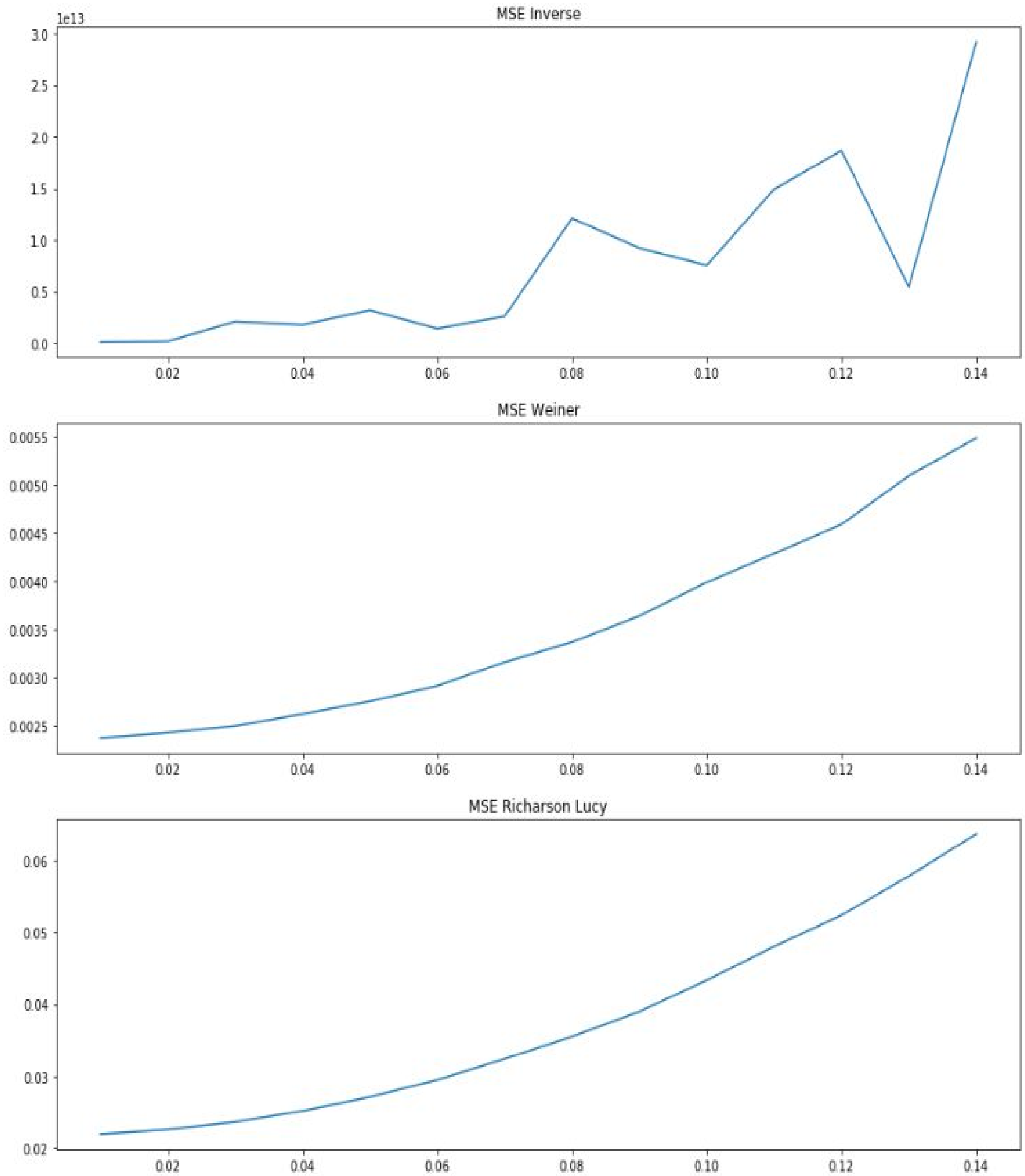
When we change the the blur for images with no noise the PSNR varies as follows:



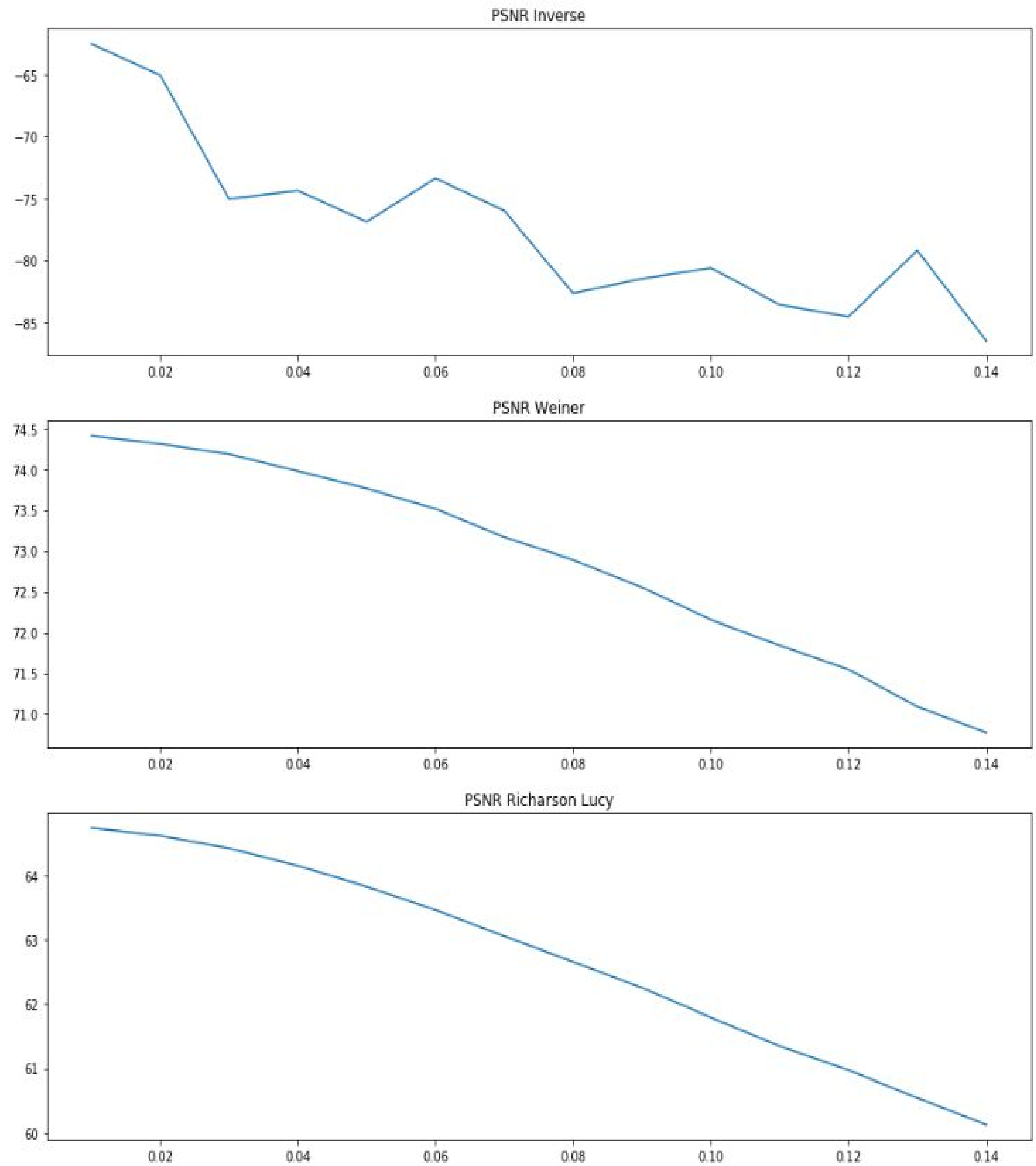




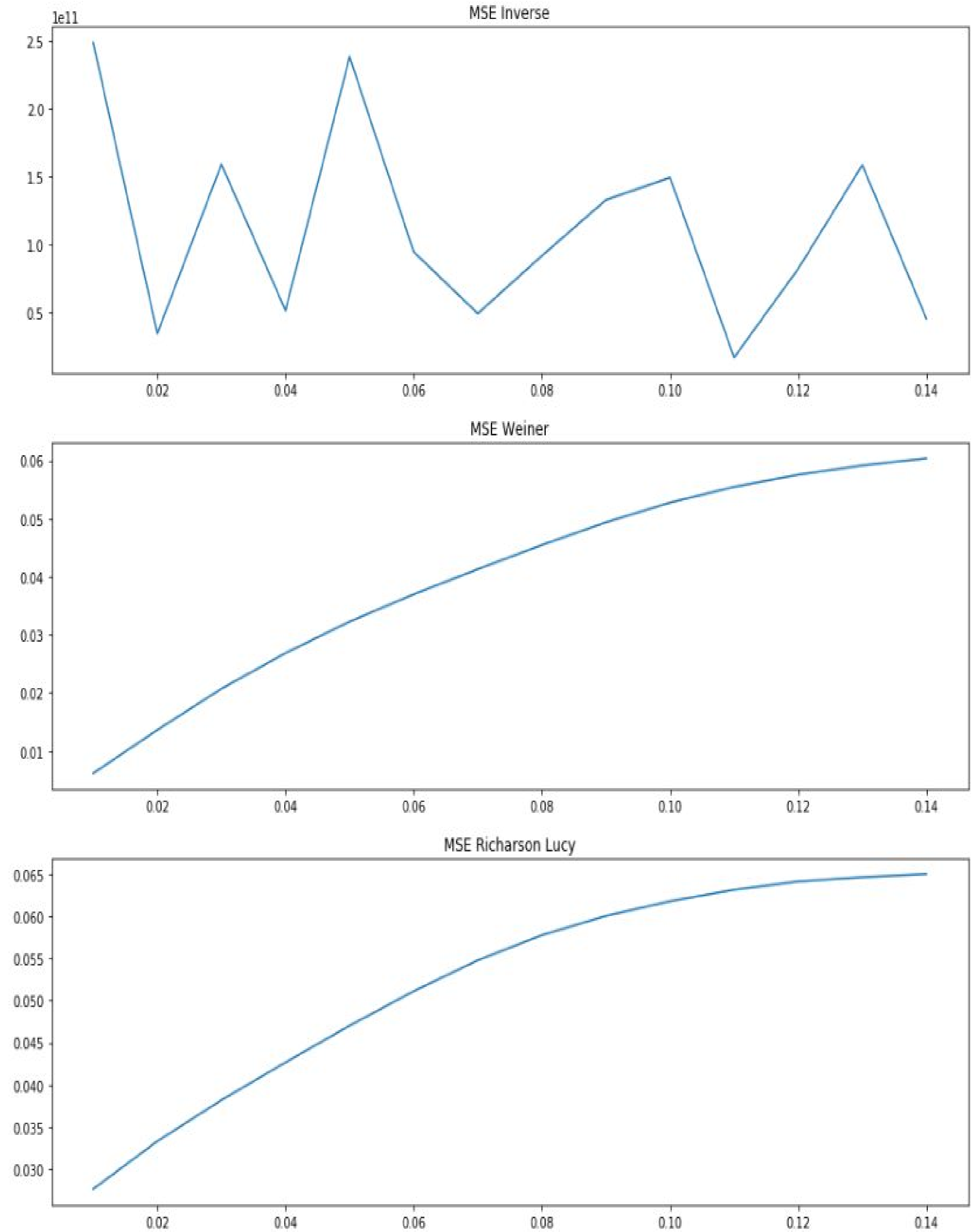
When we change the noise for images the MSE varies as follows:



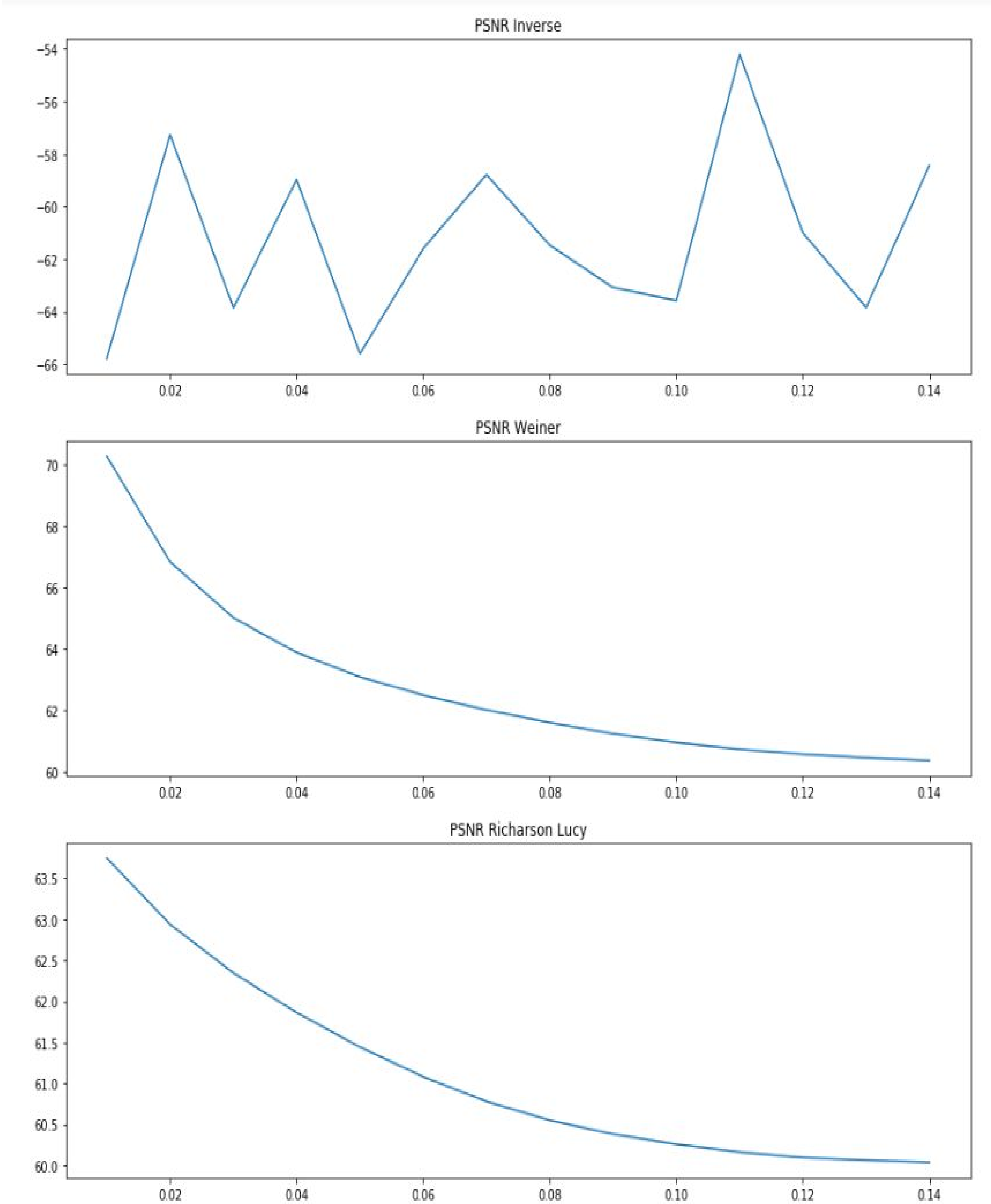
When we change the noise for images the PSNR varies as follows:



When we change the blur for images with noise the MSE varies as follows:



When we change the blur for images with noise the PSNR varies as follows:



From graphs we can see that:

- The MSE continuously increases for wiener and richardson lucy filter when the blur(with and without noise) or the noise of the images is increased.
- The PSNR continuously decreases for wiener and richardson lucy filter when the blur(with and without noise) or the noise of the images is increased.
- The MSE continuously increases for inverse filter when the blur(without noise) of the images is increased. However, there is no clear trend for noisy images. Also, for noisy images it does give good results.
- The PSNR continuously increases for inverse filter when the blur(without noise) of the images is increased. However, there is no clear trend for noisy images. Also, for noisy images it does give good results.

References:

- http://hi.cs.waseda.ac.jp/~oyamada/doc/ImageRestoration/RichardsonLucy_Gaussian.pdf
- http://yuzhikov.com/articles/BlurredImagesRestoration1.htm
- http://www.iraj.in/journal/journal_file/journal_pdf/1-399-150855999845-49.pdf