Problems using both Product rule and SUM RULE

- Many counting problems cannot be solved using just the sum rule or just the product rule.
- Many complicated counting problems can be solved using both of these rules in combination.

Example 14

Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there?

Let P be the total number of possible passwords, and let $P6$, $P7$, and $P8$ denote the number of possible passwords of length 6, 7, and 8, respectively. By the sum rule, $P = P6 + P7 + P8$.We will now find $P6$, $P7$, and $P8$.

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Example

Counting Internet Addresses In the Internet, which is made up of interconnected physical networks of computers, each computer is assigned an Internet address.

1. In Version 4 of the Internet Protocol (IPv4), now in use an address is a string of 32 bits.
2. It begins with a network number (netid).
3. The netid is followed by a host number (hostid), which identifies a computer as a member of a particular network.

<table>
<tr><th>Bit Number</th><th>0</th><th>1</th><th>2</th><th>3</th><th>4</th><th>8</th><th>16</th><th>24</th><th>31</th></tr>
<tr><td>Class A</td><td>0</td><td colspan="4">netid</td><td colspan="4">hostid</td></tr>
<tr><td>Class B</td><td>1</td><td>0</td><td colspan="4">netid</td><td colspan="3">hostid</td></tr>
<tr><td>Class C</td><td>1</td><td>1</td><td>0</td><td colspan="5">netid</td><td>hostid</td></tr>
<tr><td>Class D</td><td>1</td><td>1</td><td>1</td><td>0</td><td colspan="5">Multicast Address</td></tr>
<tr><td>Class E</td><td>1</td><td>1</td><td>1</td><td>1</td><td>0</td><td colspan="4">Address</td></tr>
</table>

Problems using both Product rule and SUM RULE

How many different IPv4 addresses are available for computers on the Internet?

Three forms of addresses are used, with different numbers of bits used for netids and hostids. **Class A addresses**, used for the largest networks, consist of 0, followed by a 7-bit netid and a 24-bit hostid. **Class B addresses**, used for medium-sized networks, consist of 10, followed by a 14-bit netid and a 16-bit hostid. **Class C addresses**, used for the smallest networks, consist of 110, followed by a 21-bit netid and an 8-bit hostid. There are several restrictions on addresses because of special uses: 1111111 is not available as the netid of a Class A network, and the hostids consisting of all 0s and all 1s are not available for use in any network. A computer on the Internet has either a Class A, a Class B, or a Class C address. (Besides Class A, B, and C addresses, there are also Class D addresses, reserved for use in multicasting when multiple computers are addressed at a single time, consisting of 1110 followed by 28 bits, and Class E addresses, reserved for future use, consisting of 11110 followed by 27 bits. Neither Class D nor Class E addresses are assigned as the IPv4 address of a computer on the Internet.) Figure 1 illustrates IPv4 addressing. (Limitations on the number of Class A and Class B netids have made IPv4 addressing inadequate; IPv6, a new version of IP, uses 128-bit addresses to solve this problem.)

Solution: Let x be the number of available addresses for computers on the Internet, and let x_A, x_B, and x_C denote the number of Class A, Class B, and Class C addresses available, respectively. By the sum rule, $x = x_A + x_B + x_C$.

The Subtraction Rule (principle of inclusion–exclusion)

THE SUBTRACTION RULE If a task can be done in either n_1 ways or n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways.

Used to count the number of elements in the union of two sets.

Suppose that $A1$ and $A2$ are sets.

Then, there are $|A1|$ ways to select an element from $A1$ and $|A2|$ ways to select an element from $A2$.

The number of ways to select an element from $A1$ or from $A2$, that is, the number of ways to select an element from their union.

- is the sum of the number of ways to select an element from $A1$ and the number of ways to select an element from $A2$, minus the number of ways to select an element that is in both $A1$ and$A2$.

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|.$$

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Example 15

How many bit strings of length eight either start with a 1 bit or end with the two bits 00?

$$1\ \underbrace{\_\ \_\ \_\ \_\ \_\ \_\ \_}_{2^7 = 128 \text{ ways}} \qquad \underbrace{\_\ \_\ \_\ \_\ \_\ \_}_{}\ 0\ 0 \qquad 1\ \underbrace{\_\ \_\ \_\ \_\ \_}_{2^5 = 32 \text{ ways}}\ 0\ 0$$