ASSIGNMENT
(RECURSIVE ALGORITHM, MATHEMATICAL INDUCTION/ STRONG INDUCTION, PIGEONHOLE PRINCIPLE)

1. Prove that $1^2+3^2+5^2+...+(2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$, whenever n is a non-negative integers.

2. **a)** Find a formula for $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\cdots+\frac{1}{n(n+1)}$ by examining the values of this expression for small values of n.
   b) Prove the formula you conjectured in part **(a)**.

3. Let $P(n)$ be the statement that $1+\frac{1}{4}+\frac{1}{9}+\cdots+\frac{1}{n^2} < 2-\frac{1}{n}$, where n is an integer greater than 1.
   (a) Show that $P(2)$ is true, completing the basis step of the proof.
   (b) What do you need to prove in the inductive step? Complete the inductive step.
   (c) Explain why these steps show that this inequality is true whenever n is an integer greater than 1.

4. Prove that 6 divides $n^3 - n$ whenever n is a nonnegative integer.

5. Which amount of money can be formed using just two dollar bills and five-dollar bills? Prove your answer using strong induction.

6. Give a recursive definition of the sequence a_n, $n = 1, 2, 3, ...$ if
   (a) $a_n = 6n$, (b) $a_n = 2n + 1$, (c) $a_n = 10^n$, (d) $a_n = 5$

7. Give a recursive algorithm for finding $n!\ mod\ m$ whenever n and m are positive integers.

8. A palindrome is a string whose reversal is identical to the string. How many bit strings of length n are palindromes?

9. Use the principle of inclusion–exclusion to find the number of positive integers less than 1,000,000 that are not divisible by either 4 or by 6.

10. What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state? (*use pigeonhole principle*).

11. How many numbers must be selected from the set $\{1, 2, 3, 4, 5, 6\}$ to guarantee that at least one pair of these numbers add up to 7? (*Can we use pigeonhole principle here to find the solution*)