20CYS111 Digital Signal Processing

Signals: Basic Operations

Dr. J. Aravinth (Mentor)

Basic Operations

Signals can be modified by applying various operations.

There are two types of basic operations:

- ***Operations Performed on the Dependent Variables***
  - *Amplitude Shifting, Amplitude Scaling, Addition, Multiplication, Differentiation/Difference, Integration/Cumulative Sum.*
- ***Operations Performed on the Independent Variables***
  - *Time Shifting, Time Scaling, Reflection.*

Operations Performed on the Dependent Variables

Amplitude Scaling

The ***amplitude scaling*** *of a signal $x(t)$ or $x[n]$ results in a signal $y(t)$ or $y[n]$, respectively, that is defined by*

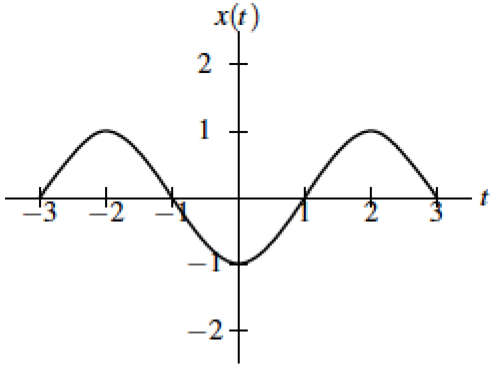
$$\boxed{\begin{aligned} y(t) &= cx(t) \\ y[n] &= cx[n] \end{aligned}},$$

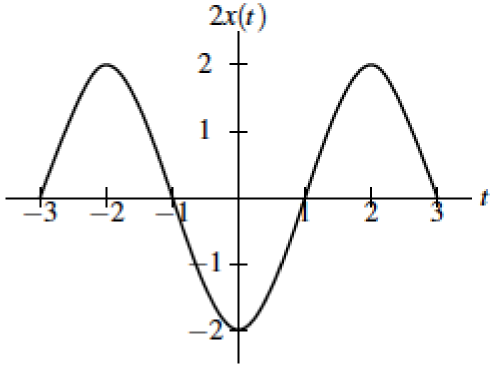
where c is called the ***scaling factor****.*

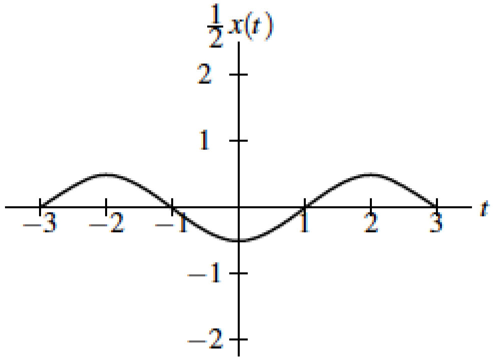
Examples: *For a voltage amplifier, we have $v_{out}(t) = cv_{in}(t)$, with $|c| > 1$; For a potentiometer, we have $v_{out}(t) = cv_{in}(t)$, with $|c| < 1$; Ohm's law: $v(t) = Ri(t)$.*

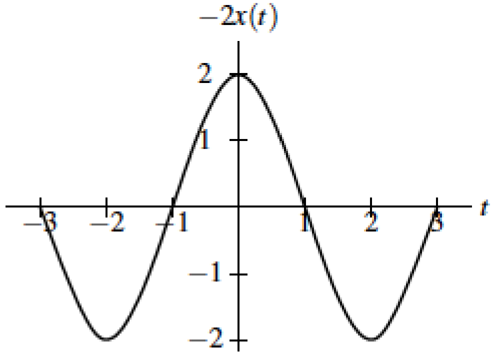
In summary, amplitude scaling implies (a) ***amplification****, or (b)* ***attenuation****, or (c)* ***reflection about the x-axis****, or (d) a combination of* ***(a) and (c)****, or of* ***(b) and (c)****.*

Amplitude Scaling



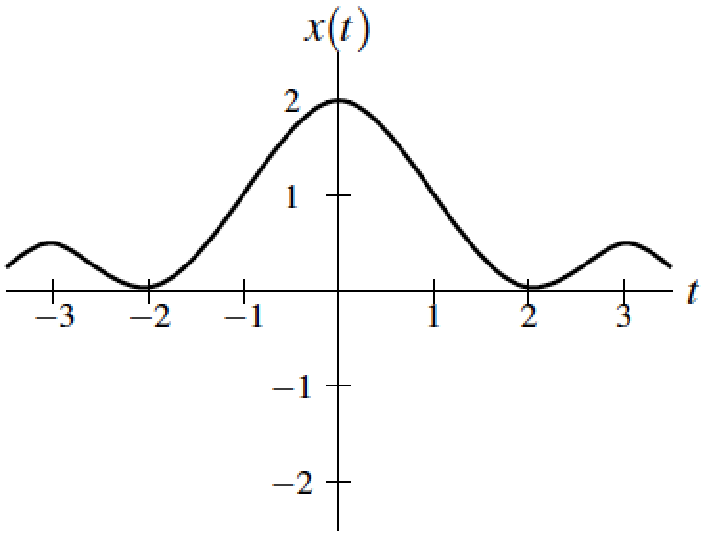


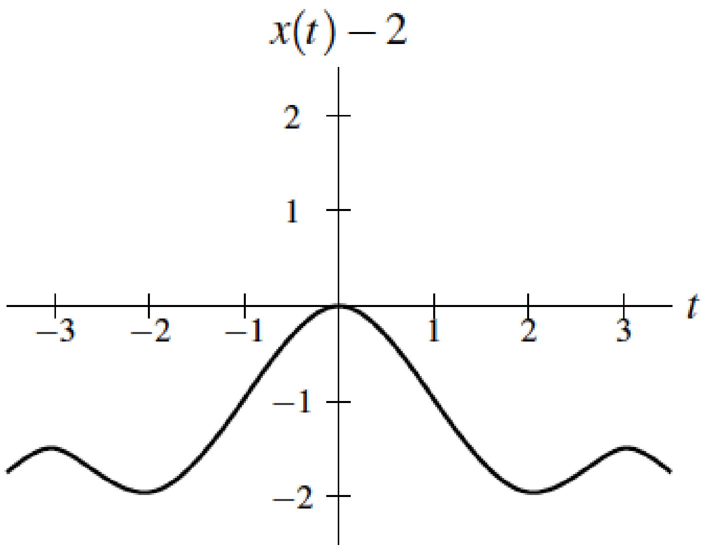




Amplitude Shifting

***Amplitude shifting** is defined by $\boxed{y(t) = x(t) + b}$, which is essentially the addition/subtraction of a DC value.*





Combined Amplitude Scaling and Amplitude Shifting

The combined amplitude scaling and amplitude shifting on a signal $x(t)$ *can be defined as*

$$y(t) = ax(t) + b = a\left(x(t) + \frac{b}{a}\right),$$

where a *and* b *are real numbers and* $a \neq 0$*; so, we can produce* $y(t)$ *starting from* $x(t)$ *in two ways:*

- *First, amplitude scaling of* $x(t)$ *by a factor* a *to get* $ax(t)$*, and then amplitude shifting of* $x(at)$ *by* b *units to get* $ax(t) + b$*, or*
- *First, amplitude shifting of* $x(t)$ *by* $\frac{b}{a}$ *units to get* $x(t) + \frac{b}{a}$*, and then amplitude scaling of* $x(t) + \frac{b}{a}$ *by a factor* a *to get* $a\left(x(t) + \frac{b}{a}\right) = ax(t) + b$.

Addition

*The **addition** of two signals $x_1(t)$ and $x_2(t)$ results in a signal $y(t)$, defined by*

$$\boxed{y(t) = x_1(t) + x_2(t)}.$$

Similarly, the the addition of two discrete-time signals $x_1[n]$ and $x_2[n]$ results in a signal $y[n]$, defined by

$$\boxed{y[n] = x_1[n] + x_2[n]}.$$

***Examples:** An audio **mixer**, which combines music and voice signals.*

Multiplication

*The **multiplication** of two signals $x_1(t)$ and $x_2(t)$ results in a signal $y(t)$, defined by*

$$\boxed{y(t) = x_1(t)x_2(t)}.$$

Similarly, the multiplication of two discrete-time signals $x_1[n]$ and $x_2[n]$ results in a signal $y[n]$, defined by

$$\boxed{y[n] = x_1[n]x_2[n]}.$$

***Examples:** Amplitude modulation (AM) signals.*

Integration/Cumulative Sum

The signal $y(t)$ obtained by the ***integration*** *of the signal $x(t)$ is defined by*

$$\boxed{y(t) = \int_{-\infty}^{t} x(t')dt'}.$$

Example: *The voltage $v(t)$ developed across a capacitance C, as a function of the current $i(t)$ flowing through it, is given by*

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(t')dt'.$$

In discrete-time, we consider the ***cumulative sum*** *operation, defined by*

$$\boxed{y[n] = \sum_{k=-\infty}^{n} x[k]}.$$

Differentiation/Difference

The signal $y(t)$ obtained by the ***differentiation*** *of the signal $x(t)$ is defined by*

$$\boxed{y(t) = \frac{d}{dt}x(t)}.$$

Example: *The voltage $v(t)$ developed across an inductance L, as a function of the current $i(t)$ flowing through it, is given by*

$$v(t) = L\frac{d}{dt}i(t).$$

In discrete-time, we consider the ***difference*** *operation, defined by*

$$\boxed{y[n] = x[n] - x[n-1]}.$$

Operations Performed on the Independent Variables

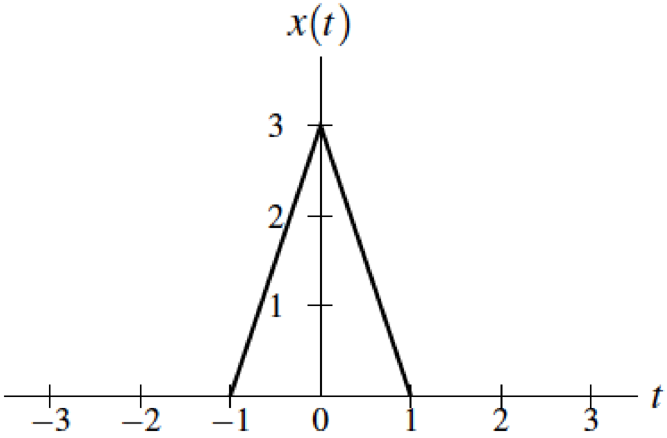
Time Shifting (Continuous-Time)

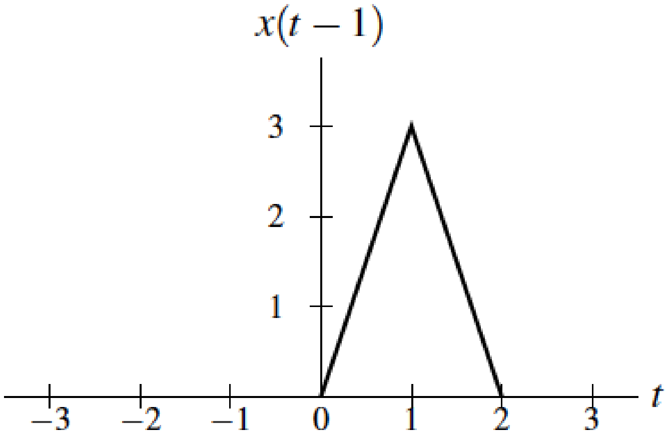
The signal $y(t)$, which is obtained from the signal $x(t)$ by a ***time shift*** *or* ***translation*** *of $x(t)$ by b units, is defined by*

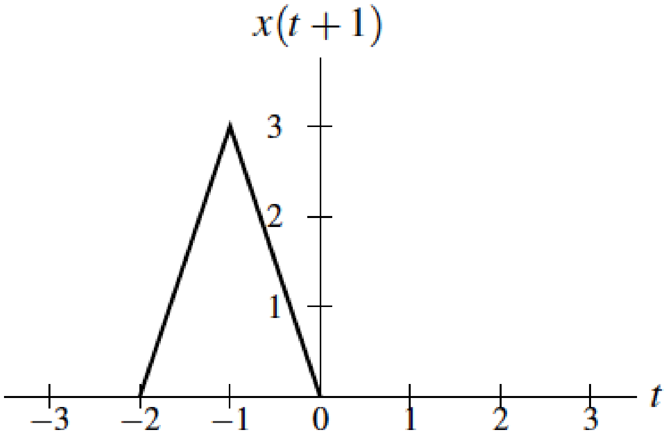
$$\boxed{y(t) = x(t - b)}.$$

Time shifting represents either a ***right shift*** *or a* ***left shift****:*

- $b > 0 \Rightarrow$ *right shift operation*
- $b < 0 \Rightarrow$ *left shift operation*


x(t)
3
2
1
t
−3
−2
−1
0
1
2
3


x(t − 1)
3
2
1
t
−3
−2
−1
0
1
2
3


x(t + 1)
3
2
1
t
−3
−2
−1
0
1
2
3

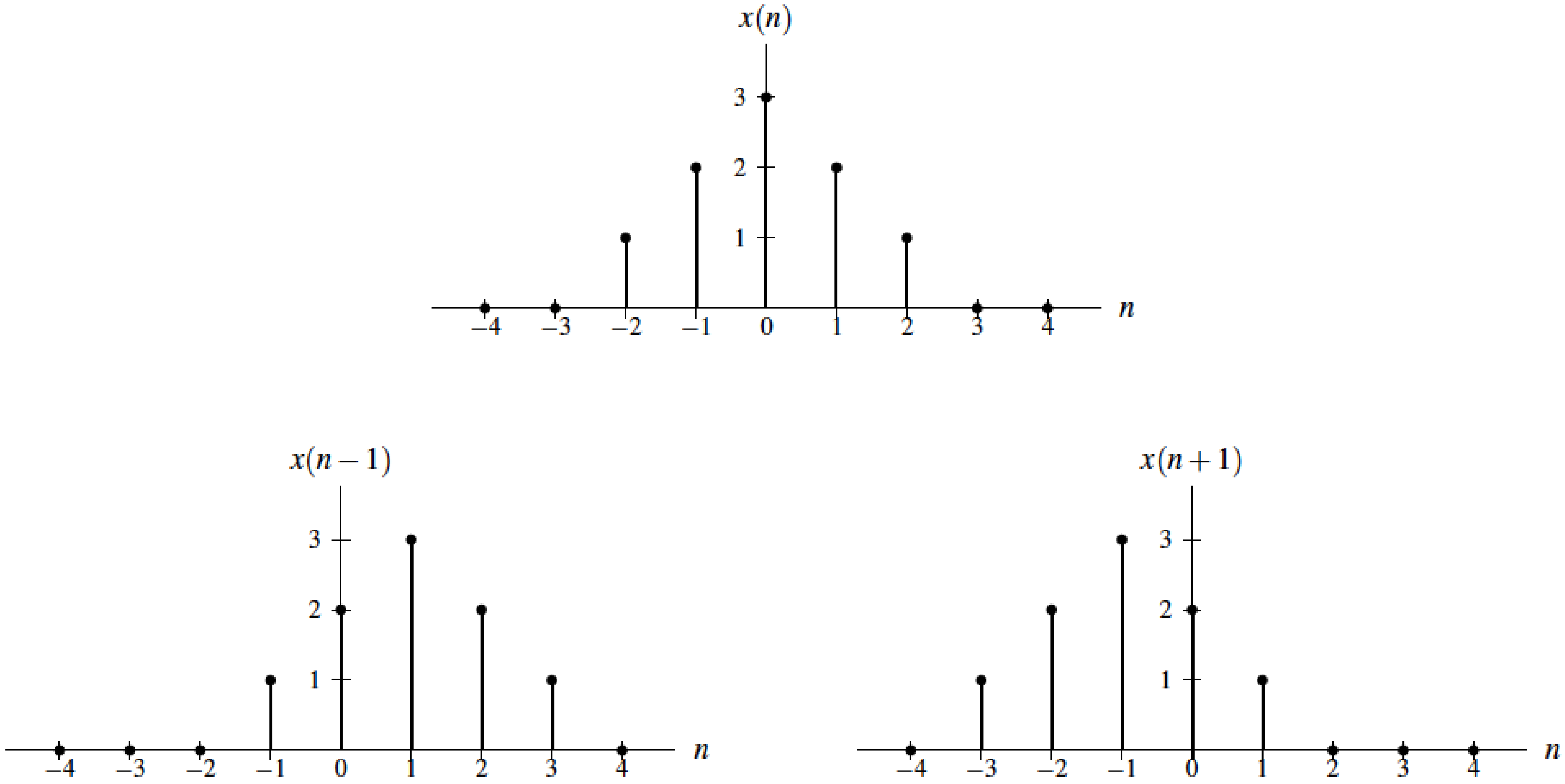
Time Shifting (Discrete-Time)

*The signal $y[n]$, which is obtained from the signal $x[n]$ by a **time shift** or **translation** of $x[n]$ by m samples, is defined by*

$$\boxed{y[n] = x[n-m]}.$$

*Time shifting represents either a **right shift** or a **left shift**:*

- $m > 0 \Rightarrow$ *right shift operation*
- $m < 0 \Rightarrow$ *left shift operation*



x(n)
x(n − 1)
x(n + 1)
n

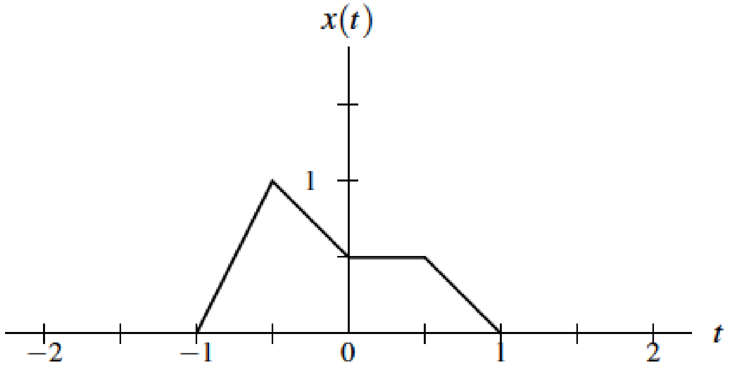
Time Scaling (Continuous-Time)

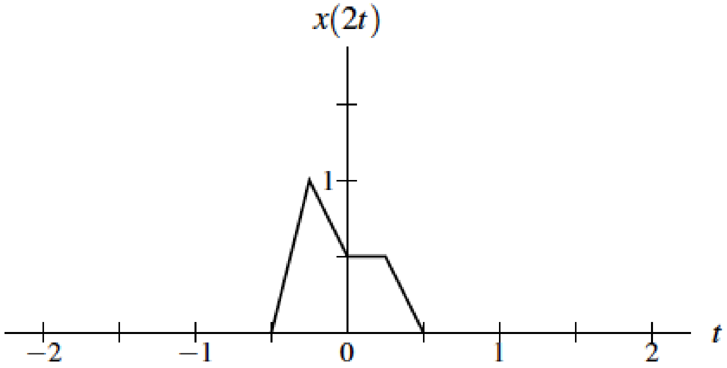
The signal $y(t)$, which is obtained from the signal $x(t)$ by ***scaling the independent variable t by a factor $a > 0$****, is defined by*

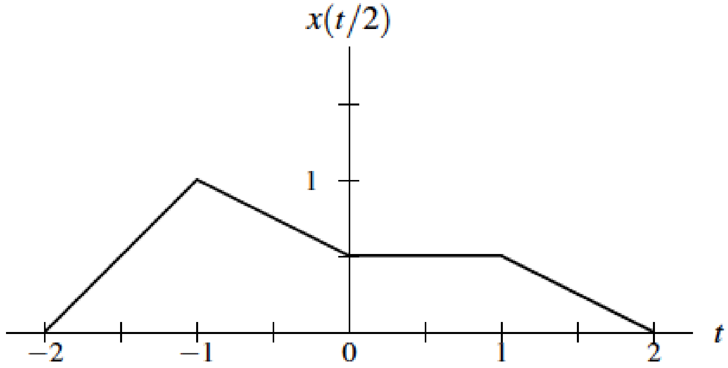
$$\boxed{y(t) = x(at)}.$$

Time scaling represents either ***compression*** *or* ***expansion*** *along the axis of the independent variable t:*

- $a > 1$ $\Rightarrow$ *compression*
- $a < 1$ $\Rightarrow$ *expansion (or* ***stretching*** *or* ***dilation****)*


x(t)
1
−2
−1
0
1
2
t


x(2t)
1
−2
−1
0
1
2
t

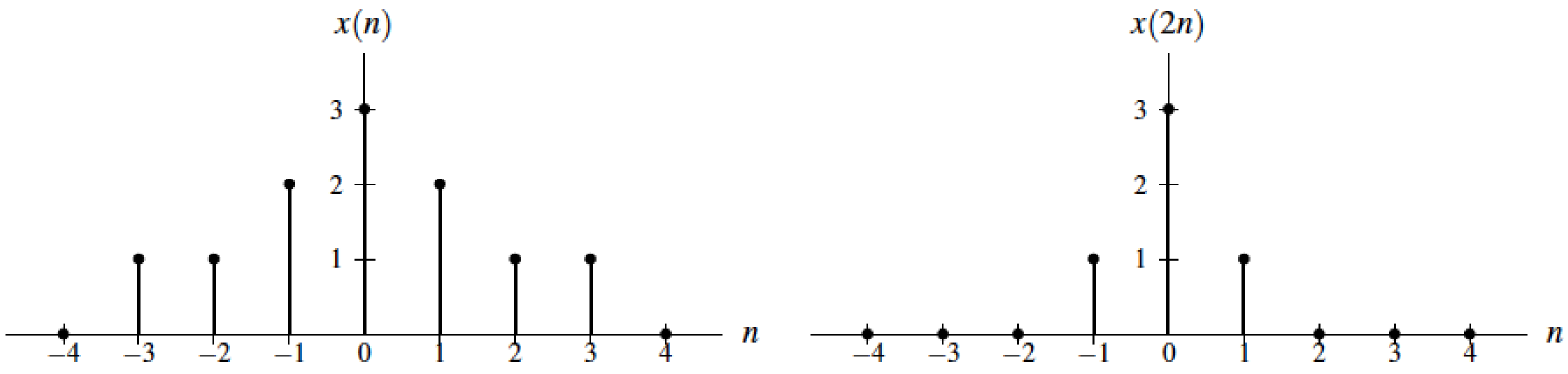

x(t/2)
1
−2
−1
0
1
2
t

Time Scaling (Discrete-Time)

The signal $y[n]$, which is obtained from the signal $x[n]$ by scaling the independent variable n by a factor $k \geq 1$, where k is a ***strictly positive integer****, is defined by*

$$\boxed{y[n] = x[kn]},$$

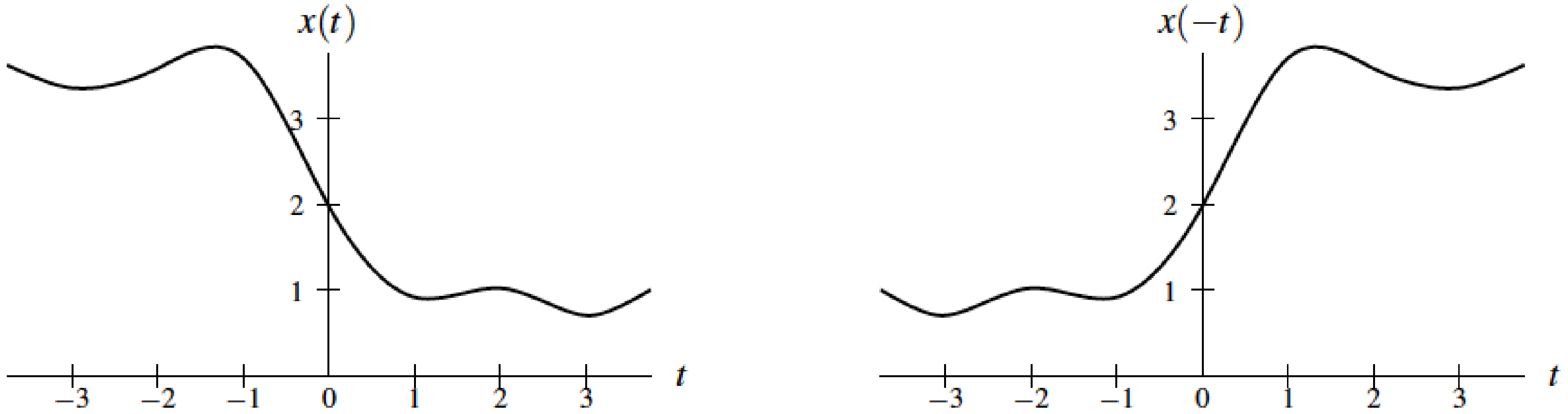
Note that we can only perform ***downsampling*** *of the original discrete-time signal.*



Reflection (Continuous-Time)

The signal $y(t)$, which is obtained from the signal $x(t)$ by ***time reversal*** *or* ***reflecting $x(t)$ about the vertical axis at $t = 0$***, *is defined by*

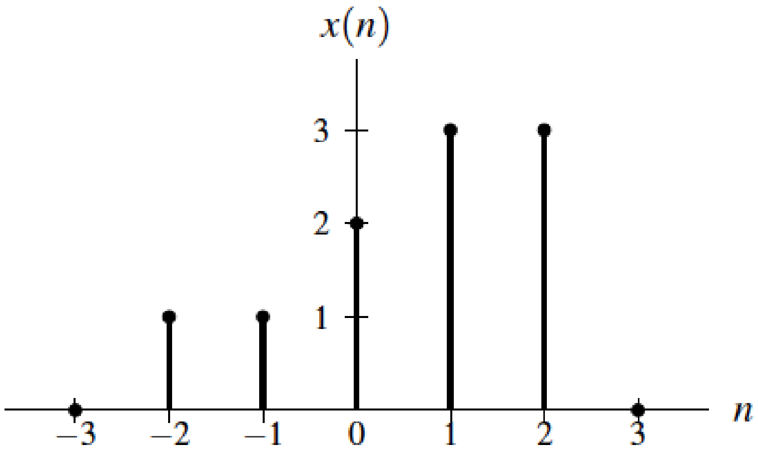
$$\boxed{y(t) = x(-t)}.$$

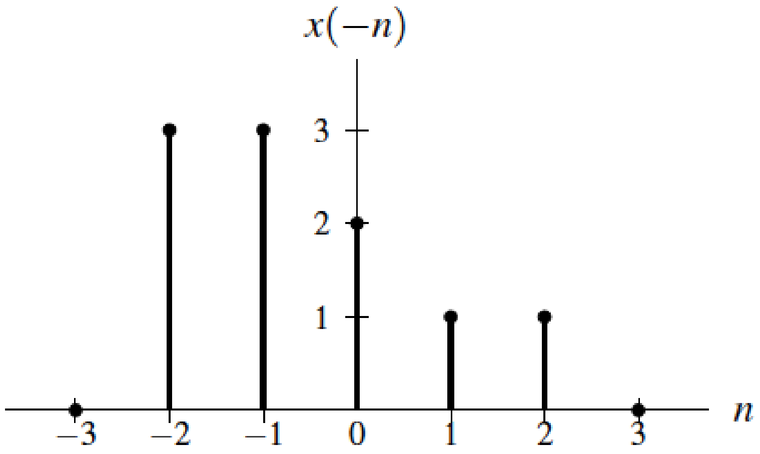


Reflection (Discrete-Time)

The signal $y[n]$, which is obtained from the signal $x[n]$ by time reversal or reflecting $x[n]$ about the vertical axis at $n = 0$, is defined by

$$\boxed{y[n] = x[-n]}.$$





Combined Time Scaling and Time Shifting

The combined time scaling and time shifting on a signal $x(t)$ can be defined as

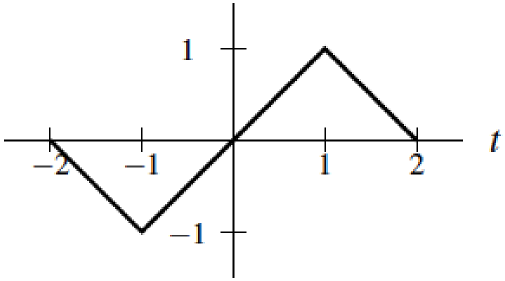
$$y(t) = x(at - b) = x\left(a\left(t - \frac{b}{a}\right)\right),$$

where a and b are real numbers and $a \neq 0$; so, we can produce $y(t)$ starting from $x(t)$ in two ways:

- *First, time shift $x(t)$ by b units to get $x(t - b)$, and then time scale $x(t - b)$ by a factor a to get $x(at - b)$, or*
- *First, time scale $x(t)$ by a factor a to get $x(at)$, and then time shift $x(at)$ by $\frac{b}{a}$ units to get $x\left(a\left(t - \frac{b}{a}\right)\right) = x(at - b)$.*

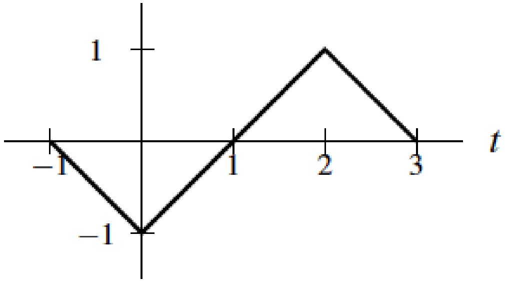
Given $x(t)$ as shown below, find $x(2t-1)$.

$x(t)$

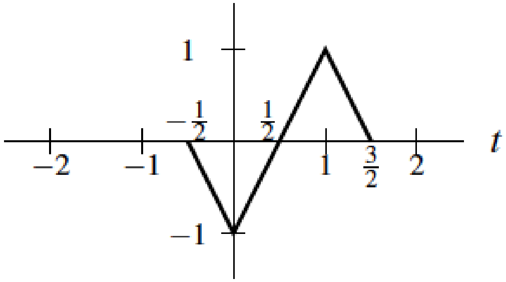


time shift by 1 and then time scale by 2

$p(t) = x(t-1)$

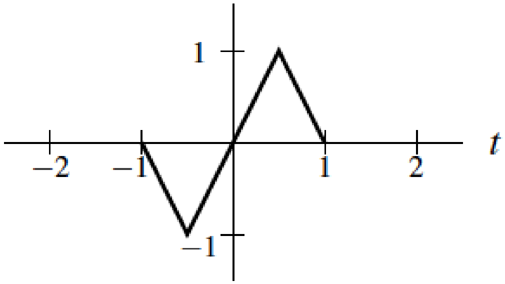


$p(2t) = x(2t-1)$

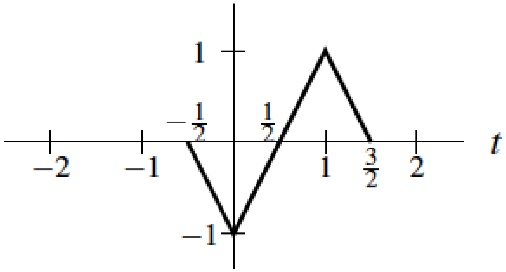


time scale by 2 and then time shift by $\frac{1}{2}$

$q(t) = x(2t)$



$q(t-1/2) = x(2(t-1/2)$
$= x(2t-1)$

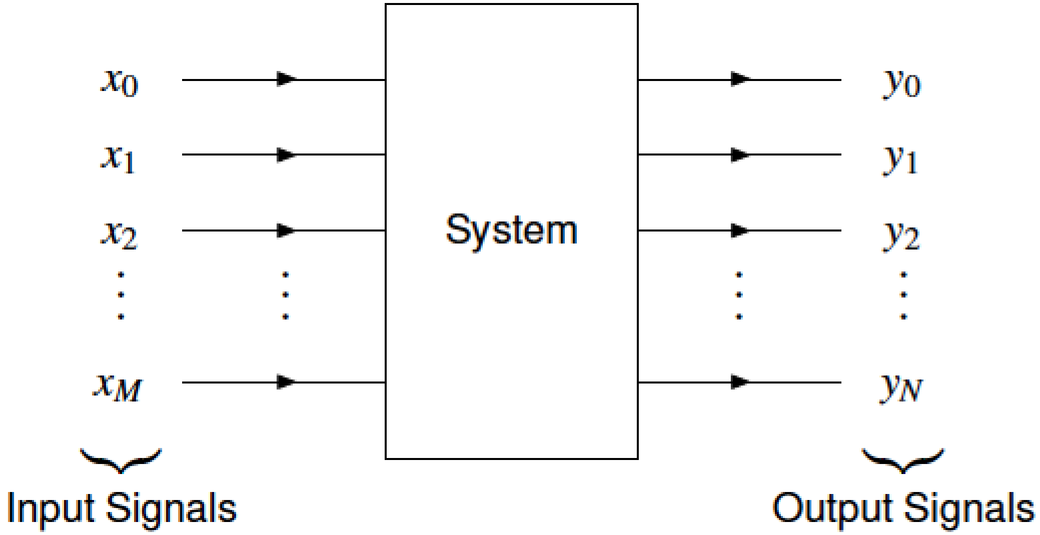


Systems As Operations

Systems as Operations

*Recall that a **system** is an entity that processes one or more input signals and produces one or more output signals.*

- *A system can have a single or multiple input(s) and output(s).*



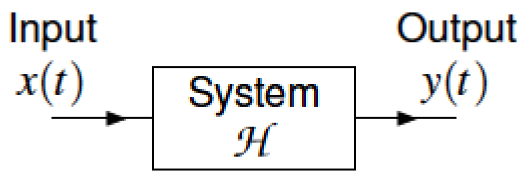
Systems as Operations

A system performs some operation(s) on the input signal(s) and produces the output signal(s).

- *$\Rightarrow$ A system with input x and output y can be described by the equation*

$$\boxed{y = \mathcal{H}\{x\}},$$

where $\mathcal{H}$ denotes an ***operator*** *(i.e., a transformation) which <u>maps a function to another function</u>.*



Systems as Operations

For given inputs, we can design a system to obtain the desired outputs by

1. *Breaking the complete set of operations to be performed into several **basic operations**.*
2. *Designing one small (sub)system per basic operation to be performed.*
3. *Interconnecting the (sub)systems in an appropriate manner to reflect the sequence/order of operations to be performed on the signal(s).*

*Two basic ways in which systems can be interconnected are **series** and **parallel**.*

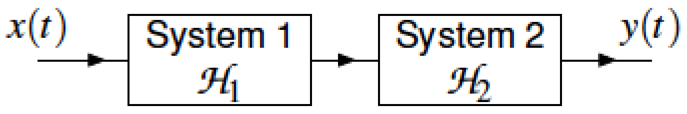
Systems as Operations

*The **series-connected** system is described by the equation*

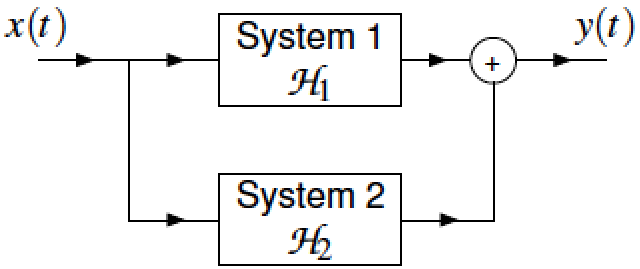
$$\boxed{y = \mathcal{H}_2\{\mathcal{H}_1\{x\}\}}.$$

*The **parallel-connected** system is described by the equation*

$$\boxed{y = \mathcal{H}_1\{x\} + \mathcal{H}_2\{x\}}.$$



Series



Parallel

References:

[1] Simon Haykin and Barry Van Veen, Signals and Systems, Second Edition, John Wiley and Sons, 2003.

[2] Lecture Notes by Michael D. Adams.
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[3] Lecture Notes by Richard Baraniuk.
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