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Tutorial Test-1
March 2022
B.Tech CSE(CYS)

Second Semester

20MAT112 NUMBER THEORY AND ALGEBRA

Time:30 Minutes Maximum : 30 Marks

Answer Any Four Questions

1. Use the definition of divisibility to prove that if $a|b$ and $b|c$ then $a|7b - 5c$.
2. Prove that if $(a, b) = 1$ then $(a + b, a - b) = 1$ *or* 2
3. Use the Euclidean algorithm to find the gcd of 1819 and 3587, and express the gcd as $d = 1819x + 3587y$.
4. (i) Prove that for every natural number $n > 2$, $(n + 1)|(n^3 + 1)$.
   (ii) Suppose that n is a natural number exceeding 1. Prove that $(n^2 - 1)|(n^3 + 1)$ if and only if $n = 2$.
5. Find all positive integer n such that $3^{n-1} + 5^{n-1}|3^n + 5^n$
6. If m, n are positive integer such that $3m + n = 3[m, n] + (m, n)$ prove that n divides m.
7. Define $T_0, T_1, T_2, ...$ by $T_1 = 2$ and $T_{n+1} = T_n^2 - T_n + 1$ for $n > 0$. Prove that $T_{n+1} = T_1T_2...T_n + 1$ and $gcd(T_i, T_j) = 1 \ \ \forall \ i \neq j$
8. If a, b, c are decimal digits prove that the six digit number $n = abcabc$ is divisible by 7 and 13.