Name : Nithin . G . R

NUMBER THEORY AND ALGEBRA

1. Given

$a|b$ and $b|c$

$b = ar$ $\quad c = bs$ $\quad (r, s \in \mathbb{Z})$

$7b - 5c = 7ar - 5bs$
$= 7ar - 5(ar)s$
$= a(7r - 5rs)$

Since $7r - 5rs$ is an integer, then $a|(7b-5c)$

3. $3587 = 1819 \cdot 1 + 1768$
$1819 = 1768 \cdot 1 + 51$
$1768 = 51 \cdot 34 + 34$
$51 = 34 \cdot 1 + 17$
$34 = 17 \cdot 2 + 0$

Hence the gcd of $(3587, 1819) = 17$

$17 = 51 - 34 \cdot 1$
$= 51 - (1768 - 51 \cdot 34)$
$= 51 \cdot 35 - 1768$
$= (1819 - 1768) \cdot 35 - 1768$
$= 1819 \cdot 35 - 1768 \cdot 36$
$= 1819 \cdot 35 - (3587 - 1819)\, 36$
$= 1819 \cdot 71 - (3587) \cdot 36$

$17 = 1819 \cdot 71 - (3587) \cdot 36$
$x = 71 \quad y = -36$

(5) Suppose $3^{n-1}+5^{n-1} \mid 3^n+5^n$

$\Rightarrow 3^n+5^n = k(3^{n-1}+5^{n-1})$. Now if $k \geq 5$

$k(3^{n-1}+5^{n-1}) \geq 5(3^{n-1}+5^{n-1}) = 5 \cdot 3^{n-1}+5 \cdot 5^{n-1} > 3^n+5^n$

This means $k \leq 4$

$3 \cdot 3^{n-1}+5 \cdot 5^{n-1} > 3(3^{n-1}+5^{n-1})$, then $k \geq 4$

and we arrive to conclusion $k=4$

In this case $3^n+5^n = 4(3^{n-1}+5^{n-1})$ which gives us $5^{n-1}=3^{n-1}$ but if $n>1$ this eq is impossible

Hence $n=1$, we can easily check $2 \mid 8$.

$n=1$ is the only solution

(2) $\gcd(a,b)=1$

Let $\gcd(a-b, a+b)=d$

$d \mid a-b$ and $d \mid a+b$

$(a-b) = rd$ —(1) $\quad a+b = pd$ —(2) $\quad r, p \in \mathbb{Z}$

(1) + (2) $\quad 2a = d(r+p) \Rightarrow d \mid 2a$

(2) − (1) $\quad 2b = d(p-r) \Rightarrow d \mid 2b$

$\gcd(a,b)=1 \quad d|2a \quad d|2b$

This is only possible if either $d=2$ or $d=1$

d cannot divide both a and b.

$\therefore \gcd(a-b, a+b) = 1$ or

If both a and b are even then $\gcd = 2$

If both a and b are odd then $\gcd = 2$

If either ~~a~~ or b is odd, the other is even then $\gcd = 1$.

Scanned with CamScanner