DISCRETE MATHEMATICS ASSIGNMENT-2

① $1, 2, 4, 8, 16, 32, 64, \ldots$

$x_n = 2^n$ where $n = 0, 1, 2, 3, \ldots$

$1, 2, 4, 7, 10, 13, 16, \ldots$

$x_n = \frac{n(n-1)}{2} + 1$ where $n = 1, 2; 3, 4, \ldots$

$1, 2, 4, 8, 16, 32, 64, \ldots$

$x_n = 2^{n-1}$ where $n = 1, 2, 3, 4, \ldots$

② a) $a_n = n a_{n-1} + n^2 a_{n-2}$

$a_0 = 1 \quad a_1 = 1$

$a_2 = 2a_1 + (2)^2 a_0$

$= 2 \times 1 + 4 \times 1 = 6$

$a_3 = 3a_2 + (3)^2 a_1$

$= 3 \times 6 + 9 \times 1 = 18 + 9 = 27$

$a_4 = 4a_3 + 16a_2$

$= 4 \times 27 + 16 \times 6 = 204$

$a_5 = 5a_4 + 25a_3$

$= 5 \times 204 + 25 \times 27 = 1695$

$a_6 = 6a_5 + 36a_4$

$= 6 \times 1695 + 36 \times 204 = 17,514$

Scanned with CamScanner

classmate

Date

Page

b) $a_n = a_{n-1} + a_{n-3}$

$a_0 = 1 \quad a_1 = 2 \quad a_2 = 0$

$a_3 = a_2 + a_0 = 0+1 = 1$

$a_4 = a_3 + a_1 = 1+2 = 3$

$a_5 = a_4 + a_2 = 3+0 = 3$

$a_6 = a_5 + a_3 = 3+1 = 4$

$a_7 = a_6 + a_4 = 4+3 = 7$

③ $a_n = 2^n + 5 \cdot 3^n$

$a_0 = 6$

$a_1 = 2 + 5\times3 = 17$

$a_2 = 4 + 5\times9 = 49$

$a_3 = 8 + 5\times27 = \cancel{127}\ 143$

Now using $a_n = 5a_{n-1} - 6a_{n-2}$

$n=2;\ a_2 = 5a_1 - 6a_0 = 5\times17 - 6\times6 = 49 \quad a_2 = 49$

$n=3;\ a_3 = 5a_2 - 6a_1 = 5\times49 - 6\times17 = 143$

Therefore $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$ is same as $a_n = 2^n + 5\cdot3^n$ for $n \geq 2$.

④ a) $\cancel{a_n} = \cancel{3n}$

$a_n = 1$

$a_0 = 1 \quad a_1 = 1 \quad a_2 = 1 \quad a_3 = 1 \quad a_4 = 1 \ldots$

$a_n = -3a_{n-1} + 4a_{n-2}$

$a_2 = -3a_1 + 4a_0$

$= -3 \times 1 + 4 \times 1 = 1$

$a_3 = -3a_2 + 4a_1$

$= -3 \times 1 + 4 \times 1 = 1$

$a_4 = -3a_3 + 4a_2$

$= -3 \times 1 + 4 \times 1 = 1$

For $n \geq 2$, we see that $-3a_{n-1} + 4a_{n-2} = -3 \times 1 + 4 \times 1 = 1 = a_n$. Therefore, $\{a_n\}$, where $a_n = 3n$ is a solution of the recurrence relation.

b) $a_n = n^2 + n \qquad a_0 = 0$

$a_1 = 1 + 1 = 2$

$a_2 = 4 + 2 = 6 \qquad a_n = 2 + (n-1)$

$a_3 = 9 + 3 = 12$

$a_4 = 16 + 4 = 20$

$a_5 = 25 + 5 = 30$

$a_n = -3a_{n-1} + 4a_{n-2}$

$a_2 = -3a_1 + 4a_0 = -3 \times \cancel{3} + 0 = -6$

$a_3 = -3a_2 + 4a_1 = -3 \times 6 + 8 = -10$

For $n \geq 2$, we see that $-3a_{n-1} + 4a_{n-2}$

$-3[(n-1)^2 + (n-1)] + 4[(n-2)^2 + (n-2)]$

$-3[n^2 + \cancel{1} - \cancel{2n} + \cancel{n} - \cancel{1}] + 4[n^2 + \cancel{4} - \cancel{3}n + \cancel{n} - \cancel{2}]$

$-3(n^2 - n) + 4(n^2 - 3n + 2)$

$(n-1)[-3n + 4(n-2)] \neq a_n$

4) b) Hence $a_n = 2(-4)^n + 3$

$a_0 = 2+3 = 5$

$a_1 = 2(-4)+3 = -5$

$a_2 = 2(16)+3 = 35$

$a_3 = 2(-64)+3 = -125$

$a_n = -3a_{n-1} + 4a_{n-2}$

$a_2 = -3a_1 + 4a_0 = -3(-5) + 4(5) = 15+20 = 35$

$a_3 = -3a_2 + 4a_1 = -3(35) + 4(-5) = -125$

For $n \geq 2$ we see that $-3a_{n-1} + 4a_{n-2}$

$-3[2(-4)^{n-1} + 3] + 4[2(-4)^{n-2} + 3]$

$-3(2(-4)^{n-1}) - 9 + 4(2(-4)^{n-2}) + 12$

$-3(2(-4)^{n-1}) + 4(2(-4)^{n-2}) + 3 = \{a_n\}$

Therefore, $\{a_n\}$, where $a_n = 2(-4)^n + 3$ is a solution recurrence relation

5) a) $a_n = 2n$

$a_0 = 0 \quad a_1 = 2 \quad a_2 = 4 \quad a_3 = 6 \quad a_4 = 8$

The recurrence relation can be

$a_n = 2a_{n-1} - a_{n-2}$

b) $a_n = n^2 + n$

$a_0 = 0 \quad a_1 = 2 \quad a_2 = 6 \quad a_3 = 12 \quad a_4 = 20$

The recurrence relation can be

$a_n = a_{n-1} + 2n$

$a_n = 3a_{n-1}$

$20 = 24 - 6 + 2$

$a_n = 3a_{n-1} -$

$a_n = a_{n-1} + 2n$

$20 = 36 - a_n$

⑥ a) $a_n = 3a_{n-1}$ $a_0 = 2$

$a_1 = 3a_0 = 6$
$a_2 = 3a_1 = 18$
$a_3 = 3a_2 = 54$
$a_4 = 4a_3 = 162$

The solution $a_n = 2 \cdot 3^n$

b) $a_n = a_{n-1} + 2n + 3$ $a_0 = 4$

$a_1 = a_0 + \cancel{2} + 3 = 9$
$a_2 = 9 + 4 + 3 = 16$
$a_3 = 16 + 6 + 3 = 25$

$a_n = (n+2)^2$

⑦ $P = 1000\$$

$t = 1$

$A = P\left(1 + r/m\right)^{mt} = P\left(1 + r/m\right)^m$

$a_n = a_{n-1} + 0.09a_{n-1}$
$= 1.09a_{n-1}$

⑧ a) 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011
The pattern running through all binary strings in order. The next three terms are 1100, 1101, 1110.

b) This sequence is made up of Fibonacci numbers, starting with one 1, then three 2's the five 3's, and so on increasing the number of copies by two each time. The next three terms are 8, 8, 8.

9) a) $a_n = 1+(-1)^n$
$a_1 = 1+(-1)^1 = 0$ $a_2 = 1+(-1)^2 = 2$ $a_3 = 0$ $a_4 = 2$

$A_n = A_{n-2}$ $\in n \geq 3$

b) The set of odd positive integers
$a_n = a_{n-1} + 2$

10) $(0,0) \in S$
$(a+2, b+3) \in S$
$\{(2,3), (4,6), (6,9), (8,12), (10,15)\} \in S$

$(0,0) \in S$
$(a+3, b+2) \in S$
$\{(3,2), (6,4), (9,6), (12,8), (15,10)\} \in S$

11) $F(n)$ = Sum of n positive integers
$F(n) = \frac{n(n+1)}{2}$

Therefore, the recursive definition is $\frac{n(n+1)}{2}$

Scanned with CamScanner