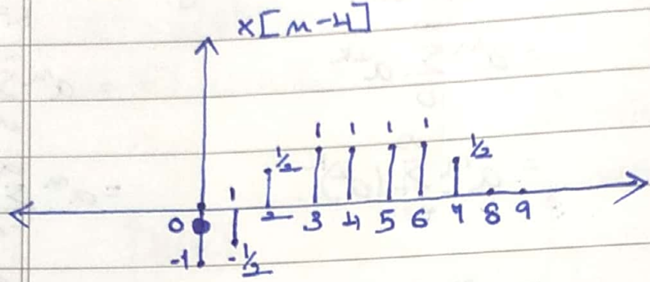
Signals and System
Assignment-1

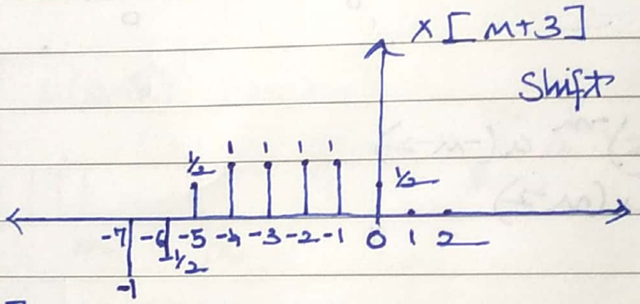
Question 1

a)

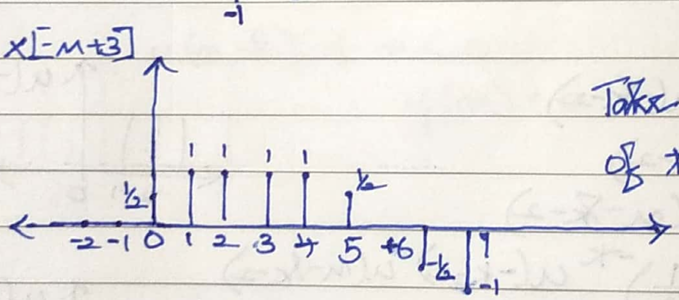


Shift 4 units to right

b) $x[3-n] = x[-n+3]$

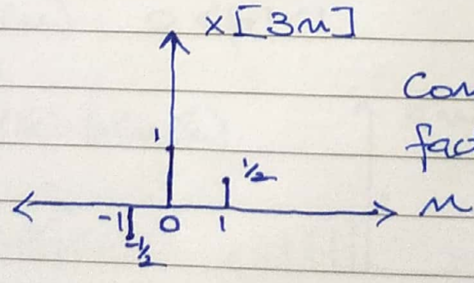


Shift 3 units to left



Take the mirror image of the above graph

c)



Compression by factor of 3 units

Since -3 and 3 is only divisible by the compression factor 3 there are only 2 value excluding 0.

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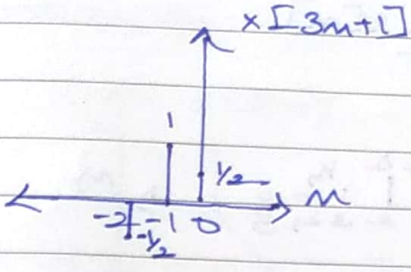
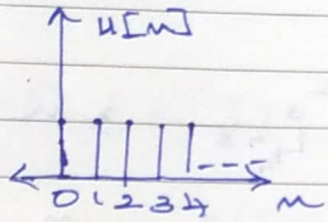
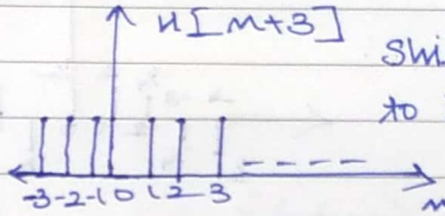
d)



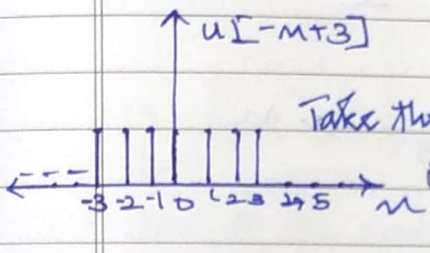
Shift the lines by 1 unit to the left obtained from the above graph (c) [Obtained c subdivision]

e)

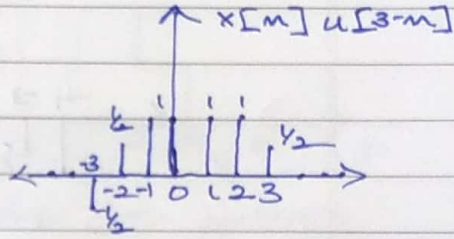




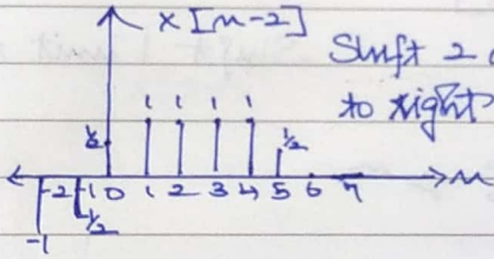
Shif 3 units to left



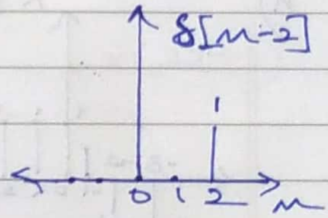
Take the mirror image of u[n+3]



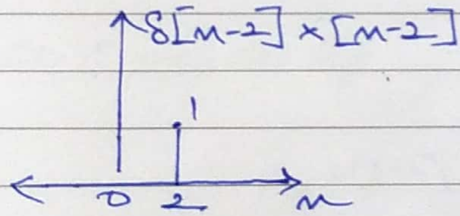
f)



Shift 2 units to right



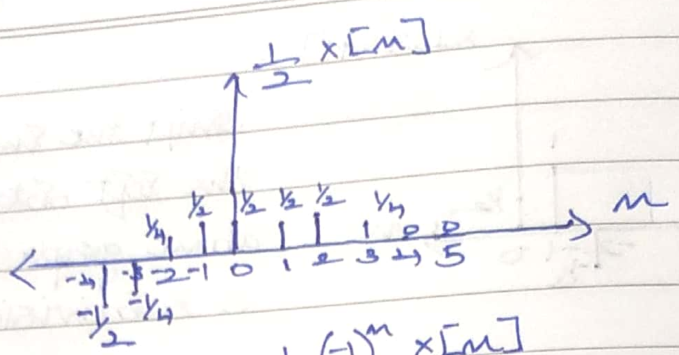
Shift δ[n] 2 units to right

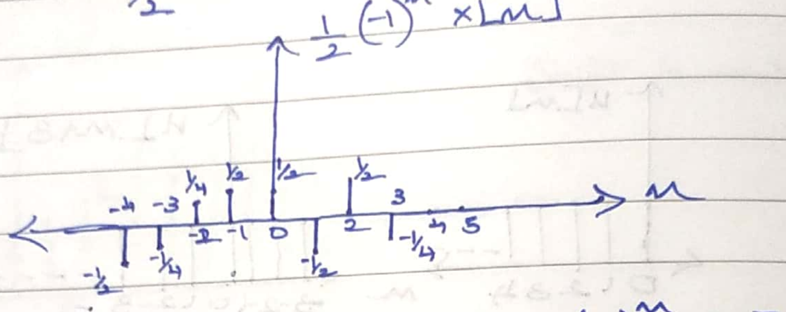


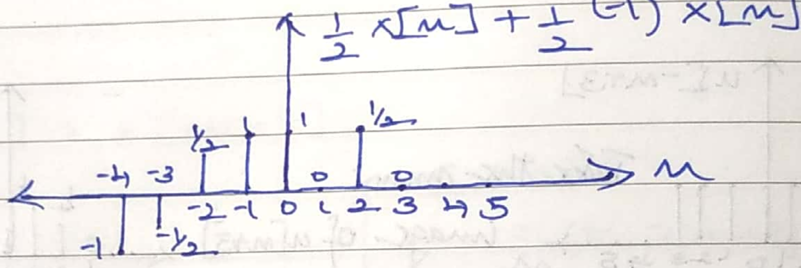
g) $\frac{1}{2}(-1)^0 x[0] = \frac{1}{2} x[0]$

$\frac{1}{2}(-1)^1 x[1] = -\frac{1}{2} x[1] = -\frac{1}{2}(-1)^3 x[3]$

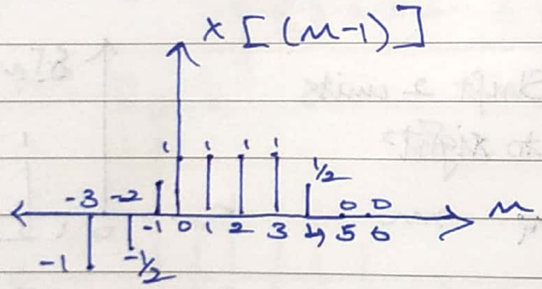
$\frac{1}{2}(-1)^2 x[2] = \frac{1}{2} x[2] = -\frac{1}{2}(-1)^4 x[5]$



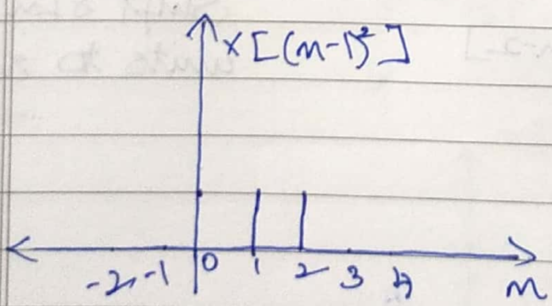




4)

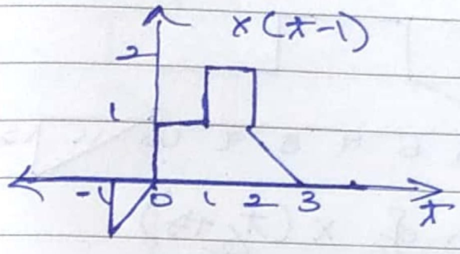


Shift 1 unit to right



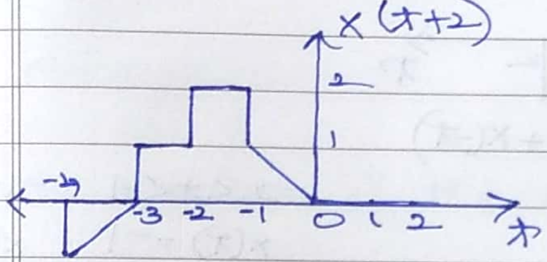
Question 2

a)

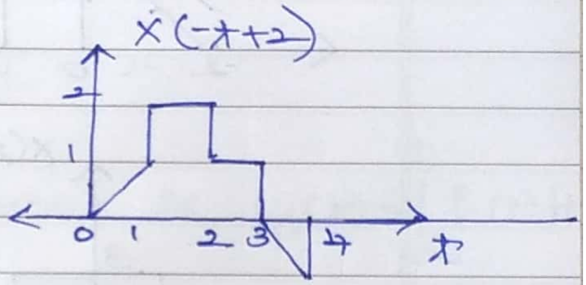


Shift 1 unit to right

b) $x(2-t) = x(-t+2)$

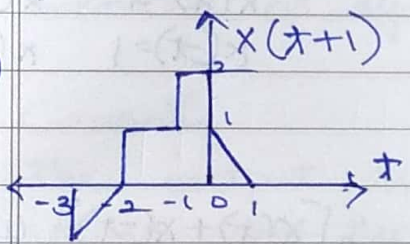


Shift 2 units to left

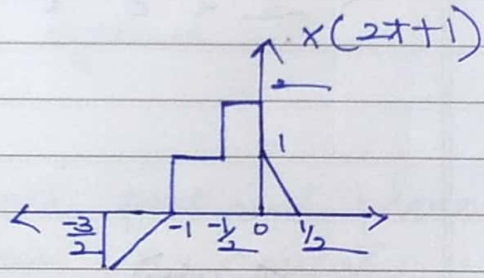


Take the mirror image of $x(t+2)$

c)

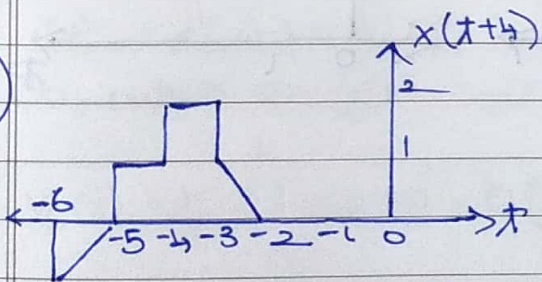


Shift 1 unit to left

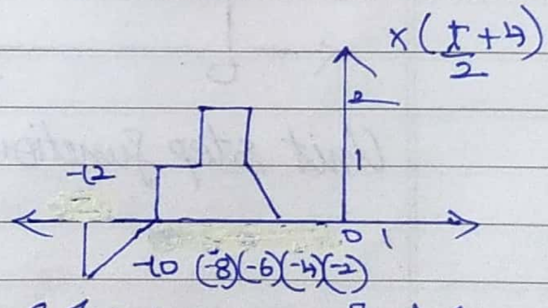


Compress by a factor 2

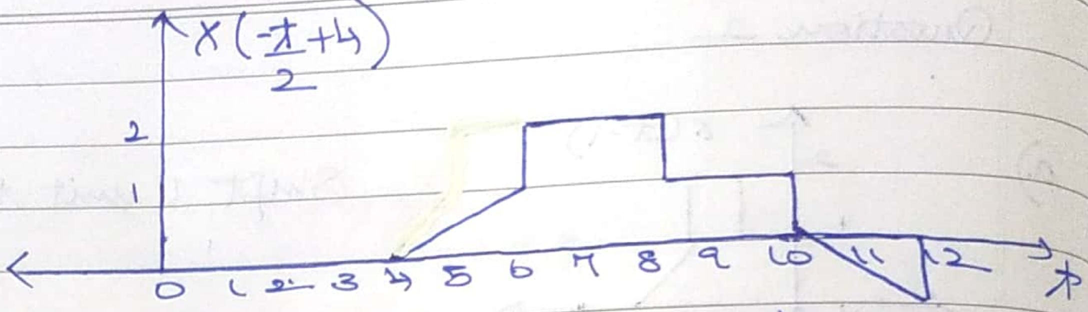
d)



Shift 4 units to left

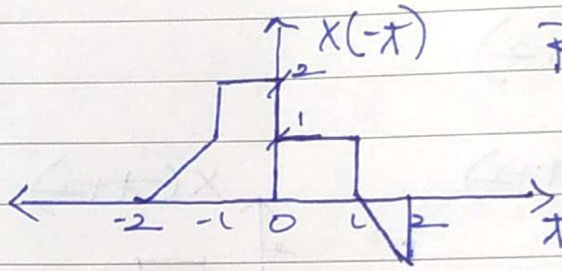


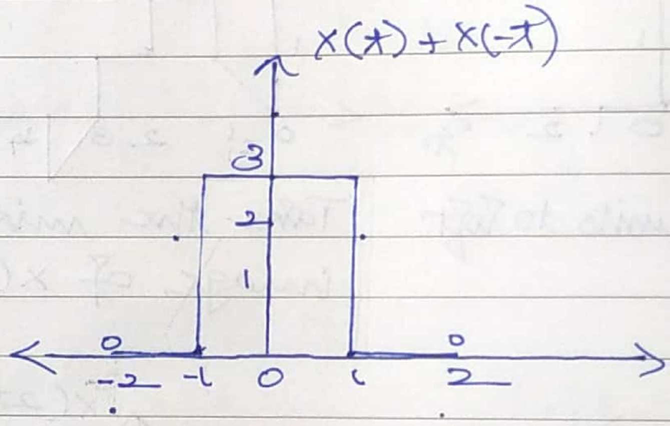
Enlarge by a factor 2



Take the reflection of $x\left(\frac{t}{2}+4\right)$

e) Reflection of $x(t)$



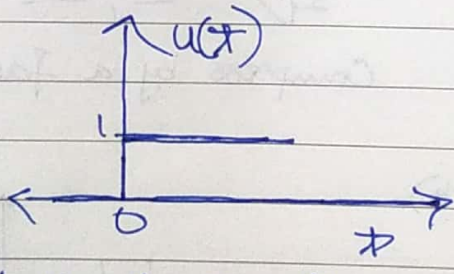


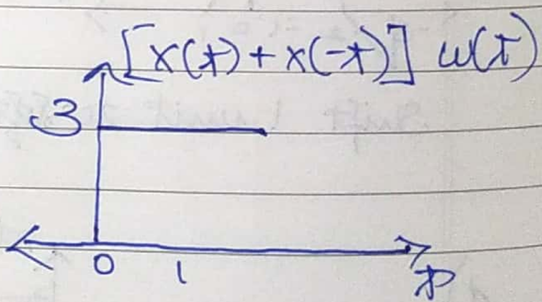
$-2 < t < -1$: $x(t) = -1$, $x(-t) = 1$

$-1 < t < 0$: $x(t) = 1$, $x(-t) = 2$

$0 < t < 1$: $x(t) = 2$, $x(-t) = 1$

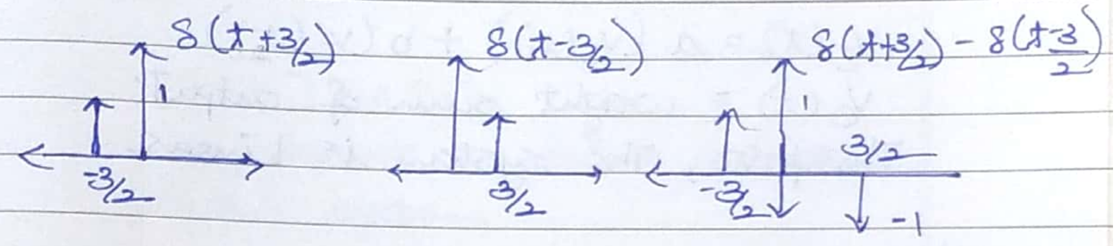
$1 < t < 2$: $x(t) = 1$, $x(-t) = -1$

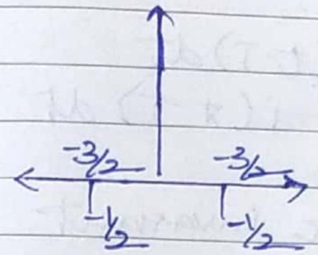




Unit step function

f)





$$\frac{y_2 - y_1}{x_2 - x_1} = -1$$

$$y_2 = \frac{1}{2}$$

Question 3

The equation of the system is $V_C(t) = \int_{-\infty}^{t} i_C(t)\,dt$

i) Memoryless

The integral $\left(\int_{-\infty}^{t}\right)$ means the system depends on past values, so the system is memory or dynamic system.

ii) Causal

The output depends on past and present values, so the system is causal.

iii) Linear

$i(t) = a i_1(t) + b i_2(t)$

So, output $\Rightarrow$ $y(t) = V_C(t)$

$$V_C(t) = \frac{1}{C}\int_{-\infty}^{t} [a i_1(t) + b i_2(t)]\,dt$$

$$= \frac{1}{C}\int_{-\infty}^{t} a i_1(t) + \frac{1}{C}\int_{-\infty}^{t} b i_2(t)\,dt$$

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$V_C(t) = a\,(V_C(t_1)) + b\,(V_C(t_2))$

$V_C(t)$ = weight sum of output

Therefore, The system is Linear

iv) Time Variant

$y(t,T) = V_C(t) = \frac{1}{C}\int_{-\infty}^{t-T} i(t-T)\,dt$

$y(t-T) = V_C(t) = \frac{1}{C}\int_{-\infty}^{t-T} i(t-T)\,dt$

$y(t,T) = y(t-T)$

So the system is Time invariant

v) Stable

A system is stable if it follows BIBO

$|x(t)| \leq M_x \leq \infty$

$y(t) = \frac{1}{C}\int_{-\infty}^{t} i(t)\,dt$

Bounded input ⇒ Unbounded output

So, unstable system

Question 4

a) $y[n] = x[-n]$

Time Invariant

$y[n,T] = [-n-T]$

$y(n-T) = x[-n-T]$

$y[n,T] = y(n-T)$

⇒ Time Invariant

Linearity

$ax_1[-n] + bx_2[-n]$ → weighted sum of input

$y[n] = ay_1[x] + by_2[x]$

$= ax_1[x] + by_2[-x]$

⇒ So the system is Linear

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Causal

$x = -1$; $y[-1] = x[1]$

The system depends on future values too

$\Rightarrow$ Non causal system

Memory

The system depends on past and future values.

$\Rightarrow$ Memory system

Stability

For M_x [input] $< \infty$; The M_y (output) $< \infty$

The system, $M_y(x[-n]) <$ Bounded

$\Rightarrow$ So the system is stable.

b) $y[n] = x[n-2] - 2x[n-8]$

Time invariant

$y[n,T] = x[n-T-2] - 2x[n-T-8]$

$y[n-T] = x[n-T-2] - 2x(n-T-8]$

$y[n,T] = y[n-T]$

$\Rightarrow$ The system is time invariant

ii) Linearity

Input $\rightarrow$ $ax_1[n-2] - 2ax_1[n-8] + bx_1[n-2] - 2bx_2[n-8]$

Output $\rightarrow$ $y[n] = ay_1[n] + by_2[n]$

$= a(x[n-2] - 2x_1[n-8]) + b(x_2[n-2] - 2x_2[n-8])$

Input = Output

$\Rightarrow$ So the system is linear

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Causal

$y[n] = x[n-2] - 2x[n-8]$

Since the system depends only on past values

The is a causal system

Memory

$y[n] = x[n-2] - 2x[n-8]$

$\downarrow$ Past values $\quad \downarrow$ Past values

Memory system

Stability

$M_x < \infty \Rightarrow M_y < \infty$

The system is stable

c) $y[n] = n x[n]$

Time invariant

$y[n,k] = n x[n-k]$

$y[n-k] = (n-k) x[n-k]$

$y[n,k] \neq y[n-k]$

$\Rightarrow$ Time variant

Linearity

Input $\rightarrow a n x_1[n] + b n x_2[n]$

Output $\rightarrow y[n] = a y_1[n] + b y_2[n]$

$= a n x_1[n] + b n x_2[n]$

Input = output

So, the system is linear

Causal / Non Causal
Depends on present value only
So it is a Causal system

Memory
Depends on present values only
It is a memory system

Stability
$M_x < \infty$
$m\,x[m]$ 'm' increases with M_x
for $M_x \to \infty$ $\quad m\,x[m] \to \infty$
So unstable system

d) $y[x] = \mathcal{E}_v \{x(m-1)\}$
$y[m] = \frac{1}{2}[x[m-1] + x[-(m-1)]]$

Time Invariant
$y[m,k] = \frac{1}{2}\{x[m-1-k] + x[1-m-k]\}$
$y[m-k] = \frac{1}{2}\{x[m-k-1] + x[1-m+k]\}$
$y[m,k] \neq y[m-k]$
Time variant

Linearity
$\text{input} \to a(x_1[m-1] + x_1[1-m]) + b(x_2[m-1] + x_2[1-m])$
$y[m] = \frac{1}{2}(ax_1[m-1] + ax_1[1-x] + bx_2(m-1) + bx_2[1-m])$

$y_1[m] = \frac{1}{2}\{x_1[m-1] + x_1[1-x]\}$
$y_2[m] = \frac{1}{2}\{x_2[m-1] + x_2[1-x]\}$

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Output $y[n] = ay_1[x] + by_2[x]$

$= \frac{1}{2}(ax_1[n-1] + ax_1[1-n] + b.x_2[n-1] + bx_2[1-n])$

Input = output

The system is linear

Causal / Non Causal

$y[n] = \frac{1}{2}(x[n-1] + x[1-n])$ (Causal + Non Causal)

The system is Non Causal

Memory

This is a memory system

Stability

$y[n] = \frac{1}{2}[x[n-1] + x[1-n]]$

$M_x < \infty$, $M_y < \infty$

So the system is stable

e) $y[n] = \begin{cases} x[n] & n \geq 1 \\ 0 & n = 0 \\ x[n+1] & n \leq -1 \end{cases}$

$y[n-k] = \begin{cases} x[n-k] & ; n \geq 1+k \\ 0 & ; n = k \\ x[n+1-k] & ; n \leq k-1 \end{cases}$

$y[n,k] = \begin{cases} x[n-k] & ; n \geq 1 \\ 0 & ; n = 0 \\ x[n-k+1] & ; n \leq -1 \end{cases}$

$y[n-k] \neq y[n,k]$

Time Variant

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Linearity

Input $y[n] = \begin{cases} a x_1[n] + b x_2[n] & ; n \geq 1 \\ 0 & ; n = 0 \\ a x_1[n+1] + b x_2[n+1] & ; n \leq -1 \end{cases}$

Output $y[n] = \begin{cases} a x_1[n] + b x_2[n] \\ 0 \\ a x_1[n+1] + b x_2[n] \end{cases}$

Input = Output
The system is linear

Causal / Non Causal
Since it depends on future values
The system is non causal

Memory
The system is memory or dynamic system
Since it depends on future input values

Stability
$M_x < \infty$ $M_y < \infty$
So the system is stable

f) $y[n] = \begin{cases} x[n] & ; n \geq 1 \\ 0 & ; n = 0 \\ x[n] & ; n \leq -1 \end{cases}$

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Time invariant

$$y(n,k) = \begin{cases} x[n-k] & n > 1 \\ 0 & n = 0 \\ x[n-k] & n \le -1 \end{cases}$$

$$y[n-k] = \begin{cases} x[n-k] & n > k+1 \\ 0 & n = k \\ x[n-k] & n \le k-1 \end{cases}$$

$y(n,k] \neq y[n-k]$

So the system time variant

Linearity

input $\rightarrow$ $$y[n] = \begin{cases} ax_1[n] + bx_2[n] & n \geq 1 \\ 0 & n = 0 \\ ax_1[n] + bx_2[n] & n \leq -1 \end{cases}$$

output $\rightarrow$ $$y[n] = \begin{cases} ax_1[n] + bx_2[n] & n \geq 1 \\ 0 & n = 0 \\ ax_1[n] + bx_2[n] & n \leq -1 \end{cases}$$

Input = output

So Linear System

Causal / Non Causal

This is a causal system

$x[n]$ depends on present values only

Memory

This a memoryless or static system.

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Stability

For all $M_x < \infty \Rightarrow M_y < \infty$ Bounded

So, this a stable system

g) $y[n] = x[4n+1]$

Time invariant

$y[n,k] = x[4n+1-k]$

$y[n-k] = x[4n+1-4k]$

$y[n,k] \neq y[n-k]$

The system is time variant

Linearity

Input $\rightarrow a x_1[4n+1] + b x_2[4n+1]$

Output $\rightarrow y[n] = a y_1[n] + b y_2[n]$

$= a x_1[4n+1]^2 + b x_2[4n+1]$

Input = Output

The system is Linear

Causal

$y = x[4n+1]$ Depends on Future inputs

Non causal system

Memory

The systems depends on future inputs, it is memory system

Stability

$M_x < \infty \Rightarrow M_y < \infty$ Bounded

The system is stable