Discrete Mathematics
Assignment-2
(Reference: read slides containing lectures 5, 6, 7 and 8)

1. Find at least three different sequences beginning with the terms 1, 2, 4 whose terms are generated by a simple formula or rule.

2. Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.

   (a) $a_n = na_{n-1} + n^2 a_{n-2},\ a_0 = 1,\ a_1 = 1$

   (b) $a_n = a_{n-1} + a_{n-3},\ a_0 = 1,\ a_1 = 2,\ a_2 = 0$

3. Let $a_n = 2^n + 5 \cdot 3^n$ for $n = 0, 1, 2 \cdots$
   Then show that $a_n = 5a_{n-1} - 6a_{n-2}$ for integers n with $n \geq 2$.

4. Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = \ -3a_{n-1} + 4a_{n-2}$

   (a) $a_n = 1$

   (b) $a_n = 2(-4)^n + 3$

5. For each of these sequences find a recurrence relation satisfied by this sequence. (The answers are not unique because there are infinitely many different recurrence relations satisfied by any sequence.)

   (a) $a_n = 2n$

   (b) $a_n = n^2 + n$

6. Find the solution to each of these recurrence relations and initial conditions.

   (a) $a_n = 3a_{n-1},\ a_0 = 2$

   (b) $a_n = a_{n-1} + 2n + 3,\ a_0 = 4$

7. A person deposits \$1000 in an account that yields 9% interest compounded annually. Set up a recurrence relation for the amount in the account at the end of n years.

8. For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list.Assuming that your formula or rule is correct, determine the next three terms of the sequence.

   (a) 1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, $\cdots$

   (b) 1, 2, 2, 2, 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, $\cdots$

9. Give a recursive definition of the sequence $\{a_n\}$, $n = 1, 2, 3, \cdots$

   (a) $a_n = 1 + (-1)^n$

   (b) The set of odd positive integers.

10. Let S be the subset of the set of ordered pairs of integers defined recursively by
    Basis step: $(0,0) \in S$
    Recursive step: If $(a,b) \in S$, then $(a+2, b+3) \in S$ and $(a+3, b+2) \in S$. Then List the elements of S produced by the first five applications of the recursive definition.

11. Let F be the function such that $F(n)$ is the sum of the first n positive integers. Give a recursive definition of $F(n)$.