LOGIC

PSAT 2021

Boolean Logic

Expressions

- An expression is simply one or more variables and/or constants joined by
- operators
- An expression is evaluated and produces a result
- The result of all arithmetic expressions are either integers or reals
- An expression can also yield ***a result that is either true or false- BOOLEAN***
- Such an expression is called a ***relational expression***
- The result reflects how something "***relates to***" something else.

For example

- **"Is the value of x greater than the value of y?"**
- Note that the preceding poses a question.
- Relational expressions are usually intended to answer yes/no, or true/false, questions.
- Obviously, boolean values and boolean variables play an important role in relational expressions.

Operators

To build relational expressions, two types of operators are used,

relational operators and logical operators

Relational operators

Operator	Meaning
==	equals
!=	does not equal
>	is greater than
>=	is greater than or equal to
<	is less than
<=	is less than or equal to

Logical operators

&&	AND	(true if both arguments are true, false otherwise)
||	OR	(true if either argument is true, false otherwise)
!	NOT	(true if argument is false, false otherwise)

Examples

Expression	Value of expression
3 < 4	True
7.6 <= 9	True
4 == 7	False
8.3 != 2.1	True

Initial values	Expression	Value of expression
a = 3 b=4 c=5 d=6	a ==b c< d (a==b) && (c<d) (a==b) || (c<d)	False True False True
	result = (a==b) && (c<d) !result	False true

Truth assignment: True or False

Let,

(a < b || (a >= b && c == d)) be **statement 1**

(a < b || c == d) be **statement 2**

In the statements 1 and 2,

a < b , c == d, a >= b are conditions to be checked for TRUE or FALSE

to determine the truth value of the entire expression

Logical expression

Let (a < b || (a >= b && c == d)) be statement 1

Let (a < b || c == d) be statement 2

 # A. Let, *a is less than b* be True

- We inspect the first of the two conditions (a < b) to see if it is true
- It is true in both statements 1 and 2
- TRUE is returned by both the statements

B. Let, *a is less than b* be FALSE

In statement 1 :

We inspect the second of the two conditions

(a >= b && c == d) to see if it is true

- We are asking whether both a >= b AND c == d are true
- If a < b is false, then a >= b is of course true

✓ *Therefore whether true or false is returned entirely depends on the condition*

c == d

- If c == d is true then **true is returned** [as it is understood a >= b is true]

  and as a < b is false, **the statement 1 returns true**
- If c == d is false and **false is returned** [as it is understood a >= b is true]
- and as a < b is false, **the statement 1 returns false**

In statement 2 :

We inspect the second of the two conditions c == d

- If c == d is true then **true is returned** and as a < b is false, **the statement 2 returns true**
- If c == d is false and **false is returned** and as a < b is false, **the statement 2 returns false**

Questions

- **Boolean expressions**

1. credits >= 120 && GPA >=2.0 (Find the truth value of the Boolean expression credits = 260, 100, GPA = 10,1 respectively.
2. Suppose you are checking whether a value is between lower and upper bound (range) : low < val < high.
3. Now suppose we want the opposite condition: that val is *not* strictly between low and high.
4. Assume a patient has a temperature of 100°C and BP of 200. Give a Boolean expression to whether the patient is normal.

[Std: Temp=37°C and BP=120]

Questions

5. Solve the given Compound Expression

   a. A==B==C==D (Given A ,B,D = true and C=false)

   b.(x > 0 && (y / x) == 3)(Given x to 0 and y to 3)

6. Is it possible to find Equivalent Expr for the given conditional Expr.

Expression	Equivalent Expression
!(a > b)	
!(a == b)	
!(a == b || c == d)	
!(a == b && c > d)	

7. A person is eligible to be a US Senator who is at least 30 years old and has been a US citizen for at least 9 years. A person is eligible to be a US Representative who is at least 25 years old and has been a US citizen for at least 7 years. Frame the Boolean expression for the Senator and Representative. Given following candidates check if he fits as Senator or Representative:

Simba: Age- 12, length of citizenship- 12

Mufasa: Age- 32, length of citizenship- 20

Scar: Age – 27, length of citizenship – 8

Answers

1.

Credit	GPA	Expression
260	10	True
260	1	False
100	10	False
100	1	False

2. low < val && val < high

3. Approach 1: low < val || val < high
   Approach 2: !(low < val && val < high)

4. temp>37 &&BP>120

5. a. false
   b. false

6.

Expression	Equivalent Expression
!(a > b)	a <= b
!(a == b)	a != b
!(a == b || c == d)	a != b && c != d
!(a == b && c > d)	a != b || c <= d

7. **Expression**

senator: age>=30 && citizenship>=9

Representative: age>= 25 && citizenship>=7

Output:

Simba :Not Eligible

Mufasa: Senator

Scar:Representative