20CYS111 Digital Signal Processing

Elementary Signals

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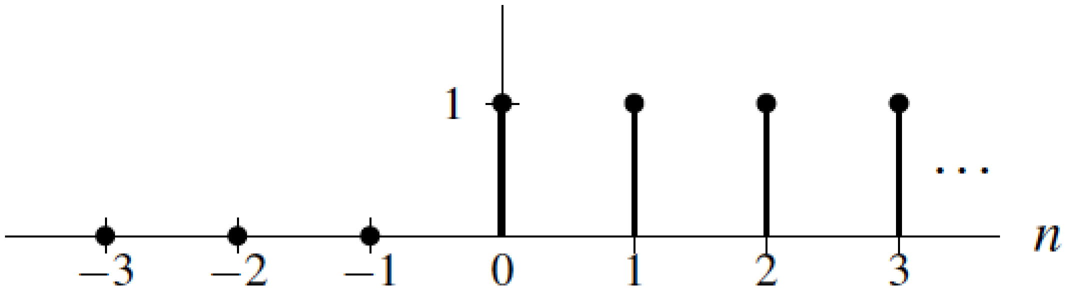
Unit Step Function (Discrete-Time)

*The **discrete-time unit step function** is defined as*

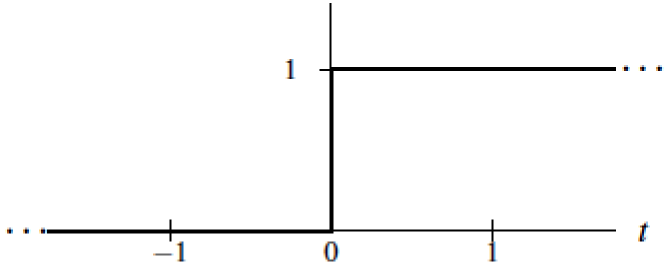
$$\boxed{u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}}.$$



Unit Step Function (Continuous-Time)

The ***continuous-time unit step function*** *is defined as*

$$\boxed{u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}}.$$



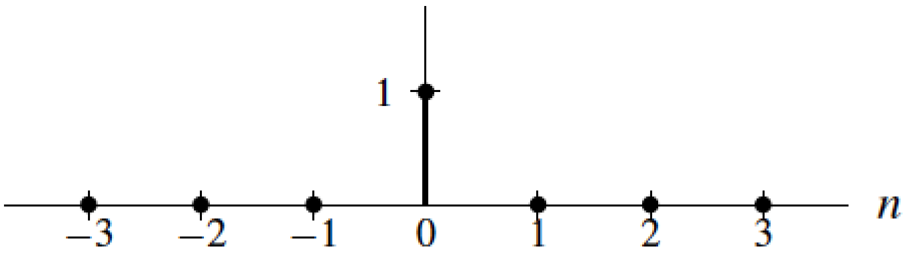
The continuous-time unit step function $u(t)$ has a discontinuity at time $t = 0$.

Unit Impulse Function (Discrete-Time)

The unit impulse function is also called the Dirac delta function.

*The **discrete-time unit impulse function** is defined as*

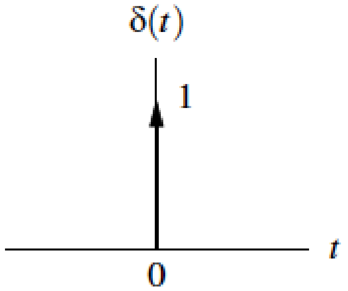
$$\boxed{\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}}.$$



Unit Impulse Function (Continuous-Time)

*The **continuous-time unit impulse function** is defined by the pair of relations:*

$$\boxed{\delta(t) = 0, \quad \text{for} \quad t \neq 0}, \qquad \text{and} \qquad \boxed{\int_{-\infty}^{\infty} \delta(t)dt = 1}.$$

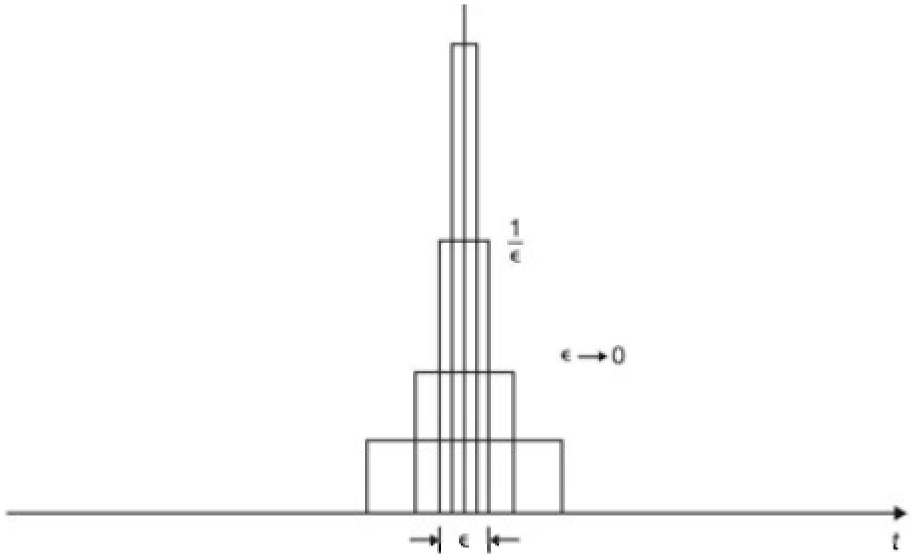


Unit Impulse Function (Continuous-Time)

Define the function $g_\epsilon(t)$ *as*

$$\boxed{g_\epsilon(t) = \begin{cases} 1/\epsilon & -\frac{\epsilon}{2} < t < \frac{\epsilon}{2} \\ 0 & \text{otherwise} \end{cases}}.$$

Then, we have $\boxed{\delta(t) = \lim_{\epsilon \to 0} g_\epsilon(t)}$.

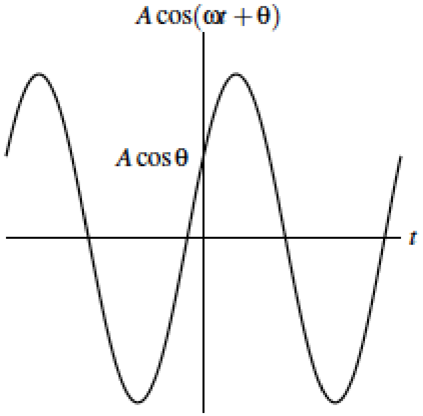


Sinusoidal Signals (Continuous-Time)

*The **continuous-time sinusoidal signal** is defined as*

$$\boxed{x[n] = A\cos(\omega t + \phi) = A\cos(2\pi f t + \phi), \qquad -\infty < t < \infty},$$

*where A is the **amplitude**, ω is the **angular frequency** (in radians per second), ϕ is the **phase** (in radians) and f is the **frequency** (in cycles per second or hertz).*

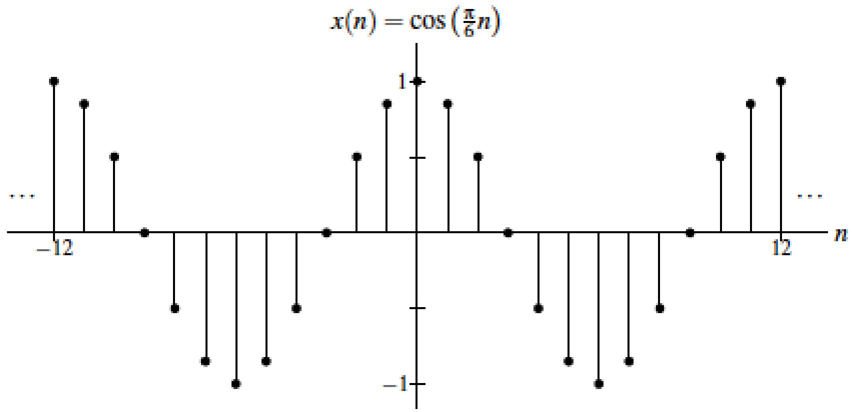


Sinusoidal Signals (Discrete-Time)

*The **discrete-time sinusoidal signal** is defined as*

$$\boxed{x(t) = A\cos(\Omega n + \phi), \qquad -\infty < n < \infty},$$

*where A is the **amplitude**, Ω is the **angular frequency** (in radians per sample) and ϕ is the **phase** (in radians).*

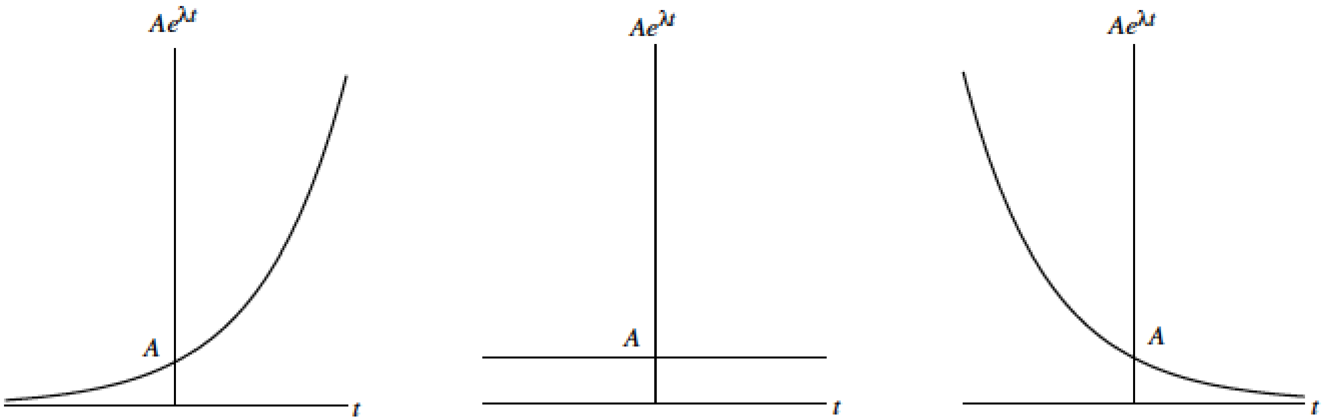


Real Exponential Signal (Continuous-Time)

The ***continuous-time (real) exponential signal*** *is defined as*

$$\boxed{x(t) = Ae^{\lambda t}, \quad -\infty < t < \infty},$$

where both A *and* λ *are real numbers.*



$\lambda > 0$ $\qquad$ $\lambda = 0$ $\qquad$ $\lambda < 0$

Real Exponential Signal (Discrete-Time)

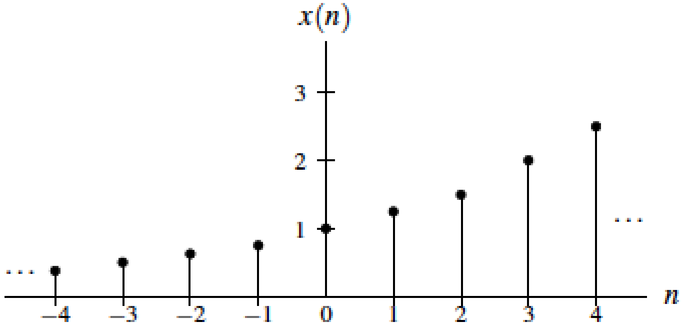
*The **discrete-time (real) exponential signal** is defined as*

$$\boxed{x[n] = ca^n, \qquad -\infty < n < \infty},$$

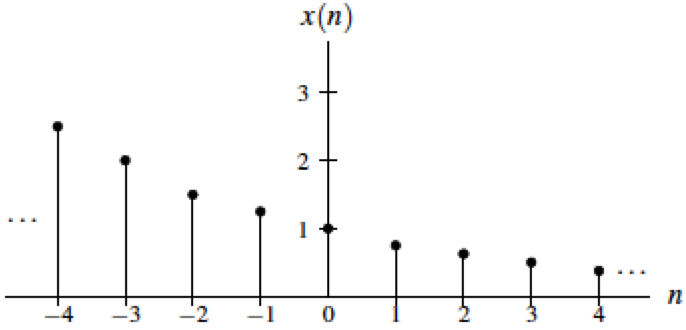
where both c and a are real numbers.

The exponential characteristic of this signal can be verified by substituting $\boxed{a = e^{\lambda}}$, leading to $\boxed{x[n] = ce^{\lambda n}}$.

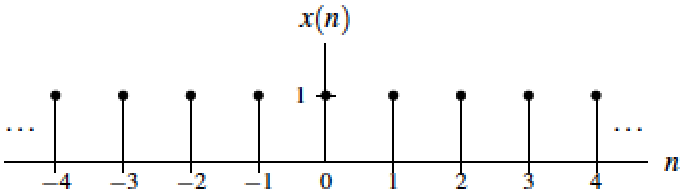
Real Exponential Signal (Discrete-Time)



$|a| > 1, a > 0$ $[a = \frac{5}{4}; c = 1]$

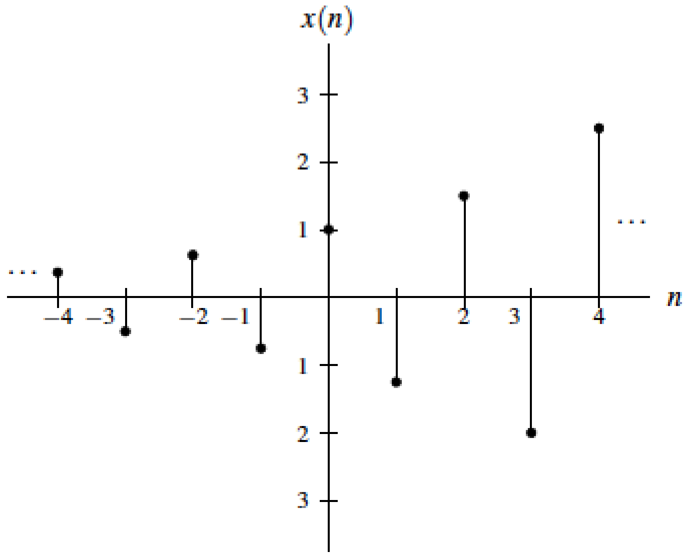


$|a| < 1, a > 0$ $[a = \frac{4}{5}; c = 1]$

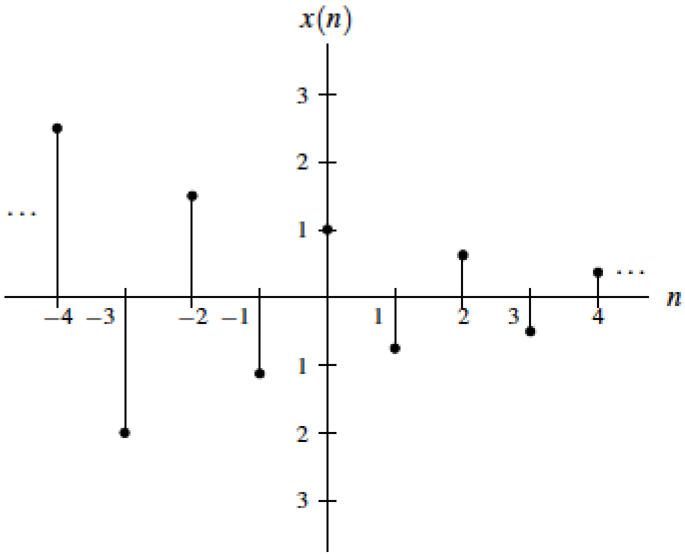


$|a| = 1, a > 0$ $[a = 1; c = 1]$

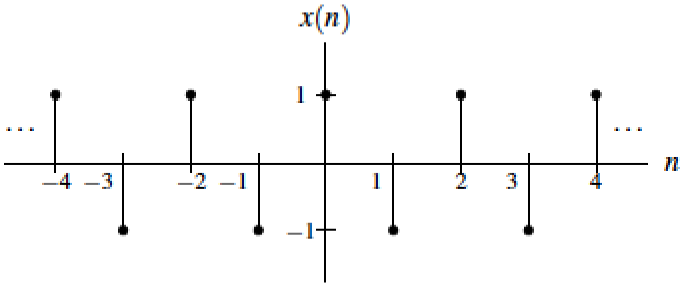
Real Exponential Signal (Discrete-Time)



$|a| > 1,\ a < 0$ $[a = -\frac{5}{4};\ c = 1]$



$|a| < 1,\ a < 0$ $[a = -\frac{4}{5};\ c = 1]$



$|a| = 1,\ a < 0$ $[a = -1;\ c = 1]$

Complex Exponential Signal (Continuous-Time)

*A **continuous-time complex exponential signal** $x(t)$ has the same form as that of the continuous-time (real) exponential signal, that is*

$$\boxed{x(t) = Ae^{\lambda t}, \qquad -\infty < t < \infty},$$

but here both A and λ are, in general, complex numbers.

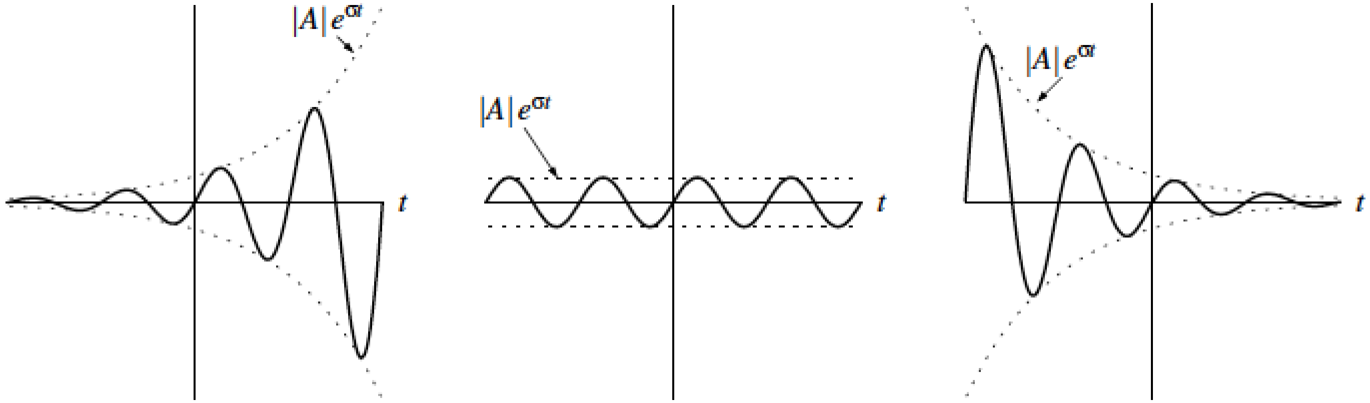
Substitute $\boxed{A = |A|e^{j\phi}}$ and $\boxed{\lambda = \sigma + j\omega}$ in the above equation, and obtain

$$x(t) = Ae^{st} = |A|e^{j\phi}e^{(\sigma+j\omega)t} = |A|e^{\sigma t}e^{j(\omega t+\phi)}.$$

Complex Exponential Signal (Continuous-Time)

Then, apply ***Euler's identity*** $\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$*, and obtain*

$$x(t) = |A|e^{\sigma t}e^{j(\omega t+\phi)} = \underbrace{|A|e^{\sigma t}\cos(\omega t+\phi)}_{Real\{x(t)\}} + j\underbrace{|A|e^{\sigma t}\sin(\omega t+\phi)}_{Imaginary\{x(t)\}}$$



$\sigma > 0$ $\sigma = 0$ $\sigma < 0$

Complex Exponential Signal (Discrete-Time)

A ***discrete-time complex exponential signal*** *$x[n]$ has the same form as that of the discrete-time (real) exponential signal, that is*

$$\boxed{x[n] = ca^n, \qquad -\infty < n < \infty},$$

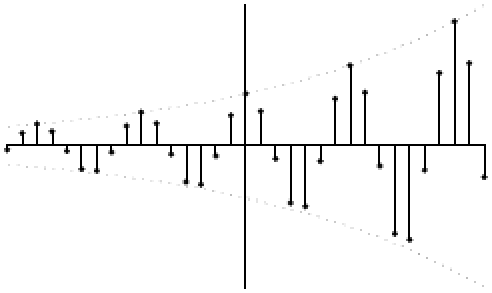
but here both c and a are, in general, complex numbers.

Substitute $\boxed{c = |c|e^{j\phi}}$ *and* $\boxed{a = |a|e^{j\Omega}}$ *in the above equation, and obtain*

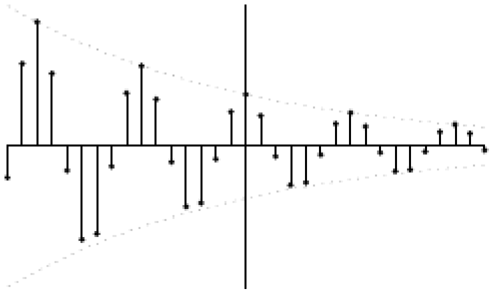
$$x[n] = ca^n = |c|e^{j\phi}\left(|a|e^{j\Omega}\right)^n = |c||a|^n e^{j(\Omega n+\phi)}$$

$$= \underbrace{|c||a|^n \cos(\Omega n + \phi)}_{Real\{x[n]\}} + j\underbrace{|c||a|^n \sin(\Omega n + \phi)}_{Imaginary\{x[n]\}}.$$

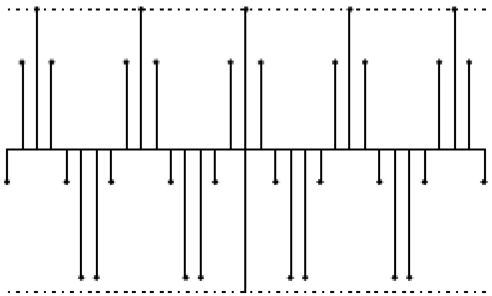
Complex Exponential Signal (Discrete-Time)



$|a| > 1$



$|a| < 1$



$|a| = 1$

References:

[1] Simon Haykin and Barry Van Veen, Signals and Systems, Second Edition, John Wiley and Sons, 2003.

[2] Lecture Notes by Richard Baraniuk.
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