The Subtraction Rule (principle of inclusion–exclusion)

THE SUBTRACTION RULE If a task can be done in either n_1 ways or n_2 ways, then the number of ways to do the task is $n_1 + n_2$ minus the number of ways to do the task that are common to the two different ways.

Example 16

A computer company receives 350 applications from computer graduates for a job. Suppose that 220 of these applicants majored in computer science, 147 majored in business, and 51 majored both in computer science and in business. How many of these applicants majored neither in computer science nor in business?

To find the number of these applicants who majored neither in computer science nor in business, we can subtract the number of students who majored either in computer science or in business (or both) from the total number of applicants.

Let A_1 be the set of students who majored in computer science and A_2 the set of students who majored in business.

Then $A_1 \cup A_2$ is the set of students who majored in computer science or business (or both)
and $A_1 \cap A_2$ is the ? By the subtraction rule

The Pigeonhole Principle

- Suppose that a flock of 20 pigeons flies into a set of 19 pigeonholes to roost.
- A least one of these 19 pigeonholes must have at least two pigeons in it.

This illustrates a general principle called the pigeonhole principle, which states that if there are more pigeons than pigeonholes, then there must be at least one pigeonhole with at least two pigeons in it.

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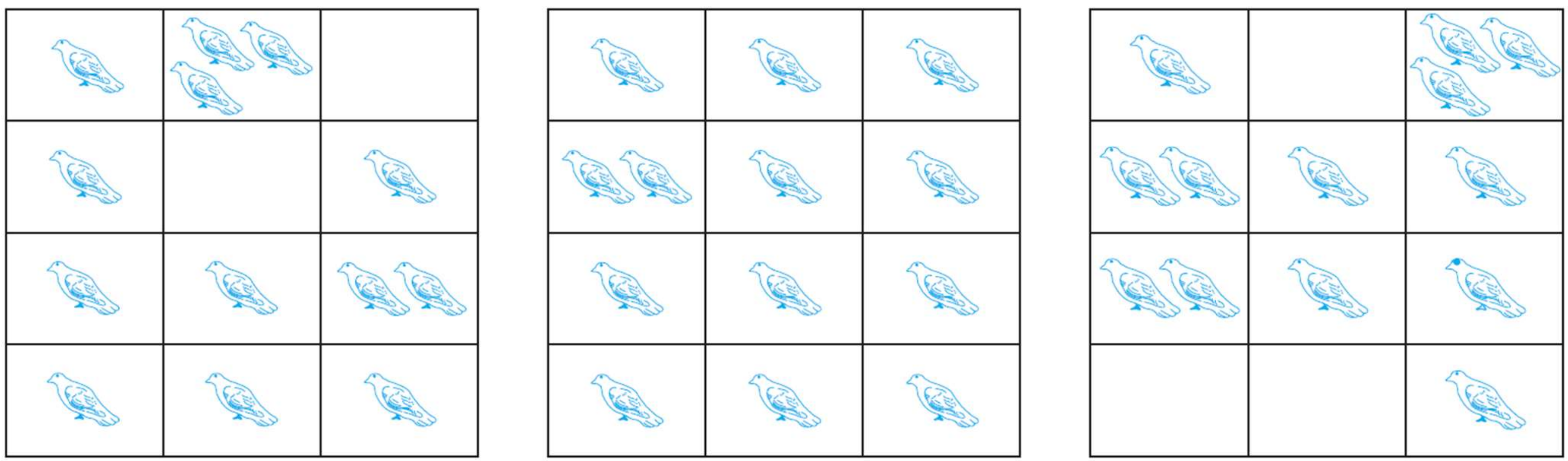

There Are More Pigeons Than Pigeonholes

Example 1

A function ***f*** from a set with **k + 1** or more elements to a set with **k** elements is not one-to-one.

Example 2

Among any group of 367 people, there must be at least two with the same birthday, because there are only 366 possible birthdays.

Example 3

In any group of 27 English words, there must be at least two that begin with the same letter, because there are 26 letters in the English alphabet.

Example 4

How many students must be in a class to guarantee that at least two students receive the same score on the final exam, if the exam is graded on a scale from 0 to 100 points?

The Generalized Pigeonhole Principle

The pigeonhole principle states that there must be at least two objects in the same box when there are more objects than boxes.

when the number of objects exceeds a multiple of the number of boxes.

THE GENERALIZED PIGEONHOLE PRINCIPLE If N objects are placed into k boxes, then there is at least one box containing at least $\lceil N/k \rceil$ objects.

Example

Among 100 people there are at least $\lceil \frac{100}{12} \rceil = 9$ who were born in the same month.

Example

What is the minimum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade, if there are five possible grades, A, B, C, D, and F?

Solution

The minimum number of students needed to ensure that at least six students receive the same grade is the smallest integer N such that $\lceil \frac{N}{5} \rceil = 6$

The smallest such integer is N = 5 · 5 + 1 = 26.

Example

How many cards must be selected from a standard deck of 52 cards to guarantee that at least three cards of the same suit are chosen?

Using the generalized pigeonhole principle, we see that if N cards are selected, there is at least one box containing at least $\lceil \frac{N}{4} \rceil$ cards

we know that at least three cards of one suit are selected if $\lceil \frac{N}{4} \rceil \geq 3$

N = 2 · 4 + 1 = 9, so nine cards suffice.

One good way to think about this is to note that after the eighth card is chosen, there is no way to avoid having a third card of some suit.