Computing Continued Fraction of 'x'

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What is Continued Fraction Algorithm?

This is an algorithm used for finding a continued fraction expression of a number through an iterative process. It is similar to Euclid's algorithm but we can use this for irrational numbers too. In case of an irrational number, the algorithm won't terminate but the number will be written as an infinite simple continued fraction.

Continued Fraction Algorithm

Let x be a real number. Let $x_0 = x$.

1. Set a_m to be the integral part of x_m.
2. Set ξ_m to be $x_m - a_m$.
3. If $\xi_m \neq 0$,set $1/\xi_m$ as x_{m+1} and go back to step1 to compute a_{m+1}.
4. If $\xi_m = 0$, terminate this algorithm.

An example will make it much more clear,

Let x = 2.875

Its integral part is 2 and so the continued fraction starts as [2, ...].

2.875 − 2 = 0.875

Calculate 1/0.875 using a calculator to get 1.14285714285714. Its integral part is 1.

So we now have [2, 1, ...].

1.14285714285714−1 = 0.14285714285714. Calculate 1/0.14285714285714 to get 7.0000000000014 whose integral part is 7.

The continued fraction is now [2, 1, 7, ...].

7.0000000000014 − 7 = 0.0000000000014 which is “almost” 0.

So, we terminate the algorithm here to get [2, 1, 7]. Therefore,

$$2.875 = 2 + \cfrac{1}{1 + \cfrac{1}{7}}$$

CF of a negative rational number:

$$\frac{-17}{12} = -2 + \frac{7}{12}$$

$$= -2 + \cfrac{1}{\cfrac{12}{7}}$$

$$= -2 + \cfrac{1}{1 + \cfrac{5}{7}} = -2 + \cfrac{1}{1 + \cfrac{1}{\cfrac{7}{5}}}$$

$$= -2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{2}{5}}} = -2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{\cfrac{5}{2}}}}$$

1

$$= -2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{2}}}}$$

$[-2, 1, 1, 2, 2]$

2

$$= -2 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{1}}}}}$$

$[-2, 1, 1, 2, 1, 1]$.

Incase of an irrational number,

Example:

$$\sqrt{6} = 2 + \cfrac{2}{4 + \cfrac{2}{4 + \cfrac{2}{4 + \ldots}}}$$

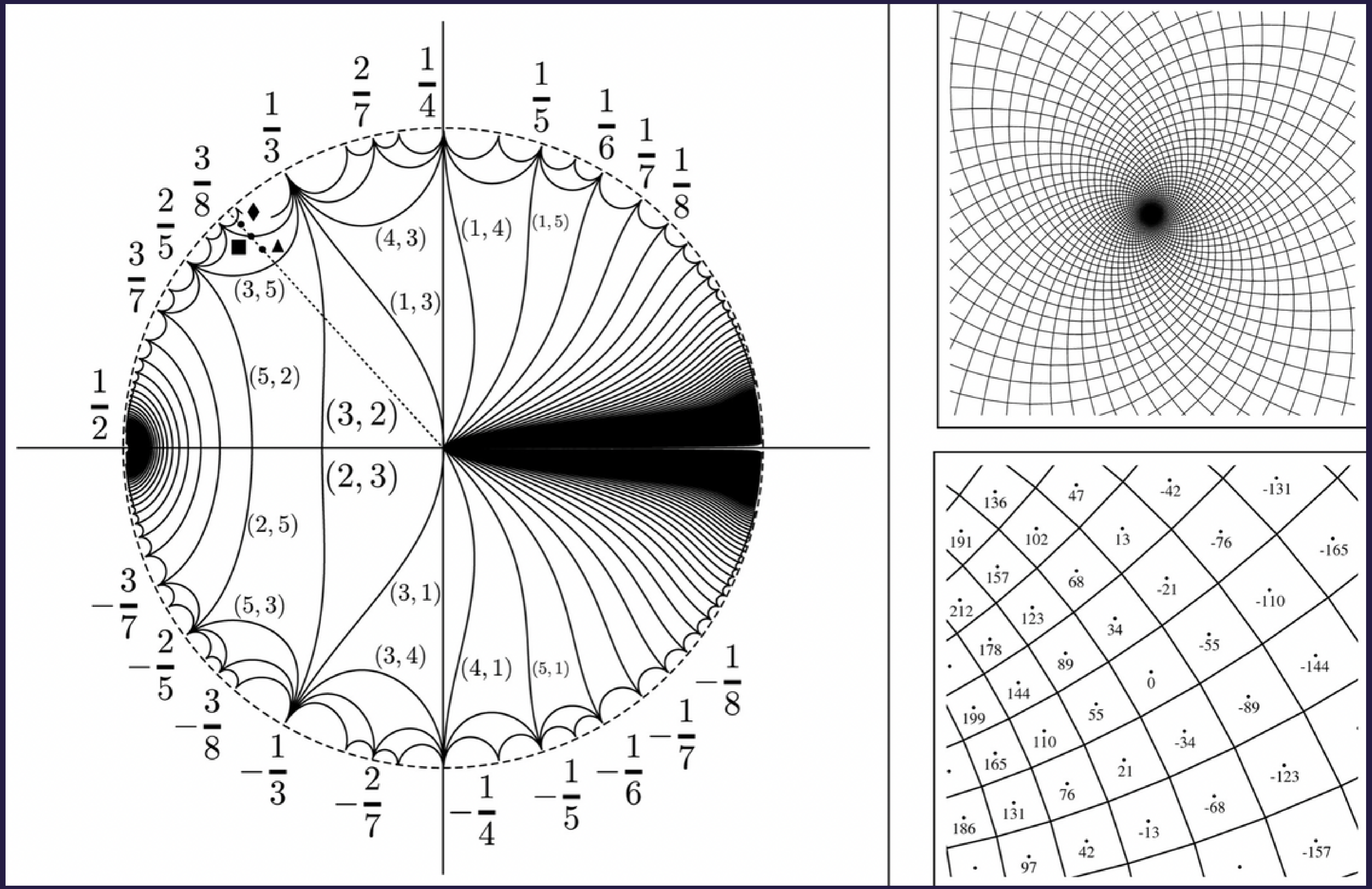
Special Constants

This is a continued fraction expansion of a special number which has been known for a long time:

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{292 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}}}}} = [3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, \ldots]$$

A relevant application:

Phyllotaxis is the study of the arrangement of leaves (or any such botanical unit) around an axis or a stem. The numbers arising from such arrangements are very closely connected to simple continued fractions and their properties.



Thank you