DSP Assignment - 3

1) $$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & ; \ \pi/4 \le |\omega| \le 3\pi/4 \\ 0 & ; \ \text{otherwise} \end{cases}$$

$N = 9$

$\alpha = \frac{9-1}{2} = 4$

① Desired frequency resp

$$H_d(e^{j\omega}) = \begin{cases} e^{-j\omega\alpha} & ; \ -3\pi/4 < \omega < -\pi/4 \\ e^{-j\omega\alpha} & ; \ \pi/4 < \omega < 3\pi/4 \\ 0 & ; \end{cases}$$

② IDTFT

$$H_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega})\, e^{j\omega n}$$

$$= \frac{1}{2\pi}\left[\int_{-3\pi/4}^{-\pi/4} e^{j\omega(n-\alpha)}\, d\omega + \int_{\pi/4}^{3\pi/4} e^{j\omega(n-\alpha)} \right].$$

$$= \frac{1}{(n-\alpha)}\left[\frac{e^{j3\pi/4(n-\alpha)} - e^{j(-3\pi/4)(n-\alpha)}}{2j} - \left\{ \frac{e^{j(\pi/4)(n-\alpha)} - e^{j(-\pi/4)(n-\alpha)}}{2j} \right\} \right]$$

$$= \frac{1}{(n-\alpha)} \sin\left(3\pi/4\,(n-\alpha)\right) - \sin\left(\pi/4\,(n-\alpha)\right)$$

$$h_d(n) = \frac{1}{\pi(n-\alpha)}\left(\sin\left(3\pi/4\,(n-\alpha)\right) - \sin\left(\pi/4\,(n-\alpha)\right)\right)$$

for $n \neq \alpha$

$$h(2) = \frac{1}{\pi}\; 3\pi/4\,(n-2) - \pi/4\,(n-2)\; \frac{1}{n-2}$$

$$= \frac{n-2}{2(n-2)} = \frac{1}{2}$$

$$h_d(n) = \begin{cases} \frac{1}{\pi(n-2)}\left(\sin(3\pi/4\,(n-4)) - \sin(\pi/4\,(n-4))\right) \\ 1/2 \;;\; n = 4 \end{cases}$$

③ Windowing Function

$$W_{hn} = \begin{cases} 0.5;\left(1-\cos\left(\frac{2n\pi}{N-1}\right)\right); & 0 \leq n \leq N-1 \; (8) \\ 0; & \text{Otherwise} \end{cases}$$

Calculating values

$W_{HG}(0) = 0 = W_{hn}(8)$
$W_{HG}(1) = 1.464 = W_{hn}(7)$
$W_{HG}(2) = 0.5 = W_{hn}(6)$
$W_{HG}(3) = 0.835 = W_{hn}(5)$
$W_{HG}(4) = 1$

(DTFT values

$h_d(0) = 0 = h_d(8)$

$h_d(1) = 0 = h_d(7)$

$h_d(2) = 0.318 = h_d(6)$

$h_d(3) = 0 = h_d(5)$

$h_d(4) = \frac{1}{2} = 0.5$

By symmetry $h(5) = 0 \quad h(7) = 0$

$h(6) = -0.159 \quad h(8) = 0$

(5) Magnitude Response

$$|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h(n)\cos\omega\left(\frac{N-1}{2} - n\right)$$

$$= h(4) + 2[h(0)\cos\omega(4-0) + h(1)\cos\omega(4-1) + h(2)\cos\omega(4-2) + h(3)\cos\omega(4-3)]$$

$$= 0.5 + 2[0 + 0 - 0.159(\cos 2\omega) + 0]$$

$$|H(e^{j\omega})| = \frac{1}{2} - \frac{\cos 2\omega}{\pi}$$

(2) $H_d(e^{j\omega}) = \begin{cases} 1 & \pi/4 \le |\omega| \le \pi/2 \\ 0 & \end{cases}$

$N = 4 \quad \alpha = \frac{4-1}{2} = 3$

(i) Desired freq

$$H_d(e^{j\omega}) = \begin{cases} 1; & -\pi/2 \le \omega < -\pi/4 \\ 1; & \pi/4 \le \omega \le \pi/2 \\ 0; & \end{cases}$$

② IDTFT

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\left[\int_{-\pi/2}^{-\pi/4} e^{j\omega n} + \int_{\pi/4}^{\pi/2} e^{j\omega n} d\omega\right]$$

$$= \frac{1}{n\pi}\left[\sin\left(\frac{\pi}{2}n\right) - \sin\left(\frac{\pi}{4}n\right)\right]$$

This is true for $n \neq 0$

If $n = 0$,

$$h_d(0) = n\frac{(\pi/2 - \pi/4)}{\pi n} = 1/4$$

Values of $h_d(n)$:

$h_d(0) = 1/4 = 0.25$

$h_d(1) = 0.0932$

$h_d(2) = -0.159$

$h_d(3) = -0.181$

$h_d(4) = 0$

$h_d(5) = 0.1086$

$h_d(6) = 0.053$

③ Windowing Function

$$W_{inn} = \begin{cases} 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right) ; & 0 \leq n \leq N-1 \\ 0 \end{cases}$$

Values

$W_{Hm}(0) = 0.08 = W_{Hm}(6)$

$W_{Hm}(1) = 0.31 = W_{Hm}(5)$

$W_{Hm}(2) = 0.77 = W_{Hm}(4)$

$W_{Hm}(3) = 1$

④ Filters coefficient

$h(0) = 0.02$

$h(1) = 0.0289$

$h(2) = -0.1224$

$h(3) = -0.181$

$h(4) = 0$

$h(5) = 0.0337$

$h(6) = 0.0042$

⑤ Magnitude

$$|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h(n)\cos\omega\left(\frac{N-1}{2}-n\right)$$

$$= h(3) + 2(h(0)\cos 3\omega + h(1)\cos 2\omega + h(2)\cos\omega$$

$$= -0.181 + 0.04\cos 3\omega + 0.0568\cos 2\omega$$

$$= -0.2448\cos\omega$$

③ $H_d(e^{j\omega}) = \begin{cases} 1 ; & \pi/4 \leq |\omega| \leq \pi/2 \\ 0 \end{cases}$

Ideal

① Desired freq

$H_d(e^{j\omega}) = \begin{cases} 1 ; & -\pi \; \omega \leq -\pi/2 \\ 1 ; & -\pi/4 < \omega < \pi/4 \\ 1 ; & \pi/2 \leq \omega < \pi \\ 0 ; \end{cases}$

② IDTFT

$$h_d(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi}\left[\int_{\pi}^{-\pi/2} e^{j\omega n} d\omega + \int_{-\pi/4}^{\pi/4} e^{j\omega n} d\omega + \int_{-\pi/2}^{\pi} e^{j\omega n} d\omega\right]$$

$$= \frac{1}{\pi n}\ \frac{e^{jn\pi} - e^{-jn\pi}}{2j} + \frac{e^{jn\pi/4} - e^{-jn\pi/4}}{2j} - \left(\frac{e^{jn\pi/2} - e^{-jn\pi/2}}{2j}\right)$$

$$= \frac{1}{n\pi}\left[\sin(n\pi) + \sin(n\pi/4) - \sin\left(\frac{n\pi}{2}\right)\right]$$

$$h_d(0) = \frac{3\pi n}{4\pi n} = 3/4$$

③ Windowing

$$W_R = \begin{cases} 1 ; & 0 \le n \le N-1 \\ 0 ; & \end{cases}$$

Since the window is rectangular $h_d(n) = h(n)$

Filter coefficient

④ $h(0) = 0.75$

$h(1) = -0.09323$

$h(2) = 0.1591$

$h(3) = 0.18113$

$h(4) = 0$

$h(5) = 0.1087$

(5) Magnitude Response

$$|H(e^{j\omega})| = h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h(n)\cos\omega\left(\frac{N-1}{2}-n\right)$$

$$= 0.1591 + 2(0.75\cos 2\omega - 0.0932\cos\omega)$$

$$= 0.1591 + 1.5\cos 2\omega - 0.1864\cos\omega$$