20CYS111 Digital Signal Processing

Signals: Classification and Properties

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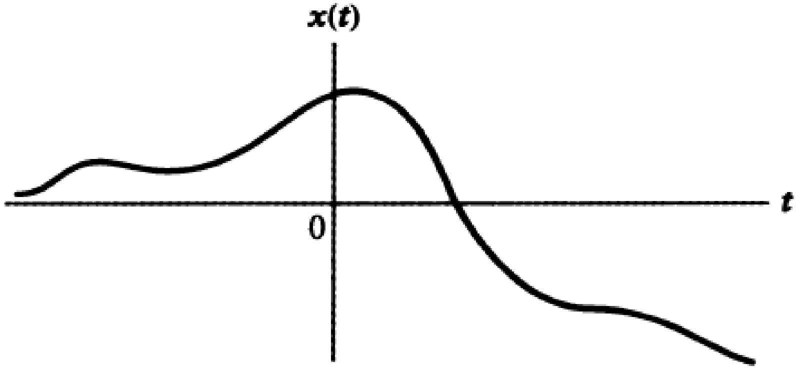
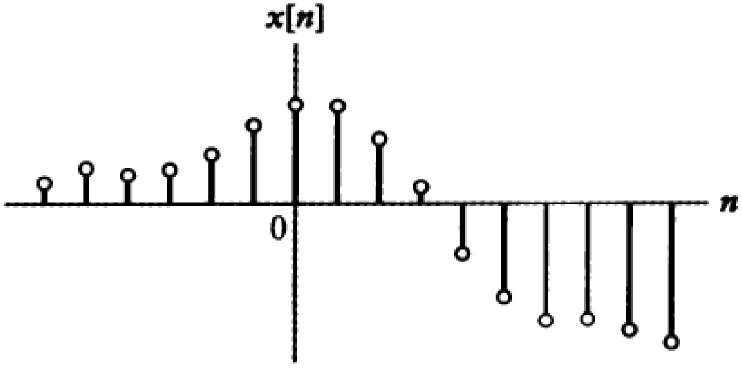
Continuous-Time Vs. Discrete-Time Signals



(a)



(b)

*A **continuous-time** signal $x(t)$ is defined for all time t,*
$-\infty < t < \infty$.

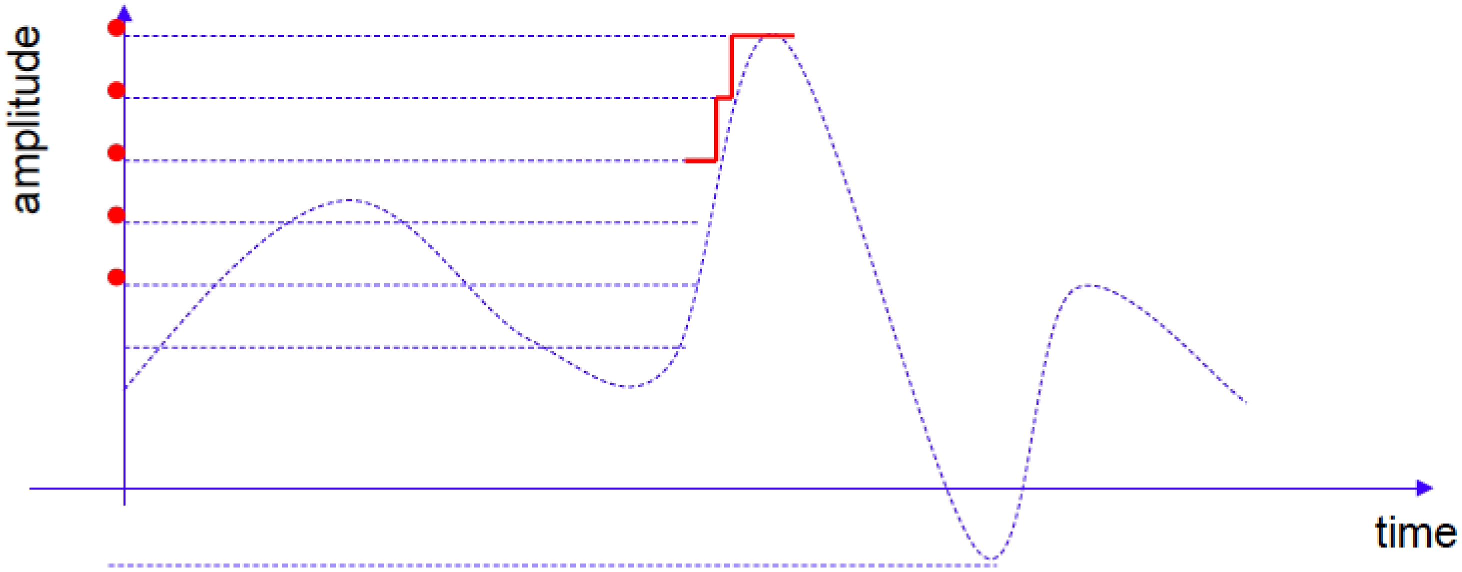
*In contrast, to specify a **discrete-time** signal, we write $\boxed{x[n] = x(nT_s)}$, $n = 0, \pm 1, \pm 2, \ldots$, where T_s is the **sampling interval**.*

***Sampling** of a continuous-time signal provides a discrete-time signal; but, some signals are naturally generated in discrete-time.*

Continuous-Valued Vs. Discrete-Valued Signals

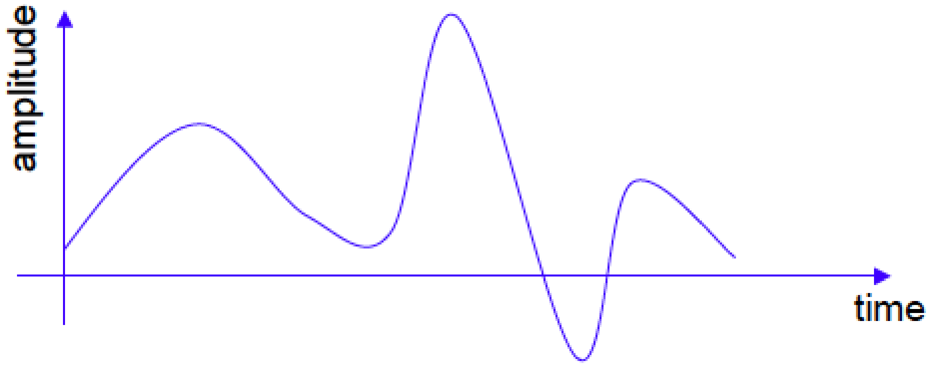
How to discretize the value of a signal?

- ***Quantization:*** *Converts a continuous-valued signal into a discrete-valued signal.*



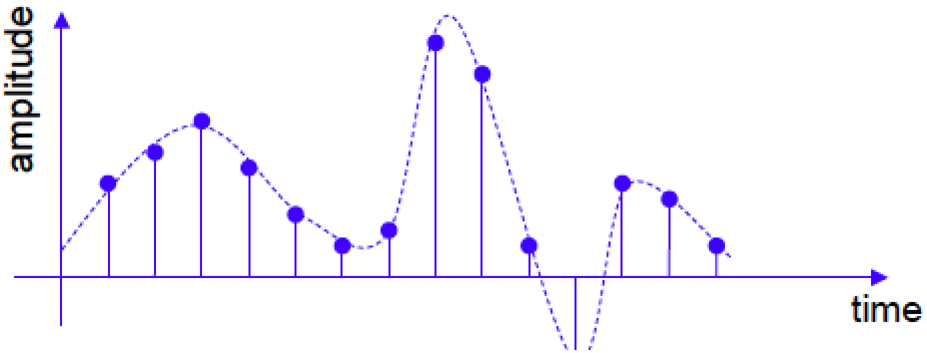
Analog Vs. Digital Signals

*A continuous-time and continuous-valued signal is called an **analog** signal.*



Analog Vs. Digital Signals

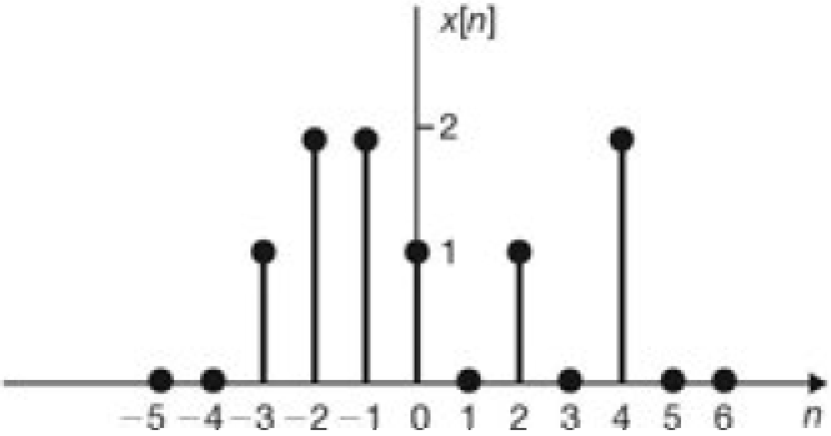
Sampling of an analog signal provides a discrete-time and continuous-valued signal.



Analog Vs. Digital Signals

Quantization of an analog signal provides a continuous-time and discrete-valued signal.

A discrete-time and discrete-valued signal is called a ***digital*** *signal.*



Real Vs. Complex Signals

*A **real** signal takes real number values.*

*A **complex** signal takes complex number values.*

- *A complex signal can also be viewed as to be taking **two-dimensional vector values**.*

*There are signals that take **multi-dimensional vector** values.*

- ***Examples??***

Deterministic Vs. Random Signals

*A **deterministic** signal is a known function of time.*

- ***Example:*** *(i) $f(\theta) = sin(\theta)$, (ii) $g(x) = log(x)$, etc.*

*A **random signal** takes random (unpredicted) values and can only be described **statistically**.*

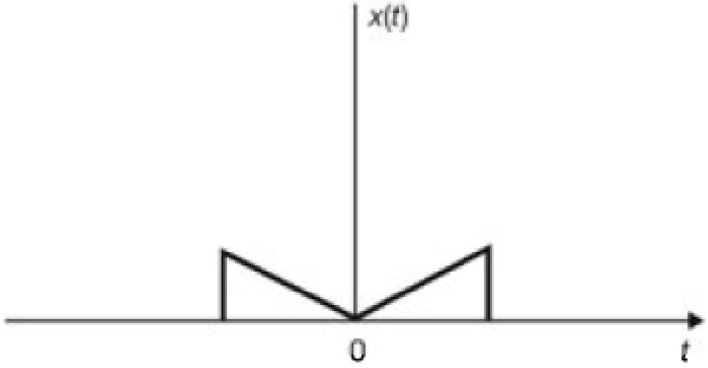
- ***Example:*** *The random **noise** in electronic and communication systems. [https://www.youtube.com/watch?v=CCnCMHNyny8 (https://www.youtube.com/watch?v=CCnCMHNyny8)](https://www.youtube.com/watch?v=CCnCMHNyny8)*

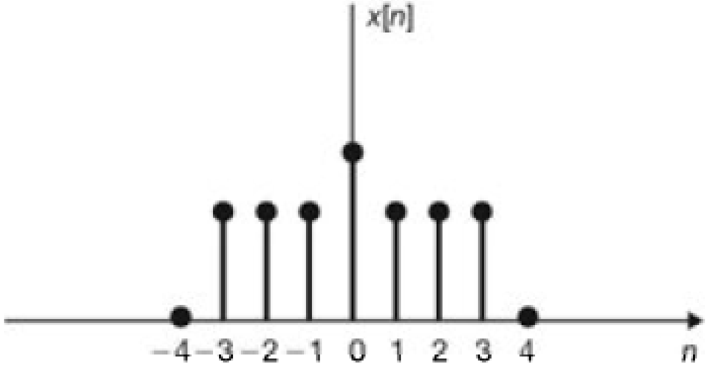
Even Vs. Odd Signals

*A signal $x(t)$ or $x[n]$ is said to be an **even** signal if*

$$\boxed{\begin{aligned} x(-t) &= x(t) \\ x[-n] &= x[n] \end{aligned}}.$$

*Thus, an even signal is **symmetric** about the vertical axis at the time origin.*



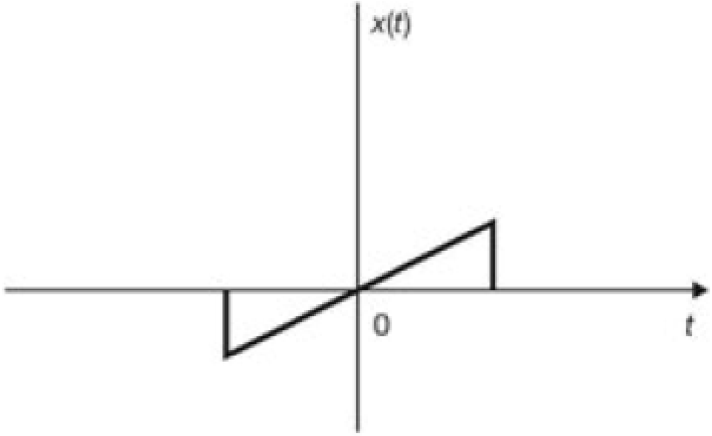


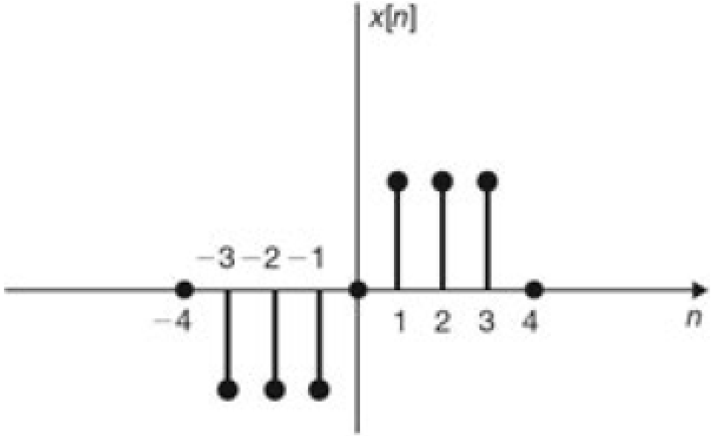
Even Vs. Odd Signals

*A signal $x(t)$ or $x[n]$ is said to be an **odd** signal if*

$$\boxed{\begin{aligned} x(-t) &= -x(t) \\ x[-n] &= -x[n] \end{aligned}}.$$

*Thus, an odd signal is **antisymmetric** about the vertical axis at the time origin.*





Even Vs. Odd Signals

It is possible for a signal to be neither even nor odd.

*Any signal $x(t)$ can be decomposed into an **even part** and an **odd part** as*

$$\boxed{x(t) = x_e(t) + x_o(t)},$$

where the even part and odd part are given by

$$\boxed{x_e(t) = \frac{x(t) + x(-t)}{2}} \quad \text{and} \quad \boxed{x_o(t) = \frac{x(t) - x(-t)}{2}}.$$

A similar result holds for any discrete-time signal $x[n]$.

Conjugate Symmetry

*For complex-valued signals, we may talk about **conjugate symmetry** instead of symmetry or anti-symmetry.*

- *Let $x(t)$ denote a complex-valued signal, i.e.,* $x(t) = a(t) + jb(t)$.
- *Let $x^*(t)$ denote the **complex conjugate** of $x(t)$, i.e.,* $x^*(t) = a(t) - jb(t)$.

*A complex-valued signal $x(t)$ is said to be **conjugate symmetric** if*

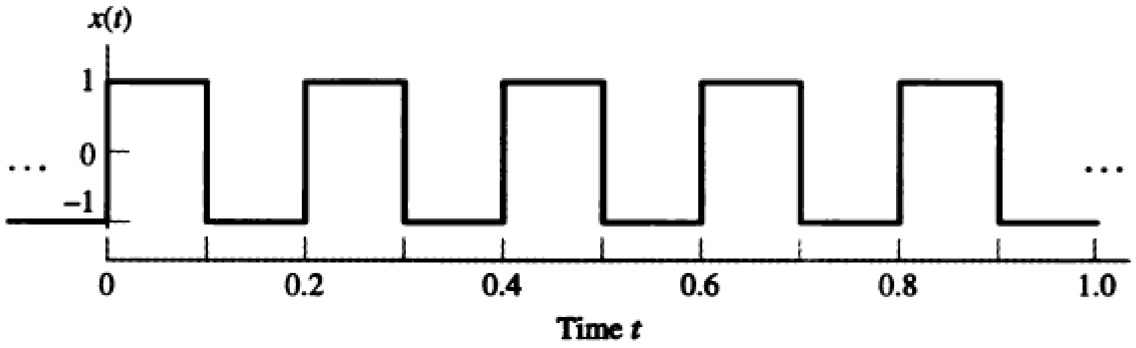
$$\boxed{x(-t) = x^*(t)}.$$

A complex-valued signal is conjugate symmetric if and only if (i) its real part is even, and (ii) imaginary part is odd.

Periodic Vs. Nonperiodic/Aperiodic Signals (Continuous-Time)

A continuous-time signal $x(t)$ is said to be ***periodic with period*** T *if there is a positive nonzero value* T *such that*

$$\boxed{x(t+T) = x(t), \qquad \text{for all } t}.$$



What is the period of the signal shown above?

Periodic Vs. Nonperiodic/Aperiodic Signals (Continuous-Time)

If $x(t)$ is periodic with a period T, then it is also periodic with periods $2T,\ 3T,\ 4T,\ \ldots$

*The **frequency** f corresponding to a period T is defined by $f = 1/T$, measured in hertz (Hz) or cycles per second.*

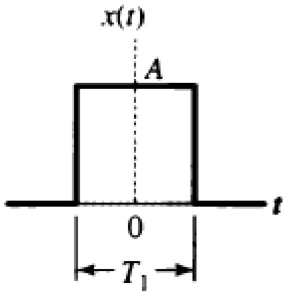
*The **angular frequency** ω correspnding to a period T is defined by $\omega = 2\pi/T$, measured in radians per second.*

*The minimum value T_0 of the period for which a signal $x(t)$ is periodic is called the **fundamental period** of that signal, and its reciprocal $f_0 = 1/T_0$ is called the **fundamental frequecy**.*

Periodic Vs. Nonperiodic/Aperiodic Signals (Continuous-Time)

A continuous-time signal $x(t)$ is said to be ***aperiodic*** *or* ***nonperiodic*** *if it is not periodic, that is, if there is no positive nonzero value of T such that*

$$x(t+T) = x(t), \qquad \text{for all } t.$$

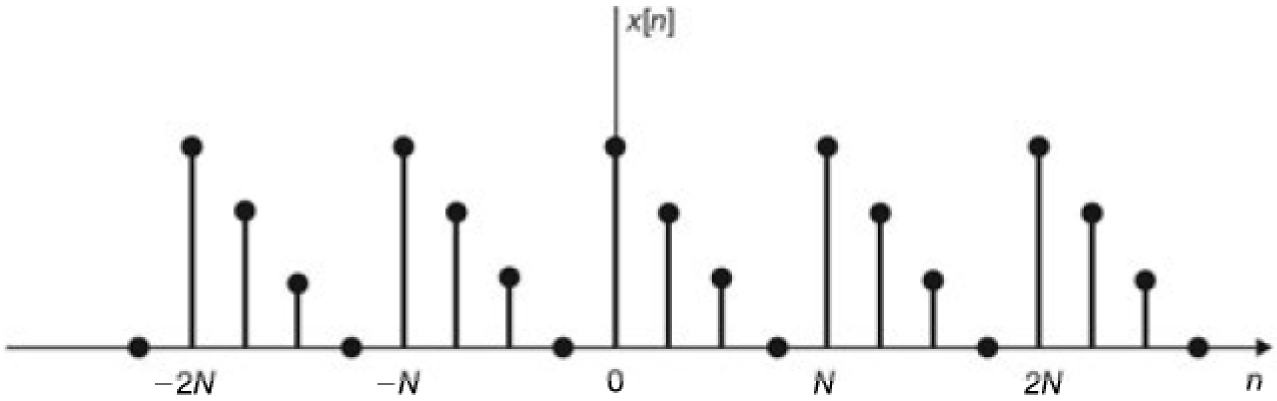


The foregoing definition of periodicity is undefined for a constant or DC signal. We will study about the frequency of a DC signal in a later lecture.

Periodic Vs. Nonperiodic/Aperiodic Signals (Discrete-Time)

A discrete-time signal $x[n]$ is said to be ***periodic with period N*** *if there is a positive integer N such that*

$$\boxed{x[n+N] = x[n], \qquad \text{for all } n}.$$



Periodic Vs. Nonperiodic/Aperiodic Signals (Discrete-Time)

If $x[n]$ is periodic with a period N, then it is also periodic with mN where m is any positive integer.

The discrete (angular) frequency Ω corresponding to the period N is defined by $\Omega = 2\pi/N$, measured in radians per sample, or simply, in radians.

The smallest period N_0 of a periodic signal $x[n]$ is called its ***fundamental period****, and the* ***fundamental (angular) frequency*** *is given by $\Omega_0 = 2\pi/N_0$.*

Energy Vs. Power Signals

In electrical/electronic systems, a signal $x(t)$ often represents a voltage $v(t)$ or current $i(t)$.

*When a voltage $v(t)$ or current $i(t)$ is applied through a resistor of R ohm, the **instantaneous power** $P(t)$ dissipated in the resistor is $v^2(t)/R$ or $i^2(t)R$.*

- *In both the cases, the instantaneous power $P(t)$ is **proportional** to the square of the signal.*

In signal analysis, we take $\boxed{R = 1}$ ohm to eliminate the dependence on the resistance.

- *Then, the instantaneous power $P_x(t)$ corresponding to a signal $x(t)$ is **equal** to the square of the signal, that is, we write*

$$\boxed{P_x(t) = x^2(t)}.$$

Energy Vs. Power Signals

*The **total energy** of the signal $x(t)$ is then given by*

$$\boxed{E_x = \int_{-\infty}^{\infty} P_x(t)dt = \int_{-\infty}^{\infty} x^2(t)dt = \lim_{T\to\infty} \int_{-T/2}^{T/2} x^2(t)dt}.$$

*The **time-averaged power** or simply **average power** of the signal $x(t)$ is given by*

$$\boxed{P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} P_x(t)dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)dt}.$$

*The square root of the average power P_x is called the **root mean square (rms)** value of the signal $x(t)$.*

Energy Vs. Power Signals

For discrete-time signals, we replace the integrals with summations.

*The **total energy** of the signal $x[n]$ is defined by*

$$\boxed{E_x = \sum_{n=-\infty}^{\infty} x^2[n]}.$$

*The **average power** of the signal $x[n]$ is defined by*

$$\boxed{P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} x^2[n]}.$$

Energy Vs. Power Signals

For periodic signals, the calculation of average power simplifies as follows:

- *For a periodic continuous-time signal $x(t)$ with fundamental period T_0, we have*

$$\boxed{P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)dt}.$$

- *For a periodic discrete-time signal $x[n]$ with fundamental period N_0, we have*

$$\boxed{P_x = \frac{1}{N_0}\sum_{n=0}^{N_0-1} x^2[n]}.$$

Energy Vs. Power Signals

A signal $x(t)$ or $x[n]$ is called an ***energy signal*** *if its total energy E_x satisfies*

$$\boxed{0 < E_x < \infty}.$$

- *For an energy signal, the average power* $\boxed{P_x = 0}$.

A signal $x(t)$ or $x[n]$ is called a ***power signal*** *if its average power P_x satisfies*

$$\boxed{0 < P_x < \infty}.$$

- *For a power signal, the total energy* $\boxed{E_x = \infty}$.

Energy Vs. Power Signals

*The energy and power classification of signals is **multually exclusive**, that is, a signal cannot be both an energy signal as well as a power signal.*

Periodic signals and random signals are usually viewed as power signals.

A signal that is both deterministic and aperiodic is an energy signal.

For complex valued signals, we must replace $x^2(t)$ with $|x(t)|^2$ and $x^2[n]$ with $|x[n]|^2$, respectively, where $|\cdot|$ denotes the modulus of the complex number.

References:

[1] Simon Haykin and Barry Van Veen, Signals and Systems, Second Edition, John Wiley and Sons, 2003.

[2] Lecture Notes by Richard Baraniuk.
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