# Discrete Mathematics Assignment - 4

1) $M_R = \begin{matrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{matrix}$ $\quad M_{R^2} = \begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{matrix}$

$M_{R^3} = \begin{matrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{matrix}$ $\quad M_{R^4} = \begin{matrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{matrix}$

$M_R = M_R \vee M_{R^2} \vee M_{R^3} \vee M_{R^4}$

$= \begin{matrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{matrix}$

$\therefore R^* = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

2) $R = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4)\}$

$M_R = \begin{matrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{matrix}$

By Warshall's algorithm, there will be 4 steps.

Step 1:
Row 1 = {2, 3, 4}
Column 1 = { $\phi$ }
$R_1 \times C_1$ = { $\phi$ }

Step 2:
Row 2 = {3, 4}
Column 2 = {1}
$R_2 \times R_2$ = {(1, 3), (1, 4)}
Since already these are present, there is no change.

Step 3:
Row 3 = {4}
Column 3 = {1, 2}
$C_3 \times R_3$ = { (1, 4), (2, 4)}

Step 4:
Row 4 = { $\phi$ }
$R_4 \times C_4$ = { $\phi$ }

This implies R is transitively closed.

3. a) Note that $(a,b) \in R$ and $(b,c) \in R$ but $(a,c) \notin R$.

$\therefore$ It is not transitive, implying it is not an equivalence relation.

b) Note that $(a,b) \in R$ and $(b,c) \in R$ but $(a,c) \in R$.

$\therefore$ It is not transitive, implying it is not an equivalence relation.

4. $A_1 = \{2,3,4\}$

$A_2 = \{1,5\}$

$A_3 = \{6\}$

$R = \{(2,2), (2,3), (2,4), (3,2), (3,3), (3,4), (4,2), (4,3), (4,4), (1,1), (1,5), (5,1), (5,5), (6,6)\}$