# DISCRETE MATHEMATICS ASSIGNMENT - 3

① $P(1) : (1)^2 + (3)^2 = \frac{2 \cdot \cancel{3} \cdot 5}{\cancel{3}}$

$10 = 10$

$P(k) : 1^2 + 3^2 + 5^2 + \ldots + (2k+1)^2 = (k+1)\frac{(2k+1)(2k+3)}{3}$

$P(k+1) : 1^2 + 3^2 + \ldots + (2(k+1)+1)^2$

$\Rightarrow 1^2 + 3^2 + \ldots + (2k+3)^2$

$\Rightarrow (k+1)\frac{(2k+1)(2k+3)}{3} + (2k+3)^2$

$\Rightarrow (2k+3)\left[\frac{(k+1)(2k+1)}{3} + (6k+9)\right]$

$\Rightarrow (2k+3)\left[\frac{2k^2+3k+1+6k+9}{3}\right]$

$\Rightarrow (2k+3)\left(\frac{2k^2+9k+10}{3}\right)$

$\therefore P(k+1) = \frac{(2k+3)(2k+5)(k+2)}{3}$

Hence Proved

② a) $\sum\limits_{n=1}^{n} \frac{1}{n(n+1)}$

$S_n = \sum\limits_{n=1}^{n} \frac{1}{n} - \frac{1}{n+1} = \sum\limits_{n=1}^{n} \frac{1}{n} - \sum\limits_{n=2}^{n+1} \frac{1}{n}$

$1 + \sum\limits_{n=2}^{n} \frac{1}{n} - \sum\limits_{n=2}^{n} \frac{1}{n} - \frac{1}{n+1} = 1 - \frac{1}{n+1}$

$= \frac{n}{n+1}$

$\therefore$ The formula is $\frac{n}{n+1}$

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b) $P(1): \frac{1}{2} = \frac{1}{2}$

$P(k): \frac{1}{1.2} + \frac{1}{2.3} + \dots + \frac{1}{k(k+1)} = \frac{k}{k+1}$

$P(k+1): \frac{1}{1.2} + \frac{1}{2.3} + \dots + \frac{1}{(k+1)(k+2)}$

$\frac{k}{(k+1)} + \frac{1}{(k+1)(k+2)} = \frac{1}{(k+1)} \frac{(k^2+2k+1)}{(k+2)}$

$\frac{(k+1)^{\cancel{2}}}{\cancel{(k+1)}(k+2)} = \frac{k+1}{k+2}$

Hence Proved.

③ a) $P(1)$ is true, because $1 = 1$.
$P(2)$ is true, because $1 + 1/4 < 2 - 1/2$
$5/4 < 3/2$

b) $P(k): 1 + 1/4 + 1/9 + \dots + 1/k^2 < 2 - 1/(k+1)$
We assume the inductive hypothesis that $P(k)$ is true for an arbitrary tve integer $k$.

$1 + 1/4 + \dots + \frac{1}{k^2} < 2 - 1/(k+1)$ By def

$\sum_{k=1}^{k} \frac{1}{k^2} < \frac{2k+1}{k+1}$

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c) $1 + \frac{1}{4} + \ldots + \frac{1}{k^2} + \frac{1}{(k+1)^2} < 2 - \frac{1}{k} + \frac{1}{(k+1)^2}$

$< 2 - \frac{1}{(k+1)}\left(\frac{k+1}{k} - \frac{1}{k+1}\right)$

$< 2 - \frac{1}{(k+1)}\left[\frac{k^2+k+1}{k(k+1)}\right]$

$< 2 - \frac{1}{k+1}$

Hence Proved.

(4) $P(1)$ : $\frac{1-1}{6} = \frac{0}{6} = 0$ True

$P(k)$ : $\frac{k^3-k}{6} \in Z$ True (Assumption)

$P(k+1)$ : $(k+1)^3 - (k+1) = k^3 + 3k^2 + 3k + 1 - k - 1$

$= (k^3 - k) + (3k^2 + 3k)$

$= (k^3 - k) + 3k(k+1)$

$k$ or $k+1$ is even so 2 divides $k(k+1)$ and 2 divides $3k(k+1)$. Clearly 3 divides $3k(k+1)$, so 6 divides $3k(k+1)$, so 6 divides $3k(k+1)$.
By inductive hypotheses 6 divides $k^3 - k$. Thus 6 divides the sum $3k(k+1) + (k^3 - k)$.

(5) $5 = 2 \cdot 0 + 5 \cdot 1$ ; $4 = 2 \cdot 2 + 5 \cdot 0$ ; $6 = 2 \cdot 3 + 5 \cdot 0$

$10 = 2 \cdot 0 + 5 \cdot 2$ ; $8 = 2 \cdot 4 + 5 \cdot 0$ ; $7 = 2 \cdot 1 + 5 \cdot 1$

$j = 2a + 5b$ $\quad j \, (4 \leq j \leq k)$

$5 = 2 \cdot 0 + 5 \cdot 1$

Assume $5 \le k$, $P(k-1)$ is true

$k-1 = 2a + 5b$

$k-1+2 = 2a + 5b + 2$

$k+1 = 2(a+1) + 5b$

$\therefore$ By PSI, $P(n)$ is true for all $n \ge 4$

6a) $a_n = 6n$

$a_{n+1} = 6(n+1) = 6n + 6 = a_n + 6$

b) $a_n = 2n+1$

$a_{n+1} = 2(n+1)+1 = 2n+3 = a_n + 2$

c) $a_n = 10^n$

$a_{n+1} = 10^{n+1} = 10 \, 10^n = 10 a_n$

d) $a_n = 5$

$a_{n+1} = 5$

7) $n!$ mod $m$

Algorithm

Procedure mod factorial (int n, int m)

if $n = 1$ then return 1

else return (n. (mod factorial (n-1, m))) mod m

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(8) Let $n$ be the length of string. When $n$ is even, For each place, we have 2 possibilities. But we need to know only half the string to write the whole string.
So, possibilities $\Rightarrow 2^{n/2}$, for $n$ is even.

When $n$ is odd,
For each place, we 2 possibilities
So, possibilities $\Rightarrow 2^{n+1/2}$

(9) Let's $n$ is the positive integers

$n = \{(1000000) - [(\frac{1000000}{4}) + (\frac{1000000}{6}) - (\frac{1000000}{12})]\}$
$n = 666667$

So there are 666667 numbers that aren't divisible by either 4 or 6

(10) States = 50
Enrollments per state = 99
Total Enrollment = $50 \times 99 = 4950$

As this is the extreme case, so if we add 1 to it, we can be assured that we will have atleast 100 from anyone of the state.
i.e. 4951 students
$\therefore$ Min students = 4951

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(11) The subsets that add upto => {3,4} {1,6} and {2,5}
The pigeon hole principle can be applied here, we require atleast 4 numbers from the given set, so that atleast 2 or 3 will get added up to be 7.
If we consider only 3 numbers, let's say that subset be {1, 2, 4} and none of these add up and gives 7.
∴, we can say that we need atleast 4 numbers from the given set to guarantee that atleast 1 pair of those numbers add upto 7.