

Strong Induction

Example 2

Show that if n is an integer greater than 1, then n can be written as the product of primes.

Solution: Let $P(n)$ be the proposition that n can be written as the product of primes.

BASIS STEP: $P(2)$ is true, because 2 can be written as the product of one prime, itself. (Note that $P(2)$ is the first case we need to establish.)

INDUCTIVE STEP: The inductive hypothesis is the assumption that $P(j)$ is true for all integers j with $2 \leq j \leq k$,

To complete the inductive step, it must be shown that $P(k + 1)$ is true under this assumption, that is, that $k + 1$ is the product of primes.

Strong Induction (example 2 continues)

- There are two cases to consider, namely, when $k + 1$ is prime and when $k + 1$ is composite.
- If $k + 1$ is prime, we immediately see that $P(k + 1)$ is true
- Otherwise, $k + 1$ is composite and can be written as the product of two positive integers a and b with $2 \leq a \leq b < k + 1$.
- A both a and b are integers at least 2 and not exceeding k , we can use the inductive hypothesis to write both a and b as the product of primes.
- Thus, if $k + 1$ is composite, it can be written as the product of primes, namely, those primes in the factorization of a and those in the factorization of b .

Strong Induction

Example 3

Prove that every amount of postage of 12 Rupees or more can be formed using just 4-Rupee and 5-Rupee stamps.

Solution: We will prove this result using the principle of mathematical induction then by Strong induction

Let $P(n)$ be the statement that postage of n rupee can be formed using 4-Rupee and 5-Rupee stamps.

BASIS STEP: Postage of 12 rupee can be formed using three 4-Rupee stamps.

INDUCTIVE STEP: The inductive hypothesis is the statement that $P(k)$ is true. That is, under this hypothesis, postage of k rupee can be formed using 4-rupee and 5-rupee stamps.

To complete the inductive step, we need to show that when we assume $P(k)$ is true, then $P(k + 1)$ is also true where $k \geq 12$.

We consider two cases, when at least one 4-rupee stamp has been used and when no 4-rupee stamps have been used.