Numbers

Every number system is associated with a base or radix

A positional notation is commonly used to express numbers

$$(a_5a_4a_3a_2a_1a_0)_r \;=\; a_5r^5+a_4r^4+a_3r^3+a_2r^2+a_1r^1+a_0r^0$$

The decimal system has a base of 10 and uses symbols (0,1,2,3,4,5,6,7,8,9) to represent numbers

$$(2009)_{10} \;=\; 2\times10^3+0\times10^2+0\times10^1+9\times10^0$$

$$(123.24)_{10} \;=\; 1\times10^2+2\times10^1+3\times10^0+2\times10^{-1}+4\times10^{-2}$$

An octal number system has a base 8 and uses symbols (0,1,2,3,4,5,6,7)

$$(2007)_8 \;=\; 2\times8^3+0\times8^2+0\times8^1+7\times8^0$$

What decimal number does it represent?

$$(2007)_8 \;=\; 2\times512+0\times64+0\times8^1+7\times8^0=1033$$

A hexadecimal system has a base of 16

Number	Symbol
0	0
1	1
2	2
3	3
4	4
5	5
6	6
7	7
8	8
9	9
10	A
11	B
12	C
13	D
14	E
15	F

$$(2BC9)_{10} = 2\times16^3 + B\times16^2 + C\times16^1 + 9\times16^0$$

How do we convert it into decimal number?

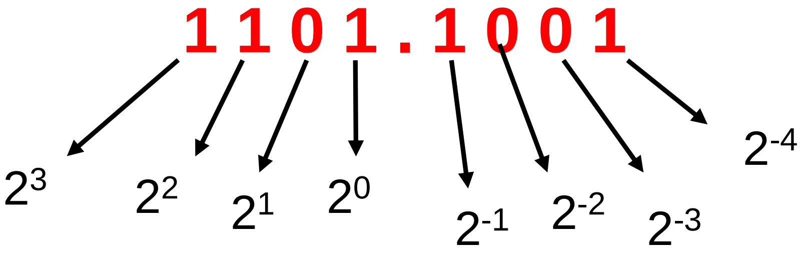
$$(2BC9)_{10} = 2\times4096+11\times256+12\times16^1+9\times16^0 = 11209$$

A Binary system has a base 2 and uses only two symbols 0, 1 to represent all the numbers

$$(1101)_2 = 1\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0$$

Which decimal number does this correspond to ?

$$(1101)_2 = 1\times 8 + 1\times 4 + 0\times 2^1 + 1\times 2^0 = 13$$



1 1 0 1 . 1 0 0 1

2^0	1
2^1	2
2^2	4
2^3	8
2^4	16
2^5	32
2^6	64
2^7	128
2^8	256
2^9	512
2^{10}	1024(K)
2^{20}	1048576(M)

2^{-1}	2^{-2}	2^{-3}	2^{-4}	2^{-5}	2^{-6}
0.5	0.25	0.125	0.0625	0.03125	0.015625

Developing Fluency with Binary Numbers

1 1 0 0 1 = ?	25
1100001 = ?	64+32+1=97
0.101 = ?	0.5+0.125=0.625
11.001 = ?	3+0.125=3.125

Converting decimal to binary number

Convert 45 to binary number

$$(45)_{10} = b_n b_{n-1} ....b_0$$

$$45 = b_n 2^n + b_{n-1} 2^{n-1} ....b_1 2^1 + b_0$$

Divide both sides by 2

$$\frac{45}{2} = 22.5 = b_n 2^{n-1} + b_{n-1} 2^{n-2} ....b_1 2^0 + b_0 \times 0.5$$

$$22 + 0.5 = b_n 2^{n-1} + b_{n-1} 2^{n-2} .... + b_1 2^0 + b_0 \times 0.5$$

$$\Rightarrow \ b_0 = 1$$

$$22+0.5=b_n2^{n-1}+b_{n-1}2^{n-2}.......+b_12^0+b_0\times 0.5$$

$$\Rightarrow\ b_0=1$$

$$22=b_n2^{n-1}+b_{n-1}2^{n-2}.......b_22^1+b_12^0$$

Divide both sides by 2

$$\frac{22}{2}=11=b_n2^{n-2}+b_{n-1}2^{n-3}.......b_22^0+b_1\times 0.5$$

$$\Rightarrow\ b_1=0$$

$$11=b_n2^{n-2}+b_{n-1}2^{n-3}......+b_32^1+b_22^0$$

$$5.5=b_n2^{n-3}+b_{n-1}2^{n-4}......+b_32^0+0.5b_2$$

$$\Rightarrow\ b_2=1$$

$$5=b_n2^{n-3}+b_{n-1}2^{n-4}......b_42^1+b_32^0$$

$$5 = b_n 2^{n-3} + b_{n-1} 2^{n-4} ......b_4 2^1 + b_3 2^0$$

$$2.5 = b_n 2^{n-4} + b_{n-1} 2^{n-5} ......b_4 2^0 + 0.5 b_3$$

$$\Rightarrow \; b_3 = 1$$

$$2 = b_n 2^{n-4} + b_{n-1} 2^{n-5} ......b_5 2^1 + b_4 2^0$$

$$1 = b_n 2^{n-5} + b_{n-1} 2^{n-6} ......b_5 2^0 + 0.5 b_4$$

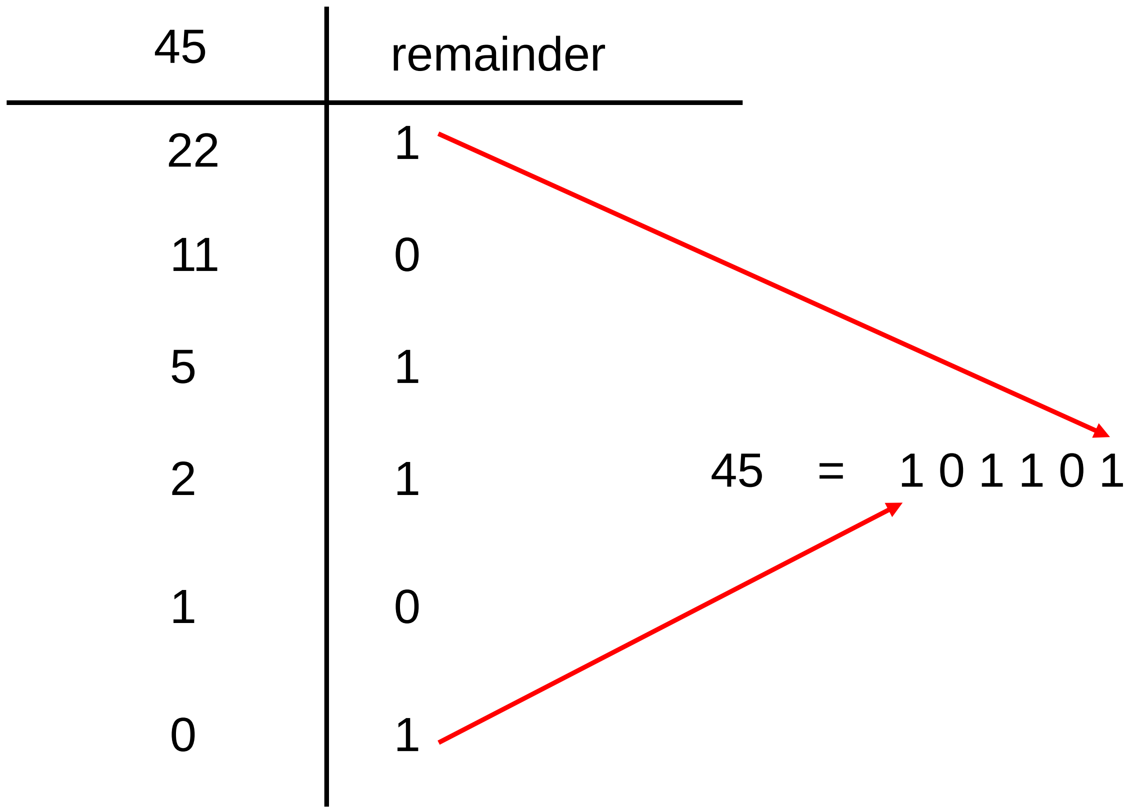
$$\Rightarrow \; b_4 = 0$$

$$\Rightarrow \; b_5 = 1$$

$$(45)_{10} = b_5 b_4 b_3 b_2 b_1 b_0 = 101101$$

Converting decimal to binary number

Method of successive division by 2



45	remainder
22	1
11	0
5	1
2	1
1	0
0	1

45 = 1 0 1 1 0 1

Convert $(153)_{10}$ to octal number system

$$(153)_{10} = (b_n b_{n-1} ....b_0)_8$$

$$(153)_{10} = b_n 8^n + b_{n-1} 8^{n-1} ....b_1 8^1 + b_0$$

Divide both sides by 8

$$\frac{153}{8} = 19.125 = b_n 8^{n-1} + b_{n-1} 8^{n-2} ....b_1 8^0 + \frac{b_0}{8}$$ $$\Rightarrow \frac{b_0}{8} = 0.125$$ $$\Rightarrow b_0 = 1$$

153	remainder
19	1
2	3
0	2

153 = $(231)_8$

Converting decimal to binary number

Convert $(0.35)_{10}$ to binary number

$$(0.35)_{10} = 0.b_{-1}b_{-2}b_{-3}.......b_{-n}$$

$$0.35 = 0 + b_{-1}2^{-1} + b_{-2}2^{-2} + ...b_{-n}2^{-n}$$

How do we find the b_{-1} b_{-2} …coefficients?

Multiply both sides by 2

$$0.7 = b_{-1} + b_{-2}2^{-1} + ...b_{-n}2^{-n+1}$$

$$\Rightarrow b_{-1} = 0$$

$$0.7 = b_{-2}2^{-1} + b_{-3}2^{-2} + ...b_{-n}2^{-n+1}$$

$$0.7 = b_{-2}2^{-1} + b_{-3}2^{-2} + ...b_{-n}2^{-n+1}$$

Multiply both sides by 2

$$1.4 = b_{-2} + b_{-3}2^{-1} + ...b_{-n}2^{-n+2}$$

Note that ½+1/4+1/8+……≤1

$$\Rightarrow b_{-2} = 1$$

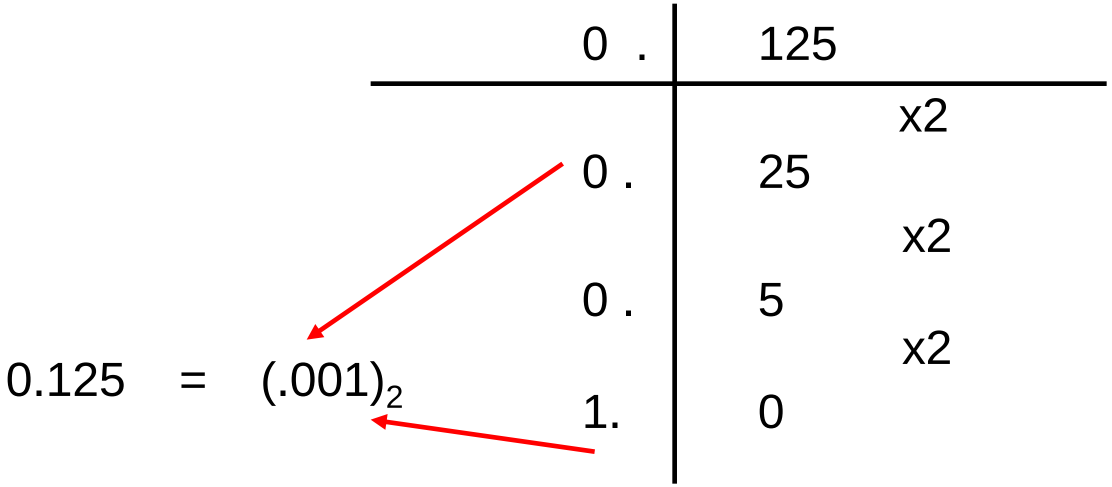
$$0.4 = b_{-3}2^{-1} + b_{-4}2^{-2}.......b_{-n}2^{-n+2}$$

$$0.8 = b_{-3} + b_{-4}2^{-1}.......b_{-n}2^{-n+3}$$

$$\Rightarrow b_{-3} = 0$$

Converting decimal to binary number

0.125 = ?



0 . 125

x2

0 . 25

x2

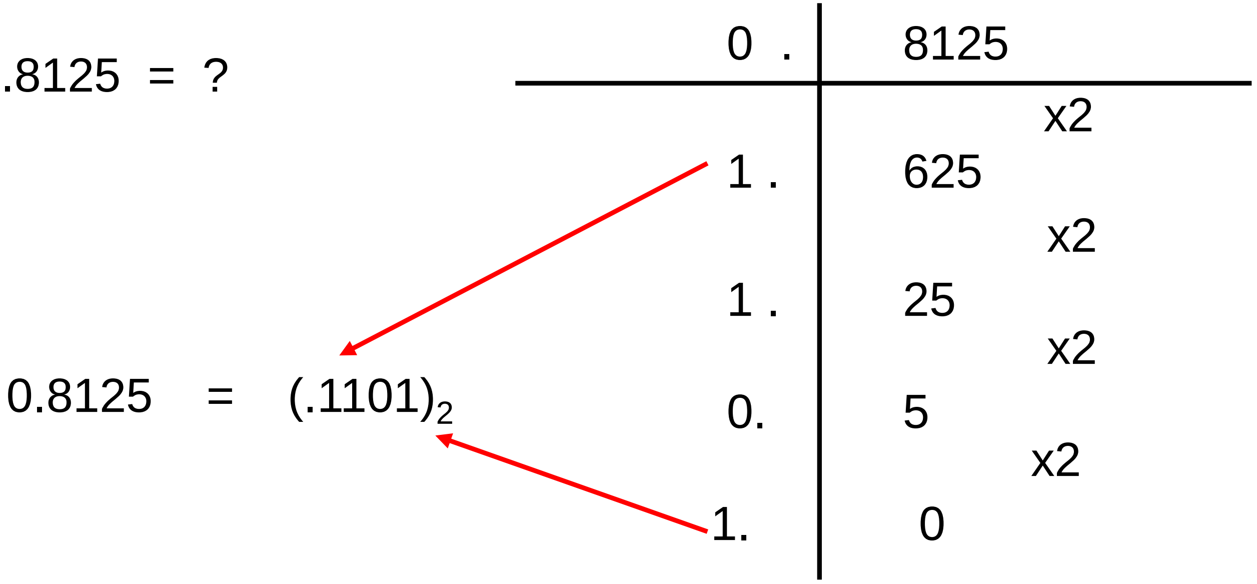
0 . 5

x2

1. 0

0.125 = $(.001)_2$

0.8125 = ?



0 . 8125

x2

1 . 625

x2

1 . 25

x2

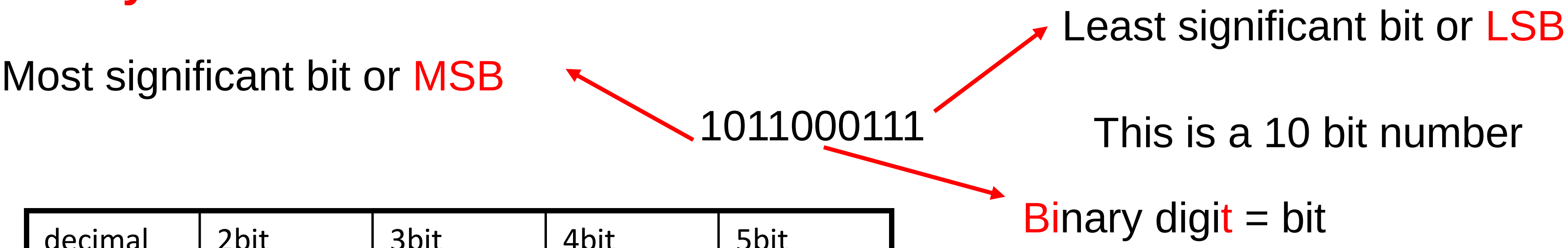
0. 5

x2

1. 0

0.8125 = $(.1101)_2$

Binary numbers



Most significant bit or MSB

1011000111

Least significant bit or LSB

This is a 10 bit number

Binary digit = bit

decimal	2bit	3bit	4bit	5bit
0	00	000	0000	00000
1	01	001	0001	00001
2	10	010	0010	00010
3	11	011	0011	00011
4		100	0100	00100
5		101	0101	00101
6		110	0110	00110
7		111	0111	00111
8			1000	01000
9			1001	01001
10			1010	01010
11			1011	01011
12			1100	01100
13			1101	01101
14			1110	01110
15			1111	01111

N-bit binary number can represent numbers from 0 to 2^N -1

Converting Binary to Hex and Hex to Binary

$$(b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0)_b = (h_1, h_0)_{Hex}$$

$$b_7 2^7 + b_6 2^6 + b_5 2^5 + b_4 2^4 + b_3 2^3 + b_2 2^2 b_1 2^1 + b_0 = h_1 16^1 + h_0$$

$$\underbrace{(b_7 2^3 + b_6 2^2 + b_5 2^1 + b_4)}_{h_1} 2^4 + \underbrace{(b_3 2^3 + b_2 2^2 b_1 2^1 + b_0)}_{h_0} = h_1 16^1 + h_0$$

$$(10110011)_b = (1011)(0011) = (B3)_{Hex}$$

$$(110011)_b = (11)(0011) = (33)_{Hex}$$

$$(EC)_{Hex} = (1110)(1100) = (11101100)_b$$

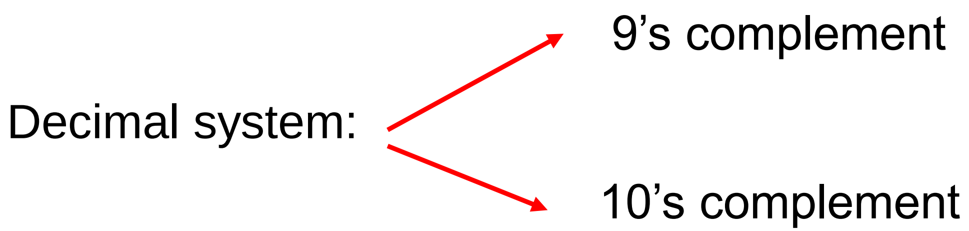
Number	Symbol
0(0000)	0
1(0001)	1
2(0010)	2
3(0011)	3
4(0100)	4
5(0101)	5
6(0110)	6
7(0111)	7
8(1000)	8
9(1001)	9
10(1010)	A
11(1011)	B
12(1100)	C
13(1101)	D
14(1110)	E
15(1111)	F

Binary Addition/Subtraction

$$\begin{array}{r} 0 \\ 0 \\ \hline 0 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ 0 \\ \hline 1 \\ \hline \end{array} \qquad \begin{array}{r} 0 \\ 1 \\ \hline 1 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ 1 \\ \hline 1\ 0 \\ \hline \end{array} \qquad \begin{array}{r} 1 \\ 1 \\ 1 \\ \hline 1\ 1 \\ \hline \end{array}$$

$$\begin{array}{r} 1\ 0\ 1 \\ 1\ 1\ 0 \\ \hline 1\ 0\ 1\ 1 \\ \hline \end{array} \qquad \begin{array}{r} 1\ 1\ 0\ 1 \\ +\ \ 1\ 1\ 1\ 0 \\ \hline 1\ 1\ 0\ 1\ 1 \\ \hline \end{array}$$

Complement of a number


Decimal system:

9’s complement

10’s complement

9’s complement of n-digit number x is $10^n - 1 - x$

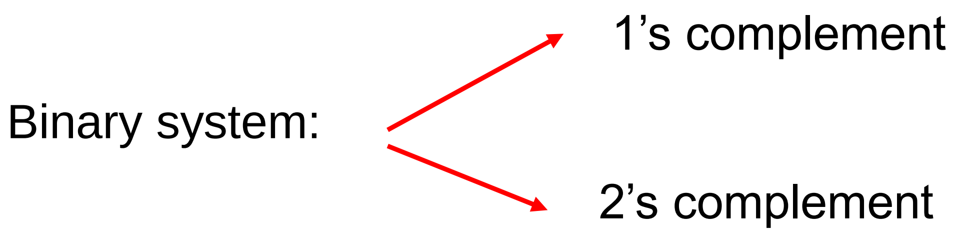
10’s complement of n-digit number x is $10^n - x$

9’s complement of 85 ? $10^2 - 1 - 85$ $99 - 85 = 14$

$$\text{9's complement of } 123 = 999 - 123 = 876$$

$$\text{10's complement of } 123 = \text{9's complement of } 123 + 1 = 877$$

Complement of a binary number



Binary system:

1’s complement

2’s complement

1’s complement of n-bit number x is $2^n - 1 - x$

2’s complement of n-bit number x is $2^n - x$

1’s complement of 1011 ? $2^4 - 1 - 1011$ $1111 - 1011 = 0100$

1’s complement is simply obtained by flipping a bit (changing 1 to 0 and 0 to 1)

1's complement of 1001101 = ?

0110010

$$\text{2's complement of 1010 = 1's complement of 1010+1} = 0110$$

$$\text{2's complement of 110010 =}$$

Leave all least significant 0’s as they are, leave first 1 unchanged and then flip all subsequent bits

$$001110$$

$$1011 \;\; \rightarrow 0101$$

$$101101100 \;\; \rightarrow 010010100$$

Representing positive and negative binary numbers

One extra bit is required to carry sign information. Sign bit = 0 represents positive number and Sign bit = 1 represents negative number

decimal	Signed Magnitude
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-0	1000
-1	1001
-2	1010
-3	1011
-4	1100
-5	1101
-6	1110
-7	1111

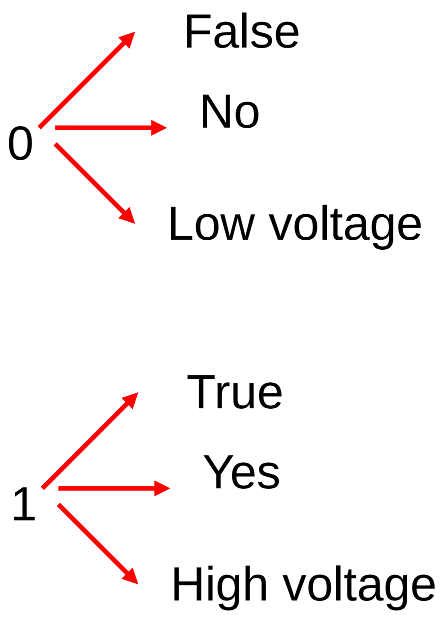
decimal	Signed 1's complement
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-0	1111
-1	1110
-2	1101
-3	1100
-4	1011
-5	1010
-6	1001
-7	1000

decimal	Signed 2's complement
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
-1	1111
-2	1110
-3	1101
-4	1100
-5	1011
-6	1010
-7	1001

Boolean Algebra

Algebra on Binary numbers

A variable x can take two values {0,1}



0 → False

0 → No

0 → Low voltage

1 → True

1 → Yes

1 → High voltage

Basic operations:

AND: $y = x_1 . x_2$

Y is 1 if and only if both x_1 and x_2 are 1, otherwise zero

Truth Table

x_1	x_2	y
0	0	0
0	1	0
1	0	0
1	1	1

Basic operations:

$$\text{OR:} \quad y = x_1 + x_2$$

Y is 1 if either x_1 and x_2 is 1. Or y= 0 if and only if both variables are zero

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$$\text{NOT:} \quad y = \overline{x}$$

x	y
0	1
1	0

Boolean Algebra

Basic Postulates

P1: $x + 0 = x$

P2: $x + y = y + x$

P3: $x.(y+z) = x.y+x.z$

P4: $x + \overline{x} = 1$

P1: $x . 1 = x$

P2: $x . y = y . x$

P3: $x+y.z = (x+y).(x+z)$

P4: $x . \overline{x} = 0$

Basic Theorems

T1: $x + x = x$

T2: $x + 1 = 1$

T3: $\overline{(\overline{x})} = x$

T4: $x + (y+z) = (x+y)+z$

T5 $\overline{(x+y)} = \overline{x} . \overline{y}$ (DeMorgan's theorem)

T6: $x + x.y = x$

T1: $x . x = x$

T2: $x . 0 = 0$

T4: $x . (y.z) = (x.y).z$

T5 $\overline{(x.y)} = \overline{x} + \overline{y}$ (DeMorgan's theorem)

T6: $x.(x+y) = x$

Proving theorems

P1: $x + 0 = x$

P2: $x + y = y + x$

P3: $x.(y+z) = x.y+x.z$

P4: $x + \bar{x} = 1$

P1: $x . 1 = x$

P2: $x . y = y . x$

P3: $x+y.z = (x+y).(x+z)$

P4: $x . \bar{x} = 0$

Prove T1: $x + x = x$

$x + x = (x+x). 1$ (P1)

$= (x+x). (x+\bar{x})$ (P4)

$= x + x.\bar{x}$ (P3)

$= x + 0$ (P4)

$= x$ (P1)

Prove T1: $x . x = x$

$x . x = x.x+ 0$ (P1)

$= x.x + x.\bar{x}$ (P4)

$= x . (x+\bar{x})$ (P3)

$= x . 1$ (P4)

$= x$ (P1)

Proving theorems

P1: $x + 0 = x$

P2: $x + y = y + x$

P3: $x.(y+z) = x.y+x.z$

P4: $x + \overline{x} = 1$

P1: $x . 1 = x$

P2: $x . y = y . x$

P3: $x+y.z = (x+y).(x+z)$

P4: $x . \overline{x} = 0$

Prove : $x + 1 = 1$

$x + 1 = x+(x+\overline{x})$

$= (x+x)+\overline{x}$

$= x + \overline{x}$

$= 1$

$x + x .y = x$

$= x . 1 + x. y$

$= x. (1+ y)$

$= x . 1$

$=x$

$x + \overline{x} .y = x+y$

$= (x + \overline{x}). (x+ y)$

$= 1. (x+ y)$

$= x + y$

DeMorgan's theorem

$$\overline{(x_1 + x_2 + x_3 + \ldots.)} = \overline{x_1} . \overline{x_2} . \overline{x_3} .$$

$$\overline{(x_1 . x_2 . x_3 \ldots..)} = (\overline{x_1} + \overline{x_2} + \overline{x_3} + \ldots..)$$

Simplification of Boolean expressions

$$\overline{(x_1 + x_2 + x_3 + ...)} = \overline{x_1} \; . \; \overline{x_2} \; . \overline{x_3} \; .$$

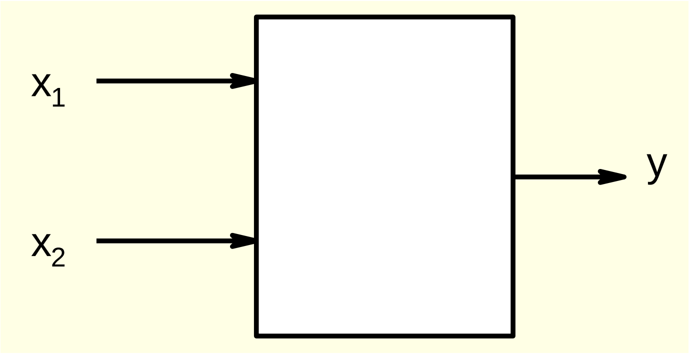
$$\overline{(\overline{x_1}.x_2 + \overline{x_2}.x_3)} = ?$$

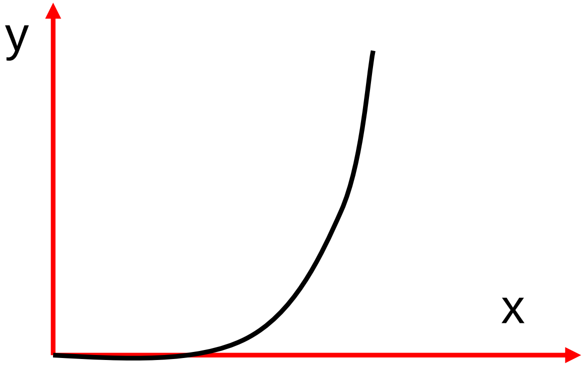
$$\overline{(x_1. \; x_2. \; x_3.....)} = (\overline{x_1} + \overline{x_2} + \overline{x_3} +.....)$$

$$= (x_1 + \overline{x_2}) \; . \; (x_2 + \overline{x_3})$$

$$= x_1. \; x_2 + x_1 \; . \; \overline{x_3} + \overline{x_2}. \; \overline{x_3}$$

Function of Boolean variables





$$y = x^2$$

x_1	x_2	y
0	0	0
0	1	1
1	0	0
1	1	0

Y = 1 when x_1 is 0 and x_2 is 1

$$y = \overline{x_1} \,.\, x_2$$

Boolean expression

Obtaining Boolean expressions from truth Table

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	0

$$y = \overline{x_1} \,.\, \overline{x_2}$$

x_1	x_2	y
0	0	0
0	1	0
1	0	1
1	1	0

$$y = x_1 \,.\, \overline{x_2}$$

x_1	x_2	y
0	0	1
0	1	0
1	0	0
1	1	1

$$\overline{x_1} \,.\, \overline{x_2}$$

$$x_1 \,.\, x_2$$

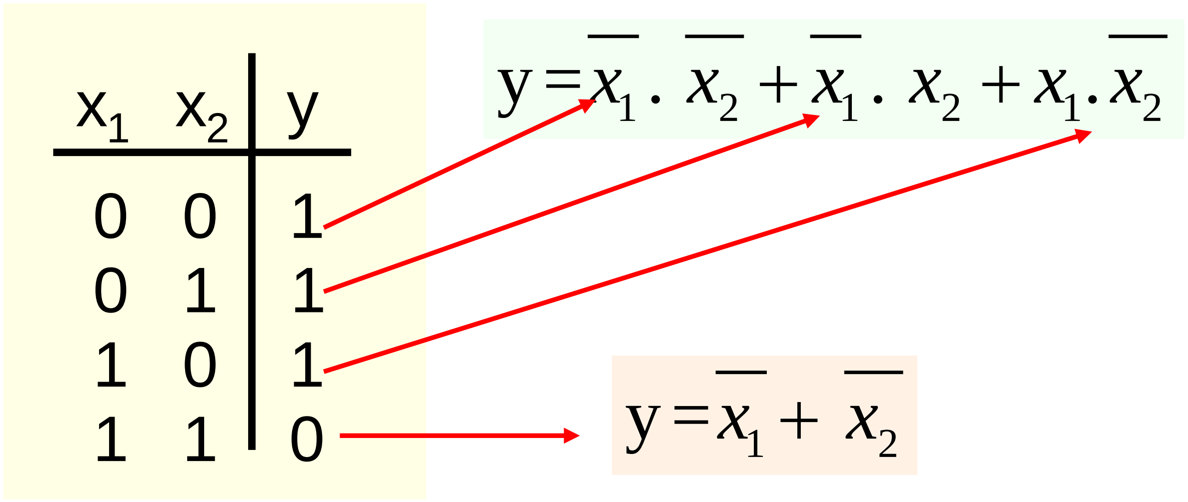
$$y = \overline{x_1} \,.\, \overline{x_2} + x_1 . x_2$$

Obtaining Boolean expressions from truth Table

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$$y = \overline{x_1} .\ x_2 + x_1 . \overline{x_2}$$

Instead of writing expressions as sum of terms that make y equal to 1, we can also write expressions using terms that make y equal to 0



x_1	x_2	y
0	0	1
0	1	1
1	0	1
1	1	0

$$y = \overline{x_1} .\ \overline{x_2} + \overline{x_1} .\ x_2 + x_1 . \overline{x_2}$$

$$y = \overline{x_1} + \overline{x_2}$$

x_1	x_2	y
0	0	1
0	1	0
1	0	1
1	1	1

$$y = x_1 + \overline{x_2}$$

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

$$y = x_1 + x_2$$

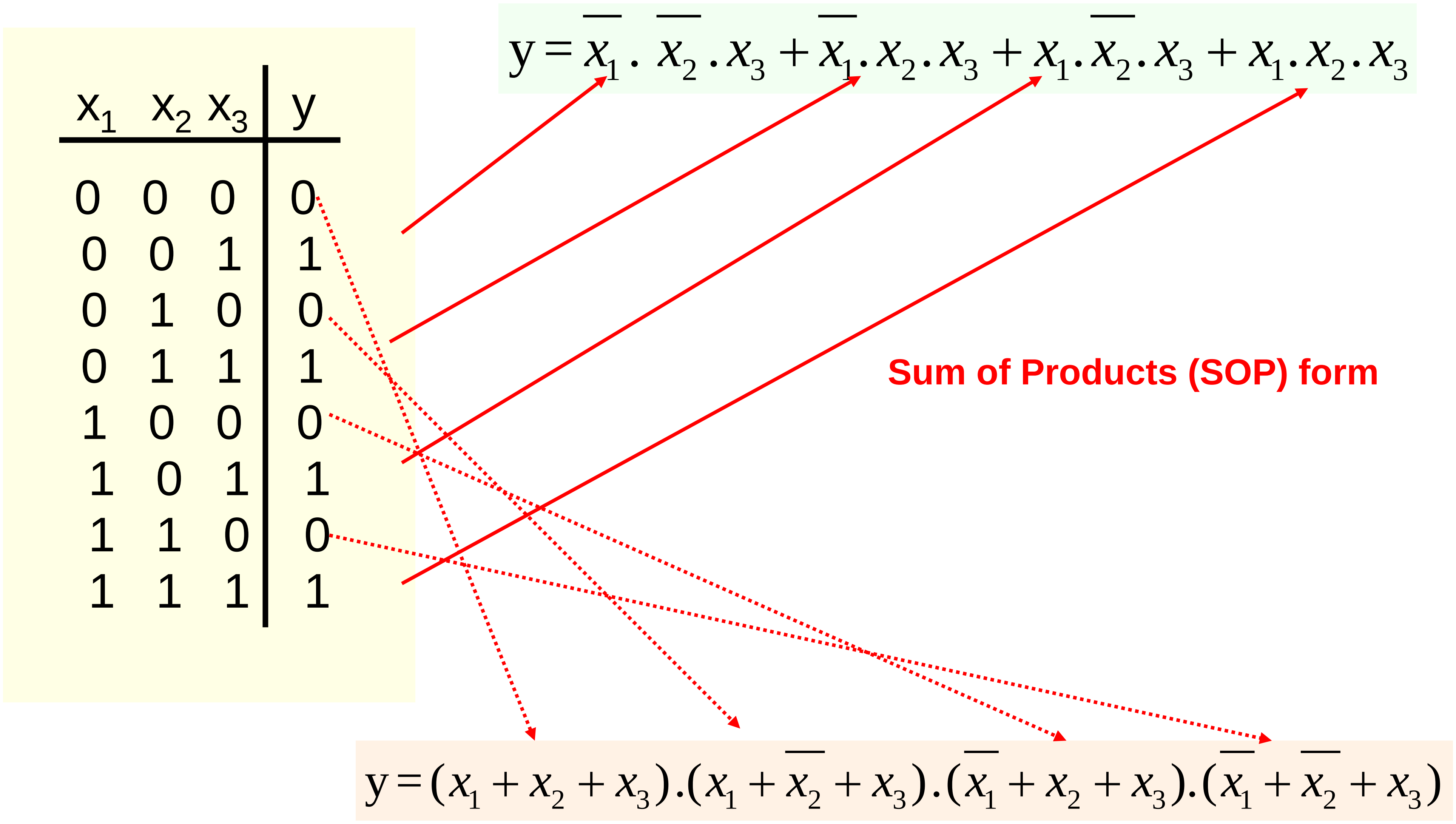
x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

$$x_1 + x_2$$

$$y = (x_1 + x_2).(\overline{x_1} + \overline{x_2})$$

$$\overline{x_1} + \overline{x_2}$$

Obtaining Boolean expressions from truth Table



x_1	x_2	x_3	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

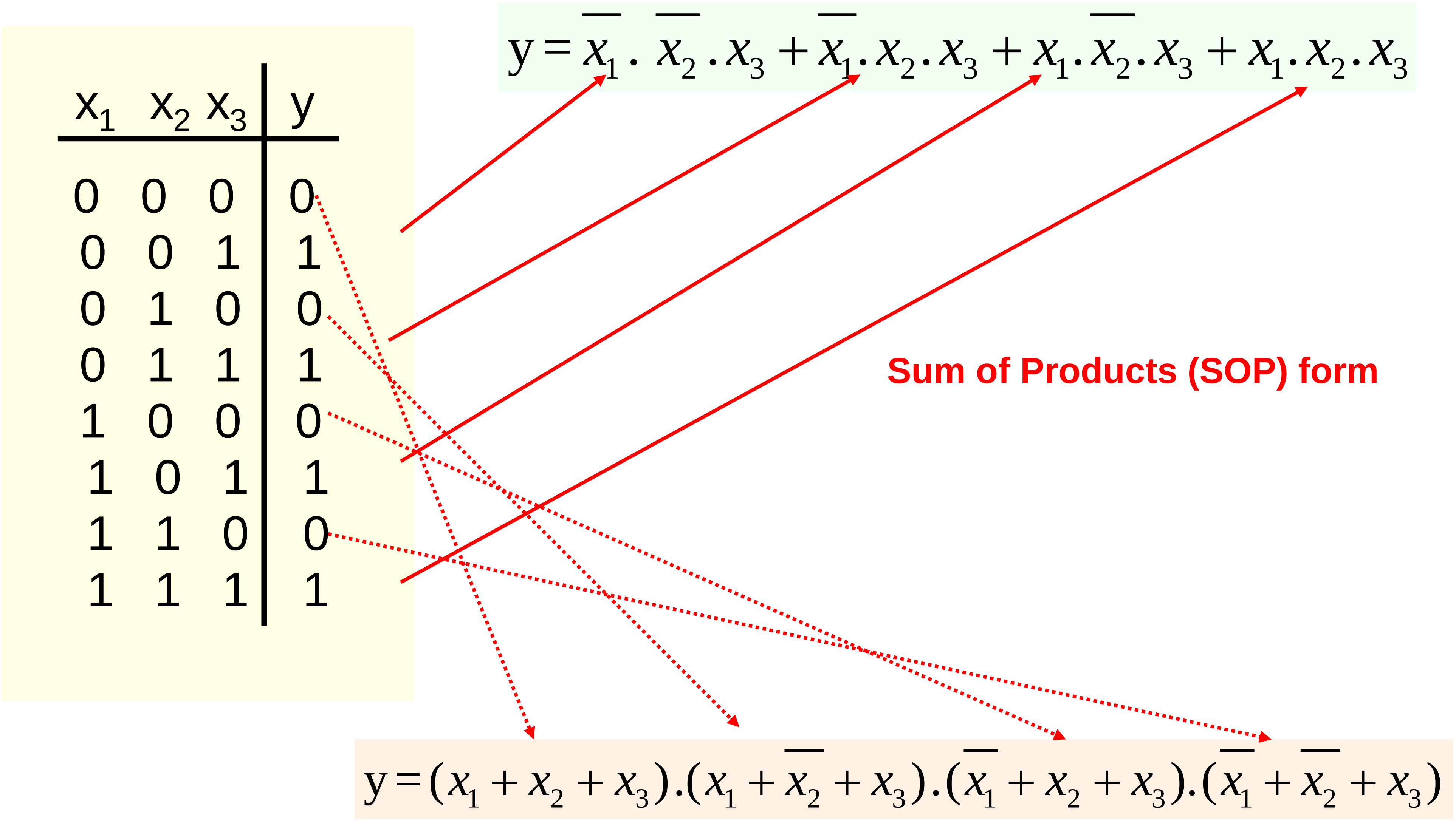
$$y = \overline{x_1}\,.\,\overline{x_2}\,.\,x_3 + \overline{x_1}\,.\,x_2\,.\,x_3 + x_1\,.\,\overline{x_2}\,.\,x_3 + x_1\,.\,x_2\,.\,x_3$$

Sum of Products (SOP) form

$$y = (x_1 + x_2 + x_3).(x_1 + \overline{x_2} + x_3).(\overline{x_1} + x_2 + x_3).(\overline{x_1} + \overline{x_2} + x_3)$$

Product of Sum (POS) form

Obtaining Boolean expressions from truth Table



x_1	x_2	x_3	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$y = \overline{x_1}\,.\,\overline{x_2}\,.x_3 + \overline{x_1}.x_2.x_3 + x_1.\overline{x_2}.x_3 + x_1.x_2.x_3$$

Sum of Products (SOP) form

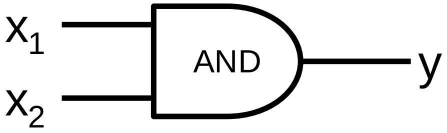
$$y = (x_1 + x_2 + x_3).(x_1 + \overline{x_2} + x_3).(\overline{x_1} + x_2 + x_3).(\overline{x_1} + \overline{x_2} + x_3)$$

Product of Sum (POS) form

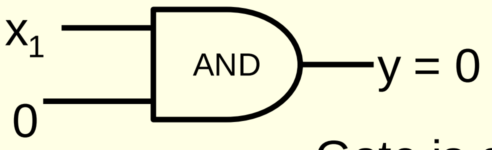
Implementing Boolean expressions

Elementary Gates

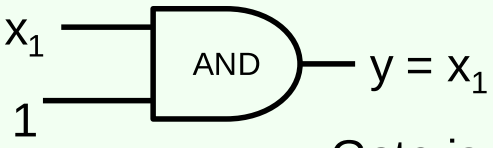
AND: $y = x_1 . x_2$



Why call it a gate?

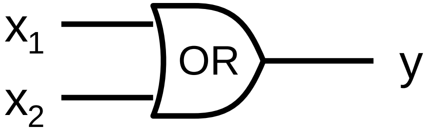


Gate is closed

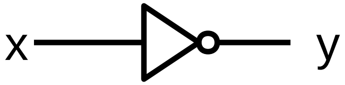


Gate is open

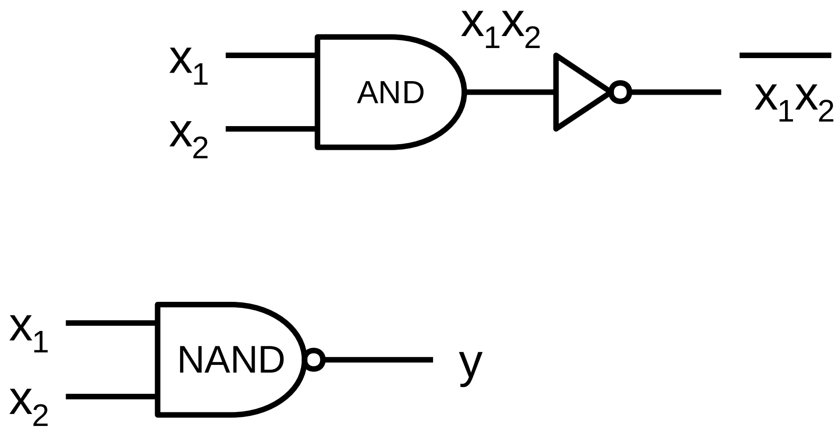
OR: $y = x_1 + x_2$



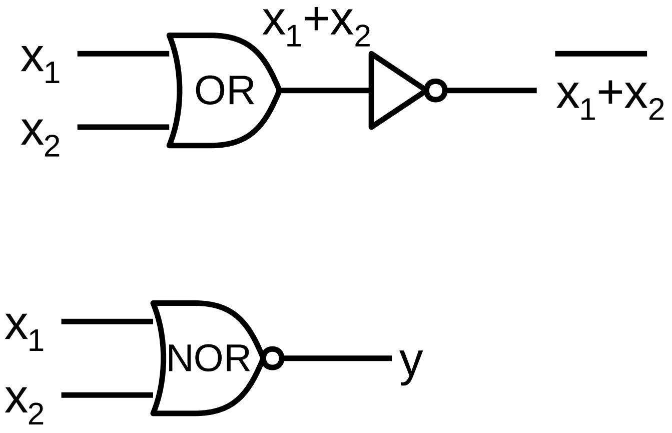
NOT: $y = \overline{x}$



NAND: $y = \overline{x_1 . \, x_2}$



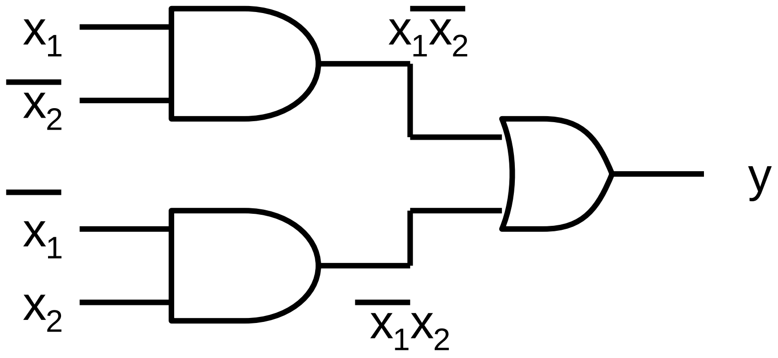
NOR: $y = \overline{x_1 + x_2}$

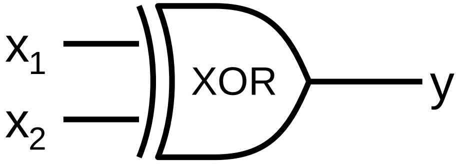


XOR: $y = x_1 \oplus x_2 = x_1 . \overline{x_2} + \overline{x_1} . x_2$

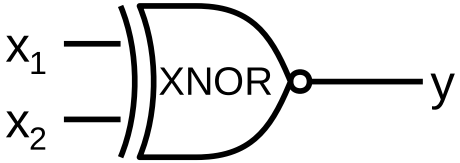
x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	0

Y is 1 if only one variable is 1 and the other is zero





XNOR: $y = x_1 \square\ x_2 = x_1 . x_2 + \overline{x_1} . \overline{x_2}$

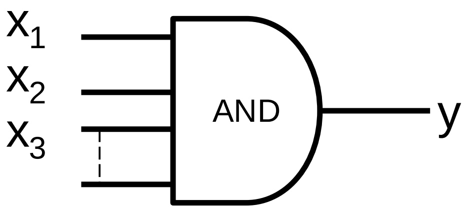


Y is 1 if only both variables are either 0 or 1

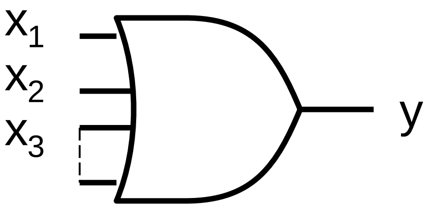
$$y = x_1 \square\ x_2 = \overline{x_1 \oplus x_2}$$

Gates with more than 2 inputs

AND: $y = x_1 . x_2 . x_3 ...$



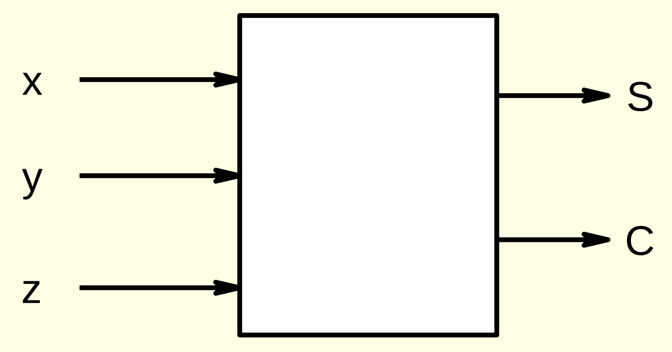
OR: $y = x_1 + x_2 + x_3 + ...$



XOR: $y = x_1 \oplus x_2 \oplus x_3 = x_1 . \overline{x_2}\,\overline{x_3} + \overline{x_1} . x_2 . \overline{x_3} + \overline{x_1} . \overline{x_2} . x_3 + x_1 . x_2 . x_3$

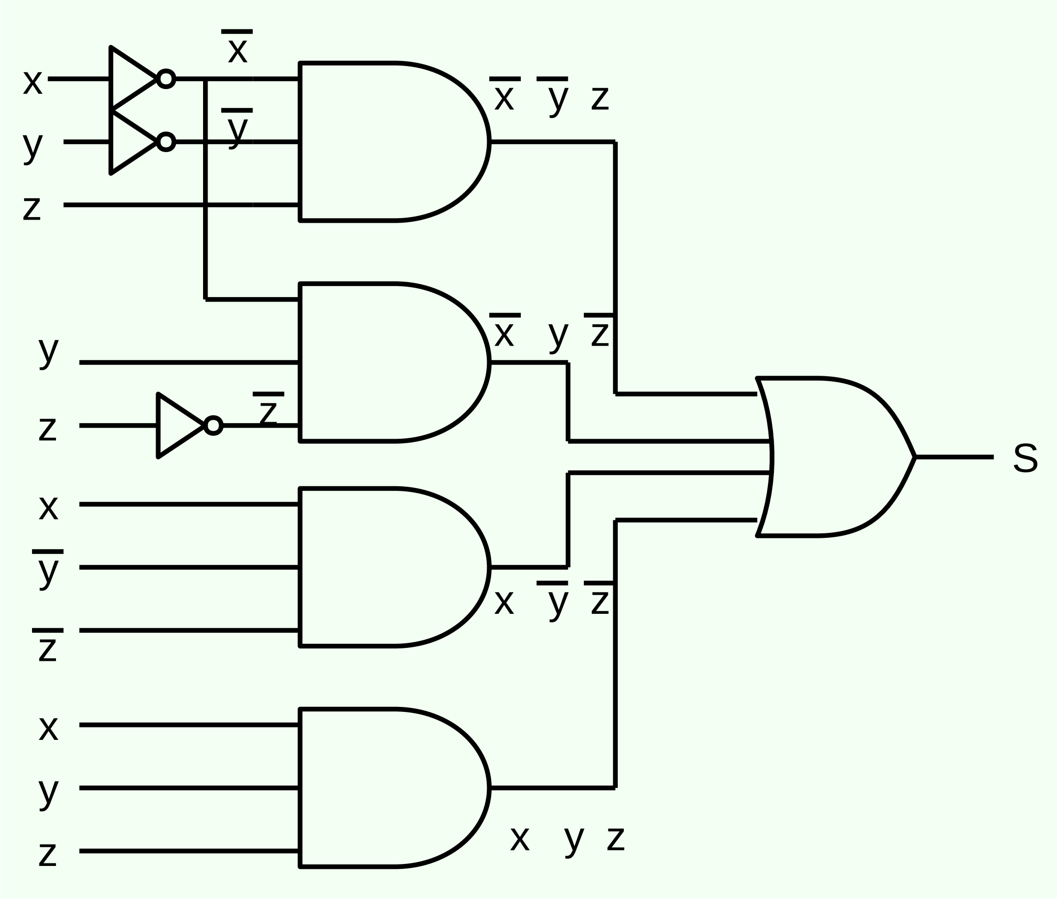
Y = 1 only if odd number of inputs is 1

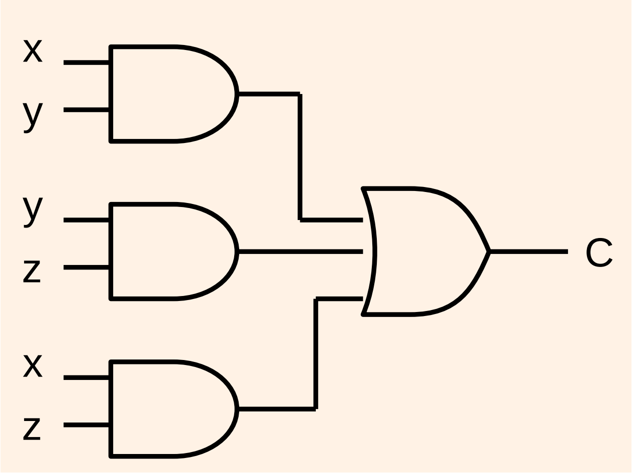
Implementing Boolean expressions using gates



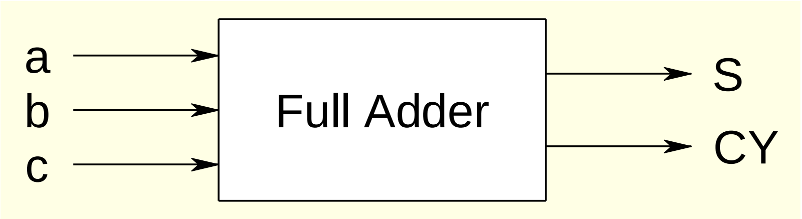
$$S = \bar{x}.\bar{y}.z + \bar{x}.y.\bar{z} + x.\bar{y}.\bar{z} + x.y.z$$

$$C = x.y + x.z + y.z$$





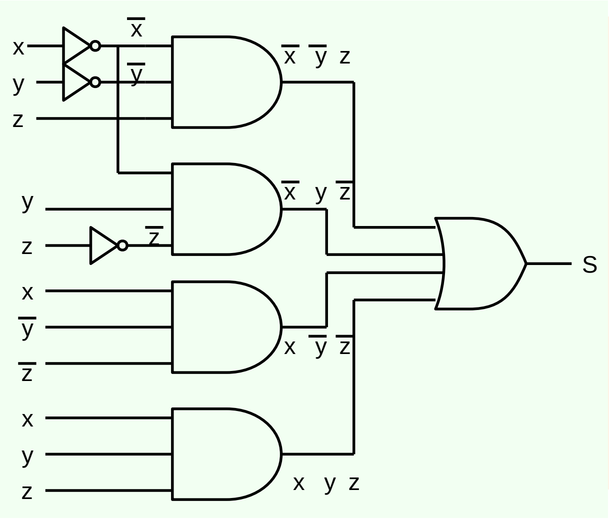
Design Overview

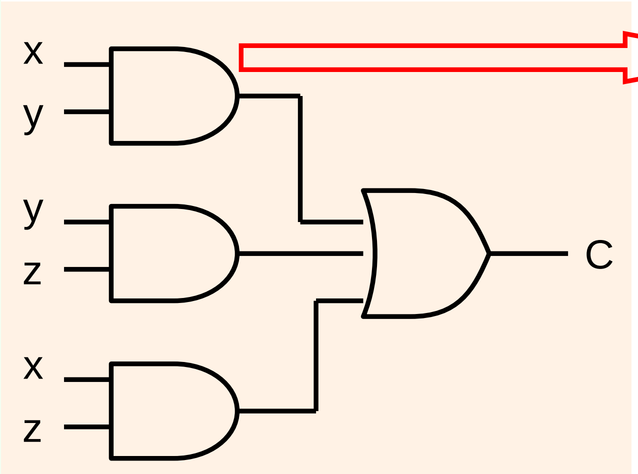


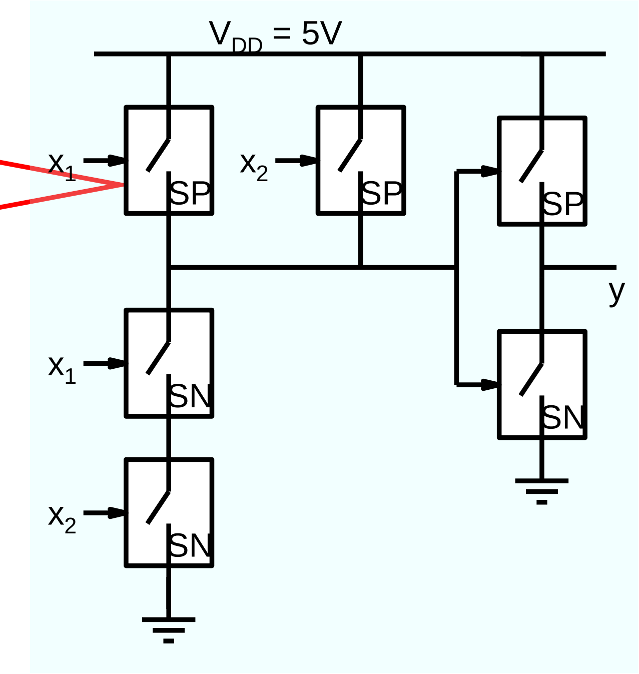
a	b	c	S	CY
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S = \bar{x}.\bar{y}.z + \bar{x}.y.\bar{z} + x.\bar{y}.\bar{z} + x.y.z$$

$$C = x.y + x.z + y.z$$







Representation of Boolean Expressions

x	y	f_1
0	0	0
0	1	1
1	0	1
1	1	0

x	y	min term	
0	0	$\overline{x}\,.\,\overline{y}$	m0
0	1	$\overline{x}\,.\,y$	m1
1	0	$x\,.\,\overline{y}$	m2
1	1	$x\,.\,y$	m3

$$f_1 = \overline{x}.\ y + x.\overline{y}$$

$$f_1 = m_1 + m_2$$

$$f_1 = \sum(1,2)$$

$$f_2 = \sum(0,2,3) = ?$$

$$f_2 = \overline{x}.\ \overline{y} + x.\overline{y} + x.y$$

A minterm is a product that contains all the variables used in a function

Three variable functions

x	y	z	min terms	
0	0	0	$\overline{x} . \overline{y} . \overline{z}$	m0
0	0	1	$\overline{x} . \overline{y} . z$	m1
0	1	0	$\overline{x} . y . \overline{z}$	m2
0	1	1	$\overline{x} . y . z$	m3
1	0	0	$x . \overline{y} . \overline{z}$	m4
1	0	1	$x . \overline{y} . z$	m5
1	1	0	$x . y . \overline{z}$	m6
1	1	1	$x . y . z$	m7

$$\mathrm{f}_2 = \sum(1, 4, 7) = ?$$

$$\mathrm{f}_2 = \overline{x} . \overline{y} . z + x . \overline{y} . \overline{z} + x . y . z$$

Product of Sum Terms Representation

x	y	f_1
0	0	1
0	1	0
1	0	0
1	1	1

x	y	Max term	
0	0	$x + \underline{y}$	M0
0	1	$\underline{x} + y$	M1
1	0	$\underline{x} + \underline{y}$	M2
1	1	$x + y$	M3

$$f_1 = (x + \overline{y}).(\overline{x} + y)$$

$$f_1 = M_1.M_2$$

$$f_1 = \Pi(1,2)$$

$$f_2 = \Pi(0,3) = ?$$

$$f_2 = (x + y).(\overline{x} + \overline{y})$$

x	y	z	Max. terms	
0	0	0	$x + y + z$	M0
0	0	1	$x + y + \overline{z}$	M1
0	1	0	$x + \overline{y} + z$	M2
0	1	1	$x + \overline{y} + \overline{z}$	M3
1	0	0	$\overline{x} + y + z$	M4
1	0	1	$\overline{x} + y + \overline{z}$	M5
1	1	0	$\overline{x} + \overline{y} + z$	M6
1	1	1	$\overline{x} + \overline{y} + \overline{z}$	M7

$$f_1 = \Pi\,(1, 5, 7) = ?$$

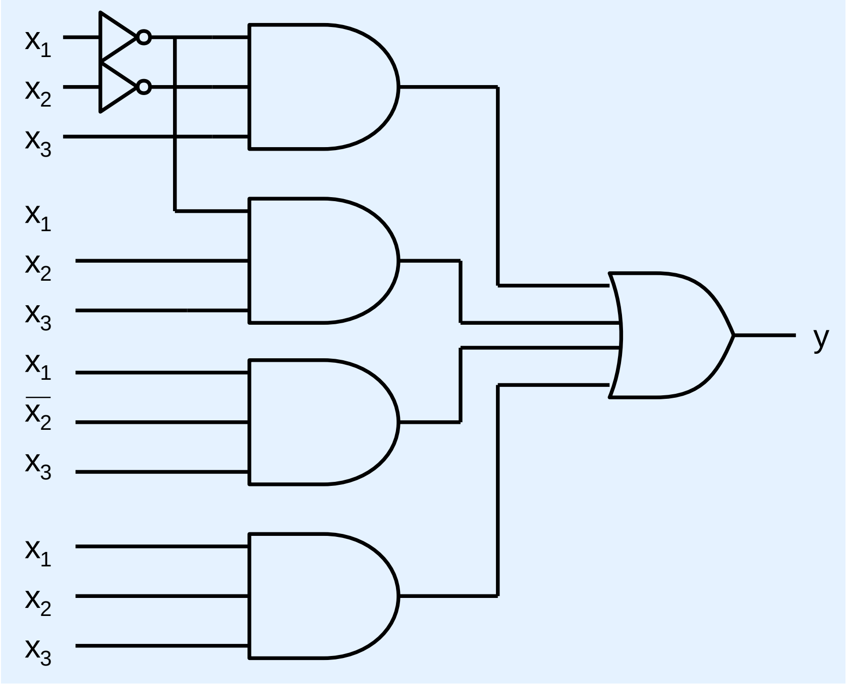
$$f_2 = (x + y + \overline{z}).(\overline{x} + y + \overline{z}).(\overline{x} + \overline{y} + \overline{z})$$

Simplification

x_1	x_2	x_3	y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

$$y = \sum (1, 3, 5, 7)$$

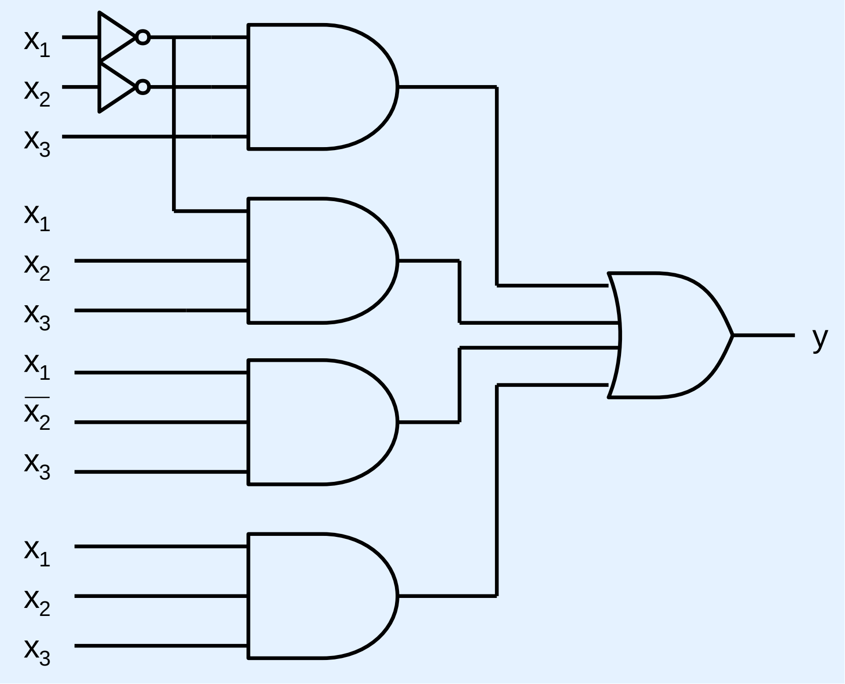
$$y = \overline{x}_1 . \ \overline{x}_2 . x_3 + \overline{x}_1 . x_2 . x_3 + x_1 . \overline{x}_2 . x_3 + x_1 . x_2 . x_3$$



Simplification of Boolean expression yields : $y = x_3$!! which does not require any gates at all !

Goal of Simplification

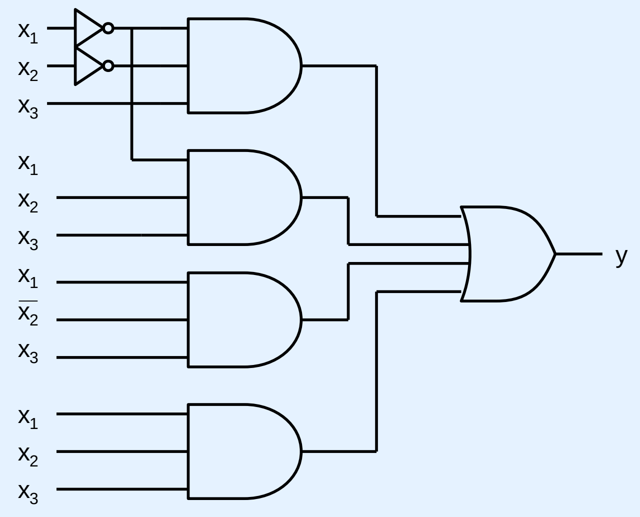
$$y = \overline{x}_1 .\ \overline{x}_2 . x_3 + \overline{x}_1 . x_2 . x_3 + x_1 . \overline{x}_2 . x_3 + x_1 . x_2 . x_3$$

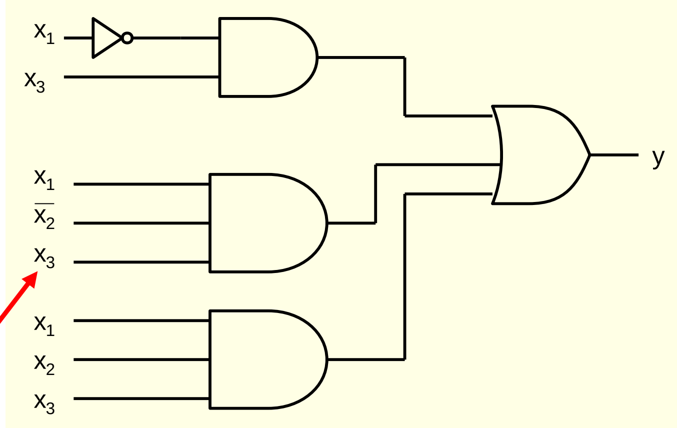


Goal of simplification is to reduce the complexity of gate circuit. This requires that we minimize the number of gates. Since number of gates depends on number of minterms, one of the goals of simplification is to **minimize the number of minterms in SOP expression**

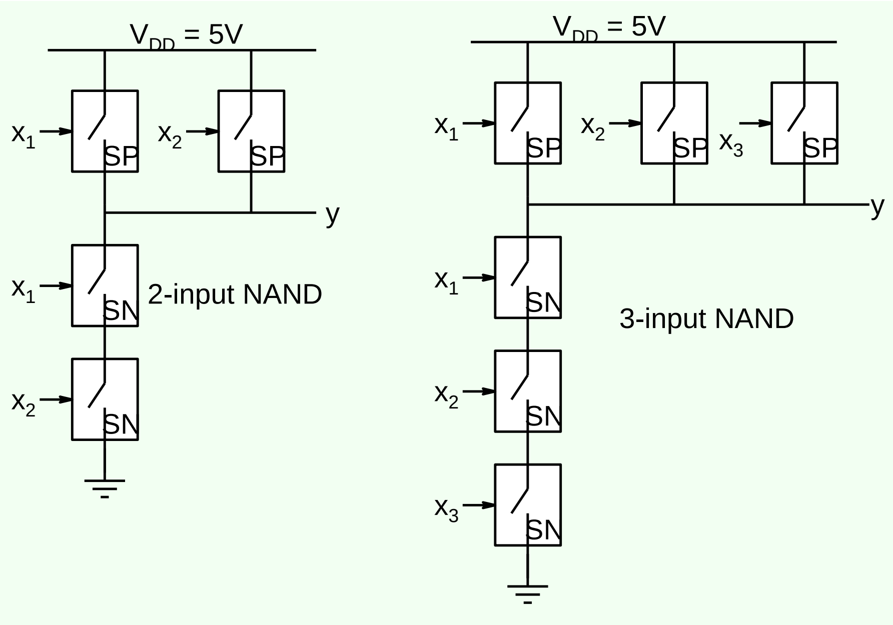
$$y = \overline{x_1} . \overline{x_2} . x_3 + \overline{x_1} . x_2 . x_3 + x_1 . \overline{x_2} . x_3 + x_1 . x_2 . x_3$$

$$\Rightarrow \quad y = \overline{x_1} . x_3 + x_1 . \overline{x_2} . x_3 + x_1 . x_2 . x_3$$





This circuit is simpler not just because it uses 4 gates instead of 5 but also because circuit-2 uses one 2-input and three 3-input gates as compared to five 3-input gates used in circuit-1



Goal of Simplification

$$y=\overline{x_1}.\overline{x_2}.x_3+\overline{x_1}.x_2.x_3+x_1.\overline{x_2}.x_3+x_1.x_2.x_3 \quad \Rightarrow \quad y=\overline{x_1}.x_3+x_1.\overline{x_2}.x_3+x_1.x_2.x_3$$

In the SOP expression:

1. Minimize number of product terms

2. Minimize number of literals in each term

$$\text{Simplification} \Rightarrow \text{Minimization}$$

Minimization

$$y = \overline{x_1} . \overline{x_2} . x_3 + \overline{x_1} . x_2 . x_3 + x_1 . \overline{x_2} . x_3 + x_1 . x_2 . x_3$$

$$y = \overline{x_1} . x_3 .(\overline{x_2} + x_2) + x_1 . x_3 .(\overline{x_2} + x_2)$$

$$y = \overline{x_1} . x_3 + x_1 . x_3$$

$$y =(\overline{x_1} + x_1). x_3$$

$$y = x_3$$

Principle used: $x + \overline{x} = 1$

$$f = \overline{x}.\,\overline{y} + \overline{x}.y + x.\overline{y}$$

Apply the Principle: $x + \overline{x} = 1$ to simplify

$$f = \overline{x}.(\overline{y} + y) + x.\overline{y}$$

$$f = \overline{x} + x.\overline{y}$$

How do we simplify further?

$$f = \overline{x}.\,\overline{y} + \overline{x}.y + x.\overline{y} = \overline{x}.\,\overline{y} + \overline{x}.\,\overline{y} + \overline{x}.y + x.\overline{y}$$

Principle used : $x + x = x$

$$\begin{aligned} f = {} & \overline{x}.\,\overline{y} + \overline{x}.y + \overline{x}.\,\overline{y} + x.\overline{y} \\ = {} & \overline{x}.\,(\overline{y} + y) + (\overline{x} + x).\,\overline{y} = \overline{x} + \overline{y} \end{aligned}$$

Simplify

$$f = \overline{x_1}.x_2.\overline{x_3}.x_4 + \overline{x_1}.x_2.x_3.x_4 + x_1.x_2.\overline{x_3}.x_4 + x_1.x_2.x_3.x_4 +$$
$$x_1.\overline{x_2}.\overline{x_3}.x_4 + x_1.\overline{x_2}.x_3.x_4$$

Principle: $x + \overline{x} = 1$ and $x + x = x$

Need a systematic and simpler method for applying these two principles

Karnaugh Map (K map) is a popular technique for carrying out simplification

It represents the information in problem in such a way that the two principles become easy to apply

K-map representation of truth table

x	y	min term
0	0	$\bar{x} . \bar{y}$ m0
0	1	$\bar{x} . y$ m1
1	0	$x . \bar{y}$ m2
1	1	$x . y$ m3

x y	0	1
0	m_0	m_1
1	m_2	m_3

x	y	f_1
0	0	0
0	1	1
1	0	1
1	1	0

x y	0	1
0	0	1
1	1	0

$$f_2 = \sum(0,2,3)$$

$x \backslash y$	0	1
0	1	0
1	1	1

$x \backslash y$	0	1
0	1	0
1	0	1

$$f = \overline{x}.\overline{y} + x.y$$

3-variable K-map representation

x	y	z	min terms	
0	0	0	$\overline{x} . \overline{y} . \overline{z}$	m0
0	0	1	$\overline{x} . \overline{y} . z$	m1
0	1	0	$\overline{x} . y . \overline{z}$	m2
0	1	1	$\overline{x} . y . z$	m3
1	0	0	$x . \overline{y} . \overline{z}$	m4
1	0	1	$x . \overline{y} . z$	m5
1	1	0	$x . y . \overline{z}$	m6
1	1	1	$x . y . z$	m7

x \ yz	00	01	11	10
0	m_0	m_1	m_3	m_2
1	m_4	m_5	m_7	m_6

x	y	z	f
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

x \ yz	00	01	11	10
0	0	1	1	0
1	0	1	1	0

x \ yz	00	01	11	10
0	1	0	1	0
1	0	1	1	0

$$\mathrm{f} = \bar{x}.\bar{y}.\bar{z} + \bar{x}.y.z + x.\bar{y}.z + x.y.z$$

4-variable K-map representation

w	x	y	z	min terms
0	0	0	0	m_0
0	0	0	1	m_1
0	0	1	0	m_2
0	0	1	1	m_3
⋮	⋮	⋮	⋮	⋮
1	1	1	0	m_{14}
1	1	1	1	m_{15}

wx \ yz	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$\text{f} = \overline{w}.\overline{x}.\overline{y}.\overline{z} + \overline{w}.\overline{x}.y.z + \overline{w}.x.\overline{y}.z + \overline{w}.x.y.z$$
$$+ w.x.\overline{y}.\overline{z} + w.x.y.\overline{z} + w.\overline{x}.\overline{y}.\overline{z}$$

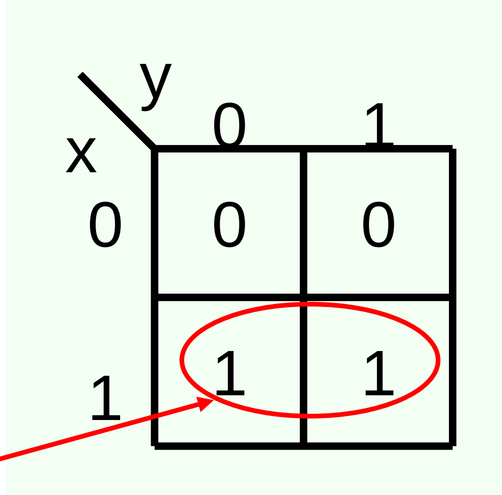
Minimization using Kmap

$$f_2 = \sum(2,3)$$

$$f = x.\bar{y} + x.y$$

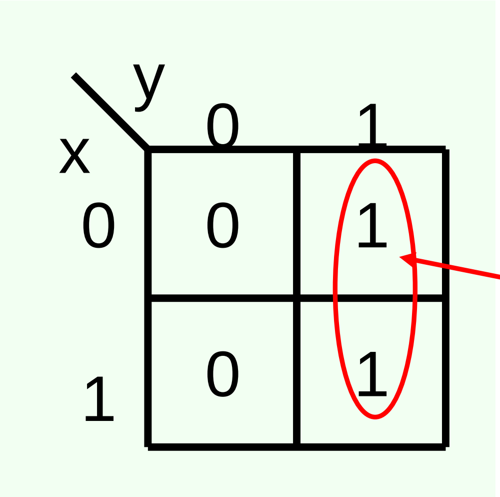
$$f = x.(\bar{y} + y)$$

$$f = x$$



x \ y	0	1
0	0	0
1	1	1

Combine terms which differ in only one bit position. As a result, whatever is common remains.

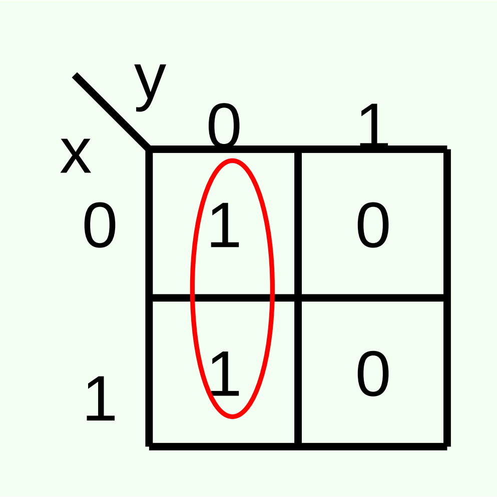


x \ y	0	1
0	0	1
1	0	1

$$f = \overline{x}.y + x.y$$

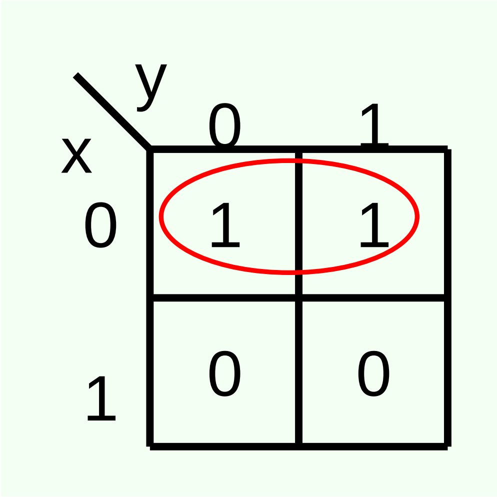
$$f = (\overline{x}. + x).y$$

$$\Rightarrow f = y$$



x \ y	0	1
0	1	0
1	1	0

$$\Rightarrow f = \overline{y}$$

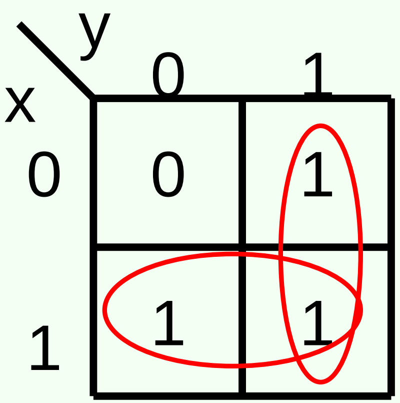


x \ y	0	1
0	1	1
1	0	0

$$\Rightarrow f = \overline{x}$$

Principle: $x + \overline{x} = 1$ and $x + x = x$

$$f_2 = \sum(0,2,3)$$



x \ y	0	1
0	0	1
1	1	1

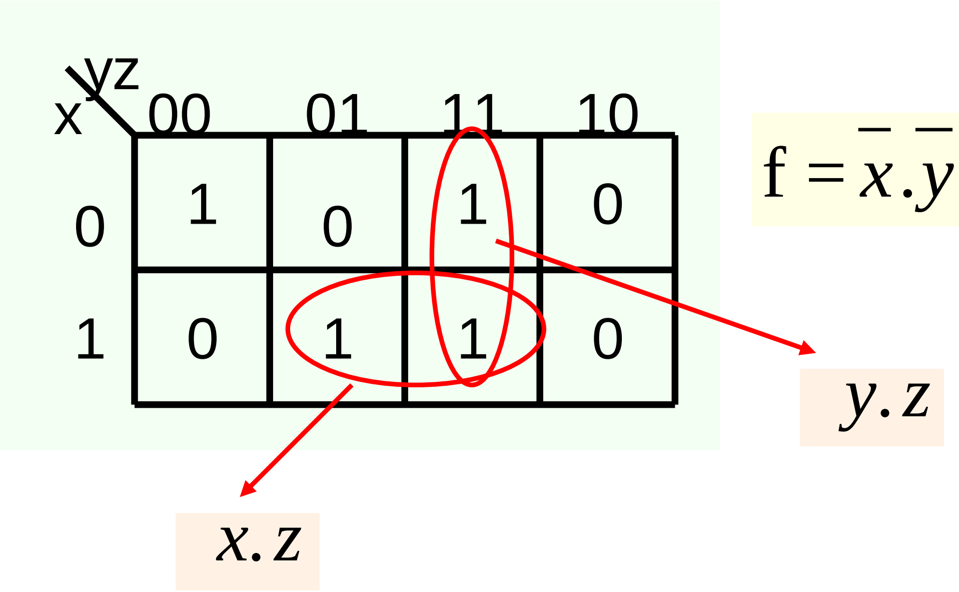
$$f = x.\overline{y} + x.y + \overline{x}.y$$

$$\begin{aligned} f &= x.(\overline{y} + y) + \overline{x}.y \\ &= x + \overline{x}.y \end{aligned}$$

$$\begin{aligned} f &= x + \overline{x}.y + x.y \\ &= x + (\overline{x} + x).y \\ &= x + y \end{aligned}$$

The idea is to cover all the 1’s with as few and as simple terms as possible

3-variable minimization



x \ yz	00	01	11	10
0	1	0	1	0
1	0	1	1	0

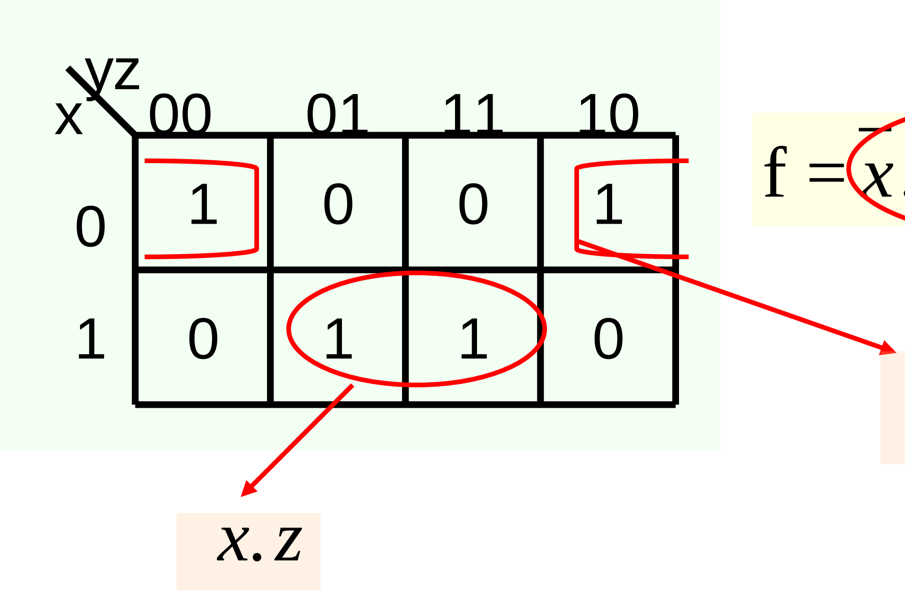
$y.z$

$x.z$

$$f = \bar{x}.\bar{y}.\bar{z} + \bar{x}.y.z + x.y.z + x.\bar{y}.z$$

$$f = \bar{x}.\bar{y}.\bar{z} + y.z + x.z$$

3-variable minimization



x \ yz	00	01	11	10
0	1	0	0	1
1	0	1	1	0

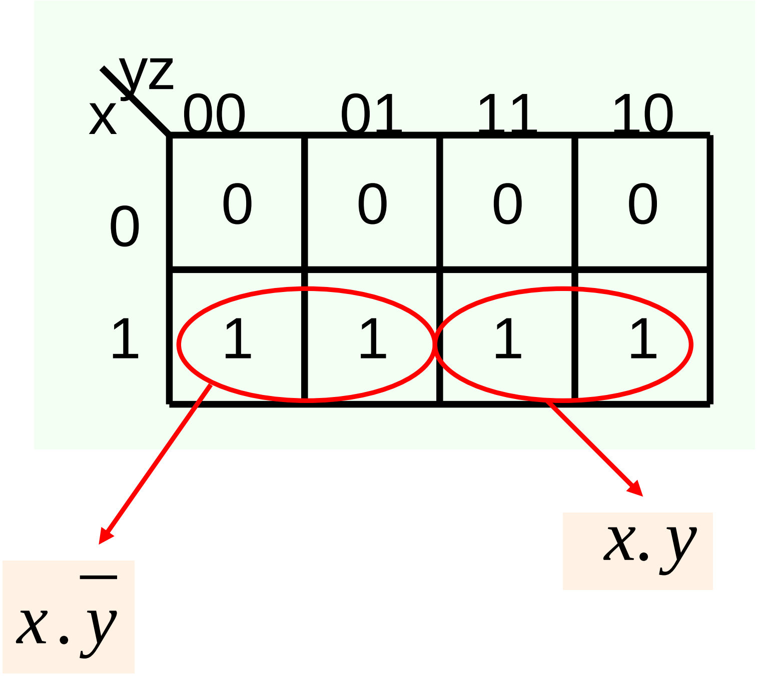
$$f = \bar{x}.\bar{y}.\bar{z} + \bar{x}.y.\bar{z} + x.y.z + x.\bar{y}.z$$

$\bar{x}.\bar{z}$

$x.z$

$$f = \bar{x}.\bar{z} + x.z$$

3-variable minimization



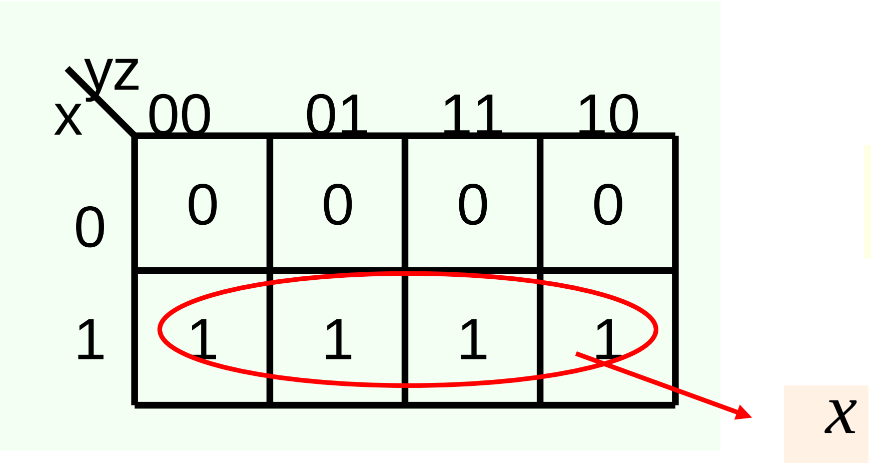
x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	1

$x.\overline{y}$

$x.y$

$$f = x.\overline{y}.\overline{z} + x.\overline{y}.z + x.y.z + x.y.\overline{z}$$

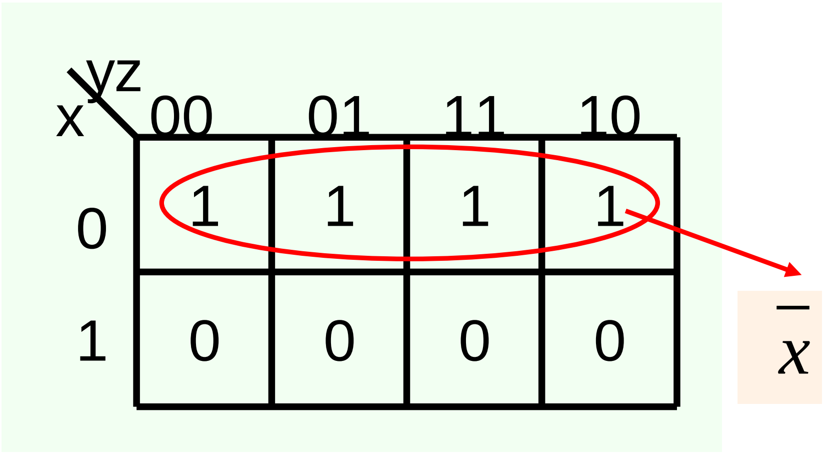
$$f = x.\overline{y} + x.y$$



x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	1

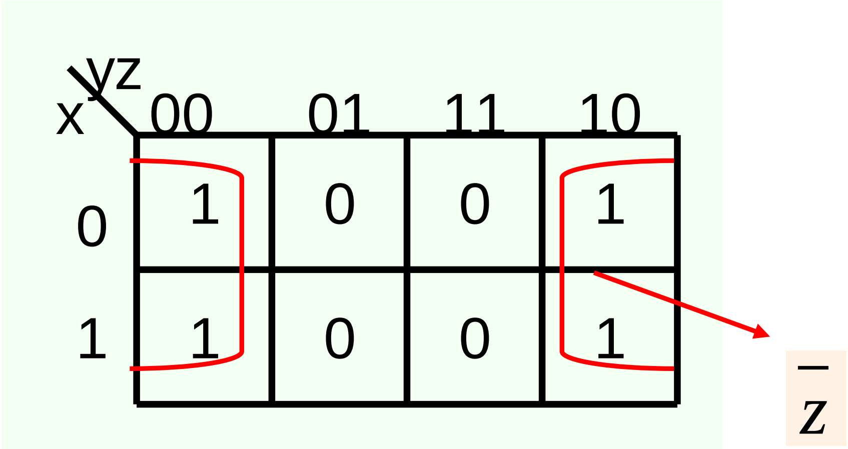
x

$$f = x.(\overline{y} + y) = x$$



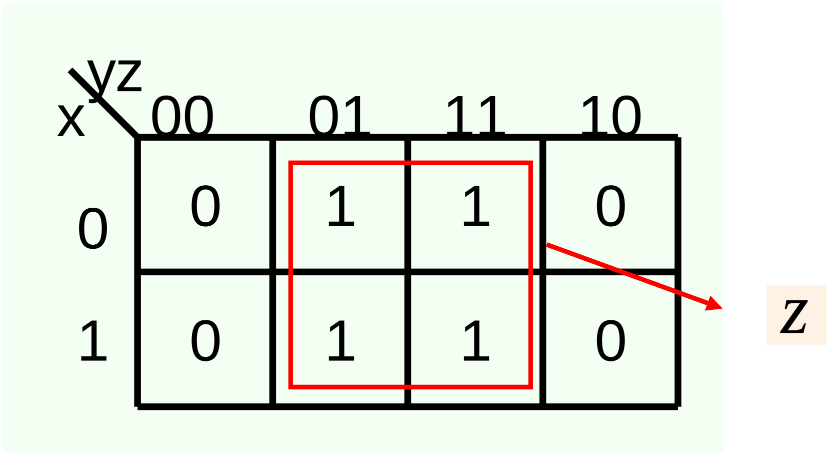
x \ yz	00	01	11	10
0	1	1	1	1
1	0	0	0	0

$\bar{x}$



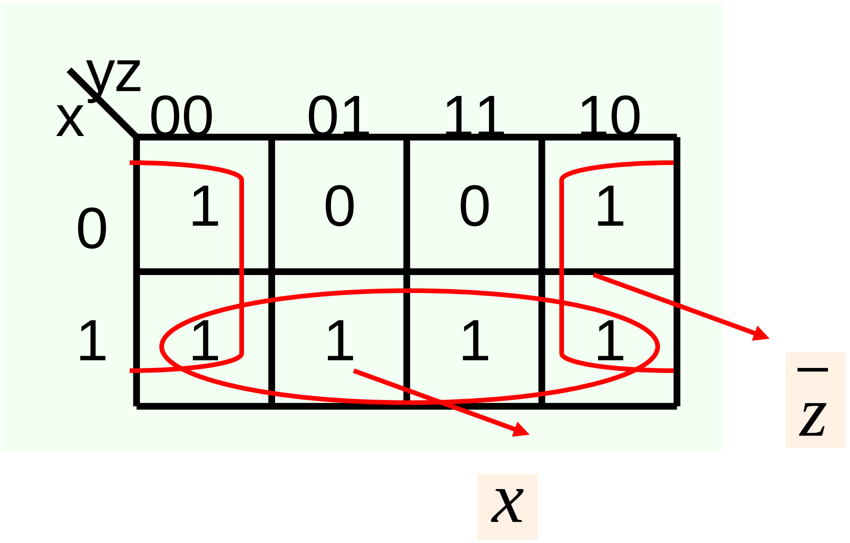
x \ yz	00	01	11	10
0	1	0	0	1
1	1	0	0	1

$\bar{z}$



x \ yz	00	01	11	10
0	0	1	1	0
1	0	1	1	0

z



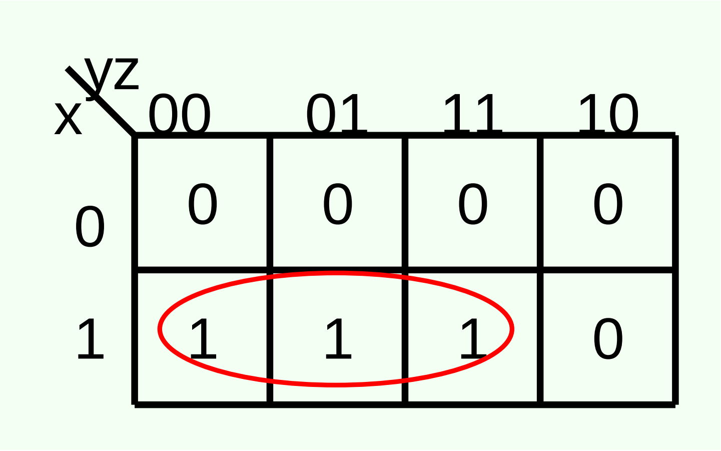
x \ yz	00	01	11	10
0	1	0	0	1
1	1	1	1	1

$\bar{z}$

x

$$f = x + \bar{z}$$

Can we do this ?



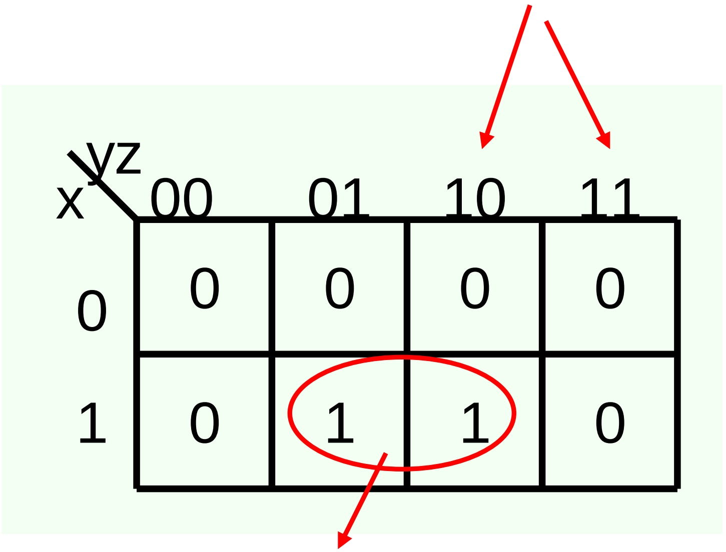
x \ yz	00	01	11	10
0	0	0	0	0
1	1	1	1	0

Note that each encirclement should represent a single product term. In this case it does not.

$$\begin{aligned} \mathrm{f} &= x.\overline{y}.\overline{z} + x.\overline{y}.z + x.y.z \\ &= x.\overline{y} + x.z \end{aligned}$$

We do not get a single product term. In general we cannot make groups of 3 terms.

Can we use kmap with the following ordering of variables?

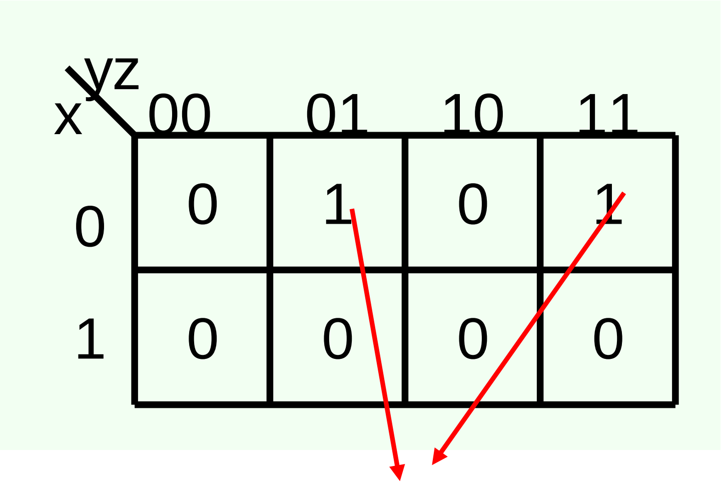


x \ yz	00	01	10	11
0	0	0	0	0
1	0	1	1	0

Can we combine these two terms into a single term ?

$$
\begin{aligned}
f &= x.\overline{y}.z + x.y.\overline{z} \\
&= x.(\overline{y}.z + y.\overline{z})
\end{aligned}
$$

Note that no simplification is possible. Kmap requires information to be represented



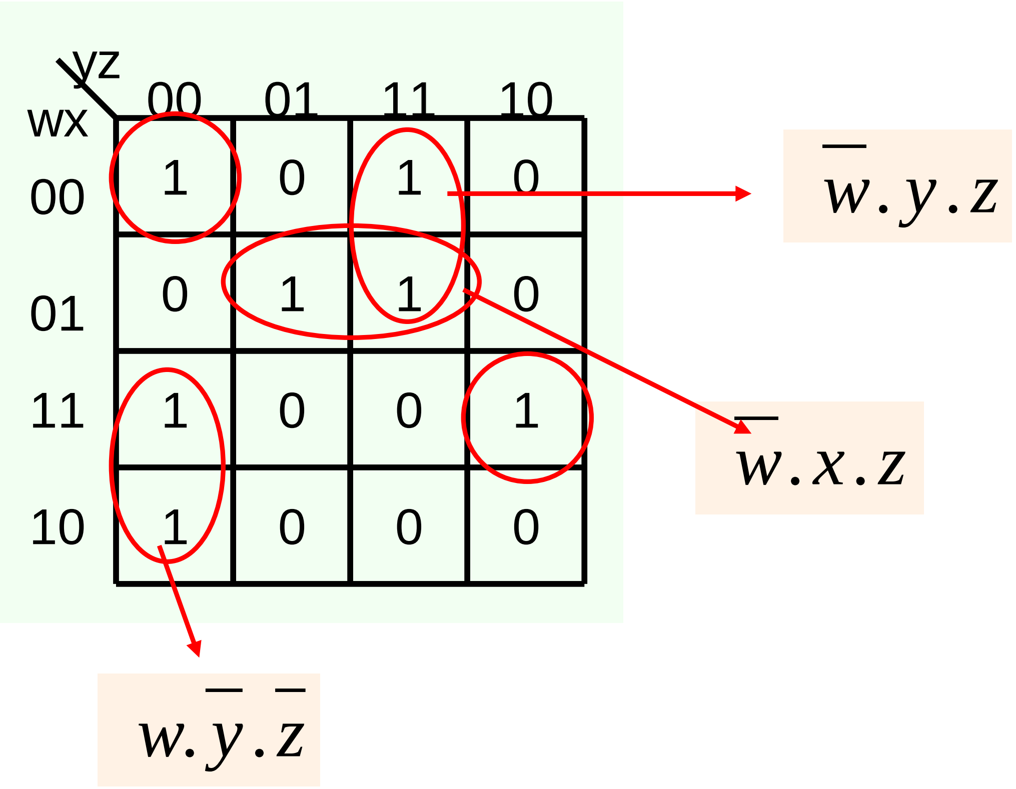
x \ yz	00	01	10	11
0	0	1	0	1
1	0	0	0	0

These two terms can be combined into a single term but it is not easy to show that on the diagram.

$$\mathrm{f} = \bar{x}.\bar{y}.z + \bar{x}.y.z$$
$$= \bar{x}.(\bar{y} + y).z = \bar{x}.z$$

Kmap requires information to be represented in such a way that it is easy to apply the principle $x + x = 1$

4-variable minimization



wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

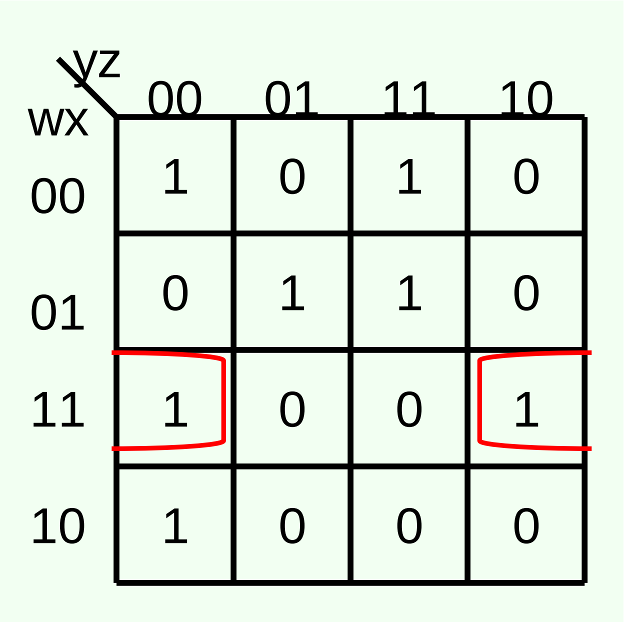
$\overline{w}.y.z$

$\overline{w}.x.z$

$w.\overline{y}.\overline{z}$

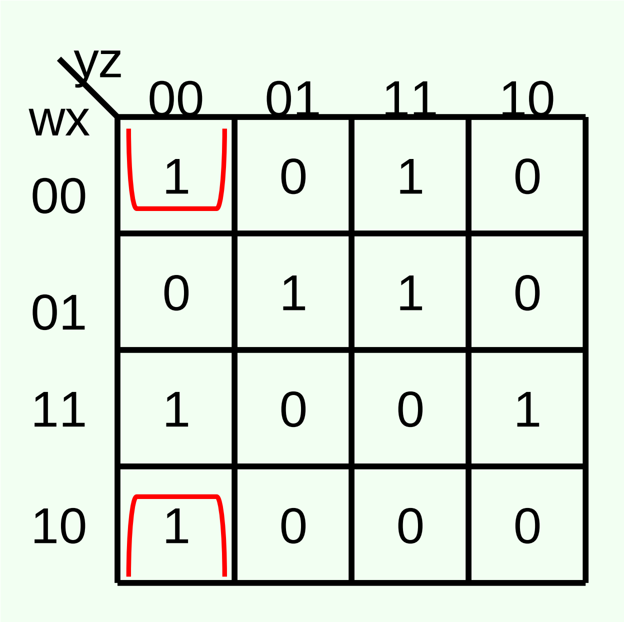
$$f = \overline{w}.y.z + \overline{w}.x.z + w.\overline{y}.\overline{z} + \boxed{\overline{w}.\overline{x}.\overline{y}.\overline{z}} + \boxed{w.x.y.\overline{z}}$$

But is this the simplest expression ?



wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

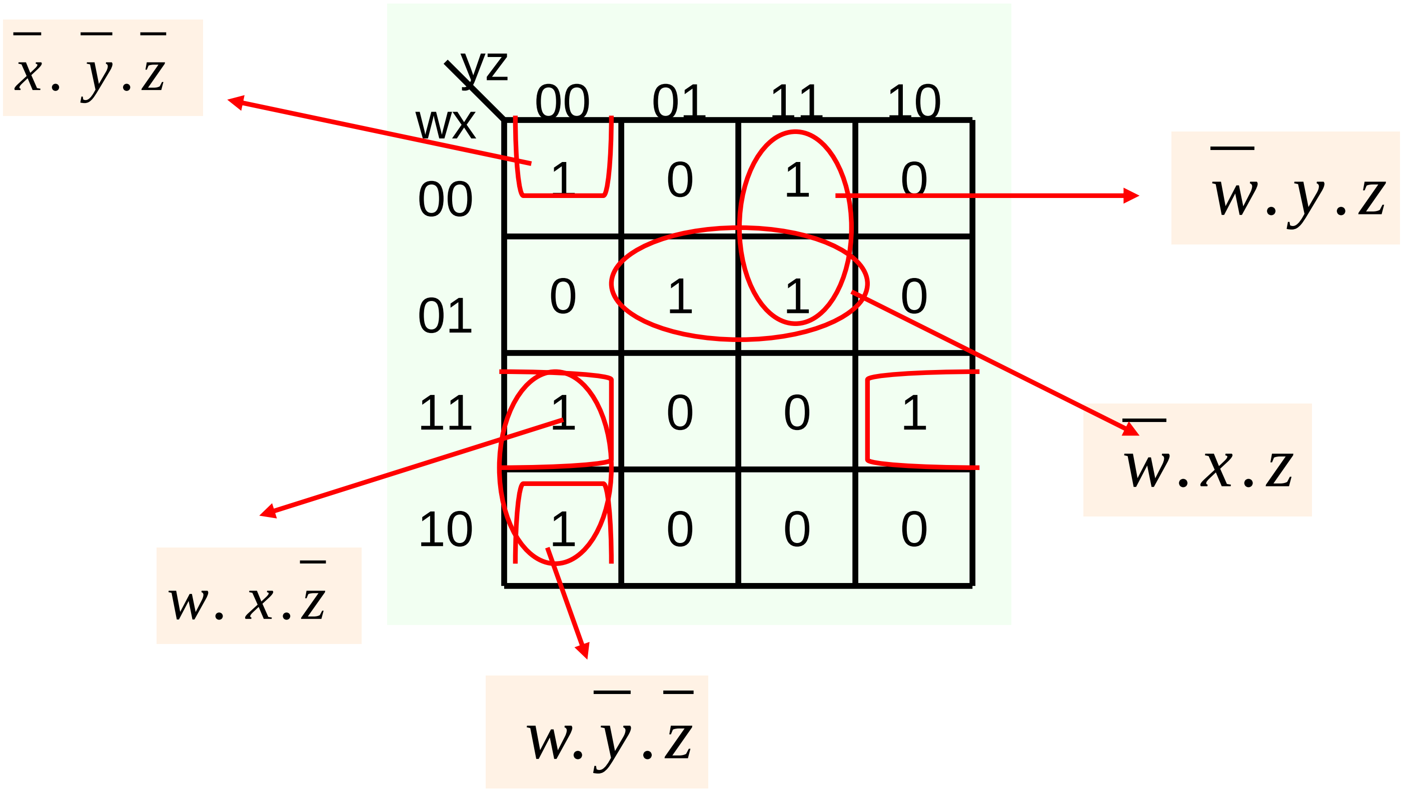
$$w.\,x.\bar{y}.\bar{z}+w.\,x.y.\bar{z}=w.\,x.\bar{z}$$



wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$w.\,\bar{x}.\bar{y}.\bar{z}+\bar{w}.\,\bar{x}.\bar{y}.\bar{z}=\bar{x}.\,\bar{y}.\bar{z}$$

4-variable minimization



wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$\bar{x}.\,\bar{y}.\bar{z}$

$\bar{w}.y.z$

$\bar{w}.x.z$

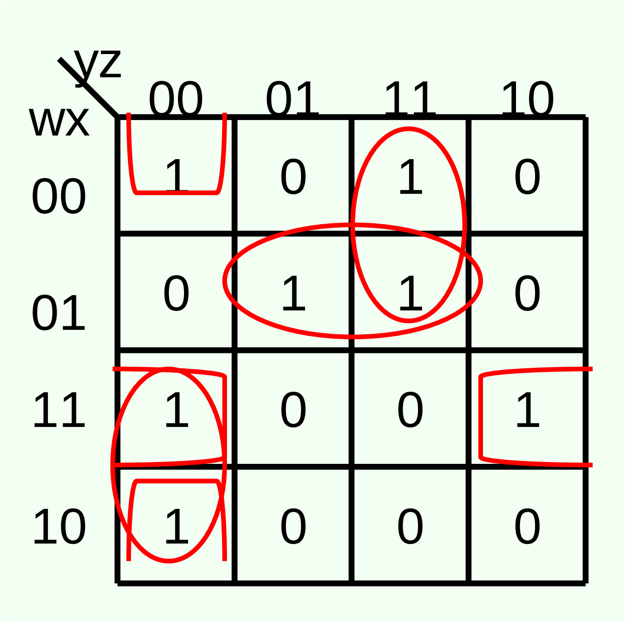
$w.\,x.\bar{z}$

$w.\bar{y}.\bar{z}$

$$f = \bar{w}.y.z + \bar{w}.x.z + w.\bar{y}.\bar{z} + w.x.\bar{z} + \bar{x}.\bar{y}.\bar{z}$$

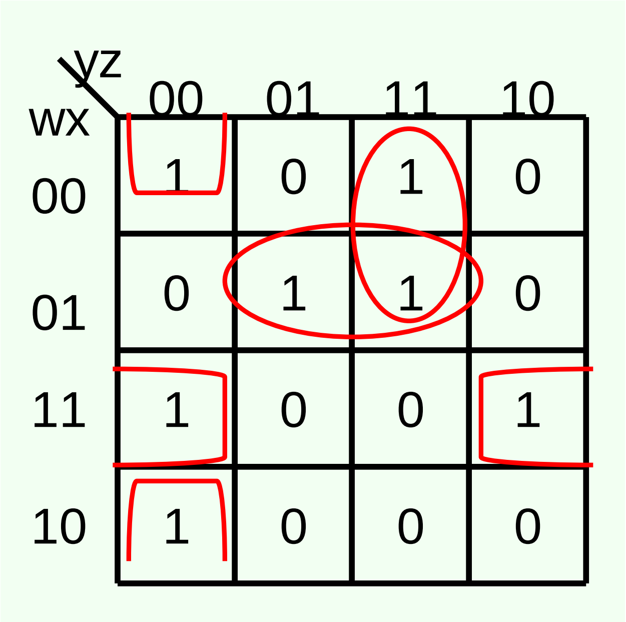
Is this the best that we can do ?

Cover the 1's with minimum number of terms



wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

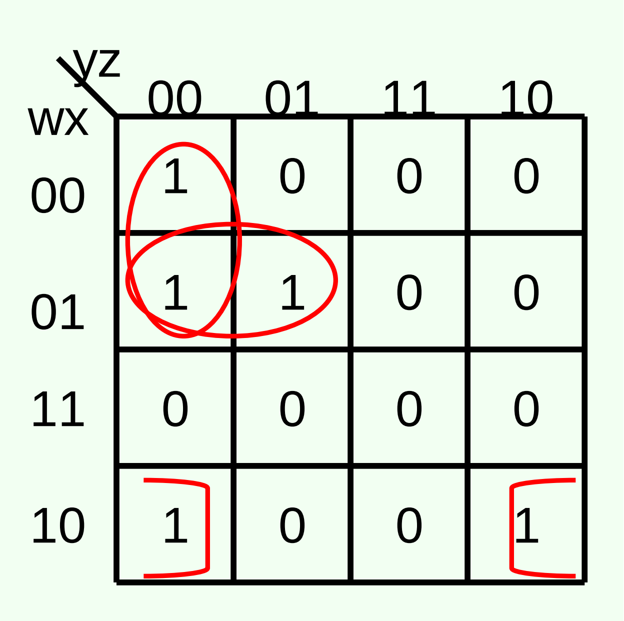
$$f = \overline{w}.y.z + \overline{w}.x.z + w.\overline{y}.\overline{z} + w.x.\overline{z} + \overline{x}.\overline{y}.\overline{z}$$



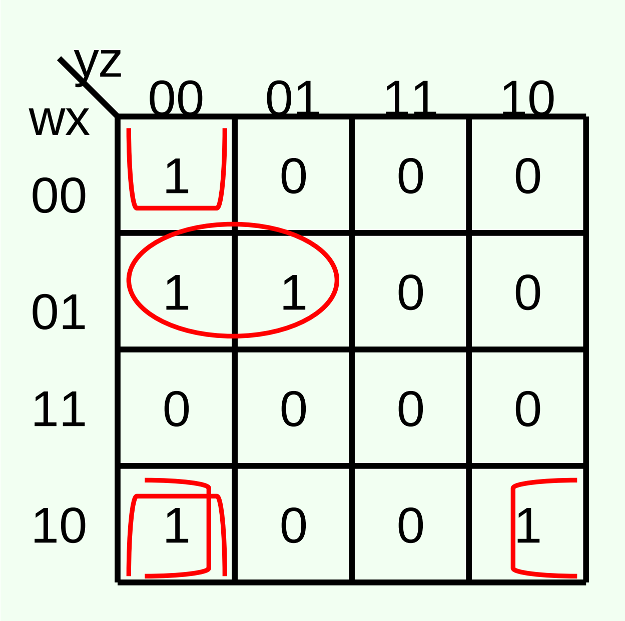
wx \ yz	00	01	11	10
00	1	0	1	0
01	0	1	1	0
11	1	0	0	1
10	1	0	0	0

$$f = \overline{w}.y.z + \overline{w}.x.z + w.x.\overline{z} + \overline{x}.\overline{y}.\overline{z}$$

4-variable minimization



wx \ yz	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	0	0	1

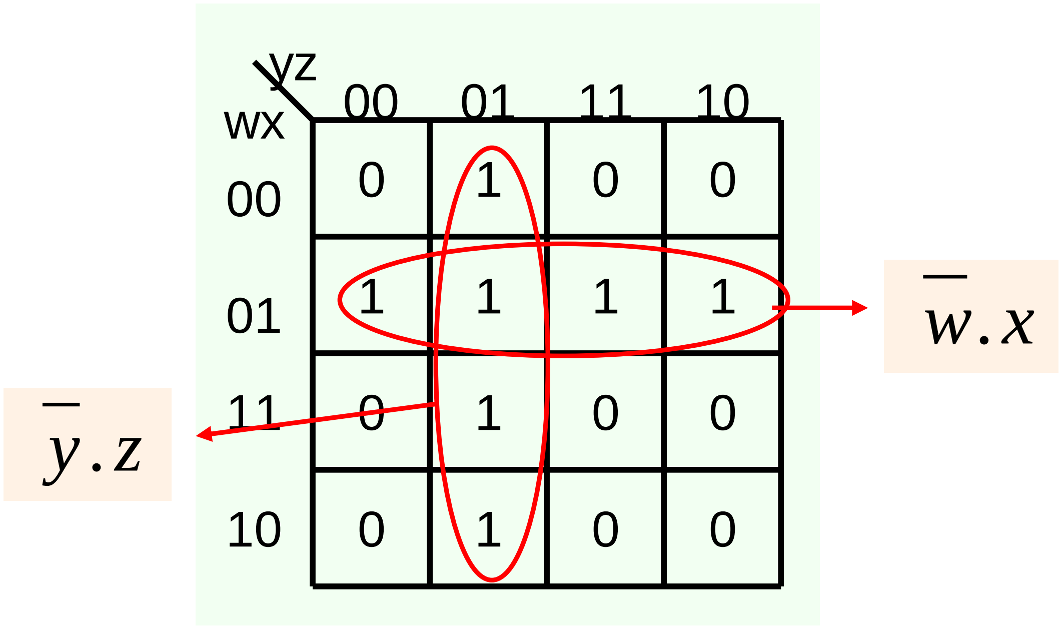


wx \ yz	00	01	11	10
00	1	0	0	0
01	1	1	0	0
11	0	0	0	0
10	1	0	0	1

$$f = \overline{w}.x.\overline{y} + w.\overline{x}.\overline{z} + \overline{w}.\overline{y}.\overline{z}$$

$$f = \overline{w}.x.\overline{y} + w.\overline{x}.\overline{z} + \overline{x}.\overline{y}.\overline{z}$$

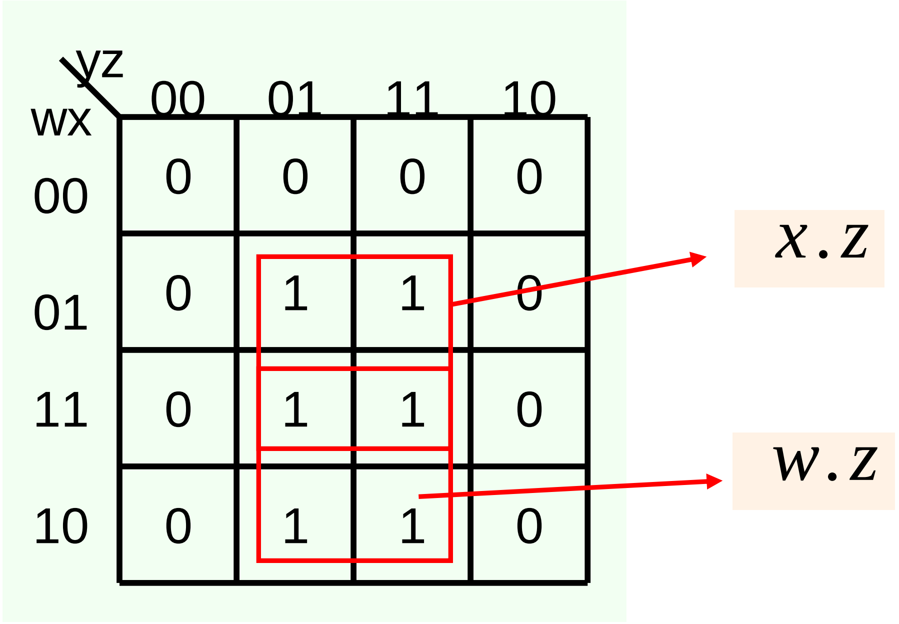
Groups of 4



yz wx	00	01	11	10
00	0	1	0	0
01	1	1	1	1
11	0	1	0	0
10	0	1	0	0

$\overline{w}.x$

$\overline{y}.z$



yz wx	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

$x.z$

$w.z$

yz wx	00	01	11	10
00	0	0	0	0
01	1	0	0	1
11	1	0	0	1
10	0	0	0	0

$x.\bar{z}$

yz wx	00	01	11	10
00	0	1	1	0
01	0	0	0	0
11	0	0	0	0
10	0	1	1	0

$\bar{x}.z$

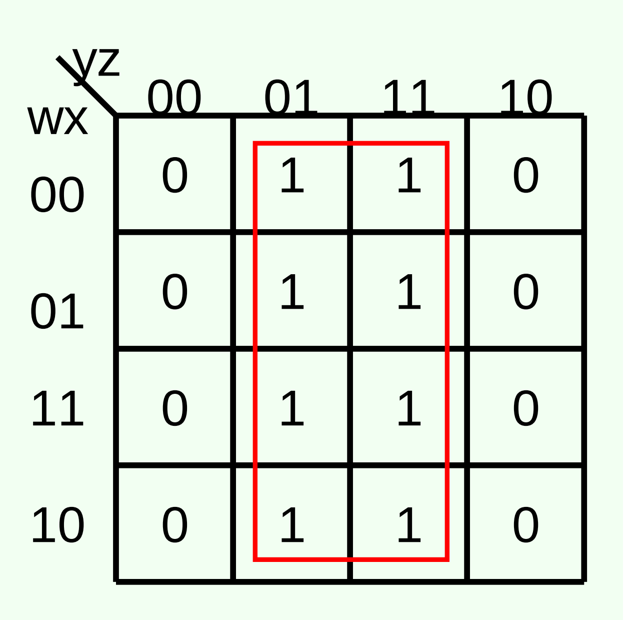
yz wx	00	01	11	10
00	1	0	0	1
01	0	0	0	0
11	0	0	0	0
10	1	0	0	1

$\bar{x}.\bar{z}$

yz wx	00	01	11	10
00	1	0	1	0
01	0	0	0	0
11	0	0	0	0
10	1	0	1	0

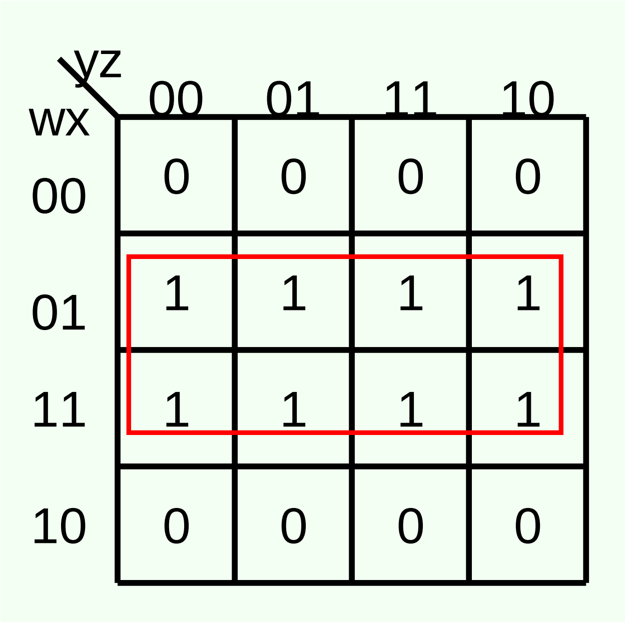
??

Groups of 8



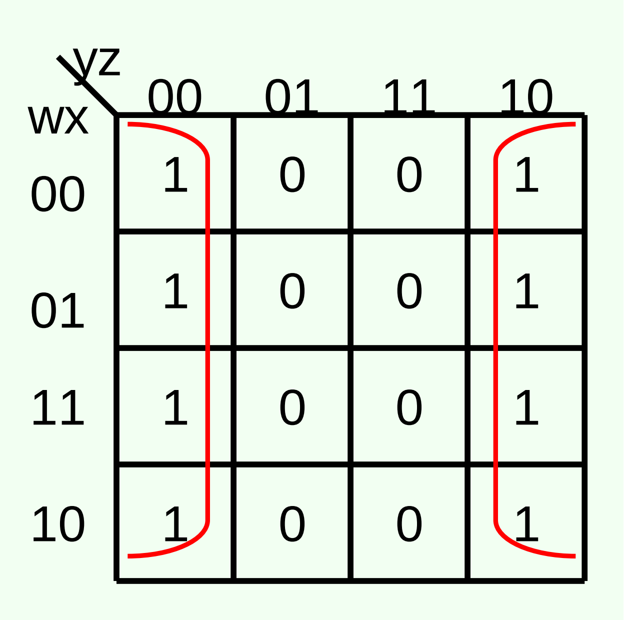
wx \ yz	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

z



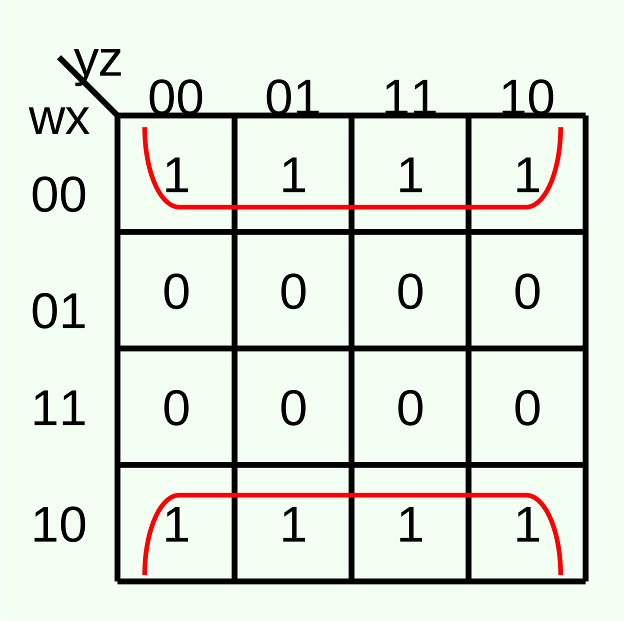
wx \ yz	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

x



wx \ yz	00	01	11	10
00	1	0	0	1
01	1	0	0	1
11	1	0	0	1
10	1	0	0	1

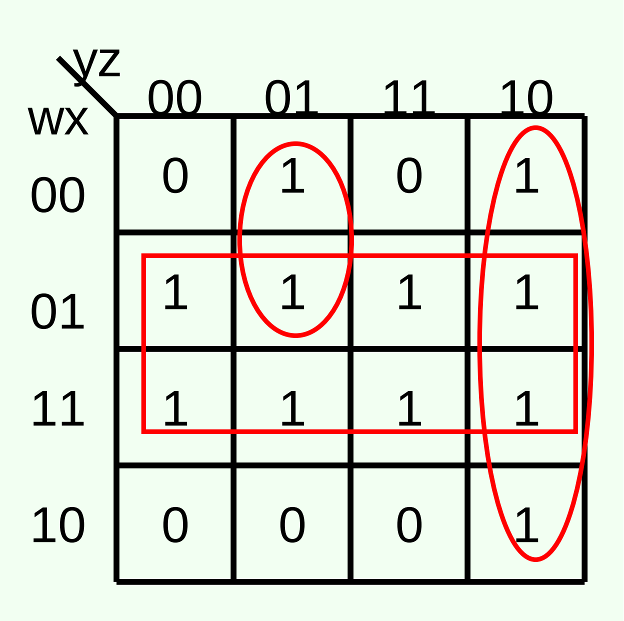
$\overline{z}$



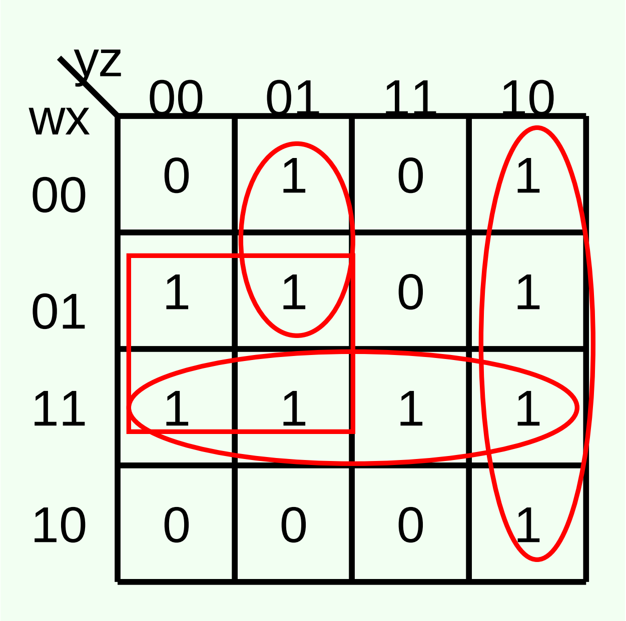
wx \ yz	00	01	11	10
00	1	1	1	1
01	0	0	0	0
11	0	0	0	0
10	1	1	1	1

$\overline{x}$

Examples

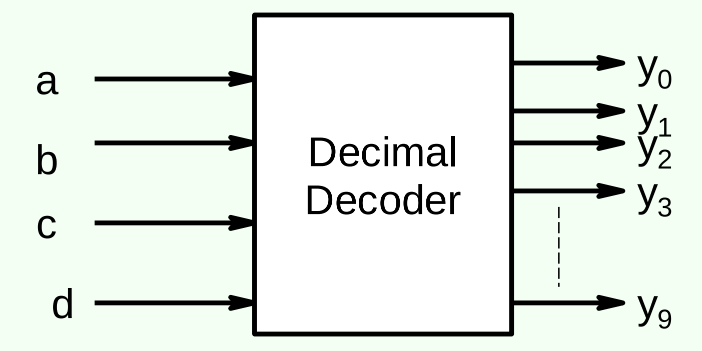


wx \ yz	00	01	11	10
00	0	1	0	1
01	1	1	1	1
11	1	1	1	1
10	0	0	0	1



wx \ yz	00	01	11	10
00	0	1	0	1
01	1	1	0	1
11	1	1	1	1
10	0	0	0	1

Don’t care terms



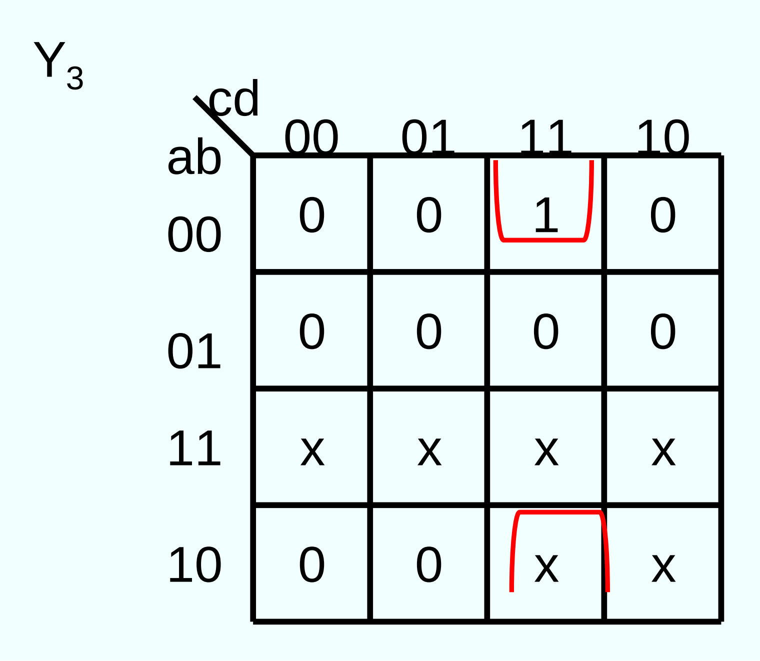
Y_3

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	x	x	x	x
10	0	0	x	x

$$y_3 = \bar{a}.\bar{b}.c.d$$

a	b	c	d	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9
0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	0
0	0	1	1	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	0	0	0
0	1	0	1	0	0	0	0	0	1	0	0	0	0
0	1	1	0	0	0	0	0	0	0	1	0	0	0
0	1	1	1	0	0	0	0	0	0	0	1	0	0
1	0	0	0	0	0	0	0	0	0	0	0	1	0
1	0	0	1	0	0	0	0	0	0	0	0	0	1
1	0	1	0	x	x	x	x	x	x	x	x	x	x
1	0	1	1	x	x	x	x	x	x	x	x	x	x
1	1	0	0	x	x	x	x	x	x	x	x	x	x
1	1	0	1	x	x	x	x	x	x	x	x	x	x
1	1	1	0	x	x	x	x	x	x	x	x	x	x
1	1	1	1	x	x	x	x	x	x	x	x	x	x

Don’t care terms can be chosen as 0 or 1. Depending on the problem, we can choose the don’t care term as 1 and use it to obtain a simpler Boolean expression



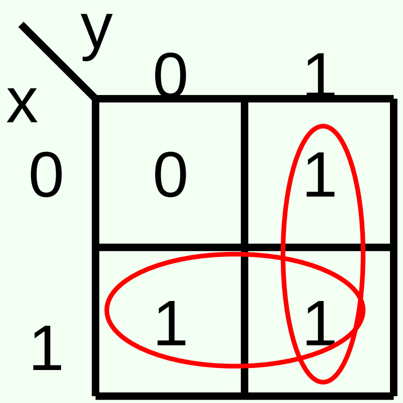
Y_3

ab \ cd	00	01	11	10
00	0	0	1	0
01	0	0	0	0
11	x	x	x	x
10	0	0	x	x

$$y_3 = \bar{b}.c.d$$

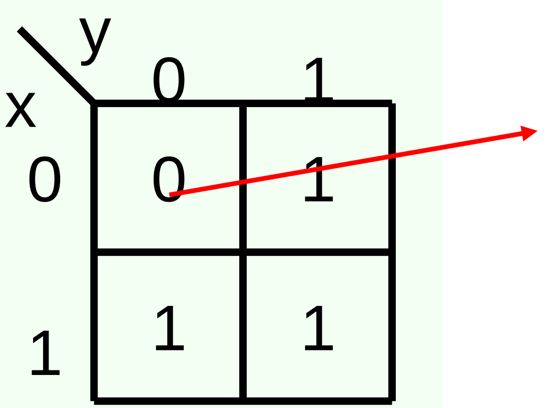
Don’t care terms should only be included in encirclements if it helps in obtaining a larger grouping or smaller number of groups.

Minimization of Product of Sum Terms using Kmap



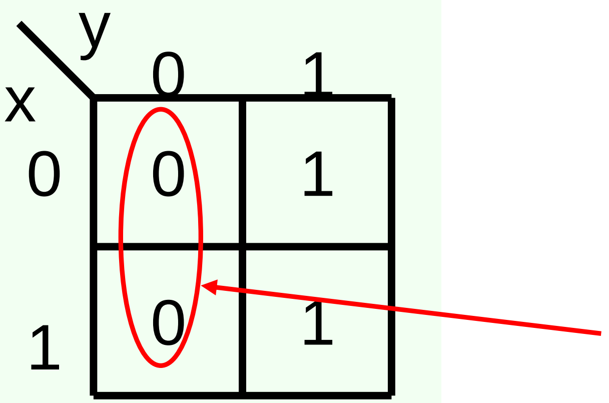
x y	0	1
0	0	1
1	1	1

$$f = x + \bar{x}.y + x.y$$
$$= x + (\bar{x} + x).y$$
$$= x + y$$



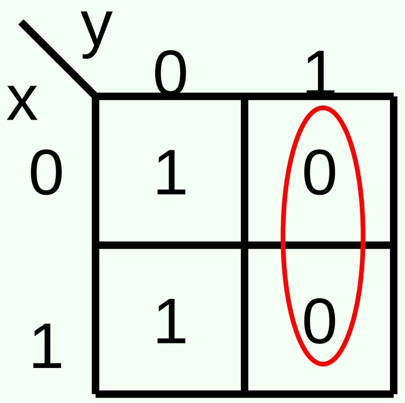
x y	0	1
0	0	1
1	1	1

$f = x + y$



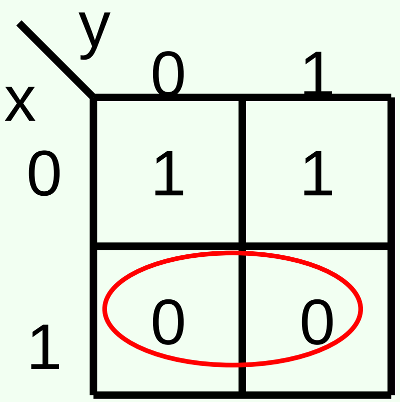
x y	0	1
0	0	1
1	0	1

$f = y$



x \ y	0	1
0	1	0
1	1	0

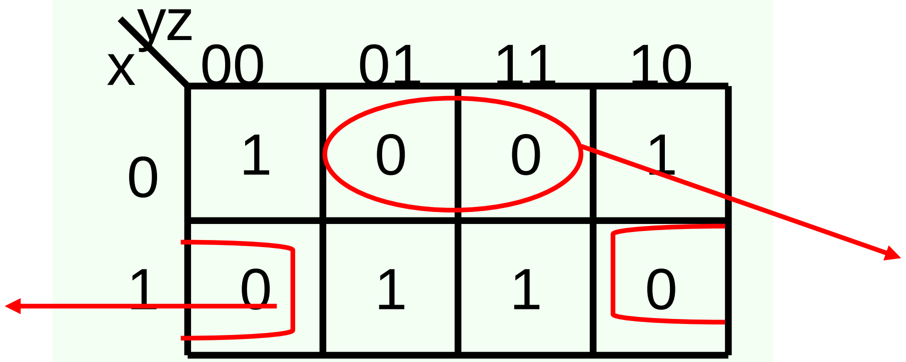
$\Rightarrow f = \overline{y}$



x \ y	0	1
0	1	1
1	0	0

$\Rightarrow f = \overline{x}$

$\overline{x}.+z$

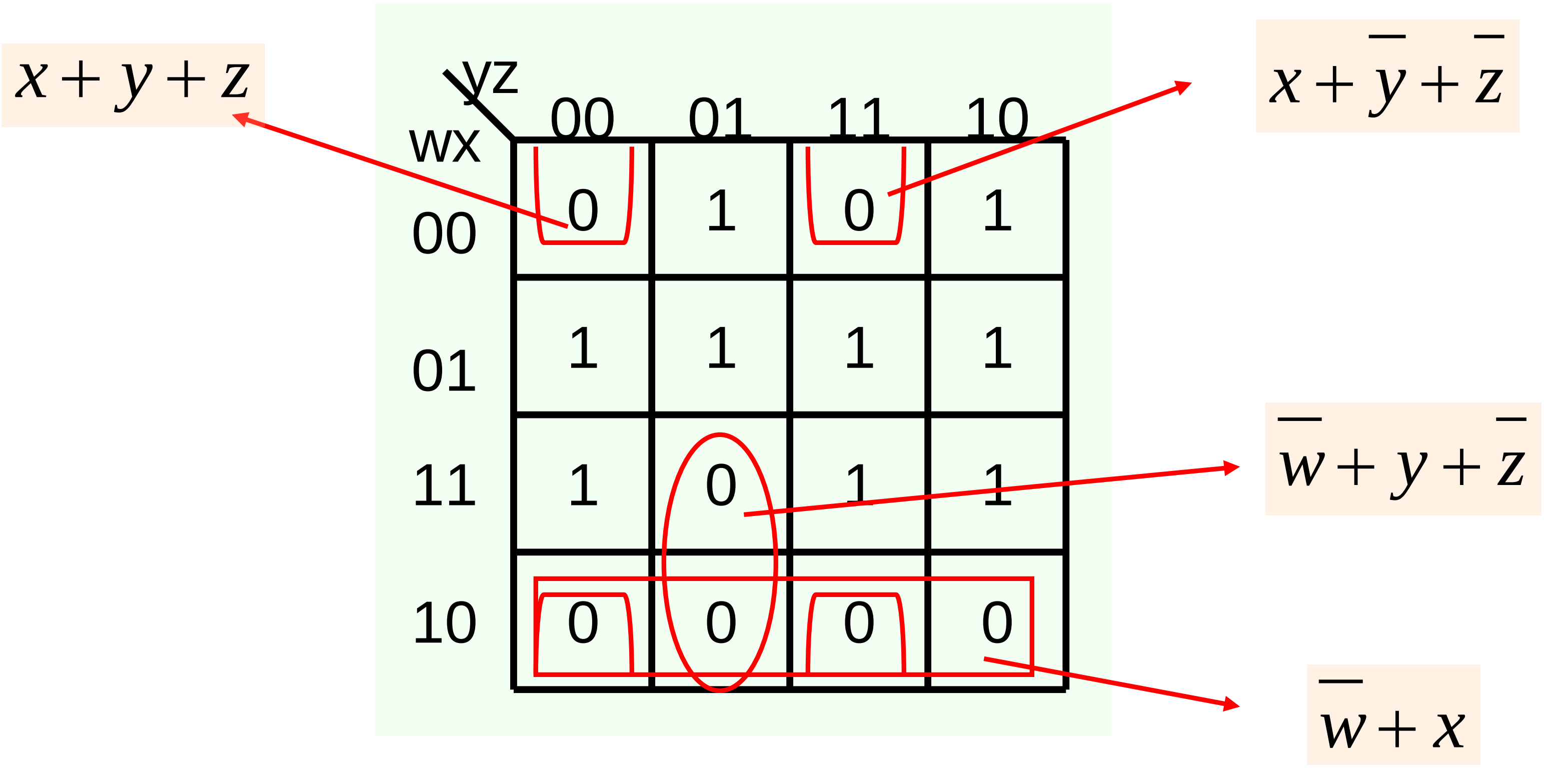


x \ yz	00	01	11	10
0	1	0	0	1
1	0	1	1	0

$x+\overline{z}$

$f =(\overline{x} .+z).(x+\overline{z})$

$\Rightarrow f = \overline{x} .\overline{z} + x.z$



wx \ yz	00	01	11	10
00	0	1	0	1
01	1	1	1	1
11	1	0	1	1
10	0	0	0	0

$x+y+z$

$x+\overline{y}+\overline{z}$

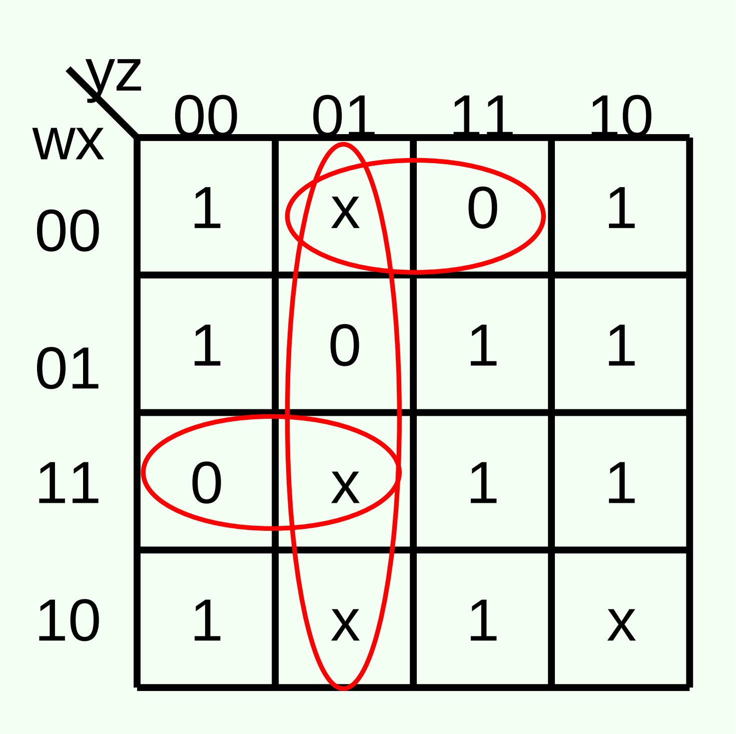
$\overline{w}+y+\overline{z}$

$\overline{w}+x$

$$f = (x+y+z).(x+\overline{y}+\overline{z}).(\overline{w}+y+\overline{z}).(\overline{w}+x)$$

Example

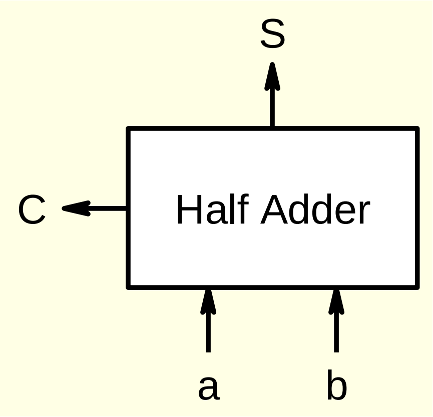
Obtain the minimized PoS by suitably using don’t care terms



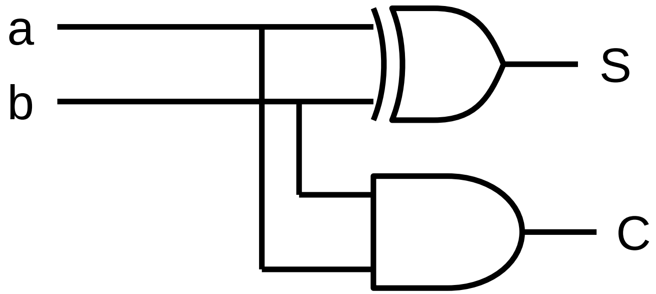
wx \ yz	00	01	11	10
00	1	x	0	1
01	1	0	1	1
11	0	x	1	1
10	1	x	1	x

$$f = (x + w + \bar{z}).(\bar{x} + \bar{w} + y).(y + \bar{z})$$

Adder/Subtractor



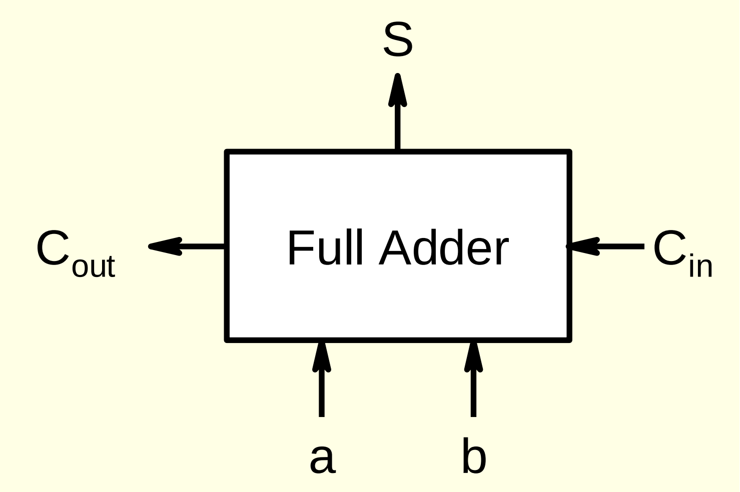
a	b	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



$$S = \overline{a}.b + a.\overline{b}; C = a.b$$

a	b	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$\begin{array}{r} 1 \\ 1\,1\,1 \\ \underline{1\,1\,0} \\ \underline{1\,1\,1\,1} \end{array}$$



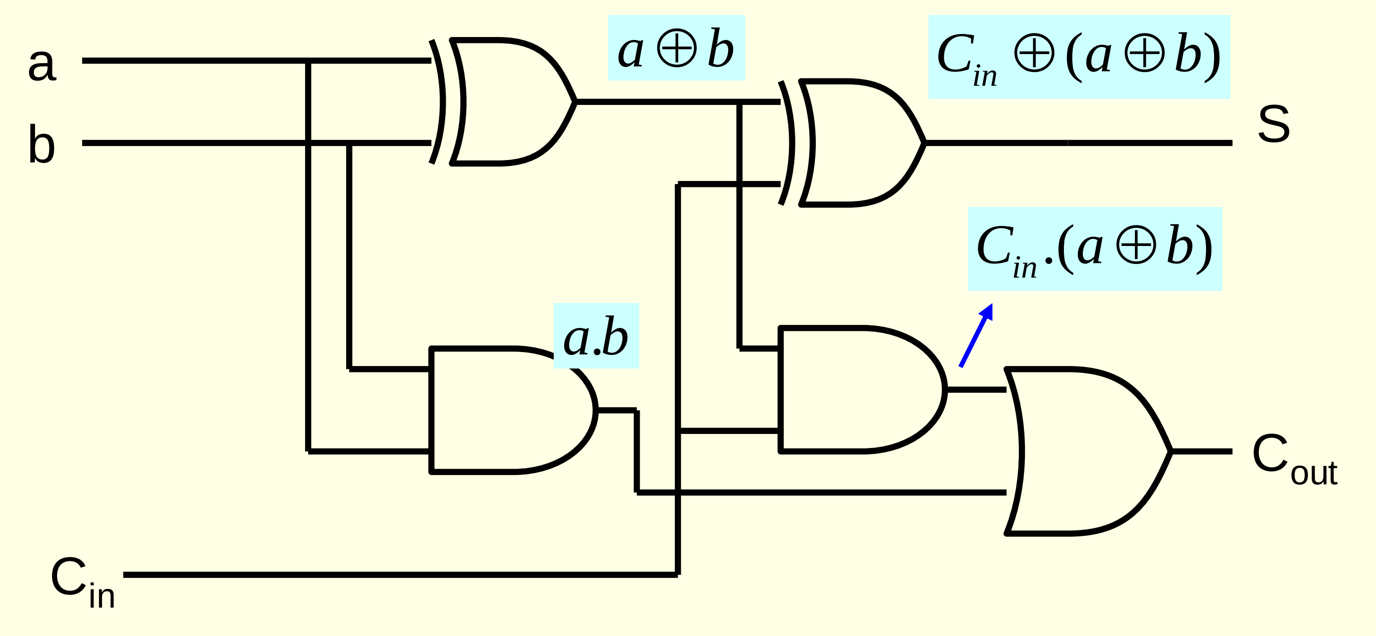
$$S = \overline{a}.\overline{b}.c_{in} + \overline{a}.b.\overline{c_{in}} + a.\overline{b}.\overline{c_{in}} + a.b.c_{in}; C_{out} = a.b + a.c_{in} + b.c_{in}$$

$$S = \bar{a}.\bar{b}.c_{in} + \bar{a}.b.\overline{c_{in}} + a.\bar{b}.\overline{c_{in}} + a.b.c_{in}$$

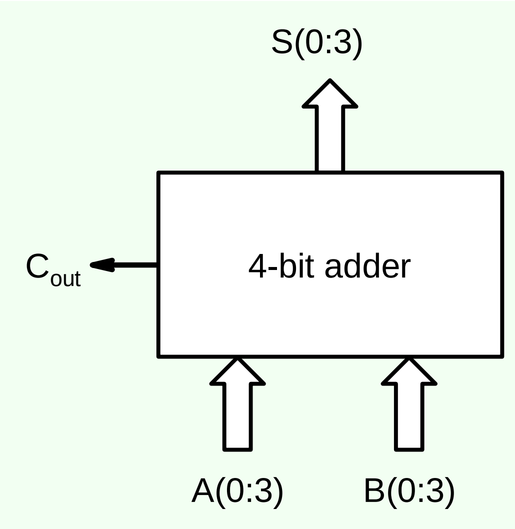
$$S = C_{in} \oplus (a \oplus b)$$

$$C_{out} = a.b + a.C_{in} + b.C_{in}$$

$$C_{out} = C_{in}(a.\bar{b} + \bar{a}.b) + a.b = C_{in}.(a \oplus b) + a.b$$

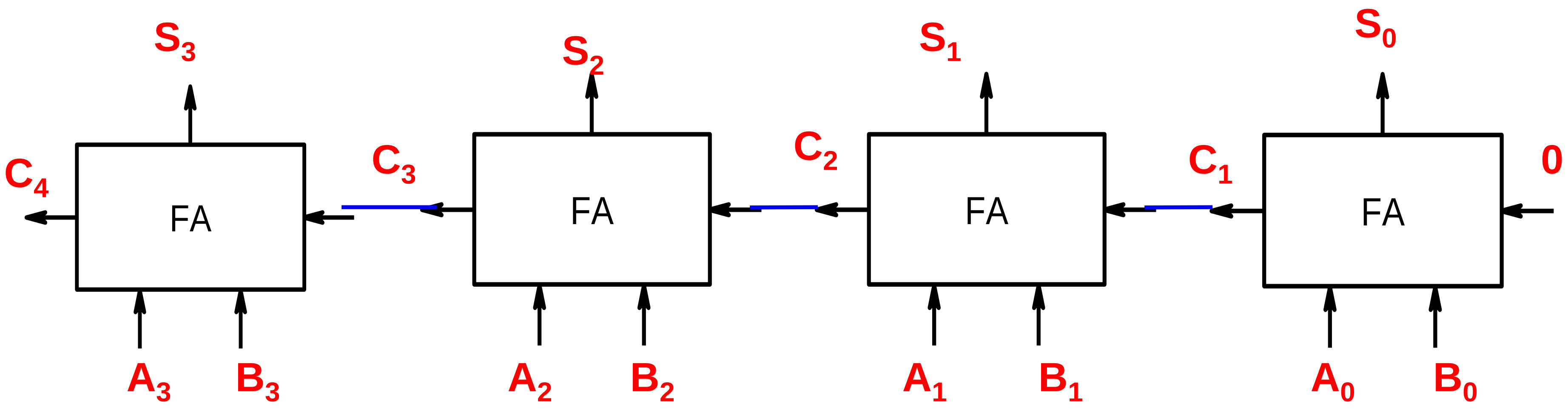


4-bit Adder



$A_3A_2A_1A_0$	$B_3B_2B_1B_0$	$S_3S_2S_1S_0$	C_{out}
0000	0000	0000	1
0000	0001	0001	0
0001	0000	0001	0
⋮	⋮	⋮	⋮

$$\begin{array}{ccccc} & C_3 & C_2 & C_1 & \\ & A_3 & A_2 & A_1 & A_0 \\ & B_3 & B_2 & B_1 & B_0 \\ \hline C_4 & S_3 & S_2 & S_1 & S_0 \\ \hline \end{array}$$



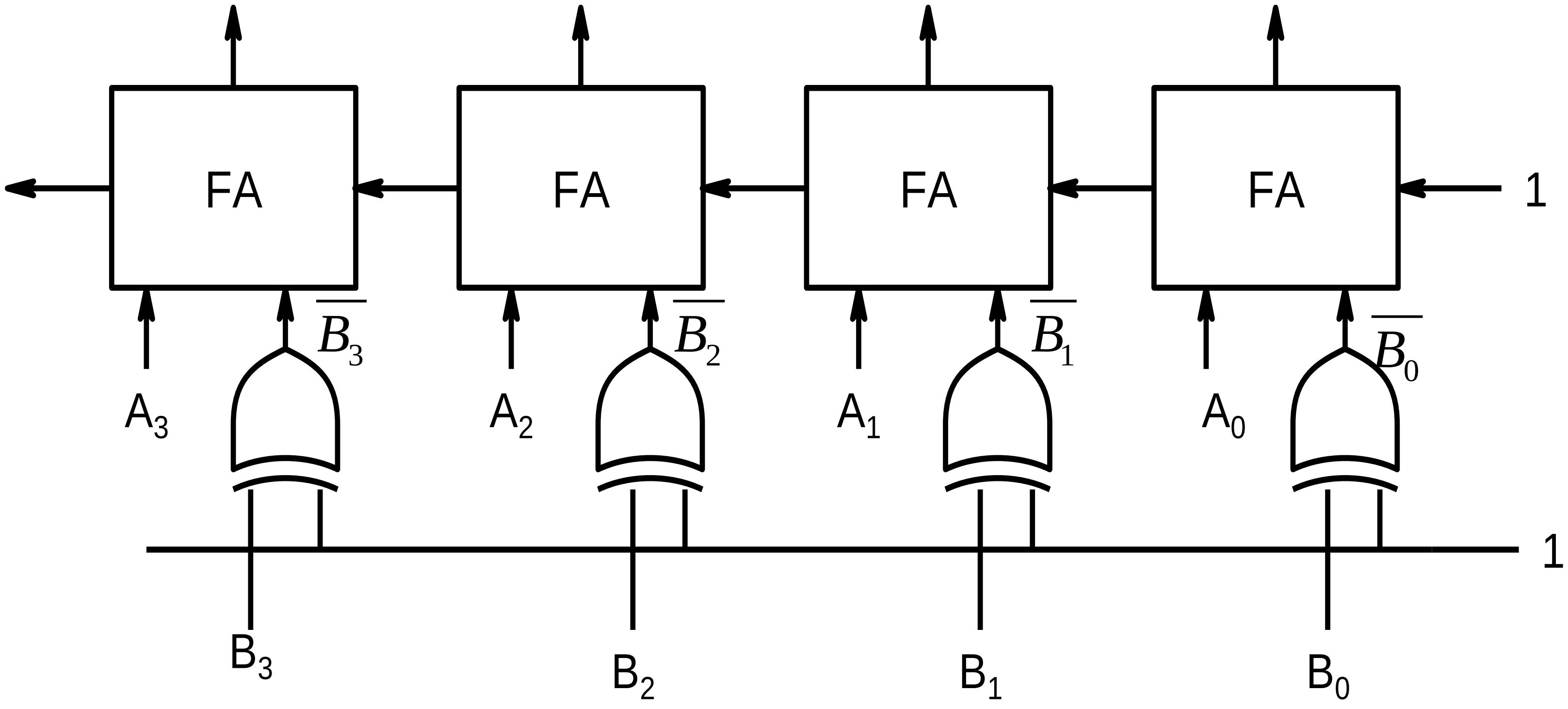
Ripple Carry Adder (20 gate circuit)

Subtraction

A - B = A + 2's complement of B

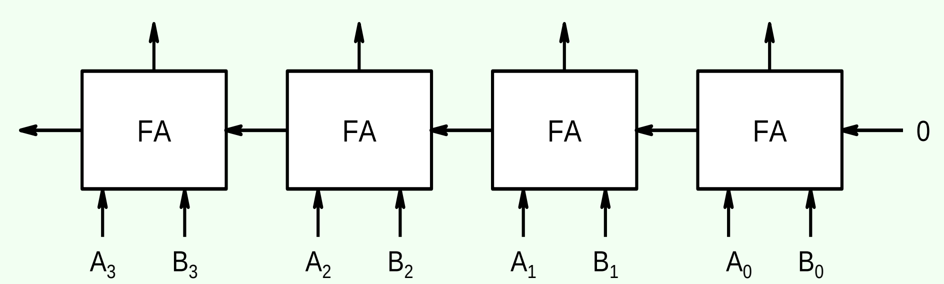
A - B = A + 1's complement of B+1

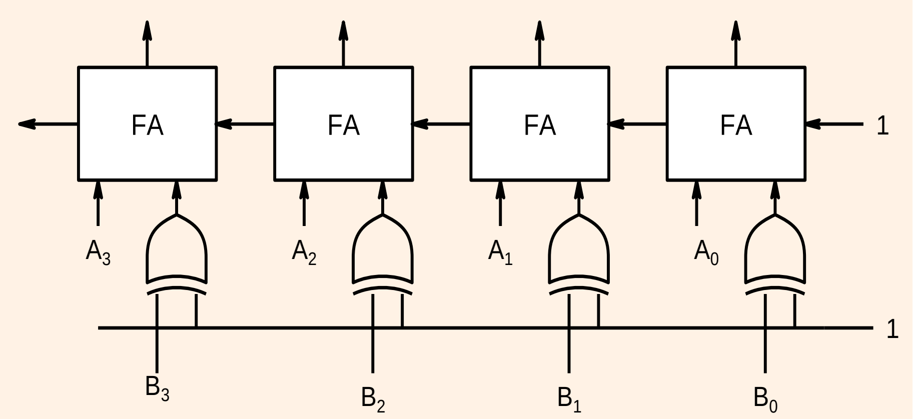
$$\boxed{A - B = A + \overline{B}+1}$$

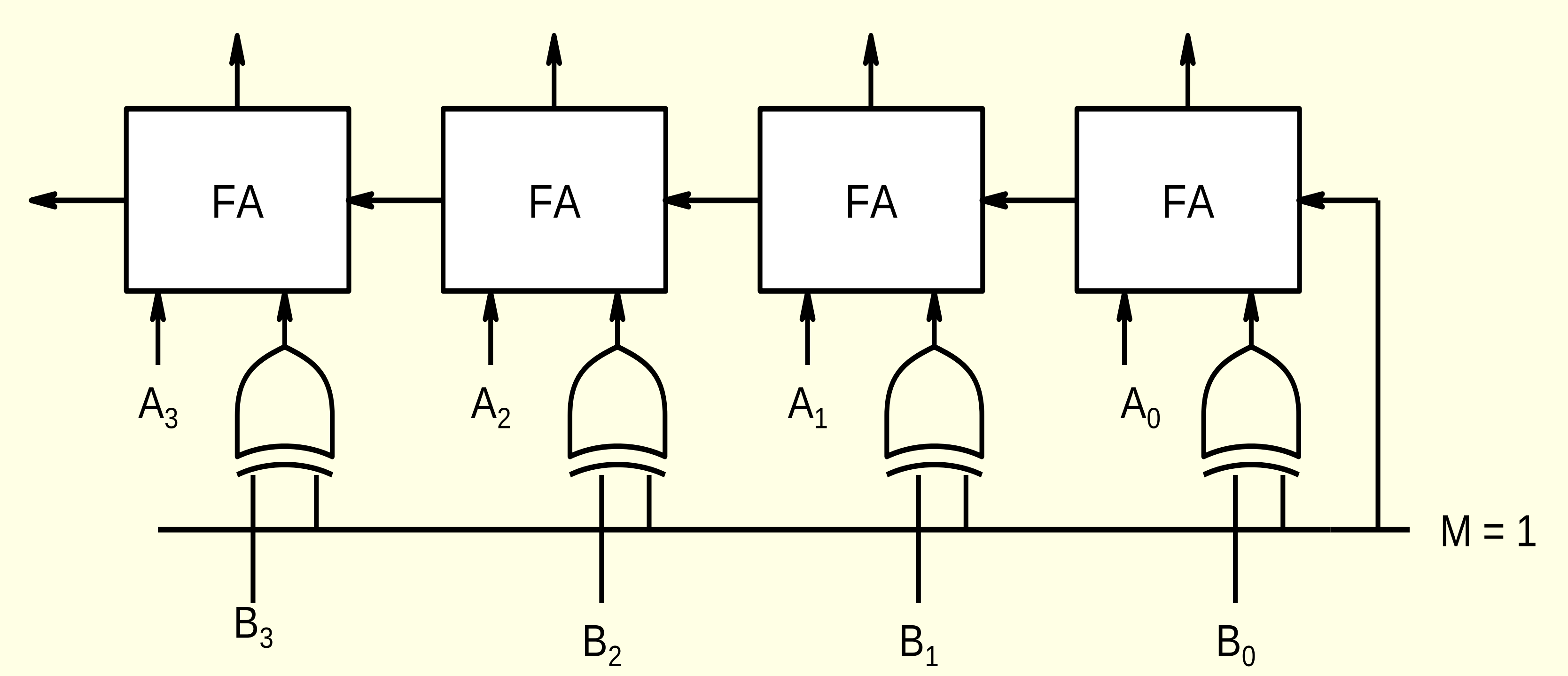


One needs add a circuit for predicting errors resulting from overflow

Adder/Subtractor

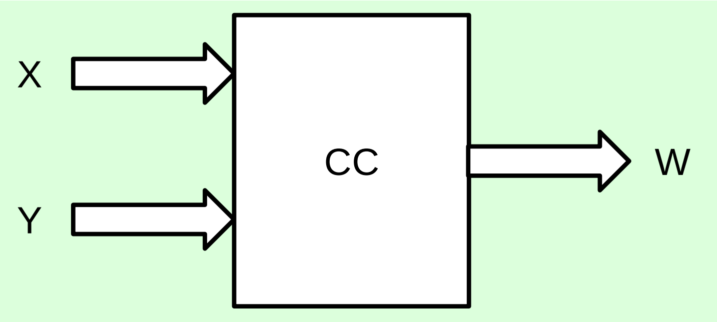






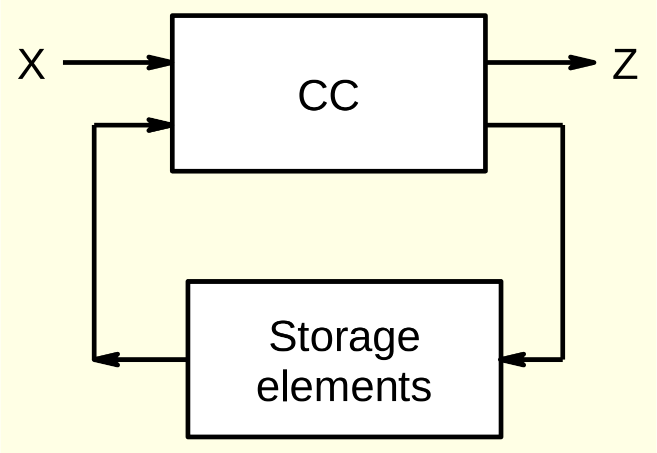
Digital Circuits

Combinational Circuits



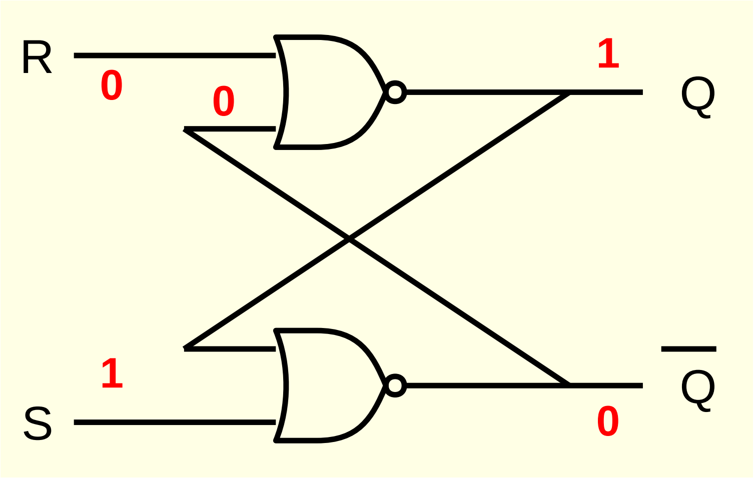
Output is determined by current values of inputs only.

Sequential Circuits



Output is determined in general by current values of inputs and past values of inputs/outputs as well.

NOR SR Latch

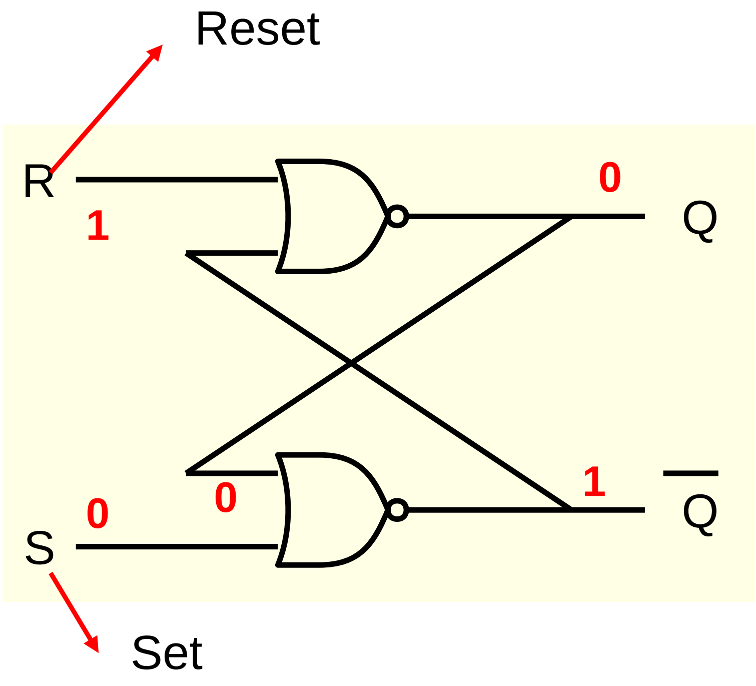


$Q = 1; \overline{Q} = 0 \quad \text{Set State}$

$Q = 0; \overline{Q} = 1 \quad \text{Re set State}$

S	R	Q	$\overline{Q}$	State
1	0	1	0	SET

NOR SR Latch

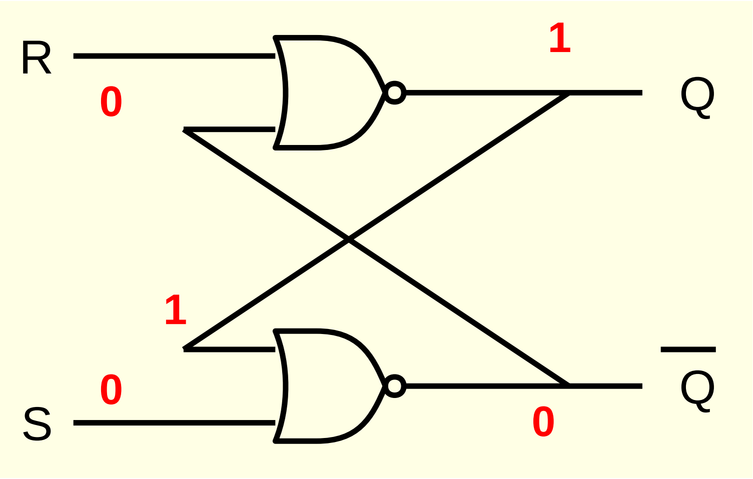


$Q = 1; \overline{Q} = 0$ Set State

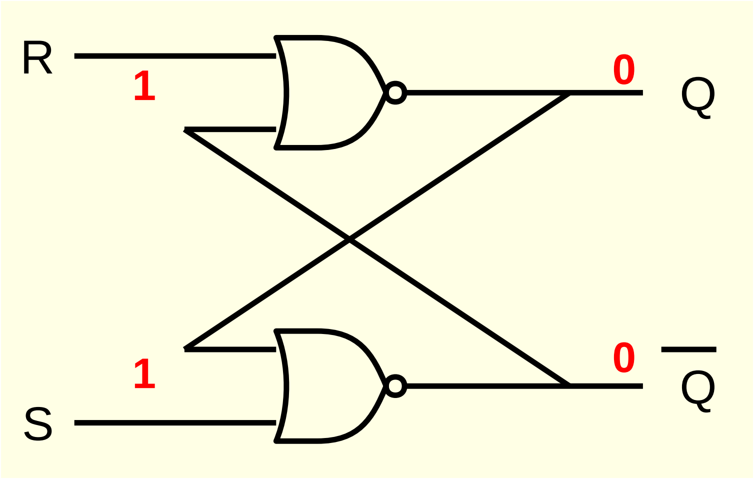
$Q = 0; \overline{Q} = 1$ Reset State

S	R	Q	$\overline{Q}$	State
1	0	1	0	SET
0	1	0	1	RESET

HOLD State

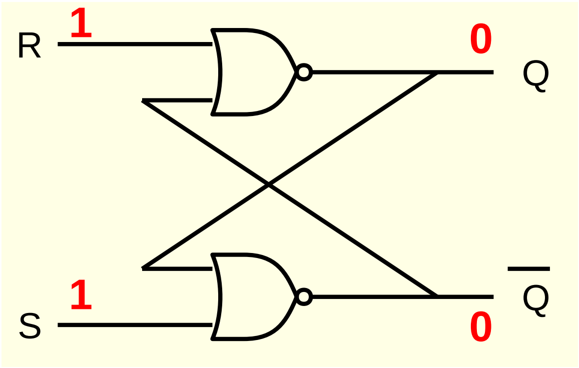


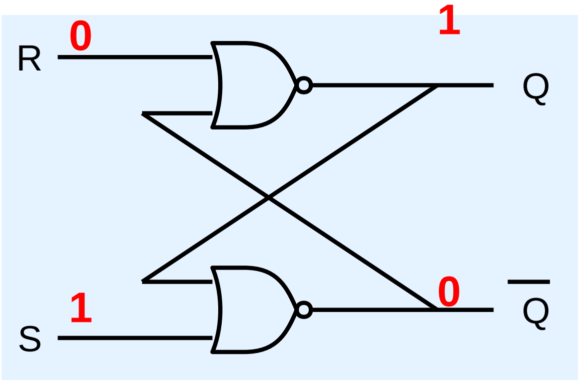
S	R	Q	$\overline{Q}$	State
1	0	1	0	SET
0	1	0	1	RESET
0	0	Q	$\overline{Q}$	HOLD
1	1	0	0	INVALID

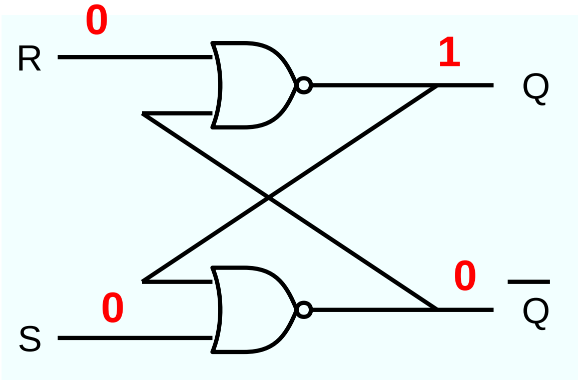


Both the outputs are well defined and 0. the first problem is that we do not get complementary output.

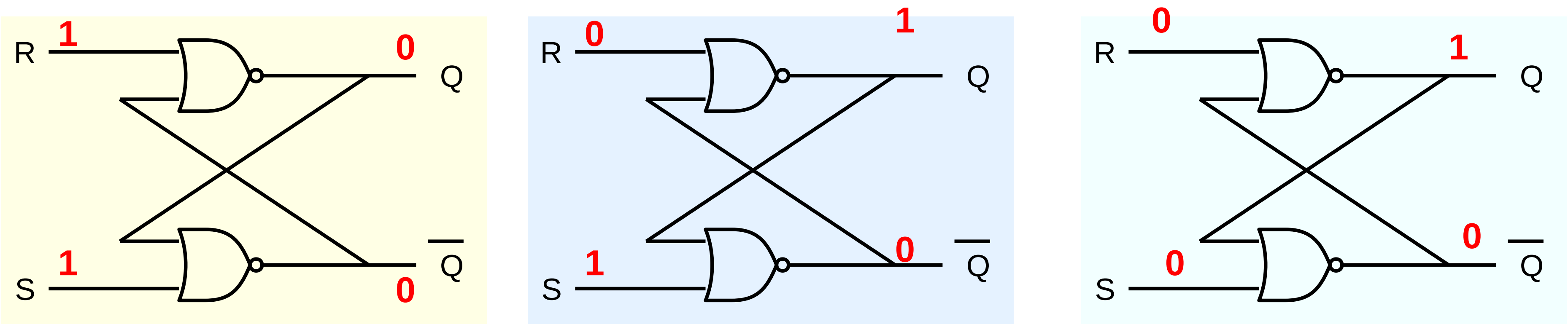
A more serious problem occurs when we switch the latch to the hold state by changing RS from 11 → 00 . Suppose the inputs do not change simultaneously and we get the situation 11 → 01* → 00





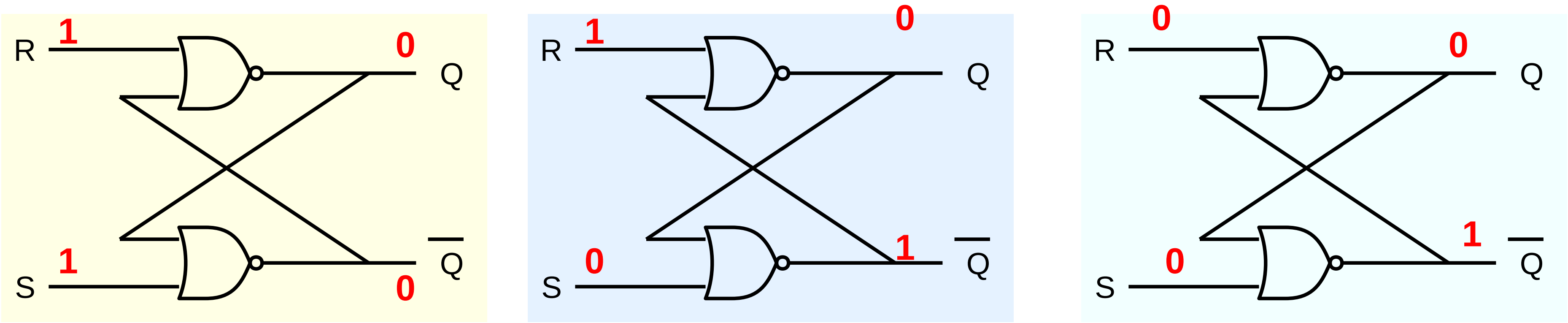


Q = 1



Q = 1

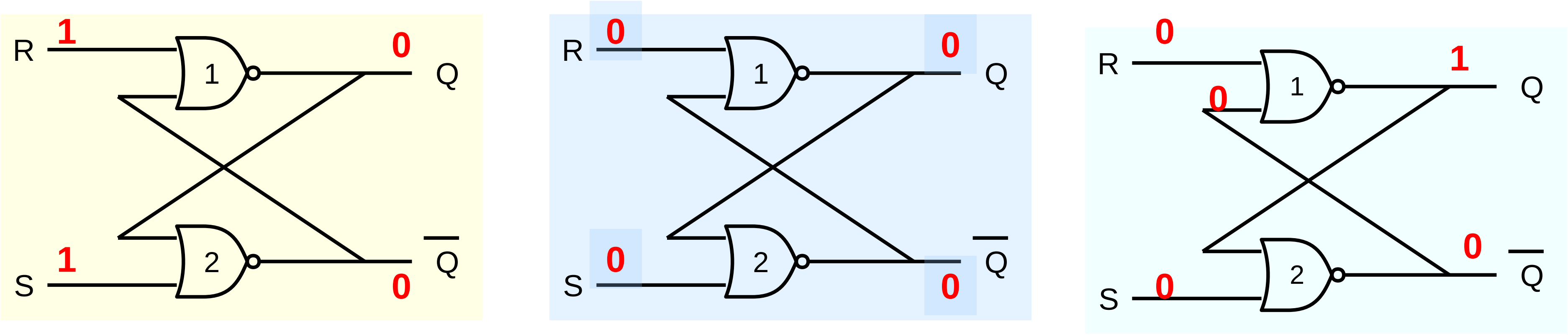
Suppose the inputs change as RS = 11 → 10* → 00



Q = 0

So although output is well defined when we apply RS = 11, it becomes unpredictable once we switch the latch to hold state by applying RS = 00. That is why RS = 11 is not used as an input combination.

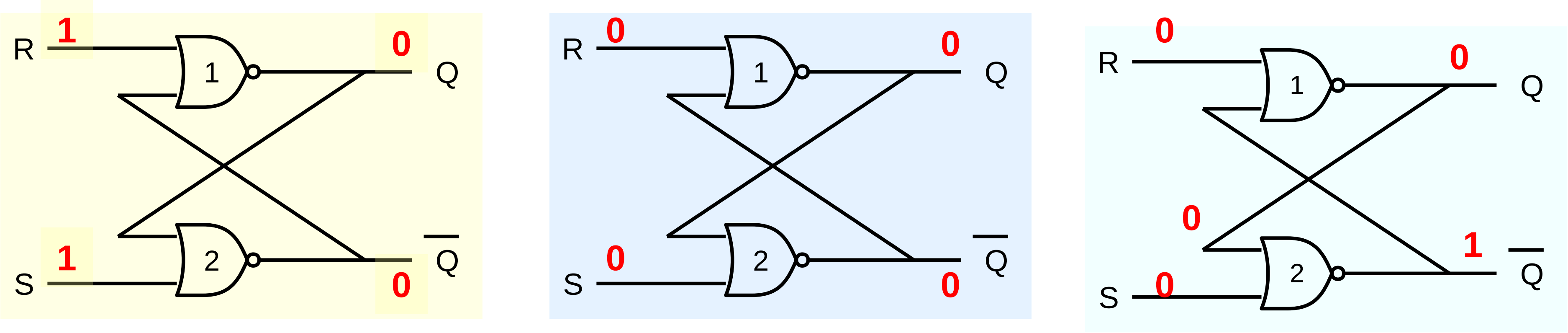
The error can occur also due to unequal gate delays.



Suppose gate-1 is faster

Q = 1

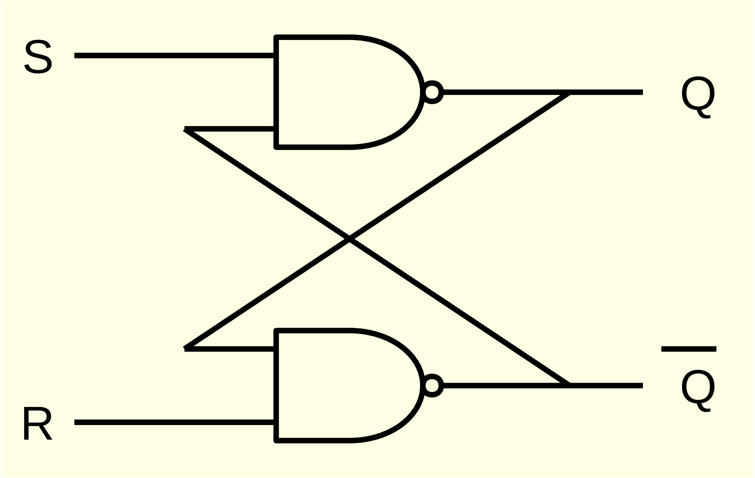
On the other hand suppose that gate-2 is faster.



Again the output is unpredictable in general

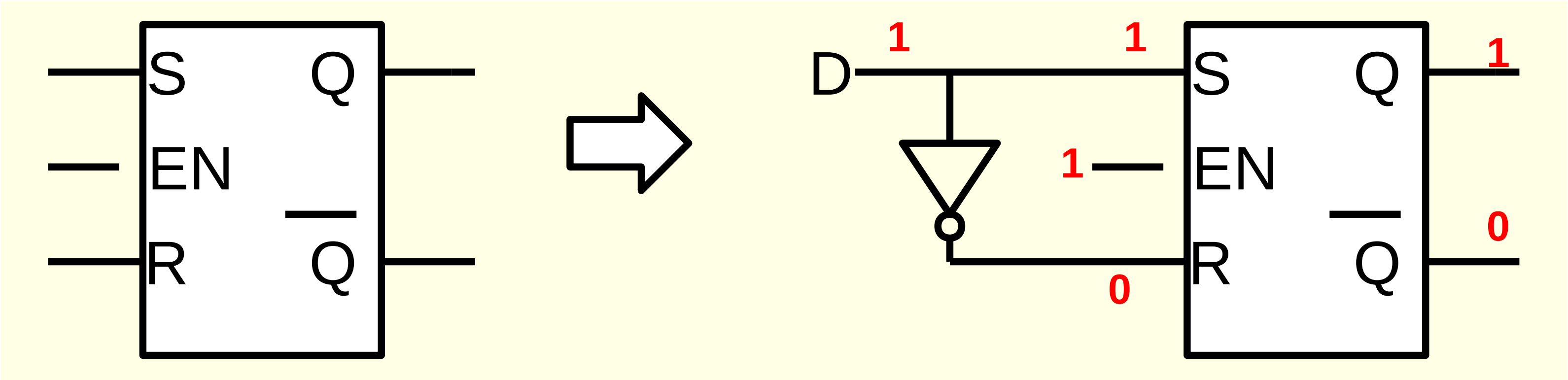
Q = 0

NAND Latch

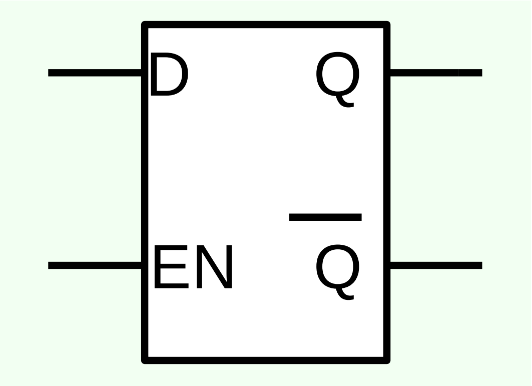


S	R	Q	$\overline{Q}$	State
0	1	1	0	SET
1	0	0	1	RESET
1	1	Q	$\overline{Q}$	HOLD
0	0	1	1	INVALID

D latch

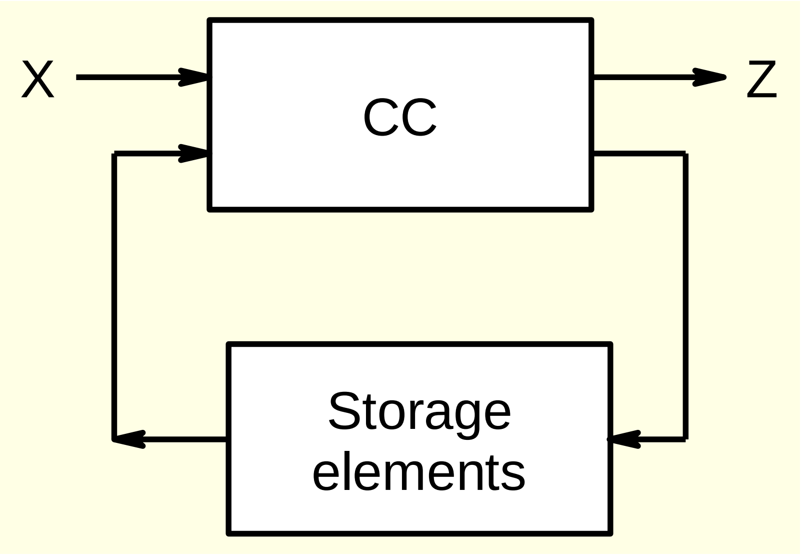


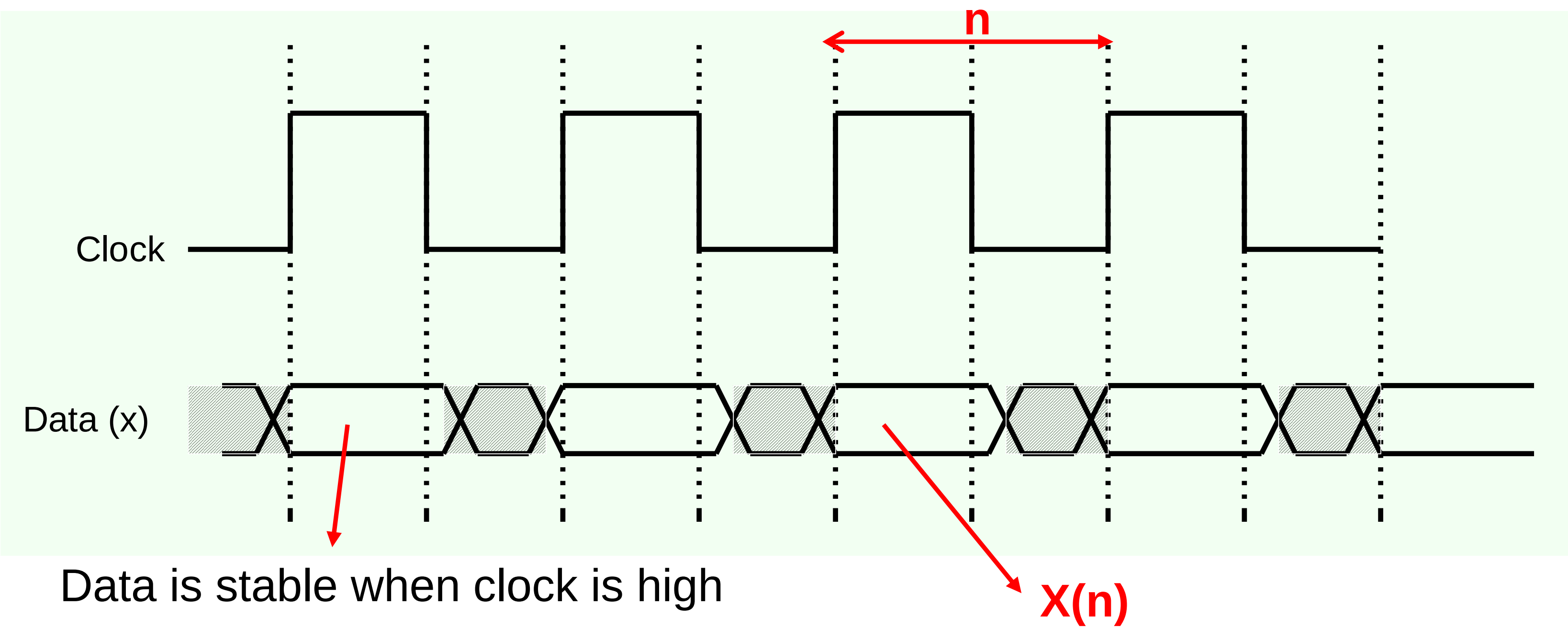
Enable	S	R	Q	$\overline{Q}$	State
0	x	x	Q	$\overline{Q}$	Hold
1	1	0	1	0	Set
1	0	1	0	1	Reset
1	0	0	Q	$\overline{Q}$	Hold
1	1	1	0	0	Invalid



If EN = 1 then Q = D otherwise the latch is in Hold state

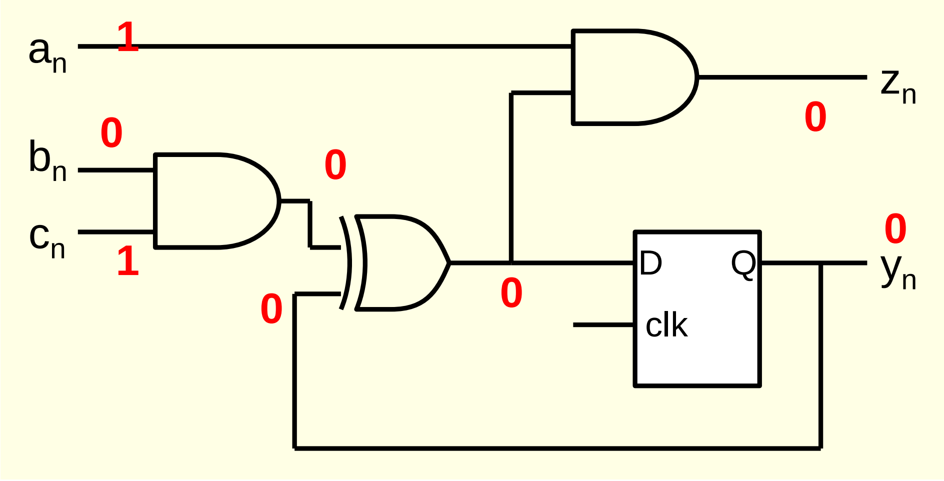
Synchronous Sequential Circuits

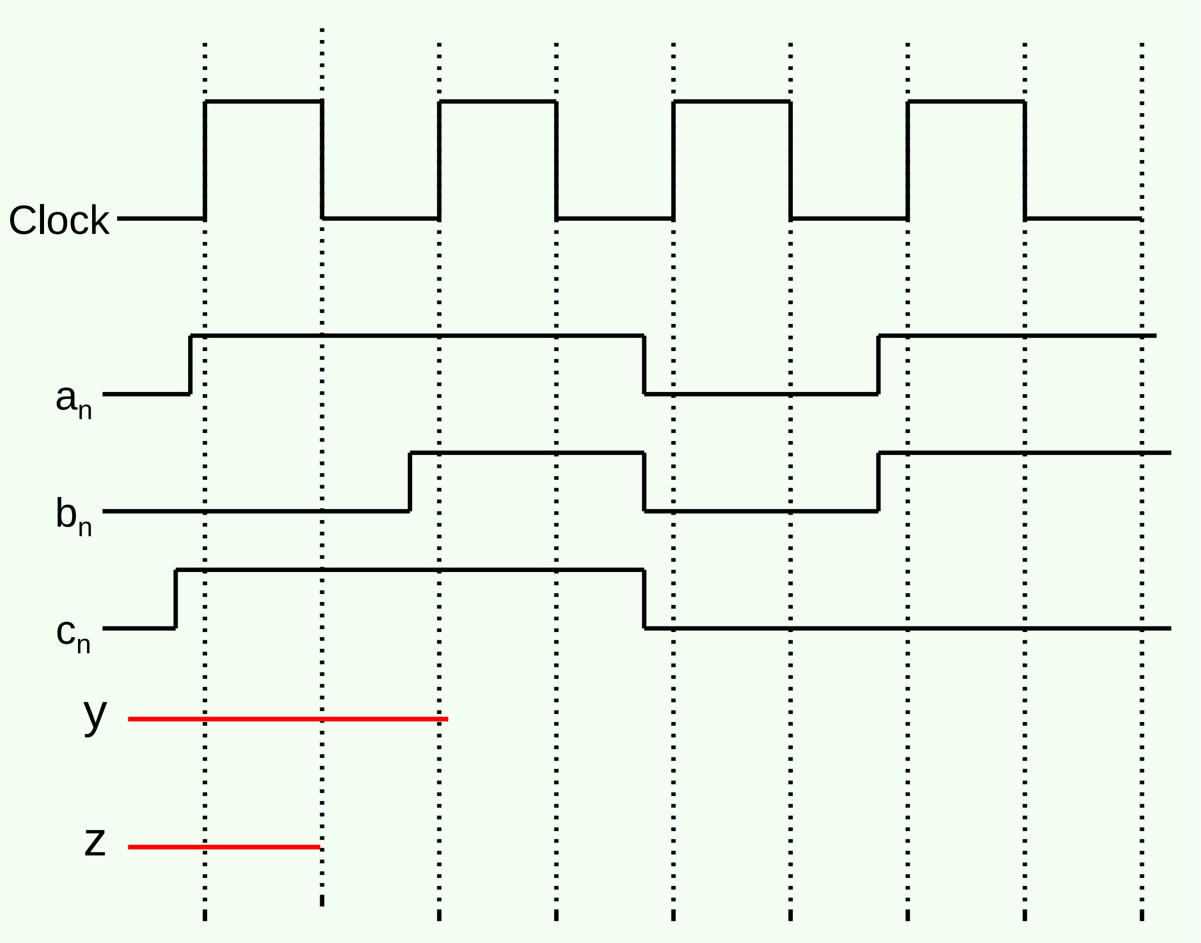




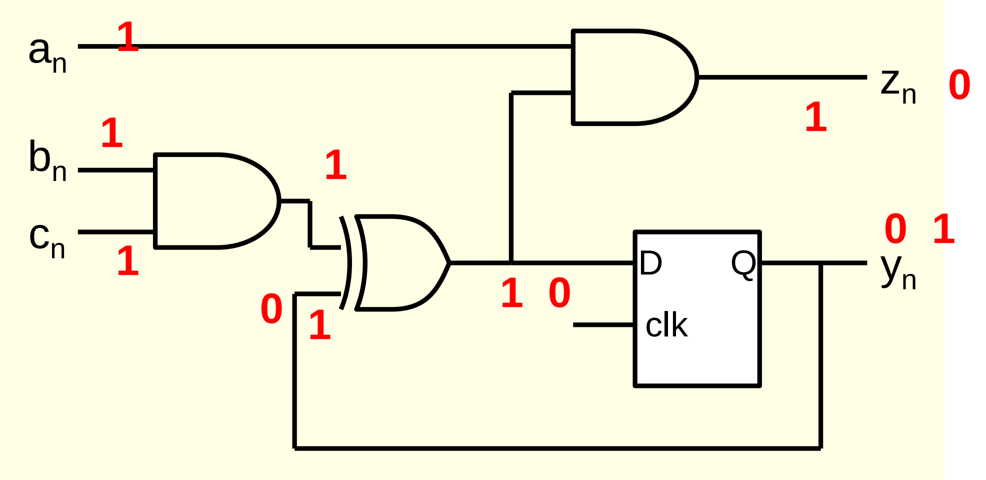
Data is stable when clock is high

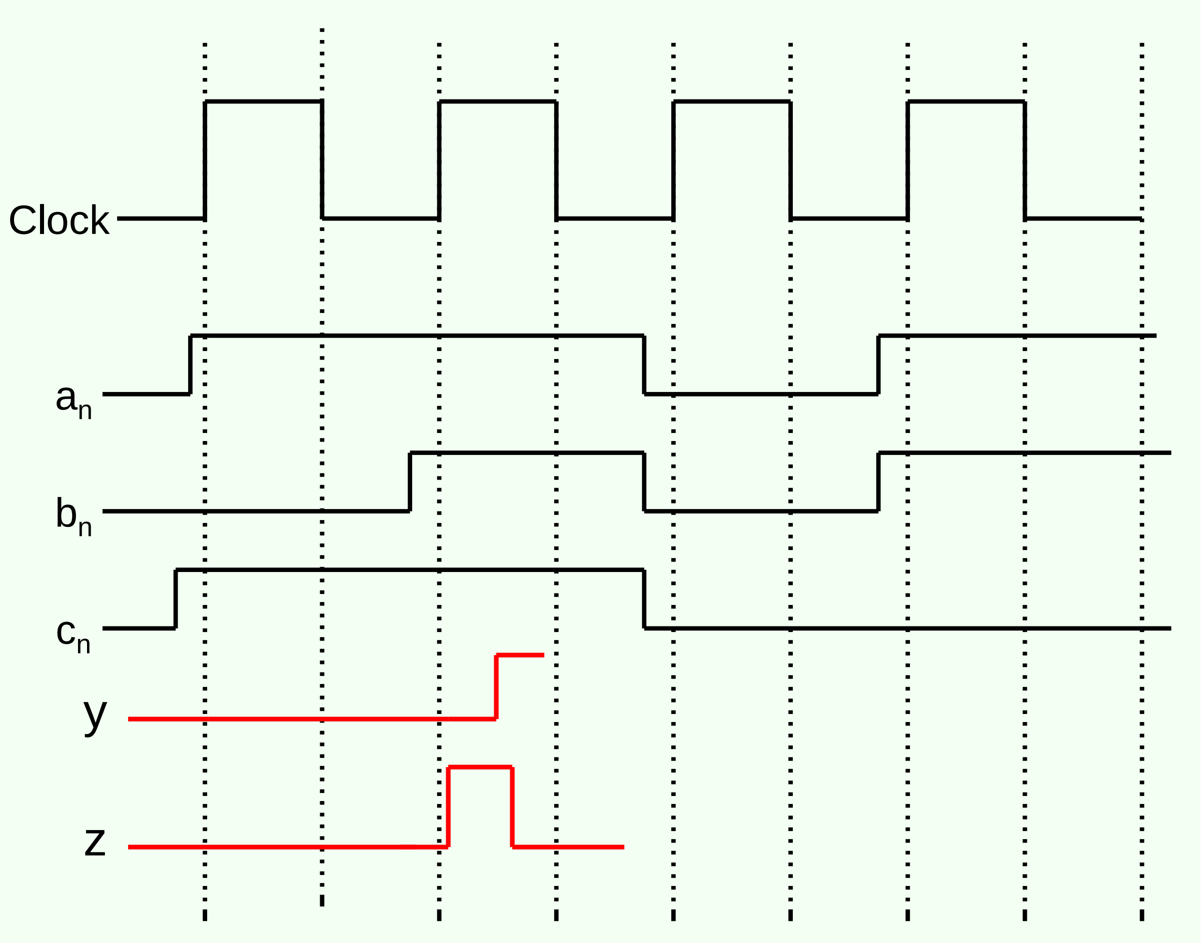
Example



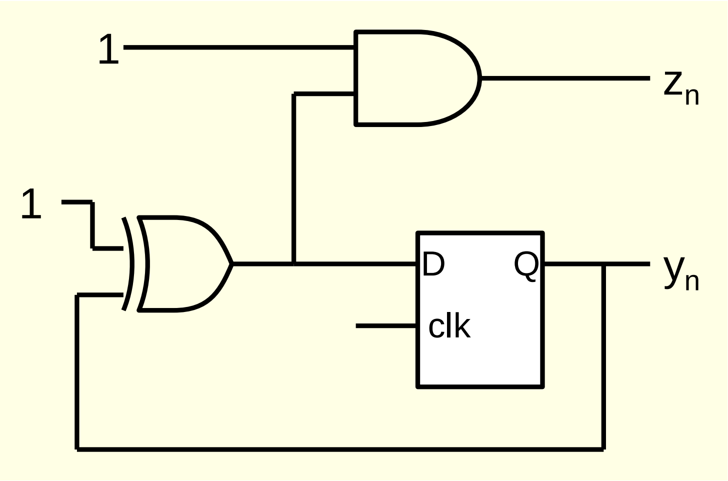


Example

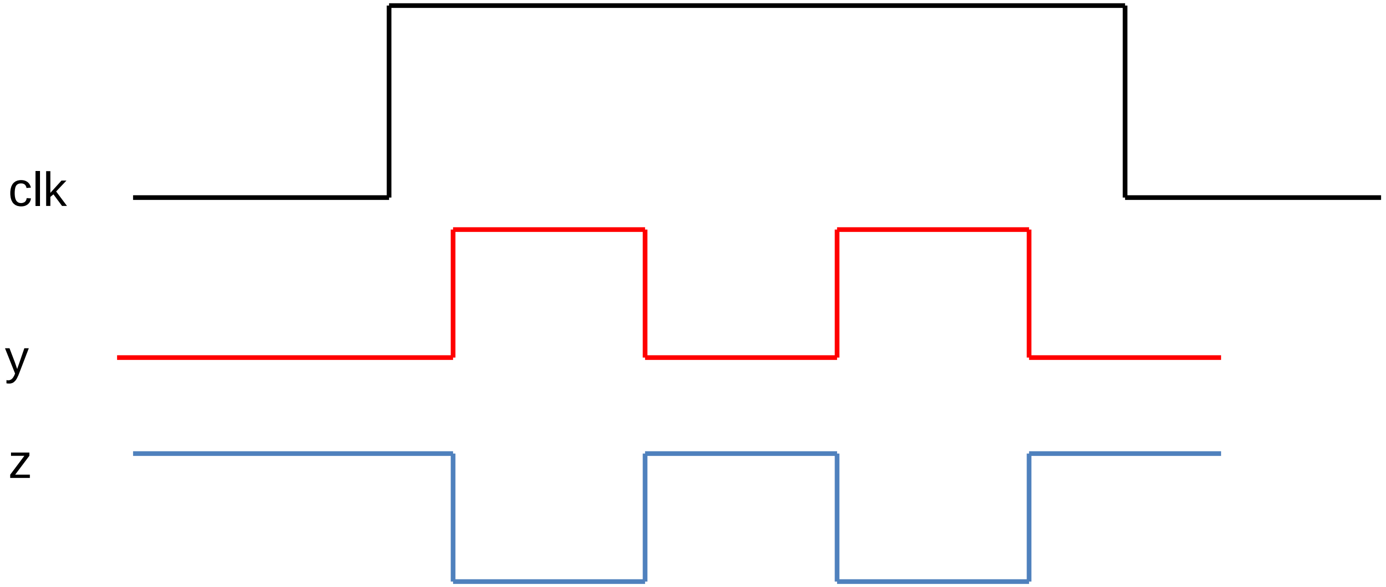




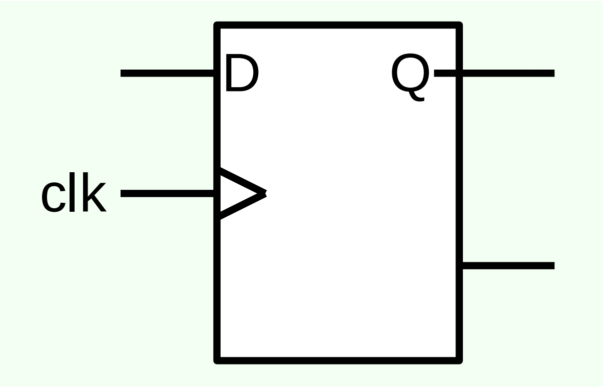
Problem with Latch

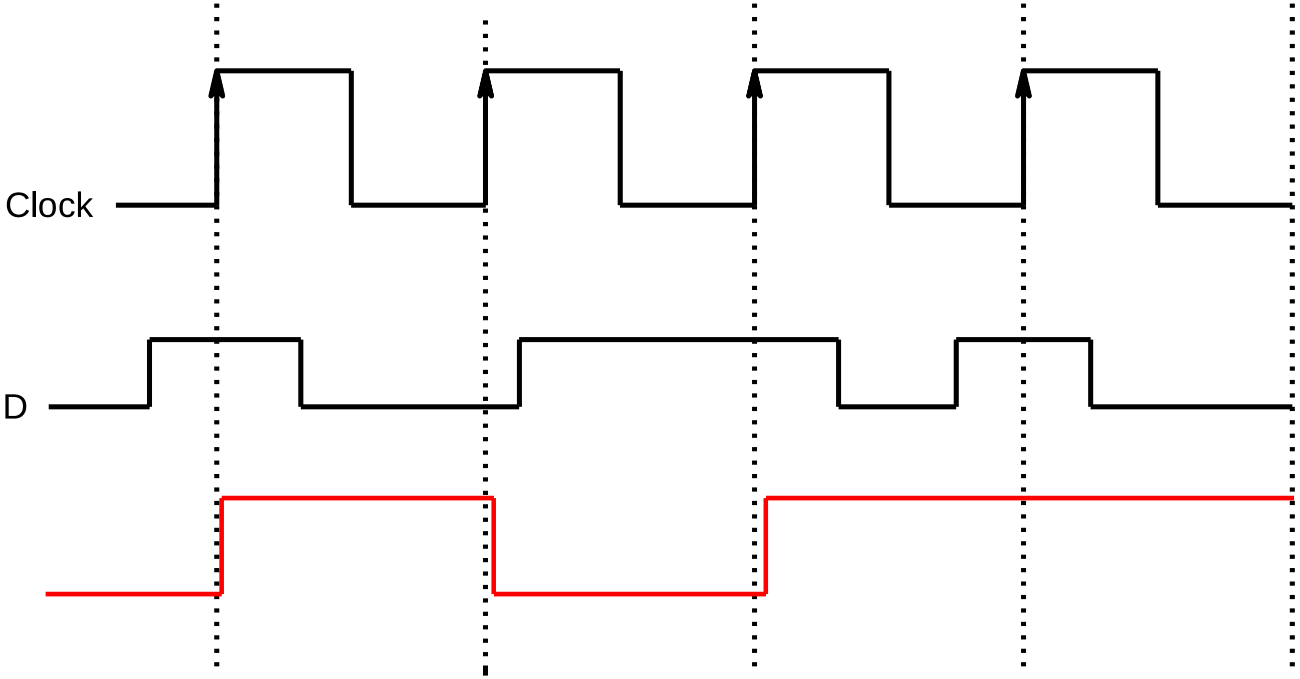


Circuits are designed with the idea there would be single change in output or memory state in single clock cycle.



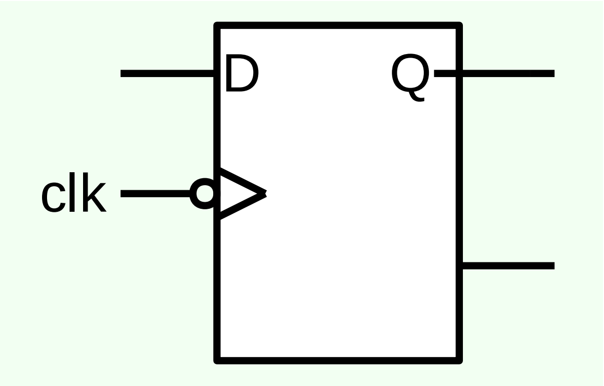
Edge Triggered Latch or Flip-flop

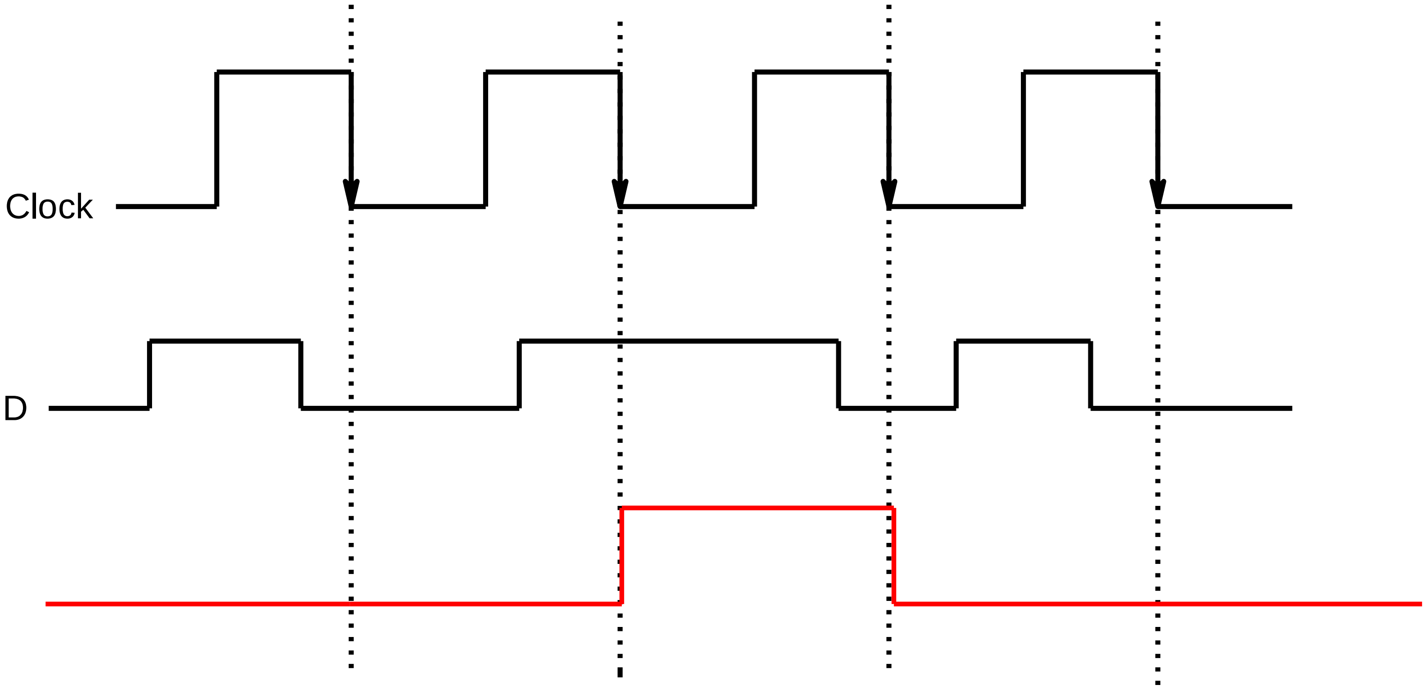




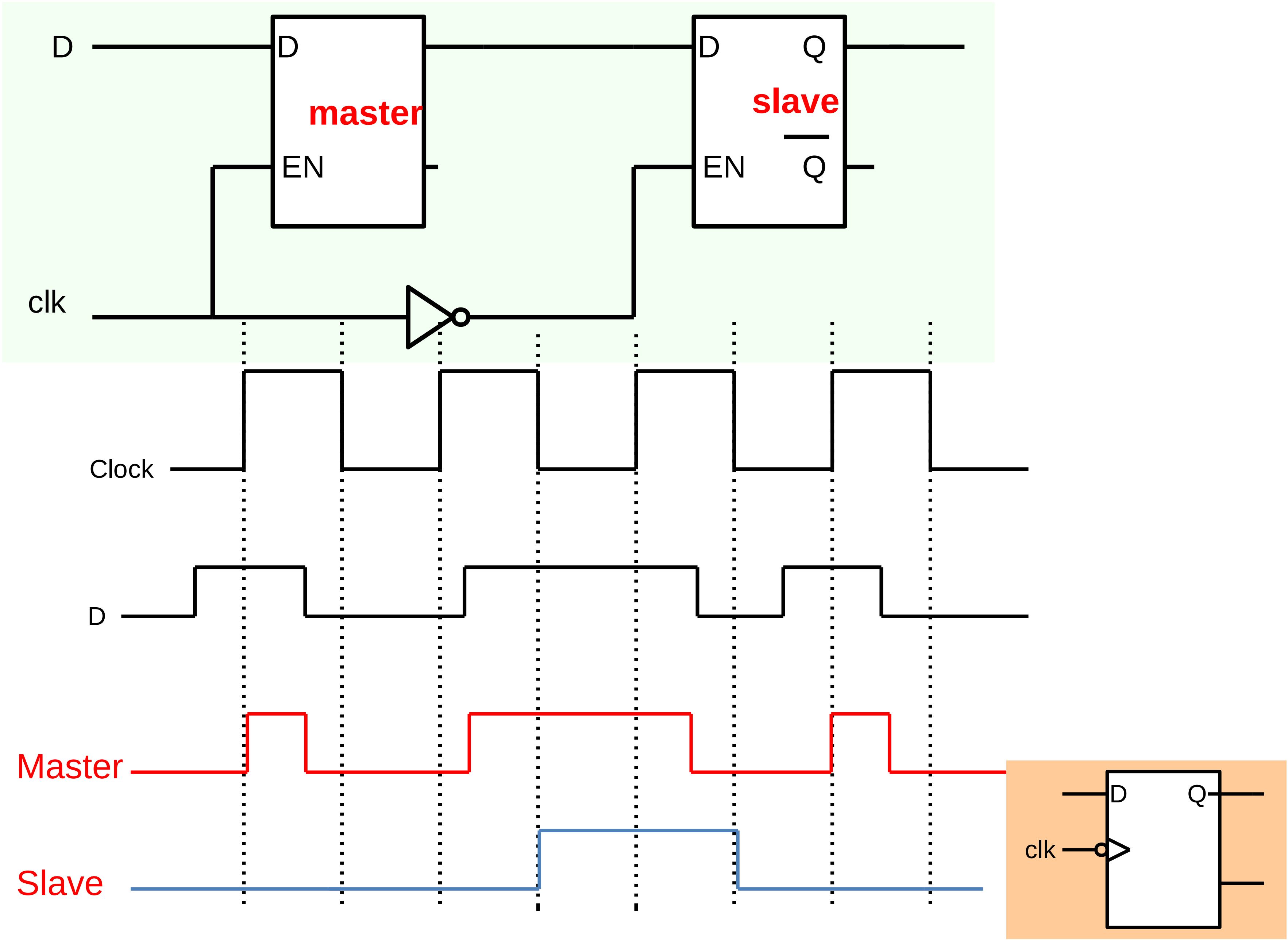
Positive edge triggered flipflop

Negative Edge Triggered Latch or Flip-flop



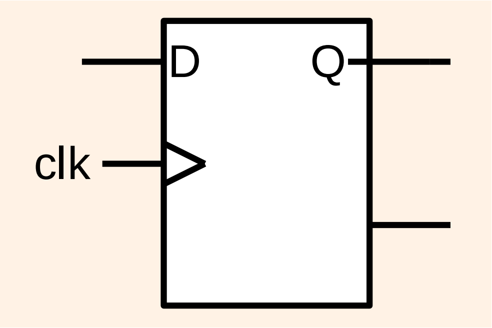


Master-Slave D Flip-flop



Characteristic table

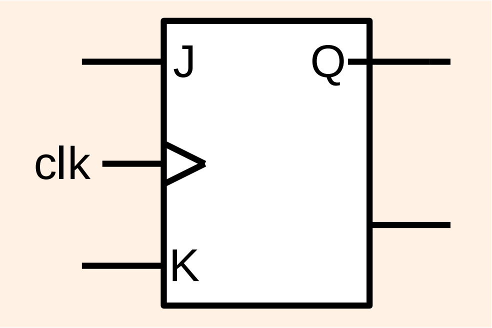
Given a input and the present state of the flip-flop, what is the next state of the flip-flop



Inputs (D)	Q(t+1)
0	0
1	1

$$Q(t+1) = D$$

JK Flip-flop

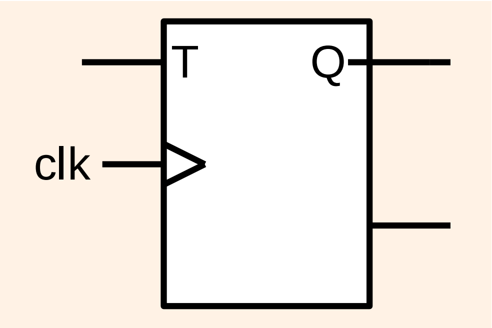


Inputs J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	$\overline{Q(t)}$

$$Q(t+1) = \overline{Q(t)}.J + Q(t).\overline{K}$$

→**Characteristic equation**

Toggle or T Flip-flop



Inputs (T)	Q(t+1)
0	$Q(t)$
1	$\overline{Q(t)}$

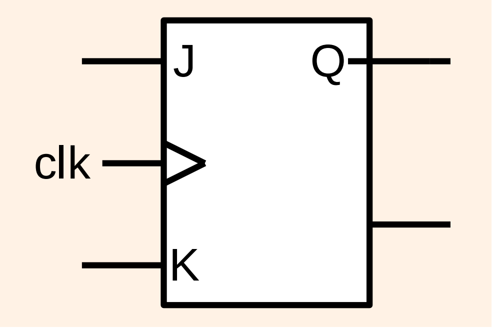
$$Q(t+1) = \overline{Q(t)}.T + Q(t).\overline{T}$$

Excitation Table

What inputs are required to effect a particular state change

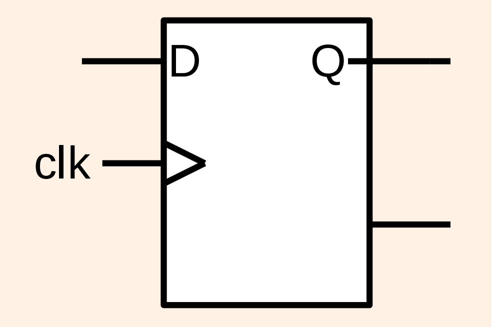
		Inputs
Q(t)	Q(t+1)	T
0	0	0
0	1	1
1	0	1
1	1	0

Excitation Table



J	K	Q(t+1)
0	0	Q(t)
0	1	0
1	0	1
1	1	$\overline{Q(t)}$

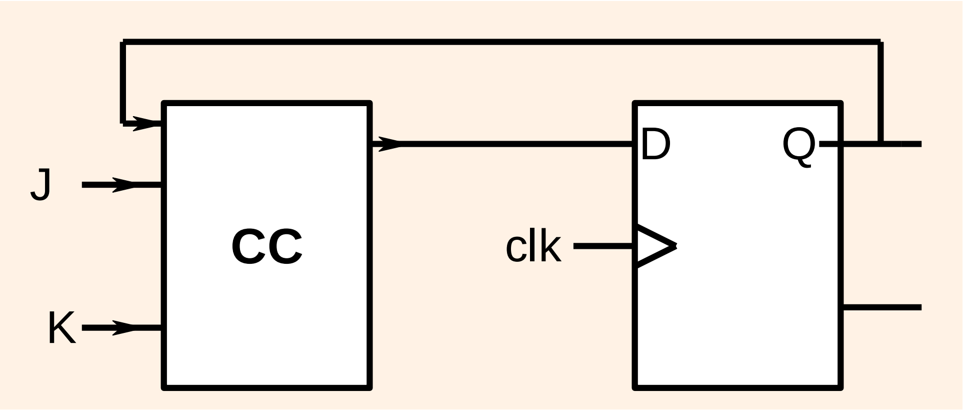
		Inputs	
Q(t)	Q(t+1)	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0



D	Q(t+1)
0	0
1	1

		Inputs
Q(t)	Q(t+1)	D
0	0	0
0	1	1
1	0	0
1	1	1

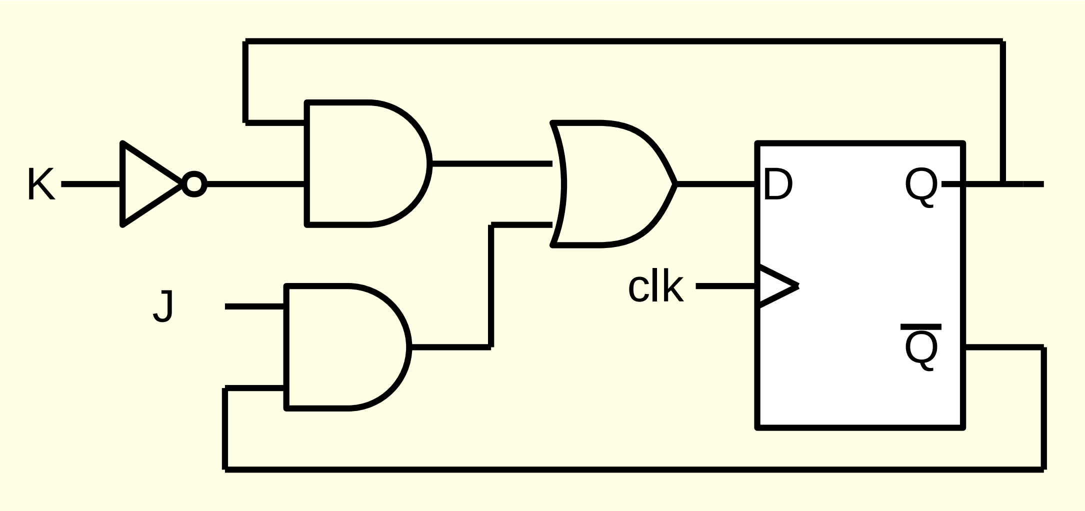
Convert a D FF to JK FF



J	K	Q(t+1)	D
0	0	Q(t)	**Q(t)**
0	1	0	**0**
1	0	1	**1**
1	1	$\overline{Q(t)}$	**$\overline{Q(t)}$**

Q \ JK	00	01	11	10
0	0	0	1	1
1	1	0	0	1

$$D = \overline{Q}.J + Q.\overline{K}$$


K
J
clk
D
Q
$\overline{Q}$