Q.

x	y
2.5	2.4
0.5	0.7
2.2	2.9
1.9	2.2
3.1	3.0
2.3	2.7
2	1.6
1	1.1
1.5	1.6
1.1	0.9
$\bar{x}=1.81$	$\bar{y}=1.91$

X =

① X-Adjust =

$(x-\bar{x})$	$(y-\bar{y})$
0.69	0.49
-1.31	-1.21
0.29	0.99
0.09	0.29
1.29	1.09
0.49	0.79
0.19	-0.31
-0.81	-0.81
-0.31	-0.31
-0.71	-1.01

② Calculate Co-variance Matrix :-

x'	y'	x'.y'	$(x')^2$	$(y')^2$
0.69	0.49	0.3381	0.4761	0.2401
-1.31	-1.21	1.5851	1.7161	1.4641
0.39	0.99	0.3861	0.1521	0.9801
0.09	0.29	0.0261	0.0081	0.0841
1.29	1.09	1.4061	1.6641	1.1881
0.49	0.79	0.3871	0.2401	0.6241
0.19	-0.31	-0.0589	0.0361	0.0961
-0.81	-0.81	0.6561	0.6561	0.6561
-0.31	-0.31	0.0961	0.0961	0.0961
-0.71	-1.01	0.7171	0.5041	1.0201
Total		~~5.4949~~ 5.539	5.549	6.449

$$\text{Cov Matrix} = \begin{pmatrix} cov(x,x), & cov(x,y) \\ cov(y,x), & cov(y,y) \end{pmatrix}$$

$$cov(x,y) = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x}).(y_i-\bar{y})$$

Now
cal

$$cov = \begin{bmatrix} 0.5549 & 0.5539 \\ 0.5539 & 0.6449 \end{bmatrix}_{2\times 2}$$

③ Calculate EigenVectors & EigenValues of CovMatrix

let CovMatrix $= A$

$$A = \begin{bmatrix} 0.5549 & 0.5539 \\ 0.5539 & 0.6449 \end{bmatrix}$$

$$\lambda I = \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

So $$A - \lambda I = \begin{bmatrix} 0.5549 & 0.5539 \\ 0.5539 & 0.6449 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 0.5549-\lambda & 0.5539 \\ 0.5539 & 0.6449-\lambda \end{bmatrix}$$

$$\det(A - \lambda I) = (0.5549-\lambda)(0.6449-\lambda) - 0.5539 \times 0.5539$$

$$= 0.5549 \times 0.6449 - 0.6449\lambda - 0.5549\lambda + \lambda^2 - (0.5539)^2$$

$$= \lambda^2 - 1.1998\lambda + 0.05705$$

$$(\lambda - 0.04908)(\lambda - 1.2840)$$

$$\text{EigenValues}, = \begin{pmatrix} 0.04908 \\ 1.2840 \end{pmatrix} \begin{matrix} = \lambda_1 \\ = \lambda_2 \end{matrix}$$

Now for $\lambda_1 = 0.04908$

Calculating Eigen Vector

$$A - \lambda I = \begin{bmatrix} 0.5549 - \lambda & 0.5539 \\ 0.5539 & 0.6449 - \lambda \end{bmatrix}$$

substitute λ by λ_1 value :

$$A - \lambda I = \begin{bmatrix} 0.5549 - 0.04908 & 0.5539 \\ 0.5539 & 0.6449 - 0.04908 \end{bmatrix}$$

$$= \underbrace{\begin{bmatrix} 0.50582 & 0.5539 \\ 0.5539 & 0.59582 \end{bmatrix}}_{B}$$

Now solve :-

$$B\vec{x} = \vec{0}$$

$$\begin{bmatrix} 0.50582 & 0.5539 \\ 0.5539 & 0.59582 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 0.50582 & 0.5539 & 0 \\ 0.5539 & 0.59582 & 0 \end{array}\right] \longrightarrow \text{Augmented Matrix}$$

Eigen Vectors = $\begin{pmatrix} -.735178656 & -0.677873 \\ 0.677873399 & -0.7351786 \end{pmatrix}$

$\underbrace{\qquad}_{\lambda_1} \quad \underbrace{\qquad}_{\lambda_2}$

(4) Choosing components & forming a feature vector

→ So we have two Eigen Vectors

→ These two Eigenvectors gives two feature vectors

→ writting according to the Eigenvalues

$$\lambda_1 > \lambda_2 > \dots \lambda_n$$

$\therefore$ $\lambda_2 > \lambda_1$

$$\begin{pmatrix} -0.67787 \\ -0.73518 \end{pmatrix} > \begin{pmatrix} -0.73518 \\ 0.67787 \end{pmatrix}$$

→ If we want only 1 feature vector than

$$\begin{pmatrix} -0.67787 \\ -0.73518 \end{pmatrix}$$

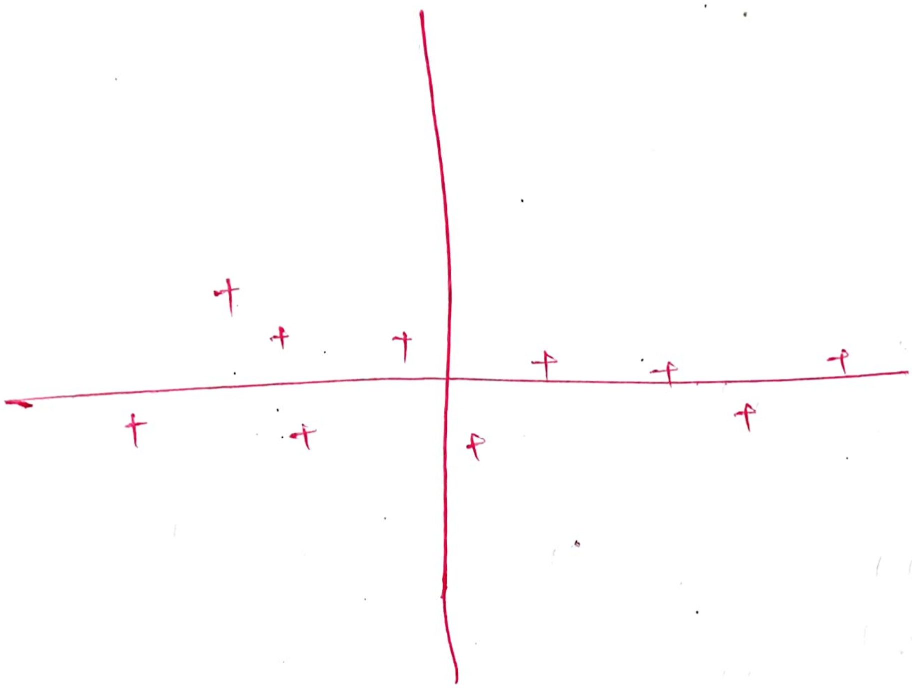
To Tramform the dataset :-

Final Data = RowFeatureVector x RowDataAdj

$$\begin{array}{c} \begin{pmatrix} x' & y' \\ 0.69 & 0.49 \\ -1.31 & -1.21 \\ 0.39 & 0.99 \\ 0.09 & 0.29 \\ 1.29 & 1.09 \\ 0.49 & 0.79 \\ 0.19 & -0.31 \\ -0.81 & -0.81 \\ -0.31 & -0.31 \\ -0.71 & -1.01 \end{pmatrix}_{10\times 2} \times \begin{pmatrix} -0.67787 \\ -0.73510 \end{pmatrix}_{2\times 1} \end{array}$$

Transformed data =

x	y
-0.82279	-0.17512
1.7776	0.14286
-0.9922	0.38437
-0.27421	0.13042
-1.6758	-0.20949
-0.91295	+0.17528
+0.09911	-0.34982
1.1446	0.04642
0.43804	0.01776
0.43805	0.01776
1.2238	-0.16267



```
from sklearn.metrics import adjusted_rand_score
    r = adjusted_rand_score(true_labels, kmeans.labels_)
from sklearn.metrics import silhouette_score
    sil = silhouette_score(scaled_features, kmeans.labels_)
```