ex I/P

	x_1	x_2	y
R1	4	1	2
R2	2	8	-14
R3	1	0	1
R4	3	2	-1
R5	1	4	-7
R6	6	7	-8

O/P (true value)

Apply linear Regression with Gradient descent.

Sol

① Hypothesis function:

$$\hat{y} = h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2$$

Predicted outcome.

Initialize and feed the data.

Assume $\theta_0 = 0$, $\theta_1 = -0.017$, $\theta_2 = -0.048$

$\alpha = 0.05$

R1: $x_1 = 4$, $x_2 = 1$, $y = 2$

$$\hat{y} = (-0.017)\times 4 + (-0.048)\times 1 + 0$$

$$\hat{y} = -0.116$$ ✓

② Cost function

$$J(\theta_0, \theta_1, \theta_2) = \frac{1}{2}(\hat{y} - y)^2$$ ✓

R1:

$$= \frac{1}{2}(-0.116 - 2)^2$$

$$= \underline{\underline{2.24}}$$ ✓

$$J(\theta_0, \theta_1, \theta_2) = \frac{1}{2}(\underline{\theta_0 + \theta_1 x_1 + \theta_2 x_2} - y)^2$$

✓ $$\frac{\partial J}{\partial \theta_0} = 1 \times (\theta_0 + \theta_1 x_1 + \theta_2 x_2 - y)$$ ✓

$$\boxed{\because \frac{\partial}{\partial x} x^n = n x^{n-1} \times \frac{\partial}{\partial x}(x)}$$

$$\frac{\partial J}{\partial \theta_0} = (\hat{y} - y)$$ ✓

$$\frac{\partial J}{\partial \theta_1} = \frac{2}{2}(\theta_0 + \theta_1 x_1 + \theta_2 x_2 - y) \times x_1$$

$$\frac{\partial J}{\partial \theta_1} = (\hat{y} - y) \times x_1$$ ✓

$$\frac{\partial J}{\partial \theta_2} = (\hat{y} - y) \times x_2$$ ✓

③ G.D (Gradient descent)

$$\boxed{\theta_j := \theta_j - \alpha * \left(\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1, \theta_2)\right)}$$

$$\theta_0 := \theta_0 - \alpha * \left(\frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1, \theta_2)\right)$$

$$\theta_1 := \theta_1 - \alpha * \left(\frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1, \theta_2)\right)$$

$$\theta_2 := \theta_2 - \alpha * \left(\frac{\partial}{\partial \theta_2} J(\theta_0, \theta_1, \theta_2)\right)$$

$$\theta_0 := 0 - 0.05 * (\hat{y} - y)$$

$$= -0.05 \times (-0.116 - 2)$$

$\theta_0 := 0.10580$

$\theta_1 := -0.017 - 0.05 \times (\hat{y} - y) \times x_1$

$= -0.017 - 0.05 \times (-0.116 - 2) \times 4$

$\theta_1 := 0.4062$

$\theta_2 := -0.048 - 0.05 \times (\hat{y} - y) \times x_2$

$= -0.048 - 0.05 \times (-0.116 - 2) \times 1$

$\theta_2 := 0.05780$

$\theta_0 = \underline{0.10580}$, $\theta_1 = 0.\underline{4062}$, $\theta_2 = 0.0\underline{5780}$

$x_1 = \underline{2}$, $x_2 = \underline{8}$, $y = -\underline{14}$

R_2: $\hat{y} = \theta_0 + \theta_1 x_1 + \theta_2 x_2$

$= 0.10580 + 0.4062 \times 2 + 0.05780 \times 8$

$= 1.3806$

R2: cost function:

$$J(\theta_0, \theta_1, \theta_2) = \frac{1}{2}(\hat{y} - y)^2$$

$$= \frac{1}{2}(1.3806 - (-14))^2$$

$$= 118.281$$

$$\frac{\partial J}{\partial \theta_0} = (\hat{y} - y) = (1.3806 + 14) = \underline{15.3806}$$

$$\frac{\partial J}{\partial \theta_1} = 15.3806 \times x_1 = 30.7612$$

$$\frac{\partial J}{\partial \theta_2} = 15.3806 \times x_2 = 123.0448$$

GD

$\theta_0 := 0.10580 - 0.05 \times (15.3806)$

$= -0.66$

$\theta_1 := 0.4062 - 0.05 \times (30.7612)$

$= -1.13$

$\theta_2 := 0.05780 - 0.05 \times (123.0448)$

$$= -6.09$$

$$\theta_0 = -0.66, \quad \theta_1 = -1.13, \quad \theta_2 = -6.09$$

$$x_1 = 1, \quad x_2 = 0, \quad y = 1$$

R_3^{e}: $$\hat{y} = -0.66 + (-1.13) \times 1 + (-6.09) \times 0$$

$$= \underline{-1.79}$$