$$Error = \sum_{i=1}^{N} (y_i - \hat{y}_i)^2$$

$\hat{y} = ax_i + b$

Substitute expression of line —

$$E = \sum_{i=1}^{N} (y_i - (ax_i + b))^2$$

E is function of 2 variable a and b. Therefore we need to use partial derivative

• Derivative of E w.r.t. a

$+ \frac{\partial y_i}{\partial a} \to 0$ constant

By Product rule

$$\frac{\partial E}{\partial a} = \sum_{i=1}^{N} 2(y_i - (ax_i + b)) \left(\frac{-\partial ax_i}{\partial a}\right) + \frac{\partial}{\partial a}(b)$$

$$\frac{\partial E}{\partial a} = \sum_{i=1}^{N} 2(y_i - (ax_i + b))(-x_i) + 0$$

set it to zero (0)

$$= \sum_{i=1}^{N} 2(y_i - (ax_i + b))(-x_i) = 0$$

$= 2\sum_{i=1}^{N} -x_i(y_i - (ax_i + b))$

$$= \sum_{i=1}^{N} (-x_i) y_i + \sum_{i=1}^{N} a x_i^2 + \sum_{i=1}^{N} b x_i = \frac{0}{2}$$

$$= -\sum_{i=1}^{N} y_i x_i + a \sum_{i=1}^{N} x_i^2 + b \sum_{i=1}^{N} x_i = 0$$

make everything positive

$$\frac{\partial E}{\partial a} \Rightarrow a \sum_{i=1}^{N} x_i^2 + b \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} y_i x_i \quad \text{— } (1)$$

• Derivative of E w.r.t. b

$$\frac{\partial E}{\partial b} = \sum_{i=1}^{N} (y_i - (ax_i + b))^2$$

$$\frac{\partial E}{\partial b} = \sum_{i=1}^{N} 2(y_i - (ax_i + b)) \left(\frac{\partial}{\partial b}(y_i) - \frac{\partial}{\partial b} ax_i - \frac{\partial b}{\partial b}\right)$$

$$\frac{\partial E}{\partial b} = \sum_{i=1}^{N} 2(y_i - (ax_i + b))(-1) = 0$$

Simplifying,

$$\frac{\partial E}{\partial b} = -\sum_{i=1}^{N} y_i + a\sum_{i=1}^{N} x_i + b\sum_{i=1}^{N} 1 = \frac{0}{2}$$

$$= a\sum_{i=1}^{N} x_i + bN = \sum_{i=1}^{N} y_i \quad \text{—— (2)}$$

eq. (1) and (2) $\longrightarrow$

Make it easier to see by replacing summation with letters :→

$$C = \sum_{i=1}^{N} x_i^2 \qquad D = \sum_{i=1}^{N} x_i$$

$$E = \sum_{i=1}^{N} y_i x_i \qquad F = \sum_{i=1}^{N} y_i$$

$$aC + bD = E \quad \text{—— (1)}$$

$$aD + bN = F \quad \text{—— (2)}$$

Now solve for a (or) b ⟶

lets solve for b ⟶

$$(aC + bD = E) \times D \qquad \Rightarrow \qquad aCD + bD^2 = ED \quad \text{— (1)}$$

$$(aD + bN = F) \times C \qquad\qquad acD + bNC = FC \quad \text{—— (2)}$$

$$\begin{array}{r} a\cancel{cD} + bD^2 = ED \\ -\cancel{acD} - bNC = -FC \\ \hline bD^2 - bNC = ED - FC \end{array} \qquad \Rightarrow \qquad \boxed{b = \frac{ED - FC}{D^2 - NC}}$$

Now solve for a :→

$$(ac + bD = E) \times N \qquad \Rightarrow \qquad aNc + \cancel{bND} = EN$$

$$(aD + bN = F) \times D \qquad\qquad - \quad aD^2 + \cancel{bND} = FD$$

$$a(Nc - D^2) = EN - FD$$

$$\boxed{a = \frac{EN - FD}{Nc - D^2}}$$

Plug The original value back :→

$$a = \frac{N \sum_{i=1}^{N} y_i x_i - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{N \sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2} \quad b = \left(\frac{\sum_{i=1}^{N} y_i x_i \sum_{i=1}^{N} x_i - \sum_{i=1}^{N} y_i \sum_{i=1}^{N} x^2}{\left(\sum_{i=1}^{N} x_i\right)^2 - N \sum_{i=1}^{N} x_i^2} \right)$$

~~denominators are same now got~~

Definition of sample mean :→

$$\boxed{\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i}$$

for multiple variable :→

$$\boxed{\overline{xy} = \frac{1}{N} \sum_{i=1}^{N} x_i y_i}$$

$$a = \frac{\overline{xy} - \bar{x}\bar{y}}{\overline{x^2} - \bar{x}^2}, \quad b = \frac{\bar{y}\,\overline{x^2} - \bar{x} \cdot \overline{xy}}{\overline{x^2} - \bar{x}^2}$$

Rearrange b so that it looks like a

multiply by -1 to both Numerator and denominator

$$b = \frac{\sum_{i=1}^{N} y_i \sum_{i=1}^{N} x^2 - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i x_i}{N \sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2}$$

Now the denominators are same for both a and b

Now divide both top and bottom of (a) and (b) by N^2 :→

$$a = \frac{\frac{N}{N^2} \sum_{i=1}^{N} y_i x_i - \frac{1}{N} \sum_{i=1}^{N} x_i \frac{1}{N} \sum_{i=1}^{N} y_i}{\frac{N}{N^2} \sum_{i=1}^{N} x^2 - \frac{1}{N^2} \left(\sum_{i=1}^{N} x_i \right)^2}$$

$$b = \frac{\frac{1}{N} \sum_{i=1}^{N} y \cdot \frac{1}{N} \sum_{i=1}^{N} x^2 - \frac{1}{N} \sum_{i=1}^{N} x_i \frac{1}{N} \sum_{i=1}^{N} y_i x_i}{\frac{N}{N^2} \sum_{i=1}^{N} x_i^2 - \frac{1}{N^2} \left(\sum_{i=1}^{N} x_i \right)^2}$$

$$\boxed{a = \frac{\overline{xy} - \bar{x} \cdot \bar{y}}{\overline{x^2} - \bar{x}^2}} \quad , \quad \boxed{b = \frac{\bar{y}\,\overline{x^2} - \bar{x} \cdot \overline{xy}}{\overline{x^2} - \bar{x}^2}}$$