Tutorial-2 Nitin Singh I.21

Ans 1. $i = 0, 1, 3, 6, 10, 15$ lets say k terms.

So general form would be $\frac{k(k+1)}{2}$

k^{th} term $= n = \frac{k(k+1)}{2} = n$

$$k^2 + k = 2n$$

$$k^2 = n = k = \sqrt{n}$$

$\therefore$ Time complexity $= O(\sqrt{n})$ Ans

Ans 2 Recurrence Relation ->

Recursion Function.

```
int fib(int n)
{  if (n<=1)         <-> O(1)=C.
      return n;
   return fib(n-1) + fib(n-2) -> T(n-1)+T(n-2)
}
```

Recurrence Relation $T(n) = T(n-1) + T(n-2) + C$.

Now $T(n-1) \simeq T(n-2)$.

$$T(n) = 2T(n-1) + C.$$

By backward substitution.

$T(n-1) = 2T(n-1-1) + C \Rightarrow 2T(n-2) + C$.

$$T(n) = 2[2T(n-2)+C]+C$$
$$= 4T(n-2)+3C.$$

Now $T(n-2) = 2T(n-2-1)+C.$
$$= 2T(n-3)+C.$$

$$\therefore T(n) = 4T(n-2)+3C.$$
$$= 4(2T(n-3)+C)+3C$$
$$T(n) = 8T(n-3)+7C$$

generalizing :- $2^k T(n-k) + (2^k - 1)C.$

assume $n-k=0 \Rightarrow n=k$

$$2^n T(0) + (2^n - 1)c.$$
$$= 2^n + (2^n - 1)c.$$
$$= 2^n(1-c) - c$$
$$= 2^n.$$

$\therefore$ Time Complexity $= O(2^n)$

• Space complexity

For Fibonacci recursion implementation, the space required is directly proportional to the maximum depth of Recursion tree, Since maximum depth is directly proportional to number of elements so $O(n)$.

Q3

(i)
```
for(i=1; i<=n; i++)
{
    for(j=1; j<=n; j=j*2)
    {
        Sum = Sum + i;
    }
}
```

(ii) n^3

```
for(i=0; i<n; i++)
{
    for(j=0; j<n; j++)
    {   for(k=0; k<n; k++)
        {   Sum = Sum+K;
        }
    }
}
```

(iii) Logn(Logn)

```
for(i=1; i<=n; i=i*2)
{
    for(k=1; k<=n; k=k*2)
    {   Sum = Sum + j;
    }
}
```

Ans 4 $T(n/4) \simeq T(n/2)$

$T(n) = 2T(n/2) + Cn^2$

as $a \geq 1$ and $b \geq 1$

by using master's method.

$T(n) = aT(n/b) + f(n)$

$c = \log_b a \Rightarrow 1$

$f(n) > n^c \Rightarrow Cn^2 > n^c$

$T(n) = O(f(n))$

$= O(n^2)$

Ans 5 for $i = 1 \rightarrow 1+2+3+ - - - - - (n+1) \Rightarrow n$

for $i = 2 \rightarrow 1+3+5+ - - - - n \Rightarrow n/2$

for $i = 3 \rightarrow 1+4+7+ - - - - n \Rightarrow n/3$

$n + \frac{n}{2} + \frac{n}{3} + - - - - - + 1$

$\Rightarrow n\left(1 + \frac{1}{2} + \frac{1}{3} + - - - - + \frac{1}{n}\right) \leq n\left(1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right)$

$n\left(1 + \frac{1}{2} + \frac{1}{3} + - - - + \frac{1}{n}\right) \leq n\left(1 + 0.5 + 0.5 + - - -\right)$

$O(n \log n)$ Ans.

Ans 8(a)

$100 < \log(\log n) < \log n < \log^2 n < \sqrt{n} < n < \log n! < n \log n <$
$\log^{2n} < n^2 < 2^n < 4^n < 2^{2mn} < n!$

(b)

$1 < \log(\log n) < \sqrt{\log(n)} < \log n < 2\log(n) < \log(2n) < n$
$< 2n < 4n < \log(n!) < n\log(n) < n^2 < 2.(2^n) < n!$