DDA (Digital Differential Analyzer) :-

- The Digital Differential Analyzer is a scan conversion line drawing algorithm based on calculating either Δx or Δy.
- We sample the line at unit intervals in one coordinate and determine correspondence integer values nearest the line path for the other co-ordinates.

$$y = mx + b$$

(x_1, y_1) & (x_2, y_2)

$|m| \leq 1$

$\Delta y \leq \Delta x$

x में unit increment होगा फिर y calculate होगा

$|m| > 1$

y में unit increment होगा फिर x calculate होगा

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\Delta y}{\Delta x}$$

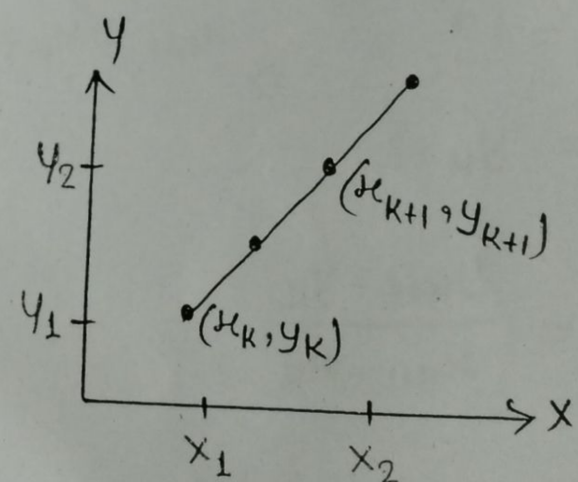
$$\boxed{\Delta y = m \Delta x}$$

$$\boxed{\Delta x = \frac{\Delta y}{m}}$$

Case. I :- $|m| \leq 1$

$$\frac{\Delta y}{\Delta x} \leq 1$$

$$\boxed{\Delta y \leq \Delta x}$$



Let (x_K, y_K) is the selected pixel, the next pixel at any time line path is (x_{K+1}, y_{K+1}) s.t

$$\begin{cases} x_{k+1} = x_k + 1 \\ y_{k+1} = ? \end{cases}$$

$$m = \frac{y_{k+1} - y_k}{x_{k+1} - x_k} \quad ; \quad m = \frac{y_{k+1} - y_k}{1}$$

→ only if $x_{k+1} = x_k + 1$

$$m = y_{k+1} - y_k$$

$$\boxed{y_{k+1} = y_k + m}$$

जो बडा होगा वो ~~Constant~~ unit increment रहेगा।

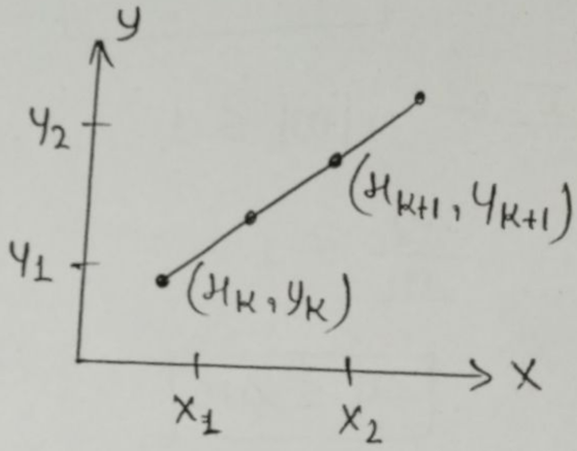
$$x_{k+1} = x_k + 1$$

$$y_{k+1} = y_k + m$$

<u>Case II</u> :— $|m| > 1$

$$x_{k+1} = ? \quad \Rightarrow \quad x_{k+1} \Rightarrow x_k + 1/m$$

$$y_{k+1} = y_k + 1$$



$$m = \frac{y_{k+1} - y_k}{x_{k+1} - x_k}$$

$$m = \frac{1}{x_{k+1} - x_k} \quad ;$$

$$x_{k+1} - x_k = \frac{1}{m}$$

$$\boxed{x_{k+1} = x_k + 1/m}$$

Positive Slope (+)

- Left to right
  - $m \le 1$
  - $m > 1$
- Right to left
  - $m \le 1$
  - $m > 1$

Negative Slope (-)

- Left to Right
  - $|m| \le 1$
  - $|m| > 1$
- Right to left
  - $|m| \le 1$
  - $|m| > 1$

$\boxed{\Delta x = 1 \quad y_{k+1} = y_k + m}$

$\boxed{x_{k+1} = x_k + 1/m \quad \Delta y = 1}$

$\boxed{\Delta x \Rightarrow -1 \quad y_{k+1} \Rightarrow y_k - m}$

$\boxed{x_{k+1} = x_k - 1/m \quad \Delta y \Rightarrow -1}$

$\boxed{\Delta x = 1 \quad y_{k+1} = y_k + m}$

$\boxed{x_{k+1} = x_k + 1/m \quad \Delta y = 1}$

$\boxed{\Delta x \Rightarrow -1 \quad y_{k+1} \Rightarrow y_k - m}$

$\boxed{x_{k+1} = x_k - 1/m \quad \Delta y \Rightarrow -1}$

Modified DDA :-

Algorithm :-

1.) Read two end points (x_1, y_1) & (x_2, y_2)

2.) Calculate $\Delta x = (x_2 - x_1)$ & $\Delta y = (y_2 - y_1)$

3.) If $abs(\Delta x) \ge abs(\Delta y)$ then

Step = Δx

else

Step = Δy

4.) $X_{increment} = \frac{\Delta x}{Step}$

5.) $Y_{increment} = \frac{\Delta y}{Step}$

5.) $X = x_1$ & $Y = y_1$

6.) put pixel (X, Y, Color)

7.) $x = x + x_{increment}$

$y = y + y_{increment}$

8.) put pixel (x, y, color)

9.) Repeat step 7 & 8 until $x = x_2$ and $y = y_2$

To Set the Graphics mode :—

int gm, gd = DETECT

initgraph (&gd, &gm, "c:\\tc\\bgi");

path

To Display pixel :—

put pixel (x, y, I) → Intensity (Color)

26/07/024

Que) Calculate the pixel positions along a straight line between A(10, 12) and B(20, 20) using DDA algorithm.

Soln :—

$\Delta x = x_2 - x_1$
$\Rightarrow 20 - 10$
$\Rightarrow 10$

$\Delta y = y_2 - y_1$
$\Rightarrow 20 - 12$
$\Rightarrow 8$

If abs(Δx) ≥ abs(Δy)

$10 \geq 8$

$\boxed{\text{Step} = 10}$

$X_{increment} = \frac{\cancel{10}}{\cancel{10}} \Rightarrow 1$, $Y_{increment} = \frac{8}{10} \Rightarrow 0.8$

$X = 10 + 1 = 11$
$Y = 12 + 0.8 = 12.8$

$x = 11 + 1 \Rightarrow 12$
$y = 12.8 + 8 \Rightarrow 13.6$

(12, 13.6)
(11, 12.8)
(10, 12)

$x = 12 + 1 \Rightarrow 13$

$y = 13.6 + 8 \Rightarrow 21.6$

$x = 13 + 1 \Rightarrow 14$

$y = 21.6 + 8 \Rightarrow 29.8$

Que) $A(20, 20)$, $B(10, 12)$

Soln :- $\Delta x = 10 - 20$ $\qquad$ $\Delta y = 12 - 20$

$\Rightarrow -10$ $\qquad$ $\Delta y = -8$

$abs(\Delta x) \geq abs(\Delta y)$

$10 \geq 8$

$step = 10$

$x_{incr} \Rightarrow \frac{-10}{10} \Rightarrow -1$ $\qquad$ $y_{incr} \Rightarrow \frac{-8}{10} \Rightarrow -0.8$

Que) $(1, 1)$, $(5, 6)$

Que) $(0, 0)$, $(-6, -6)$

(AB)

Bresenham's Line Drawing Algorithm :— $|m| \le 1$

1) Read two End point (x_1, y_1) & (x_2, y_2)
2) Calculate Δx, Δy
3) put pixel (x_1, y_1, Color)
4) Calculate the initial value of decision parameter as
$$\boxed{P_0 = 2\Delta y - \Delta x}$$
5) If $P_K < 0$ then plot the pixel at position (x_K, y_K) and Compute next parameter as
$$\boxed{P_{K+1} = P_K + 2\Delta y}$$
else plot the pixel at position $(x_K + 1, y_K + 1)$ and Compute the next decision parameter as -
$$\boxed{P_{K+1} = P_K + 2\Delta y - 2\Delta x}$$

* 6) Repeat step 5 until $x = x_2$ & $y = y_2$

Ques) $A(10, 10)$, $B = (20, 20)$

Soln :— $\Delta x = 10$

$\Delta y = 10$ $\quad m = \frac{\Delta y}{\Delta x} \Rightarrow \frac{10}{10} \Rightarrow 1$

$P_0 \Rightarrow 20 - 10$

$\boxed{P_0 \Rightarrow 10}$

$P_0 \geq 0$

$(x_k + 1, y_k + 1) \Rightarrow (11, 11)$

$P_{k+1} = P_k + 2\Delta y - 2\Delta x$

$P_{0+1} = P_0 + 2 \times 10 - 2 \times 10$

$\Rightarrow 10 + 20 - 20$

$\boxed{P_1 \Rightarrow 10}$

Ques) $A(1, 1)$, $B(8, 7)$

Soln :— $\Delta x \Rightarrow 7$

$\Delta y \Rightarrow 6$ $\quad m = \frac{6}{7} \Rightarrow 0.8 < 1$

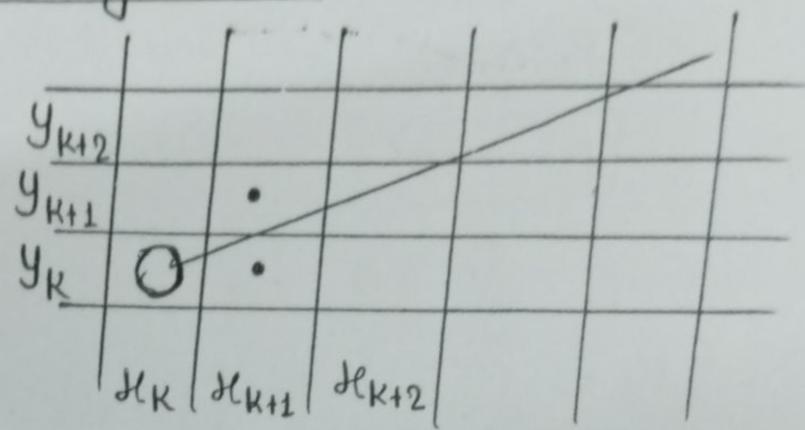
$P_0 = 5$ $\quad \boxed{5 > 0}$

$P_1 = 5 + 12 - 14$

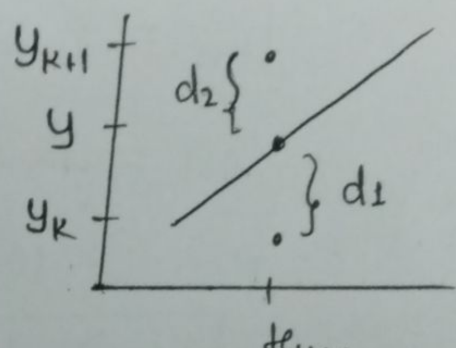
$\boxed{P_1 \Rightarrow 3}$ $\quad 3 > 0$

$(3, 3)$

Bresenhem's Line Drawing Algorithm :– $|m| \le 1$



Pixel at position (x_k, y_k) is selected. then for next pixel position there are two choices at (x_{k+1}, y_k) and (x_{k+1}, y_{k+1})



$$y = mx + b$$
$$= m(x_k + 1) + b$$

$$d_1 = y_1 - y_2$$
$$= m(x_k+1) + b - y_k$$

$$d_2 = y_{k+1} - y$$
$$= y_k + 1 - y$$
$$= y_k + 1 - m(x_k+1) - b$$

$$d_1 - d_2 = 2m(x_k+1) + 2b - 2y_k - 1$$

$$P_k = \Delta x(d_1 - d_2) = 2\Delta y(x_k+1) + 2b\Delta x - 2y_k\Delta x - \Delta x$$

$$P_k \Rightarrow 2\Delta y x_k - 2\Delta x y_k + 2\Delta y + 2\Delta x b - \Delta x$$

$$P_k \Rightarrow 2\Delta y x_k - 2\Delta x y_k + 2\Delta y + \Delta x(2b-1)$$

$$P_k \Rightarrow 2\Delta y x_k - 2\Delta x y_k + c \quad \text{———} \; (1)$$

where $c = 2\Delta y + \Delta x(2b-1)$

$$P_{K+1} \Rightarrow 2\Delta y x_{K+1} - 2\Delta x y_{K+1} + c \quad \text{—— (iii)}$$

(iii) − (i)

$$P_{K+1} - P_K \Rightarrow 2\Delta y x_{K+1} - 2\Delta x y_{K+1} + \cancel{c} - 2\Delta y x_K + 2\Delta x y_K - \cancel{c}$$

~~2Δyx₁~~

$$\Rightarrow 2\Delta y(x_{K+1} - x_K) - 2\Delta x(y_{K+1} - y_K)$$

Now $x_{K+1} = x_K + 1$

$$P_{K+1} - P_K \Rightarrow 2\Delta y(\cancel{x_K} + 1 - \cancel{x_K}) - 2\Delta x(y_{K+1} - y_K)$$

$$P_{K+1} - P_K \Rightarrow 2\Delta y - 2\Delta x(y_{K+1} - y_K)$$

If Pixel (X_{K+1}, y_{K+1}) is selected then

$$y_{K+1} = y_K + 1$$

$$P_{K+1} - P_K = 2\Delta y - 2\Delta x(\cancel{y_K} + 1 - \cancel{y_K})$$

$$\boxed{P_{K+1} = P_K + 2\Delta y - 2\Delta x}$$

If Pixel at position (x_{K+1}, y_K) is selected then $y_{K+1} = y_K$

$$P_{K+1} - P_K = 2\Delta y - 2\Delta x(\cancel{y_K} - \cancel{y_K})$$

$$\boxed{P_{K+1} = P_K + 2\Delta y}$$

$$P_0 = 2\Delta y x_k - 2\Delta x y_k + 2\Delta y + 2\Delta x b - \Delta x$$

$$\Rightarrow -2\Delta x\left[\frac{-\Delta y}{\Delta x} x_k + y_k\right] + 2\Delta y + 2\Delta x b - \Delta x$$

$$\Rightarrow -2\Delta x\left[-m\Delta x_k + y_k\right] + 2\Delta y + 2\Delta x b$$

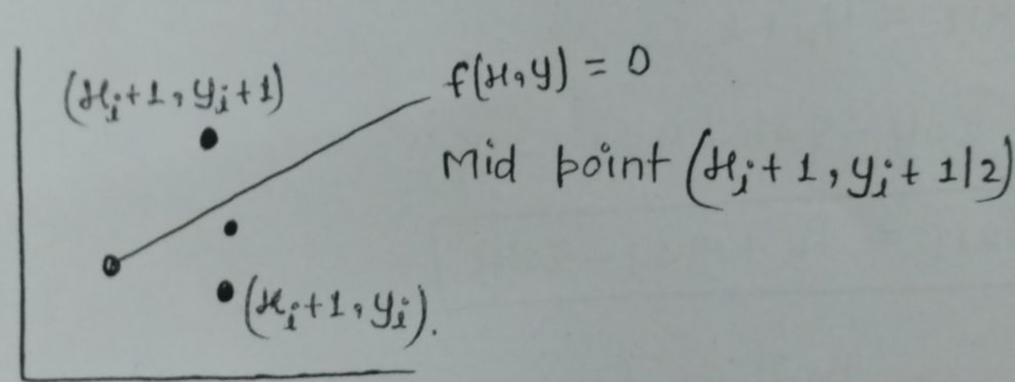
$$\Rightarrow -2\Delta x\left[y_k - m x_k\right] + 2\Delta y + 2\Delta x b - \Delta x$$

$$\Rightarrow -2\Delta x b + 2\Delta y + 2\Delta x b - \Delta x$$

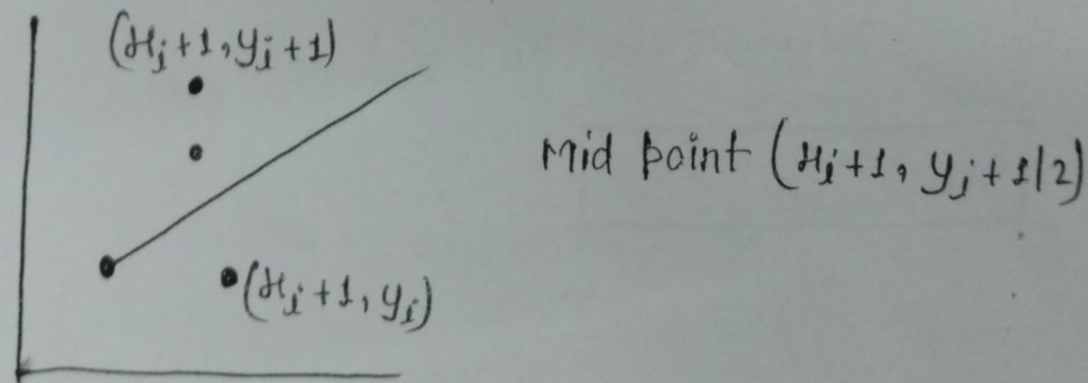
$$\boxed{P_0 = 2\Delta y - \Delta x} \quad \#$$

$$\left\{\begin{array}{l} y = mx + b \\ y - mx = b \end{array}\right.$$

29/07/024 **MID POINT**



+ve pixel (x_i+1, y_i+1) will be selected.



(mid point) > 0

DERIVATION :-

$y = mx + b$

$y - mx - b = 0$ [Mid point $(x_i+1, y_i+1/2)$]

$y_i + 1/2 - m(x_i+1) - b = 0$

$y_i - mx_i - m - b + 1/2 = 0$

$d_i \Rightarrow m(x_i+1) + b - (y_i + 1/2)$ [d_i = decision parameter]

$d_i \Rightarrow \frac{\Delta y}{\Delta x}(x_i+1) + b - (y_i + 1/2)$

$d_i \Rightarrow \Delta y(x_i+1) + \Delta x b - (y_i + 1/2)$ ----- (I)

$d_{i+1} \Rightarrow \Delta y(x_{i+1}+1) + \Delta x b - \Delta x(y_{i+1} + 1/2)$

$\Rightarrow \Delta y(x_i+1) + \Delta y + \Delta x b - \Delta x(y_{i+1} + 1/2)$ --- (II)

(II) - (I)

$d_{i+1} - d_i \Rightarrow \Delta y\cancel{(x_i+1)} + \Delta y + \cancel{\Delta x b} - \Delta x(y_{i+1} + 1/2) - \Delta y\cancel{(x_i+1)}$
$- \cancel{\Delta x b} + \Delta x(y_i + 1/2)$

$d_{i+1} - d_i \Rightarrow \Delta y - \Delta x(y_{i+1} + 1/2) + \Delta x(y_i + 1/2)$

$\Rightarrow \Delta y - \Delta x(y_{i+1} - y_i) - \cancel{1/2\Delta x} + \cancel{1/2\Delta x}$

$d_{i+1} - d_i \Rightarrow \Delta y - \Delta x(y_{i+1} - y_i)$

$d_{i+1} \Rightarrow d_i + \Delta y - \Delta x(y_{i+1} - y_i)$

If $d_i \leq 0$ then $y_{i+1} = y_i$

$\boxed{d_{i+1} = d_i + \Delta y}$

If $d_i > 0$ the $y_{i+1} = y_i + 1$

$$d_{i+1} \Rightarrow d_i + \Delta y - \Delta x(\cancel{y_i} + 1 - \cancel{y_i})$$

$$\boxed{d_{i+1} = d_i + \Delta y - \Delta x} \quad \#$$

$$d_i \Rightarrow \Delta y(x_i + 1) + \Delta x b - \Delta x(y_i + 1/2)$$

$$\Rightarrow \Delta y x_i + \Delta y + \Delta x b - \Delta x y_i - 1/2\,\Delta x$$

$$\Rightarrow \Delta y x_i - \Delta x y_i + \Delta y + \Delta x b - 1/2\,\Delta x$$

$$\Rightarrow -\Delta x\left[y_i - \frac{\Delta y}{\Delta x} x_i\right] + \Delta y + \Delta x b - 1/2\,\Delta x$$

$$\Rightarrow -\Delta x\left[y_i - m x_i\right] + \Delta y + \Delta x b - 1/2\,\Delta x$$

$$\Rightarrow -\cancel{\Delta x b} + \Delta y + \cancel{\Delta x b} - 1/2\,\Delta x$$

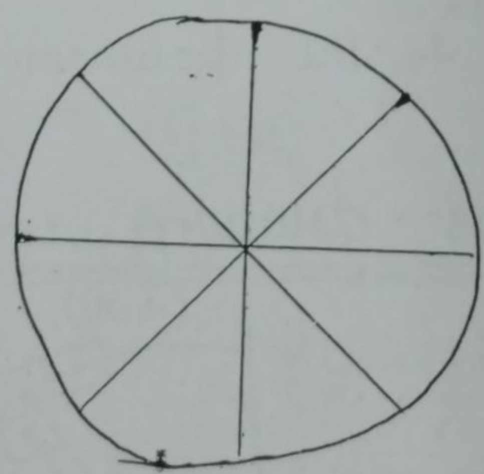
$$\boxed{d_i \Rightarrow \Delta y - 1/2\,\Delta x} \quad \#$$

02/08/021

Scan Converting a Circle

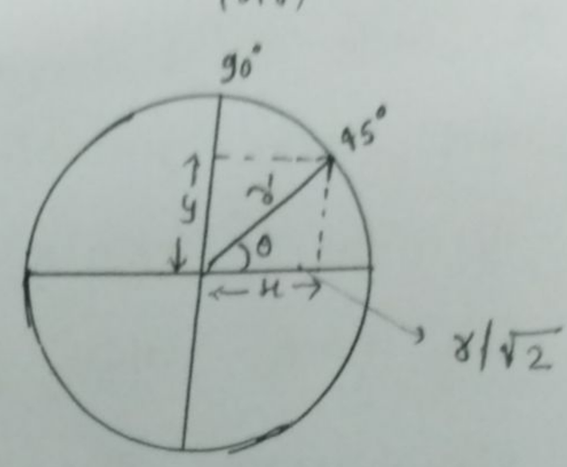
Ques) Explain the Circle Symmetric property ?

- A Circle is a Symmetrical figure.
- Any Circle generating algorithm Can take advantage of the Circle's Symmetry to plot eight point for each Value that the algorithm Calculates.
- Eight way Symmetry is used by reflecting each Calculated point around each 45° axis.
- therefore we need to Compute any 45° arc to determine the Circle Completely.



Defining a Circle :—

1) Polynomial Methods :—



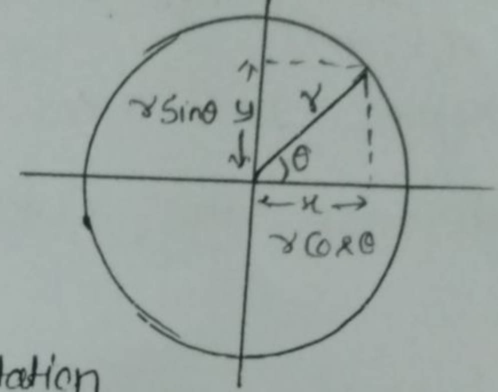
$$x^2+y^2=r^2$$

$$y^2=r^2-x^2$$

$$y=\sqrt{r^2-x^2}$$

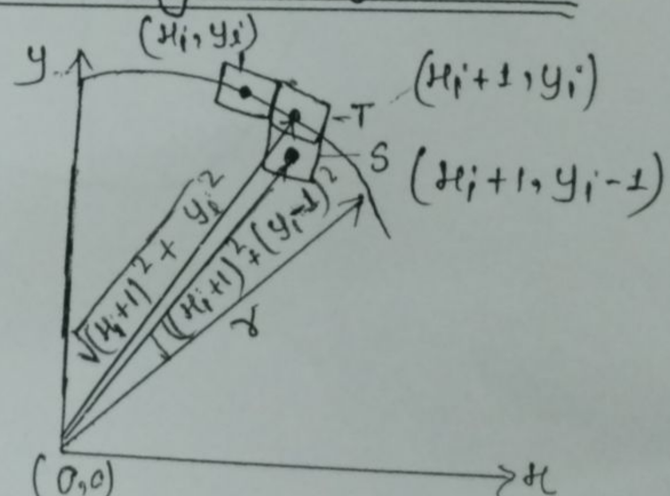
2) Parametric Methods :—

$x = r\cos\theta$, $y = r\sin\theta$



To Compute the trignometry Computation too hard as Compare to the polynomial method.

Bresenhem's Circle drawing Algorithm :—



$$D(T) = (x_i+1)^2 + y_i^2 - r^2$$

$$D(S) = (x_i+1)^2 + (y_i-1)^2 - r^2$$

D(T) will always be positive (T is outside the true Circle)

D(S) will always be negative (S is inside the true Circle)

$d_i = D(T) + D(S)$

$\Rightarrow 2(x_i+1)^2 + y_i^2 + (y_i-1)^2 - 2r^2$ —— (I)

→ when $d_i < 0$ we have $|D(T) < D(S)|$ and pixel T is selected.

→ when $d_i \geq 0$ we have $|D(T) \geq D(S)$ and pixel S is selected.

$d_{i+1} \Rightarrow 2(x_i+1)^2 + y_{i+1}^2 + (y_{i+1}-1)^2 - 2r^2$ —— (II)

(II) − (I)

$$d_{i+1}-d_i \Rightarrow 2(x_{i+1}+1)^2 + y_{i+1}^2 + (y_{i+1}-1)^2 - \cancel{2r^2} - 2(x_i+1)^2 \mp y_i^2 \mp (y_i-1)^2 + \cancel{2r^2}$$

Since $x_{i+1} = x_i + 1$

$$d_{i+1}-d_i \Rightarrow 2(x_i+2)^2 + y_{i+1}^2 + (y_{i+1}-1)^2 - 2(x_i+1)^2 + y_i^2 + (y_i-1)^2$$

$$\Rightarrow 2(x_i^2+4+4x_i) + y_{i+1}^2 + (y_{i+1}^2+1-2y_{i+1}) - 2(x_i^2+1+2x) - y_i^2 - (y_i^2+1-2y_i)$$

$$d_{i+1}-d_i \Rightarrow 2x_i^2 + 8 + 8x_i + y_{i+1}^2 + y_{i+1}^2 + \cancel{1} - 2y_{i+1} - \cancel{2x_i^2} - 2 - 4x_i - y_i^2 - y_i^2 - \cancel{1} + 2y_i$$

$\Rightarrow 6 + 4x_i + 2(y_{i+1}^2 - y_i^2) - 2(y_{i+1} - y_i)$

$d_{i+1} \Rightarrow d_i + 4x_i + 2(y_{i+1}^2 - y_i^2) - 2(y_{i+1} - y_i) + 6$

If $d_i < 0$ then T is the selected pixel then $y_{i+1} = y_i$

$d_{i+1} \Rightarrow d_i + 4x_i + 2(\cancel{y_i^2} - \cancel{y_i^2}) - 2(\cancel{y_i} - \cancel{y_i}) + 6$

$$\boxed{d_{i+1} = d_i + 4x_i + 6}$$

If $d_i \geq 0$ then S is the selected pixel then

$y_{i+1} = y_i - 1$

$d_{i+1} \Rightarrow d_i + 4x_i + 2((y_i - 1)^2 - y_i^2) - 2(y_i - 1 - y_i) + 6$

$d_{i+1} \Rightarrow d_i + 4x_i + 2(y_i^2 + 1 - 2y_i - y^2) + 2 + 6$

$d_{i+1} \Rightarrow d_i + 4x_i + \cancel{2y_i^2} + 2 - 4y_i - \cancel{2y_i^2} + 2 + 6$

$$\boxed{d_{i+1} \Rightarrow d_i + 4(x_i - y_i) + 10}$$

put $x = 0$, $y = r$ in eqn ①, we get

$d_0 \Rightarrow 2(0+1)^2 + r^2 + (r-1)^2 - 2r^2$ $(x = 0, y = r)$

$\Rightarrow 2 + \cancel{r^2} + \cancel{r^2} + 1 - 2r - \cancel{2r^2}$

$$\boxed{d_0 = 3 - 2r}$$

05/08/024

Bresenhem's Circle Drawing Algorithm :-

The algorithm for generating all the pixel co-ordinates in the 90° to 45° octant that are needed when scan converting a circle of radius γ.

```
int x = 0 ; y = r , d = 3 - 2r .
while (x ≤ y) {
        Put-pixel (x, y, Color) ;
        if (d<0)
            d = d + 4x + 6 ;
    else
            d = d + 4(x-y) + 10
              y --
      }
        x++ ;
}
```

Que 1.) Calculate the points to draw circle and ~~drawing~~ having radius $\gamma = 5$ and centre $(0,0)$

Soln :- ① $x = 0$, $y = 5$, $d = 3 - 2 \times 5$ $\quad (0,5)$

$\Rightarrow -7 < 0$

$d_1 \Rightarrow -7 + 4 \times 0 + 6 \Rightarrow -1$

② $x = 1$, $y = 5$, $d_1 \Rightarrow -1$ $\quad (1,5)$ $\quad \cdot (3,4)$

$(2,5)$

$d_2 \Rightarrow -1 \times 4 \times 1 + 6 \Rightarrow 9$ $\quad (1,5)$

③ $x = 2$, $y = 5$, $d \Rightarrow 9$ $\quad (2,5)$ $\quad (0,5)$

$d_3 \Rightarrow 9 + 4 \times (2 - 5) + 10$

$\Rightarrow 7$

$x = 3, y = 4$ (3, 4)

$d_4 \Rightarrow 7 + 4(3-4) + 10$

$\Rightarrow 13$

$x = 4, y = 3$, $d \Rightarrow 13$ (4, 3)

$d_5 \Rightarrow 13 +$

(x, y)	(0, 5)	(1, 5)	(2, 5)	(3, 4)
(x, -y)	(0, -5)	(1, -5)	(2, -5)	(3, -4)
(-x, -y)	(-0, -5)	(-1, -5)	(-2, -5)	(-3, -4)
(-x, y)	(0, 5)	(-1, 5)	(-2, 5)	(-3, 4)
(y, x)	(5, 0)	(5, 1)	(5, 2)	(4, 3)
(y, -x)	(5, 0)	(5, -1)	(5, -2)	(4, -3)
(-y, -x)	(-5, 0)	(-5, -1)	(-5, -2)	(-4, -3)
(-y, x)	(-5, 0)	(-5, 1)	(-5, 2)	(-4, 3)

Que) Calculate the points to draw circle having radius = 8 and Centre (0, 0)

Soln :-

MID POINT CIRCLE DRAWING ALGORITHM :-

1.) Read the radius (r) of the Circle.

2) Initialize the starting position as $x = 0$, $y = r$

3) Calculate initial Value of decision parameter as

$\boxed{d = 1.25 - r}$ or $\boxed{d = 1 - r}$

4) do {

put pixel (x, y, Color)

if (d<0) {

$x = x + 1$

$y = y$

$\boxed{d = d + 2x + 1}$

}

else {

$x = x + 1$

$y = y - 1$

$\boxed{d = d + 2x + 2y + 1}$

}

while (x<y)

5.) Determine Symmetry points.

6) Stop.

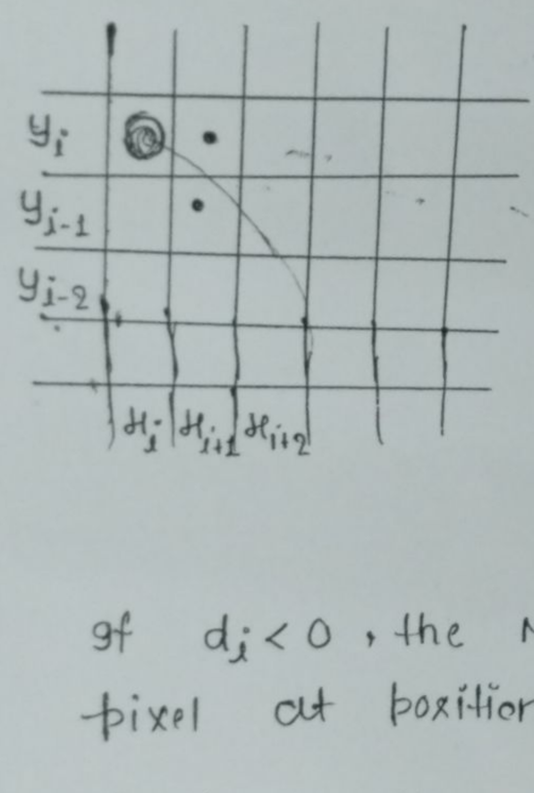
Ques) $\gamma = 10$ $\quad d_0 \Rightarrow 1-\gamma$

$d \Rightarrow 1-10 \quad , \quad d \Rightarrow -9 + 2\times 0 + 1$

$\Rightarrow -9 \qquad\qquad \Rightarrow -8$

09/08/024

Mid point Circle Drawing Algorithm :-



Next pixel will be at the position

$(x_i+1, y_i) \quad (x_i+1, y_i-1)$

Mid point $= (x_i + 1, y_i - \frac{1}{2})$

$$x^2+y^2-\gamma^2 = 0$$

$$d_i = (x_i+1)^2 + (y_i - 1/2)^2 - \gamma^2$$

$$\Rightarrow (x_i+1)^2 + y_i^2 + 1/4 - y_i - \gamma^2$$

If $d_i < 0$, the Mid point is Inside circle and the pixel at position (x_{i+1}, y_i) is selected.

If $d_i \geq 0$ the mid point is outside circle and the pixel at position (x_{i+1}, y_{i-1}) is selected.

$$d_i \Rightarrow x_i^2 + 1 + 2x_i + y_i^2 + 1/4 - y_i - \gamma^2 \quad \text{— (I)}$$

$$d_{i+1} \Rightarrow x_{i+1}^2 + 1 + 2x_{i+1} + y_{i+1}^2 + 1/4 - y_{i+1} - \gamma^2$$

— (II)

$$d_{i+1} - d_i \Rightarrow x_{i+1}^2 + 1 + 2x_{i+1} + y_{i+1}^2 + \cancel{1/4} - y_{i+1} - \cancel{r^2} - x_i^2 - \cancel{1}$$

$$- 2x_i - y_i^2 - \cancel{1/4} + y_i + \cancel{r^2}$$

$$\Rightarrow x_{i+1}^2 - x_i^2 + 2x_{i+1} - 2x_i + y_{i+1}^2 - y_i^2 - y_{i+1} + y_i$$

$$x_{i+1} \Rightarrow x_i + 1$$

$$\Rightarrow (x_i + 1)^2 - x_i^2 + 2(x_i + 1) - 2x_i + y_{i+1}^2 - y_i^2 - y_{i+1} + y_i$$

$$\Rightarrow \cancel{x_i^2} + 1 + 2x_i - \cancel{x_i^2} + 2x_i + 2 - \cancel{2x_i} + y_{i+1}^2 - y_i^2 - y_{i+1} + y_i$$

$$d_{i+1} - d_i \Rightarrow y_{i+1}^2 - y_i^2 + 2x_i + 3 - y_{i+1} + y_i$$

If $d_i < 0$ then $y_{i+1} = y_i$

$$d_{i+1} = d_i + \cancel{y_i^2} - \cancel{y_i^2} + 2x_i + 3 - \cancel{y_i} + \cancel{y_i}$$

$$\boxed{d_{i+1} = d_i + 2x_i + 3}$$

If $d_i \geq 0$ then $y_{i+1} = y_i - 1$

$$d_i \Rightarrow (x_i + 1)^2 + (y_i - 1/2)^2 - r^2$$

$$\Rightarrow (x_i + 1)^2 + y_i^2 + 1/4 - y_i - r^2$$

$$d_{i+1} \Rightarrow d_i + (y_i - 1)^2 - y_i^2 + 2x_i + 3 - y_i + 1 + y_i$$

$$\Rightarrow d_i + \cancel{y_i^2} + 1 - 2y_i - \cancel{y_i^2} + 2x_i + 3 - \cancel{y_i} + 1 + \cancel{y_i}$$

$$\boxed{d_{i+1} \Rightarrow d_i + 2(x_i - y_i) + 5}$$

$$d_i \Rightarrow x_i^2 + 1 + 2x_i + y_i^2 + 1/4 - y_i - r^2 \quad \text{— (ii)}$$

$$d_{i+1} \Rightarrow x_{i+1}^2 + 1 + 2x_{i+1} + y_{i+1}^2 + 1/4 - y_{i+1} - r^2 \quad \text{— (iii)}$$

for initial decision parameter the first pixel will be Located at $(0, r)$

$$d_0 \Rightarrow 0 + 1 + 0 + \cancel{r^2} + 1/4 - r - \cancel{r^2}$$

$$\boxed{d_0 \Rightarrow \frac{5}{4} - r}$$

$$\boxed{d_0 \Rightarrow 1 - r}$$

Ques 1) $r = 3$ and $(0,0)$

Soln:— $c = (0,0)$

$r \Rightarrow 3$

$x = 0, \; y = 0$

$d_0 \Rightarrow 1 - r$

$\Rightarrow 1 - 3 \Rightarrow -2$

$d_0 < 0 \Rightarrow \boxed{-2 < 0}$

$x_1 \Rightarrow x_0 + 1 \Rightarrow 1$

$y_1 \Rightarrow y_0 \Rightarrow 10$

$d_{0+1} \Rightarrow d_0 + 2x_0 + 3$

$d_{0+1} \Rightarrow -2 + 2 \times 0 + 3$

$\boxed{d_1 \Rightarrow 1}$

$d_1 > 0$

$\boxed{1 > 0}$

$x_2 \Rightarrow x_1 + 1 \Rightarrow 1 + 1 \Rightarrow 2$

$y_2 \Rightarrow y_1 \Rightarrow 10$

(2) $r = 8$ and Centre $(0,0)$

$\boxed{d_1 > 0}$

$d_{i+1} \Rightarrow d_i + 2(x_i - y_i) + 5$

$d_2 \Rightarrow 1 + 2(x_1 - y_1) + 5$

$d_2 \Rightarrow 1 + 2(1 - 10) + 5 \Rightarrow 1 + 2(-9) + 5$

$\boxed{d_2 \Rightarrow -12}$

$\boxed{-12 < 0}$

$x_3 \Rightarrow 2 + 1 \Rightarrow 3 \quad y_3 \Rightarrow y_2 \Rightarrow 10$

$d_3 \Rightarrow -12 + 2 \times 3 + 3$

$d_3 \Rightarrow -12 + 6 + 3 \Rightarrow -12 + 9$

$\boxed{d_3 \Rightarrow -3}$

TRANSFORMATION → changes

1) Geometric Transformation
2) Co-Ordinate Transformation

1) Translation Transformation :-

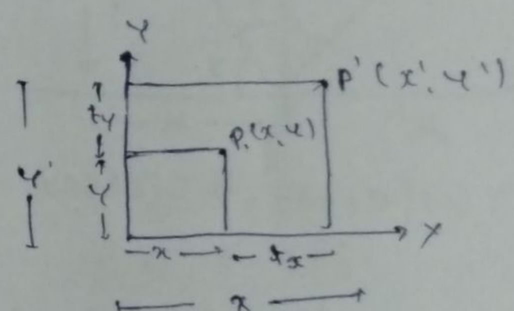
In translation, an object is displaced a given distance and direction from its orignal position. If the displacement is given by the vector $\underline{V = t_x i + t_y j}$

the new object point $P'(x', y')$ Canbe found by applying the transformation T_v to $P(x, y)$

$$\boxed{P' = T_v(P)}$$

$x + 0\cdot y + t_x = x' = x + t_x$

$0\cdot x + y + t_y = y' = y + t_y$



Matrix form :-

$$T_v = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{pmatrix} x + t_x \\ y + t_y \\ 1 \end{pmatrix}$$

$$\boxed{\begin{aligned} x' &= x + t_x \\ y' &= y + t_y \end{aligned}}$$

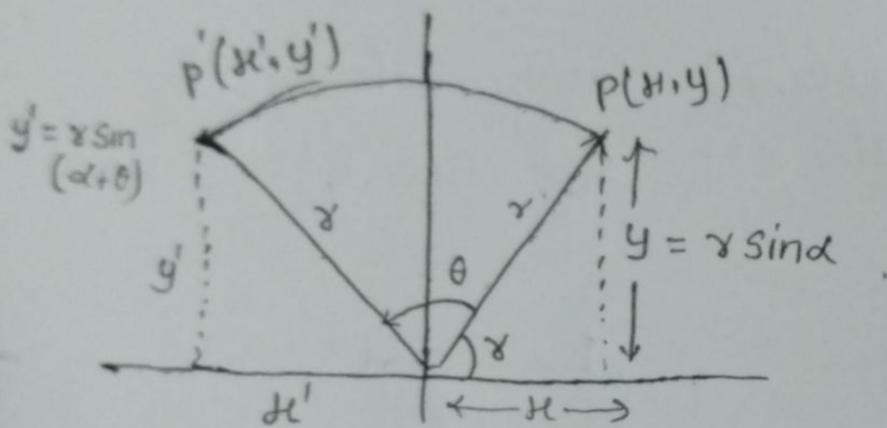
2) Rotation Transformation :—

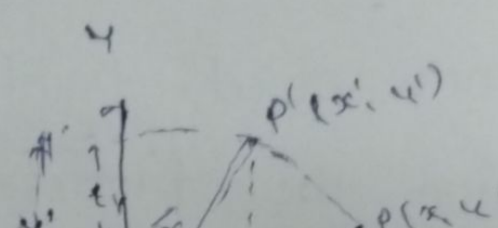
Rotation about Origin :—

In Rotation the object is rotated θ° about the origin. the direction of rotation is Counter clockwise (conti) if θ is positive angle and clockwise if θ is negative angle.

the transformation of Rotation R_θ is :

$$\boxed{P' = R_\theta \cdot P}$$





$x' = r\cos(\alpha+\theta)$ $\quad x = r\cos\alpha$

$$x' = r\cos\alpha \cdot \cos\theta - r\sin\alpha \cdot \sin\theta$$

$$\boxed{x' = x\cos\theta - y\sin\theta}$$

$$y' = r\sin(\alpha+\theta)$$

$$= r\sin\alpha \cdot \cos\theta + r\cos\alpha \cdot \sin\theta$$

$$y' = y\cos\theta + x\sin\theta$$

$$\boxed{y' = x\sin\theta + y\cos\theta}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{array}{c|cc} & \text{coff } x & \text{coff } y \\ \hline x' & \cos\theta & -\sin\theta \\ y' & \sin\theta & \cos\theta \end{array} \begin{bmatrix} x \\ y \end{bmatrix} ; R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

जब rotation clockwise करना हो तो sin के चिन्ह बदल देंगे i.e. '−' का '+', '+' का '−'

12/08/024

Que) a) Find the matrix that represents rotation about of an objects by 30° about the Origin.

b) what are the new Co-ordinates of the points (2, -4)

Solⁿ :- a) $R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$R_{30^\circ} = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix} \Rightarrow \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix}$

$\boxed{p' = R_\theta \cdot P}$ $P = \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

$p' \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 2 \\ -4 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} \sqrt{3} + 2 \\ 1 - 2\sqrt{3} \end{bmatrix}$ ## $x' \Rightarrow \sqrt{3} + 2$

$y' \Rightarrow 1 - 2\sqrt{3}$

means to say किसी चीज का size increase or decrease करना

3.) Scalling Transformation :-

Scaling with respect to Origin :-

→ Scaling is the process of Expanding or Compressing the dimensions of an object.

→ There are two factors used in scaling transformation i.e S_x and S_y. where S_x is scale factor for x Coordinate where S_y is scale factor for the y- Co-ordinate.

→ A Scaling Constant greater than one indicates an Expansion of length and less than one dimension of length.

→ the scaling transformation $S_{Sx,Sy}$ is given by

$$\boxed{P' = S_{Sx,Sy} \cdot P}$$

where $\boxed{\begin{matrix} x' \Rightarrow S_x \cdot x \\ y' \Rightarrow S_y \cdot y \end{matrix}} \begin{matrix} + 0y \\ + 0x \end{matrix}$

$$S_{Sx,Sy} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix}$$

In homogenous form

$$\Rightarrow \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

↑ Homoginious form

4) Mirror Reflection :-

about an axis :-

* If x-axis work as a mirror

$$\boxed{P' = M_x \cdot P}$$

where $x' \Rightarrow x$
$y' \Rightarrow -y$

$$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

* If y-axis work as a mirror

$$\boxed{P' = M_y \cdot P}$$

where $x' \Rightarrow -x$
$y' \Rightarrow y$

$$M_y \Rightarrow \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

Inverse Geometric transformation

1.) <u>Translation</u> $T_V^{-1} = T_{-V}$ i.e translation in opposite direction.

2.) <u>Rotation</u> $R_\theta^{-1} = R_{-\theta}$ i.e Rotation in opposite direction.

3.) <u>Scaling</u> $S_{S_x S_y}^{-1} = S_{1/S_x \cdot 1/S_y}$

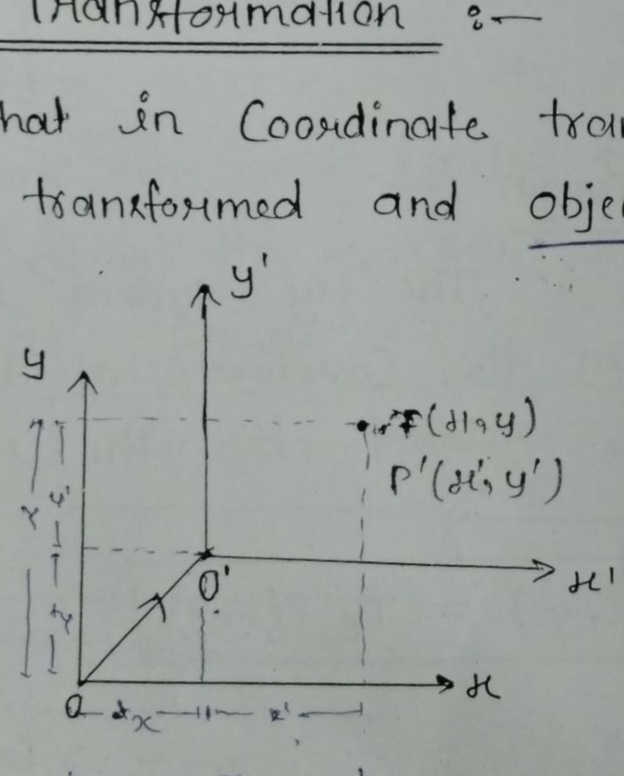
4.) <u>Mirror Reflection</u> $M_x^{-1} = M_x$ and $M_y^{-1} = M_y$

$$T_V = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}, \quad R_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$\boxed{16/08/024}$

Co-ordinate Transformation :-

we know that in Coordinate transformation Coordinate System is transformed and object is made Stationery.



1) Translation :— If xy Coordinate system is displaced to a new position, where the direction and distance of the displacement is given by the vector $V = t_x i + t_y j$, the Coordinate of a point in new position is given by following relation.

$$\boxed{P'(x', y') = \overline{T}_V \cdot P(x, y)}$$

where

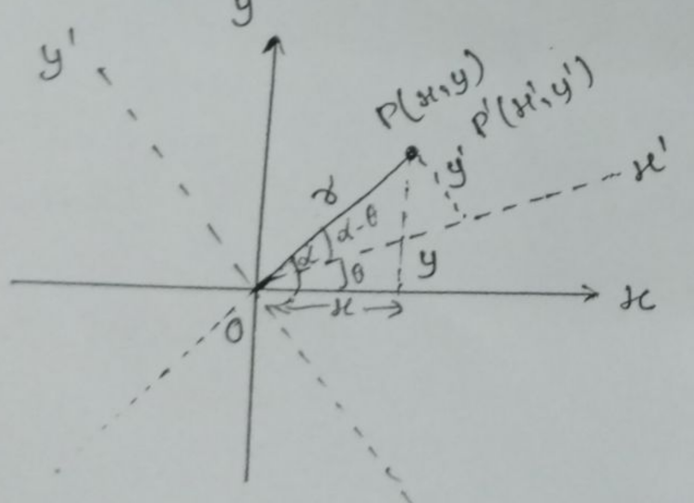
$$x' = x - t_x$$
$$y' = y - t_y$$

$$\overline{T}_V = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \\ 0 & 0 & 1 \end{bmatrix}$$

2) Rotation about the Origin :—

The xy system is rotated θ° about the Origin, then the Coordinate of the point in both system are related by the rotation transformation $\overline{R}_\theta$

$$\boxed{P'(x', y') = \overline{R}_\theta \, P(x, y)}$$



$$x = r \cos\alpha$$
$$y = r \sin\alpha$$

$$x' = r \cos(\alpha - \theta)$$
$$y' = r \sin(\alpha - \theta)$$

$x' = r\cos\alpha \cdot \cos\theta + r\sin\alpha \sin\theta$

$$\boxed{x' = x\cos\theta + y\sin\theta}$$

Matrix multi-plication हमेशा Right to left होगा।

$y' = r\sin\alpha \cos\theta - r\cos\alpha \cdot \sin\theta$

$y' = y\cos\theta - x\sin\theta$

$y' = -x\sin\theta + y\cos\theta$

$$\bar{R}_\theta = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

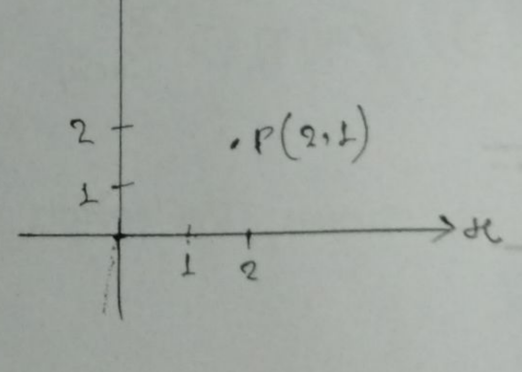
3.) <u>Scaling with respect to Origin</u> :-

Suppose that a new Co-ordinate System is formed by leaving the Origin and Coordinate axes unchanged, but introducing different unit of measurement along the x and y axes. If the new units are obtained from the old units by a Scaling of S_x units along the x axis and S_y units along the y axes as follows :-

$$\boxed{P'(x', y') = \bar{S}_{S_x, S_y}\ P(x, y)}$$

where

$$\boxed{x' = \frac{1}{S_x} \cdot x\ ,\ y' = \frac{1}{S_y} \cdot y}$$



$S_x = 2$
$S_y = 1/2$

$x' = \frac{1}{2} \times 2 = 1\ ,\ y' = 2 \times 1 = 2$

4) Mirror reflection about axis :-

If the new Coordinate system is obtained by reflecting the old system about either x or y axis the relationship between Coordinates is given by the Coordinate transformation $\overline{M}_x$ and $\overline{M}_y$.

→ for reflection about the x-axis ⇒

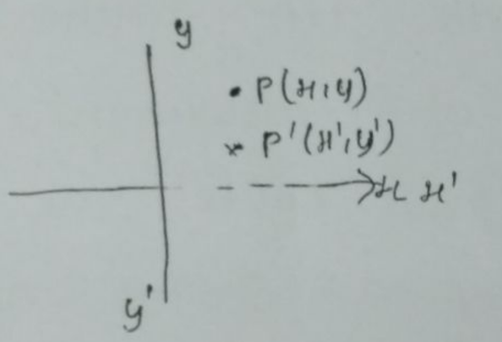
$$\boxed{P'(x', y') = \overline{M}_x \, P(x, y)}$$

where $x' \Rightarrow x$ & $y' \Rightarrow -y$

- for reflection about the y-axis ⇒

$$\boxed{P'(x', y') = \overline{M}_y \, P(x, y)}$$

where $x' \Rightarrow -x$ & $y' = y$



Inverse Coordinate transformation :-

1) Translation $\overline{T}_V^{-1} = \overline{T}_{-V}$ transformation is the opposite direction.

2) Rotation $\overline{R}_\theta^{-1} = \overline{R}_{-\theta}$ Rotation in opposite direction.

3) Scaling $\overline{S}_{S_x S_y}^{-1} = \overline{S}_{1/S_x \, 1/S_y}$

4) Mirror Reflection $\overline{M}_x^{-1} = \overline{M}_x$ and $\overline{M}_y^{-1} = \overline{M}_y$

Ques) Describe the transformation that rotates an object point $Q(x, y)$, θ degree about a fixed Centre of rotation $P(h, k)$.

Soln :- point shift to origin

Rotation about origin   Soln in three step

1.) Translate so that the Centre of rotation $P(h,k)$ ~~is~~ at the position. Origin (T_{-v})

2) Perform a rotation of θ degree about the Origin. (R_θ)

3) Translate the Origin back to $P(T_v)$

$$\boxed{R_{\theta,P} = \overset{3}{T_v}\ \overset{2}{R_\theta}\ \overset{1}{T_{-v}}}$$

$$R_{\theta,P} = \underset{T_v}{\begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix}} \underset{R_\theta}{\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{T_{-v}}{\begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix}}$$

$$R_{\theta,P} \Rightarrow \begin{bmatrix} \cos\theta & -\sin\theta & [-h\cos\theta + k\sin\theta + h] \\ \sin\theta & \cos\theta & [-h\sin\theta - k\cos\theta + k] \\ 0 & 0 & 1 \end{bmatrix} \quad \#$$

Trick

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \\ & \end{bmatrix} \begin{bmatrix} x - h \\ y - k \end{bmatrix} = \begin{bmatrix} \cancel{(\cos\theta - \sin\theta)}\cdot \\ \cos\theta(x-h) - \sin\theta(y-k) + h \\ \sin\theta(x-h) + \cos\theta(y-k) + k \end{bmatrix}$$

(h,k) को (0,0) बनाने के लिए point में हटाये

Point

Last में जो हटाया गया है, उसे add कर दें।

Que 1) Perform a 45° rotation of triangle A(0,0), B(1,1), C(5,2)

a) about the Origin

b) about the P(-1,-1)

1) point to origin
2) Origin to rotation
3) one step

Soln:—

a) Matrix ta traingle

$$P = \begin{array}{c} \begin{matrix} A & B & C \end{matrix} \\ \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

$$\boxed{P' = R_\theta P}$$

$$R_{45^\circ} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ & 0 \\ \sin 45^\circ & \cos 45^\circ & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix} = \begin{array}{c} \begin{matrix} A' & B' & C' \end{matrix} \\ \begin{bmatrix} 0 & 0 & 3/\sqrt{2} \\ 0 & \sqrt{2} & 7/\sqrt{2} \\ 1 & 1 & 1 \end{bmatrix} \end{array}$$

$A'(0,0)$ $B'(0,\sqrt{2})$ $C'(3/\sqrt{2}, 7/\sqrt{2})$ Ans

b) about the P(-1,-1) :-

$$R_{\theta P} \Rightarrow \overset{3}{T_V}\ \overset{2}{R_\theta}\ \overset{1}{T_{-V}} \quad (\text{Steps})$$

(First Inverse translatation then Rotation then again translatation)

$$\boxed{P' \Rightarrow R_{\theta,p} \cdot P}$$

P(-1,-1)

$tx \Rightarrow -1 \quad ty \Rightarrow -1$

$$R_{45,p} \Rightarrow \overset{T_V}{\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}} \overset{R_\theta}{\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{T_{-V}}{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\Rightarrow \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & (\sqrt{2}-1) \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{P' = R_{45,p}, P}$$

$$P' = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} & -1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & (\sqrt{2}-1) \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$$

$$P' \Rightarrow \begin{bmatrix} -1 & -1 & \left(\frac{3}{\sqrt{2}}-1\right) \\ (\sqrt{2}-1) & (2\sqrt{2}-1) & \left(\frac{9}{\sqrt{2}}-1\right) \\ 1 & 1 & 1 \end{bmatrix}$$

$A' \Rightarrow (-1, \sqrt{2}-1)$

$B' \Rightarrow (-1, 2\sqrt{2}-1)$

$C' \Rightarrow \left(\frac{3}{\sqrt{2}}-1, \frac{9}{\sqrt{2}}-1\right)$

Ans

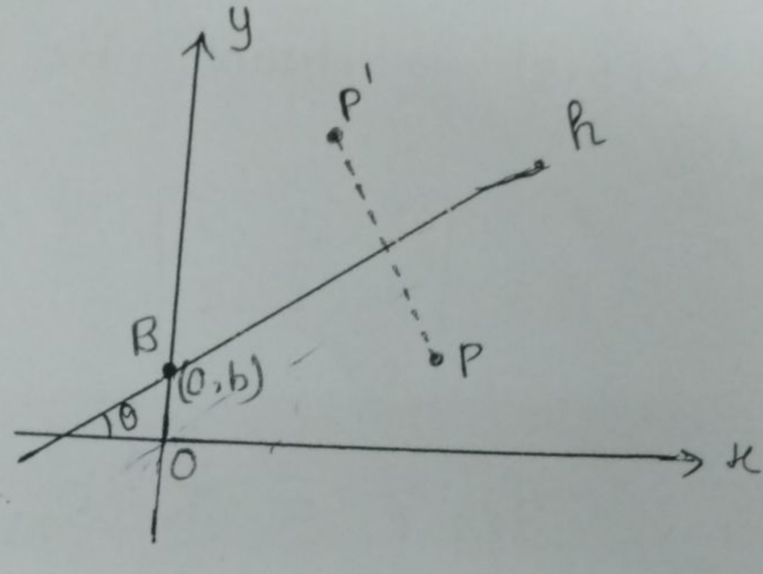
$$\boxed{P' = S_{2,2,C} \cdot P}$$

$$P' = \begin{bmatrix} 2 & 0 & -5 \\ 0 & 2 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$$P' = \begin{bmatrix} -5 & -3 & 5 \\ -2 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

$A' = (-5, 2)$
$B' = (-3, 0)$
$C' = (5, 2)$ Ans

Que 5) Describe the transformation M_1 which reflects an object about a line L, let line L have a y intercept (0, b) and an angle of inclination $\theta°$ (with respect to the x-axis)



Soln :-

1.) Translate the Intersection point B to the Origin.
2.) Rotate by $-\theta°$, so that the line L aligns with the x-axis.
3.) Mirror reflects about the x-axis.
4.) Rotate back by $\theta°$
5.) Translate B Back to (0, b)

$$\boxed{M_L \Rightarrow \overset{5}{T_V}\ \overset{4}{R_\theta}\ \overset{3}{M_x}\ \overset{2}{R_{-\theta}}\ \overset{1}{T_{-V}}}$$