# Assignment 5: CS 663, Fundamentals of Digital Image Processing

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# Answer 1(a):

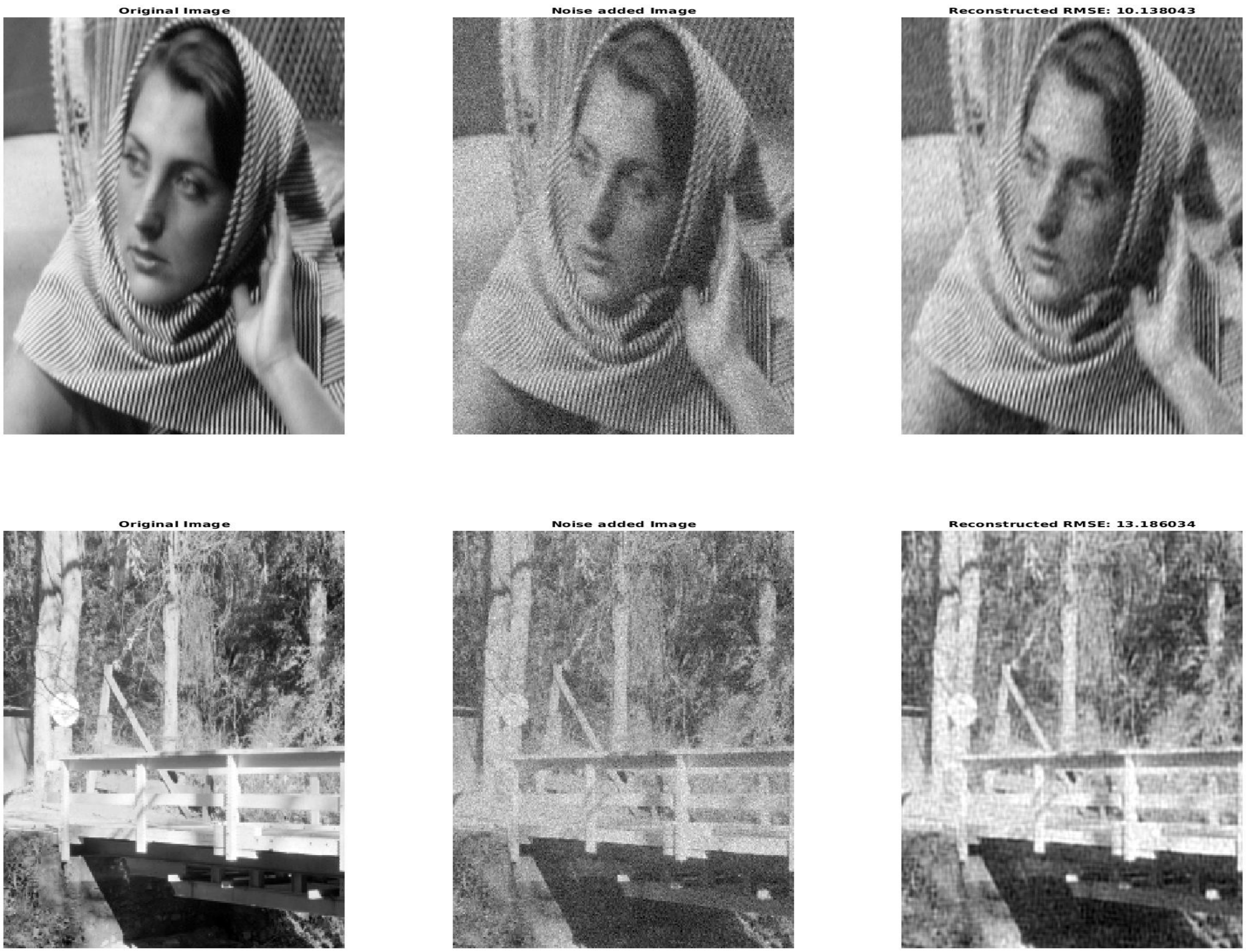


Figure 1: Figure from left to right represents original, noise added and reconstructed using all patches.

## 1(b):


Figure 2: Figure from left to right represents original, noise added and reconstructed using 200 nearest neighbours patches.

## 1(c):

(a) First figure represents the reconstruction performed using the first method,second image using first 200 patches and third one using the bilateral filter over Barbara image.

(b) First figure represents the reconstruction performed using the first method,second image using first 200 patches and third one using the bilateral filter over the patch of stream image.

Figure 3: Some grouped images

## 0.1 1(d)

Yes this , procedure of the clipping of the values may help , As we are adding the noise to the Existing image having the value ranged in [0,255] , therefore the noise affected pixels will be there whose value will be outside the given window .
so by clipping, we can at least be sure that that the noise affected pixels value having the near to boundary value will be having less noisy effect . Obs: the RMSE is reduced by some amount with with respect to the non-clipping case .(previously it was 12.37 , and after clipping it is 12.36) so we may roughly say that the ,there

will be slight difference in denoised in both cases . (Please find the clipped_denoised result for this part inside **directory** 1/images/clamped_output_barbara.jpg)

## Answer 2:

### (a)

Method uses the translation property of the Fourier transform,i.e. Fourier shift theorem.So, if there are two images say $f_1$ and $f_2$ that differ by some displacement say($x_0$ and $y_0$) then,

$$f_2(x, y) = f_1(x - x_0, y - y_0).$$

and on using Fourier transform we can represent them using $F_1$ and $F_2$.

$$F_2(\xi, \eta) = e^{-j2\pi(\xi x_0 + \eta y_0)} * F_1(\xi, \eta)$$

. Then cross-power spectrum of the two images $f_1$ and $f'$ with their respective Fourier transform F and F' is defined as:

$$\frac{F(\xi, \eta)F^{'*}(\xi, \eta)}{|F(\xi, \eta)F'(\xi, \eta)|} = e^{j2\pi(\xi x_0 + \eta y_0)}$$

where $F^*$ denotes the complex conjugate of F. Then the shift theorem is used which guarantees that the phase of the cross power spectrum is equivalent to the phase difference between the images. Then, by taking the inverse Fourier transform of the representation in the frequency domain we get the function that has an impulse which is zero everywhere except at the displacement that is needed to register the given two images.
The time complexity to predict the translation for the image of size N*N is $N^2 logN$. Time complexity associated with Pixel wise classification for an window size of W*W is O($W^2N^2$).

### (b)

As we have $f_1(x, y)$ and $f_2(x, y)$ and corresponding Fourier transforms and their Magnitudes are $F_1(\xi, \eta)$ , $F_2(\xi, \eta)$ and $M_1(\xi.\eta)$ ,$M_2(\xi, \eta)$ ,respectively both .

rotation of the image $f_1$ say by $_0$, then,

$$f_2(x, y) = f_1(x \cos\theta_0 + y \sin\theta_0, -x \sin\theta_0 + y \cos\theta_0)$$

Taking the Fourier transform both side , we will get

$$F_2(x, y) = F_1(u \cos\theta_0 + v \sin\theta_0, -u \sin\theta_0 + v \cos\theta_0)$$

, their corresponding magnitude can be given as ,

$$|F_1(\xi, \eta)| = M_1(\xi, \eta)$$

$$|F_2(\xi, \eta)| = M_2(\xi, \eta)$$

Converting the given spatial coordinates into polar coordinates , ($\xi$ ,$\eta$ )to ($\rho$ , $\theta$) , and we also know that the magnitude will be same .
Therefore the rotation problem in the frequency spatial domain will be converted to the frequency polar domain ,

$$M_2(\rho, \theta) = M_1(\rho, \theta - \theta_0)$$

Where the calculation of the rotation $\theta_0$ problem is converted to now the translation problem ,
Here, we can see that calculate the rotation by using the cross-power spectrum by converting the magnitudes of $F_1$ and $F_2$ to polar coordinates .

# Answer 3:

Given a rectanguar matrix $A$ $s.t.$ $A_{m\times n}$ $(m \leq n)$ and given that $P = A^T A$ and $Q = AA^T$ ,

## a

to prove that $y^t Py \geq 0$ and $z^t Qz \geq 0$

*Proof.*

$$\begin{aligned} y^T Py &= y^T A^T Ay \\ &= (Ay)^T Ay \\ &= ||Ay||_2^2 \\ &\geq 0 \end{aligned}$$

□

Similarly , to prove that $z^t Qz \geq 0$

*Proof.*

$$\begin{aligned} z^T Qz &= z^T AA^T z \\ &= (A^T z)^T A^T z \\ &= ||A^T z||_2^2 \\ &\geq 0 \end{aligned}$$

□

To Prove the Non-negative eigenvalue , for P and Q
since $(\lambda, y) = eigenvalue(P), eigvector(P)$
and $(\mu, z) = eigenvalue(Q), eigvector(Q)$ ,

*Proof.*

$$
\begin{aligned}
ForP& \\
Py &= \lambda y \\
AA^T y &= \lambda y \\
&\text{Multiply both side with with } y^t \\
y^t Py &= \lambda y^t y \\
y^t Py &= \lambda ||y||_2^2 \\
&\text{since } y^t Py \geq 0 \text{ and } ||y|||_2^2 \geq 0 \\
&\Rightarrow \lambda \geq 0 \\
Similarly\ for\ Q& \\
Qz &= \mu z \\
&\text{Multiply both side with with } z^t \\
z^t Qz &= \lambda ||z||_2^2 \\
&\Rightarrow \mu \geq 0
\end{aligned}
$$

□

## 3(b)

Since , $\lambda, u = eigenvalue(P), eigvector(P)$ ,
TPT , $\lambda, Au = eigenvalue(Q), eigvector(Q)$ ,

*Proof.*

$$
\begin{aligned}
Pu &= \lambda u \\
A^T Au &= \lambda u \\
&\text{Multiply both side with with } A \\
(AA^T)Au &= \lambda Au \\
Q(Au) &= \lambda Au \\
&\text{From above we can see that } (lambda, Au) = (eigenvalue(Q), eigenvector(Q))
\end{aligned}
$$

□

Since , $\mu, v = eigenvalue(Q), eigvector(Q)$ ,
TPT , $\mu, A^T v = eigenvalue(P), eigvector(P)$ ,

*Proof.*

$$
\begin{aligned}
Qv &= \mu v \\
AA^T v &= \mu v \\
&\quad \text{Multiply both side with with } A^T \\
(A^T A) A^T v &= \mu A^T v \\
P(A^T v) &= \mu A^T v \\
&\quad \text{From above we can see that } (\mu, A^T v) = (eigenvalue(P), eigenvector(P))
\end{aligned}
$$

□

Since size($P$) ,size($Q$) =$n \times n$ ,$m \times m$ ,
therefore , size($u$) , size($v$) =$n \times 1$ ,$m \times 1$ , Therefore number of elements in $u$ ,$v$ are $n$ and $m$ respectively .

## 3(c)

Given :

$$
u_i = \frac{A^T v_i}{\|A^T v_i\|_2} \tag{1}
$$

TPT , $Au_i = \gamma_i v_i$ for non negative $\gamma_i$
Let $\mu$ and $v_i$ be ith eigenvalue and eigenvector of Q

*Proof.*

$$
\begin{aligned}
Qv_i &= \mu_i v_i \\
AA^T v_i &= \mu_i v_i \\
&\quad \text{since} \\
Au_i &= \frac{AA^T v_i}{\|A^T v_i\|_2} \\
&\quad \text{therefore above can be written as , using Q relation ,} \\
Au_i &= \frac{\mu_i v_i}{\|A^T v_i\|_2} \\
&\quad \text{put } \gamma_i = \mu_i / \|A^T v_i\|_2
\end{aligned}
$$

$\square$

Using above we can write as $Au_i = \gamma_i v_i$ ,
And as $\gamma_i \geq 0$ and real , as we proved $\mu_i$ the eigenvalue of $Q$ will be non-negative , therefore $\gamma$ is non-negative , Hence proved .

## 3(d)

Given : $u_i$ ith eigenvector of $A^T A$
$v_i$ ith eigenvector of $AA^T$
$V = [u_1|u_2|u_3|...|u_m]$ and $U = [v_1|v_2|v_3|...|v_m]$ and $\Gamma$ is diagonal matrix of the non negative values $\gamma_i$

As we know that , and $u_i^t u_j = 0$ and $v_i^t v_j = 0$ for $i \neq j$

To prove that , $A = U\Gamma V^T$ ,

*Proof* :

Let $\mathbf{U}\Gamma$ given as ,

$$\mathbf{U}\Gamma = [v_1\,|v_2|\cdots|\,v_m]\begin{pmatrix} \gamma_1 & 0 & \cdots & 0 \\ 0 & \gamma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \gamma_m \end{pmatrix}$$

SImplifying furthur ,

$$\mathbf{U}\Gamma = [\gamma_1 v_1\,|\gamma_2 v_2|\cdots|\,\gamma_m v_m]$$

from part c i.e. above as we got $Au_i = \gamma_i v_i$
therefore

$$\mathbf{U}\Gamma = [Au_1\,|Au_2|\cdots|\,Au_n]$$
$$\mathbf{U}\Gamma = \mathbf{A}\,[u_1\,|u_2|\cdots|\,u_n]$$

$$\mathbf{U}\Gamma = \mathbf{AV}$$

multiplying both side by $V^T$ , we get

$$\mathbf{U\Gamma V = AVV^T}$$

from above part c, it can be seen that the $v_i$ are normalised vector with respect to its own magnitude therefore corrosponding stacked matrix of $u_i$ which is given as $\mathbf{V}$, can be evaluated to , $\mathbf{VV^T = I_n}$

$$\mathbf{U\Gamma V^T = AI_n}$$

$$\mathbf{U\Gamma V^T = A}$$

As , we have already proved that the $\gamma_i$ are the non-negative values .
hence proved .