Assignment 1: CS 663, Fundamentals of Digital Image Processing

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Answer 1:

The function is given is:

$$v(x,y) = ax + by + cxy + d$$

We have to prove or disprove the linearity of the of function

$$v(x,y)$$

,
first replacing x with $(px_1 + x_2)$, and treating y as constant, we get :

$$\begin{align} v(px_1 + x2, y) &= a(px_1 + x_2) + by + c(px_1 + x_2)y + d \tag{1} \\ &= apx_1 + ax_2 + by + cpx_1y + cx_2y + d \tag{2} \end{align}$$

Now calculating $pv(x_1, y) + v(x_2, y)$ we get

$$\begin{align} pv(x_1, y) + v(x_2, y) &= p(ax_1 + by + cx_1y + d) + ax_2 + by + cx_2 + d \tag{3} \\ &= a(px_1 + x_2) + by(p+1) + cy(px_1 + x_2) + d(p+1) \tag{4} \end{align}$$

from above we can see that $v(px_1 + x_2, y)$, and $pv(x_1, y) + v(x_2, y)$ are not equal, therefore, $v(x, y)$ is non-linear with respect to x keeping y constant

similarly we can see that the given function function $v(x, y)$ with respect to y keeping x constant is also non-linear.

Answer 2:

We need to prove that another round of the histogram equalisation will produce the same result,
for original image consider random variable R having L intensity levels , and $S = T(r)$ as the transformed random variable.
Let the values taken by S and R as s and r respectively in the range 0 to $(L-1)$, and the $p_S(s)$ and $p_R(r)$ are the corresponding probability density function corresponding to transformed(first equalised) and original image, ,
for transformation we have the relation,

$$p_S(s) = p_S(T(r)) = \frac{p_R(r)}{T'(r)} = \frac{p_R(r)}{d(T(r)/dr} \tag{5}$$

and the transformated variable is given as,

$$s = T(r) = (L-1)\int_0^r p_R(w)dw \tag{6}$$

Now using equation (6) in equation (5), we get,

$$p_S(s) = \frac{1}{(L-1)} \tag{7}$$

Considering the new transformation over the previously transformed image. The new random variable as S' with the density function $p_{S'}(s')$,
on the Equalisation of equalised image we get density as(from(6)):

$$p_{S'}(s') = \frac{p_S(s)}{T'(s)} \tag{8}$$

and,

$$s' = T(s) = (L-1)\int_0^s p_S(w)dw \tag{9}$$

Put the equation (9) in equation (8), we get

$$p_{S'}(s') = \frac{p_S(s)}{T'(s)} = \frac{p_S(s)}{d(T(s))/ds} = \frac{1}{(L-1)} \tag{10}$$

Observation: from equation (10) it is clear that after transforming the transformed image we got the same probability density function as the first transformed image to histogram equalisation. Which is the same result round.

Answer 3:

We have image I with PMF $p_I(i)$, PMF of image J is $p_J(j)$
To get the expression for the image $I + J$,
Let K be the new random variable as $Z = I + J$, and the corresponding probability mass function $p_{I,J}(.)$ we will charectrise it as $p_Z(.)$:

$$\begin{aligned} p_Z(z) &= P(Z = z) \\ &= \sum_{i+j=z} p_{I,J}(i, j) \\ &= \sum_i P(I = i, J = z - i) \\ &= \sum_i p_{I,J}(i, z - i) \end{aligned}$$

therefore we get pmf expression random variable $Z = I + J$ as:

$$p_Z(z) =_i p_{I,J}(i, z - i) \tag{11}$$

If we consider the images I and J as independant then we get the expression for the discrete joint probability mass function as:

$$p_Z(z) = \sum_i p_I(i)p_Y(z - i) = \sum_j p_I(z - j)p_J(j) = \tag{12}$$

The above expression also can be considered as the discrete convolution of dicrete RV.

Answer 4:

Plotting the measure of dependence between $J1$ and $J4$,
Observation: by measuring parameter of dependence between the image $J1$ and

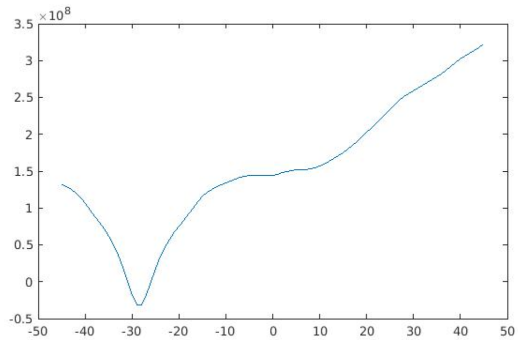

Figure 1: theta on x and NCC on y axis

$J4$ we can observe that we have the optimal roation with minimum cross correlation value, at -28. degrees shown in Figure.1 .

for Joint entropy we have it around -30 degree Figure(2)
for Quadratic Mutual information we got two optimum values at -32.5 degrees and -28.0 degrees approx figure(3).

Intuition Regarding QMI: As we know that M can be given as,usual Information can be given as weighted sum over the bun pair of the images,here QMI allows the efficient calculation o f the mutual information over the range of image values, whereas in case of the trivial calculation of the Mutual information we may face the range of the values where the value is undefined.
As the QMI is the arithmetic function it can be efficient method than the logarithmic Mutual information function.

C

Colorbar of the image$J1$ and $J4$ ($J3$ optimally roatated at -28 degrees0).

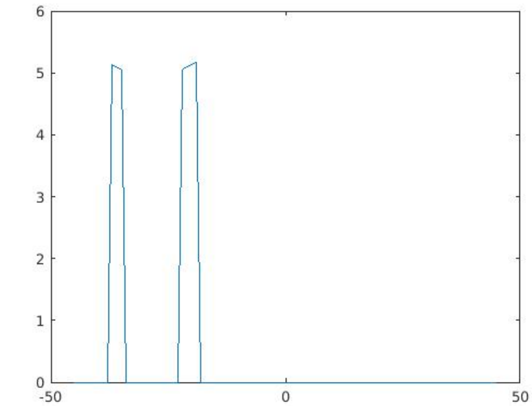

Figure 2: theta vs Joint entropy

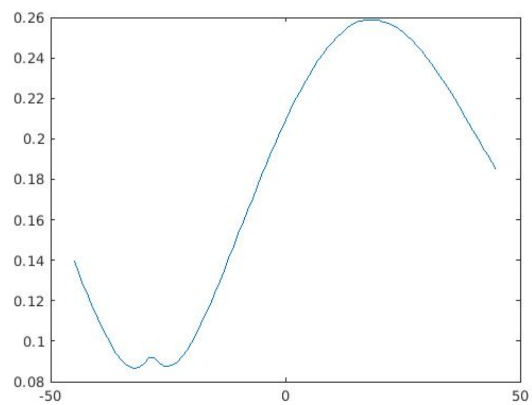

Figure 3: theta vs QMI

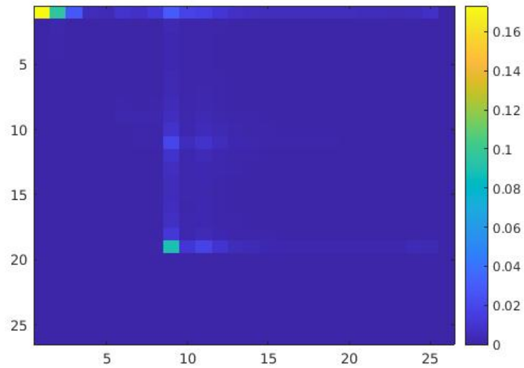

Figure 4: Colorbar of image J1 and J4

Answer 5

Figure 5: goi1

Figure 6: goi2

Figure 7: Nearest Neighbour interpolation

Figure 8: goi1

Figure 9: goi2

Figure 10: Bilinear interpolation

Part d

Question: In the first step, suppose that the n points you chose in the first image happened to be collinear. Explain the effect on the estimation of the affine transformation matrix?

Answer: Since affine transformation preserves the collinearity,parallelism and convexity so after the affine transformation the points will still remain collinear. i.e. after the implication of affine transformation points that were collinear will still lie in the same line.