Project : CS 754, Advanced Image Processing

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sLLE (Spherical Locally Linear Embedding)

We know the problem of tomographic reconstruction done using projections taken at some random unknown angles. We want to estimate these view angles robustly through the random observations of the projections. LLE is a technique which provides the non linear embedding of points on a flat manifold. Here, in this project this flat manifold is replaced by spherical manifold. Hence the LLE is extended to sLLE.

sLLE

1. In LLE, we firstly compute the neighbours of each data point , X_i.

2. We compute the weights W_{ij} that best represents or reconstructs each data point X_i from its neighbour.i.e.,

$$E(w) = \sum_i |X_i - \sum_j W_{ij} X_j|^2 \tag{1}$$

   here 1 is being minimized for computing W_{ij}.

3. Vector Y_i is computed using the W_{ij}

$$\phi(y) = \sum_i |Y_i - \sum_j W_{ij} Y_j|^2 \tag{2}$$

   and 2 is minimized using the bottom non zero eigen vectors.

Standard code of LLE is available and is further updated to obtain the functionality of sLLE. sLLE contains the same first two steps as that of LLE but last step is little different as the flat manifold has been replaced with spherical manifold.
further elaborating the 2

$$tr((Y - WY)(Y - WY)^T)$$

which can be rewritten as :

$$\phi(Y) = tr((Y - WY)^T(Y - WY))$$

$$= tr(Y^TY - Y^TW^TY - Y^TWY - Y^TW^TWY)$$

$$= tr(Y^T(I - W^T - W - W^TW)Y)$$
$$= tr(Y^T(I - W)^T(I - W)Y)$$
$$= tr(Y^TMY)$$

above objective function is similar to LLE but we want to include that the embedding points to lie on an unit sphere.Hence the constraint becomes

$$min_{Y,B}\ tr(Y^TMY) \quad s.t. \quad Y^TBY = Z^TZ = I \tag{3}$$

Now this 3 is solved in the following steps:

1. Compute Y by minimizing

$$min_Y\ tr(Y^TMY) \quad s.t. \quad Y^TBY = Z^TZ = I \tag{4}$$

   solution to 4 is given from Theorem 1.2 in just equivalent to first D eigen values of the generalized eigenvalue problem MY=γBY where the eigenvalues of the problem are given by $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$.

2. we want to compute B with the following constraint to be followed that the diagonal elements of $\sqrt{B}YY^T\sqrt{B}$ are all unit ones and this is accomplished by setting

$$B = diag(\frac{1}{y_i'y_i})$$

the output obtained is Z=$\sqrt{B}Y$.

Determining View Angles

We have the 2 dimensional vector obtained after dimensionality reduction in the fourier domain.
for calculating the initial set of view angles we use the inverse trignometric function to the coordinates of Z_i which denotes the i^{th} embedding point.

$$\phi_i = arctan(\frac{Z_i(2)}{Z_i(1)}) \tag{5}$$

Now the obtained angles are sorted based on the values of $\phi_1, \phi_2, \phi_3, ..., \phi_n$ and they are rearranged along the circle.

Noise Robustness

$$SNR = 10log_{10}(\frac{Signal}{Noise}) \tag{6}$$

where,
signal denotes the standard deviation of original projections without noise.
Noise denotes the standard deviation of the noise added to the projection vectors.

Results

Formulas used for reporting the results are:

$$MSE = \frac{1}{mn}\sum_{i=0}^{m-1}\sum_{j=0}^{n-1}[I_1(i,j) - I_2(i,j)]^2 \tag{7}$$

where,
I_1 and I_2 represents two images.
m and n represents the size of image.

$$PSNR = 20log_{10}(\frac{MAX_I}{\sqrt{MSE}}) \tag{8}$$

MAX_I represents the maximum possible pixel value of input image.

Results shown below are as follows:
first 4 set of images are reconstructed using LLE.
second set of 4 images are reconstructed using sLLE.
Third set of 4 images are reconstructed using sLLE with additive noise added to projections.

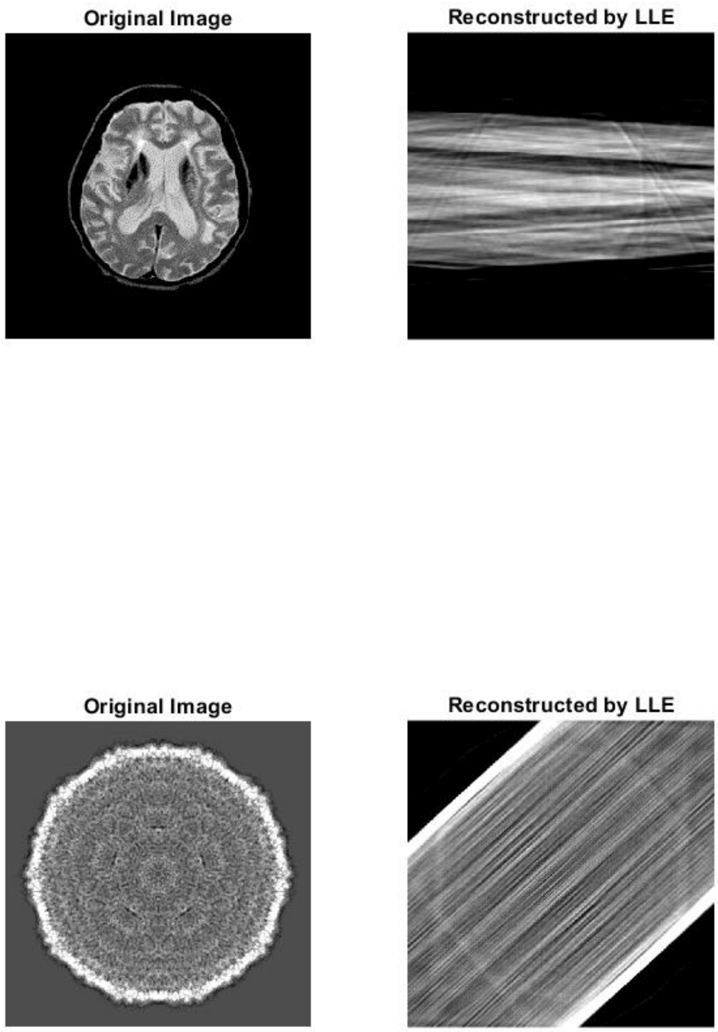

Figure 1: original image and reconstructed image of Brain MR and Cyanophage using LLE.

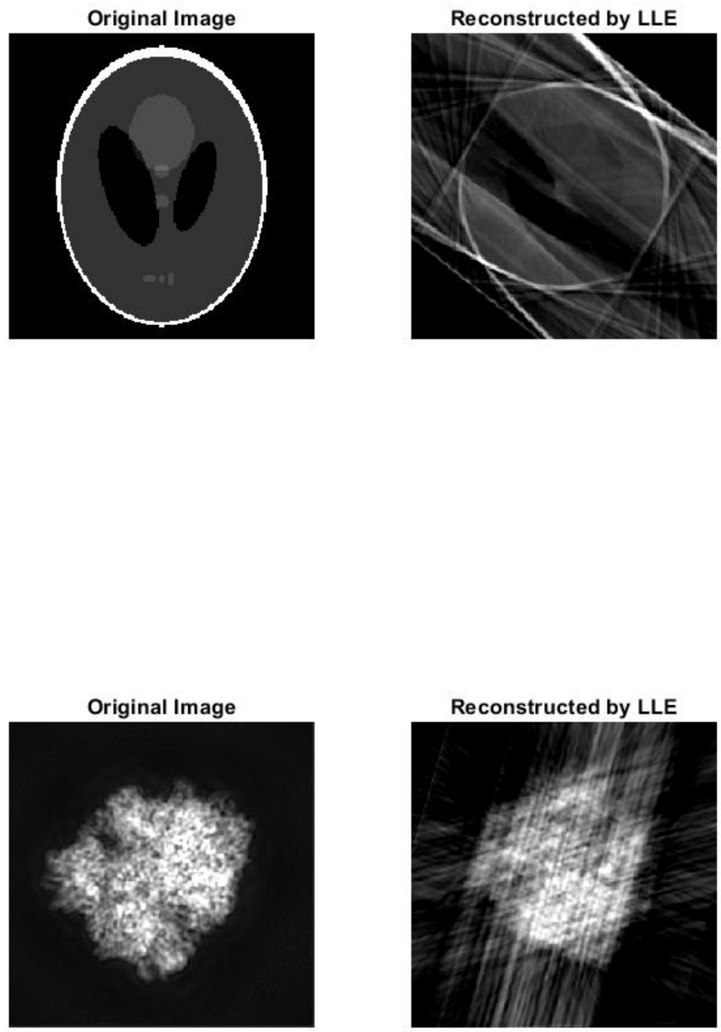

Figure 2: original image and reconstructed image of Phantom and Ribosome using LLE.

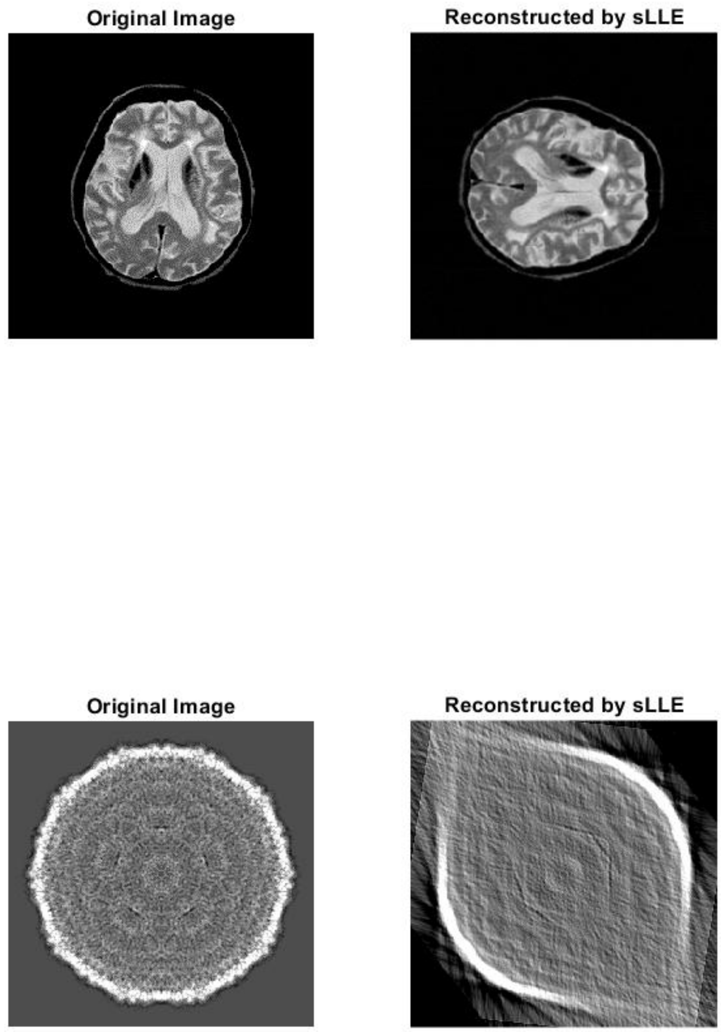

Figure 3: original image and reconstructed image of Brain MR and Cyanophage using sLLE.

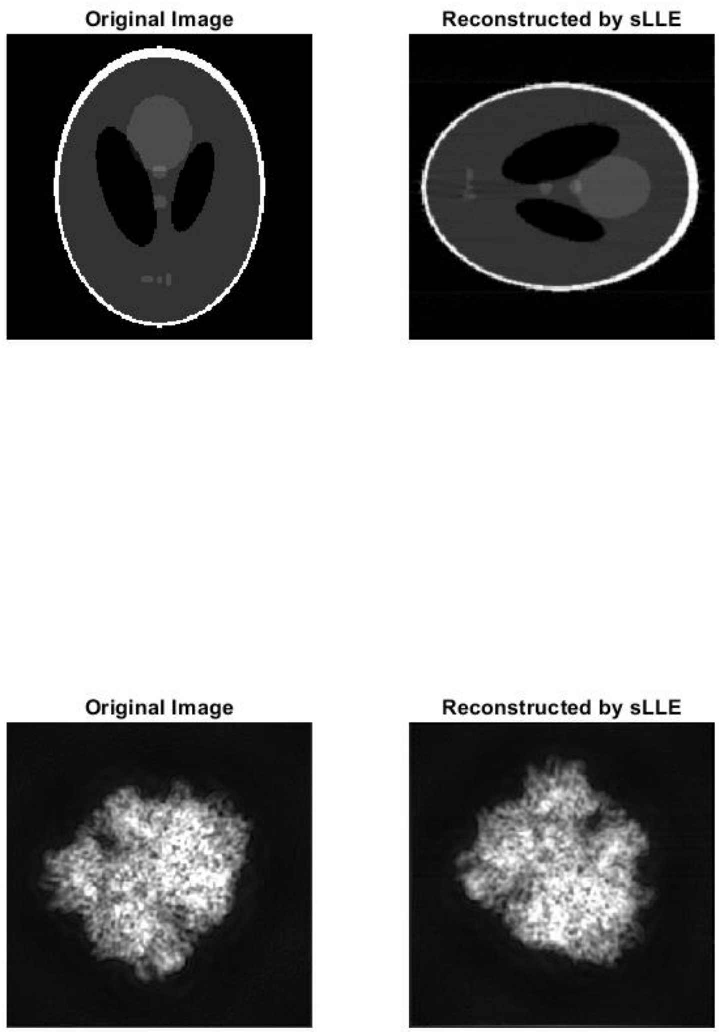

Figure 4: original image and reconstructed image of Phantom and Ribosome using sLLE.

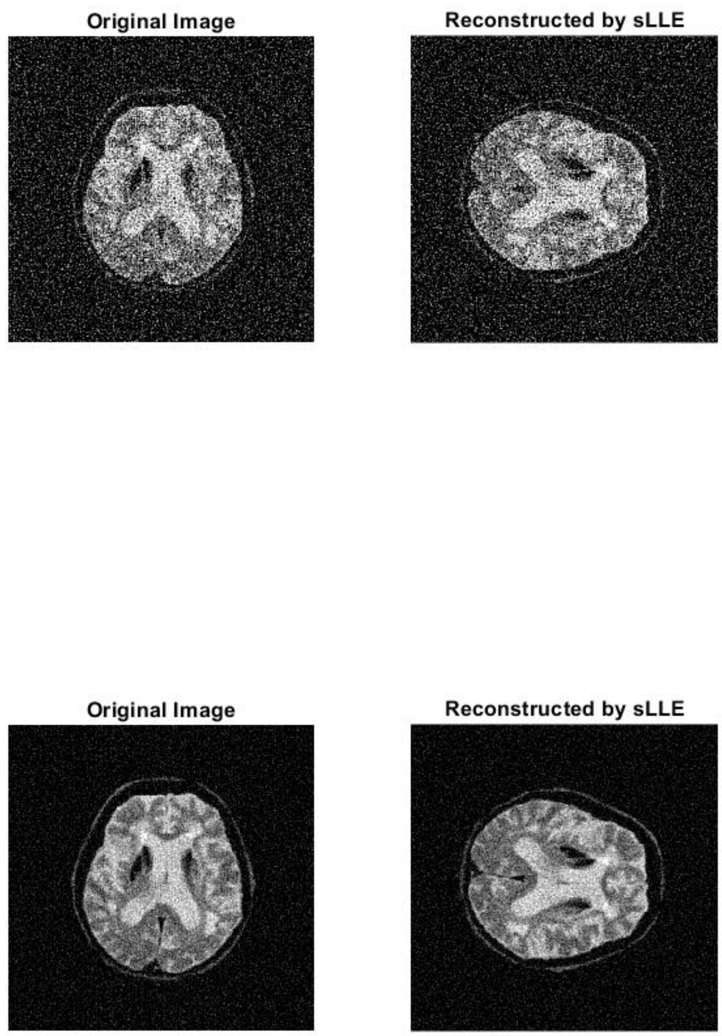

Figure 5: original image and reconstructed image of Brain MR image having SNR values of 1.08dB and 5.06dB using sLLE.

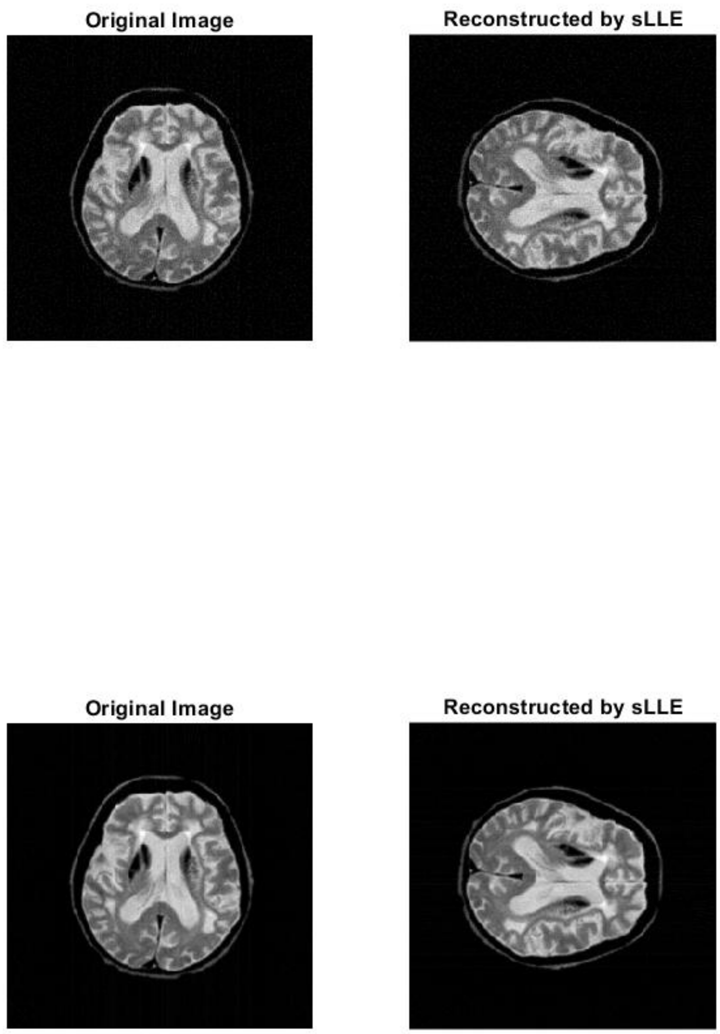

Figure 6: original image and reconstructed image of Brain MR image having SNR value of 10.28dB and 18.07dB using sLLE.

References

1. Yi Fang, Mengtian Sun, S.V.N. Vishwanathan, and Karthik Ramani. slle: Spher-ical locally linear embedding with applications to tomography. InCVPR 2011,pages 1129–1136, 2011

2. Ahmed H. Sameh and John A. Wisniewski. A trace minimization algorithm-for the generalized eigenvalue problem.SIAM Journal on Numerical Analy-sis,19(6):1243–1259, 1982

3. Roweis ST, Saul LK. Nonlinear dimensionality reduction by locally linear embedding. Science. 2000 Dec 22;290(5500):2323-6. doi: 10.1126/science.290.5500.2323. PMID: 11125150.

4. https://cs.nyu.edu/ roweis/lle/code.html

5. https://www.programmersought.com/article/41805168377/

6. https://www.stat.cmu.edu/ cshalizi/350/lectures/14/lecture-14.pdf

7. https://cs.nyu.edu/ roweis/lle/papers/lleintroa4.pdf