

FLAT :- formal language & automata theory

① Introduction to automata Theory.

② Regular languages & expressions.

③ CFG \rightarrow context free grammar & languages

④ PDA & TM \rightarrow Turing machine
↓
push down automata

⑤ Lex & Yacc

PDA
DPDA - Deterministic (one state)
NPDA - Non deterministic

* INTRODUCTION TO AUTOMATA THEORY :-

* Basic Terminologies.

* NDFA \rightarrow Non-deterministic finite automata.

* DFA \rightarrow Deterministic finite automata

* Equivalence of DFA & NDFA

* E-NFA

① ALPHABET : \rightarrow A finite non empty set of symbols / elements (Σ)

e.g. $\Sigma = \{0, 1\}$, $\Sigma = \{a, b, c, d, \dots, z\}$

$\Sigma = \{0, 1\} \rightarrow$ binary alphabet

$\Sigma = \{a, b, c, d, \dots, z\} \rightarrow$ set of lowercase english letters

$\Sigma = \{aa, aaaa, aaaaa, \dots\}$

② STRINGS : \rightarrow A finite sequence of symbols

(word)

$\epsilon, \lambda \rightarrow$ empty string

e.g. ① 01101

② ababac

③ {B, 011010, 0011}

length of string (cardinality / size) : + 1

$$\text{Q) } 1011011 = ? \quad (\text{aabbabc}) \quad \begin{array}{r} \text{using} \\ \text{0} \quad 2 \quad 4 \quad 6 \\ | \quad | \quad | \quad | \\ 1 & 0 & 1 & 1 & 1 0 & 0 & 1 & 1 \end{array}$$

* Concatenation of strings:-

x is a string having i symbols in it. y is also a string having j number of symbols in it such that

$$\chi = a_1 a_2 a_3 \dots a_l$$

$$y = b_1 b_2 b_3 \dots b_j$$

then concatenation of two strings $xy = aaaa\dots abbb\dots b$

⑧ power of an alphabet :-

set of all strings of length k is denoted by Σ^k where k is length of the string.

$$\Sigma^1 = \{0, 1\}, \Sigma^2 = \{G\}, \Sigma^3 = \{00, 01\}; \Sigma^4 = \{000, 111\}$$

power of an alphabet is also denoted with \cdot

$\Sigma^3 = \{0, 1\}^3$

$$S^+ = \sum U \Sigma^4 U \Sigma^3 - \dots$$

Non empty finite set of alphabets)

④ Language : \rightarrow set of all strings

$$L = \{0, 1, 100, 111, 1010, 0011, 11001, 11001100, \dots\}$$

Hierarchy or classification of languages

- ① Type 0 → Recursively enumerable → TM (Turing machine)
- ② Type 1 → Context sensitive Language → LBA (Linear Bounded automata)
- ③ Type 2 → Context free Language → PDA (Push down automata)
- ④ Type 3 → Regular Language → FA (Finite automata)
∅ → empty language

Type 0 → phrase structured | Recursively enumerable → TM (Turing machine)

Type 1 → context sensitive Language → LBA → Linear Bounded automata.

Type 2 → Context free language → PDA → push down automata

Type 3 → Regular language → finite automata

→ DFA
→ NFA / NDFA } abstract machines / computational models
→ E-NFA

TOC → Theory of computation

abstract → brief summary

finite automata is defined as abstract machine mathematical computational model which comprises both hardware & software in reality these kind of machine don't have physical existence.

Type :-

- ① DFA : → Deterministic final automata
- ② NDFA : → Non-Deterministic final automata
- ③ E-NFA

* Representation of finite automata :-

states

Input alphabets

Transition

start state

final state

q_0 or A

state

TD → Transition diagram

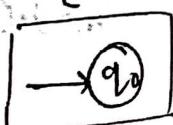
TT → Transition Table



Arc / edge of Transition

from q_0 on
to q_1

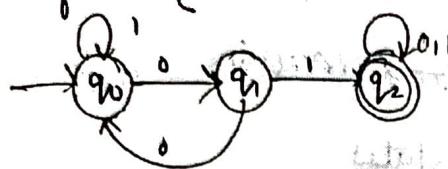
The start state is represented by :-



start stat.

final state is denoted by \textcircled{F}

* Example for DFA. Identify the start state, final state & transition.



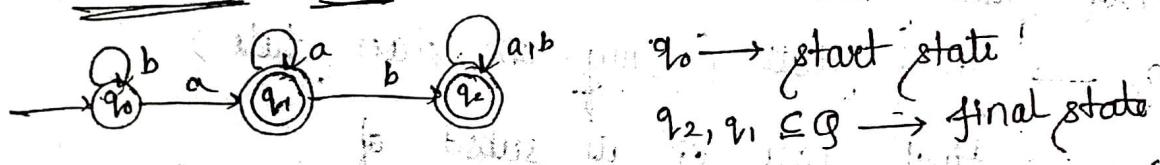
start state $\rightarrow q_0 / q_s$

final state $\rightarrow q_2 / q_f$

Input alphabets $\rightarrow \Sigma = \{0, 1\}$

Transitions $\rightarrow \delta(q_0, 0) = q_1, \quad \delta(q_0, 1) = q_2$
 $\delta(q_1, 0) = q_0, \quad \delta(q_1, 1) = q_2$
 $\delta(q_2, 1) = q_2, \quad \delta(q_2, 0) = q_2$

* Transition Table :-



* Transition Table :-

	δ	a	b
$\rightarrow q_0$	q_1	q_0	
$\# q_1$	q_1	q_2	
$\# q_2$	q_2	q_2	

Input alphabets

$q_0 \rightarrow$ start state

$q_2, q_1 \subseteq Q \rightarrow$ final state

$\Sigma = \{a, b\}$

$\delta(q_0, a) = q_1$

$\delta(q_0, b) = q_2$

$\delta(q_1, a) = q_0$

$\delta(q_1, b) = q_2$

$\delta(q_2, a) = q_2$

$\delta(q_2, b) = q_2$

here \rightarrow indicates start state

* indicates final state

DEFINITION:

DFA : \rightarrow Deterministic finite automata

$$\text{Mdfa} = (\mathcal{Q}, \Sigma, \delta, q_0, q_f) \quad M! \xrightarrow{\text{Machine}}$$

\mathcal{Q} : \rightarrow finite non empty set of states

Σ : \rightarrow finite non empty set of input alphabets

δ : \rightarrow Transition function

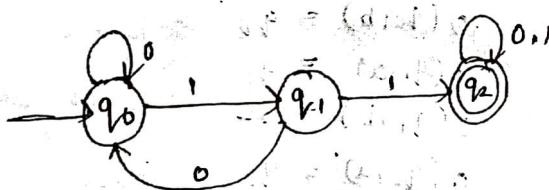
$$\delta : \mathcal{Q} \times \Sigma = \mathcal{Q}$$

transition is mapping from state to state
which yields state

$q_0 \rightarrow$ start state (Only one start state)

$q_f \rightarrow$ final state & is subset of \mathcal{Q}

(i.e more than one final state)



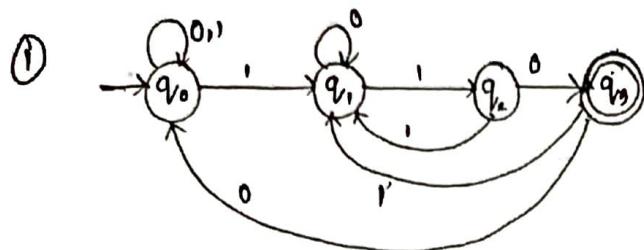
$$\mathcal{Q} = \{q_0, q_1, q_2\}$$

$$\Sigma = \{0, 1\}$$

$q_2 \rightarrow$ final state

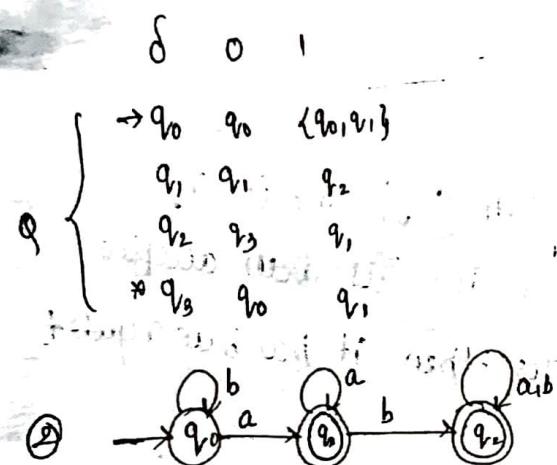
$q_0 \rightarrow$ start state

* Check whether the following transitions constitute DFA or not

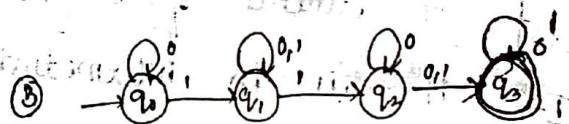


$$\begin{aligned}\delta(q_0, 0) &= q_0 \\ \delta(q_0, 1) &= \{q_0, q_3\} \\ \delta(q_1, 0) &= q_1 \\ \delta(q_1, 1) &= q_2 \\ \delta(q_2, 0) &= q_3 \\ \delta(q_2, 1) &= q_1 \\ \delta(q_3, 0) &= q_0 \\ \delta(q_3, 1) &= q_1\end{aligned}$$

Transition table



As for same input alphabet
there are multiple states
it is not DFA



$$\Sigma = \{0, 1\}$$

$$Q = \{q_0, q_1, q_2, q_3\}$$

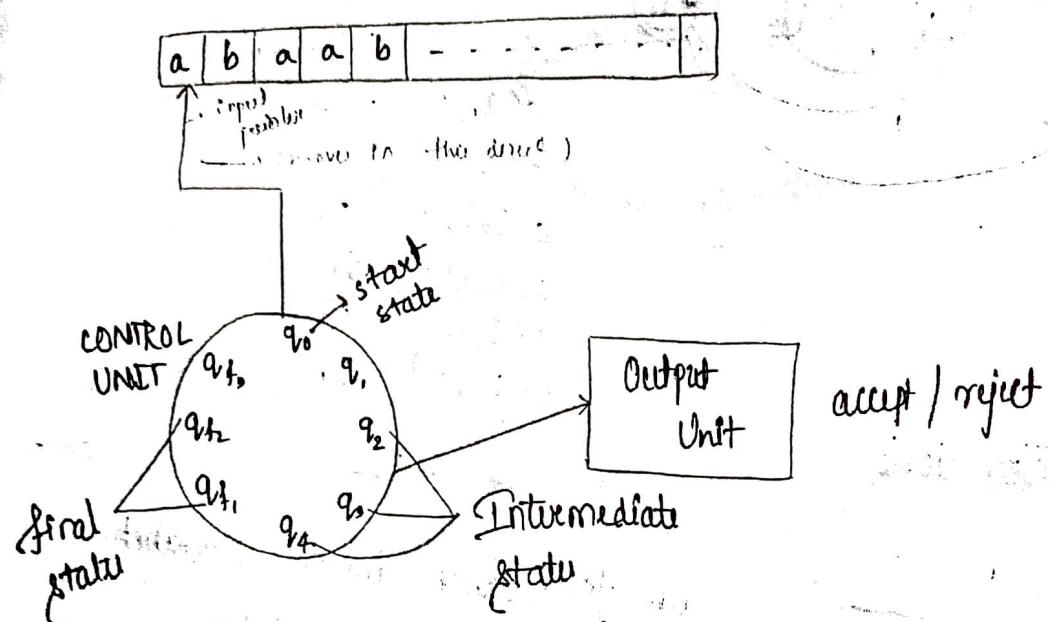
Transitions

$$\begin{aligned}\delta(q_0, 0) &= q_0 \\ \delta(q_0, 1) &= q_1 \\ \delta(q_1, 0) &= q_1 \\ \delta(q_1, 1) &= \{q_1, q_2\} \\ \delta(q_2, 0) &= \{q_2, q_3\} \\ \delta(q_2, 1) &= q_3 \\ \delta(q_3, 0) &= q_3 \\ \delta(q_3, 1) &= \emptyset\end{aligned}$$

It is DFA

* Working procedure of finite automata :-

Input Tape

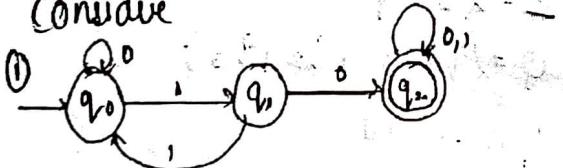


The machine will be in start state (assumption).

If it reaches final state then it will be accepted.

If it is in intermediate state then it has been rejected.

Consider



$$q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_3 \xrightarrow{1} q_4$$

$$\begin{aligned}
 W_0 &= 011010 \\
 \delta(q_0, 0) &= q_0 \\
 \delta(q_0, 1) &= q_1 \\
 \delta(q_1, 1) &= q_0 \\
 \delta(q_0, 0) &= q_0 \\
 \delta(q_0, 1) &= q_1 \\
 \delta(q_1, 0) &= q_2
 \end{aligned}$$

OR

$$W_1 = 0110101$$

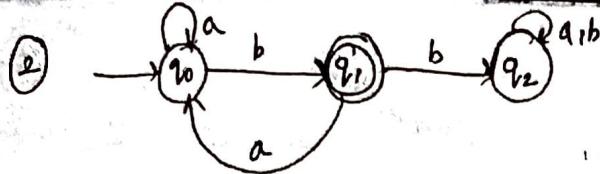
$$q_0 \xrightarrow{0} q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_3 \xrightarrow{0} q_4 \xrightarrow{1} q_5 \xrightarrow{0} q_6 \xrightarrow{1} q_7 \xrightarrow{0} \text{final state}$$

$\therefore \text{accepted}$

Final state

$$W_2 = 11001101$$

$$q_0 \xrightarrow{1} q_1 \xrightarrow{1} q_2 \xrightarrow{0} q_3 \xrightarrow{0} q_4 \xrightarrow{0} q_5 \xrightarrow{1} q_6 \xrightarrow{1} q_7 \xrightarrow{0} q_8 \xrightarrow{1} q_9 \xrightarrow{0} \text{Rejected}$$



$$w = abba$$

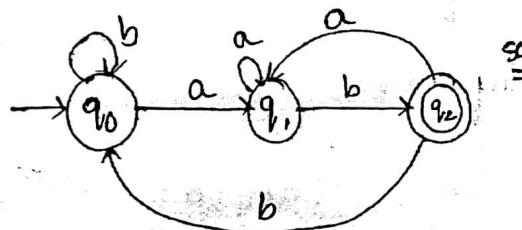
$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_2$ Rejected

$$w = abbabb$$

$q_0 \xrightarrow{a} q_0 \xrightarrow{b} q_1 \xrightarrow{b} q_2 \xrightarrow{a} q_2 \xrightarrow{b} q_2$

* What are the moves that are made by the following DFA to accept the following strings?

- ① abaab ② abb ③ abaa.



$$\Omega = \{q_0, q_1, q_2\}$$

$q_0 \rightarrow$ start state

$q_2 \rightarrow$ final state

$$\Sigma = \{a, b\}$$

$\left. \begin{array}{l} \\ \\ \end{array} \right\} \text{V. A. to work}$

$$\delta(q_0, a) = q_1, \quad \delta(q_1, a) = q_1, \quad \delta(q_2, a) = q_1,$$

$$\delta(q_0, b) = q_0, \quad \delta(q_1, b) = q_2, \quad \delta(q_2, b) = q_0$$

i) ~~w~~ w : abaab

$$\delta(q_0, a) = q_1, \quad \delta(q_1, b) = q_2, \quad \delta(q_2, a) = q_1, \quad \delta(q_1, a) = q_1, \quad \delta(q_1, b) = q_2$$

since the machine is in final state the string

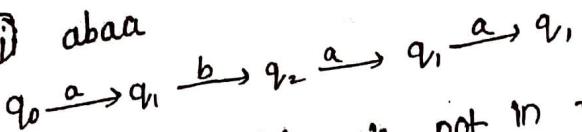
is accepted by DFA.

$$\text{ii) } w = abb \quad \delta(q_0, a) = q_1, \quad \delta(q_1, b) = q_2, \quad \delta(q_2, b) = q_0$$

since the machine is not in final state the string

is rejected or $q_0 \xrightarrow{a} q_1 \xrightarrow{b} q_2 \xrightarrow{b} q_0$

⑩ abaa



since the machine is not in final state it is rejected.

* Extended transition function

It is denoted by $\delta^*(q_1, w) = p$

δ^* denotes extended transition function.

q is the first parameter which represents current state.

w is a second parameter which represents the input string.

p is a new state obtained after transition.

Let, $w = za$ w is input string.

$$\delta(\delta^*(q_1, z), a)$$

$$\delta^*(q_1, e) = q$$

* Using extended transition function write the moves that are made by DFA for the following-

① abaab ② abb ③ abaa.

$$① \delta^*(q_0, \epsilon) = q_0$$

$$\begin{aligned} \therefore \delta^*(q_0, a) &= \delta(\delta^*(q_0, \epsilon), a) \\ &= \delta(q_0, a) \\ &= q_1 \end{aligned}$$

$$\begin{aligned} \underline{ab} \rightarrow \delta^*(q_0, ab) &= \delta(\delta^*(q_0, a), b) \\ &= \delta(q_1, b) \\ &= q_2 \end{aligned}$$

for the prefix

$$\begin{aligned} \underline{aba} \rightarrow \delta^*(q_0, aba) &= \delta(\delta^*(q_0, ab), a) \\ &= \delta(q_2, a) \\ &= q_1 \end{aligned}$$

$$\begin{aligned} \underline{abaa} \rightarrow \delta^*(q_0, abaa) &= \delta(\delta^*(q_0, aba), a) \\ &= \delta(q_1, a) \\ &= q_2 \end{aligned}$$

$$\begin{aligned} \underline{abaab} \rightarrow \delta^*(q_0, abaab) &= \delta(\delta^*(q_0, abaa), b) \\ &= \delta(q_2, b) \\ &= q_2 \end{aligned}$$

$$\textcircled{2} \quad \delta^*(q_0, \epsilon) = q_0$$

for prefix a $\delta^*(q_0, a) = \delta(\delta^*(q_0, \epsilon), a)$
= $\delta(q_0, a)$
= q_1

for prefix b $\delta^*(q_0, ab) = \delta(\delta^*(q_0, a), b)$
= $\delta(q_1, b)$
= q_2

for prefix b $\delta^*(q_0, abb) = \delta(\delta^*(q_0, ab), b)$
= $\delta(q_2, b)$
= q_0

$$\textcircled{3} \quad \delta^*(q_0, \epsilon) = q_0$$

$$\delta^*(q_0, a) = \delta(\delta^*(q_0, \epsilon), a)$$

= $\delta(q_0, a)$
= q_1

$$\delta^*(q_0, ab) = \delta(\delta^*(q_0, a), b)$$

= $\delta(q_1, b)$
= q_2

$$\delta^*(q_0, aba) = \delta(\delta^*(q_0, ab), a)$$

= $\delta(q_2, a)$
= q_1

$$\delta^*(q_0, abaa) = \delta(\delta^*(q_0, aba), a)$$

= $\delta(q_1, a)$
= q_1

* Transition diagram for the following language.

① $\emptyset \rightarrow$ Empty language.



② $\epsilon \rightarrow$ Empty string.



③ DFA to accept only one a



④ DFA to accept one a or one b.



⑤ DFA to accept zero or more number of a's or b's



DFA :-

① pattern recognition problem

② Divisible by k

③ Modulo k counter

To solve a problem

STEP :-

① Identify minimum string.

② Identify input alphabet

③ Construct a base DFA having start state q₀ & final state -

④ Identify the transitions that are not defined in the 3rd step.

⑤ Write complete transition diagram with all ~~transitions~~
 & hence write transition table.

* Construct DFA to accept strings of a having
 atleast one a .

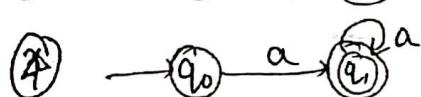
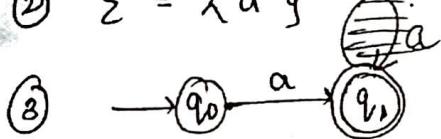
$$\Rightarrow L = \{a, aa, aaa, aaaa, \dots\}$$

or

$$L = \{w : |na| \geq 1, w \in \{a^*\}\}$$

① a

② $\Sigma = \{a\}$



③ $Q = \{q_0, q_1\}$ $\Sigma = \{a\}$ $q_0 \rightarrow$ start state $q_1 \rightarrow$ final state
 $\delta : \delta(q_0, a) = q_1, \quad \delta(q_1, a) = q_1,$

δ	a
$\rightarrow q_0$	q_1
$\times q_1$	q_1

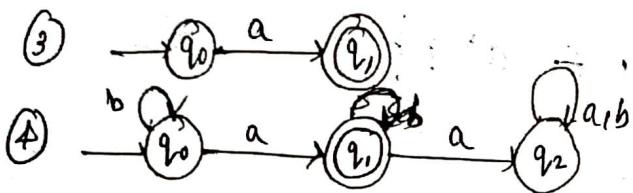
② Construct a DFA to accept strings of a's & b's having exactly one a.

$$\Rightarrow L = \{ a, ab, abb, abbb \dots \}$$

$$L = \{ ba, bba, bbbba, bab \dots \}$$

① a

② $\Sigma = \{a, b\}$



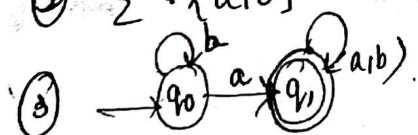
Trap/dead state

δ	a	b
$\rightarrow q_0$	q_1	q_2
$\rightarrow q_1$	q_2	q_1
$\rightarrow q_2$	q_2	q_2

③ Construct DFA to accept strings of a's & b's having atleast one a.

\Rightarrow ① a

② $\Sigma = \{a, b\}$



$$L = \{ a, ab, abb, bab, bba, bbaa, baa, baaa, bbaa, bba \dots \}$$

$$\delta(q_0, b) = \varnothing$$

$$\delta(q_1, a) = \varnothing$$

$$\delta(q_1, b) = \varnothing$$

δ	a	b
$\rightarrow q_0$	q_1	q_1
$\rightarrow q_1$	q_1	q_1

$$Q = \{q_0, q_1\}$$

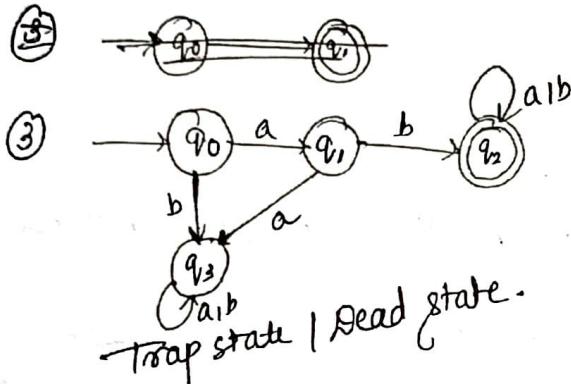
$$\Sigma = \{a, b\}$$

$q_0 \rightarrow$ start state

$q_1 \rightarrow$ final state

④ Construct a DFA of strings $a \& b$ such that sequence should begin with ab .

- ⇒ ① ab $L = \{ ab, ababba, abba, abbb, \dots \}$
 ② $\Sigma = \{a, b\}$



$$Q = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{a, b\}$$

$q_0 \rightarrow$ start state

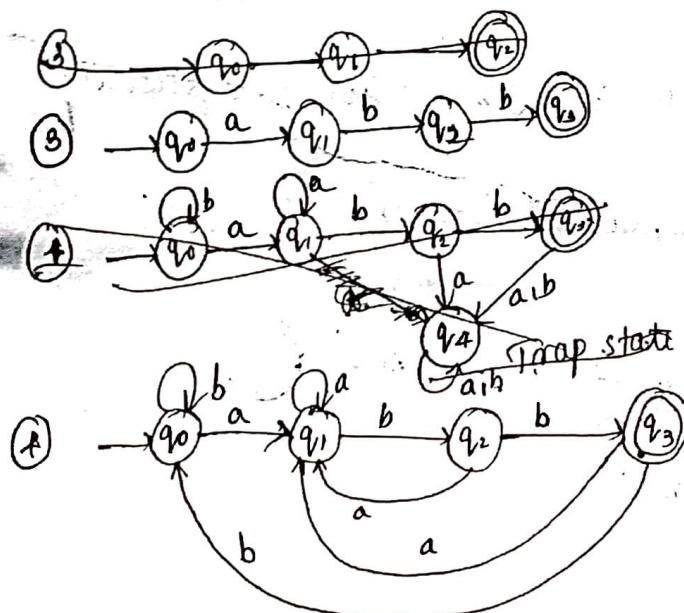
$q_2 \rightarrow$ final state

δ	a	b
$\rightarrow q_0$	q_1	q_3
$\rightarrow q_1$	q_3	q_2
$\rightarrow q_2$	q_2	q_2
q_3	q_3	q_3

⑤ Construct DFA of strings of $a \& b$ such that each & every string should end with abb .

- ① abb
 ② $\Sigma = \{a, b\}$

$$L = \{ abb, ababb, aaabb, babb, aababb, bbabb, aaabb, \dots \}$$



$$Q = \{q_0, q_1, q_2, q_3, q_4\}$$

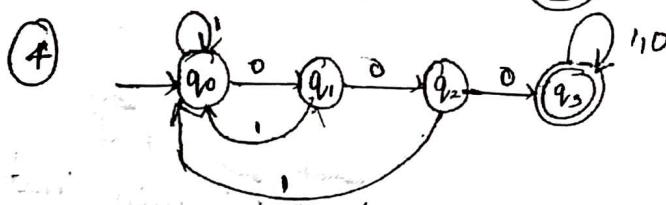
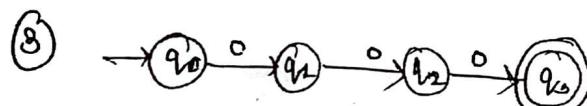
$$\Sigma = \{a, b\}$$

$q_0 \rightarrow$ start state

$q_3 \rightarrow$ final state

- ③ Construct a DFA of strings 0's & 1's, such that it should have three consecutive zeros.
- $\Rightarrow L = \{0001100011, 10000, 00011, \dots\}$

① $\Sigma = \{0, 1\}$ ② 000

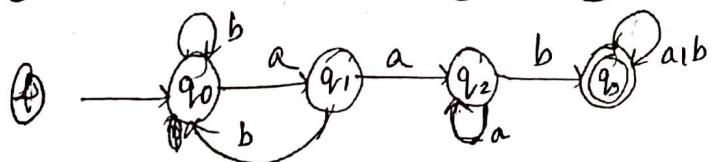
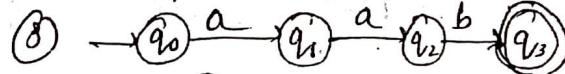


- ⑤ Write a DFA of strings a's & b's having a substring aab.

$\Rightarrow L = \{aab, baabaa, aabb, aaaabb, \dots\}$

① aab

② $\Sigma = \{a, b\}$



δ : a, b

$\rightarrow q_0$

q_1

q_2

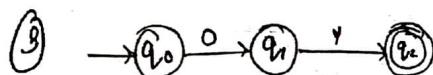
$\rightarrow q_3$

- ⑧ Construct a DFA to accept strings of 0's & 1's such that somewhere in the sequence 01 should appear in the string.

$$\Rightarrow L = \{01, 1011, 00101, 110100, 000101, 0011\}$$

① Q

② $\Sigma = \{0, 1\}$



δ : 0, 1

q_0

q_1

q_2

- ⑨ Construct a DFA to accept strings of 0's & 1's such that strings should not have a substring 01.

* NOTE: If not keyword appears

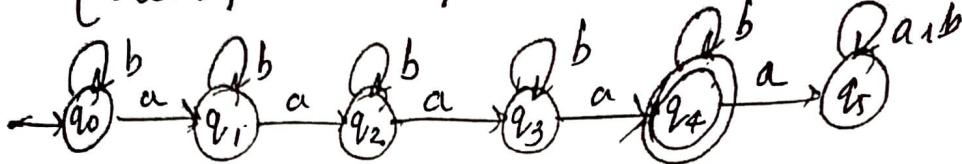
change final state \rightarrow Non-final state

Nonfinal state \rightarrow final state



(10) Construct DFA of ~~str~~ having ~~a~~ & ~~a's~~ for the input alphabet $\Sigma = \{a, b\}$.

$$\Rightarrow L = \{aaa, bababa, aabbaabb, abababa \dots\}$$

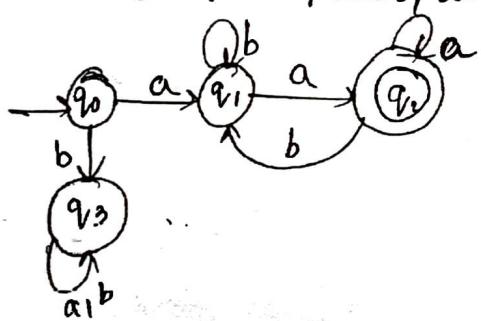


(11) * construct a DFA to accept the following language

$$L = \{awaw \mid w \in \{a, b\}\}$$

→ Minimum string $\rightarrow aa$

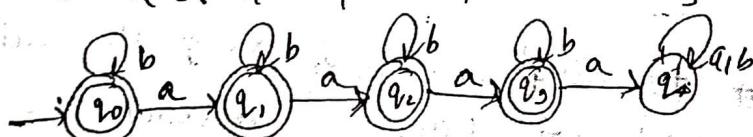
$$L = \{aa, aba, aaa, abaa \dots\}$$



(12) a's & b's having atleast 3 a's.

$$\Rightarrow L = \{aaa, baaa, abaa\}$$

$$L = \{\epsilon, a, aa, aaa, aaaa \dots\}$$

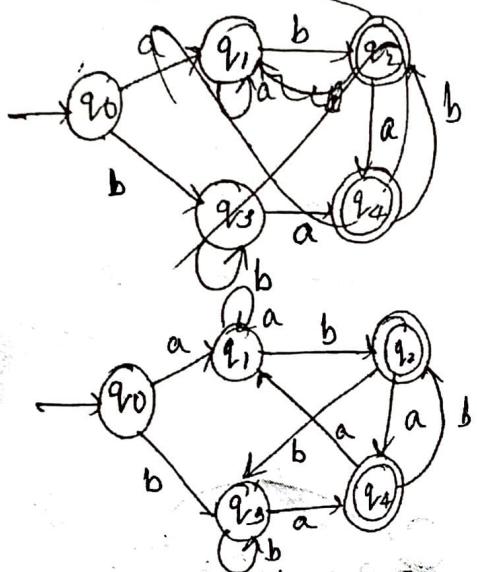


Here they are made final states because to accept zero a's, 1a, 2a ...

(13) $L = \{w \in a, b \mid w(ab + ba)\}$

\Rightarrow This implies the string should either end with ab or ba.

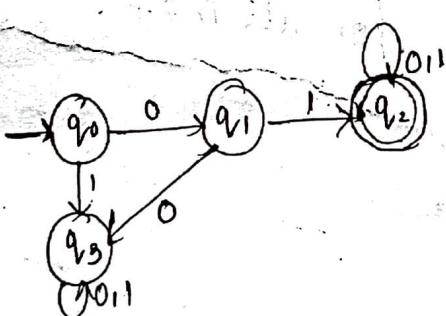
$L = \{aab, aba, aab, bab, bba, aaba\} - \{ \}$



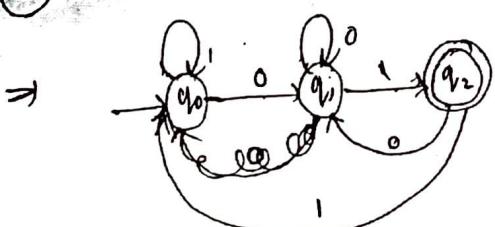
$$\delta(q_{11}, a) =$$

(14) Construct a DFA of strings o's & 1's which begins with 01.

$\Rightarrow L \subset \{01, 0100, 0110, \dots\}$



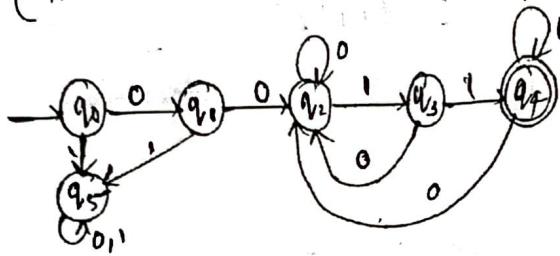
(15) Ends with 01.



⑩ Construct DFA for following languages:

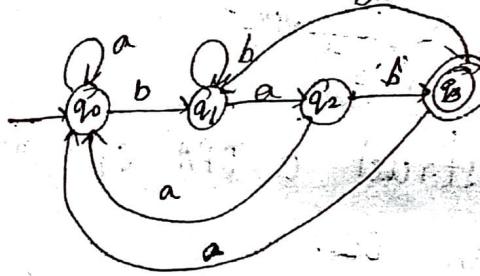
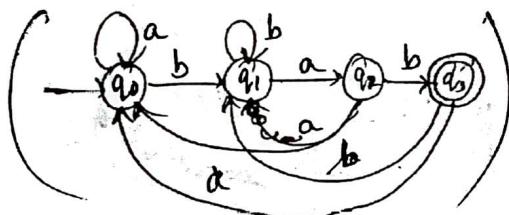
$$\textcircled{1} \quad L = \left\{ w_6(0+1)^* 00(0+1)^* 11 \right\}$$

$\text{① } L = \{w \in (0+)^* \mid 00(0+)^*\}$
 \Rightarrow Begins with atleast two zeros & ends
 with atleast two ones.
 (* indicates zero or any time)



$$\textcircled{2} \quad L = \{ w \in \{a, b\}^* \mid wba \in \}$$

\Rightarrow strings ending with bab .



Q1) DFA's of a's & b's which accepts all the strings of length a^n for all $n \neq 4$.

1

(18) Multiples of two of a's $\Sigma = \{a, b\}$

DIVISIBLE BY k PROBLEMS :-

$$\delta(q_i, a) = q_j \quad Q \times \Sigma = Q$$

$$j = (r * i + d) \bmod k$$

r : \rightarrow radix input

i : \rightarrow remainder obtained after dividing by k

d : \rightarrow digit

k : \rightarrow divisor

for binary radix input : $\rightarrow 2$

decimal radix input : $\rightarrow 10$

: $\rightarrow 3, 4, 5, 6, 7, 8, 9$

{0, 1, 1, 2}

NOTE : \rightarrow start state & final state will be same.

① Construct a DFA of strings '0's & '1's where each string is represented as decimal numbers which are divisible by 5.

$$\Rightarrow n = 2$$

$$t = 011121314$$

$$d = \{0, 1\}$$

$$k = 5$$

$$i=0 \quad d \quad j \text{ s.t. } (r \times i + d) \bmod k = j \quad \delta(q_1, a) = q_j$$

$$0 \quad q_0 = (2 \times 0 + 0) \bmod 5 = 0 \quad \delta(q_0, 0) = q_0$$

$$1 \quad q_0 = (2 \times 0 + 1) \bmod 5 = 1 \quad \delta(q_0, 1) = q_1$$

$$i=1 \quad 0 \quad q_1 = (2 \times 1 + 0) \bmod 5 = 2 \quad \delta(q_1, 0) = q_2$$

$$1 \quad q_1 = (2 \times 1 + 1) \bmod 5 = 3 \quad \delta(q_1, 1) = q_3$$

$$i=2 \quad 0 \quad q_2 = (2 \times 2 + 0) \bmod 5 = 4 \quad \delta(q_2, 0) = q_4$$

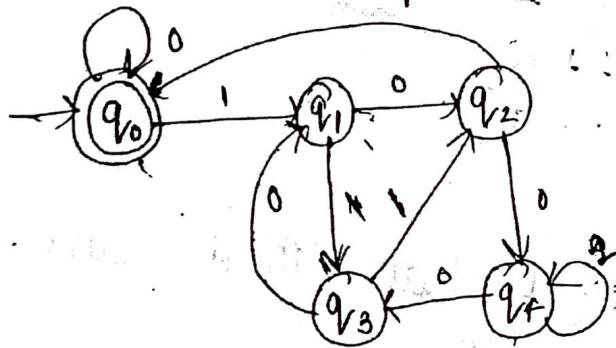
$$1 \quad q_2 = (2 \times 2 + 1) \bmod 5 = 0 \quad \delta(q_2, 1) = q_0$$

$$i=3 \quad 0 \quad q_3 = (2 \times 3 + 0) \bmod 5 = 1 \quad \delta(q_3, 0) = q_1$$

$$1 \quad q_3 = (2 \times 3 + 1) \bmod 5 = 2 \quad \delta(q_3, 1) = q_2$$

$$i=4 \quad 0 \quad q_4 = (2 \times 4 + 0) \bmod 5 = 3 \quad \delta(q_4, 0) = q_3$$

$$1 \quad q_4 = (2 \times 4 + 1) \bmod 5 = 4 \quad \delta(q_4, 1) = q_0$$



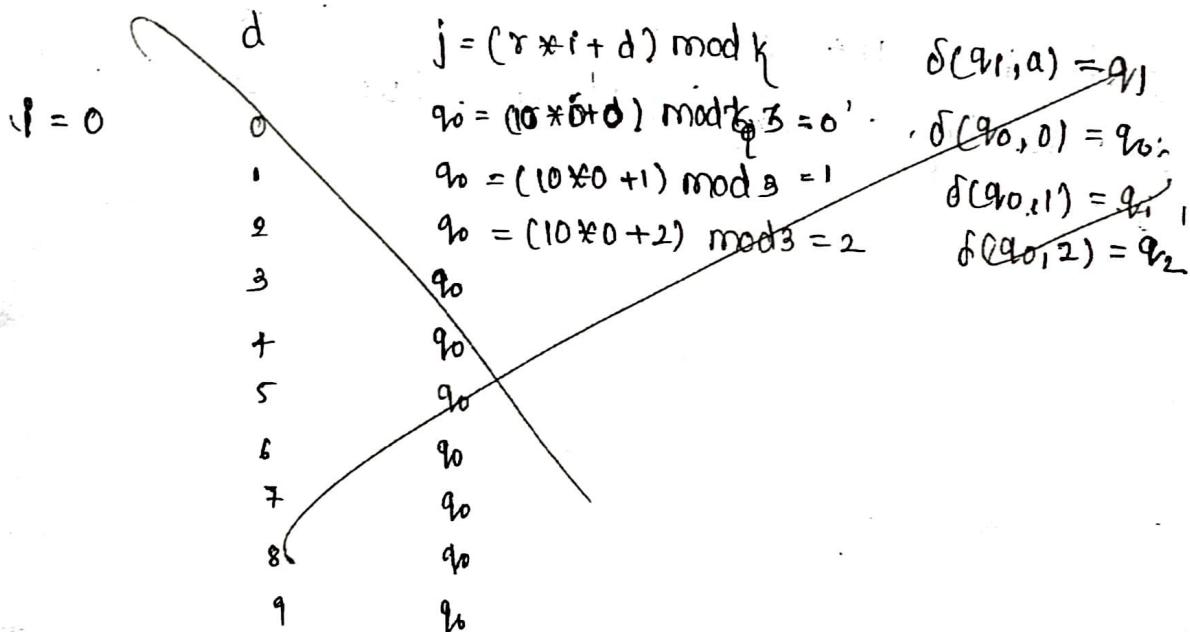
② Construct a DFA of strings of decimal numbers that are divisible by 3.

$$\Rightarrow r = 10$$

$$i = 0, 1, 2$$

$$d = \{0_{11}, 2_{13}, 4_{15}, 6_{17}, 8_{19}\}$$

$$k = 3$$



$$j = (r * i + d) \bmod k$$

$$\delta(q_1, a) = q_1$$

$$q_0 = (10 * 0 + 0) \bmod 3 = 0$$

$$\delta(q_0, 0) = q_0$$

$$q_0 = (10 * 0 + 1) \bmod 3 = 1$$

$$\delta(q_0, 1) = q_1$$

$$q_0 = (10 * 0 + 2) \bmod 3 = 2$$

$$\delta(q_0, 2) = q_2$$

$$i = 0$$

$$0$$

$$1$$

$$2$$

$$3$$

$$4$$

$$5$$

$$6$$

$$7$$

$$8$$

$$9$$

$$d$$

$$(r * i + d) \bmod k = j$$

$$i = 0$$

$$(0_{13}, 6_{19})$$

$$(10 * 0 + 0) \bmod 3 = q_0$$

$$(1_{14}, 1_7)$$

$$(10 * 0 + 1) \bmod 3 = q_1$$

$$(2_{15}, 8)$$

$$(10 * 0 + 2) \bmod 3 = q_2$$

$$i = 1$$

$$(-0_{13}, 6_{19})$$

$$(10 * 1 + 0) \bmod 3 = q_1$$

$$(1_{14}, 1_7)$$

$$(10 * 1 + 1) \bmod 3 = q_2$$

$$(2_{15}, 8)$$

$$(10 * 1 + 2) \bmod 3 = q_0$$

$$i = 2$$

$$(0_{13}, 6_{19})$$

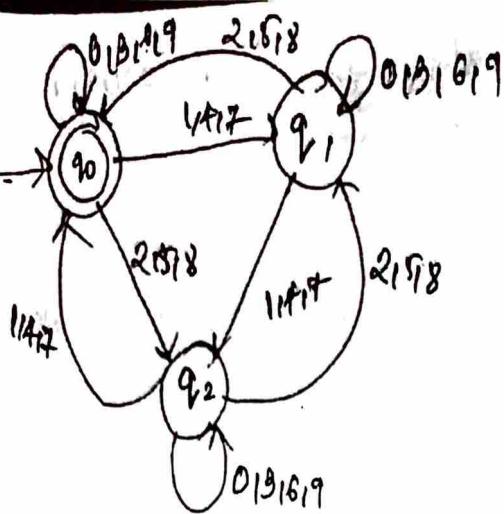
$$(10 * 2 + 0) \bmod 3 = q_2$$

$$(1_{14}, 1_7)$$

$$(10 * 2 + 1) \bmod 3 = q_0$$

$$(2_{15}, 8)$$

$$(10 * 2 + 2) \bmod 3 = q_1$$



- Q) Construct a DFA of strings of 0's & 1's which are divisible by 5 such that DFA should not accept binary number 0000.

④ Construct a DFA of string of 0's, 1's & 2's which are represented as combination of 0's, 1's & 2's divisible by 4.

Divisibility problem :-

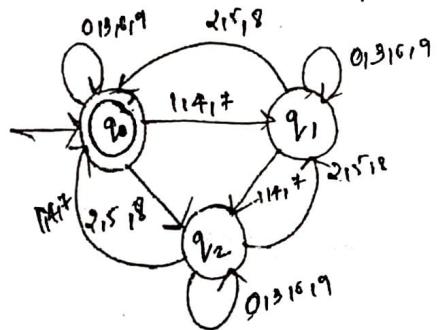
① divisible by 3 of decimal number.

→ Remainders are 0, 1, 2

$$\begin{array}{l} 0 \rightarrow q_0 \\ 1 \rightarrow q_1 \\ 2 \rightarrow q_2 \end{array}$$

$$d = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.$$

δ	(0, 1, 3, 6, 9)	(1, 4, 7)	(2, 5, 8)
q_0	q_0	q_1	q_2
q_1	q_1	q_2	q_0
q_2	q_2	q_0	q_1



② Construct a DFA all strings over $\Sigma = \{0, 1, 4\}$ ends with 100.

→ q_0 is start state or initial state. q_1 is a state ending with one. q_2 is a state ending with 10; q_3 100.

$$q_1 \rightarrow 1 \quad q_2 \rightarrow 10 \quad q_3 \rightarrow 100$$

$$M = (\mathcal{Q}, \Sigma, \delta, q_0, \mathcal{F})$$

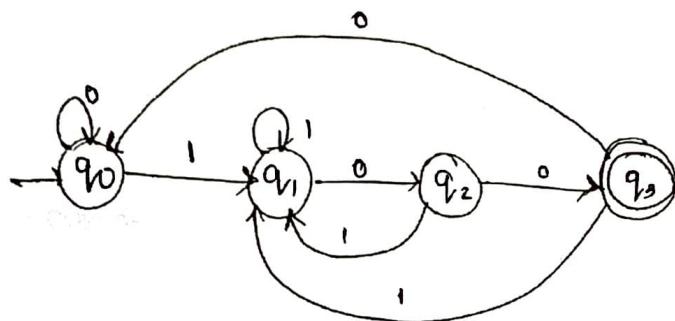
$$\mathcal{Q} = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{0, 1, 4\}$$

$$q_0 \rightarrow ss \quad q_3 \rightarrow fs$$

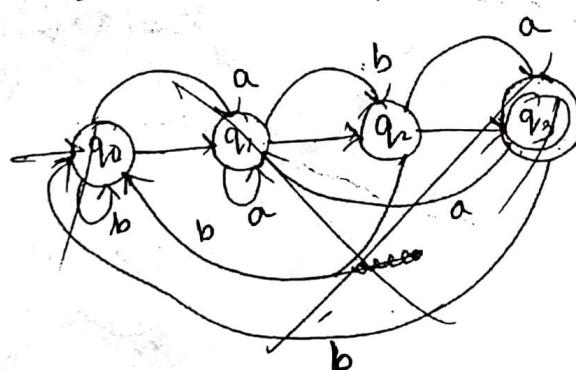
δ	$\delta = 0$	$\delta \neq 0$
$E \rightarrow q_0$	q_0	q_1
q_1	q_2	q_1
q_2	q_3	q_1
q_3	q_0	q_1

$\emptyset\emptyset, \emptyset\emptyset$
 $\emptyset 10, \emptyset 11$
 $\emptyset 100, \emptyset 101$
 $1000, 1001$



② Ending with aba.

δ	a	b	
$E \rightarrow q_0$	q_1	q_0	$\emptyset a, \emptyset b$
a	q_1	q_1	$a a, a b$
ab	q_2	q_0	$a b a, a b b$
aba	q_1	q_0	$a b a a, a b a b$



③ Construct a DFA of strings of 'a's & 'b's starting with aba.

NFA
MIN
δ

NFA \Rightarrow Non deterministic finite automata (N/A/F)

$$MNFA = \{Q, \Sigma, \delta, q_0, F\}$$

$$\delta: \Sigma \times Q = Q \text{ (DFA)}$$

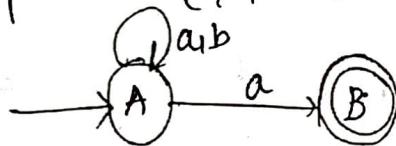
$$\delta: Q \times \Sigma = 2^Q \text{ (powerset)} \rightarrow NFA$$

Q is set of finite states

powerset \rightarrow set of all subsets.

$$A = \{a, b, c\}$$

$$-P(A) = \{\emptyset, \{a, b, c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a\}, \{b\}, \{c\}\}$$



$$\delta(A, a) = A$$

$$\delta(A, b) = A$$

$$\delta(B, a) =$$

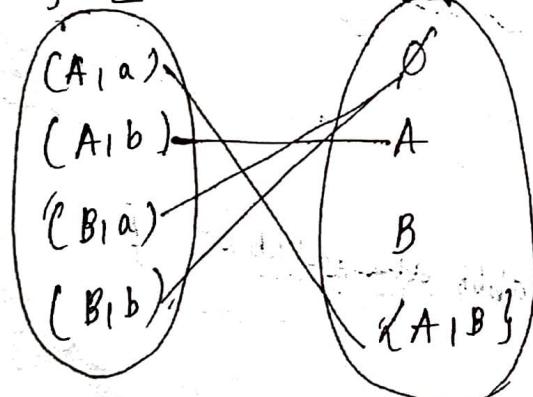
$$\delta(B, b) =$$

δ	a	b
$\rightarrow A$	$\{A, B\}$	A
$\rightarrow B$	\emptyset	\emptyset

$$Q \times \Sigma$$

$$= 2^Q$$

$$\delta:$$



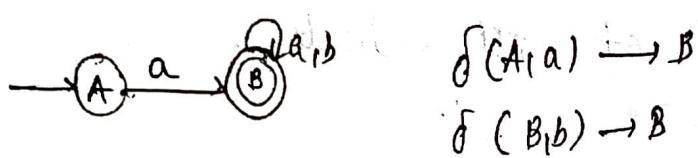
~~Example of NFA~~

EXAMPLES:

① Obtain an NFA over strings a^* where all the strings starts with a .

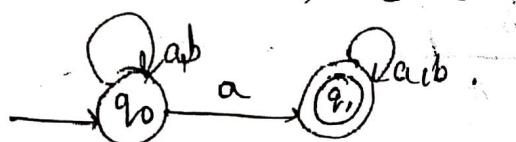
\Rightarrow ① $L = \{ \text{ starts with } a \}$

$$L = \{ a, ab, abb, aabb, \dots \}$$

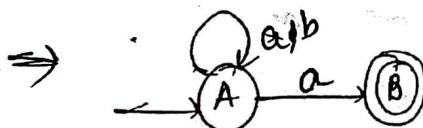


② $L = \{ \text{ containing } a \}$ over a, b

$\Rightarrow L = \{ \text{ aba, bab, bba, ab, ba } \}$



③ $L = \{ \text{ ends with } a \}$ over a, b



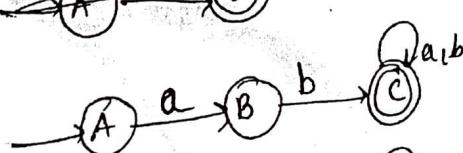
④ $L = \{ \text{ starting with ab } \}$

⑤ $L = \{ \text{ containing ab } \}$

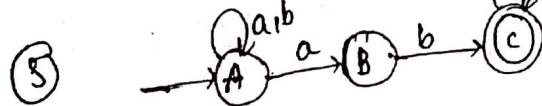
⑥ $L = \{ \text{ ends with ab } \}$



④ $L = \{ abbb, aba, aabb \}$



$L = \{ abab, abbb, babb \}$

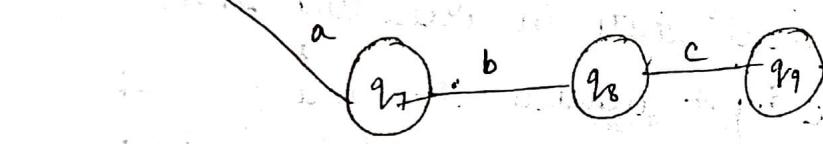
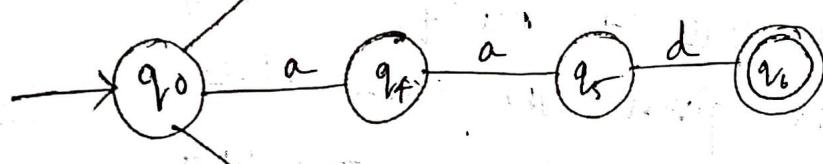
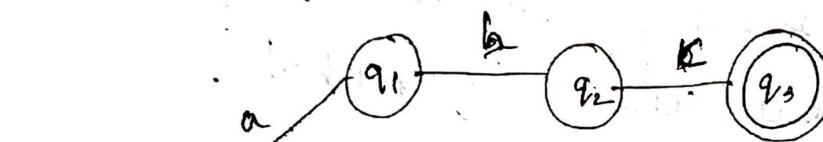
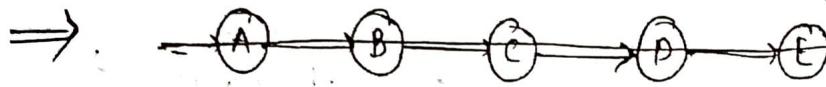


⑦ In order to accept the following set of strings

abc

aad

abcd

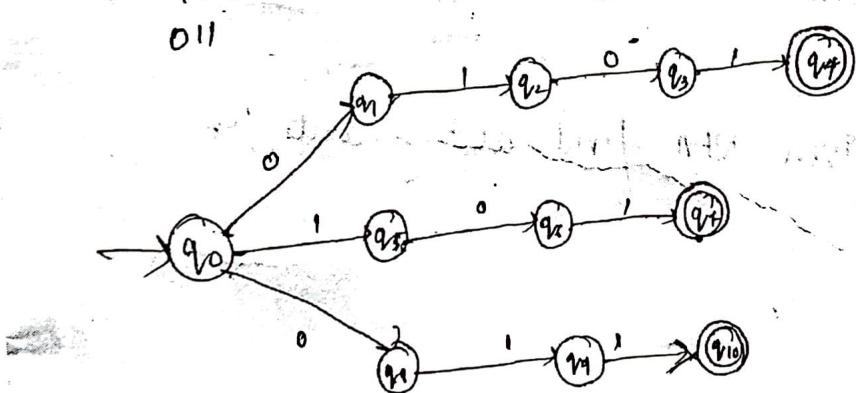


⑧ Construct NFA for given set of strings over
In order to accept the given set of strings

0101

101

011



NOTE : While processing the string or any sequence in a vector or in a set of states atleast one state should be accepting state so that the string or sequence is accepted by NFA