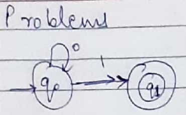
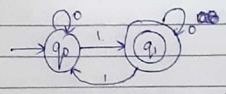
* Problems

1. 

$Q = \{q_0, q_1\}$ $\quad \delta(q_0, 0) = q_0$
$\Sigma = \{0, 1\}$ $\quad \delta(q_0, 1) = q_1$
(Transition should be defined $\delta(q_1, 0) = ND$
for every state. Since q_1 states $\delta(q_1, \cancel{1}) = ND$

Language $L = \{01, 001, 0001, 00001 \cdots\}$ is accepted
~~011~~

2. 

It's DFA

$L = \{01, 0100, 100, 10, 1011 \cdots\}$
(is accepted)

~~Language is accepted~~
$\delta(q_0, w) \in F$.
PGF
L accepted

Question to be asked

* $\Sigma^* \rightarrow$ Considers empty string
  $\Sigma \rightarrow$ Does not consider empty string
  } The difference is presence of epsilon.

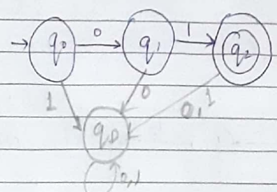
* Pattern Recognition   * ends.
* length   * having a substring
* starts   * divisible by counter 'k'

[Eg.] Construct a DFA, over an ~~over~~ input alphabet $\{0, 1\}$ which accepts the string only '01'.
$\Sigma = \{0, 1\}$ $\quad L = \{01\}$

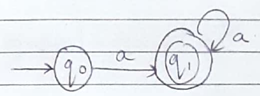
Min string = 2 (n)
No of states = 2+1
(n+1)

To construct:
- Identify L, Σ
- Basic/Initial DFA for min string

$\delta(q_0, 0) = q_1$
$\delta(q_1, 1) = q_2$
$\delta(q_0, 1) = q_D$
$\delta(q_1, \cancel{1}) = q_D$
$\delta(q_2, 0) = q_D$
$\delta(q_2, 1) = q_D$

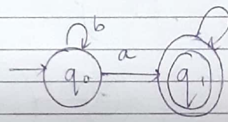


[Eg 2] Construct a DFA having atleast one 'a'
$\Sigma = \{a\}$ $\quad L = \{a, aa, aaa, aaaa \cdots\}$ $\quad \epsilon$ - denotes zero number of symbol

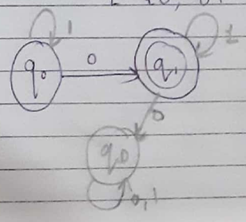


δ	a
q_0	q_1
* q_1	q_1

[Eg 3] Construct a DFA $\Sigma = \{a, b\}$ atleast one 'a'.
$L = \{ab, aab, bab, a, abb \cdots\}$



[Eg 4] Construct a DFA over an input alphabet $\Sigma = \{0, 1\}$ having exactly one '0'
$L = \{0, 01, 011, 0111, 01111 \cdots\}$

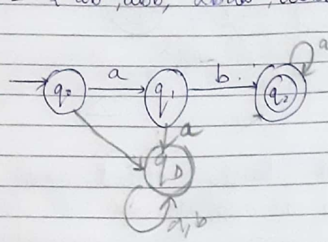


δ	0	1
q_0	q_1	q_0
* q_1	q_D	q_1
q_D	q_D	q_D

Eg 5. Construct a DFA over an input alphabet $\{a, b\}$ which starts with the string ab.

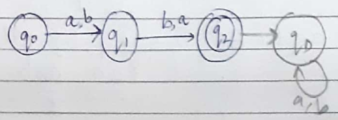
$L = \{ ab, abb, abab, abbb \dots \}$

$\delta(q_0, a) = q_1$
$\delta(q_1, b) = q_2$
$\delta(q_1, b)$



Eg 6. Construct a DFA $\Sigma = \{a, b\}$ having length = 2.

$L = \{a, b, ba, bb, aa\}$

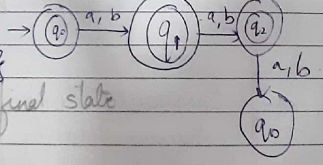


δ	a	b
q_0	q_1	q_1
q_1	q_2	q_2
$*q_2$	q_D	q_D

7. Having length ≤ 2

$L = \{ \epsilon, a, b, ba, bb, aa \}$

↳ Initial state as final state

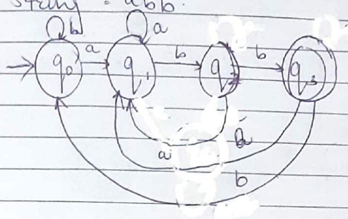


δ	a	b
$*q_0$	q_1	q_1
$*q_1$	q_2	q_2
$*q_2$	q_D	q_D
q_D	q_D	q_D

8. $\Sigma = \{a, b\}$ which ends with $\{a, bb\}$.

$L = \{ aabb, aababb, babb \dots \}$

min string = abb.



δ	a	b
ϵ q_0	q_1	q_0
a q_1	q_1	q_2
ab q_2	q_1	q_3
abb $*q_3$	q_1	q_0

- Power of an alphabet – Σ^K – Set of strings with length K. For eg: $\Sigma = \{0, 1\}$
  $\Sigma^1 = \{0, 1\}$
  $\Sigma^2 = \{00, 01, 10, 11\}$
  $\Sigma^3 = \{000, 001, 010, 011, 100, 101, 110, 111\}$

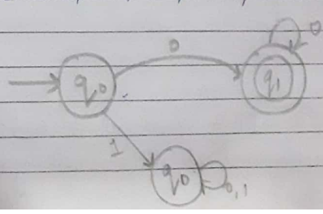
- $\Sigma^* = \Sigma^0 \cup \Sigma^+$
  $= \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \dots$

→ $\Sigma^0 = \epsilon$, $\therefore \Sigma^*$ contains ϵ (epsilon – empty string or zero ")

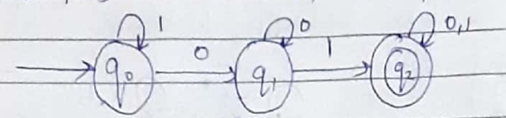
9 Construct a DFA over input alphabet $\{0, 1\}$ which starts with string 0.

→ Minimum string acceptable by the DFA = 0.

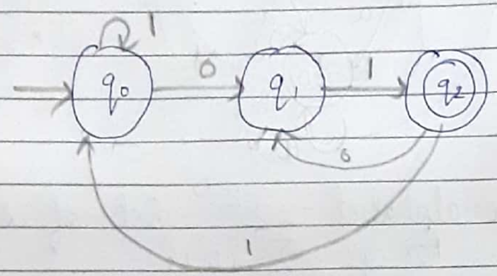
$L = \{0, 01, 00, 000, 0011, \dots\}$



10. a) Construct / Design a DFA that has substring 01,
$\Sigma = \{0,1\}$ $L = \{01, 1001, 00001, 100001, \ldots\}$

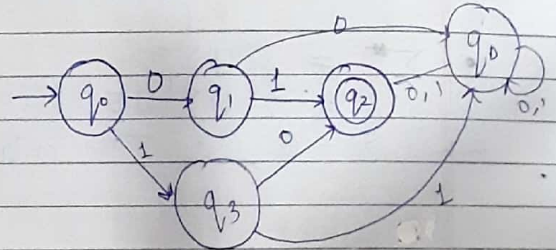


b) Ends with '01' $L = \{01, 0001, 11101, 0101, \ldots\}$



11. Construct a DFA which read strings only '01' or '10' over Input $\Sigma = \{0,1\}$.

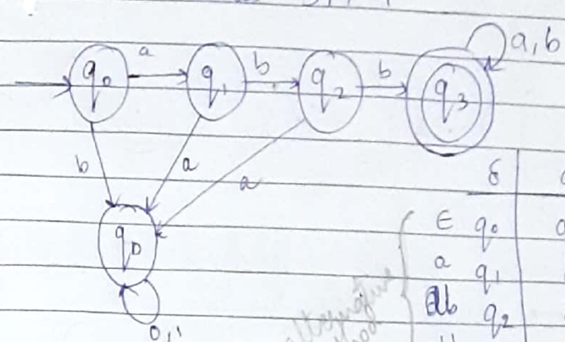
Language = $\{01, 10\}$
States = $2+1 = 3$



Transition table.

δ	0	1
q_0	q_1	q_3
q_1	q_D	q_2
$*q_2$	q_D	q_D
q_3	q_1	q_D
q_D	q_D	q_D

12. Construct a DFA which accepts strings over $\{a, b\}$ such that all strings should start with abb. $L = \{abb, abba, abab, abbaa, \ldots\}$
States = $3+1 = 4$

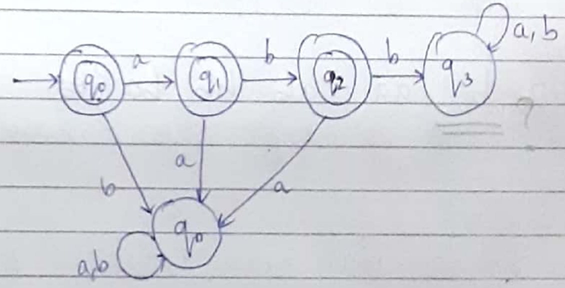


alternative method

	δ	a	b
ϵ	q_0	$q_1(\epsilon a \to a)$	$q_D(\epsilon b \to b)$
a	q_1	$q_D(aa)$	$q_2(ab)$
ab	q_2	$q_D(aba)$	$q_3(abb)$
abb	q_3	$q_3(abba)$	$q_3(abbb)$
	q_D	q_D	q_D

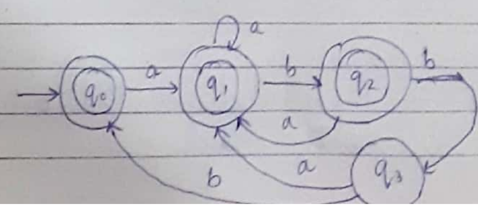
13. CDFA over input, $\Sigma = \{a, b\}$ such that all string should 'not' start with abb.

→ When 'not' is encountered (referring to above problem)
Soln:– 1. Construct DFA
2. Change final state to non-final state
3. Change non-final state to final state

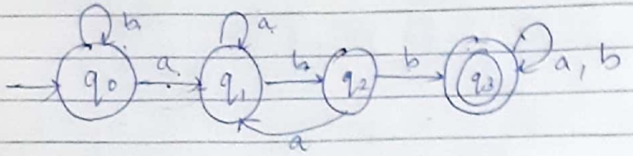


Confusing

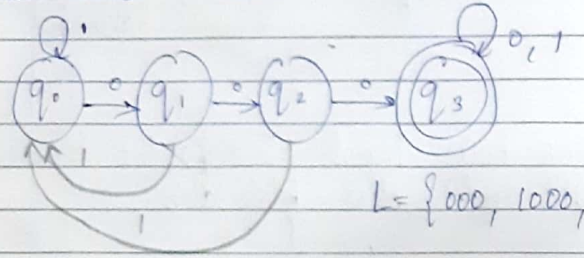
14. CDFA over $\Sigma = \{a,b\}$ that does not ends with abb



15 CDFA $\Sigma = \{a, b\}$ having a substring abb.
Language = {abb, aabbb, babba, bbabba ...}



16. CDFA over input alphabets $\Sigma = \{0, 1\}$ having consecutive zeros '000'.

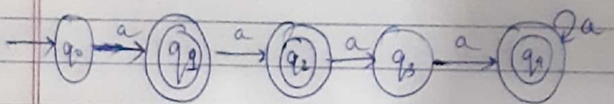


L = {000, 1000, 10001, 11000}

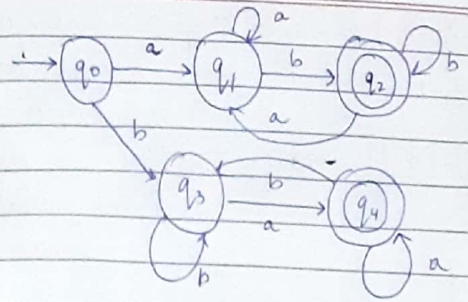
W = length of string

17 Construct DFA over $\Sigma = \{a\}$ such that $|W| \bmod 3 = 0$
18. having a substring 100, $\Sigma = \{0, 1\}$
19 Ends with aba {a, b}.
20 Begins with 101 {0, 1}.

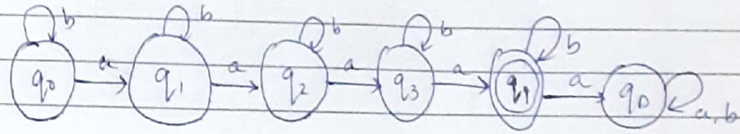
21 Construct a DFA to accept the language, $L = \{a^n / n \geq 1, n \neq 3\}$
$\Sigma = \{a\}$
L = {a, aa, aaa, aaaa, aaaaa ...}



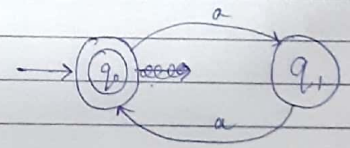
23 CDFA over ~~an~~ input alphabet $\Sigma = \{a, b\}$ starts and ends with different symbol



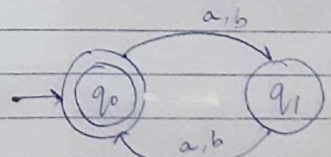
24 CDFA over an input alphabet $\Sigma = \{a, b\}$ which accepts 4 a's
L = {aaaa, aaaab, aaaba ...}



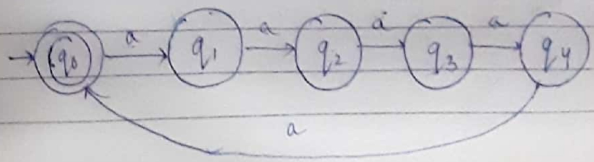
25 a) CDFA over an input alphabet $\Sigma = \{a\}$, which accepts even number of a's / $|W| \bmod 2 = 0$ {a} / divisible by 2
→ 0, 1 → ② → 2 states



b) $\Sigma = \{a, b\}$, even number of a's and b's.

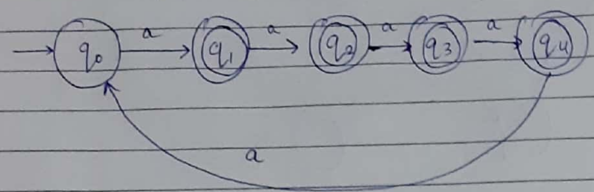


26 $|W| \bmod 5 = 0$ $\Sigma = \{a\}$, 5 states : 0, 1, 2, 3, 4



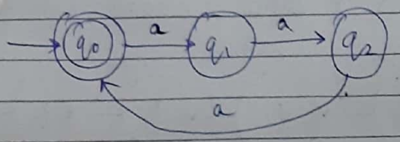
27. Construct a DFA $L = \{ |W| \bmod 5 \neq 0,\ W \in \{a\} \}$

$\Rightarrow$ Remainders $= 0,1,2,3,4$ and it can accept all excluding '0', rest all are made as final state.



17.

$|W| \bmod 3 = 0$.

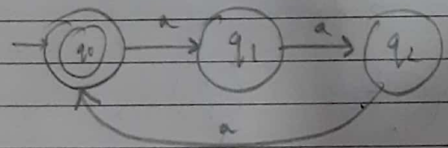


28. $|W| \bmod 3 > |W| \bmod 2$. $\Sigma = \{a\}$

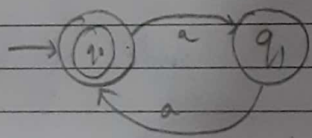
1st Machine 2nd Machine

0, 1, 2 0, 1

1st M_1



2nd M_2



Take cross product of both the machines

$\{ \underset{M_1\ M_2}{(0,0)}, (0,1)\ (1,0)\ (1,1)\ (2,0)\ (2,1) \}$

$\delta\left(q_0 \underset{M_1\ M_2}{(0,0)}, a \right) = (1,1) \rightarrow q_1$

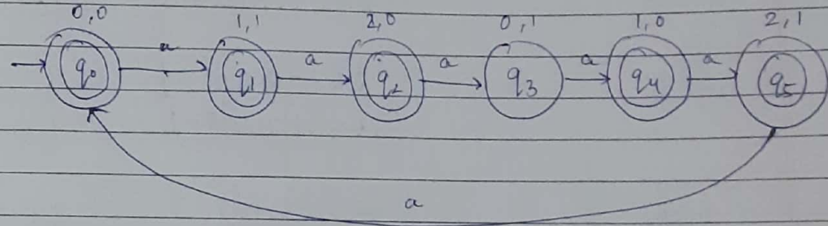
$\delta(q_1(1,1), a) = (2,0) \rightarrow q_2$

$\delta(q_2(2,0), a) = (0,1) \rightarrow q_3$

$\delta(q_3(0,1), a) = (1,0) \rightarrow q_4$

$\delta(q_4(1,0), a) = (2,1) \rightarrow q_5$

$\delta(q_5(2,1), a) = (0,0) \rightarrow q_0$



29. Construct a DFA over input alphabet $\{0,1\}$ where ~~which~~ the strings are binary number divisible by 5.

Divisibility $-$ $\delta(q_i, a) = q_j$

$j = (r * i + d) \bmod k$

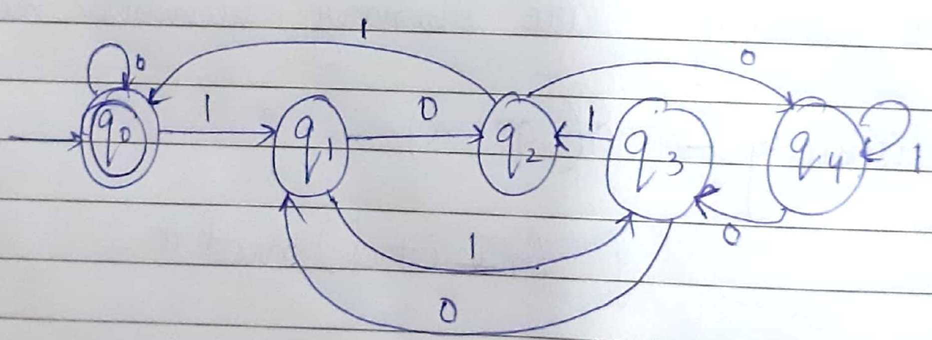
$r \rightarrow$ input binary 2, decimal 10

$i \rightarrow$ remainder k - divisor

$d \rightarrow$ digits

$r = 2$ $i = 0,1,2,3,4$ $k = 5$ $d = \{0,1\}$

remainder	digits (d)	$(r * i + d) \bmod k = j$	$\delta(q_i, d) = q_j$
$i = 0$	0	$(2*0+0) \bmod 5 = 0$	$\delta(q_0, 0) = q_0$
	1	$(2*0+1) \bmod 5 = 1$	$\delta(q_0, 1) = q_1$
$i = 1$	0	$(2*1+0) \bmod 5 = 2$	$\delta(q_1, 0) = q_2$
	1	$(2*1+1) \bmod 5 = 3$	$\delta(q_1, 1) = q_3$
$i = 2$	0	$(2*2+0) \bmod 5 = 4$	$\delta(q_2, 0) = q_4$
	1	$(2*2+1) \bmod 5 = 0$	$\delta(q_2, 1) = q_0$
$i = 3$	0	$(2*3+0) \bmod 5 = 1$	$\delta(q_3, 0) = q_1$
	1	$(2*3+1) \bmod 5 = 2$	$\delta(q_3, 1) = q_2$
$i = 4$	0	$(2*4+0) \bmod 5 = 3$	$\delta(q_4, 0) = q_3$
	1	$(2*4+1) \bmod 5 = 4$	$\delta(q_4, 1) = q_4$



30. Construct a DFA which is interpreted as binary $\{0,1\}$ divisible by 4

$r = 2$ $\quad i = 0, 1, 2, 3$ $\quad k = 4$ $\quad d = \{0, 1\}$