

Example 1: What is the probability of getting a 2 or a 5 when a die is rolled?

Solution:

Taking the individual probabilities of each number, getting a 2 is $1/6$ and so is getting a 5.

Applying the formula of compound probability,

Probability of getting a 2 **or** a 5,

$$P(2 \text{ or } 5) = P(2) + P(5) - P(2 \text{ and } 5)$$

$$\implies 1/6 + 1/6 - 0$$

$$\implies 2/6 = 1/3.$$

Example 2: Consider the example of finding the probability of selecting a black card or a 6 from a deck of 52 cards.

Solution:

We need to find out $P(B \text{ or } 6)$

Probability of selecting a black card =
 $26/52$

Probability of selecting a 6 = $4/52$

Probability of selecting both a black card
and a 6 = $2/52$

$$\begin{aligned} P(B \text{ or } 6) &= P(B) + P(6) - P(B \text{ and } 6) \\ &= 26/52 + 4/52 - 2/52 \\ &= 28/52 \\ &= 7/13. \end{aligned}$$

Example 1: Say, a coin is tossed twice.
What is the probability of getting two
consecutive tails ?

Probability of getting a tail in one toss = $1/2$
The coin is tossed twice. So $1/2 * 1/2 = 1/4$
is the answer.

Here's the verification of the above answer
with the help of sample space.

When a coin is tossed twice, the sample
space is $\{(H,H), (H,T), (T,H), (T,T)\}$.

Our desired event is (T,T) whose
occurrence is only once out of four
possible outcomes and hence, our answer
is $1/4$.

Example 2: Consider another example where a pack contains 4 blue, 2 red and 3 black pens. If a pen is drawn at random from the pack, replaced and the process repeated 2 more times, What is the probability of drawing 2 blue pens and 1 black pen?

Solution

Here, total number of pens = 9

Probability of drawing 1 blue pen = $\frac{4}{9}$

Probability of drawing another blue pen = $\frac{4}{9}$

Probability of drawing 1 black pen = $\frac{3}{9}$

Probability of drawing 2 blue pens and 1 black pen = $\frac{4}{9} * \frac{4}{9} * \frac{3}{9} = \frac{48}{729} = \frac{16}{243}$

Example 1: A pack contains 4 blue, 2 red and 3 black pens. If 2 pens are drawn at random from the pack, NOT replaced and then another pen is drawn. What is the probability of drawing 2 blue pens and 1 black pen?

Solution:

Probability of drawing 1 blue pen = $\frac{4}{9}$

Probability of drawing another blue pen = $\frac{3}{8}$

Probability of drawing 1 black pen = $\frac{3}{7}$

Probability of drawing 2 blue pens and 1 black pen = $\frac{4}{9} * \frac{3}{8} * \frac{3}{7} = \frac{1}{14}$

Example 2: What is the probability of drawing a king and a queen consecutively from a deck of 52 cards, without replacement.

Probability of drawing a king = $4/52 = 1/13$
After drawing one card, the number of cards are 51.

Probability of drawing a queen = $4/51$.

Now, the probability of drawing a king and queen consecutively is $1/13 * 4/51 = 4/663$

Example: In a class, 40% of the students study math and science. 60% of the students study math. What is the probability of a student studying science given he/she is already studying math?

Solution

$$P(\text{M and S}) = 0.40$$

$$P(\text{M}) = 0.60$$

$$P(\text{S}|\text{M}) = P(\text{M and S})/P(\text{S}) = 0.40/0.60 = 2/3 = 0.67$$

Example: A single coin is tossed 5 times.
What is the probability of getting at least one head?

Solution:

Consider solving this using complement.

Probability of getting no head = $P(\text{all tails})$
 $= 1/32$

$P(\text{at least one head}) = 1 - P(\text{all tails}) = 1 - 1/32 = 31/32.$

Example 1

What is the probability of the occurrence of a number that is odd or less than 5 when a fair die is rolled.

Solution

Let the event of the occurrence of a number that is odd be 'A' and the event of the occurrence of a number that is less than 5 be 'B'. We need to find $P(A \text{ or } B)$.

$P(A) = 3/6$ (odd numbers = 1, 3 and 5)

$P(B) = 4/6$ (numbers less than 5 = 1, 2, 3 and 4)

$P(A \text{ and } B) = 2/6$ (numbers that are both odd and less than 5 = 1 and 3)

Now, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$= 3/6 + 4/6 - 2/6$$

$$P(A \text{ or } B) = 5/6.$$

Probability Example 2

A box contains 4 chocobars and 4 ice creams. Tom eats 3 of them, by randomly choosing. What is the probability of choosing 2 chocobars and 1 icecream?

Solution

Probability of choosing 1 chocobar = $\frac{4}{8} = \frac{1}{2}$

After taking out 1 chocobar, the total number is 7.

Probability of choosing 2nd chocobar = $\frac{3}{7}$
Probability of choosing 1 icecream out of a total of 6 = $\frac{4}{6} = \frac{2}{3}$

So the final probability of choosing 2 chocobars and 1 icecream = $\frac{1}{2} * \frac{3}{7} * \frac{2}{3} = \frac{1}{7}$

Probability Example

When two dice are rolled, find the probability of getting a greater number on the first die than the one on the second, given that the sum should equal 8.

Solution

Let the event of getting a greater number on the first die be G .

There are 5 ways to get a sum of 8 when two dice are rolled = $\{(2,6),(3,5),(4,4), (5,3),(6,2)\}$.

And there are two ways where the number on the first die is greater than the one on the second given that the sum should equal 8, $G = \{(5,3), (6,2)\}$.

Therefore, $P(\text{Sum equals } 8) = 5/36$ and $P(G) = 2/36$.

Now, $P(G|\text{sum equals } 8) = P(G \text{ and sum equals } 8)/P(\text{sum equals } 8)$
 $= (2/36)/(5/36)$
 $= 2/5$