Date
04-10-22

Assignment - 1

Q1 Define the following :

(i) Function :- let A and B be any two non-empty sets. A function 'f' from A to B is a set of ordered pair such that every element of set A has one and only one image in set B
i.e $f: A \to B$ a function
such that $y = f(x)$ for $x \in A$ and $y \in B$

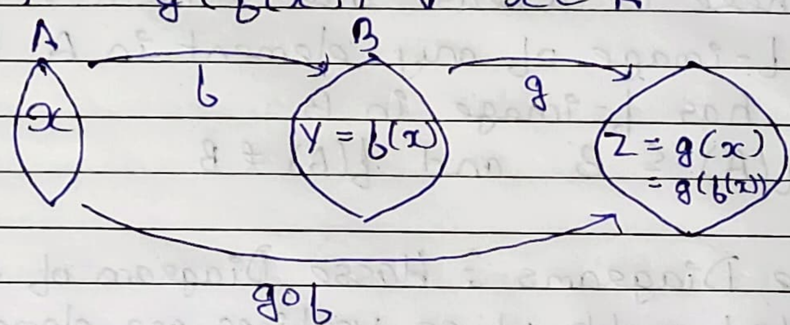
(ii) Into Function :- A function $f: A \to B$ which is not onto function is called into function or if there is atleast one element in B which is not the f-image of any element in A, but every element of A has f-image in B.
i.e $f(A) \subseteq B$ and $f(A) \neq B$

(iii) Hasse Diagrams :- Hasse Diagram of a POSET A is a directed graph whose vertices are elements of A and there is a directed edges from a and b whenever $a \ll b$
In Hasse diagram we will place b higher than 'a' draw a line between them

(iv) Equivalence Relation :- A relation 'R' on a set 'A' is called equivalence (~) relation if it specifies three condition.
(i) R is reflexive
(ii) R is symmetric
(iii) R is transitive

(v) Partial Order Relation :- A relation R on a set 'A' is called a partial order relation if it follows three conditions.
1) R is reflexive i.e $aRa \ \forall \ a \in A$
2) R is anti-symmetric i.e aRb, bRa if $a = b \ \forall$ $a, b \in A$
3) R is transitive i.e $aRb, bRc \Rightarrow aRc \ \forall \ a, b, c \in A$

(vi) Composite Function :- Let $f : A \to B$ and $g : B \to C$ be two function then the composite of function f and g denoted by (gof) is a function from A to C is defined as
$(gof) : A \to C$ such that
$(gof)(x) = g(f(x)) \ \forall \ x \in A$



Composite of Position

(vii) Lattice with Properties :- A lattice is a poset $(L, \leq)$ in which every subset $\{a, b\}$ consisting of 2 elements has least upper bound (lub) and greatest lower bound (glb)
Lub of a subset $\{a, b\}$ is denoted by $a \vee b$
Glb of a subset $\{a, b\}$ is denoted by $a \wedge b$
Properties of Lattice :- let L be a lattice and $a, b \in L$ then

1) Identity Property : (i) $a \vee a = a$
(ii) $a \wedge a = a$

2) Commutative Property : (i) $a \vee b = b \vee a$
(ii) $a \wedge b = b \wedge a$

3) Associative Property : (i) $a \vee (b \vee c) = (a \vee b) \vee c$
(ii) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

4) Absorption Property : (i) $a \vee (a \wedge b) = a$
(ii) $a \wedge (a \vee b) = a$

5) (i) $a \vee b = b$ iff $a \leq b$
(ii) $a \wedge b = a$ iff $a \leq b$
(iii) $a \wedge b = a$ iff $a \vee b = b$

6) Distributive Property : (i) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
(ii) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

7) Isotonicity Property : Let $a, b, c \in L$ and $a \leq b$, then
(i) $a \vee c \leq b \vee c$
(ii) $a \wedge c \leq b \wedge c$

(viii) Bijective Function : A function $f: A \to B$ is said to be one-one onto function or Bijective function if it is one-one & onto function both.

(ix) Invertible Function : Let $f: A \to B$ be an one-one onto function and let $y \in B$ be an arbitrary element then the inverse relation $f^{-1}: B \to A$ is called the inverse function of f.
i.e if $f: A \to B$ is one-one onto function such that $y = f(x)$ $\forall$ $y \in B$
then $f^{-1}: B \to A$ is also one-one onto function such that $x = f^{-1}(y)$ $\forall$ $x \in A$
then $f^{-1}: B \to A$ is also one-one onto function such that $x = f^{-1}(y)$ $\forall$ $x \in A$, then f^{-1} is invertible

(x) Types of Relation :-

(a) Inverse Relation :- let R be any relation from a set A to a set B, then the inverse relation denoted by R^{-1} and defined as

$$R^{-1} : B \to A$$

$R^{-1} = \{(y,x) : x \in A \text{ and } y \in B \; (x,y) \in R\}$

i.e $(x,y) \in R \Leftrightarrow (y,x) \in R^{-1}$

$$xRy \Leftrightarrow yR^{-1}x$$

(b) Identity Relation :- let R be any relation from set A to itself is called identity relation. It is denoted by I_A, $I_A : A \to A$

$$I_A = \{x,y\} : x \in A, y \in A, x = y\}$$

(c) Universal Relation :- A relation R in a set A is said to be universal relation if $R = A \times A$

(d) Void Relation :- A relation $R : A \to A$ is said to be void relation, if $R = \phi$

(xii) Properties of Relation :-

(a) Reflexive Relation :- A relation R on set A is said to be reflexive if and only if each element in A is related to itself.

i.e $aRa \; \forall \; a \in A$

(b) In notes ...

Q2 Let $f : R^+ \to R^+$ and $g : R^+ \to R^+$ be function defined by $f(x) = \sqrt{x}$ & $g(x) = 3x+1$ $\forall$ $x \in R^+$. Find fog & gof

Ans $fog = [f[g(x)]]$

$= f(3x+1) = \sqrt{3x+1}$

$gof = g[f(x)]$

$= g(\sqrt{x}) = 3\sqrt{x}+1$

$fog \neq gof$, No.

Q3 If $f(x) = x^2$, $g(x) = (x+1)$ & $h(x) = (x-1)$ are the function & then find $gofoh$ & $fogoh$.

Ans $gofoh = gof[h(x)]$

$= gof(x-1) = g[f(x-1)]$

$= g[(x-1)^2]$

$= (x-1)^2+1$

$= x^2+1-2x+1 = x^2-2x+2$

$fogoh \Rightarrow fog[h(x)]$

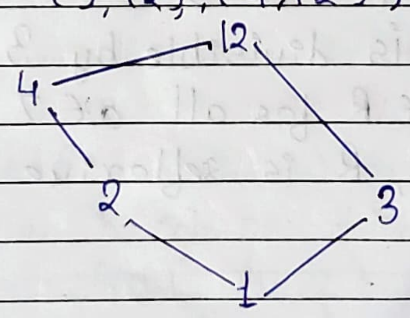
$= fog(x-1)$

$= f[g(x-1)] = f(x-1+1)$

$= f(x) = x^2$

Q4 Draw the Hasse diagram for the poset $A = \{1,2,3,4,12\}$ under the rule R = "a divides b" when $a \leq b$.

Ans $f(x,y) = \{(1,2), (1,3), (1,4), (1,12), (2,4), (2,12), (3,12), (4,12)\}$



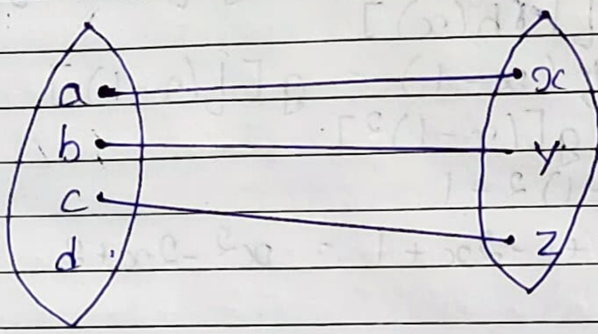
Q5 Define pictorial representation of relation with an example.

Ans A relation R from A to B can be depicted pictorially using arrow diagram. In arrow diagram, we write down the elements of two set A and B in two disjoint circle. Then, we draw arrow from set A to set B whenever $(a,b) \in R$

Let $A = \{a, b, c, d\}$ and $B = \{x, y, z\}$

and $R = \{(a,x), (b,y), (c,z)\}$

Then this will be represented in arrow diagrams.



Q6 Prove that the relation "(a-b) is divisible by 3" $\forall a, b \in Z$ is an equivalence relation.

Ans To prove equivalence relation, the given relation should be reflexive, symmetric & transitive. We have to check these properties of R

Reflexivity :- let a be an arbitary element of R

then $a - a = 0 = 0 \times 3$

$a - a$ is divisible by 3

$\Rightarrow (a,a) \in R$ for all $a \in Z$

Therefore, R is reflexive on Z

Symmetry : let $(a,b) \in R$
$\Rightarrow$ $a-b$ is divisible by 3
$a-b = 3p$ for some $p \in Z$
~~b-a~~ $b-a = 3(-p)$
Here, $-p \in Z$
$b-a$ is divisible by 3
$\Rightarrow$ $(b,a) \in R$ for all $a,b \in Z$
Clearly R is symmetric on Z.

Transitivity : let $(a,b) \in R$ and $(b,c) \in R$
$\Rightarrow$ $a-b$ and $b-c$ are divisible by 3
$a-b = 3p$ for some $p \in Z$ — (i)
$b-c = 3q$ for some $q \in Z$ — (ii)
On adding ~~to~~ eq (i) & (ii), we get
$a-b+b-c = 3p+3q$
$a-c = 3(p+q)$
Here, $p+q \in Z$
$a-c$ is divisible by 3
$\Rightarrow$ $(a,c) \in R$ for all $a,c \in Z$
Thus R is transitive on Z
$\therefore$ R is reflexive, symmetric & transitive
Clearly R is an equivalence relation on Z.

Q7 Show that if R is an equivalence relation on set A then R^{-1} is also equivalence relation.

Ans let us assume three elements x,y,z belongs to a relation R on set A that is, $A = \{x,y,z\}$
Given that R is an equivalence relation on a set A then relation R can be defined as
~~$R = \{(x,x),(y,y),(z,z),(y,x),(x,y),(z,x),(x,z),(z,y)(y,z)\}$~~

$R = \{(x,x), (y,y), (z,z), (x,y), (y,x), (x,z) (z,x)$
$(y,z) (z,y)\}$

Therefore R^{-1} can be written as :-

$R^{-1} = \{(x,x), (y,y), (z,z), (y,x), (x,y), (z,x),$
$(x,z), (z,y), (y,z)\}$

which shows that R^{-1} is also an equivalence relation on set A

$\{(x,x), (y,y), (z,z)\} \in R^{-1}$ shows reflexive

$\{(x,y) (x,z)\} \in R^{-1}$ then $\{(y,z)\} \in R^{-1}$ shows transitive

$\{(y,x), (x,y), (z,x) (x,z), (z,y), (y,z)\} \in R^{-1}$ shows symmetric

Therefore R^{-1} is equivalence

Q8 If R and S are two equivalence relation, then show that $R \cap S$ is also an equivalence relation

Ans. let an element $a \in A$. Since R and S are equivalence relations they are reflexive

Therefore $(a,a) \in R$ and $(a,a) \in S$

So $(a,a) \in R \cap S$

$\therefore$ $R \cap S$ is reflexive — (I)

- Let $(a,b) \in R \cap S$

$\Rightarrow (a,b) \in R$ and $(a,b) \in S$

$(b,a) \in R$ and $(b,a) \in S$

(Since R & S are Symmetric)

$\Rightarrow (b,a) \in R \cap S$

$\therefore$ R is symmetric — (II)

- Let $(a,b), (b,c) \in R \cap S$

$(a,b) (b,c) \in R \Rightarrow (a,c) \in R$

$(a,b) (b,c) \in S \Rightarrow (a,c) \in S$

Since R and S are transitive, $\Rightarrow (a,c) \in R \cap S$

$\Rightarrow R\cap S$ is transitive — (iii)

From eq (i) (ii) & (iii), $R\cap S$ is an equivalnce relation

Q9 ~~Show that~~ State and Prove De-Morgan's Law

Ans In set theory, De-Morgan's laws describe the complement of the union of two sets is always equals to the intersection of their complements. And the complement of the intersection of two sets is always equal to the union of their complements.

Proof :- $(A\cap B)' = A'\cup B'$

let $P = (A\cap B)'$ and $Q = A'\cup B'$

let's s be an arbitary element of M

then $s\in P = s\in (A\cap B)'$

$= s\notin (A\cap B)$

$= s\notin A$ or $s\notin B$

$= s\in A'$ or $s\in B'$

$= s\in A'\cup B'$

$= s\in N$

Therefore, $P\subset Q$ — (i)

Again, let t be an arbitary element of Q then $t\in N = t\in A'\cup B'$

$= t\in A'$ or $t\in B'$

$= t\notin A$ or $t\notin B$

$= t\notin (A\cap B)$

$= t\in (A\cap B)'$

$= t\in M$

Therefore, $Q\subset P$ — (ii)

Now combine (i) and (ii) we get, $P = Q$

i.e $(A\cap B)' = A'\cup B'$

Q10 Let R be the relation on the set of ordered pairs of positive integers such that $(a,b)\ R\ (c,d) \iff ad = bc$

Show that R is an equivalence relation.

Ans (i) Since $(a,b)\ R\ (a,b)\ \forall\ (a,b) \in A$ as $ab = ba$

R is reflexive.

(ii) Again $(a,b)\ R\ (c,d)$

$ac = bd \Rightarrow db = ca$ and so $(c,d)\ R\ (a,b)$

$\therefore$ R is symmetric.

(iii) Again $(a,b)\ R\ (c,d)$ and $(c,d)\ R\ (u,v)$

$\Rightarrow ad = bc$ and $cv = du$

$\Rightarrow \dfrac{adv}{d} = \dfrac{bcu}{c} \Rightarrow av = bu$

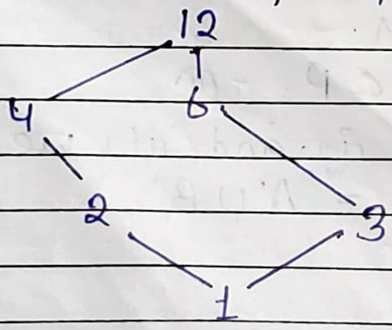
$\therefore (a,b)\ R\ (u,v)$

$\therefore$ R is transistive

From (i), (ii) & (iii), it follows that R is a equivalence relation.

Q11 Draw that Hasse diagram for the Poset $A = \{1,2,3,4,6,12\}$ under the relation "a divides b", when $a \le b$.

Ans $f(a,b) = \{(1,1)\ (1,2), (1,3), (1,4), (1,6), (1,12), (2,4), (2,6), (2,12), (3,6), (3,12)\ (4,12), (6,12)\}$



Q12 If $f : R \to R$ defined by $f(x) = \frac{7-2x}{3}$, then

(i) Show that f is investible

(ii) find f^{-1}

Ans (i) one-one test of f:

Let x and y be two elements of domain (R), such that

$$f(x) = f(y)$$

$$\frac{7-2x}{3} = \frac{7-2y}{3}$$

$$-2x = -2y$$

$$\boxed{x = y}$$

$\therefore$ f is one-one

onto test for f:

Let y be in the co-domain (R), such that $f(x) = y$

$$\Rightarrow \frac{7-2x}{3} = y$$

$$-2x = 3y - 7$$

$$x = \frac{7-3y}{2} \in R \text{ (Domain)}$$

$\Rightarrow$ f is onto

So, f is a bijection and hence it is investible

(ii) Now we have to find f^{-1}

let $f^{-1}(x) = y$ — (1)

$$x = f(y)$$

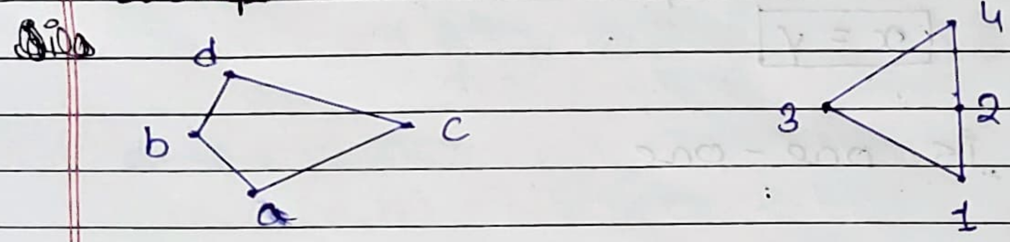
$$x = \frac{7-2y}{3}$$

$$y = \frac{7-3x}{2}$$

From eq (1)
So, $f^{-1}(x) = \frac{7-3x}{2}$

Q14 Define the following :

(i) Isomorphic Lattice :- Two lattice L_1 and L_2 are called isomorphic lattices if there is a bijection from L_1 to L_2 i.e $f: L_1 \to L_2$, such that
$f(a \wedge b) = f(a) \wedge f(b)$ and $f(a \vee b) = f(a) \vee f(b)$
Example :



The lattice shown in fig are isomorphic. Consider the mapping $f = \{(a,1), (b,2), (c,3), (d,4)\}$
For example : $f(b \wedge c) = f(a) = 1$, also we have $f(b) \wedge f(c) = 2 \wedge 3 = 1$

(ii) POSET :- Partially ordered set, A set 'A' together with relation 'R' (partial order relation) is called partially ordered set or poset denoted by (A, R)
A partially ~~ord~~ ordered relation is ~~developed~~ denoted by $\leq$,
$a \leq b$ is read as "a precedes b"

(iii) Sublattices :- Let $(L, \leq)$ be a lattice. A non-empty subset say L is called a sublattice of L if $a \vee b \in S$ & $a \wedge b \in S$ ~~A~~ $\forall$ ~~a,b~~ $a, b \in S$

(iv) Hasse Diagram :- Hasse diagram of a poset A is a directed graph whose vertices are elements of A and there is a directed edges ~~for~~ from a and b whenever $a << b$. In Hasse diagram we place b higher than 'a' draw a line between them