Practical 13: - Euler's Method

Approximate the function values for the given differential equation.

Q1:
$$\frac{dx}{dt} = 1 + \frac{x}{t}$$
, $1 \le t \le 6$, $x(1) = 1$

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In[191]:= Clear[t, x, a, b, n, h, i, w];
      f[t_{-}, x_{-}] := 1 + \frac{x}{t};
      n = 10; a = 1; b = 6;
      h = \frac{b-a}{n};
      Array[w, n, 0];
      w[0] = 1;
      For [i = 1, i \le n, ++i,
       w[i] = w[i-1] + h * f[(a + (i-1) * h), w[i-1]];
      For [i = 0, i \le n, ++i,
      Print["x : ", N[a+i*h, 2], " = ", N[w[i], 10]]
      x : 1.0 = 1.000000000
      x : 1.5 = 2.000000000
      x : 2.0 = 3.166666667
      x : 2.5 = 4.458333333
      x : 3.0 = 5.850000000
      x : 3.5 = 7.325000000
      x : 4.0 = 8.871428571
      x : 4.5 = 10.48035714
      x : 5.0 = 12.14484127
      x : 5.5 = 13.85932540
      x : 6.0 = 15.61926407
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Q2:
$$\frac{dx}{dt} = \frac{t}{x}$$
, $0 \le t \le 5$, $x (0) = 1$

In[207]= Clear[t, x, a, b, n, h, i, w];

$$f[t_{-}, x_{-}] := \frac{t}{x};$$

$$n = 10; a = 0; b = 5;$$

$$h = \frac{b-a}{n};$$

Array[w, n, 0];
$$w[0] = 1;$$

$$For[i = 1, i \le n, ++i,$$

$$w[i] = w[i-1] + h * f[(a + (i-1) * h), w[i-1]];$$

$$for[i = 0, i \le n, ++i,$$

$$print["x : ", N[a + i * h, 2], " = ", N[w[i], 10]]$$

$$x : 0 = 1.000000000$$

$$x : 1.0 = 1.250000000$$

$$x : 1.5 = 1.650000000$$

$$x : 2.0 = 2.104545455$$

$$x : 2.5 = 2.579707442$$

$$x : 3.0 = 3.064258511$$

$$x : 3.5 = 3.553773346$$

$$x : 4.0 = 4.046207677$$

$$x : 4.5 = 4.540497679$$

$$x : 5.0 = 5.036038071$$

$$Q3 : \frac{dx}{dt} = tx^3 - x, 0 \le t \le 1, x (0) = 1$$
In[216]= Clear[t, x, a, b, n, h, i, w];
$$f[t_{-}, x_{-}] := \frac{t}{x};$$

$$n = 4; a = 0; b = 1;$$

$$h = \frac{b-a}{n};$$

$$Array[w, n, 0];$$

$$w[0] = 1;$$

$$For[i = 1, i \le n, ++i,$$

$$w[1] = w[i-1] + h * f[(a + (i-1) * h), w[i-1]];$$

$$For[i = 0, i \le n, ++i,$$

Print["x : ", N[a+i*h, 2], " = ", N[w[i], 10]]

x : 0 = 1.000000000

x : 0.25 = 1.000000000

x : 0.50 = 1.062500000

x : 0.75 = 1.180147059

x : 1.0 = 1.339025563