Practical 14 : - Second Order Runge - Kutta Methods

1) Modified Euler Method.

$$Q1 : - \frac{dx}{dt} = 1 + \frac{x}{t}, \quad 1 \le t \le 6, \quad x(1) = 1$$

In[223]:=
```
Clear[t, x, a, b, n, h, i, w];
f[t_, x_] := 1 + x/t;
n = 10; a = 1; b = 6;
h = (b - a)/n;
Array[w, n, 0];
w[0] = 1;
For[i = 1, i ≤ n, ++i,
 temp = w[i - 1] + h/2 * f[a + (i - 1) * h, w[i - 1]];
 w[i] = w[i - 1] + h * f[a + (i - 1) * h + h/2, temp];
]
For[i = 0, i ≤ n, ++i,
 Print["x : ", N[a + i * h, 2], " = ", N[w[i], 10]]
]
```

```
x : 1.0 = 1.000000000
x : 1.5 = 2.100000000
x : 2.0 = 3.371428571
x : 2.5 = 4.769841270
x : 3.0 = 6.269264069
x : 3.5 = 7.852602953
x : 4.0 = 9.507736708
x : 4.5 = 11.22561556
x : 5.0 = 12.99922197
x : 5.5 = 14.82295369
x : 6.0 = 16.69223406
```

Q2 : $\frac{dx}{dt} = \frac{t}{x}$, $0 \le t \le 5$, $x(0) = 1$

In[231]:=
```
Clear[t, x, a, b, n, h, i, w];
f[t_, x_] := t/x;
n = 10; a = 0; b = 5;
h = (b - a)/n;
Array[w, n, 0];
w[0] = 1;
For[i = 1, i ≤ n, ++i,
 temp = w[i - 1] + h/2 * f[a + (i - 1) * h, w[i - 1]];
 w[i] = w[i - 1] + h * f[a + (i - 1) * h + h/2, temp];
]
For[i = 0, i ≤ n, ++i,
 Print["x : ", N[a + i * h, 2], " = ", N[w[i], 10]]
]
```

```
x : 0 = 1.000000000

x : 0.50 = 1.125000000

x : 1.0 = 1.428370787

x : 1.5 = 1.818168594

x : 2.0 = 2.250391137

x : 2.5 = 2.705382440

x : 3.0 = 3.173642355

x : 3.5 = 3.650186993

x : 4.0 = 4.132204434

x : 4.5 = 4.618006884

x : 5.0 = 5.106527325
```

Q3 : $\frac{dx}{dt} = tx^3 - x, \; 0 \le t \le 1, \; x(0) = 1$

In[239]:=
```
Clear[t, x, a, b, n, h, i, w];
f[t_, x_] := t/x;
n = 4; a = 0; b = 1;
h = (b - a)/n;
Array[w, n, 0];
w[0] = 1;
For[i = 1, i ≤ n, ++i,
 temp = w[i - 1] + h/2 * f[a + (i - 1) * h, w[i - 1]];
 w[i] = w[i - 1] + h * f[a + (i - 1) * h + h/2, temp];
]
For[i = 0, i ≤ n, ++i,
 Print["x : ", N[a + i * h, 2], " = ", N[w[i], 10]]
]
```

```
x : 0 = 1.000000000

x : 0.25 = 1.031250000

x : 0.50 = 1.119564005

x : 0.75 = 1.252498684

x : 1.0 = 1.417300850
```

2) Heun Method.

Q1 : $-\frac{dx}{dt} = 1 + \frac{x}{t}, \; 1 \le t \le 6, \; x(1) = 1$

In[255]:=
```
Clear[t, x, a, b, n, h, i, w];
f[t_, x_] := 1 + x/t;
n = 10; a = 1; b = 6;
h = (b - a)/n;
Array[w, n, 0];
w[0] = 1;
For[i = 1, i ≤ n, ++i,
 temp = w[i - 1] + h * f[a + (i - 1) * h, w[i - 1]];
 w[i] = w[i - 1] + h/2 * (f[a + (i - 1) * h, w[i - 1]] + f[a + (i - 1) * h + h , temp]);
]
For[i = 0, i ≤ n, ++i,
 Print["x : ", N[a + i * h, 2], " = ", N[w[i], 8]]
]
```

```
x : 1.0 = 1.0000000
x : 1.5 = 2.0833333
x : 2.0 = 3.3402778
x : 2.5 = 4.7253472
x : 3.0 = 6.2120833
x : 3.5 = 7.7831448
x : 4.0 = 9.4262727
x : 4.5 = 11.132335
x : 5.0 = 12.894261
x : 5.5 = 14.706414
x : 6.0 = 16.564194
```

Q2 : $\frac{dx}{dt} = \frac{t}{x}$, $0 \le t \le 5$, $x(0) = 1$

In[263]:=

```
Clear[t, x, a, b, n, h, i, w];
f[t_, x_] := t/x;
n = 10; a = 0; b = 5;
h = (b - a)/n;
Array[w, n, 0];
w[0] = 1;
For[i = 1, i ≤ n, ++i,
 temp = w[i - 1] + h * f[a + (i - 1) * h, w[i - 1]];
 w[i] = w[i - 1] + h/2 * (f[a + (i - 1) * h, w[i - 1]] + f[a + (i - 1) * h + h , temp]);
]
For[i = 0, i ≤ n, ++i,
 Print["x : ", N[a + i * h, 2], " = ", N[w[i], 10]]
]
```

```
x : 0 = 1.000000000
x : 0.50 = 1.125000000
x : 1.0 = 1.421678121
x : 1.5 = 1.808987822
x : 2.0 = 2.241148240
x : 2.5 = 2.696819449
x : 3.0 = 3.165891041
x : 3.5 = 3.643196175
x : 4.0 = 4.125879764
x : 4.5 = 4.612253948
x : 5.0 = 5.101263360
```

Q3 : $\frac{dx}{dt} = tx^3 - x,\ 0 \le t \le 1,\ x(0) = 1$

In[271]:=
```
Clear[t, x, a, b, n, h, i, w];
f[t_, x_] := t/x;
n = 4; a = 0; b = 1;
h = (b - a)/n;
Array[w, n, 0];
w[0] = 1;
For[i = 1, i ≤ n, ++i,
 temp = w[i - 1] + h * f[a + (i - 1) * h, w[i - 1]];
 w[i] = w[i - 1] + h/2 * (f[a + (i - 1) * h, w[i - 1]] + f[a + (i - 1) * h + h , temp]);
]
For[i = 0, i ≤ n, ++i,
 Print["x : ", N[a + i * h, 2], " = ", N[w[i], 10]]
]
```

```
x : 0 = 1.000000000

x : 0.25 = 1.031250000

x : 0.50 = 1.118795008

x : 0.75 = 1.250845839

x : 1.0 = 1.415033388
```