Practical 6 : - Gauss - Jacobi Method

Solve the given system of equation using the iterative method Gauss – Jacobi Method with tollerance 10^–6.

Q1 :-

5x1 + x2 + 2x3 = 10

-3x1 + 9x2 + 4x3 = -14

x1 + 2x2 - 7x3 = -33

In[ ]:=
```
A = {{5, 1, 2}, {-3, 9, 4}, {1, 2, -7}};
b = {{10}, {-14}, {-33}};
d = DiagonalMatrix[Diagonal[A]];
L = LowerTriangularize[A] - d;
U = UpperTriangularize[A] - d;
t = -Inverse[d].(L + U);
c = Inverse[d].b;
xold = {{0}, {0}, {0}}; xnew = {{0}, {0}, {0}};
For[i = 1, i ≤ 50, i++,
 xnew = t.xold + c;
 If[Max[Abs[xnew - xold]] < 10^-6, Break[]];
 xold = xnew;
 Print["Iteration ", i, " ", N[xnew]]
]
```

```
Iteration 1 {{2.}, {-1.55556}, {4.71429}}
Iteration 2 {{0.425397}, {-2.98413}, {4.55556}}
Iteration 3 {{0.774603}, {-3.43845}, {3.92245}}
Iteration 4 {{1.11871}, {-3.04067}, {3.84253}}
Iteration 5 {{1.07112}, {-2.89044}, {4.00534}}
Iteration 6 {{0.975953}, {-2.97867}, {4.04146}}
Iteration 7 {{0.979148}, {-3.02644}, {4.00266}}
Iteration 8 {{1.00422}, {-3.00813}, {3.98947}}
Iteration 9 {{1.00584}, {-2.99391}, {3.99828}}
Iteration 10 {{0.99947}, {-2.99729}, {4.00257}}
Iteration 11 {{0.998428}, {-3.00132}, {4.0007}}
Iteration 12 {{0.999985}, {-3.00083}, {3.9994}}
Iteration 13 {{1.00041}, {-2.99974}, {3.99976}}
Iteration 14 {{1.00004}, {-2.99976}, {4.00013}}
Iteration 15 {{0.999898}, {-3.00004}, {4.00008}}
Iteration 16 {{0.999979}, {-3.00007}, {3.99997}}
Iteration 17 {{1.00002}, {-2.99999}, {3.99998}}
Iteration 18 {{1.00001}, {-2.99998}, {4.}}
Iteration 19 {{0.999994}, {-3.}, {4.00001}}
Iteration 20 {{0.999997}, {-3.}, {4.}}
Iteration 21 {{1.}, {-3.}, {4.}}
Iteration 22 {{1.}, {-3.}, {4.}}
Iteration 23 {{1.}, {-3.}, {4.}}
```

Q2 :-

4 x1 + x2 + x3 + x4 = -5

1 x1 + 8 x2 + 2 x3 + 3 x4 = 23

x1 + 2 x2 - 5 x3 = 9

-x1 + + 2 x3 + 4 x4 = 4

```
In[1]:= Clear["Global*`"];
A = (4   1   1   1
     1   8   2   3
     1   2  -5   0
    -1   0   2   4);
b = (-5
     23
      9
      4);
d = DiagonalMatrix[Diagonal[A]];
L = LowerTriangularize[A] - d;
U = UpperTriangularize[A] - d;
t = -Inverse[d].(L + U);
c = Inverse[d].b;
xold = (0
        0
        0
        0); xnew = (0
                    0
                    0
                    0);
For[i = 1, i ≤ 50, i++,
 xnew = t.xold + c;
 If[Max[Abs[xnew - xold]] < 10^-6, Break[]];
 xold = xnew;
 Print["Iteration ", i, " ", N[xnew]]
]
```

```
Iteration 1 {{-1.25}, {2.875}, {-1.8}, {1.}}
Iteration 2 {{-1.76875}, {3.10625}, {-0.9}, {1.5875}}
Iteration 3 {{-2.19844}, {2.72578}, {-0.91125}, {1.00781}}
Iteration 4 {{-1.95559}, {2.99969}, {-1.14938}, {0.906016}}
Iteration 5 {{-1.93908}, {3.06704}, {-0.991242}, {1.08579}}
Iteration 6 {{-2.0404}, {2.95802}, {-0.961002}, {1.01085}}
Iteration 7 {{-2.00197}, {2.99123}, {-1.02487}, {0.970402}}
Iteration 8 {{-1.98419}, {3.01756}, {-1.0039}, {1.01194}}
Iteration 9 {{-2.0064}, {2.99452}, {-0.989813}, {1.0059}}
Iteration 10 {{-2.00265}, {2.99604}, {-1.00347}, {0.993306}}
Iteration 11 {{-1.99647}, {3.00371}, {-1.00211}, {1.00107}}
Iteration 12 {{-2.00067}, {2.99968}, {-0.99781}, {1.00194}}
Iteration 13 {{-2.00095}, {2.99881}, {-1.00026}, {0.998738}}
Iteration 14 {{-1.99932}, {3.00066}, {-1.00067}, {0.999891}}
Iteration 15 {{-1.99997}, {3.00012}, {-0.999601}, {1.0005}}
Iteration 16 {{-2.00026}, {2.99971}, {-0.999945}, {0.999808}}
Iteration 17 {{-1.99989}, {3.00009}, {-1.00017}, {0.999908}}
Iteration 18 {{-1.99996}, {3.00006}, {-0.999942}, {1.00011}}
Iteration 19 {{-2.00006}, {2.99994}, {-0.999966}, {0.999982}}
Iteration 20 {{-1.99999}, {3.00001}, {-1.00004}, {0.999969}}
Iteration 21 {{-1.99998}, {3.00002}, {-0.999995}, {1.00002}}
Iteration 22 {{-2.00001}, {2.99999}, {-0.999989}, {1.}}
Iteration 23 {{-2.}, {3.}, {-1.00001}, {0.999992}}
Iteration 24 {{-2.}, {3.}, {-1.}, {1.}}
Iteration 25 {{-2.}, {3.}, {-0.999997}, {1.}}
Iteration 26 {{-2.}, {3.}, {-1.}, {0.999998}}
Iteration 27 {{-2.}, {3.}, {-1.}, {1.}}
Iteration 28 {{-2.}, {3.}, {-0.999999}, {1.}}
```

Q3 :-

$4 x1 - x2 = 2$

$-x1 + 4 x2 - x3 = 4$

$- x2 + 4 x3 = 10$

In[11]:=
```
Clear["Global*`"];
A = (4  -1  0
     -1  4  -1
     0  -1  4);
b = (2
     4
     10);
d = DiagonalMatrix[Diagonal[A]];
L = LowerTriangularize[A] - d;
U = UpperTriangularize[A] - d;
t = -Inverse[d].(L + U);
c = Inverse[d].b;
xold = (0
        0
        0); xnew = (0
                    0
                    0);
For[i = 1, i ≤ 50, i++,
 xnew = t.xold + c;
 If[Max[Abs[xnew - xold]] < 10^-6, Break[]];
 xold = xnew;
 Print["Iteration ", i, " ", N[xnew]]
]
```

```
Iteration 1 {{0.5}, {1.}, {2.5}}
Iteration 2 {{0.75}, {1.75}, {2.75}}
Iteration 3 {{0.9375}, {1.875}, {2.9375}}
Iteration 4 {{0.96875}, {1.96875}, {2.96875}}
Iteration 5 {{0.992188}, {1.98438}, {2.99219}}
Iteration 6 {{0.996094}, {1.99609}, {2.99609}}
Iteration 7 {{0.999023}, {1.99805}, {2.99902}}
Iteration 8 {{0.999512}, {1.99951}, {2.99951}}
Iteration 9 {{0.999878}, {1.99976}, {2.99988}}
Iteration 10 {{0.999939}, {1.99994}, {2.99994}}
Iteration 11 {{0.999985}, {1.99997}, {2.99998}}
Iteration 12 {{0.999992}, {1.99999}, {2.99999}}
Iteration 13 {{0.999998}, {2.}, {3.}}
Iteration 14 {{0.999999}, {2.}, {3.}}
```