Practical 4 :

Solution of vibration of semi – infinite string with fixed end.

$$u_{tt} = c^2 u_{xx}\,,\quad 0 < x < \infty\,,\quad t > 0,$$

$$u\,(x,\,0) = f\,(x)\,,\; 0 \le x \le \infty,$$

$$u_t\,(x,\,0) = g\,(x)\,,\quad 0 \le x \le \infty,$$

$$u_x\,(0,\,t\,) = 0$$

In[∘]:=
```
ClearAll;
weqn = D[u[x, t], {t, 2}] == c^2 * D[u[x, t], {x, 2}]
ic = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == g[x], Derivative[1, 0][u][0, t] == 0}
```

In[∘]:=
```
dsol = DSolveValue[{weqn, ic }, u[x, t], {x, t}]
```

Out[∘]=

$$\begin{cases} \frac{1}{2}\left(f\left[-\sqrt{c^2}\ t+x\right]+f\left[\sqrt{c^2}\ t+x\right]\right)+\frac{\int_{-\sqrt{c^2}\ t+x}^{\sqrt{c^2}\ t+x} g[K[1]]\,dK[1]}{2\sqrt{c^2}} & x > \sqrt{c^2}\ t \ge 0 \\ \frac{1}{2}\left(-f\left[\sqrt{c^2}\ t-x\right]+f\left[\sqrt{c^2}\ t+x\right]\right)+\frac{\int_{\sqrt{c^2}\ t-x}^{\sqrt{c^2}\ t+x} g[K[1]]\,dK[1]}{2\sqrt{c^2}} & 0 \le x \le \sqrt{c^2}\ t \\ \text{Indeterminate} & \text{True} \end{cases}$$

In[∘]:=
```
dsol /. {f[x_] → Sin[x], g[x_] → x^2}
```

Out[∘]=

$$\begin{cases} \frac{-\frac{1}{3}\left(-\sqrt{c^2}\ t+x\right)^3+\frac{1}{3}\left(\sqrt{c^2}\ t+x\right)^3}{2\sqrt{c^2}}+\frac{1}{2}\left(-\mathrm{Sin}\left[\sqrt{c^2}\ t-x\right]+\mathrm{Sin}\left[\sqrt{c^2}\ t+x\right]\right) & x > \sqrt{c^2}\ t \ge 0 \\ \frac{-\frac{1}{3}\left(\sqrt{c^2}\ t-x\right)^3+\frac{1}{3}\left(\sqrt{c^2}\ t+x\right)^3}{2\sqrt{c^2}}+\frac{1}{2}\left(-\mathrm{Sin}\left[\sqrt{c^2}\ t-x\right]+\mathrm{Sin}\left[\sqrt{c^2}\ t+x\right]\right) & 0 \le x \le \sqrt{c^2}\ t \\ \text{Indeterminate} & \text{True} \end{cases}$$

In[ ]:=
```
Manipulate[Plot3D[%32, {t, 0, 5.28319}, {x, 0, 10}], {c, -2, 2}]
```

Out[ ]=