Practical 6 :

Solution of one dimensional heat equation $u_{tt} = c^2 u_{xx}$, for a homogeneous rod of length l, that is to solve the IBVP :

$$u_t = k\,u_{xx},\quad 0 < x < l,\quad t > 0,$$
$$u(x, 0) = f(x),\quad 0 \le x < l,$$
$$u(0, t) = 0,$$
$$u(l, t) = 0.$$

```
In[•]:= ClearAll;

In[•]:= eq = D[u[x, t], {t}] == k * D[u[x, t], {x, 2}] /. {k → 1}
Out[•]= u^(0,1)[x, t] == u^(2,0)[x, t]

In[•]:= cond = {u[x, 0] == x^2, u[0, t] == 0, u[Pi, t] == 0}
Out[•]= {u[x, 0] == x^2, u[0, t] == 0, u[π, t] == 0}

In[•]:= dsol = DSolveValue[{eq, cond}, u[x, t], {x, t}] /. {K[1] → m}
Out[•]= Sum_{m=1}^{∞} -(2 E^(-m^2 t) (2 - 2 (-1)^m + (-1)^m m^2 π^2) Sin[m x]) / (m^3 π)

In[•]:= dsol /. {Infinity → 4} // Activate
Out[•]= -(2 E^-t (4 - π^2) Sin[x]) / π - E^-4t π Sin[2 x] - (2 E^-9t (4 - 9 π^2) Sin[3 x]) / (27 π) - 1/2 E^-16t π Sin[4 x]
```

In[ ]:=
```
Plot3D[-(2 E^-t (4 - π^2) Sin[x])/π - E^-4 t π Sin[2 x] - (2 E^-9 t (4 - 9 π^2) Sin[3 x])/(27 π) - 1/2 E^-16 t π Sin[4 x],
 {t, 0, 10}, {x, 0, Pi}, AxesLabel → Automatic]
```

Out[ ]=