Practical 7 : - Gauss - Seidel Method

Solve the given system of equation using the iterative method Gauss
   – Seidel Method with tollerance 10^–6.

Q1 :–
$4x1 - x2 - 2x3 = -4$
$-x1 + 3x2 + = 5$
$-x2 + 3x3 = 7$

```
In[*]:= Clear["Global*`"];
       A = ( 4  -1  -2 )
           ( -1  3   0 );
           ( 0  -1   3 )

       b = ( -4 )
           (  5 );
           (  7 )

       d = DiagonalMatrix[Diagonal[A]];
       L = LowerTriangularize[A] - d;
       U = UpperTriangularize[A] - d;
       t = -Inverse[d + L].U;
       c = Inverse[d + L].b;
       xold = ( 0 )      ( 0 )
              ( 0 ); xnew = ( 0 );
              ( 0 )      ( 0 )

       For[i = 1, i ≤ Infinity, i++,
        xnew = t.xold + c;
        If[Max[Abs[xnew - xold]] < 10^-6, Break[]];
        xold = xnew;
        Print["Iteration ", i, " ", N[xnew]]
       ]

Iteration 1 {{-1.}, {1.33333}, {2.77778}}

Iteration 2 {{0.722222}, {1.90741}, {2.96914}}

Iteration 3 {{0.96142}, {1.98714}, {2.99571}}

Iteration 4 {{0.994642}, {1.99821}, {2.9994}}

Iteration 5 {{0.999256}, {1.99975}, {2.99992}}

Iteration 6 {{0.999897}, {1.99997}, {2.99999}}

Iteration 7 {{0.999986}, {2.}, {3.}}

Iteration 8 {{0.999998}, {2.}, {3.}}

Iteration 9 {{1.}, {2.}, {3.}}
```

Q2 :−
$4 x1 - x2 - 2 x4 = -1$
$-x1 + 4 x2 - x3 - 2 x5 = 0$
$-x2 + 4 x3 - 2 x6 = 1$
$-x1 + 4 x4 - x5 = -2$
$-x2 - x4 + 4 x5 - x6 = 1$
$-x3 - x5 + 4 x6 = 2$

In[ ]:=
```mathematica
Clear["Global*`"];
A = (  4  -1   0  -2   0   0
      -1   4  -1   0  -2   0
       0  -1   4   0   0  -2
      -1   0   0   4  -1   0
       0  -1   0  -1   4  -1
       0   0  -1   0  -1   4 );

b = ( -1
       0
       1
      -2
       1
       2 );

d = DiagonalMatrix[Diagonal[A]];
L = LowerTriangularize[A] - d;
U = UpperTriangularize[A] - d;
t = -Inverse[d + L].U;
c = Inverse[d + L].b;

xold = ( 0       xnew = ( 0
         0                0
         0                0
         0     ;          0   );
         0                0
         0 );             0

For[i = 1, i ≤ Infinity, i++,
 xnew = t.xold + c;
 If[Max[Abs[xnew - xold]] < 10^-6, Break[]];
 xold = xnew;
 Print["Iteration ", i, " ", N[xnew]]
 ]
```

```
Iteration 1 {{-0.25}, {-0.0625}, {0.234375}, {-0.5625}, {0.09375}, {0.582031}}

Iteration 2 {{-0.546875}, {-0.03125}, {0.533203}, {-0.613281}, {0.234375}, {0.691895}}

Iteration 3 {{-0.564453}, {0.109375}, {0.623291}, {-0.58252}, {0.304688}, {0.731995}}

Iteration 4 {{-0.513916}, {0.179688}, {0.660919}, {-0.552307}, {0.339844}, {0.750191}}

Iteration 5 {{-0.481232}, {0.214844}, {0.678806}, {-0.535347}, {0.357422}, {0.759057}}

Iteration 6 {{-0.463963}, {0.232422}, {0.687634}, {-0.526635}, {0.366211}, {0.763461}}

Iteration 7 {{-0.455212}, {0.241211}, {0.692033}, {-0.52225}, {0.370605}, {0.76566}}

Iteration 8 {{-0.450822}, {0.245605}, {0.694231}, {-0.520054}, {0.372803}, {0.766758}}

Iteration 9 {{-0.448626}, {0.247803}, {0.69533}, {-0.518956}, {0.373901}, {0.767308}}

Iteration 10 {{-0.447527}, {0.248901}, {0.695879}, {-0.518406}, {0.374451}, {0.767582}}

Iteration 11 {{-0.446978}, {0.249451}, {0.696154}, {-0.518132}, {0.374725}, {0.76772}}

Iteration 12 {{-0.446703}, {0.249725}, {0.696291}, {-0.517994}, {0.374863}, {0.767788}}

Iteration 13 {{-0.446566}, {0.249863}, {0.69636}, {-0.517926}, {0.374931}, {0.767823}}

Iteration 14 {{-0.446497}, {0.249931}, {0.696394}, {-0.517891}, {0.374966}, {0.76784}}

Iteration 15 {{-0.446463}, {0.249966}, {0.696411}, {-0.517874}, {0.374983}, {0.767849}}

Iteration 16 {{-0.446446}, {0.249983}, {0.69642}, {-0.517866}, {0.374991}, {0.767853}}

Iteration 17 {{-0.446437}, {0.249991}, {0.696424}, {-0.517861}, {0.374996}, {0.767855}}

Iteration 18 {{-0.446433}, {0.249996}, {0.696426}, {-0.517859}, {0.374998}, {0.767856}}

Iteration 19 {{-0.446431}, {0.249998}, {0.696427}, {-0.517858}, {0.374999}, {0.767857}}

Iteration 20 {{-0.44643}, {0.249999}, {0.696428}, {-0.517858}, {0.374999}, {0.767857}}
```

Q3 :-
$5 x_1 + x_2 + 2 x_3 = 10$
$-3 x_1 + 9 x_2 + 4 x_3 = -14$
$x_1 + 2 x_2 - 7 x_3 = -33$

```
In[ ]:= Clear["Global`*"];
      A = ( 5   1   2
           -3   9   4 );
            1   2  -7

      b = ( 10
           -14 );
           -33

      d = DiagonalMatrix[Diagonal[A]];
      L = LowerTriangularize[A] - d;
      U = UpperTriangularize[A] - d;
      t = -Inverse[d + L].U;
      c = Inverse[d + L].b;

      xold = ( 0        0
               0 ); xnew = ( 0 );
               0        0

      For[i = 1, i ≤ Infinity, i++,
       xnew = t.xold + c;
       If[Max[Abs[xnew - xold]] < 10^-6, Break[]];
       xold = xnew;
       Print["Iteration ", i, " ", N[xnew]]
      ]
```

```
Iteration 1 {{2.}, {-0.888889}, {4.74603}}
Iteration 2 {{0.279365}, {-3.57178}, {3.73369}}
Iteration 3 {{1.22088}, {-2.80801}, {4.08641}}
Iteration 4 {{0.927039}, {-3.06272}, {3.97166}}
Iteration 5 {{1.02388}, {-2.97944}, {4.00929}}
Iteration 6 {{0.992174}, {-3.00674}, {3.99696}}
Iteration 7 {{1.00256}, {-2.99779}, {4.001}}
Iteration 8 {{0.99916}, {-3.00072}, {3.99967}}
Iteration 9 {{1.00028}, {-2.99976}, {4.00011}}
Iteration 10 {{0.99991}, {-3.00008}, {3.99996}}
Iteration 11 {{1.00003}, {-2.99997}, {4.00001}}
Iteration 12 {{0.99999}, {-3.00001}, {4.}}
Iteration 13 {{1.}, {-3.}, {4.}}
Iteration 14 {{0.999999}, {-3.}, {4.}}
Iteration 15 {{1.}, {-3.}, {4.}}
```