Aim : - Find the Solution of the Cauchy Problem with non - parametric form
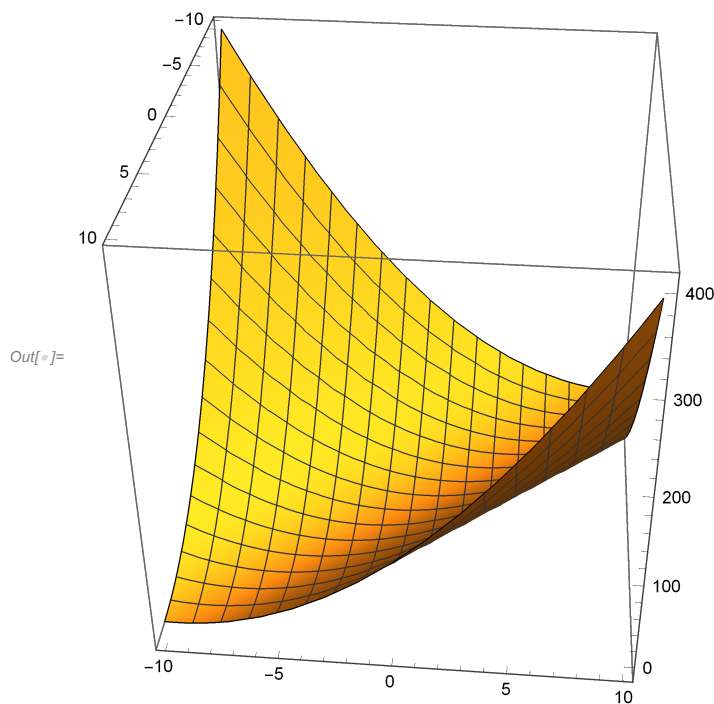
In[ ]:= **pde1 = D[u[x, y], x] - D[u[x, y], y] == 1**

Out[ ]= $-u^{(0,1)}[x, y] + u^{(1,0)}[x, y] == 1$

In[ ]:= **solutionSurface = DSolve[{pde1, u[x, 0] == x^2}, u[x, y], {x, y}]**

Out[ ]= $\{\{u[x, y] \rightarrow x^2 - y + 2 x y + y^2\}\}$

In[ ]:= **Plot3D[x^2 - y + 2 x y + y^2, {x, -10, 10}, {y, -10, 10}, BoxRatios → {1, 1, 1}]**

Out[ ]=

In[ ]:= **ch1 = D[y[x], x] == -1**

Out[ ]= $y'[x] == -1$

In[ ]:= **sol = DSolve[ch1, {y[x]}, {x}]**

Out[ ]= $\{\{y[x] \rightarrow -x + C[1]\}\}$

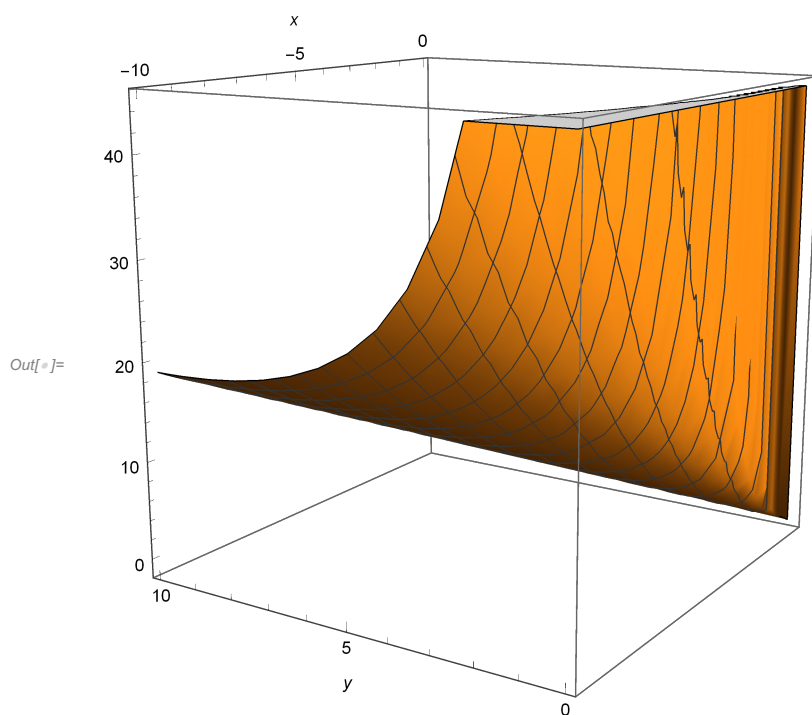In[ ]:= `Plot[y[x] /. sol /. C[1] → Range[0, 4], {x, -4, 4}]`

Out[ ]=
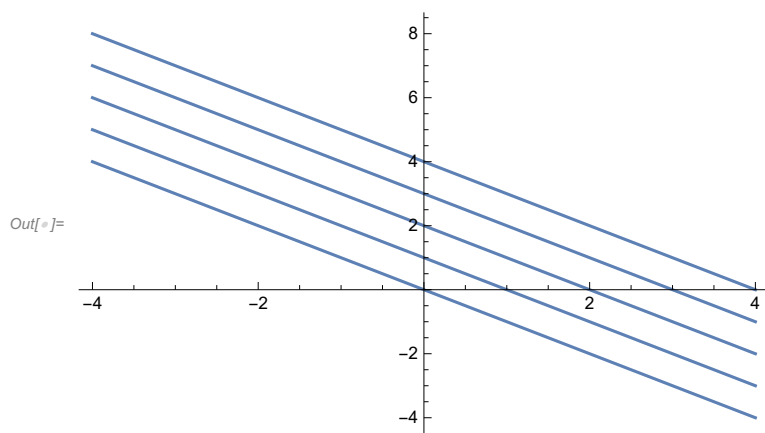
In[ ]:= `pde2 = D[u[x, y], x] * x + y * D[u[x, y], y] == u[x, y] + 1`

Out[ ]= $y\, u^{(0,1)}[x, y] + x\, u^{(1,0)}[x, y] == 1 + u[x, y]$

In[ ]:= `DSolve[{pde2, u[x, x^2] == x^2}, {u[x, y]}, {x, y}]`

Out[ ]= $\left\{\left\{u[x, y] \to \dfrac{x^2 - y + y^2}{y}\right\}\right\}$

In[ ]:= `Plot3D[`$\dfrac{x^2 - y + y^2}{y}$`, {x, -10, 0}, {y, 0, 10}, BoxRatios → {1, 1, 1}, AxesLabel → Automatic]`

Out[ ]=

In[12]:= `ch2 = D[y[x], x] ==` $\dfrac{y[x]}{x}$

Out[12]= $y'[x] == \dfrac{y[x]}{x}$

In[14]:= **sol2 = DSolve[ch2, {y[x]}, {x}]**

Out[14]= $\{\{y[x] \rightarrow x\, C[1]\}\}$

In[15]:= **Plot[y[x] /. sol2 /. C[1] → Range[0, 4], {x, -4, 4}]**

Out[15]=