Practical 2 :

Aim:- Solution of Cauchy problem for homogeneous Wave Equations

$u_{tt} = c^2 u_{xx}$, x ϵ ℝ, t > 0,

u(x,0) = f(x), x ϵ ℝ,

u_t(x,0) = g(x), x ϵ ℝ.

In[∘]:=
```
ClearAll;
```

In[∘]:=
```
weqn = D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}]
```

Out[∘]= $u^{(0,2)}[x, t] == u^{(2,0)}[x, t]$

In[∘]:=
```
ic = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == g[x]}
```

Out[∘]= $\{u[x, 0] == f[x], u^{(0,1)}[x, 0] == g[x]\}$

In[∘]:=
```
dsol = DSolveValue[{weqn, ic}, u[x, t], {x, t}]
```

Out[∘]= $\frac{1}{2}\left(f[-t+x] + f[t+x]\right) + \frac{1}{2}\int_{-t+x}^{t+x} g[K[1]]\, dK[1]$

```
(* Above found solution is known as D Alembert
 solution for cauchy problem for homogeneous Wave Equation. *)
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In[∘]:=
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dsol /. {f[x_] → Exp[-x^2], g[x_] → 1}
```
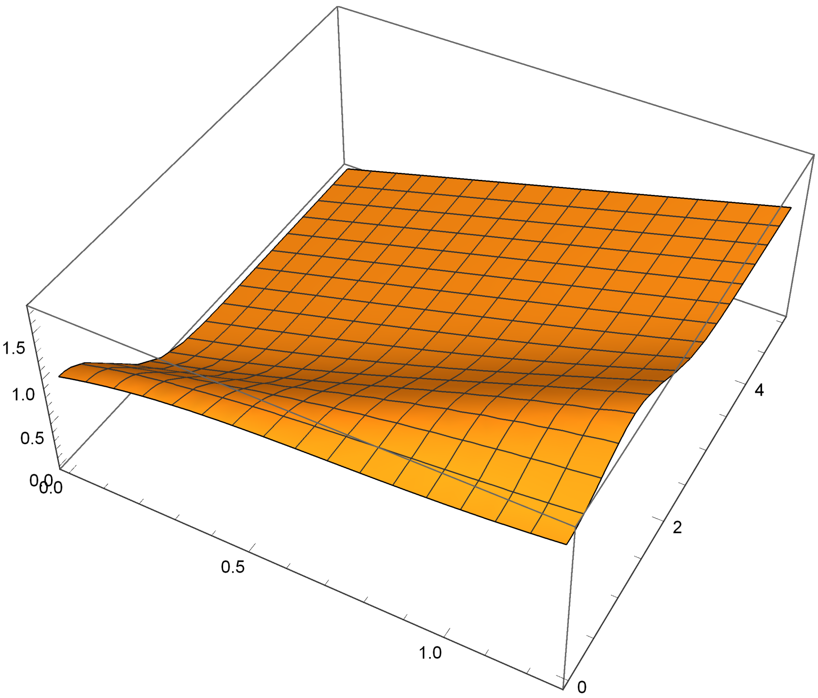

Out[∘]= $\frac{1}{2}\left(e^{-(-t+x)^2} + e^{-(t+x)^2}\right) + t$

In[∘]:= Plot3D$\left[\frac{1}{2}\left(e^{-(-t+x)^2}+e^{-(t+x)^2}\right)+t, \{t, 0, 1.25947\}, \{x, 0, 5\}\right]$

Out[∘]=



In[∘]:= dsol /. $\{f[x\_] \to Sin[x], g[x\_] \to x^2\}$

Out[∘]= $\frac{1}{2}\left(-\frac{1}{3}(-t+x)^3+\frac{1}{3}(t+x)^3\right)+\frac{1}{2}(-Sin[t-x]+Sin[t+x])$

In[∘]:= Plot3D$\left[\frac{1}{2}\left(-\frac{1}{3}(-t+x)^3+\frac{1}{3}(t+x)^3\right)+\frac{1}{2}(-Sin[t-x]+Sin[t+x]),\right.$
$\left.\{t, 0, 5.28319\}, \{x, -7.28319, 5.28319\}\right]$

Out[∘]=