Practical 8 : - SOR Method

Solve the given system of equation using the iterative method SOR Method with tollerance 10^−6 and weight w = 0.9 .

Q1 : −

$4x1 - x2 - 2x3 = -4$

$-x1 + 3x2 + = 5$

$-x2 + 3x3 = 7$

In[ ]:=
```
Clear["Global*`"];
A = {{4, -1, -2}, {-1, 3, 0}, {0, -1, 3}};
b = {{-4}, {5}, {7}};
d = DiagonalMatrix[Diagonal[A]];
L = LowerTriangularize[A] - d;
U = UpperTriangularize[A] - d;
w = 0.9;
t = Inverse[(1/w) * d + L].(((1/w) - 1) * d - U);
c = Inverse[(1/w) * d + L].b;
xold = {{0}, {0}, {0}}; xnew = {{0}, {0}, {0}};
For[i = 1, i ≤ 50, i++,
 xnew = t.xold + c;
 If[Max[Abs[xnew - xold]] < 10^-6, Break[]];
 xold = xnew;
 Print["Iteration ", i, " ", N[xnew]]
]
```

```
Iteration 1 {{-0.9}, {1.23}, {2.469}}
Iteration 2 {{0.3978}, {1.74234}, {2.8696}}
Iteration 3 {{0.823127}, {1.92117}, {2.96331}}
Iteration 4 {{0.948067}, {1.97654}, {2.98929}}
Iteration 5 {{0.984709}, {1.99307}, {2.99685}}
Iteration 6 {{0.995493}, {1.99795}, {2.99907}}
Iteration 7 {{0.998671}, {1.9994}, {2.99973}}
Iteration 8 {{0.999608}, {1.99982}, {2.99992}}
Iteration 9 {{0.999884}, {1.99995}, {2.99998}}
Iteration 10 {{0.999966}, {1.99998}, {2.99999}}
Iteration 11 {{0.99999}, {2.}, {3.}}
Iteration 12 {{0.999997}, {2.}, {3.}}
Iteration 13 {{0.999999}, {2.}, {3.}}
```

Q2 :-

$$4x1 - x2 - 2x4 = -1$$
$$-x1 + 4x2 - x3 - 2x5 = 0$$
$$-x2 + 4x3 - 2x6 = 1$$
$$-x1 + 4x4 - x5 = -2$$
$$-x2 - x4 + 4x5 - x6 = 1$$
$$-x3 - x5 + 4x6 = 2$$

In[•]:=
```
Clear["Global*`"];
```

$$A = \begin{pmatrix} 4 & -1 & 0 & -2 & 0 & 0 \\ -1 & 4 & -1 & 0 & -2 & 0 \\ 0 & -1 & 4 & 0 & 0 & -2 \\ -1 & 0 & 0 & 4 & -1 & 0 \\ 0 & -1 & 0 & -1 & 4 & -1 \\ 0 & 0 & -1 & 0 & -1 & 4 \end{pmatrix};$$

$$b = \begin{pmatrix} -1 \\ 0 \\ 1 \\ -2 \\ 1 \\ 2 \end{pmatrix};$$

```
d = DiagonalMatrix[Diagonal[A]];
L = LowerTriangularize[A] - d;
U = UpperTriangularize[A] - d;
w = 0.9;
t = Inverse[(1/w) * d + L].(((1/w) - 1) * d - U);
c = Inverse[(1/w) * d + L].b;
```

$$xold = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}; \quad xnew = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix};$$

```
For[i = 1, i ≤ Infinity, i++,
 xnew = t.xold + c;
 If[Max[Abs[xnew - xold]] < 10^-6, Break[]];
 xold = xnew;
 Print["Iteration ", i, " ", N[xnew]]
]
```

```
Iteration 1 {{-0.225}, {-0.050625}, {0.213609}, {-0.500625}, {0.100969}, {0.52078}}
Iteration 2 {{-0.484172}, {-0.0205031}, {0.476099}, {-0.586283}, {0.215745}, {0.657743}}
Iteration 3 {{-0.541858}, {0.0802394}, {0.586648}, {-0.582004}, {0.28167}, {0.711146}}
Iteration 4 {{-0.523034}, {0.149089}, {0.637225}, {-0.562507}, {0.320156}, {0.736525}}
Iteration 5 {{-0.496887}, {0.190555}, {0.663034}, {-0.546015}, {0.342755}, {0.749955}}
Iteration 6 {{-0.477521}, {0.215036}, {0.677166}, {-0.534924}, {0.356041}, {0.757467}}
Iteration 7 {{-0.465085}, {0.22944}, {0.685201}, {-0.528027}, {0.363852}, {0.761784}}
Iteration 8 {{-0.457497}, {0.237911}, {0.689853}, {-0.523873}, {0.368445}, {0.764295}}
Iteration 9 {{-0.452962}, {0.242892}, {0.692569}, {-0.521404}, {0.371146}, {0.765765}}
Iteration 10 {{-0.450277}, {0.24582}, {0.694161}, {-0.519945}, {0.372734}, {0.766628}}
Iteration 11 {{-0.448693}, {0.247542}, {0.695096}, {-0.519085}, {0.373667}, {0.767134}}
Iteration 12 {{-0.447761}, {0.248555}, {0.695645}, {-0.51858}, {0.374216}, {0.767432}}
Iteration 13 {{-0.447212}, {0.24915}, {0.695968}, {-0.518282}, {0.374539}, {0.767607}}
Iteration 14 {{-0.446889}, {0.2495}, {0.696158}, {-0.518107}, {0.374729}, {0.76771}}
Iteration 15 {{-0.446699}, {0.249706}, {0.696269}, {-0.518004}, {0.374841}, {0.767771}}
Iteration 16 {{-0.446588}, {0.249827}, {0.696335}, {-0.517944}, {0.374906}, {0.767806}}
Iteration 17 {{-0.446522}, {0.249898}, {0.696374}, {-0.517908}, {0.374945}, {0.767827}}
Iteration 18 {{-0.446484}, {0.24994}, {0.696396}, {-0.517887}, {0.374968}, {0.76784}}
Iteration 19 {{-0.446461}, {0.249965}, {0.69641}, {-0.517875}, {0.374981}, {0.767847}}
Iteration 20 {{-0.446448}, {0.249979}, {0.696417}, {-0.517867}, {0.374989}, {0.767851}}
Iteration 21 {{-0.44644}, {0.249988}, {0.696422}, {-0.517863}, {0.374993}, {0.767854}}
Iteration 22 {{-0.446435}, {0.249993}, {0.696425}, {-0.517861}, {0.374996}, {0.767855}}
Iteration 23 {{-0.446432}, {0.249996}, {0.696426}, {-0.517859}, {0.374998}, {0.767856}}
Iteration 24 {{-0.446431}, {0.249998}, {0.696427}, {-0.517858}, {0.374999}, {0.767856}}
Iteration 25 {{-0.44643}, {0.249999}, {0.696428}, {-0.517858}, {0.374999}, {0.767857}}
```

Q3 :-

$$5x1 + x2 + 2x3 = 10$$
$$-3x1 + 9x2 + 4x3 = -14$$
$$x1 + 2x2 - 7x3 = -33$$

In[•]:=
```
Clear["Global*`"];
A = {{5, 1, 2}, {-3, 9, 4}, {1, 2, -7}};
b = {{10}, {-14}, {-33}};
d = DiagonalMatrix[Diagonal[A]];
L = LowerTriangularize[A] - d;
U = UpperTriangularize[A] - d;
w = 0.9;
t = Inverse[(1/w) * d + L].(((1/w) - 1) * d - U);
c = Inverse[(1/w) * d + L].b;
xold = {{0}, {0}, {0}}; xnew = {{0}, {0}, {0}};
For[i = 1, i ≤ Infinity, i++,
 xnew = t.xold + c;
 If[Max[Abs[xnew - xold]] < 10^-6, Break[]];
 xold = xnew;
 Print["Iteration ", i, " ", N[xnew]]
]
```

```
Iteration 1 {{1.8}, {-0.86}, {4.25314}}

Iteration 2 {{0.603669}, {-3.00616}, {3.97277}}

Iteration 3 {{0.971276}, {-2.99834}, {3.99401}}

Iteration 4 {{0.998985}, {-2.99774}, {3.99985}}

Iteration 5 {{0.999546}, {-2.99985}, {3.99997}}

Iteration 6 {{0.99994}, {-2.99999}, {3.99999}}

Iteration 7 {{0.999995}, {-3.}, {4.}}

Iteration 8 {{0.999999}, {-3.}, {4.}}
```