Practical – 5

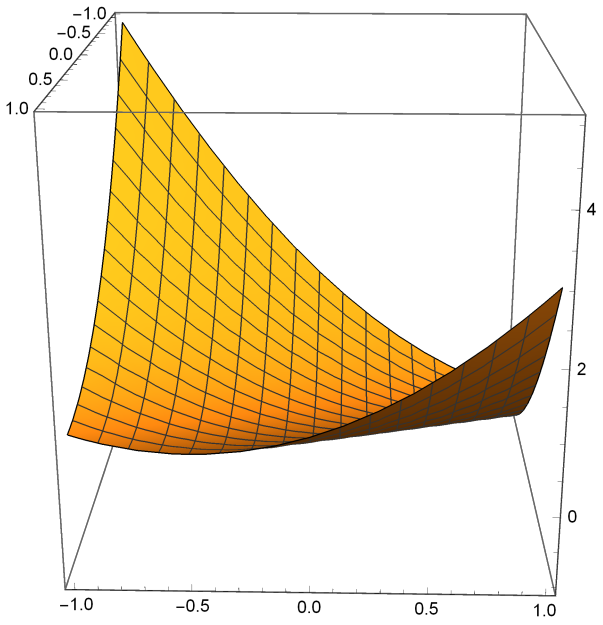
Cauchy problem with parametric initial problem

In[4]:=
```
eq1 = {x'[t] == 1, y'[t] == -1, u'[t] == 1, x[0] == s, y[0] == 0, u[0] == s^2}
```
Out[4]=
$$\{x'[t] == 1,\ y'[t] == -1,\ u'[t] == 1,\ x[0] == s,\ y[0] == 0,\ u[0] == s^2\}$$

In[8]:=
```
sol1 = DSolve[eq1, {u[t], x[t], y[t]}, {t}]
```
Out[8]=
$$\{\{x[t] \to s + t,\ y[t] \to -t,\ u[t] \to s^2 + t\}\}$$

In[18]:=
```
Eliminate[{x == s + t, y == -t, u == s^2 + t}, {s, t}]
```
Out[18]=
$$x^2 - y + 2xy + y^2 == u$$

In[47]:=
```
Plot3D[x^2 - y + 2 x y + y^2, {x, -1, 1}, {y, -1, 1}, BoxRatios -> {1, 1, 1}]
```
Out[47]=



In[28]:=
```
Clear[x, y, z]
```

In[29]:=
```
eq2 = {x'[t] == x[t], y'[t] == y[t], u'[t] == u[t] + 1, x[0] == s, y[0] == s^2, u[0] == s^2}
```
Out[29]=
$$\{x'[t] == x[t],\ y'[t] == y[t],\ u'[t] == 1 + u[t],\ x[0] == s,\ y[0] == s^2,\ u[0] == s^2\}$$

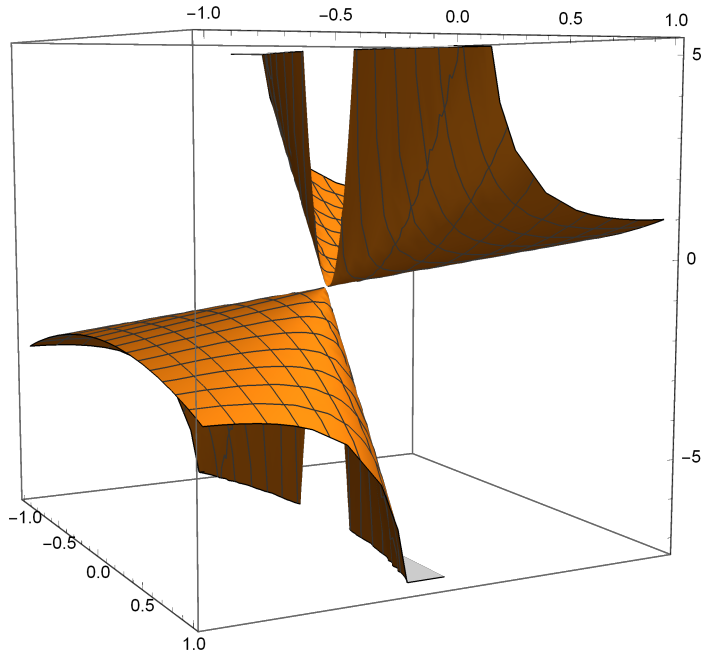
In[30]:=
```
DSolve[eq2, {u[t], x[t], y[t]}, {t}]
```
Out[30]=
$$\{\{x[t] \to e^t s,\ y[t] \to e^t s^2,\ u[t] \to -1 + e^t + e^t s^2\}\}$$

In[32]:=
```
Eliminate[{x == Exp[t] * s, y == Exp[t] * s^2, u == -1 + Exp[t] + Exp[t] * s^2}, {s, t}]
```
Eliminate: Inverse functions are being used by Eliminate, so some solutions may not be found; use Reduce for complete solution information.

Out[32]=
$$x^2 - y + y^2 == u\,y$$

In[48]:=
```
Plot3D[(x^2 - y + y^2)/y, {x, -1, 1}, {y, -1, 1}, BoxRatios → {1, 1, 1}]
```

Out[48]=




```
(* uux + uy = 1, x[0] =s^2, y[0] = 2s, u[0] = s*)
```

In[33]:=
```
eq3 = {x'[t] == u[t], y'[t] == 1, u'[t] == 1, x[0] == s^2, y[0] == 2 * s, u[0] == s}
```

Out[33]=
$$\{x'[t] == u[t],\ y'[t] == 1,\ u'[t] == 1,\ x[0] == s^2,\ y[0] == 2\,s,\ u[0] == s\}$$

In[34]:=
```
DSolve[eq3, {u[t], x[t], y[t]}, {t}]
```

Out[34]=
$$\left\{\left\{u[t] \to s + t,\ x[t] \to \frac{1}{2}\left(2\,s^2 + 2\,s\,t + t^2\right),\ y[t] \to 2\,s + t\right\}\right\}$$

In[39]:=
```
Clear[x, y, u, s, t];
```

In[42]:=
```
Eliminate[{x == s^2 + s * t + t^2/2, y == 2 s + t, u == s + t}, {s, t}]
```

Out[42]=
$$2\,x - y^2 == 2\,u^2 - 2\,u\,y$$

In[53]:=
```
Plot3D[{1/2 (y - Sqrt[4 x - y^2]), 1/2 (y + Sqrt[4 x - y^2])},
 {x, -1, 1}, {y, -1, 1}, BoxRatios → {1, 1, 1}]
```

Out[53]=