

Practical 3 :

Solution of vibration of semi –
infinite string with fixed end.

$$u_{tt} = c^2 u_{xx}, \quad 0 < x < \infty, \quad t > 0,$$

$$u(x, 0) = f(x), \quad 0 \leq x < \infty,$$

$$u_t(x, 0) = g(x), \quad 0 \leq x < \infty,$$

$$u(0, t) = 0$$

`In[]:= ClearAll;`

`In[]:= weqn = D[u[x, t], {t, 2}] == c^2 * D[u[x, t], {x, 2}]`

`Out[]:= u^{(0,2)}[x, t] == c^2 u^{(2,0)}[x, t]`

`In[]:= ic = {u[x, 0] == f[x], Derivative[0, 1][u][x, 0] == g[x], u[0, t] == 0}`

`Out[]:= {u[x, 0] == f[x], u^{(0,1)}[x, 0] == g[x], u[0, t] == 0}`

`In[]:= dsol = DSolveValue[{weqn, ic}, u[x, t], {x, t}]`

$$\text{Out[]:= } \begin{cases} \frac{1}{2} \left(f[-\sqrt{c^2} t + x] + f[\sqrt{c^2} t + x] \right) + \frac{\int_{-\sqrt{c^2} t + x}^{\sqrt{c^2} t + x} g[K[1]] \, dK[1]}{2\sqrt{c^2}} & x > \sqrt{c^2} t \geq 0 \\ \frac{1}{2} \left(-f[\sqrt{c^2} t - x] + f[\sqrt{c^2} t + x] \right) + \frac{\int_{\sqrt{c^2} t - x}^{\sqrt{c^2} t + x} g[K[1]] \, dK[1]}{2\sqrt{c^2}} & 0 \leq x \leq \sqrt{c^2} t \\ \text{Indeterminate} & \text{True} \end{cases}$$

`In[]:= PiecewiseExpand[dsol]`

$$\text{Out[]:= } \begin{cases} \frac{1}{2} \left(f[-\sqrt{c^2} t + x] + f[\sqrt{c^2} t + x] \right) + \frac{\int_{-\sqrt{c^2} t + x}^{\sqrt{c^2} t + x} g[K[1]] \, dK[1]}{2\sqrt{c^2}} & \sqrt{c^2} t - x < 0 \&\& \sqrt{c^2} t \geq 0 \\ \frac{1}{2} \left(-f[\sqrt{c^2} t - x] + f[\sqrt{c^2} t + x] \right) + \frac{\int_{\sqrt{c^2} t - x}^{\sqrt{c^2} t + x} g[K[1]] \, dK[1]}{2\sqrt{c^2}} & x \geq 0 \&\& \sqrt{c^2} t - x \geq 0 \\ \text{Indeterminate} & \text{True} \end{cases}$$

`In[]:= dsol /. {f[x_] -> Exp[-x^2], g[x_] -> 1}`

$$\text{Out[]:= } \begin{cases} \frac{1}{2} \left(e^{-(-\sqrt{c^2} t + x)^2} + e^{-(\sqrt{c^2} t + x)^2} \right) + t & x > \sqrt{c^2} t \geq 0 \\ \frac{1}{2} \left(-e^{-(\sqrt{c^2} t - x)^2} + e^{-(\sqrt{c^2} t + x)^2} \right) + \frac{x}{\sqrt{c^2}} & 0 \leq x \leq \sqrt{c^2} t \\ \text{Indeterminate} & \text{True} \end{cases}$$

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In[ ]:= Manipulate[Plot3D[%21, {t, 0, 2.66223}, {x, 0, 5}], {c, -2, 2}]
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Out[]:=

