# Practical 5 :

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Solution of one dimensional wave equation $u_{tt} = c^2 u_{xx}$,
for finite string of length l,
that is to solve the IBVP ;

$u_{tt} = c^2 u_{xx}$ , $0 < x < l$ , $t > 0$,

$u(x, 0) = f(x)$, $0 \le x \le l$,

$u_t(x, 0) = g(x)$, $0 \le x \le l$,

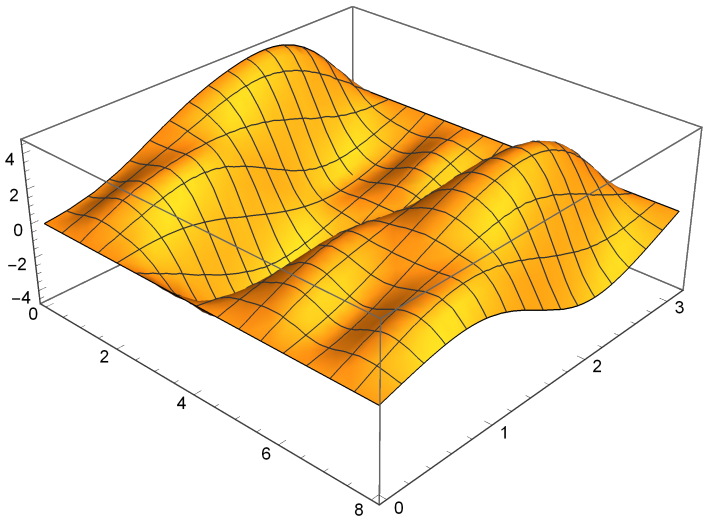
$u(0, t) = 0$,

$u(l, t) = 0$ .

```
In[•]:= ClearAll;

In[•]:= weqn = D[u[x, t], {t, 2}] == D[u[x, t], {x, 2}];

In[•]:= ic = {u[x, 0] == x^2 (π - x), u^(0,1)[x, 0] == 0};
        bc = {u[0, t] == 0, u[π, t] == 0};

In[•]:= dsol = DSolve[{weqn, bc, ic}, u, {x, t}] /. {K[1] → m}

Out[•]= {{u → Function[{x, t}, Sum[-(4 (1 + 2 (-1)^m) Cos[t m] Sin[x m])/m^3, {m, 1, ∞}]]}}

In[•]:= asol[x_, t_] = u[x, t] /. dsol[[1]] /. {∞ → 4} // Activate

Out[•]= 4 Cos[t] Sin[x] - 3/2 Cos[2 t] Sin[2 x] + 4/27 Cos[3 t] Sin[3 x] - 3/16 Cos[4 t] Sin[4 x]
```

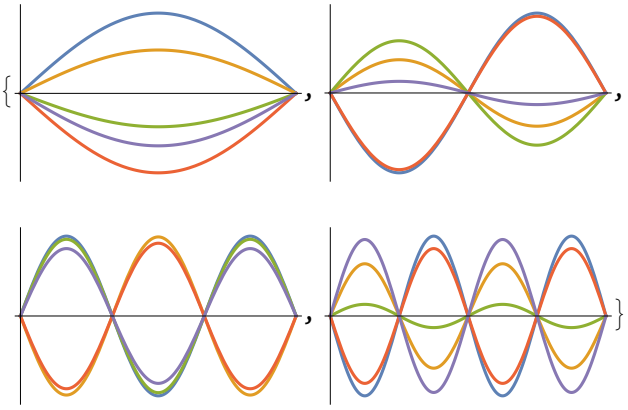
In[ ]:=
```
Plot3D[4 Cos[t] Sin[x] - 3/2 Cos[2 t] Sin[2 x] +
  4/27 Cos[3 t] Sin[3 x] - 3/16 Cos[4 t] Sin[4 x], {t, 0, 8}, {x, 0, Pi}]
```

Out[ ]=



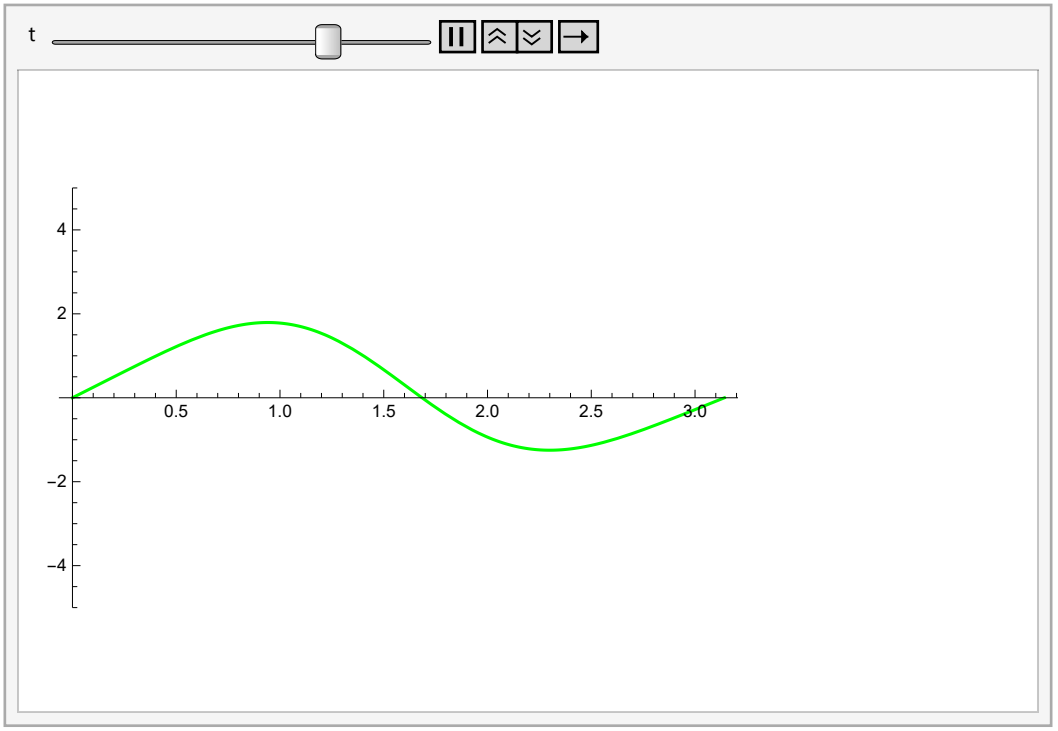

In[ ]:=
```
Table[Show[Plot[Table[asol[x, t][[m]], {t, 0, 4}] // Evaluate,
    {x, 0, Pi}, Ticks → False], ImageSize → 150], {m, 4}]
```

Out[ ]=



In[•]:=
```
Animate[Plot[asol[x, t], {x, 0, π}, PlotRange → {-5, 5},
  ImageSize → Medium, PlotStyle → Green], {t, 0, 2 Pi}, SaveDefinitions → True]
```

Out[•]=




```
(* Note : In the above wave equation the length of the string is considered π (Pi).*)
```