Assignment Solutions : Graphs - 1

Q1 For a graph with n nodes, how many edges are there in a complete graph?

Solution : nC2: We can choose any two nodes out of n nodes and then connect them

Q2 Minimum number of edges to make a graph on n nodes is?

Solution : 0, since the nodes which are not at all connected still make a graph. It is known as a void graph

Q3 Minimum number of edges to make a connected graph of n nodes?

Solution : n-1 nodes, which make up a tree as well.

Q4 Insertion and deletion of edges is fastest in ___ ?

Solution : Adjacency Matrix, bs it takes O(1) in oth insertion and deletionL

Q5 Which representation should be used if we have a graph with a large number of nodes (>108) but with a few edges between them?

Solution : Adjacency Matrix, as it takes O(1) in O(1) insertion and deletion.

Q6 Can adjacencyMatrix[i][j] != adjacencyMatrix[j][i] in any case? If yes, what would be the type of graph called?

Solution : It is possible when the graph is directed.

Q7 For a graph with n nodes, how many edges are there in a complete graph?

Solution : nC2: We can choose any two nodes out of n nodes and then connect them.

Q8 Minimum number of edges to make a graph on n nodes is?

Solution : 0, since the nodes which are not at all connected still make a graph. It is known as a void graph.

Q9 Minimum number of edges to make a connected graph of n nodes?

Solution : n-1 nodes, which make up a tree as well.

Q10 Given the following adjacency list for an undirected graph:

```
[
  [1, 2],
  [0, 3],
  [0, 3],
  [1, 2]
]
```

What are the adjacent vertices of vertex 1?

Solution : The adjacent vertices of a vertex are the vertices that are connected to it by an edge. In the given adjacency list, the adjacent vertices of vertex 1 are vertices 0 and 2.

Q11 Given the following adjacency matrix for a directed graph:

```
[[0, 1, 0, 0],
[0, 0, 1, 0],
[0, 0, 0, 1],
[0, 0, 0, 0]]
```

What is the indegree of vertex 3?

Solution : The indegree of a vertex in a directed graph is the number of edges that point to it. In the given adjacency matrix, there is one edge that points to vertex 3, namely edge (2, 3). Therefore, the indegree of vertex 3 is 1.

Q12 Consider a directed graph with 4 nodes (1, 2, 3, 4) and the following edges: (1->2), (2->3), (3->4), (4->1). Represent this graph using an Edge List.

Solution : The Edge List representation for the given directed graph would be:

(1, 2)
(2, 3)
(3, 4)
(4, 1)

Q13 What is the maximum number of edges in a simple undirected graph with n nodes?

Solution : The maximum number of edges in a simple undirected graph with n nodes is (n * (n-1)) / 2. This is achieved in a complete graph.

Q14 What is the minimum number of edges in a simple undirected connected graph with n nodes?

Solution : The minimum number of edges in a simple undirected connected graph with n nodes is n-1. This forms a tree.

Q15 What is the minimum number of colors needed to color the nodes of a graph such that no two adjacent nodes have the same color?

Solution : The minimum number of colors needed to color the nodes of a graph such that no two adjacent nodes have the same color is called the chromatic number of the graph.

Q16 What is the minimum cut of a graph?

Solution : The minimum cut of a graph is the smallest number of edges that must be removed to disconnect the graph into two separate components.

Q17 What is the minimum number of nodes required to create a cycle in a directed graph?

Solution : At least 3 nodes are required to create a cycle in a directed graph.

Q18 What is the degree of a vertex in a graph?

Solution : The degree of a vertex is the number of edges incident to that vertex.

Q19 Consider a graph with 5 nodes, A, B, C, D, and E. The following edges are present in the graph:
A -> B
B -> C
C -> D
D -> E
E -> A

What is the degree of vertex C?

Solution : The degree of a vertex is the number of edges connected to it. The degree of vertex C is 2, since it is connected to two other vertices, B and D.

Q20 What is the time complexity of finding the minimum spanning tree of a graph using Prim's algorithm?

Solution : The time complexity of finding the minimum spanning tree of a graph using Prim's algorithm is O(|E| + |V| log |V|), where |E| is the number of edges in the graph and |V| is the number of vertices in the graph.

Q21 In a complete graph, what is the sum of the degrees of all the vertices?

Solution : It is 2 times the number of vertices minus 2.

Q22 If a graph has 10 vertices and 25 edges, is it a tree?

Solution : No (A tree with n vertices has n-1 edges)

Q23 How many edges can be removed from a complete graph with 6 vertices to make it a tree?

Solution : 9 (A complete graph with n vertices has n(n-1)/2 edges, and a tree with n vertices has n-1 edges)

Q24 How many vertices are there in a graph with 12 edges and a maximum degree of 4?

Solution : 7 (You can use the Handshaking Lemma to find the number of vertices: 2 * number of edges = sum of degrees)

Q25 What is the diameter of a complete graph with 10 vertices?

Solution : 1 (In a complete graph, any two vertices are connected by an edge)

Q26 In a simple graph with 4 vertices and 3 edges, is there a cycle?

Solution : No (A graph with n vertices and n-1 edges is a tree, and trees have no cycles)