Contents

1 Data structures

1.1 Segment Tree

```
struct Node{
  ll val = 0;
};
Node operator+(Node a,Node b){return {a.val+b.val};}
class Segment_Tree{
public:
  int n;
  vector<Node> tree;

  Segment_Tree(int x = 1e5+10){
    n = x;
    tree.resize(2*n+1);
  }

  void build(vi &arr){
    for(int i=0;i<n;i++)tree[n+i] = {arr[i]};
    for(int i = n-1; i>0;i--)tree[i] = tree[i*2]+tree[i*2+1];
  }

  void update(int p, ll val){
    for(tree[p+=n] = {val};p>1;p>>=1)tree[p>>1] = tree[p]+tree[p^1];
  }

  ll query(int l,int r){
    Node s;
    for(l+=n,r+=n ; l<r ; l>>=1 , r>>=1)
      if(l&1)s = s+ tree[l++];
      if(r&1)s = s+ tree[--r];
    }
    return s.val;
  }
};
```

1.2 Merge Sort Tree

```
struct Node{
  vi val;

  int search(int x){
    int l = 0, r = sz(val)-1;
    while(l<=r){
      int m = l+(r-l)/2;
      if(val[m] < x)l =m+1;
      else r = m-1;
    }
    return sz(val)-l;
  }
};

Node operator+(Node a,Node b){
  int i=0,j=0;
  Node aux;
  while (i< sz(a.val) && j<sz(b.val)){
    if (a.val[i]< b.val[j] ) aux.val.pb(a.val[i++]);
    else aux.val.pb(b.val[j++]);
  }
  while (i<sz(a.val)) aux.val.pb(a.val[i++]);
  while (j<sz(b.val)) aux.val.pb(b.val[j++]);
  return aux;
}
class Segment_Tree{
  public:
  int n,k=0;
  vector<Node> tree;
  Segment_Tree(int x = 1e5+10){
    n = x;
    while((1<<k)<n)k++;
    tree.resize(2*(1<<k)+1);
  }
  void build(vi &arr){
    for(int i=0;i<n;i++)tree[(1<<k)+i].val.pb(arr[i]);
    for(int i = (1<<k)-1; i>0;i--)tree[i] = tree[i*2]+tree[i*2+1];
  }

  void query(int a,int b,ll &ans,int i){
    a += (1<<k);
    b += (1<<k);
    while(a<=b){
      if(a%2 == 1)ans += tree[a++].search(i);
      if(b%2 == 0)ans += tree[b--].search(i);
      a/=2;
      b/=2;
    }
  }
};
```

1.3 Fenwick Tree

```
struct Node{
  ll n = 0;
  Node operator+(Node b){return {n+b.n};}
};

class Fenwick_Tree{
public:
  vector<Node> tree;
  int n;
  Fenwick_Tree(int x){
    n = x+1;tree.resize(n);
  }

  ll sum(int r){
    r++;
    Node ans = {0};
    while(r){
      ans = ans + tree[r];
      r -= r&-r;
    }
    return ans.n;
  }

  void update(int id, Node val){
    id++;
    while(id < n){
      tree[id]  = tree[id]+val;
      id += id&-id;
    }
  }
};
```

1.4 Sparse Table

```cpp
struct Node{
  int val = 0;
  Node operator+(Node b){return {__gcd(val,b.val)};}
};

class SparseTable{
  public:
  vector<vector<Node>> sparse;

  SparseTable(int n=2e5+10){
    sparse.assign(n , vector<Node> (log2(n)+1));
  }

  void build(const vi &a){
    for(int i=0;i<sz(a);i++)
      sparse[i][0] = {a[i]};

    for(int j=1;(1<<j)<=sz(a);j++)
      for(int i=0;i+(1<<j)-1 <sz(a);i++)
        sparse[i][j]= sparse[i][j-1]+sparse[i+(1<<(j-1))][j-1];
  }

  int query(int l, int r){
    int len = r-l+1;
    int k = log2(len);
    return (sparse[l][k]+sparse[r-(1<<k)+1][k]).val;
  }

};
```

1.5 Disjoint Set Union

```cpp
class DSU{
  private:
  vi parent,rank,size;
  int c;
  public:
  int mx = 0;
  DSU(int n):parent(n+1),rank(n+1,0),size(n+1,1),c(n){
    iota(all(parent),0);
  }

  int find(int u){return parent[u] == u?u:parent[u] = find(parent[u]) ;}

  bool same(int u,int v){return find(u)==find(v);}

  int get_size(int u){return size[find(u)];}

  int count(){return c;}

  void merge(int u,int v){
    u = find(u);
    v = find(v);
    if(u!=v){
      c--;
      if(rank[u] > rank[v])swap(u,v);
      parent[u] = v;
      size[v] += size[u];
      if(rank[u] == rank[v])rank[v]++;
      mx = max(mx,size[v]);
    }
  }
};
```

2 Graphs

2.1 DFS

```
void dfs(int u){
  vis[u] = 1;
  for(auto v:adj[u]){
    if(!vis[v])dfs(v);
  }
}
```

2.2 BFS

```
queue<int> q;
q.push(0);
while(!q.empty()){
  int u = q.front();
  q.pop();
  for(int v:adj[u]){
    if (!vis[v]) {
      vis[v] = 1;
      q.push(v);
    }
  }
}
```

2.3 Topological Sort

```
vi ts;
queue<int> q;
for (int i = 1; i < n+1; i++)
  if(!in[i])q.push(i);

while (!q.empty()) {
  int u=q.front();
  q.pop();
  ts.pb(u);
  for (int v : adj[u]) {
    in[v]--;
    if(!in[v])q.push(i);
  }
}

if(sz(ts)!=n)cout <<"IMPOSSIBLE";
```

2.4 Dijkstra

2.5 Bellman-Ford

2.6 Floyd-Warshall

2.7 SCC Kosaraju

```
class SCC{
private:
  int n;
  vector<vi> adj;
  vector<vi> adj_t;
  vector<vi> adj_cond;
  vi order;
  vector<bool> vis;
  vi who;
  int comp;
public:
  SCC(int n,vector<vi> &a,vector<vi>&b):n(n),adj(a),adj_t(b){
    comp = -1;
    who.resize(n);
    vis.resize(n);
    for(int i=0;i<n;i++)
      if(!vis[i])dfs1(i);

    vis.assign(n,0);
    for(int i=n-1;i>=0;i--)
      if(!vis[order[i]]){
        comp++;
        dfs2(order[i]);
      }
    adj_cond.resize(comp+1);
    for(int u=0;u<n;u++)
      for(auto v:adj[u])
        if(who[v]!=who[u])
          adj_cond[who[u]].pb(who[v]);
  }

  void dfs1(int u) {
    vis[u] = true;
    for (int v:adj[u])
      if (!vis[v])dfs1(v);
    order.pb(u);
```

```
  }

  void dfs2(int u) {
    vis[u] = 1;
    for (int v : adj_t[u])
      if (!vis[v])dfs2(v);
    who[u] = comp;
  }
};
```

2.8 2-SAT

```
class SAT{
private:
  int n;
  vector<vi> adj;
  vector<vi> adj_t;
  vector<vi> adj_cond;
  vi order;
  vector<bool> vis;
  vi who;
  vector<bool> ans;
  int comp;
public:

  SAT(int m=0){
    n = (m<<1);
    adj.resize(n);
    adj_t.resize(n);
    vis.assign(n,0);
    who.assign(n,-1);
  }

  void dfs1(int u) {
    vis[u] = true;
    for (int v:adj[u])
      if (!vis[v])dfs1(v);
    order.pb(u);
  }

  void dfs2(int u) {
    vis[u] = 1;
    for (int v : adj_t[u])
      if (!vis[v])dfs2(v);
    who[u] = comp;
  }

  //~a->b ~b->a
  void addOR(int a, bool af, int b, bool bf) {
    a += a + (af ^ 1);
    b += b + (bf ^ 1);
    adj[a ^ 1].push_back(b);
    adj[b ^ 1].push_back(a);
    adj_t[b].push_back(a ^ 1);
    adj_t[a].push_back(b ^ 1);
  }

  //( a OR b ) ^ (~a or ~b)
  void addXOR(int a, bool af, int b, bool bf) {
    addOR(a, af, b, bf);
    addOR(a, !af, b, !bf);
  }
  //(a -> b)
  void _add(int a,bool af,int b,bool bf) {
    a += a + (af ^ 1);
    b += b + (bf ^ 1);
    adj[a].push_back(b);
    adj_t[b].push_back(a);
  }

  //(a->b)^(b->a)
  void add(int a,bool af,int b,bool bf) {
    _add(a, af, b, bf);
    _add(b, !bf, a, !af);
  }

  bool solve_SAT(){
    for(int i=0;i<n;i++)
      if(!vis[i])dfs1(i);

    comp = -1;
    vis.assign(n,0);
    for(int i=sz(order)-1;i>=0;i--)
      if(!vis[order[i]]){
        comp++;
        dfs2(order[i]);
```

```
      }
    ans.assign(n/2,0);
    for(int i=0;i<n;i+=2){
      if(who[i] == who[i+1])return 0;
      ans[i/2] = (who[i] > who[i+1]);
    }
    return 1;
  }

  bool operator[](int i){
    return ans[i];
  }

};
```

2.9 Euler Paths and cycles

2.9.1 Directed Graphs

```
struct EulerSolver{
private:
  vector<vector<pii>> adj;
  int n,edges;
  vi in,out;
  vi ans,edge_ans;
  int root;
  vi done;
  void dfs(int u){
    while(done[u] < sz(adj[u])){
      auto [v,id] = adj[u][done[u]++];
      dfs(v);
      edge_ans.pb(id);
    }
    ans.pb(u);
  }
public:
  EulerSolver(vector<vector<pii>> &a,int t):adj(a){
    n = t;
    edges = 0;
    in.assign(n,0);
    out.assign(n,0);
    done.assign(n,0);
    root = -1;
    for(int u=0;u<n;u++)
      for(auto v:adj[u]){
        edges++;
        in[v.F]++;
        out[u]++;
      }

  }

  bool possible(){
    int cnt1 =0,cnt2 =0;
    bool ok = 1;
    for(int i=0;i<n;i++){
      if (in[i] - out[i] == 1) cnt1++;
      if (out[i] - in[i] == 1) cnt2++, root = i;
      if (abs(in[i] - out[i]) > 1) ok = 0;
    }
    if(cnt1 > 1 || cnt2 > 1)ok = 0;
    if(!ok)return 0;

    if(root == -1)
      for(int i=0;i<n;i++)
        if(out[i])root = i;

    if(root == -1)return 1;
    dfs(root);

    if(sz(ans) != edges+1)return 0;
    reverse(all(ans));
    reverse(all(edge_ans));
    return 1;
  }

  int size(){
    return ans.size();
  }

  int operator()(int i,bool x){
    if(x)return ans[i];
    else return edge_ans[i];
  }
};
```

2.9.2 Unirected Graphs

```
struct EulerSolver{
private:
  vector<vector<pii>> adj;
  int n,edges;
  vi ans,edge_ans;
  vi deg;
  int root;
  vi done;
  vector<bool> vis;

  void dfs(int u){
    while(done[u] < sz(adj[u])){
      auto [v,id] = adj[u][done[u]++];
      if(vis[id])continue;
      vis[id]=1;
      dfs(v);
      edge_ans.pb(id);
    }
    ans.pb(u);
  }
public:
  EulerSolver(vector<vector<pii>> &a,int t):adj(a),n(t){
    edges = 0;
    done.assign(n,0);
    deg.assign(n,0);
    root = -1;
    for(int u=0;u<n;u++)
      for(auto v:adj[u]){
        edges++;
        deg[v.F]++;
        deg[u]++;
      }
    vis.assign(edges/2,0);
  }

  bool possible(){
    int odd = 0;
    for(int i=0;i<n;i++)
      if((deg[i]/2)&1){
        odd++;
        root = i;
      }

    if(odd>2)return 0;

    if(root == -1)
      for(int i=0;i<n;i++)
        if(deg[i]){root = i;break;}

    if(root == -1)return 1;
    dfs(root);

    if(sz(ans) != edges/2+1)return 0;
    reverse(all(ans));
    reverse(all(edge_ans));
    return 1;
  }

  int size(){
    return ans.size();
  }

  int operator()(int i,bool x){
    if(x)return ans[i];
    else return edge_ans[i];
  }
};
```

3 Trees

3.1 LCA - Binary Lifting

3.1.1 Normal LCA

```cpp
const int MAX = 1e5+5,LG=18;
vi deep(MAX);//Si tiene peso vi cost(MAX)
int par[MAX][LG+1];

void dfs(int u = 1,int p=0){// U = raiz del arbol
  par[u][0] = p;
  deep[u] = deep[p]+1;//cost[u] += cost[p]
  for(int i=1;i<=LG;i++)par[u][i]=par[par[u][i-1]][i-1];
  for(int v:adj[u]){
    if(v!=p){
      dfs(v,u);
    }
  }
}

int lca(int u,int v){
  if(deep[u] < deep[v])swap(u,v);
  for(int k=LG;k>=0;k--)
    if(deep[par[u][k]] >= deep[v])
      u = par[u][k];
  if(u==v)return u;
  for(int k=LG;k>=0;k--)
    if(par[u][k]!=par[v][k])
      u=par[u][k],v=par[v][k];
  return par[u][0];
}

int dist(int u,int v){
  int lc = lca(u,v);
  return deep[u]+deep[v]-(deep[lc]<<1);
}

int kth(int u,int k){
  assert(k>=0);
  for(int i=0;i<=LG;i++)
    if(k&(1<<i))u = par[u][i];
  return u;
}
```

3.1.2 Min/Max weight on the path

```cpp
const int MAX = 1e5+5,LG=18;
vector<vector<pii>> adj(MAX,vector<pii> ());
vi deep(MAX);
int par[MAX][LG+1];
int mn[MAX][LG+1];
int mx[MAX][LG+1];

void dfs(int u=1,int p=0,int costmn = INF,int costmx = -INF){
  par[u][0] = p;
  deep[u] = deep[p]+1;

  if(p!=0){
    mn[u][0]  = costmn;
    mx[u][0]  = costmx;
  }
  for(int i=1;i<=LG;i++){
    par[u][i]=par[par[u][i-1]][i-1];
    mn[u][i]=min(mn[u][i-1],mn[par[u][i-1]][i-1]);
    mx[u][i]=max(mx[u][i-1],mx[par[u][i-1]][i-1]);
  }

  for(auto [v,t]:adj[u]){
    if(v!=p){
      dfs(v,u,t,t);
    }
  }
}

pii minmax(int u,int v){
  pii ans = {INF,-INF};
  if(deep[u] < deep[v])swap(u,v);
  for(int k=LG;k>=0;k--)if(deep[par[u][k]] >= deep[v]){
    ans.F = min(ans.F,mn[u][k]);
    ans.S = max(ans.S,mx[u][k]);
    u = par[u][k];
  }
  if(u==v)return ans;
  for(int k=LG;k>=0;k--)if(par[u][k]!=par[v][k]){
    ans.F = min({ans.F,mn[u][k],mn[v][k]});
}
```

```
    ans.S = max({ans.S,mx[u][k],mx[v][k]});
    u=par[u][k],v=par[v][k];
  }
  ans.F = min({ans.F,mn[u][0],mn[v][0]});
  ans.S = max({ans.S,mx[u][0],mx[v][0]});

  return ans;
}
```

3.2 Centroid Decomposition

```
class Centroid{
private:
  vector<vi> tree;
  vi par;
  vi sub;
  vector<bool> check;
  int n;
public:
  Centroid(vector<vi> &t):tree(t){
    n = tree.size();
    par.resize(n);
    sub.resize(n);
    check.resize(n);
    build();
  }

  void build(int u = 0, int p = -1){
    dfs(u, p);
    int c = get_centroid(u,p,sub[u]);
    check[c] = 1;
    par[c] = p+1;

    for (auto v : tree[c]){
      if(!check[v])
        build(v, c);
    }

  }

  int dfs(int u,int p){
    if(check[u])return 0;
    sub[u] = 1;
    for(int v:tree[u])
      if(v!=p)sub[u]+=dfs(v,u);

    return sub[u];
  }

  int get_centroid(int u,int p,int x){
    for(auto v:tree[u])
      if(v!=p && sub[v]*2>x && !check[v])return get_centroid(v,u,x);

    return u;
  }

  int operator[](int i){
    return par[i];
  }
};
```

3.3 Euler Tour Technique

3.4 Tree Matching

3.5 Diameter

```
vector<vector<pii>> adj;
vector<bool> vis;

int bfs(int n,vll &d,int v = 0){
  vis.assign(n,0);
  d.assign(n,0);
  d[v] = 0;
  queue<int> q;
  q.push(v);
  pll last = {v,0};
  vis[v] = 1;
  while(!q.empty()){
    int u = q.front();q.pop();
    for(auto [w,c]:adj[u]){
      if(vis[w])continue;
      d[w] = d[u]+c;
      q.push(w);
      vis[w] = 1;
      if(d[w]>last.S)last = {w,d[w]};
    }
```

```
  }
  return int(last.F);
}

void solve(){
  int n;cin >> n;
  adj.assign(n,vector<pii> ());
  for(int i=1;i<n;i++){
    int a,b;ll c;
    cin >> a >> b >> c;
    adj[--a].pb({--b,c});
    adj[b].pb({a,c});
  }
  vll dx(n),dy(n);
  int a =bfs(n,dx);
  int b =bfs(n,dy,a);
  bfs(n,dx,b);

  for(int i=0;i<n;i++){
    cout << max(dx[i],dy[i]) << "␣\n"[i==(n-1)];
  }

}
```

3.6 Heavy Light Decomposition

```
class HLD{
private:
  int n;
  vector<vi> adj;
  vi parent;
  vi heavy;
  vi depth;
  vi root;
  vi treePos;
  Segment_Tree tree;

  int dfs(int u =0){
    int size = 1, mx_sub = 0;
    for(auto v:adj[u])
      if(v!=parent[u]){
        parent[v] = u;
        depth[v] = depth[u]+1;
        int sub = dfs(v);
        if(sub > mx_sub){
          heavy[u] = v;
          mx_sub = sub;
        }
        size += sub;
      }
    return size;
  }

  template <class BinaryOperation>
  void processPath(int u, int v, BinaryOperation op) {
    for (; root[u] != root[v]; v = parent[root[v]]) {
      if (depth[root[u]] > depth[root[v]]) swap(u, v);
      op(treePos[root[v]], treePos[v] + 1);
    }
    if (depth[u] > depth[v]) swap(u, v);
    op(treePos[u], treePos[v] + 1);
  }

  void init(vi &a){
    for(int i=0,crt=0;i<n;++i){
      if(parent[i] == -1 || heavy[parent[i]]!=i){
        for(int j = i;j!=-1;j = heavy[j]){
          root[j] = i;
          treePos[j] = crt++;
        }
      }
    }
    tree = Segment_Tree(n);
    for(int i=0;i<n;i++){
      tree.update(treePos[i],a[i]);
    }
  }

public:

  HLD(int n,vector<vi> &vale,vi &a):n(n),adj(vale){
    parent.assign(n,0);
    heavy.assign(n,-1);
    depth.assign(n,0);
    root.resize(n);
    treePos.assign(n,0);
```

```
    parent[0] = -1;
    dfs();
    init(a);
  }

  void update(int u,int &val){
    tree.update(treePos[u],val);
  }

  void updatePath(int u, int v, int value) {
    processPath(u, v, [this, &value](int l, int r) {
      tree.update_range(l, r, value);
    });
  }

  int query(int u,int v){
    int ans = 0;
    processPath(u,v,[this,&ans](int l,int r){
      ans = max(ans,tree.query(l,r));
    });
    return ans;
  }
};
```

4 Math

4.1 Identities

$C_n = \frac{2(2n-1)}{n+1} C_{n-1}$
$C_n = \frac{1}{n+1}\binom{2n}{n}$
$C_n \sim \frac{4^n}{n^{3/2}\sqrt{\pi}}$
$\sigma(n) = O(\log(\log(n)))$ (number of divisors of n)
$F_{2n+1} = F_n^2 + F_{n+1}^2$
$F_{2n} = F_{n+1}^2 - F_{n-1}^2$
$\sum_{i=1}^{n} F_i = F_{n+2} - 1$
$F_{n+i}F_{n+j} - F_nF_{n+i+j} = (-1)^n F_iF_j$
(Möbius Inv. Formula) Let $g(n) = \sum_{d|n} f(d)$, then $f(n) = \sum d \mid ng(d)\mu\left(\frac{n}{d}\right)$.

4.2 Theorems

5 Strings

5.1 Suffix Array

```
struct suffix{
  int index;
  pii rank;
  bool operator<(suffix b){return rank<b.rank;}
  bool operator>(suffix b){return rank>b.rank;}
};

class Suffix_Array{
  public:
  string s;int n;
  vector<suffix> suffixes;
  vi lcp;
  Suffix_Array(string x){
    s = x+"$";
    n = sz(s);
    suffixes.resize(n);
  }


  void radix_sort(vector<suffix> &suff){
    for(int i: vi{2,1}){
      auto key = [&](const suffix &x){
        return i == 1?x.rank.F:x.rank.S;
      };

      int mx = 0;
      for (const auto &i:suff) { mx = max(mx, key(i)); }
      vector<int> occs(mx + 1);
      for (const auto &i:suff)occs[key(i)]++;
      vector<int> start(mx + 1);
      for (int i=1;i<=mx; i++) start[i] = start[i-1]+occs[i-1];

      vector<suffix> new_arr(suff.size());
      for (const auto &i : suff) {
        new_arr[start[key(i)]] = i;
        start[key(i)]++;
      }
```

```
      suff = new_arr;
    }
  }

  void build(){
    for(int i=0;i<n;i++)suffixes[i].index = i , suffixes[i].rank = {s[i
        ],(i+1<n?s[i+1]:-1)};
    sort(all(suffixes));
    vi equiv(n);

    for(int i=1;i<n;i++){
      auto c_val = suffixes[i].rank;
      int cs = suffixes[i].index;
      auto  p_val = suffixes[i - 1].rank;
      int ps = suffixes[i-1].index;
      equiv[cs] = equiv[ps] + (c_val > p_val);
    }

    for(int cmp_ant = 1;cmp_ant <n; cmp_ant<<=1){
      for(auto &x:suffixes)
        x.rank = {equiv[x.index],equiv[(x.index+cmp_ant)%n]};

      //Std sort O(nlognlogn) Radix Sort O(nlogn)
      radix_sort(suffixes);

      for(int i=1;i<n;i++){
        auto c_val = suffixes[i].rank;
        int cs = suffixes[i].index;
        auto  p_val = suffixes[i - 1].rank;
        int ps = suffixes[i-1].index;
        equiv[cs] = equiv[ps] + (c_val > p_val);
      }
    }
  }

  void build_lcp(){
    vector<int> suff_ind(n);
    for (int i = 0; i < n; i++)suff_ind[suffixes[i].index] = i;
    lcp.resize(n-1);
    int start_at = 0;
    for (int i = 0; i < n - 1; i++) {
      int prev = suffixes[suff_ind[i] - 1].index;
      int curr_cmp = start_at;
      while (s[i + curr_cmp] == s[prev + curr_cmp])curr_cmp++;
      lcp[suff_ind[i] - 1] = curr_cmp;
      start_at = max(curr_cmp - 1, 0);
    }
  }

  void show(){
    for(int i=0;i<n;i++)cout << suffixes[i].index << "␣\n"[(i+1)==n];
  }
};
```

6 Others