Assignment 1 (ICSE 2017)

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2 (c) Jaya borrowed ₹50,000 for 2 years. The rates of interest for two successive years are 12% and 15% respectively. She repays ₹33,000 at the end of the first year. Find the amount she must pay at the end of the second year to clear her debt.

Solution:

Total number of years over which the debt spans = n

Principal amount = ₹P

Rate of interest for the 1^{st} year = $R_1\%$ p.a

Rate of interest for the 2^{nd} year = $R_2\%$ p.a

Amount paid at the end of 1^{st} year = ₹q_1

Amount due at the end of 1^{st} year

$$= P\left(1+\frac{R_1}{100}\right) \tag{1}$$

The principal for the 2^{nd} year

$$= P\left(1+\frac{R_1}{100}\right) - q_1 \tag{2}$$

Amount due at the end of 2^{nd} year

$$= \left(P\left(1+\frac{R_1}{100}\right) - q_1\right)\left(1+\frac{R_2}{100}\right) \tag{3}$$

$$= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right) - q_1\left(1+\frac{R_2}{100}\right) \tag{4}$$

In (4),

a) the left term corresponds to P + the interest on P (the principal)
b) the right term corresponds to the amount paid in the previous year + interest on it

we can directly substitute the values in eq(4) to get the desired value

Similarly generalising over n years (instead of 2 years),

we can say that the amount due at the end of the n^{th} year

$$= P\prod_{k=1}^{n}\left(1+\frac{R_k}{100}\right) - \sum_{i=1}^{n-1}\left(q_i\prod_{l=i+1}^{n}\left(1+\frac{R_l}{100}\right)\right) \tag{5}$$

Here, the loan of ₹P spans over n years. The rate of interests (per annum) corresponding to the 1^{st}, 2^{nd}, …, n^{th} year being $R_1, R_2, \ldots, R_n$ respectively. ₹q_i is the amount paid at the end of the i^{th} year ($\forall i \in \{1, 2, \ldots, n-1\}$)

substituting the values,

$n = 2$, $P = 50000$, $R_1 = 12$, $R_2 = 15$, $q_1 = 33000$

$$= 50000\prod_{k=1}^{2}\left(1+\frac{R_k}{100}\right) - \sum_{i=1}^{1}\left(q_i\prod_{l=i+1}^{2}\left(1+\frac{R_l}{100}\right)\right)$$

$$= 50000\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right) - q_1\left(1+\frac{R_2}{100}\right)$$

$$= 50000\left(1+\frac{12}{100}\right)\left(1+\frac{15}{100}\right) - 33000\left(1+\frac{15}{100}\right)$$

$$= 26450$$

Hence, Jaya must pay ₹26,450 at the end of the second year to clear her debt.