

# Assignment 1 (ICSE 2017)

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- 2 (c) Jaya borrowed ₹50,000 for 2 years. The rates of interest for two successive years are 12% and 15% respectively. She repays ₹33,000 at the end of the first year. Find the amount she must pay at the end of the second year to clear her debt.

**Solution:**

The various parameters considered in this problem are listed in Table I.

TABLE I

Symbol	Value	Description
$n$	2	Total number of years over which the debt spans
$P$	₹50,000	Principal
$R_1$	12%	Rate of interest for the 1 <sup>st</sup> year (per annum)
$R_2$	15%	Rate of interest for the 2 <sup>nd</sup> year (per annum)
$q_1$	₹33,000	Amount paid at the end of 1 <sup>st</sup> year
$A_n$	?	Amount due at the end of $n^{\text{th}}$ year

Amount due at the end of 1<sup>st</sup> year

$$= P \left( 1 + \frac{R_1}{100} \right) \quad (1)$$

The principal for the 2<sup>nd</sup> year

$$= P \left( 1 + \frac{R_1}{100} \right) - q_1 \quad (2)$$

Amount due at the end of 2<sup>nd</sup> year

$$= \left( P \left( 1 + \frac{R_1}{100} \right) - q_1 \right) \left( 1 + \frac{R_2}{100} \right) \quad (3)$$

$$= P \left( 1 + \frac{R_1}{100} \right) \left( 1 + \frac{R_2}{100} \right) - q_1 \left( 1 + \frac{R_2}{100} \right) \quad (4)$$

∴  $n = 2$ , we can directly substitute the values in eq(4) to get the desired value

Also note that in eq(4):

- The left term corresponds to  $P$  + the interest on  $P$
- The right term corresponds to  $q_1$  + interest on  $q_1$ .

So for  $n$  years,

$$A_n = P \prod_{k=1}^n \left( 1 + \frac{R_k}{100} \right) - \sum_{i=1}^{n-1} \left( q_i \prod_{l=i+1}^n \left( 1 + \frac{R_l}{100} \right) \right) \quad (5)$$

TABLE II  
VARIABLES OF THE EQ(5)

Symbol	Description
$n$	Total number of years the loan spans over
$P$	Principal
$R_1, R_2, \dots, R_n$	The rate of interests (per annum) corresponding to the 1 <sup>st</sup> , 2 <sup>nd</sup> , ..., $n^{\text{th}}$ year respectively
$q_i$	Amount paid at the end of the $i^{\text{th}}$ year ( $\forall i \in \{1, 2, \dots, n-1\}$ )

Table II describes all the variables in eq(5). Substituting the values of  $n$ ,  $P$ ,  $R_1$ ,  $R_2$ ,  $q_1$  in eq(5) we get

$$A_2 = 50000 \prod_{k=1}^2 \left( 1 + \frac{R_k}{100} \right) - \sum_{i=1}^1 \left( q_i \prod_{l=i+1}^2 \left( 1 + \frac{R_l}{100} \right) \right) \quad (6)$$

$$A_2 = 50000 \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) - q_1 \left(1 + \frac{R_2}{100}\right) \quad (7)$$

$$A_2 = 50000 \left(1 + \frac{12}{100}\right) \left(1 + \frac{15}{100}\right) - 33000 \left(1 + \frac{15}{100}\right) \quad (8)$$

$$\implies A_2 = 26450 \quad (9)$$

Hence, Jaya must pay ₹26,450 at the end of the second year to clear her debt.