

Assignment 1 (ICSE 2017)

Nitya Seshagiri Bhamidipaty (cs21btech11041)

- 2 (c) Jaya borrowed ₹50,000 for 2 years. The rates of interest for two successive years are 12% and 15% respectively. She repays ₹33,000 at the end of the first year. Find the amount she must pay at the end of the second year to clear her debt.

Solution:

The various parameters considered in this problem are listed in Table I.

TABLE I

Symbol	Value	Description
n	2	Total number of years over which the debt spans
P	₹50,000	Principal
R_1	12%	Rate of interest for the 1 st year (per annum)
R_2	15%	Rate of interest for the 2 nd year (per annum)
q_1	₹33,000	Amount paid at the end of 1 st year
A_n	?	Amount due at the end of n^{th} year

Amount due at the end of 1st year

$$= P \left(1 + \frac{R_1}{100} \right) \quad (1)$$

The principal for the 2nd year

$$= P \left(1 + \frac{R_1}{100} \right) - q_1 \quad (2)$$

Amount due at the end of 2nd year

$$= \left(P \left(1 + \frac{R_1}{100} \right) - q_1 \right) \left(1 + \frac{R_2}{100} \right) \quad (3)$$

$$= P \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_2}{100} \right) - q_1 \left(1 + \frac{R_2}{100} \right) \quad (4)$$

∴ $n = 2$, we can directly substitute the values in eq(4) to get the desired value

Also note that in eq(4):

- The left term corresponds to P + the interest on P
- The right term corresponds to q_1 + interest on q_1 .

So for n years,

$$A_n = P \prod_{k=1}^n \left(1 + \frac{R_k}{100} \right) - \sum_{i=1}^{n-1} \left(q_i \prod_{l=i+1}^n \left(1 + \frac{R_l}{100} \right) \right) \quad (5)$$

TABLE II
VARIABLES OF THE EQ(5)

Symbol	Description
n	Total number of years the loan spans over
P	Principal
R_1, R_2, \dots, R_n	The rate of interests (per annum) corresponding to the 1 st , 2 nd , ..., n^{th} year respectively
q_i	Amount paid at the end of the i^{th} year ($\forall i \in \{1, 2, \dots, n-1\}$)

Table II describes all the variables in eq(5). Substituting the values of n , P , R_1 , R_2 , q_1 in eq(5) we get

$$A_2 = 50000 \prod_{k=1}^2 \left(1 + \frac{R_k}{100} \right) - \sum_{i=1}^1 \left(q_i \prod_{l=i+1}^2 \left(1 + \frac{R_l}{100} \right) \right) \quad (6)$$

$$A_2 = 50000 \left(1 + \frac{R_1}{100}\right) \left(1 + \frac{R_2}{100}\right) - q_1 \left(1 + \frac{R_2}{100}\right) \quad (7)$$

$$A_2 = 50000 \left(1 + \frac{12}{100}\right) \left(1 + \frac{15}{100}\right) - 33000 \left(1 + \frac{15}{100}\right) \quad (8)$$

$$\implies A_2 = 26450 \quad (9)$$

Hence, Jaya must pay ₹26,450 at the end of the second year to clear her debt.