Assignment 2 (ICSE 2018)

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13 (a) Evaluate: $\int \frac{x-1}{\sqrt{x^2-x}}dx$

Solution:

$$\frac{x-1}{\sqrt{x^2-x}} = \frac{1}{2}\frac{2x-1}{\sqrt{x^2-x}} - \frac{1}{2}\frac{1}{\sqrt{x^2-x}} \quad (1)$$

Integrating both sides

$$\int \frac{x-1}{\sqrt{x^2-x}}\,dx = \frac{1}{2}\int \frac{2x-1}{\sqrt{x^2-x}}\,dx - \frac{1}{2}\int \frac{1}{\sqrt{x^2-x}}\,dx \quad (2)$$

Let $t = x^2 - x$

$$dt = (2x-1)dx \quad (3)$$

$$\int \frac{2x-1}{\sqrt{x^2-x}}\,dx = \int \frac{dt}{\sqrt{t}} \quad (4)$$

$$= 2\sqrt{t} + C \quad (5)$$

$$= 2\sqrt{x^2-x} + C \quad (6)$$

Also let $u = x - \frac{1}{2}$

$$du = dx \quad (7)$$

$$\int \frac{1}{\sqrt{x^2-x}}\,dx = \int \frac{1}{\sqrt{\left(x-\frac{1}{2}\right)^2 - \frac{1}{4}}}\,dx \quad (8)$$

$$= \int \frac{1}{\sqrt{u^2 - \left(\frac{1}{2}\right)^2}}\,du \quad (9)$$

$$= \ln\left|u + \sqrt{u^2 - \left(\frac{1}{2}\right)^2}\right| + C \quad (10)$$

$$= \ln\left|x - \frac{1}{2} + \sqrt{x^2-x}\right| + C \quad (11)$$

From eq(2), eq(6), eq(11) we get

$$\int \frac{x-1}{\sqrt{x^2-x}}\,dx = \frac{1}{2}\times 2\sqrt{x^2-x} - \frac{1}{2}\ln\left|x - \frac{1}{2} + \sqrt{x^2-x}\right| + C \quad (12)$$

Hence,

$$\int \frac{x-1}{\sqrt{x^2-x}}\,dx = \sqrt{x^2-x} - \frac{1}{2}\ln\left|2x - 1 + \sqrt{4x^2-4x}\right| + C \quad (13)$$