Assignment 1 (ICSE 2017)

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2 (c) Jaya borrowed ₹50,000 for 2 years. The rates of interest for two successive years are 12% and 15% respectively. She repays ₹33,000 at the end of the first year. Find the amount she must pay at the end of the second year to clear her debt.

Solution:

The various parameters considered in this problem are listed in Table I.

TABLE I

Symbol	Value	Description
n	2	Total number of years the debt spans
P	₹50,000	Principal
R_1	12%	Rate of interest for the 1^{st} year (per annum)
R_2	15%	Rate of interest for the 2^{nd} year (per annum)
q_1	₹33,000	Amount paid at the end of 1^{st} year
A_n	?	Amount due at the end of n^{th} year

Amount due at the end of 1^{st} year

$$= P\left(1+\frac{R_1}{100}\right) \tag{1}$$

The principal for the 2^{nd} year

$$= P\left(1+\frac{R_1}{100}\right) - q_1 \tag{2}$$

Amount due at the end of 2^{nd} year

$$= \left(P\left(1+\frac{R_1}{100}\right) - q_1\right)\left(1+\frac{R_2}{100}\right) \tag{3}$$

$$= P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right) - q_1\left(1+\frac{R_2}{100}\right) \tag{4}$$

$\because n = 2$, we can directly substitute the values in eq(4) to get the desired value

Also note that in eq(4):

a) The left term corresponds to P + the interest on P

b) The right term corresponds to q_1 + interest on q_1.

So for n years,

$$A_n = P\prod_{k=1}^{n}\left(1+\frac{R_k}{100}\right) - \sum_{i=1}^{n-1}\left(q_i\prod_{l=i+1}^{n}\left(1+\frac{R_l}{100}\right)\right) \tag{5}$$

TABLE II
VARIABLES OF THE EQ(5)

Symbol	Description
n	Total number of years the loan spans over
P	Principal
$R_1, R_2, \ldots, R_n$	The rate of interests (per annum) corresponding to the 1^{st}, 2^{nd}, …, n^{th} year respectively
q_i	Amount paid at the end of the i^{th} year ($\forall i \in \{1, 2, \ldots, n-1\}$)

Table II describes all the variables in eq(5) Substituting the values of n, P, R_1, R_2, q_1 in eq(5) we get

$$A_2 = 50000\prod_{k=1}^{2}\left(1+\frac{R_k}{100}\right) - \sum_{i=1}^{1}\left(q_i\prod_{l=i+1}^{2}\left(1+\frac{R_l}{100}\right)\right) \tag{6}$$

$$A_2 = 50000\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right) - q_1\left(1+\frac{R_2}{100}\right) \quad (7)$$

$$A_2 = 50000\left(1+\frac{12}{100}\right)\left(1+\frac{15}{100}\right) - 33000\left(1+\frac{15}{100}\right) \quad (8)$$

$$\implies A_2 = 26450 \quad (9)$$

Hence, Jaya must pay ₹26,450 at the end of the second year to clear her debt.