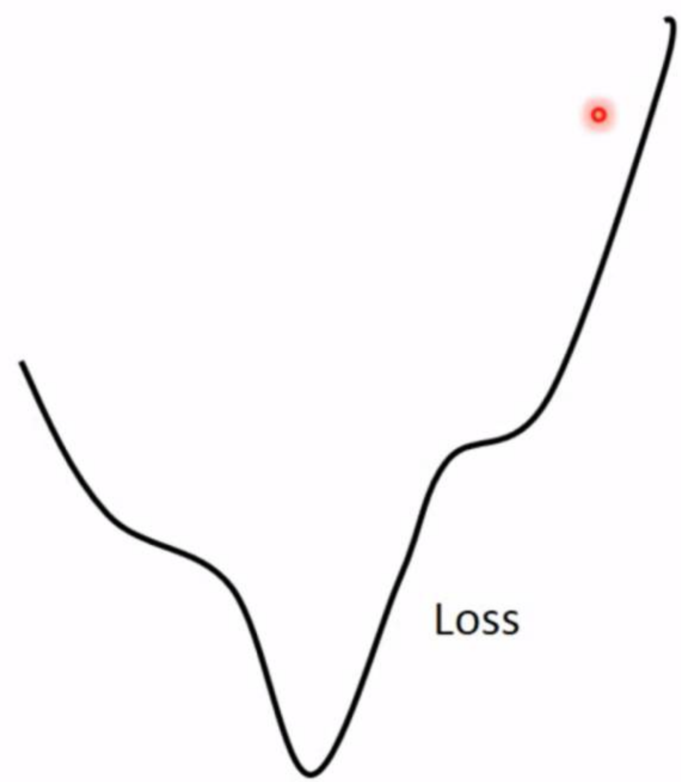
Gradient Descent


Loss

Suppose that θ has two variables $\{\theta_1, \theta_2\}$

Randomly start at $\theta^0 = \begin{bmatrix} \theta_1^0 \\ \theta_2^0 \end{bmatrix}$

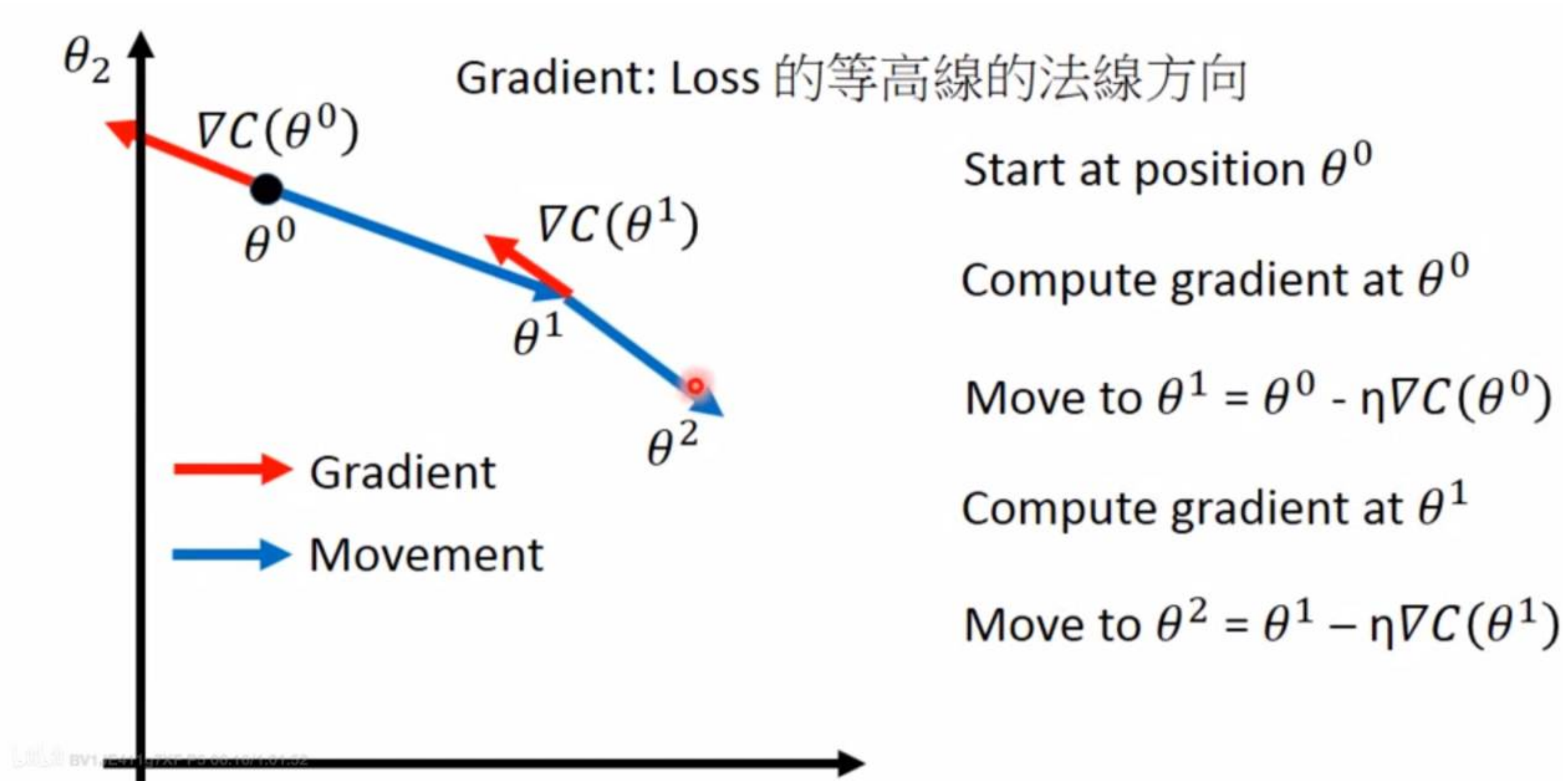
$$\nabla L(\theta) = \begin{bmatrix} \partial C(\theta_1)/\partial\theta_1 \\ \partial C(\theta_2)/\partial\theta_2 \end{bmatrix}$$

$$\begin{bmatrix} \theta_1^1 \\ \theta_2^1 \end{bmatrix} = \begin{bmatrix} \theta_1^0 \\ \theta_2^0 \end{bmatrix} - \eta \begin{bmatrix} \partial L(\theta_1^0)/\partial\theta_1 \\ \partial L(\theta_2^0)/\partial\theta_2 \end{bmatrix} \Rightarrow \theta^1 = \theta^0 - \eta\nabla L(\theta^0)$$

$$\begin{bmatrix} \theta_1^2 \\ \theta_2^2 \end{bmatrix} = \begin{bmatrix} \theta_1^1 \\ \theta_2^1 \end{bmatrix} - \eta \begin{bmatrix} \partial L(\theta_1^1)/\partial\theta_1 \\ \partial L(\theta_2^1)/\partial\theta_2 \end{bmatrix} \Rightarrow \theta^2 = \theta^1 - \eta\nabla L(\theta^1)$$

Gradient :vector

Visualization

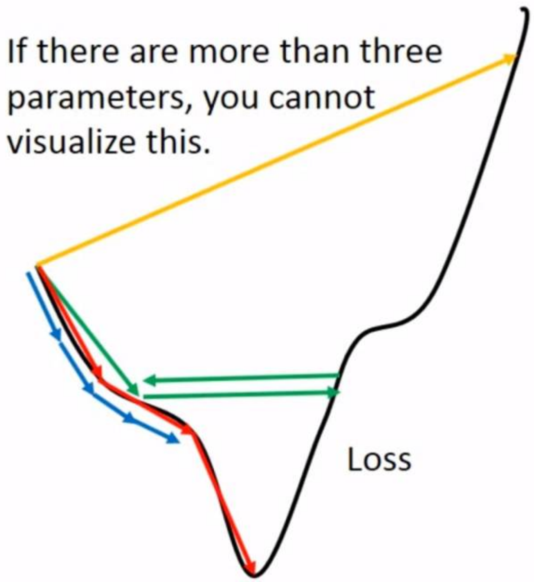


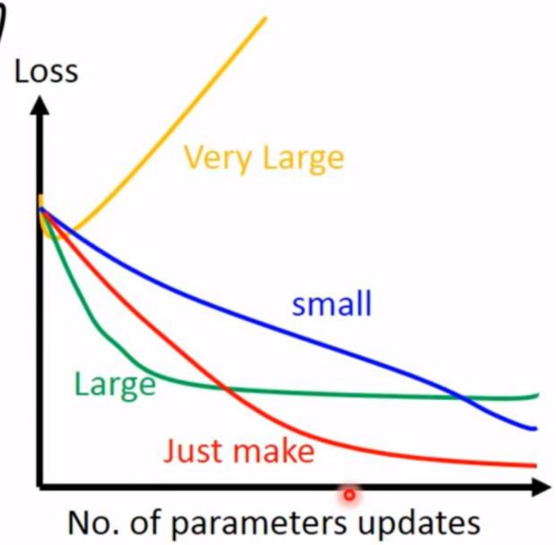
Gradient: vector，红色箭头，更新参数

Learning Rate

$$\theta^i = \theta^{i-1} - \eta \nabla C\left(\theta^{i-1}\right)$$

Set the learning rate η carefully





But you can always visualize this.

- E.g. 1/t decay: $\eta^t = \eta / \sqrt{t+1}$

- Learning rate cannot be one-size-fits-all
  - Giving different parameters different learning rates

Adagrad

$$w^{t+1} \leftarrow w^t - \frac{\eta^t}{\sigma^t} g^t$$

σ^t: ***root mean square*** of the previous derivatives of parameter w

$$\eta^t = \frac{\eta}{\sqrt{t+1}} \qquad g^t = \frac{\partial C(\theta^t)}{\partial w}$$

- Divide the learning rate of each parameter by the ***root mean square of its previous derivatives***

$$\eta^t = \frac{\eta}{\sqrt{\cancel{t+1}}}$$

1/t decay

$$w^{t+1} \leftarrow w^t - \frac{\boxed{\eta^t}}{\boxed{\sigma^t}} g^t$$

$$\sigma^t = \sqrt{\frac{1}{\cancel{t+1}} \sum_{i=0}^{t} (g^i)^2}$$

$$w^{t+1} \leftarrow w^t - \frac{\eta}{\sqrt{\sum_{i=0}^{t} (g^i)^2}} g^t$$

THERE ARE MANY OTHER METHODS…

Contradiction? $\eta^t = \frac{\eta}{\sqrt{t+1}}$ $g^t = \frac{\partial C(\theta^t)}{\partial w}$

Vanilla Gradient descent

$$w^{t+1} \leftarrow w^t - \eta^t \underline{g^t}$$

Larger gradient, larger step

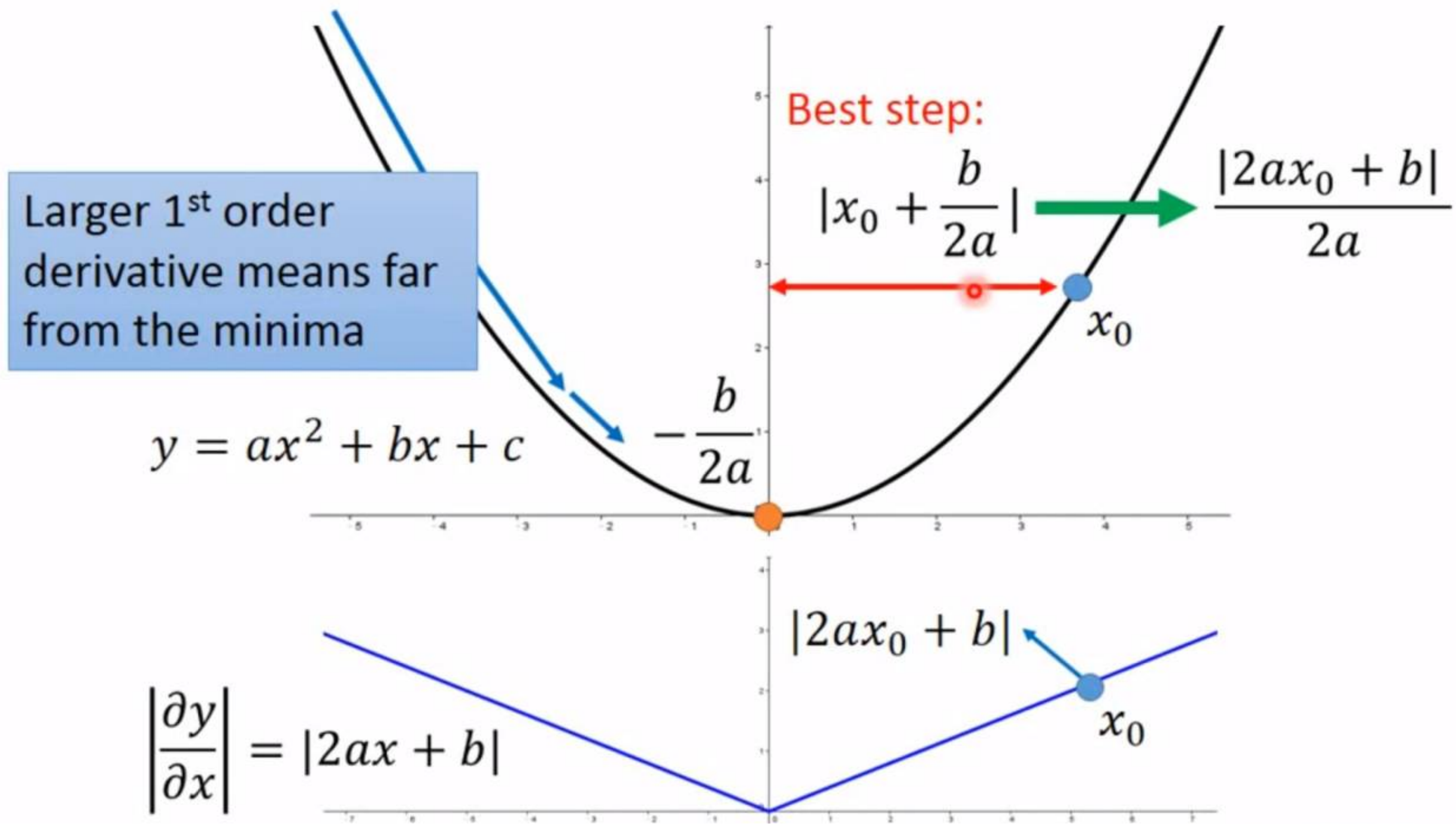
Adagrad

$$w^{t+1} \leftarrow w^t - \frac{\eta}{\sqrt{\underline{\sum_{i=0}^{t} (g^i)^2}}} \underline{g^t}$$

Larger gradient, larger step

Larger gradient, smaller step

Larger gradient, larger steps?



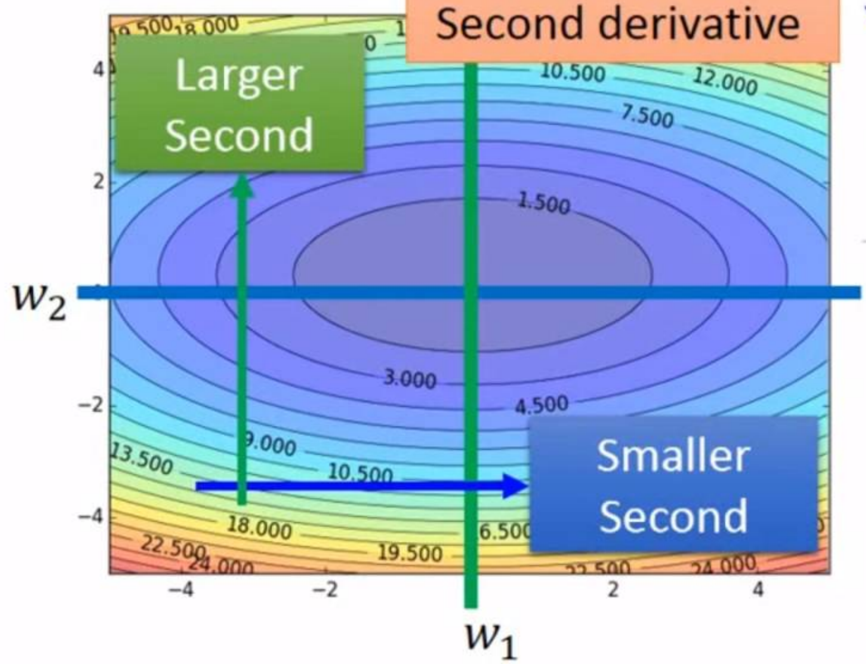
只考虑一个参数时：
距离与微分的大小成
正比

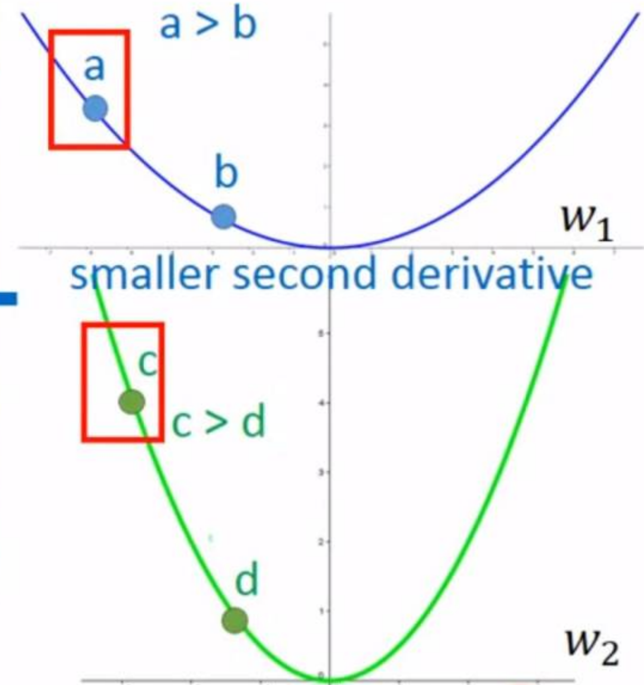
Comparison between different parameters

~~Larger 1^{st} order derivative means far from the minima~~

Do not cross parameters

The best step is $\dfrac{|\text{First derivative}|}{\text{Second derivative}}$





Stochastic Gradient Descent (SGD)

——faster

$$L = \sum_n \left(\hat{y}^n - \left(b + \sum w_i x_i^n \right) \right)^2$$

Loss is the summation over all training examples

◆ ***Gradient Descent*** $\theta^i = \theta^{i-1} - \eta \nabla L\left(\theta^{i-1}\right)$

◆ ***Stochastic Gradient Descent***

Pick an example x^n

$$L^n = \left(\hat{y}^n - \left(b + \sum w_i x_i^n \right) \right)^2 \qquad \theta^i = \theta^{i-1} - \eta \nabla L^n\left(\theta^{i-1}\right)$$

Loss for only one example

看完所有example，更新参数；每看到一个example，update一次参数，更新20次参数。

Feature Scaling

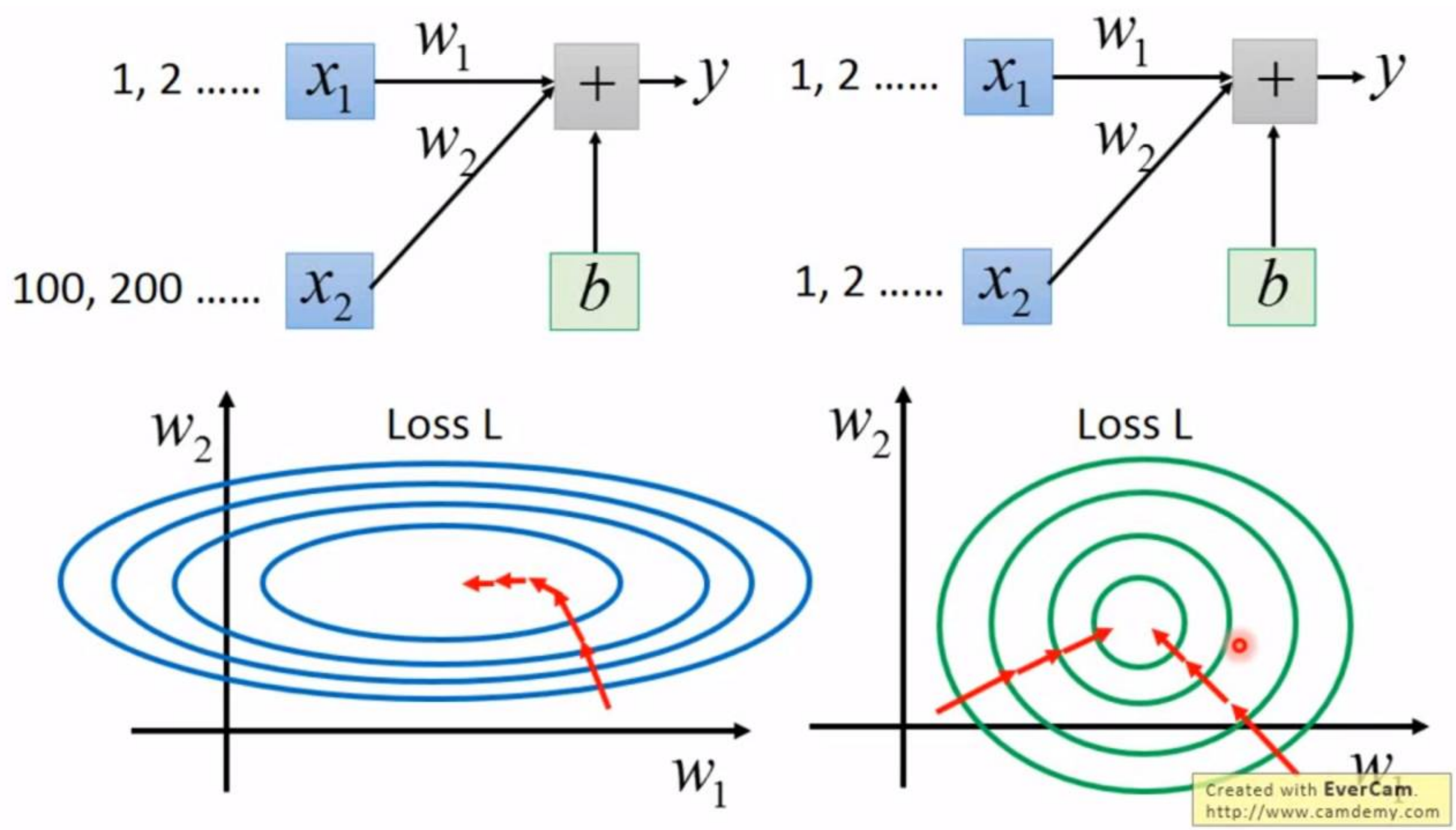
$$x_i^r \leftarrow \frac{x_i^r - m_i}{\sigma_i}$$

The means of all dimensions are 0, and the variances are all 1

For each dimension i:

mean: m_i

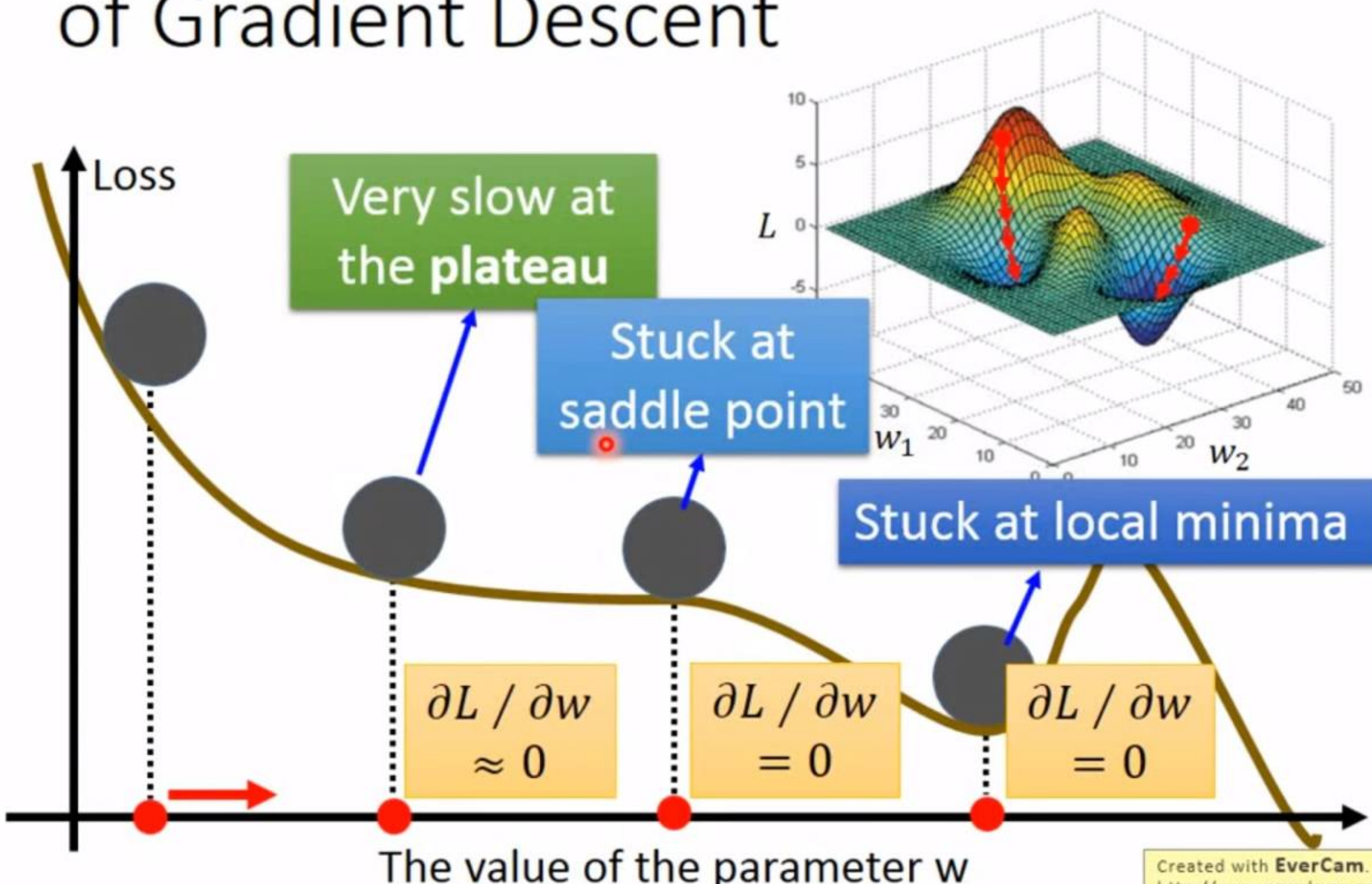
standard deviation: σ_i



1, 2 ......
x1
w1
+
y
w2
100, 200 ......
x2
b
1, 2 ......
x1
w1
+
y
w2
1, 2 ......
x2
b
Loss L
w2
w1
Loss L
w2
Created with EverCam.
http://www.camdemy.com

Gradient Descent Theory ——Taylor Series

More Limitation of Gradient Descent



Thanks!